Consumption risk sharing and adjustment costs

Fanelli, Luca and Cavaliere, Giuseppe and Gardini, Attilio

Department of Statistics, University of Bologna

October 2004

Consumption risk sharing and adjustment costs

Giuseppe Cavaliere,* Luca Fanelli, Attilio Gardini

University of Bologna

First draft: October 2004; November 2006

Abstract

We show that full risk sharing may not be at odd with the idea that changes in regional consumption display error-correcting dynamics, in line with the idea that information and transaction costs stemming from interregional portfolio diversification and labor movements induced by permanent income shocks may delay the adjustment process. Using Italian data over the period 1960-2001 it is found that regional per capita consumptions match the proposed error-correcting structure.

Keywords: Consumption risk sharing, Adjustment costs, Forward-looking behavior.

J.E.L. Classification: C32, E21.

1 Introduction

Common wisdom contends that under complete markets changes in per capita consumptions of a set of regions should be related to changes in aggregate per capita consumption only. Conventional risk sharing tests and/or techniques aimed at measuring the different channels of consumption insurance are based on this requirement, see e.g. Asdrubali et al. (1996). However, several empirical tests have shown substantial departures from this proposition – the so-called 'full risk sharing hypothesis' (FRS) – both on individual and aggregate data, see Lewis (1999) and reference therein. Such tests are usually based on the idea that changes in regional

---

*Corresponding author: University of Bologna, Department of Statistical Sciences, via Belle Arti 41, I-40126 Bologna. e-mail: cavaliere@stat.unibo.it.

consumptions, once corrected for changes in aggregate consumption, are not predictable on the basis of the available information set, see also Canova and Ravn (1996).

In this paper we consider a simple stylized model which shows that, in the presence of fatto dei cambi consumption adjustment costs, even in the presence of FRS regional consumption changes may qui display an error-correcting structure involving not only changes in aggregate consumption, but also lagged departures from the optimal risk sharing position. Hence, consumption growth rates, rather than co-moving perfectly over time, reflect a dynamic adjustment process toward FRS.

We use this framework to investigate the dynamic adjustment of Italian regional consumption data over the period 1960-2001. It is shown that the results match quite closely the proposed interrelated error-correcting model.

2 Model

As in e.g. Canova and Ravn (1996) we assume that the risk sharing pool is composed by $j = 1, ..., N$ regions endowed with a stochastic amount of a single consumption good.[1] In each region a representative consumer exists and her expected lifetime utility is given by the expected sum of her future discounted utility. Under complete markets and standard conditions on the utility functions which we omit for brevity, it can be shown that in the presence of FRS (Canova and Ravn, 1996; Cavaliere et al., 2006) the optimal per capita consumption in region j at time t satisfies

$$c_t^{*j} = \theta_j c_t^W + \eta_t^j, \tag{1}$$

where c_t^W is aggregate (national) per capita consumption, θ_j is a strictly positive coefficient (inversely related to the j-th agent's relative risk aversion) and the η_t^j term captures idiosyncratic preference shocks. In compact notation

$$\mathbf{c}_t^* = \boldsymbol{\theta} \, c_t^W + \boldsymbol{\eta}_t \tag{2}$$

where $\mathbf{c}_t^* = (c_t^{*1}, ..., c_t^{*N})'$ is the $N \times 1$ vector containing the optimal levels of per capita consumptions in the N regions, see eq. (1); here $\boldsymbol{\eta}_t = (\eta_t^1, ..., \eta_t^N)'$, and $\boldsymbol{\theta} = (\theta_1, ..., \theta_N)'$. If (i) preference shocks are stationary and (ii) deviations of $\mathbf{c}_t$ (with $\mathbf{c}_t = (c_t^1, ..., c_t^N)'$ denoting the

---

[1]Although we use the term 'region' throughout the paper, the approach discussed in the paper also holds for a coalition of nations sharing the same currency.

$N \times 1$ vector of observed per capita consumptions) from the optimal level $\mathbf{c}_t^*$ are transitory, then a testable implication of (2) is that c_t^i and c_t^W must be cointegrated with cointegrating vector $(1 \, , \, -\theta_i)$ for $i = 1, ..., N$, see e.g. Obstfeld (1994), Kollmann (1995), Canova and Ravn (1996), Cavaliere et al. (2006a, 2006b).

Apart from habit persistence, the presence of nontradeable components in the utility function and/or incomplete markets (Lewis, 1999), temporary deviations of actual consumption from the optimal FRS position (2) may be due to the frictions characterizing the markets which provide consumption insurance against long term income fluctuations. Examples are the information and trading costs implied regional portfolio diversification, and regional labor market stickiness.

More precisely, the equilibrium relation (2) could not be satisfied in the short term due to frictions in cross-regional portfolio investment relevant for the dynamics of the consumption process. In fact, interregional portfolio diversification implies additional information costs, trading costs and requires time to manage portfolio investment (indirect costs), which could generate delays in cross-regional investment decisions. These delays generate departures from the equilibrium consumption solution. Moreover, movements of labor in response to permanent income shocks may represent a further cause for the slow adjustment to (2). Consider, for instance, an autonomous permanent income regional shock which reduces the demand for the product of a given region. In this case one expects that workers move from such a region to the other regions to compensate the effects of the adverse shock on their consumption streams. Yet, the implied cross-regional migration process may require time before the effects of the shock are fully compensated.

In light of these considerations, we now consider a simple stylized model aimed at capturing, on empirical grounds, the situation in which, given the necessity of adjusting regional portfolios and moving labor in the face of permanent income shocks, agents minimize the (dis)utility costs of being away from the FRS position. Formally, each representative agents of region j is assumed to solve the following intertemporal optimization problem:

$$\min_{\{c_{t+h}^j\}} E_t \sum_{h=0}^{\infty} \rho^h \left[d_{0j}(c_{t+h}^j - c_{t+h}^{*j})^2 + d_{1j}(c_{t+h}^j - c_{t+h-1}^j)^2 \right] \tag{3}$$

where $E_t \cdot = E(\cdot \mid \Omega_t)$ denotes expectations conditional on the information set available at time t, Ω_t, ρ $(0 < \rho < 1)$ is a time-invariant discount factor (assumed to be common across regions),[2]

---

[2]Observe that the assumption of common discount factors can be relaxed without changing the main result of

and $d_{0j} > 0$ and $d_{1j} > 0$ are two scalar parameters. There are two types of costs embedded in the discounted present value of the cost function in (3): the first term, weighted by d_{0j} measures the cost of being away from the optimal consumption level, the second term, weighted by d_{1j}, measures the cost of changing consumption levels to restore equilibrium.

By considering the problem (3) jointly for all regions, one gets the expression

$$\min_{\{\mathbf{c}_{t+h}\}} E_t \sum_{h=0}^{\infty} \rho^h \left[(\mathbf{c}_{t+h} - \mathbf{c}_{t+h}^*)' \mathbf{D}_0 (\mathbf{c}_{t+h} - \mathbf{c}_{t+h}^*) + (\mathbf{c}_{t+h} - \mathbf{c}_{t+h-1})' \mathbf{D}_1 (\mathbf{c}_{t+h} - \mathbf{c}_{t+h-1}) \right] \qquad (4)$$

where $\mathbf{D}_0 = diag(d_{01}, ..., d_{0N})$ and $\mathbf{D}_1 = diag(d_{11}, ..., d_{1N})$ are $N \times N$ symmetric diagonal positive definite matrices.

The optimization problem in (4) can be further generalized, however, to the case where either $\mathbf{D}_0$ or $\mathbf{D}_1$, or both, are non-diagonal. Taking $\mathbf{D}_0$ and/or $\mathbf{D}_1$ as non-diagonal entails cross-adjustment terms. The representative agent of each region potentially diversifies her asset holdings across all the N regions of the risk sharing coalition, with the aim of shelding her consumption streams from idiosyncratic permanent income fluctuations. This means that, in principle, deviations from FRS occurring in the other regions might engage an overall (and costly) process of portfolio revision and adjustment. Likewise, cross-regional labor movements induced by permanent income shocks affecting a given region, might spread the costs of smoothing consumption to the other regions if the labour market is sticky. For these reasons we specify $\mathbf{D}_0$ and $\mathbf{D}_1$ in (4) as non-diagonal, leaving to the econometric analysis of the model the task of supporting or rejecting the chosen model.

3 Implications

We now show that allowing for non-instantaneous consumption adjustment in the risk sharing analysis implies a more involved dynamic structure than commonly argued in the risk sharing literature.

First order conditions. The first-order conditions for (4) reads as a system of Euler equations that, following Fanelli (2006), can be written as

$$\Delta \mathbf{c}_t = \rho E_t \Delta \mathbf{c}_{t+1} - \mathbf{D}(\mathbf{c}_t - \boldsymbol{\theta} c_t^W) + \widetilde{\boldsymbol{\eta}}_t \qquad (5)$$

the paper.

where $\widetilde{\boldsymbol{\eta}}_t = \mathbf{D}\boldsymbol{\eta}_t$, $\Delta\mathbf{c}_t = \mathbf{c}_t - \mathbf{c}_{t-1}$ and the elements of the matrix $\mathbf{D} = \mathbf{D}_1^{-1}\mathbf{D}_0$ measure the relative importance of disequilibria, adjustment and cross-adjustment costs. The j-th equation of (5) (i.e. that relative to region j) is given by

$$\Delta c_t^j = \rho E_t \Delta c_{t+1}^j - d_{jj}(c_t^j - \theta_j c_t^W) - \sum_{i=1, i \neq j}^{N} d_{ji}(c_t^i - \theta_i c_t^W) + \widetilde{\eta}_t^j \qquad (6)$$

where $\widetilde{\eta}_t^j$ is the j-th element of $\widetilde{\boldsymbol{\eta}}_t$, d_{jj} is the j-th element on the main diagonal of $\mathbf{D}$, and the d_{ji} for $j \neq i$ are the corresponding off-diagonal elements. Unless $d_{ji} = 0$ (i.e. $\mathbf{D}$ is diagonal), consumption changes in region j depend not only on their own future expected changes but also on how much region j and the other regions deviate from the optimal risk sharing position.

Forward solution. Using techniques in Hansen and Sargent (1981) and Binder and Pesaran (1995) the level version of (5) can be solved forward as:

$$\mathbf{c}_t = \mathbf{K}\mathbf{c}_{t-1} + \sum_{h=0}^{\infty} (\rho\mathbf{K})^h (\mathbf{I}_N - \rho\mathbf{K})(\mathbf{I}_N - \mathbf{K})\boldsymbol{\theta} E_t c_{t+h}^W + \mathbf{v}_t \qquad (7)$$

where $\mathbf{v}_t = (\mathbf{I}_N - \mathbf{K})\boldsymbol{\eta}_t$, $\mathbf{K}$ is a $N \times N$ matrix with stable eigenvalues which depends on ρ and $\mathbf{D}$ only (Fanelli, 2006). The representation (7) highlights that for region j, consumption at time t is a combination of consumption at time $t-1$ of all regions in the risk sharing pool and expected future values of aggregate consumption, with weights declining geometrically over time.

By using the equality $\sum_{h=0}^{\infty} (\rho\mathbf{K})^h (\mathbf{I}_N - \rho\mathbf{K}) = \sum_{h=0}^{\infty} (\rho\mathbf{K})^h - \sum_{h=0}^{\infty} (\rho\mathbf{K})^{h+1}$, adding $(-\mathbf{c}_{t-1})$ to both sides and $\pm (\mathbf{I}_N - \mathbf{K})\boldsymbol{\theta} c_{t-1}^W$ to the right hand side, after rearranging terms the model can be reparameterized in the error-correcting format

$$\Delta\mathbf{c}_t = (\mathbf{K} - \mathbf{I}_N)(\mathbf{c}_{t-1} - \boldsymbol{\theta} c_{t-1}^W) + \sum_{h=0}^{\infty} (\rho\mathbf{K})^h (\mathbf{I}_N - \mathbf{K})\theta E_t \Delta c_{t+h}^W + \mathbf{v}_t \qquad (8)$$

which shows that the dynamics of consumption in each region depends on past deviations from the optimal risk sharing position and future expected changes of aggregate consumption. The $\mathbf{K}$ matrix in (8) plays a role which is very similar to that of the adjustment matrix $\mathbf{D}$ in the system (6); indeed the elements of $\mathbf{K}$ are function of $\mathbf{D}$ and, in general, if $\mathbf{D}$ is non-diagonal, $\mathbf{K}$ will be non-diagonal too. The model (8) must be solved for future expected values of Δc_t^W in order to derive a closed-form solution and testable restrictions, as shown below.

VAR dynamics. As is standard in the literature, we assume that the process generating Δc_t^W can be approximated as an ARIMA(p,1,0) model, see e.g. Attanasio (1999). By setting $p = 1$,

5

Δc_t^W obeys the following equation

$$\Delta c_t^W = \phi_0 + \phi_1 \Delta c_{t-1}^W + u_t \quad , \; u_t \sim WN(0, \sigma_u^2) \tag{9}$$

where ϕ_0 is a constant growth rate and $|\phi_1| < 1$; note that setting $p = 1$ does not imply a loss of generality since extensions to data generating processes with richer dynamics, albeit notationally more involved, can be obtained as well. From eq. (9), $E_t \Delta c_{t+h}^W = E(\Delta c_{t+h}^W \mid \Omega_t) = \widetilde{\phi}_0^h + \phi_1^h \Delta c_t^W$, where $\widetilde{\phi}_0^h = \phi_0 \left(\frac{1 - \phi_1^h}{1 - \phi_1} \right)$. Therefore, by substituting this expression into the forward-solution (8) and rearranging terms we obtain

$$\Delta \mathbf{c}_t = \mathbf{\Gamma} \, \boldsymbol{\theta} \Delta c_t^W + (\mathbf{K} - \mathbf{I}_N)(\mathbf{c}_{t-1} - \boldsymbol{\theta} c_{t-1}^W) + \boldsymbol{\mu} + \mathbf{v}_t \tag{10}$$

where the $N \times N$ matrix $\mathbf{\Gamma}$ is defined as $\mathbf{\Gamma} = (\mathbf{I}_N - \rho \phi_1 \mathbf{K})^{-1}(\mathbf{I}_N - \mathbf{K})$, and $\boldsymbol{\mu}$ is a constant which depends on the parameters ρ, $\mathbf{K}$, $\boldsymbol{\theta}$, ϕ_0 and ϕ_1. The j-th equation of (10) (i.e. that relative to the region j) is then given by

$$\Delta c_t^j = \boldsymbol{\gamma}_j' \, \boldsymbol{\theta} \Delta c_t^W + (k_{jj} - 1)(c_{t-1}^j - \theta_j c_{t-1}^W) + \sum_{i=1, i \neq j}^{N} k_{ji}(c_{t-1}^i - \theta_i c_{t-1}^W) + \mu_j + v_t^j \tag{11}$$

where the $1 \times N$ vector $\boldsymbol{\gamma}_j'$ is the j-th row of $\mathbf{\Gamma}$, $(k_{jj} - 1)$ is the j-th element on the principal diagonal of $(\mathbf{K} - \mathbf{I}_N)$, k_{ji} $(i \neq j)$ are the corresponding off-diagonal elements, and μ_j and v_t^j are the j-th elements of $\boldsymbol{\mu}$ and $\mathbf{v}_t$ respectively. Again, in this framework consumption changes in region j depend on the contemporaneous aggregate consumption (with a coefficient that involves the relative risk aversion coefficients of all N regions), on the past deviations from the optimal risk sharing levels in region j and on the past deviations from the optimal risk sharing levels in all the other regions (provided $k_{ji} \neq 0$, $j \neq i$, i.e. $\mathbf{K}$ non-diagonal). Hence, in this framework consumption growth rates, rather than co-moving perfectly over time, reflect an interrelated process of adjustment towards FRS. Notice also that empirical risk sharing tests based on testing the orthogonality of Δc_t^j (corrected for Δc_t^W; see e.g. Obstfeld, 1994) to the information set Ω_t might erroneously reject the FRS hypothesis because the dynamic adjustment towards equilibrium is not properly accounted for. Indeed, eq. (11) suggests that Δc_t^j (corrected for Δc_t^W and $(\mathbf{c}_{t-1} - \boldsymbol{\theta} c_{t-1}^W)$) must be orthogonal with respect to the information set Ω_{t-1}.

4 Empirical results

In this section we aim at assessing to what extent regional consumption data in Italy match the (testable) implications of the stylized model depicted in the previous section. It has been widely recognized that Italy is characterized by a remarkable degree of consumption risk sharing: for instance, using the same approach as in Asdrubali et al. (1996), Dedola et al. (1999) show that in the period 1960–1995 shocks are smoothed almost completely. Recently, Cavaliere et al. (2006) show that over the period 1960–2001 and for all the 20 Italian regions, (real) regional per capita consumption (c_t^j) cointegrates with the (real) aggregate per capita consumption (c_t^W) as implied by eq. (1), with cointegrating vector $(1, \theta^j)$. Estimates of θ^j obtained by using Johansen's (1996) maximum likelihood approach are reported in Table 1, first row (see Cavaliere et al., 2006, for details on data sources and definitions).

Recall that the basic implication of the adjustment cost model introduced in the previous sections is that consumption consumption growth rate in region j, Δc_t^j, once corrected for the aggregate consumption growt rate, Δc_t^W, should be correlated with the regional (lagged) disequilibrium, $c_{t-1}^j - \theta^j c_{t-1}^W$ and, if cross-adjustment costs occur, with the other regions' (lagged) disequilibria, $c_{t-1}^i - \theta^i c_{t-1}^W$, $i \neq j$, as well. Furthermore, Δc_t^j should not depend e.g. on its lagged values or on lagged values of Δc_t^W. To test these implications we estimate, for each of the twenty Italian regions, the error correcting model[3]

$$
\begin{aligned}
\Delta c_t^j = \; & \delta_{j1} \Delta c_t^W + \delta_{j2} \Delta c_{t-1}^j + \delta_{j3} \Delta c_{t-1}^W \\
& + \alpha_j (c_{t-1}^j - \hat{\theta}^j c_{t-1}^W) + \sum_{\substack{i=1 \\ i \neq j}}^{20} \alpha_{ji} (c_{t-1}^i - \hat{\theta}^i c_{t-1}^W) + \mu_j + v_t^j
\end{aligned}
\tag{12}
$$

where $\hat{\theta}^j$ denotes the (superconsistent) estimates of the cointegrating vectors (as reported in Table 1)[4]. The parameters δ_{j2} and δ_{j2} associated with lagged regional and national consumption growth rates have been included in order to control for possible exogenous habit persistence effects (Fuhrer and Klein, 1998; Ravn, 2001) which are not directly accounted in our stylized model. Notice that (12) reduces to the forward solution (11) in the special case $\delta_{j2} = 0$ and

---

[3] Full estimates are available from the authors upon request.

[4] For regions where the linear combination $(c_t^j - \hat{\theta}^j c_t^W)$, instead of being level-stationary is found to be trend stationary, $(c_t^j - \hat{\theta}^j c_t^W)$ of eq. (12) is replaced by $(c_t^j - \hat{\theta}^j c_t^W - \hat{\varphi}^j t)$, $\hat{\varphi}^j$ denoting the estimate of the slope of the deterministic trend.

$\delta_{j3} = 0$.

In the second row of Table 1 we test whether lagged regional consumption changes do not help to explain future consumption changes ($\delta_{j2} = 0$), while in the third row we tested the significance of lagged aggregate consumption ($\delta_{j3} = 0$). Overall, these restrictions find empirical support for the majority of the regions, hence showing that neglected habit persistence effects do not seem to be significant.

In the fourth and fifth rows of Table 1 we test the main implications of the risk sharing model with adjustment costs. Specifically, in the fourth row we analyze whether regional consumption adjusts to its own lagged disequilibrium: as expected, the null hypothesis of no adjustment ($\alpha_j = 0$) is rejected for the large majority of regions. Is the adjustment process interrelated across regions? In the fifth row we investigate this issue by checking whether consumption changes in region j depend on the disequilibria in the other regions. The null hypothesis of no cross-adjustments ($\delta_{ij} = 0$, all $i \neq j$) is strongly rejected for *all* regions.[5]

Finally, in order to shed light on the geographical structure of the dynamic adjustment, in Table 1, rows 6–10, we evaluate whether regional consumption adjusts to disequilibria characterizing (i) north-western regions (NW), (ii) north-eastern regions (NE), (iii) central regions (C), (iv) southern regions (S). We also test the adjustment of regional consumption with respect to disequilibria in the neighbooring regions. Interestingly, the results suggest that although adjustment with respect to neighboring regions plays a relevant role in almost half of the regions, a remarkable amount of cross-adjustment involving the poorer regions in the South and the richer regions in the North is detected, providing further empirical evidence on the presence of interrelated consumption risk sharing across the Italian regions.

References

Asdrubali, P., Sørensen, B. E. and Yosha, O. (1996), Channels of interstate risk sharing: United States 1963-90, *Quarterly Journal of Economics* 111, 1081–1110.

---

[5]It could reasonably be argued that, once regional income is included in the cointegrating relations and/or in the growth rate regressions (12), the evidence of interrelated consumption adjustment may disappear. However, for the great majority of the Italian regions income does not appear to be significant either in the long run (i.e., in the cointegrating relations) or in the short run (i.e., in the consumption growth rate regressions), see Cavaliere et al. (2006).

Attanasio, O.P. (1999), Consumption, in Taylor J.B. and Woodford M. (eds.), *Handbook of Macroeconomics*, 1B, Amsterdam: North-Holland, 741–812.

Attfield, C.L.F. (1995), A Bartlett adjustment to the likelihood ratio test for a system of equations, *Journal of Econometrics* 66, 207-223.

Binder, M., Pesaran, M. H. (1995), Multivariate rational expectations models and macroeconomic modelling: a review and some new results, in Pesaran, M. H. and M. Wickens (eds.), *Handbook of Applied Econometrics*, Oxford: Blackwell, 139–187.

Cavaliere, G., Fanelli, L. and Gardini, A. (2006), Regional consumption dynamics and risk sharing in Italy, *International Review of Economics and Finance,* 15, 525–542.

Canova, F., Ravn, M. (1996), International consumption risk sharing, *International Economic Review,* 37, 573–601.

Dedola, L., Usai, S. and Vannini, M. (1999), An assessment on risk sharing in Italy and the United Kingdom, in Adams, J. and Pigliaru, F. (eds.), Economic growth and change. *National and regional patterns of convergence and divergence*, Cheltenham, Edward Elgar, 417–447.

Fanelli, L. (2006), Dynamic adjustment cost models with forward-looking behaviour, *Econometrics Journal* 9, 23-47.

Fuhrer, J. C., and Klein, M. W. (1998), Risky habits: on risk sharing, habit formation, and the interpretation of international consumption correlations, *NBER Working Paper* No. 6735.

Hansen, L.P., Sargent, T.J. (1981), Linear rational expectations models for dynamically interrelated variables, in Lucas R.E.Jr. and Sargent, T.J. (eds.), *Rational Expectations and Econometric Practise.* University of Minnesota Press, Minneapolis, 127-156.

Johansen, S. (1996), *Likelihood-based inference in cointegrated vector autoregressive models*, Oxford University Press, Oxford.

Lewis, K.K. (1999), Trying to explain home bias in equities and consumption, *Journal of Economic Literature*, 37, 571–608.

9

Kollmann, R. (1995), Consumption, real exchange rates and the structure of international asset markets, *Journal of International Money and Finance*, 14, 191–211.

Obstfeld, M. (1994), *Are industrial-country consumption risks globally diversified?*, in Leiderman, L. and Razin, A. (eds.), Capital Mobility; The Impact on Consumption, Investment and Growth, New York: Cambridge University Press.

Ravn, M.O. (2001), Consumption dynamics and real exchange rates, *CEPR Discussion Paper* 2940.