On Mental Transformations

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Abstract

The paper presents an alternative interpretation of the experimental data published by Kahneman and Tversky in their 1992 study "Advances in Prospect Theory", which describes the Cumulative version of their Prospect Theory from 1979. It was assumed that, apart from the operations made during the initial stage of problem resolution, which Prospect Theory defines as Editing (here generalized as Mental Adaptation), other mental transformations such as Prospect Scaling (resulting from Focused Attention) and Logarithmic Perception of Financial Stimuli should be considered when analyzing the experimental data. This led to the design of an explicit, simple and symmetric solution without the use of the probability weighting function. The double S-type function obtained (the aspiration function) resembles the utility curve specified by the Markowitz hypothesis (1952) and substitutes the fourfold pattern of risk attitudes introduced by Cumulative Prospect Theory. The results presented constitute a basis for negating Prospect Theory as a theory which correctly describes how decisions are made under conditions of risk and may signal a return to a description of people's behavior that only relies on the utility-like function.

Keywords: *Prospect/Cumulative Prospect Theory, Markowitz Utility Hypothesis, Utility & Aspiration Functions, Mental Processes, Adaptation & Attention Focus*

JEL classification: *D03, D81, C91*

1. Introduction

The first approach based on a utility curve was proposed by Nicolas Bernoulli as early as 1734. However, it was von Neumann and Morgenstern (1944) who showed that the expected utility hypothesis could be derived from several axioms which assumed that human decisions are rational. Since then, expected utility theory has become the dominant hypothesis in the economic thought of that time. As early as 1948, Friedman and Savage argued that the curvature of the utility function varies according to individual wealth. Further developments were proposed by Markowitz (1952), who considered the shape of the utility function around the "customary" level of wealth. Later on, in Subjective Utility Theory (Savage, 1954) the classical definition of probability was replaced with subjective and personal probability.

However, the growing amount of experimental data indicated that no utility function could correctly explain human behavior. These experiments included the Allais paradox (1953), the Ellsberg paradox (1961), the preference reversal effect (Lichtenstein and Slovic, 1971), framing effects (Kahneman and Tversky, 1979) and others. This led to the creation of several theories collectively referred to as Non-Expected Utility Theories. These include Prospect Theory (Kahneman and Tversky, 1979), Regret Theory (Bell, 1982; Loomes and Sudgen, 1982) and Rank - Dependent (Expected) Utility Theory (Quiggin, 1982). As Prospect Theory was met with objections from a mathematical point of view, a corrected version was created - Cumulative Prospect Theory, CPT (Kahneman and Tversky, 1992).

Prospect Theory, and its extended version, gave rise to the concepts of the value function and the probability weighting function. The value function is supposed to evidence risk aversion for gain prospects and risk seeking for loss prospects, as well as a general aversion to losses. The probability weighting function is supposed to show the non-linear transformation of probabilities when making decisions, which would explain people's inclination to participate in lotteries as well as the tendency towards less risky investments in the case of average probabilities. Prospect Theory gave rise to new research trends. Much attention was focused on the probability weighting function (Camerer and Ho, 1994; Wu and Gonzalez, 1996, 1999; Prelec, 1998; Tversky and Wakker, 1995). The theory was used to explain financial market phenomena such as the Equity Premium Puzzle (Benartzi and Thaler, 1995), the Disposition Effect (Shefrin and Statman, 1985), Mental Accounting (Thaler, 1985), and the Endowment Effect (Kahneman, Knetsch, and Thaler, 1990). In 2002, Daniel Kahneman was awarded the Nobel Prize in Economics for his work on Prospect Theory[1].

Prospect Theory has also met with criticism. Nwogugu (2006) has compiled a large collection of objections and draws on a bibliography of 131 titles to support his claims. The author asserts that Prospect Theory was derived using improper methods and calculations and that it is not consonant with natural mental processes. Shu (1995) shows that it is wrong to assume the existence of probability weights. Neilson and Stowe (2002) demonstrate that Prospect Theory cannot simultaneously explain participation in lotteries and the Allais paradox. Blavatsky (2005) claims that the theory does not explain the St. Petersburg paradox, a classic problem of decision making under conditions of risk. Levy and Levy (2002) state that their experimental results negate Prospect Theory and confirm the Markowitz hypothesis. These results have been criticized by Wakker (2003).[2]

The present paper, too, is critical of Prospect Theory. However, it is not criticizing individual components or individual methodological assumptions, but is rather focused on analyzing the entire process of how the end results of the 1992 study were obtained from the experimental data that were presented. It has been stated that apart from the operations made at the initial stage of problem resolution, which Prospect Theory defines as Editing (in this study generalized as Mental Adaptation), any analysis of the experimental data should include other mental transformations such as Prospect Scaling (resulting from Focused Attention) and Logarithmic Perception of Financial Stimuli. This assumption finds its explanation in psychology, in particular cognitive psychology, and in research at the sensory and neuronal levels.

On the basis of the assumptions stated above and using exactly the same experimental data that were used to derive Cumulative Prospect Theory, an explicit, simple and symmetric

[1] Milton Friedman (in 1976), Maurice Allais (in 1988) and the previously mentioned Harry Markowitz (in 1990) were also awarded the Nobel Prize, albeit for achievements in other areas of economics.

[2] Whereas Levy and Levy analyzed the shape of the value function without regard to the probability weighting function, Wakker, who developed CPT axiomatization, pointed out that the only correct course is to consider both functions together. This statement from the guardian of the Theory is of paramount importance to understanding the conclusions of the present study. It is also interesting to note that Wakker only makes use of this argument in view of the serious criticism of the value function of Prospect Theory. In fact a great deal of the publications supporting Prospect Theory only refer to the utility function or the probability weighting function. This very flexible (not to say free) treatment of the assumptions underlying Prospect Theory, can lead (often in an unauthorized manner) to the scope of its effectiveness being broadened. However, authors that do not criticize Prospect Theory are not exposed to pointing out that obvious mistake. This concerns, for example, those authors who explain the Disposition Effect by only using the value function of Prospect Theory.

solution was obtained without the use of the probability weighting function. A function was obtained which describes a direct relationship between the probability and relative certainty equivalent (or its normalized logarithm). The resulting curve (named the aspiration function) has a symmetric double S-type shape consistent with the Markowitz hypothesis (1952). The study shows that the relationship obtained may be derived using simple transformations of the value function and the probability weighting function, i.e. those functions which are the end result of Prospect Theory. As the resulting relationship is of a general nature, the value and probability weighting functions are in this situation only one of the many ways of representing the general solution. This proves that these functions should no longer be treated as the correct interpretation of the experimental data. More importantly, the aspiration function explains how attitude towards risk is dependant on state of mind. The risk seeking is present when the expected value of outcomes lies far away from the aspiration target defined by the attention focus. This is implied by the convex shape of this part of the curve. On the other hand, when the expected value of outcomes is close to the aspiration target, the concave shape of the curve indicates risk aversion. In case of losses, the pattern is reversed. The explanation of risk attitudes given by the convex-concave-convex-concave shape of the aspiration function substitutes the fourfold pattern introduced by CPT. Summarizing the results presented in this study provide a basis of negating Prospect Theory as a theory that correctly describes how decisions are made under conditions of risk.

The paper is organized as follows. Section 2 briefly describes the mental transformations which form the basis of the derivation presented in the following part of the study. These transformations include Probability Weighting, Mental Adaptation, Prospect Scaling and Logarithmic Perception of Financial Stimuli. Section 3 provides a solution using the Mental Adaptation and Prospect Scaling transformations. Direct S-shaped relationships between the probability and relative certainty equivalents are obtained separately for gain and loss prospects. Section 4 presents the solution with the additional consideration of the Stimuli Logarithmic Transformation. In Section 5, the results for gain and loss prospects are combined to produce a single solution named the “aspiration function”. The obtained curve strongly resembles the utility function specified by the Markowitz hypothesis. Section 6 of the paper is devoted to comparing the achieved result with that hypothesis in more detail. Section 7 shows that the relationship obtained may be derived by transforming the end results of Prospect Theory. Section 8 indicates that this relationship is of a general nature while those functions which are the end result of Prospect Theory are only one of many ways of representing the general solution. Section 9 summarizes the study.

2. Short Review of Mental Transformations[3]

2.1. Transformation of Probabilities.

That perception of probabilities is distorted is simultaneously one of the key assumptions and key results of Prospect Theory. The concept of decision weights was introduced into the first version of Prospect Theory in 1979. Even at that early stage, Kahneman and Tversky were stating that decision weights were not probabilities and did not comply with the axioms of probability. This led to serious mathematical objections (failure to comply with the First Order Stochastic Dominance). As a result, Rank-Dependent Expected Utility Theory (Quiggin) was developed as early as 1982 to remedy the shortcomings of its predecessor. The key concepts of that theory were later adopted by Cumulative Prospect Theory (Kahneman, Tversky, 1992). The probability weights are treated as a probabilistic measure in CPT, even though its axiomatization is based on complex topological models and Choquet integrals (e.g. Schmeidler, 1989, Wakker 1989, 1990; Kahneman and Tversky, 1992 and appendix to their publication written by Wakker).

It is important to note that Kahneman and Tversky distinguish *overestimation* (often encountered when assessing the probability of rare events) and *overweighting* (as a feature of decision weights) (Prospect Theory, 1979). The latter phenomenon lacks psychological justification to the extent that the former has it (for instance by dint of insufficient knowledge). It is difficult to explain in psychological terms how a decision regarding an event whose probability is known seems to assume a different probability value. This is what the probability weighting function addresses. Furthermore no mechanism was posited to explain why this effect of probability transformation only manifests itself at the moment a decision is made. A failure to distinguish between *overestimation* (which can be referred to as a kind of *subjective* view of events whose probabilities are not known) and *overweighting* (an artificial concept to explain the results of experiments regarding events whose probabilities are known) leads to the commonly accepted view that the probability weighting function has a profound psychological justification. I cannot concur with this view. More importantly, the next part of this study shows that the

[3] Because of the breadth of the argumentation, these transformations are described in more detail in other, as yet unpublished, works of the author of this publication. The review presented here mainly refers to those works.

probability weighting function (i.e. the entire probability transformation concept) is not necessary to explain the results of the experiments conducted by Kahneman and Tversky.

2.2. Mental Adaptation

The concept of mental adaptation is discussed in more detail in another, as yet unpublished study by this author. From the definition of "adaptation" at the evolutionary, sensory, psychological and mathematical levels, it is shown that the primary purpose of the initial phase of problem resolution, known as Editing in Prospect Theory, is to transform and simplify the problem of making a decision, and that this may be referred to as the problem adaptation stage. Furthermore, since mental adaptation to a certain phenomenon[4] may be succinctly described as "the state of not thinking about this phenomenon" (Sulavik, 1997), and since translation (shift) is a mathematical representation of adaptation, it can be seen that the major operations of the Editing phase, viz. *Coding*, *Segregation*, *Cancellation* and *Detection of Dominance,* may all come under concept of mental adaptation. This generalization is not merely of linguistic significance; it enables a common connection to be found between seemingly different mental operations carried out at an early stage of decision making. Moreover, it has a significant meaning when discussing more complex interactions between adaptation and attention processes.[5]

2.3. Prospect Scaling

Prospect Scaling, as the mental transformation resulting from focusing attention, is of key significance for deriving the solution presented in the following part of this study. The springboard for discussion presented in more detail in an as yet unpublished work by this author is the Weber law[6], one of the fundamental laws of psychophysics. It has been shown that the human sensory system adapts itself to financial quantities, just as it does to physical ones. The result is that the ratio of Just Noticeable Difference (jnd) to reference value remains roughly constant for different financial amounts.[7] This means that when looking at financial prospects

[4] Originally to death in case of rescuers.

[5] Mental Adaptation should not be confused with Hedonic Adaptation – a term also used in the psychological literature – despite the many similarities between the two. Frederick and Loewenstein summarize in their 1999 review of the subject that Hedonic Adaptation consists of two different processes which need to be clearly distinguished. The first is *shifting adaptation level* (which we understand as mental, sensory etc. adaptation) and the second is *sensitization* or *desensitization* which correlates with the Scaling described in 2.3.

[6] Not to be confused with the Weber-Fechner Law discussed in 2.4.

[7] For instance, jnd is approximately 1 PLN (Polish Zloty) when shopping with 100 PLN, whereas negotiations are conducted in units of thousands of PLN when purchasing a 1 million PLN house. A purchase offer of 979 538 PLN

(projects, investments, lotteries etc.), the reference value (size of the investment, major lottery prize) becomes a point of reference in the entire mental process, causing an absolute amount of money (say 10 USD) to be relevant or irrelevant depending on the context. This conclusion constitutes a fundamental difference to Prospect Theory, which regards profits and losses in absolute terms, and tries to draw a value function as a function of absolute amounts of money[8].

The mechanism responsible for this mental transformation is attention – one of the most thoroughly examined concepts in cognitive psychology. According to one of the definitions, attention is the process of selectively concentrating on a single perceived object, source of stimulation, or topic from among the many available options (Nęcka, 2007). The existence of attention is indispensable on account of a living organism's need to adapt to the demands of the environment (Broadbent, 1958) and on account of the finite ability of the brain to process information (Duncan, Humphreys, 1989). Several models of attention division are discussed, especially in relation to Focused Attention. The entire mechanism can be explained by such aspects of attention as Selection and Gain (Amplification) Control*,* the existence of which is evidenced by attention research at the neuronal level. Among others, Hillyard et al. (1998) state that attention has a gain (amplification) control character which aims to increase the signal to noise ratio of the stimuli on which attention is focused. The signal of most interest to the brain is maintained at a stable and optimal level as a result. Further, it is assumed that the amplification control mechanism operates at a higher mental level as well. This leads to problems differing in scale being perceived as equally significant when attention is focused. It is not difficult to conclude that the mathematical equivalent of amplification is homothety or scaling.

Since attention and its degree of concentration decides the choice of reference point, and since there are other signals and issues vying for attention, it may be assumed that other quantities may be potential points of reference. One of the most important points for a human being is the value of his/her wealth. If there is any likelihood that the problem under consideration will have serious wealth consequences, then the choice of reference point will definitely be influenced by that fact (even if there is an attempt to focus attention on some other value). On the other hand, it can safely be assumed that it will not be possible to focus attention

is hard to imagine; 980 thousand PLN seems far more likely. One zloty, a significant amount in the former case, is completely insignificant in the latter. Even 100 PLN, the sum total of a person's expenditure in a shop, is of no significance in a house purchase. (1 EUR = 4 PLN, 1 USD = 3 PLN).

[8] This does not even seem possible according to the Weber law.

on minor problems. The context of the problem (framing) is of vital significance. This is what induces people to make different choices depending on the manner in which an issue is presented. Finally, the choice of reference point may also be influenced by random events (anchoring). The arguments regarding attention mechanisms (and especially focused attention) provide a basis for explaining other effects known in the literature, such as mental accounting, which describes the fact that people keep mentally separate accounts for their expenditures and investments[9].

The arguments cited indicate that the attention focused on a specific payment in the conducted experiments seems to be a natural effect that has to be factored into any analysis of the results[10]. However, the authors of Prospect Theory have failed to do this, even though Kahneman has written some important studies on the topic of attention (1973). The assumption that the value of a prospect payment becomes a reference point in the conducted experiments leads to a completely different solution than that which Prospect Theory proposes.

2.4. Logarithmic Perception of Financial Stimuli

Logarithmic perception of financial stimuli is the last mental transformation significant to deriving the results presented in the following part of the study. Here, the point of reference for a detailed discussion in a yet unpublished work is also a fundamental psychophysical law, viz. the Weber-Fechner law, which concerns the logarithmic perception of stimuli.[11] Logarithmic perception of financial figures is not considered in Prospect Theory despite there being known examples of logarithmic and exponential functions being used in financial applications.[12] Instead, the authors of Prospect Theory used the relationship introduced by Stevens (1957), who stated that stimuli perception was determined by a power function. That type of function was included in Prospect Theory to describe the value function. Surprisingly the difference in approach turns out to be insignificant since both functions (logarithmic and power) have an almost identical

[9] One of the most frequently repeated assertions found in the behavioral literature is that Prospect Theory explains the phenomenon of Mental Accounting (Thaler, 1985). In fact, Thaler only shows how some calculations on separate mental accounts are performed using the value function of Prospect Theory and devotes only a minor part of the publication to the subject. What really fascinates Thaler is that people actually keep such separate accounts. Prospect Theory provides no answer as to why they do. The real explanation of this phenomenon lies in the process of focusing attention.

[10] This is especially the case under experimental conditions as those surveyed are remunerated for their participation; it means they are paid to focus their whole attention on the analyzed problems.

[11] Hearing described using the decibel scale is an example of this sort of perception.

[12] Suffice it to mention compound interest, logarithmic index charts and logarithmic rates of return.

shape for low x-coordinate values.[13] This leads to the conclusion that the value function in Prospect Theory is actually a logarithmic curve. Further arguments in favor of a logarithmic perception of financial stimuli are provided by other results presented in CPT, which states that mixed prospects are accepted when gains are at least twice as high as losses. This effect may be easily explained by noticing that in logarithmic terms, a 100% profit corresponds to a 50% loss. The experimental results presented in the CPT article also show that, for a probability of 0.5, the certainty equivalents appear to be around 0.41[14] of the payment value irrespective of the type of prospect (gain or loss), the presence or absence of riskless components, or the payment amount after deducting the riskless components. This effect is interpreted as being a result of a logarithmic perception of payment value. The final conclusion is that the value curve in Prospect Theory is more a reflection of a logarithmic perception of financial figures than any real representation of attitude to risk. Taking the logarithmic perception of financial stimuli into consideration when analyzing experimental data leads to some interesting conclusions. These are presented in point 4 of this study.

3. Solution Using Prospect Scaling Transformation

This section contains the alternative analysis of the experimental data presented by Kahneman and Tversky in their 1992 paper. This analysis is based on the assumption that apart from operations defined by Prospect Theory as Editing (and to a large extent generalized here as Mental Adaptation Transformations), Prospect Scaling should be also considered when analyzing the experimental data. It is here assumed that the reference points for the certainty equivalents under examination were the prospect payments (outcomes) themselves. It should be emphasized that the methodology presented below does not assume either the existence or the absence of the value function or the utility function. Similarly, it does not assume either the existence or the absence of the probability weighting function. Even if these do exist, no certainty equivalent (*CE*) calculation method is assumed. This means that the results obtained are of a general nature.

During the experiment conducted by Kahneman and Tversky, certainty equivalents *CE* were collected for the prospects of payment *\$A* with probability 1 - *p* or payment *\$P* with probability *p*, where:

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[13] Within the range [0, 0.6], $x^{0.88}/1.34$ is the best approximation of the *ln(1+x)* function using a power function. The coefficient 0.88 is exactly the same as the power coefficient of the value function in Prospect Theory.

[14] This is exactly $\sqrt{2}-1$.

$|A| < |P|$ (1)

Due to (1), the payment *\$A* should be interpreted as the riskless component. The experimental results are presented in Table 3.3 of the original CPT publication (1992). It is assumed that there is a function *f* such that:

$CE = f(A, P, p)$ (2)

The variables *CE'* and *P'* are now introduced to account for the mental adaptation process. These are an *A* translation of *CE* and *P*:

$CE' = CE - A$ (3)

$P' = P - A$ (4)

If $A = 0$, we refer to the prospect as having no riskless component and then $CE' = CE$ and $P' = P$. Introducing these new variables presupposes the existence of a function *g* such that:

$CE' = g(P', p)$ (5)

At this point Equation (5) is transformed in such a way that probability *p*, and not *CE'*, becomes the value to be determined. Due to the fact that *CE'* is monotonic with respect to *p*, it may be assumed that there is an inverse function *h* such that:

$p = h(CE', P')$ (6)

In order to take Prospect Scaling (an effect resulting from Focused Attention) into account, it is assumed that the value of payment *P'* becomes the reference point for the certainty equivalent *CE'* and that the equivalent values are scaled by a coefficient $1/P'$. As a result, a variable $r = CE' / P'$ is introduced as the relative certainty equivalent with a value in the range [0,1]. This also supports the existence of the following *q* function defined over the range [0,1]:

$p = q(CE' / P') = q(r)$ (7)

For example, for the specific values listed in Table 3.3 of Kahneman and Tversky's paper, the relationships $q(9/50) = 0.10$, $q(21/50) = 0.50$, and $q(37/50) = 0.90$ are obtained for the prospect (0, 50), and the relationships $q(14/100) = 0.05$, $q(25/100) = 0.25$ are obtained for the prospect (0, 100). For the prospect with the riskless component (50, 150), the relationships $q((64-50)/100) = q(14/100) = 0.05$, $q((72.5-50)/100) = q(22.5/100) = 0.25$, and $q((86-50)/100) = q(36/100) = 0.5$, are obtained after the Mental Adaptation Transformation.

The obtained values are plotted on the graph $p = q(r)$ and approximated using the least squares method with the assistance of the Cumulative Beta Distribution $I_s(\alpha,\beta)$ (i.e. regularized incomplete beta function). This particular function was selected because it is defined in the domain [0,1] and because of the extraordinary flexibility the two parameters α and β give its shape.[15] Approximations were made separately for the loss ($P < 0$)[16] and gain ($P > 0$) prospects. The results are presented in Fig. 1.

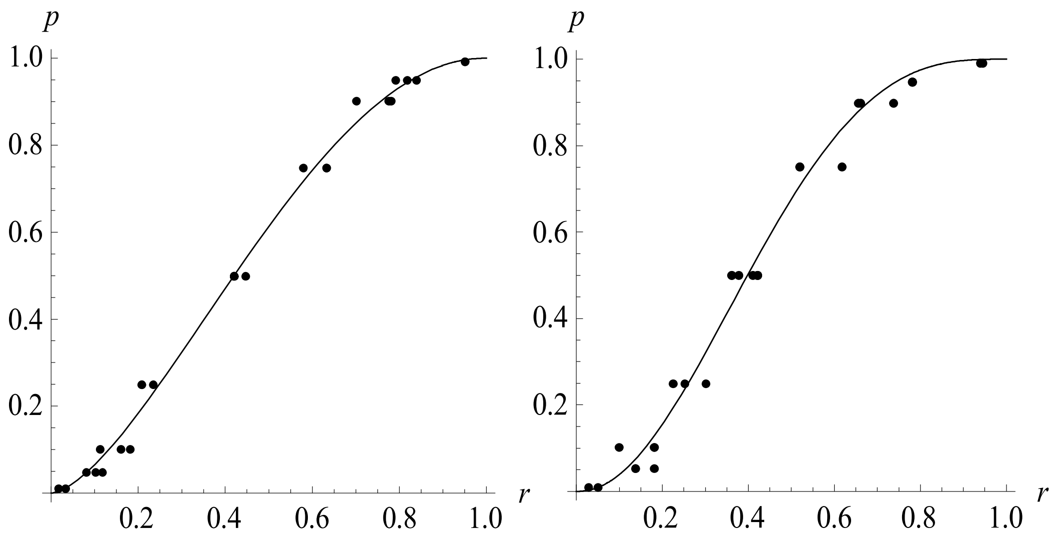

Fig.1. Transformed experimental points and approximation $p = q(r)$ using cumulative beta distribution function for loss prospects (left) and for gain prospects (right).

The approximations obtained for the function $p = q(r)$ for loss and gain prospects allow the following conclusions to be drawn:

1. The function $p = q(r)$ has an S-type shape both for loss and gain prospects.
2. The intersection of the approximation functions $p = q(r)$ with the straight line $p = r$ occurs when r has a value of approximately 0.25.
3. The respective values of the parameters α and β are 2.24 and 3.22 for gain prospects and 1.59 and 2.09 for loss prospects. The disparity between the parameters α and β in both cases confirms the asymmetry of the function $q(r)$ with respect to the center point $(p, r) = (1/2, 1/2)$.

Assuming Focused Attention and the resulting Prospect Scaling Transformation led to a different solution than that presented by Prospect Theory. First of all, the entire description has been reduced to the relationship $p = q(r)$, and the value function and the probability weighting

[15] The function is a cumulative distribution function, although there is no distribution of the random variable in the classical sense. Neither the function nor the method and accuracy of the approximation are the most important considerations at this point as this is only the first approximation of the solution. More detailed calculations will be conducted in section 4.

[16] It should be noted that for the loss prospects, the value of relative certainty equivalent r is also positive, as *CE'* and *P'* are both negative in this case.

function have disappeared altogether as they are not needed to describe the experimental results. Secondly, the relative certainty equivalent r is directly transformed into probability p (and vice versa). This means that in order to determine the certainty equivalent CE for probability p, the value of r need only be read directly from the graph (Fig. 2) and multiplied by the value of payment P'. For example, $r = 0.75$ for $p = 0.95$. Hence, $CE' = 75$ for $P' = 100$ (the value obtained experimentally was 78).. In case of prospects with riskless components, e.g. (50, 150), the value of the certainty equivalent $CE = CE' + A = 75 + 50 = 125$ (the value 128 was obtained in the experiment).

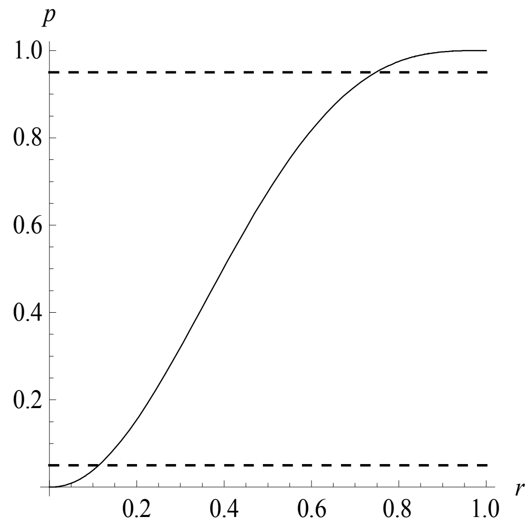

Fig. 2. Relationship $p = q(r)$ for gain prospects with plotted lines $p = 0.05$ and $p = 0.95$.

Finally, nonlinear changes of the certainty equivalents (especially within high and low probability ranges) can be presented simply (see Figure 2). Increasing probability from 0 to 0.05 causes the relative certainty equivalent r to increase from 0 to 0.11. Increasing probability from 0.95 to 1 causes the relative certainty equivalent r to increase from 0.75 to 1. A similar explanation could be presented for loss prospects.

4. Solution Using Stimuli Logarithmic Transformation

Section 3 presented the initial solution to the problem. In this section, a more extensive result will be obtained by considering the Stimuli Logarithmic Transformation, and the approximation itself will be conducted in a more methodic and systematic way. The procedure is similar to the one in Section 3, but instead of the relative certainty equivalent r, its normalized logarithm s, defined below, will be used for the approximation:

$$s = \frac{\ln(1+r)}{\ln(2)} \quad (8)$$

The denominator ln(2) is introduced to normalize the value of s to the range [0,1]. Introducing the variable s presupposes that there is a function u such that:

$$p = u(s) \quad (9)$$

The mean value of the approximation error, defined below, was taken to be the minimization function:

$$AvgErr = \sqrt{\frac{\sum_{i=1}^{n}\left(\frac{MCE_i^{'}}{CE_i^{'}} - 1\right)^2}{n-1}} \quad (10)$$

where: MCE'_i - value resulted from the model, CE'_i - value obtained in the experiment, n – number of data (28).

The following functions were selected for approximation:

Cumulative Kumaraswamy Distribution:

$$KUM = 1 - \left(1 - s^{\alpha}\right)^{\beta} \quad (11)$$

Cumulative Beta Distribution - regularized incomplete beta function:

$$BET = I_s(\alpha, \beta) \quad (12)$$

Furthermore, three functions usually used to determine the reverse S-type shaped probability weighting function were used here to approximate an S-type shaped function.

Function used by Kahneman and Tversky:

$$KTW = \frac{s^{\alpha}}{\left((1-s)^{\alpha} + s^{\alpha}\right)^{\frac{1}{\alpha}}} \quad (13)$$

Prelec’s function with one parameter:

$$PR1 = e^{-(-\ln(s))^{\alpha}} \quad (14)$$

Prelec’s function with two parameters:

$$PR2 = e^{-(-\beta\ln(s))^{\alpha}} \quad (15)$$

Cumulative Prospect Theory calculations were also performed in order to check its correctness and accuracy in comparison against the presented approach. As Prospect Theory uses two functions (value and probability weighting) it should help find a better approximation than all the other tested functions. The results of the calculations are presented in Table 1. The column AvgErr contains mean approximation errors for loss and gain prospects. The next columns contain the obtained parameters.

	$AvgErr_n$	$AvgErr_p$	α_n	β_n	α_p	β_p
CPT	0.174	0.233				
KUM	0.151	0.164	1.756	1.882	2.307	3.065
BET	0.153	0.167	1.827	1.754	2.603	2.630
KTW	0.156	0.195	1.502		1.747	
PR1	0.183	0.194	1.495		1.883	
PR2	0.162	0.177	1.277	1.142	1.621	1.116

Table 1. Results of approximation of function $p = u(s)$. The rows contain the approximation function (see description). The columns contain approximation errors and obtained coefficients for loss (n) and gain (p) prospects.

Prospect Theory provides the greatest approximation error (especially for gain prospects) despite using two functions. The lowest errors were obtained for the Cumulative Kumaraswamy Distribution, and then for the Cumulative Beta Distribution. The *KTW*, *PR1* and *PR2* functions produce worse results than the *KUM* and *BET* functions, but still do better than CPT (with the exception of *PR1* for loss prospects). Paradoxically then, the *KTW*, *PR1* and *PR2* functions used by other authors to confirm Prospect Theory were successfully tested here to negate it. Surprisingly, they gave better results when used by themselves than in conjunction with the value function of the Theory. The determined Cumulative Kumaraswamy and Cumulative Beta distribution shapes for loss and gain prospects are presented in Figure 3. These curves are practically identical and, because the approximation errors are very close, the two functions may be used interchangeably.

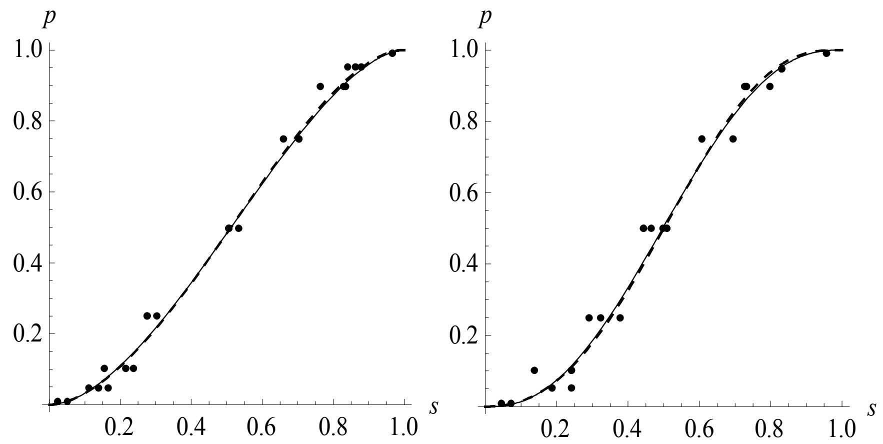

Fig. 3. Approximation of function $p = u(s)$ for loss (left) and gain (right) prospects using Cumulative Beta Distribution function (solid) and Kumaraswamy function (dashed).

It should be emphasized that these curves are extraordinarily symmetric with respect to the center of the graph. Table 2 presents the values of probability p for $s = 1/2$ for loss and gain prospects.

	p_n	p_p
KUM	0.484	0.499
BET	0.483	0.505

Table 2. Values of probability p for $s = 1/2$. The rows contain the approximation function, the columns contain the respective probabilities for the loss (n) and gain (p) prospects.

Both curves virtually go through the point $(p, s) = (1/2, 1/2)$. This symmetry is also evidenced by the similarity of the values of the α and β parameters of the determined Cumulative Beta functions (Tab.1).

5. Putting the Pieces Together

The solutions obtained so far comprise two $p = u(s)$ functions with one describing losses, the other gains[17]. The loss and gain prospects need to be scaled before the two functions can be presented together. The simplest assumption has been adopted, similar to the Prospect Theory approach when defining the value function, namely:

$$u_p = \lambda\, u_n \tag{16}$$

where u_p is the curve for gains, and u_n is the curve for losses. In order to determine the scale

[17] Of course there are also two $p = q\ (r)$ functions with one describing losses, the other gains. These could alternatively be used in Section 5.

factor λ, Kahneman and Tversky conducted additional experiments, the results of which are presented in Table 3.6 of the original publication. The obtained results indicate that the mixed prospects are accepted if the profit is at least twice as great as the loss.[18] In order to scale both $p = u(s)$ curves, a *ratio* Θ of 2.07 is assumed for further calculations as the mean value of Θ resulting from problems 1-6[19]. We can now write:

$$u_p(s_p) = \lambda\, u_n(s_n) \tag{17}$$

where: $s_p = \ln(1+1)/\ln(2) = 1$ and $s_n = \ln(1+1/2.07)/\ln(2) \approx 0.568$. Hence

$$u_p(1) = \lambda\, u_n(0.568) \tag{18}$$

Taking into account that $u_p(1) = 1$, and $u_n(0.568) \approx 0.582$ (using the Kumaraswamy function approximation) we obtain:

$$\lambda = 1/0.582 \approx 1.72 \tag{19}$$

Now, let us present this result graphically. Fig. 4a shows functions u_p and u_n (the latter multiplied by λ). It is evident that u_n is now equal to 1 for $s = 0.568$, and that the loss and gain curves are scaled. Fig. 4b presents both functions in different form. The function u_n for the loss prospects is presented within a range of [-1,0], and the scale factor λ has a value of -1.72.

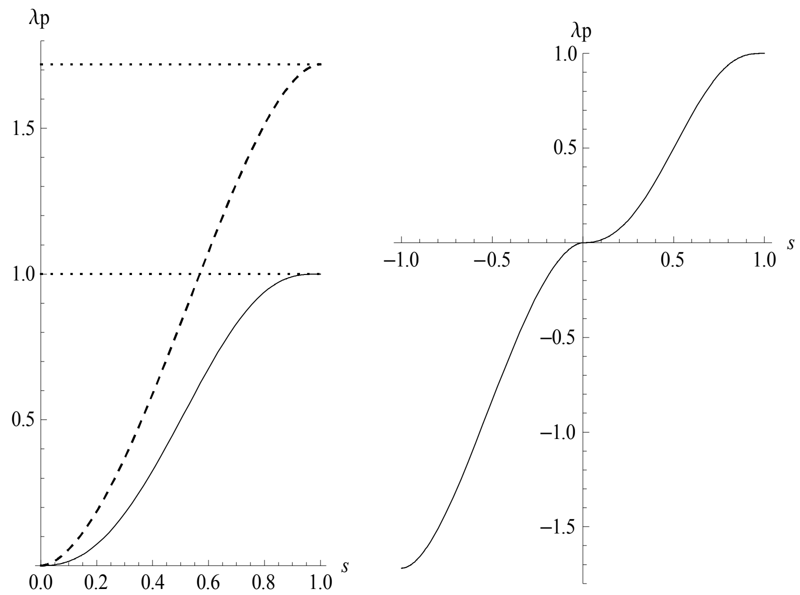

Fig. 4. Functions $u_p(s)$ and $u_n(s)$ presented together on a single graph. (Left) within a range of s [0,1]; function $u_n(s)$ multiplied by the constant $\lambda = 1.72$, (right) function $u_n(s)$ within a range of s [-1,0] and multiplied by the constant $\lambda = -1.72$.

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[18] On this basis, Kahneman and Tversky established the value of λ in the value function to be 2.25. Unfortunately, the method of establishing this value was not provided.

[19] See "Advances in Prospect Theory" for more details.

The question may be posed, how to interpret the curve presented in figure 4b? Those accustomed to the term "value function" may use it. On the other hand, I strongly prefer the term "aspiration function" (which will be explained in more detail in the next section). Whatever the accepted terminology, it needs to be stated that this curve presents the sum total of all the knowledge that has come out of Prospect Theory and its cumulative version.

1. The fourfold pattern of risk attitudes, which was presented by CPT (but not by the earlier PT) and confirmed in other studies, is evident:

    a). in case of gain prospects, the curve is convex for probabilities below 30% (corresponding to risk taking), and becomes concave for probabilities above 30% (corresponding to risk aversion);

    b). in case of loss prospects, the curve is concave for probabilities below 20% (corresponding to risk aversion), and becomes convex for probabilities above 20% (corresponding to risk seeking).

2. The convex-concave-convex-concave shape of the aspiration function substitutes therefore the fourfold pattern of risk attitudes described by CPT.

3. The function's more linear shape for loss prospects confirms the results of other studies that people's attitude to risk for losses is rather neutral in nature[20].

4. Both parts of the curve (for loss and gain prospects) describe the results of experiments without having to resort to the probability weighting function. They also describe these results with better precision than Prospect Theory.

5. Both parts of the curve are scaled, which means that mixed prospects can also be analyzed.

6. The Aspiration Function and the Markowitz Utility Function Hypothesis

In 1952, Markowitz published an article "The Utility of Wealth" presenting his hypothesis on the shape of the utility function. While this article was known to Kahneman and Tversky, they believed that neither this nor any other utility function could explain certain psychological experiments. This led to the development of Prospect Theory as an alternative to classical economic theories based on utility functions. That the aspiration function so closely resembles the curve presented in the Markowitz article (Fig. 5) is highly surprising given the result obtained

[20] See Wakker (2003) for reference, who confirms that the pattern for losses is less clear than in the case of gains.

comes out from the experimental data used to derive Cumulative Prospect Theory.

Markowitz specified the utility function as follows: *The utility function has three inflection points. The middle inflection point is defined to be at the "customary" level of wealth. The first inflection point is below customary wealth and the third inflection point is above it. The distance between the inflection points is a non-decreasing function of wealth. The curve is monotonically increasing but <u>bounded from above and from below</u>; it is first concave, then convex, then concave, and finally convex. We may also assume that $|U(-X)| > U(X)$, $X > o$ (where $X = o$ is customary wealth).*[21]

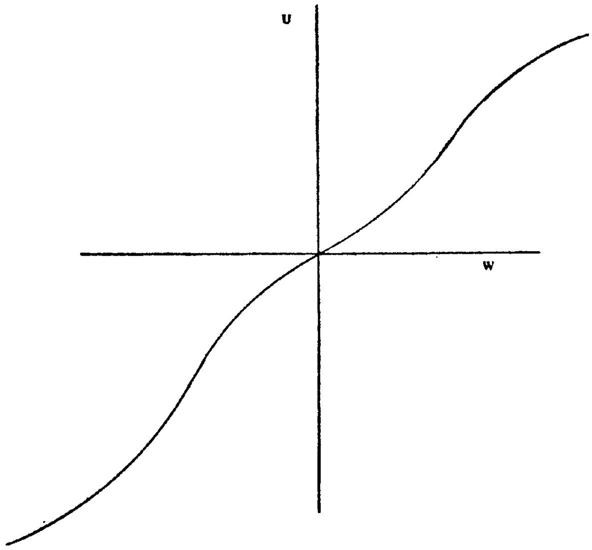

Fig. 5 The shape of the utility function according to the Markowitz hypothesis of 1952.

It is clear that all bar one of the requirements of the utility curve expressed by Markowitz in his hypothesis are met by the curve presented in Fig. 4b. The aspiration function has three inflection points right where Markowitz predicted they would be. The function is monotonically increasing and <u>is limited from the top and from the bottom.</u> Concavities and convexities occur in the order assumed by Markowitz. The condition related to the function value for X values having opposite signs is also met (which Fig. 4a verifies). The only condition, which is not met, is that the distances between the inflection points depend on people's wealth. Markowitz noted: *If the chooser were rather rich, my guess is that he would act as if his first and third inflection points were farther from the origin. Conversely, if the chooser were rather poor, I should expect him to*

[21] One more requirement was important for Markowitz: *In the case of recent windfall gains or losses the second inflection point may, temporarily, deviate from present wealth.* This requirement does not influence the shape of the curve but is important when considering the dynamics of people's behavior.

act as if his first and third inflection points were closer to the origin. In the Markowitz hypothesis the position of inflection points changes because the *w* (wealth)-axis is expressed in absolute terms. This, however is only a minor difference to the aspiration function, where the *s*-axis[22] is expressed relatively. As Markowitz assumed a correspondence between inflection points and wealth, he could have expressed as well the *w*-axis in relation to the wealth value. This way he would have "fixed" the position of inflection points on the graph.

However, what really differentiates the aspiration function is that the value in relation to which wealth changes are considered does not depend on wealth but on state of mind, or strictly speaking, on whatever value the subject's attention happens to be focused on. For this reason gains and losses (using the Prospect Theory terminology) are considered in relation to a reference value which differs from wealth in most of cases. Therefore people commonly say "I have gained 15% on my stock investments" rather than "I have gained 5% *of my wealth* on my stock investments". It is clear enough that the former sentence assumes the value of the stock investment as the reference for gain/losses considerations.

There is also another psychological reason why the shape of the aspiration function does not depend on the total value of wealth (as in Markowitz hypothesis). According to Thaler (1985) people keep mentally separate accounts, so that investments and expenditures are considered as separate parts rather than as a whole. As a result, instead of saying "I have lost 2% of my wealth on my stock and real estate investments" people typically consider "I have lost 5% on my house but I have gained 15% on stocks" despite the fact that the absolute values of stock and house investments may differ substantially. This follows that the aspiration functions applies for each separate account, however with different reference values set up by the attention focus.

Important to note that this reference value may be considered here as a subject's aspiration target (in the positive and the negative sense), which, depending on circumstances, can be mentally set up on different levels. This explains the name of the aspiration function and helps to understand the risk attitude associated with its different parts. Let us consider gain prospects first. When the expected value of outcomes lies far away from the aspiration target (e.g. close to the origin), this part of the curve is convex what implies the risk seeking attitude. This could easily be explained by saying that the subject has much to win but has not that much to lose. On the other hand, when the expected value of outcomes is close to the aspiration target the concave

[22] Or *r*-axis if we draw a similar curve for relative certainty equivalents.

shape of the curve indicates risk aversion. Here the subject does not have much to win but definitely has a lot to lose. Conversely, small losses cause risk aversion for negative prospects as there is still a lot to lose (this part of the curve is concave) whereas big losses imply a risk seeking attitude, as there is nothing more to lose but everything to win (the curve is convex here).

The risk attitude pattern described above may simultaneously work for different mental accounts with different aspiration targets assigned to them. This may also mean people could be risk seeking and risk averse at the same time depending on the status and prospects of each account.[23]

Markowitz's assumption that the shape of the utility curve corresponds with the value of wealth precluded his curve (however tempting its shape) from being able to explain experiments on financial payments which were not directly related to the wealth of the people being studied. This is what led Kahneman and Tversky to reject the Markowitz hypothesis and develop Prospect Theory. The result presented here, however, may signal a return to an approach based on the utility-like function and lead to a negation of Prospect Theory. Accepting that gains and losses need not to be considered in relation to wealth, but to any other value depending on where a person's attention is focused, is all that it would take to come back to this earlier concept. The payoff is a simpler and more accurate description of people's behavior.[24]

7. Solution Resulting From Prospect Theory

In section 3, a direct relationship $p = q(r)$ between probability p and relative certainty equivalent r was obtained. Section 4 presented a similar relationship $p = u(s)$ between probability

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[23] This may look pretty complex, but once it is correctly understood how attention works and what attention focus is, decisions commonly perceived as inconsistent start to become more rational.

[24] At this point I have to express my highest esteem to prof. Markowitz. In most references, his hypothesis from 1952 is presented (when presented at all) as the last Expected Utility-based theory before the era of Non-Expected Utility Theories, which alone could explain the results of psychological experiments. This view is however unfounded. It was Markowitz who first considered the shape of the utility function around the "customary" level of wealth (which reappeared as the gain/loss concept in Prospect Theory 27 years later). It was Markowitz who noticed that *Generally people avoid symmetric bets. This suggests that the curve falls faster to the left of the origin than it rises to the right of the origin* (which reappeared as the general loss aversion principle in Prospect Theory). Markowitz was able to use his curve to explain the purchase of both insurance and lottery tickets (one of the most prominent achievements of Prospect Theory 27 years later). The risk seeking/aversion pattern evinced by the shape of his curve is equivalent to what Cumulative Prospect Theory called the "fourfold" pattern 40 years later (and what the earlier version of the Theory had failed to discover). Due to the fact that the curve was bounded from above and below, Markowitz was able to avoid the St. Petersburg Paradox, something which eluded Prospect Theory. Finally, Markowitz presented important remarks about consequent choices and the dynamics of people's behavior. This subject is not addressed by Prospect Theory at all. I am absolutely certain that had Markowitz had not devoted his entire attention to Portfolio Theory (for which he was later awarded the Nobel Prize) after publishing his hypothesis, he would have presented a complete behavioral theory long before Kahneman and Tversky.

p and s, the normalized logarithm of r. This section is going to demonstrate that similar relationships can be obtained by Prospect Theory. First, however, the flowchart of both methodologies will be analyzed.

Figure 6 presents the flowchart of the CPT methodology together with that of mine in order to systematize both approaches. The right side presents the respective transformations, described in sections 3 and 4. The values of the certainty equivalents *CE* were recorded in the experiment as a function of payment *$P* or *$A* with a respective probability of p or 1-p. The original set of data is subjected to Mental Adaptation Transformation. The problem is then transformed to determine probability p (instead of certainty equivalent *CE'*). Prospect Scaling and Stimuli Logarithmic Transformation come next. These steps finally lead to determining the relationship $p = u(s)$. The best functions to describe this transformation seem to be the Cumulative Kumaraswamy Distribution and the Cumulative Beta Distribution.

Prospect Theory takes the very same first step, i. e. data are subjected to Editing operations equivalent to Mental Adaptation Transformations.[25] The probability weighting function is then derived and the two-variable function g(P', p)/P' is arbitrarily transformed into the single-variable function $w(p)$. This step is unclear and may be mathematically unsound. Moreover the authors do not explain how to derive the value function $v(x)$ from the experimental data (they only state they use a nonlinear regression procedure to estimate the parameters of assumed functions). The curve $v(x)$ has been placed near the Stimuli Logarithmic Transformation as these operations are equivalent.

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[25] As Cumulative Prospect Theory uses a rank-dependent (cumulative) representation it does not require explicit editing operations in order to avoid predicted violations of stochastic dominance. In the special case where $\$P>\$A>0$ the representation of the lottery (P, p; A, 1-p) is: $w(p)v(P)+[1-w(p)]v(A)$ where w is a probability weighting function and v is a value function. It could be rearranged to: $v(A)+w(p)[v(P)-v(A)]$, which shows that in case of Cumulative Prospect Theory it is not needed to perform any special editing operations required by its older version. It should however be noted that the editing operation presented above differs from the Segregation operation described by Prospect Theory. It states that the prospect (P, p; A, 1-p) is naturally decomposed into a sure gain of $A and the risky prospect (P-A, p; 0, 1-p), so that the riskless component $A is not further considered during $w(p)v(P-A)$ evaluation. This representation is obviously different from the CPT one. Whereas in case of PT adding a riskless component to both outcomes increases the certainty equivalents by the riskless component value, in case of CPT the value of CE should grow more (for a power factor α approaching 1 the difference between both results diminishes). However the experimental data of Kahneman & Tversky presented in their table 3.3. confirm rather PT approach (which is also used in this paper). In 11 out of 26 cases of prospects *with* a riskless component their CE differ from CE of prospects *without* a riskless component exactly by the riskless component value. For instance for the probability of 0.50 the certainty equivalent for outcomes ($0, $50) is $21 and for ($50, $100) – $71, which is exactly $21 + $50. Similarly the CE is $36 for ($0, $100), and $86 for ($50, $150). In 7 cases the difference is bigger and in 8 cases – smaller (absolute values are considered for negative prospects).

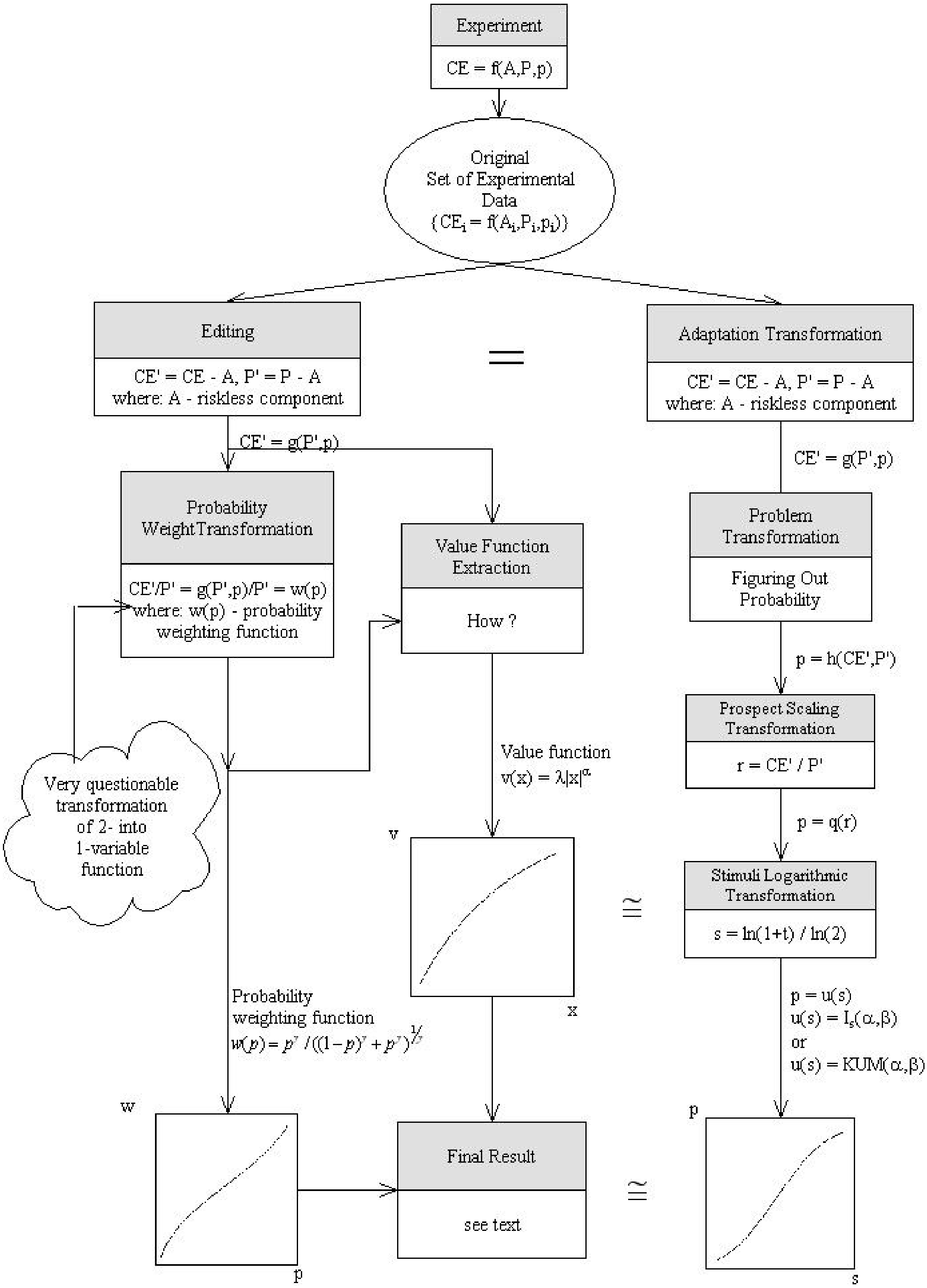

Fig. 6. Methodology Flowchart. The consecutive stages of Prospect Theory data analysis are on the left. The analysis stages of the author of this publication are on the right.

The end result of Prospect Theory is two functions, the value function and the probability weighting function, both of which need to be used together to provide correct calculations (Wakker, 2003). It should be expected that both methodologies lead to a similar final solution as they both use the same experimental data. For this reason, the box "Final Result" has been placed near my solution in the form of $p = u(s)$.

Using the value function and the probability weighting function in tandem leads to a surprising result, once a few simple transformations have been performed. In Prospect Theory, the certainty equivalent is obtained by applying the formula:

$$v(CE') = v(P')w(p) \tag{20}$$

where *w* is the probability weighting function (defined by Formula 13 as *KTW*) and *v* is the value function defined as the power function. The following is therefore obtained:

$$(CE')^{\alpha} = (P')^{\alpha} w(p) \tag{21}$$

hence

$$w(p) = \frac{(CE')^{\alpha}}{(P')^{\alpha}} = \left(\frac{CE'}{P'}\right)^{\alpha} = r^{\alpha} \tag{22}$$

In this way, the two functions that come out of Prospect Theory are reduced to a single relationship after a few simple transformations. This relationship uses the relative certainty equivalent *r* instead of the certainty equivalent *CE'* and payment *P'*. This therefore demonstrates that the values of the certainty equivalents *CE'* are also treated by Prospect Theory as relative although this is hidden and not intended by its creators. In this way, the function $w(p)$ takes on a whole new significance. This means that no longer is it the probability weighting function, but it defines the value r^{α}. The following is obtained after further transformations:

$$r = w(p)^{\frac{1}{\alpha}} \tag{23}$$

i.e. a direct relationship between the variable *r* and probability *p* (Refer Section 3 for a determination of the inverse relationship $p = q(r)$). Adding 1 to both sides of the equation and obtaining the logarithm gives:

$$\ln(1 + r) = \ln\left(1 + w(p)^{\frac{1}{\alpha}}\right) \tag{24}$$

Finally, by dividing both sides of the equation by ln(2) gives the final expression of the variable *s*:

$$s = w_m(p) = \frac{\ln\left(1 + w(p)^{\frac{1}{\alpha}}\right)}{\ln(2)} \tag{25}$$

In this way, the variable s, the normalized logarithm of r, is obtained as a function of probability p (Refer Section 4 for a determination of the inverse relationship $p = u(s)$). We remark that the function $x^{0.88}$ may be approximated by 1.34 $\ln(1+x)$ (See Footnote 13). Substituting this for (22) gives the approximate solution for s:

$$w(p) = r^{\alpha} \approx 1.34\ln(1+r) \tag{26}$$

which after further reduction gives:

$$s = \frac{\ln(1+r)}{\ln(2)} \approx \frac{w(p)}{1.34\ln(2)} \approx w(p) \tag{27}$$

Therefore, the function $w(p)$, which Prospect Theory defines as the probability weighting function, actually determines the value of r^{α} on the one hand, and the value of the variable s (with a certain degree of accuracy) on the other. The inverse relationship, i.e. probability p as a function of the variable s, should be determined so as to compare Solution (25) with the derived function $p = u(s)$ (9):

$$p = w_m^{-1}(s) \tag{28}$$

This can only be done numerically due to Function Type (13). Figure 7 presents a comparison of both solutions.

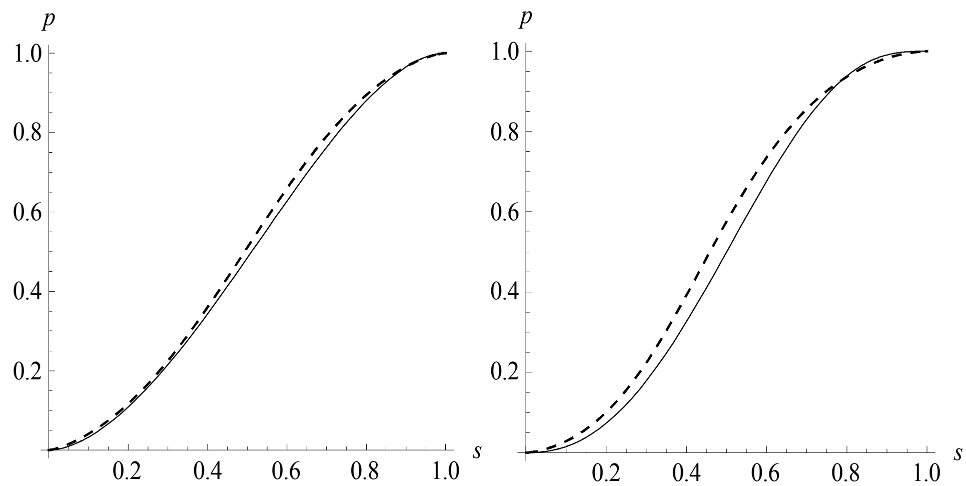

Fig. 7. Comparison of solutions $p = u(s)$ (Solid) and $p = w_m^{-1}(s)$ (Dashed) for the loss prospect (left) and gain prospects (right).

That both solutions are similar in shape should come as no surprise as they were obtained on the basis of the same experimental data. The differences between them owes to the fact that transformation $p = u(s)$ was obtained by a direct approximation of the experimental data (Refer Section 4) whereas Relationship (28) was obtained "indirectly" by deriving the value function $v(x)$ and the probability weighting function $w(p)$, combining them into a single equation (22), further transforming it into (25), and finally deriving the inverse function numerically. It is evident that the similarity in shape (especially for the loss prospect) is significant. The differences may be accounted for by the following:

1. Different functions were used for the approximation;
2. Prospect Theory's derivation of two functions from the data may be an additional source of errors;
3. Arbitrarily transforming a two-variable function into a single-variable function when deriving the probability weighting function;
4. Kahneman and Tversky used the median *CE* value for identical probabilities.

All of this was reflected in the much higher error of approximation found in Prospect Theory, especially for the gain prospect.

8. Which Solution is Correct?

The discussion presented in Section 7 confirms that the solutions presented in sections 3 and 4 defining the direct relationships between probability p and the variables r and s, are entirely correct, and are even confirmed by Prospect Theory itself.[26] If both methods eventually produce (almost) identical results, then the question as to whether both solutions are correct naturally presents itself. The answer is: No. As the results from the methodology presented in Section 3 expanded in Section 4, and repeated in Figure 6 show, the only correct interpretation of the experimental data is the direct relationship between probability p and the variables r and s (i.e. the relative certainty equivalent and its normalized logarithm). The value of the certainty equivalent *CE* for probability p (or the inverse relation) can be calculated directly from the determined transformation without having to use the value function or the probability weighting function. It is a general solution, which does not require any initial assumptions regarding the existence of the value, utility or probability weighting functions, and nor does it require the

[26] This article would probably never have been written had Kahneman and Tversky performed further transformations on their results.

methodology of calculating certainty equivalents. The solution is based on natural Mental Adaptation, Focused Attention and Stimuli Logarithmic Perception Transformations, which are known from psychology, psychophysics and neuroscience. It uses the basic mathematical transformations – translation, scaling and natural logarithms. It does not require rejecting the axioms of classical probability theory and nor does it call for complex topology concepts. As for Prospect Theory, it should first of all be stated that it does not provide the relationships $p = q(r)$ and $p = u(s)$ presented in this paper. Prospect Theory provides the value function and the probability weighting function as its end result. As Wakker has shown (2003), using only one of these functions to interpret data produces incorrect results (see also footnote 2). However, if both functions have to be used together anyway, then the principle of Occam's razor requires that the simpler solution be adopted, especially if that solution more accurately describes the experimental data. There is, however, a mental barrier to rejecting both functions, namely a commonly held view that each in its own way offers interesting psychological interpretations. The arguments presented in this paper leave no doubt that the value function and the probability weighting function are only one of the many ways of representing the general solution. The interpretations assigned to them may therefore only seem to be correct.[27]

9. Summary

The article presents an alternative interpretation of the experimental data published by Kahneman and Tversky in their 1992 paper "Advances in Prospect Theory". Mental transformations, crucial to deriving the results, were discussed in the introduction section. Later, the solution was derived without using the probability weighting function. The obtained function has a double S-type shape that strongly resembles the utility curve specified by the Markowitz hypothesis (1952). The presented aspiration function shows that risk seeking appears while being far from the aspiration target defined by the focused attention process. On the other hand risk aversion is present when this target is close. In case of losses the pattern is reversed. The

[27] It could be possible to provide many ways of resolving the transformation $p = u$(s) into several functions, each offering "deep" psychological insights into real world phenomena. Hypothetically, it may also be possible to create a general psychology theory, in which $g_1 = \sqrt{x}$ was defined as the utility function (because it is increasing and concave), $g_2 = x^{\frac{3}{2}}$ the motivation function (because it is increasing and convex) and $g_3 = \frac{1}{x^2}$ the regret function (because it is decreasing), and the end result of the theory would be the product of these functions. It is not difficult to confirm that $y = g_1 g_2 g_3 = 1$, what means that even a constant value may be resolved into many appealing, but at the same time completely random and incorrect ways of interpreting real life and behaviors.

explanation of risk attitudes given by the convex-concave-convex-concave shape of the aspiration function substitutes the fourfold pattern introduced by CPT. The results presented provide a basis for negating Prospect Theory as the theory which best describes decision-making under conditions of risk and may foreshadow a return to describing people's behavior only using utility-like functions.

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