"An Agent Based Cournot Simulation with Innovation: Identifying the Determinants of Market Concentration"

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Abstract

In this paper, I develop a hybrid model that contains elements of both agent based simulations (ABS) as well as analytic Cournot models, to study the effects of firm characteristics, market characteristics, and innovation on market concentration, as measured by a Herfindahl-Hirschman Index (HHI). The model accommodates the following components: multiple firms with heterogeneous marginal costs, market entry and exit, barriers to entry, low or high cost industries, changing demand, varying levels of marginal cost reducing returns-to-innovation, varying costs associated with innovation, increased returns to innovation from past experience innovating, and varying propensities to innovate within the market. The components mentioned above are commonly cited as determinants of market concentration. A sensitivity analysis which is robust to high degrees of model complexity demonstrates that the model provides results that are consistent with economic theories of markets.

Key words. Agent-based simulation, Cournot, Heterogeneous agents, Repeated games, Sensitivity analysis.

1 Introduction

In this paper, I develop a hybrid model that contains elements of both agent based simulations (ABS) as well as analytic Cournot models, to study the effects of firm characteristics, market characteristics, and innovation on market concentration, as measured by a Herfindahl-Hirschman Index (HHI). One contribution of this paper is that it demonstrates that ABS and analytic models are not necessarily competing approaches to modeling; rather, they can be complementary. Cournot models are game theory models involving firms who make strategic output decisions that take into consideration the output choices of competing firms in the market. These game theory models are especially useful since most real world markets can be characterized as imperfectly competitive, where the actions of one firm affect the profitability and actions of other firms. The Cournot model is one of the primary models of imperfect competition usually studied in the context of oligopoly. Game theory provides a tool by which such interdependencies can be examined and in a dynamic setting where games are repeated computational methods such as ABS are often employed in the analysis.

The model developed in this paper is a flexible n-firm Cournot model that when $n=1$ produces monopoly price and quantity and as $n \to \infty$ produces the perfectly competitive market price and quantity. The model accommodates the following components: multiple firms with heterogeneous marginal costs, market entry and exit, barriers to entry, low or high cost industries, changing demand, varying levels of marginal cost reducing returns-to-innovation, varying costs associated with innovation, increased returns to innovation from past experience innovating, and varying propensities to innovate within the market. The components mentioned above are commonly cited as determinants of market concentration.

To study the effects that each model parameter has on market concentration a sensitivity analysis similar to that developed by Brenner [1] is employed. While the analysis is similar to Brenner's novel approach to model verification, I do provide a full presentation of the regression analysis which provides more clarity on the approach and allows a discussion of heterogeneity as a measure of uncertainty at various parameter settings, a topic which is not present in the Brenner article. The sensitivity analysis method is robust for understanding and untangling the various layers of complexity inherent in agent based simulations containing many parameters.

The parameters are divided into two groups. The first includes firm and market characteristics with a theoretical background well established in the neo-classical framework of economics. The results from the sensitivity analysis for this group are consistent with economic theory and statistically significant. The second group of parameters relates specifically to the innovation aspect of the simulation and the associated economic theories are in many cases contradictory. The results for this group are discussed in terms of Schumpeterian theory, endogeneity theory, and evolutionary theories proposed by simulation researchers.

2 Literature Review

ABS models, although scant in the literature, are especially important to those studying how industries organize themselves [1], [2], [3], [4], [5], [6]. Brenner [1], for example, shows the benefits of simulation in understanding generative aspects of mechanisms associated with the evolution of industrial clusters. With an ABS approach the researcher gains both a dynamic modeling tool and a high degree of heterogeneity not found in static models or dynamic models that are based on systems of ordinary differential equations. In reality, industries have both a high degree of heterogeneity in terms of cost structure and develop through complex dynamic processes.

Two primary concerns of early work in simulation models has been 1) to establish that the models are consistent with their analytic counterparts and 2) that the dynamic processes by which an equilibrium is or isn't attained serves as a rich source of information regarding the individual agent's behavior and motivations. Price [7], for example, compares evolutionary programming to several classic analytic models to demonstrate that analytic model results can be achieved via simulation. Additionally he demonstrates that the method provides unique information by allowing learning-agents to modify their behaviors and evolve strategies as they interact within the model.

Within this area of ABS there are three key phenomena of interest in a dynamic setting that have been explored in the previous literature. The research might focus on one component alone or some combination of components. The first is process innovation which reduces costs, affects the innovating firms profitability, the distribution of market share, and hence the concentration of the market. Second is product innovation, advertising, and external shocks that affect consumer demand which may be volatile and may be increasing or decreasing over time. This can either force firms out of a market or bring more firms in which ultimately affects market concentration. Third, firms can adapt to their environment and learn from experience. Essentially, in terms of learning, firms may either react to some measure of their own performance such as profitability, may react to the behavior of competitors, or may react to changes in the environment such as changing demand. This too affects the way and rate at which market concentration changes.

Barr and Saraceno [8] examine learning in a Cournot duopoly setting. Their study focuses on how firms learn to react to environmental changes (changing demand) in determining optimal output. They characterize firms as artificial neural networks that make output decisions based on environmental factors. As the environment changes firms change their behavior and converge toward an optimal behavior. Firms learn which behavior reaps the greatest reward given the environment. Bischi and Kopel [9] explore a similar scenario of agent learning based on adaptive expectations. Kimbrough, Lu, and Murphy [10] and Kimbrough and Murphy [11] develop models of agent learning in repeated Cournot games that allow the development of collusion. Finally, Thomas Riechmann [12] studies agent learning in oligopoly games and the effect that learning has on producing Cournot or Walrasian outcomes. He distinguishes individual learning from social learning and explores parameters for agent rationality. In models of individual learning agents interact in the market but update their strategies independent of other firms. The social learning models are characterized by firms learning from one another after interacting in the market.

Learning in agent based computational economic models, whether in a Cournot setting or not, often involves innovation. In this sense, firms might learn to innovate thru research and development, learn to imitate competitors, or learn about how to improve their own processes by doing what it is they do (learning-by-doing). Kerber and Saam [13] model a Hayekian learning mechanism based on experimentation where firms imitate rivals' technologies creating a knowledge generating market process. Pajares, Lopez-Paredes, and Hernandez-Iglesias [3] use an ABS approach to model market evolution when firms are allowed to invest in either process innovation (learning-by-doing) or product innovation which allows them to create a new product and enter a new market and/or to exit a market they are already in.

Other research that has to do with firms innovating over time include studies such as Athey and Schmutzler [4] which looks at how market dominance emerges under various types of continuous investment schemes such as incremental investment, patent races, learning-by-doing, and network externalities. As firms self-invest or innovate they reduce costs and as a result gain market share and increased profits. Similarly Aghion, Harris, Howitt, and Vickers [5] examine market structure under innovation where one firm is an innovation leader and other firms imitate. Based on their results they make suggestions for anti-trust policy and patent policy. One of the first papers in this vein of research was Vickers [6] which looked at the effects of innovation on market share in a Cournot duopoly game. He finds that highly competitive product market behavior leads to market dominance from innovation but that Cournot behavior leads to action-reaction on the part of the firms resulting in diminished gains in market share.

In addition to learning in innovation studies, ABS based Cournot game research focuses on firm reactions to a changing environment as is the case in oligopoly pricing under changing demand conditions. Wilson and Reynolds [14] in their empirical piece find that during demand expansion price is a pure strategy Nash equilibrium but that in a recession firms set prices above the competitive price. Genc, Reynolds, and Sen [15] find that in markets with volatile demand it is possible for oligopoly firms to increase profits. Finally, Sznajd-Weron and Weron [16] use an agent based model to look at how advertising affects demand and performance in a duopoly market.

The model developed in this paper accommodates marginal cost reducing innovations and demand shocks. In its present form the model does not contain a learning component in the terms discussed above though does allow firms to gain experience from previous innovations which increases the returns to future innovations. After developing the model in the next section several hypotheses will be tested to determine if the simulation provides results for the first group of parameters that are consistent with mainstream market theory. Namely:

- Markets with high costs of production tend to be more concentrated than markets with low costs of production.
- As demand expands all firms become more profitable and more firms enter the market which decreases market concentration.
- As barriers to entry increase the number of firms entering a market decreases and the market becomes more concentrated.

For the second group of parameter, which is specific to the innovation aspect of the model, the results from the sensitivity analysis will be discussed in the context of, and compared to, several contradictory theories relating innovation to market structure. These include: Schumpeterian theory, which hypothesizes large firms in concentrated industries drive innovation, endogeneity theory, which hypothesizes that both market concentration and innovation are endogenous and determined by external factors such as technology, demand, institutional arrangement, strategic behavior, and chance, and finally evolutionary theories which developed out of the work of simulation researchers. Although no formal hypotheses are tested here the parameters are explored to determine their effect, statistical significance, and relation to the above mentioned theories. The parameter effects examined include:

- Changing costs for innovation for all firms in a market.
- Changing the cost reductions experienced from innovations for all firms in a market.

- Changing the propensity for firms to innovate in a market.

3 Model

As mentioned above, the model developed here combines aspects of an analytic Cournot model and an ABS model. Given the complexity of the ABS systems described above, namely heterogeneous agents (i.e. differing marginal costs), multiple agents, and multiple parameters of interest it would be very difficult if not impossible to solve the time paths as discussed above in true Cournot fashion. This demonstrates one of the key limitations of traditional analytic models especially in a dynamic setting. That said, Sarkar, Gupta, and Pal [17] have developed an analytic heterogeneous multi-firm model that relies on a marginal cost averaging method for computing the Cournot solution, and so from a purist perspective is not technically a Cournot solution, but is a close approximation. This averaging method also allows some non-linear analyses that are not available in the true Cournot solution though they are not explored here. The ABS model developed for this paper relies on this same marginal cost averaging process.

3.1 The n-Firm Cournot Element of the Model

To derive the n-firm Cournot solution Sarkar, Gupta, and Pal begin with inverse demand $P(Q) = a - bQ,$ where a>0 and b>0. Each firm chooses q_i to maximize profit, $\pi_i = q_i[P(Q) - mc_i]$. In words, profit equals total revenue minus total cost where mc_i is each firm's marginal cost. Firm $i's$ first order condition for profit maximization is

$$P(Q) - mc_i - bq_i = 0. \qquad (1)$$

Summing first order conditions for all firms gives $NP(Q) - bQ = \sum_{i=1}^{N} mc_i$. Dividing this by N produces $P(Q) - \frac{bQ}{N} = \overline{mc}$. Where $\overline{mc} = (\sum_{i=1}^{N} mc_i)/N$, the average marginal cost in the market. Substituting $P(Q) = a - bQ$ into the previous equation produces

$$a - \left[\frac{N+1}{N}\right] bQ = \overline{mc}. \qquad (2)$$

In equilibrium Q^*, market equilibrium can be found by setting $\overline{mc}$ equal to the linear function with the same intercept as demand but with a slope of $-\left[\frac{N+1}{N}\right] b$. So, $Q^* = \frac{(a-\overline{mc})}{b}(\frac{N}{N+1})$. From here $P^* = a - bQ^*$. Substituting Q^* yields $P^* = a - b[\frac{(a-\overline{mc})}{b}\left(\frac{N}{N+1}\right)]$ which reduces to $P^* = a - (a - \overline{mc})\left(\frac{N}{N+1}\right)$. When N=1 the monopoly $Q^*\ and\ P^*$ are produced and as N increases the perfect competition $Q^*\ and\ P^*$ are produced. Equation (1) above implies that for each individual firm $q_i^* = (P^* - mc_i)/b$. Plugging P^* into q_i^* yields the reaction function for the individual firm where each firm's output is dependent on its own marginal cost as well as the average marginal cost of the market.

$$q_i^* = \frac{a}{b} - \frac{a-\overline{mc}}{b}(\frac{N}{N+1}) - \frac{mc_i}{b} \qquad (3)$$

Using the variables computed above I calculate among other things firm market share and market HHI where $HHI = \sum_{i=1}^{n}\left[\frac{q_i^*}{Q^*}\right]^2$ and ranges from 0.0-1.0. The parameter effects can then be determined via repeated experiments to test the previously stated hypotheses.

Important underlying assumptions of the Cournot solution are that firms have perfect information about all other firms in a market and that they choose their output simultaneously based on that perfect information to maximize individual profit. So the firms are not interacting in a localized neighborhood based on limited information as is the case with many ABS models. Neighborhood interaction in an ABS context is however a logical extension.

3.2 The ABS Element of the Model

Here I am beginning at the Cournot equilibrium resulting from the analytic model described above and generating a dynamic process from that equilibrium solution via parameter changes to the model. While the firms are not interacting with one another other than trivially in solving the Cournot equilibrium, collectively, the firms are interacting with the environment. This interaction is what gives rise to the dynamics of this particular model. While there are typically far too many elements in oligopoly markets to be collected in one analytic model, the ABS approach is distinct as a modeling framework from analytic approaches in that it is modular by nature and quite easily accommodates a great deal of complexity that results from adding parameters and components to the model. The method of sensitivity analysis presented in section 4 is a robust tool for the analysis of such complex ABS models and allows a means of isolating the effects of each parameter.

Model Parameters – initial settings			
Description	Symbol	Lower bound	Upper bound
Demand intercept	a	20	20
Demand slope	b	-1	-1
Number of firms	N	10	10
Marginal cost	mc	2	14
Barriers to entry	-	1	7
Returns to innovation	-	.001	.01
Cost of innovation	-	5	75
Propensity to innovate	-	1%	68%
experience	experience	0	0

Table 0: Ranges for parameters in the model.

The model was implemented in a two dimensional space in Netlogo. While necessity of such spatial aspects of the model in its present form are at present limited, future modifications, as discussed throughout the paper, would certainly benefit from this two dimensional environment. In building the model I began by creating an innovation region in the center of the world, the size of which can be adjusted within the world dimension of 17x17. As the size of the innovation region relative to the world increases all firms have a greater likelihood of being able to innovate. The region is represented as a square but since the world wraps around from one edge to the other it might be best to think of it as something resembling a continent on a globe.

Propensity to innovate in the table ranges from a 2x2 to a 14x14 region of innovation. The ratio of the area of the innovation region to the area of the world are the percentages listed in table 0.

Next, I created the firms. In all of my simulations I begin with 10 firms randomly placed in the world. A value for average market marginal cost is assigned between 2 and 14 as well as a standard deviation for those costs (the source of agent heterogeneity in this model) randomly assigned from 0 to 1. These parameter settings are appropriate for the initial demand equation slope and intercept settings. In general, when marginal costs are high supply intersects the elastic end of the demand equation and when marginal costs are low supply intersects the inelastic end of the demand equation. Demand can be stationary, increasing, or decreasing. Growth in the intercept parameter at each iteration is set between -.005 and +.005. Changing demand slope is not explored in this paper.

Firms move one-step, in a 360 degree random direction at each iteration. By making firm movement completely random I assume away the possibility that firms who innovate will be more likely to innovate in the future. A logical extension would be to increase the future likelihood of innovation for firms that innovate. After each iteration, firms collectively produce output based on the n-firm heterogeneous agent Cournot solution derived from the reaction function produced from equation (1). If a firm is located on the innovation region and its cumulated profits are sufficiently positive[1] to cover innovation costs then the firm will innovate. Innovation costs vary from 5 to 75. Although innovation success is predetermined a useful extension would be to assign a probability of innovation success and a distribution of innovation costs. Results of innovations are determined to be a marginal cost reduction between .001 and .01 of current marginal cost at any given iteration. Returns could be normally distributed as well so while some firms will gain from innovation others would gain more and still others could lose their innovation investment. Finally, firms gain a point of experience for each innovation attempt which improves the effectiveness of future innovations.[2]

When a firm's cumulative profit falls below zero it exits the market. Similarly, when the average profit of the market in a given iteration reaches a threshold then a new firm will enter the market in a random position on the world map. The threshold is determined by the barriers to entry for the market. At a low setting firms enter often while at a high setting firms enter the market less readily. The model assumes the new entrant will have a marginal cost equal to the market average though it would be possible to assign a distribution of possible initial marginal costs for new entrants.[3]

Some final simplifying assumptions of the model include the fact that market products are homogeneous in this model. In reality firms have some degree of product differentiation. I am not aware of a paper that models a n-firm Cournot game with differentiated products or even if a solution is computable in such a case. As mentioned above, previous researchers have modeled product differentiation in this setting using a purely multi agent system [3]. Additionally, in reality firms often merge into one larger firm. This model does not allow mergers. Each firm is an independent entity. Although, this feature could be added with reasonable ease and would be

[1] Since there is a cost to innovation firms cumulative profits must be sufficiently positive to cover the costs of innovation. Firms cannot borrow in this model.

[2] The change in marginal cost from innovation is defined by the following formula:

$$mc_{t+1} = mc_t - (1 + ln(1 + experience \wedge 2)) * (innovation\_returns) * mc_t)$$

where mc is marginal cost, experience refers to the number of times the firm has innovated in the past, the exponent on experience is simply a scalar for innovation effectiveness set at 2.0.

[3] There may be a more theoretically accurate way to model such barriers allowing technology hoarding and a diffusion process for example.

an interesting extension to the model. I have also considered creating vertical firms in the same environment, in which case vertical mergers could occur.

4 Sensitivity Analysis of the Parameters

In a simulation all parameters are varied randomly except one which is varied systematically. The systematically varied parameter is adjusted incrementally so that samples for a number of settings are generated. At each setting of the systematically varied parameter 200 experiments are conducted with each experiment consisting of 300 iterations; the mean HHI is computed and collected. For each simulation the mean HHI's are regressed against the systematically varied parameter.

I present the simulation results as Brenner does by progressing through each parameter discussing its relevance to economic theory and impact on market concentration as measured by the mean HHI. Regression analysis is used to estimate the effect of the systematically varied parameter on the mean HHI from each experiment in the simulations. I also report the magnitude of the parameter (its total impact over the range of parameter settings) given random variation in the other parameters. This provides a measure of leverage for the parameter not obtainable from the coefficient estimate alone. From a policy perspective high leverage parameters are preferred to low leverage parameters as affecting them will yield greater change on the output variable of interest within the system. Finally, I perform a Breusch-Pagan / Cook-Weisberg (BP/CW) test for heteroscedasticity, examine the variance of the mean HHI's at each setting, and discuss the pattern of heteroscedasticity or lack thereof as evidenced by a visual inspection of residual plots. Though indicative of heteroscedasticity in an econometric context, the results are useful from a policy perspective as a measure of parameter uncertainty in the model. That said the regression results reported adjust for heteroscedasticity and include standard errors with a bias correction of the form $\frac{n}{n-k}$.

4.1 Mainstream Market Theory Parameters

4.1.1 High Cost vs. Low Cost

For aviation or automobile manufacturing the start up and operating costs tend to be very high. These markets typically consist of a small number of large firms. Cost reducing innovations in this setting can provide a large effect when distributed on a large scale. Low cost industries tend to have many smaller firms and such innovations do not have as dramatic an effect on costs.[4] The simulation produces results consistent with these theories and in general producers with higher initial costs tend to generate more concentrated markets than do low cost producers.

Regression results suggest that initial marginal cost has a positive effect on mean HHI. The results are statistically significant and the total variation in HHI across the parameter space is 0.18 suggesting from a policy perspective that this is a parameter of moderate leverage. The

[4] Mathematically this effect is achieved in the model by defining marginal cost reductions as a % (between .001 and .01) of the current level. So, higher cost industries experience greater dollar reductions in this framework. In terms of footnote 3 it can be show that the formula reduces to the linear difference equation:
$mc_{t+1} = mc_t - \rho * mc_t$, where $0 < \rho < .1$, creating a balancing feedback loop. This follows from the parameter constraints: $experience < 300$ and $innovation\_returns < .01$.

BP/CW p-value = .0000, which suggests heteroscedasticity. A visual inspection of residual plots, suggests a functional form for the error term that is increasing in the independent variable suggesting an increasing uncertainty in the HHI outcomes at higher initial marginal costs.

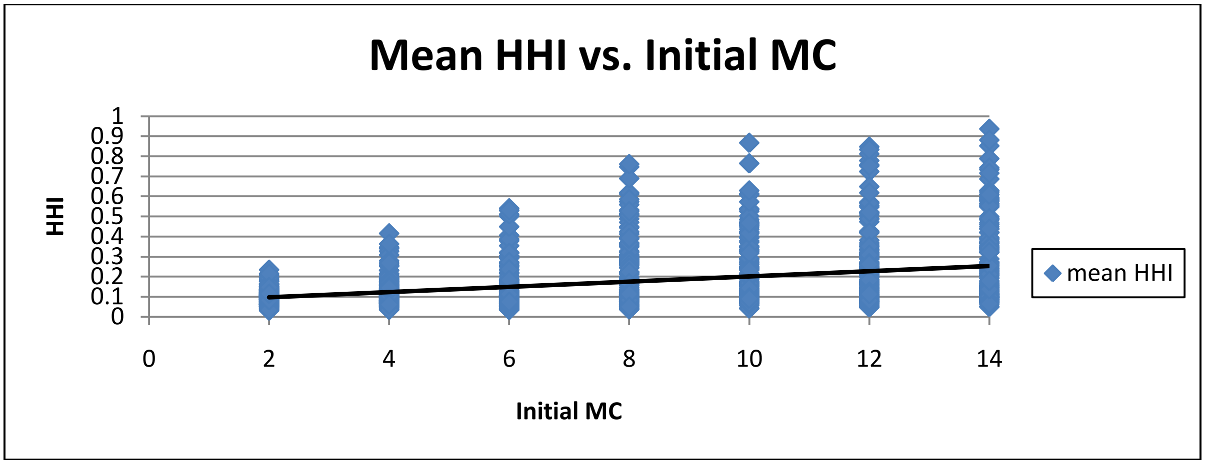

Figure 1: plot of HHI and Initial mc with regression line.

Table 1: Regression results of HHI on initial mc.

					Number of obs =	1252
					F(1, 1250) =	175.64
					Prob > F =	0.0000
					R-Squared =	.1257
					Root MSE =	.13885
HHI	Coef.	Robust Std. Err.	t	P>|t|	95% Conf. Interval	
Initial mc	.0130067	.0009814	13.25	0.000	.0110813	.0149321
constant	.070787	.0057297	12.35	0.000	.059546	.0820279

4.1.2 Demand growth

As demand grows within a market, profitability increases, more firms enter the market, and market concentration falls over time. Similarly, as demand decreases profitability falls, firms exit the market, market shares increase, and market concentration increases. The simulation produces results consistent with theory.

Regression results suggest that demand growth has a negative effect on market concentration. The results are statistically significant and the total variation in HHI across the parameter space is 0.26, suggesting demand growth is a parameter of high leverage from a policy perspective. The BP/CW p-value = .0000, which suggests heteroscedasticity. An inspection of the residual plots suggests a functional form for the error term that is decreasing in the independent variable suggesting decreasing uncertainty in the HHI outcomes as demand increases.[5]

[5] An interesting extension would be to change the slope (elasticity) of demand over time combined with falling demand or growing demand. Another extension would be to simulate different levels of demand volatility, as discussed in the literature review [15] T. S. Genc, *et al.*, "Dynamic Oligopolistic Games under Uncertainty: A Stochastic Programming Approach," *Journal of Economic Dynamics and Control,* vol. 31, pp. 55-80, 01 2007.,

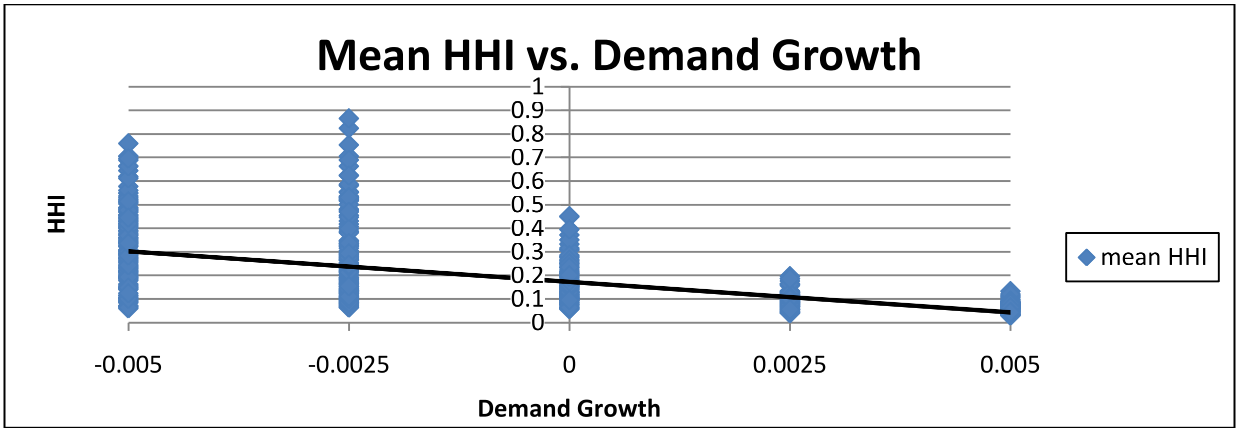

Figure 2: plot of HHI and demand growth with regression line.

Table 2: Regression results of HHI on demand growth.

					Number of obs =	911
					F(1, 909) =	465.95
					Prob > F =	0.0000
					R-Squared =	0.4055
					Root MSE =	.10827
HHI	Coef.	Robust Std. Err.	t	P>|t|	95% Conf. Interval	
Demand Growth	-25.82638	1.19645	-21.59	0.000	-28.1745	-23.47825
constant	.1721315	.0039804	43.25	0.000	.1643198	.1799433

4.1.3 Barriers to Entry

Markets with high barriers to entry tend to be highly concentrated. It is not easy for other firms to enter the market even if it is a very profitable one to enter. Oligopoly firms erect barriers; they purchase patents so that other firms cannot license a particular technology; they construct legal obstacles for new entrants, and lobby for protection from government, just to name a few. That said if a market is sufficiently profitable then firms will find ways to enter. Barriers to entry secure market power for those already in the market. As barriers fall via deregulation or other means, firms enter the market more readily and market concentration on average falls. The simulation produces results that are consistent with theory.

With no barriers to entry, firms rapidly enter the market driving market concentration to very low levels characteristic of perfect competition. In general, as the barriers to entry increase from 1 to 7 the HHI tends to increase as well. Regression results suggest barriers to entry have a positive effect on mean HHI. The results are statistically significant and the total variation in HHI across the parameter space is 0.09 suggesting the parameter is one of low leverage. The BP/CW p-value = .0002, which suggests heterogeneity. A visual inspection of residual plots suggests no functional form for the error term as a function of the independent variable. It is

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combining various patterns of demand expansions and contractions over time as is the case with seasonal demand changes.

interesting to note that, relative to previous parameters, variance in mean HHI is quite large across many values in the parameter space.

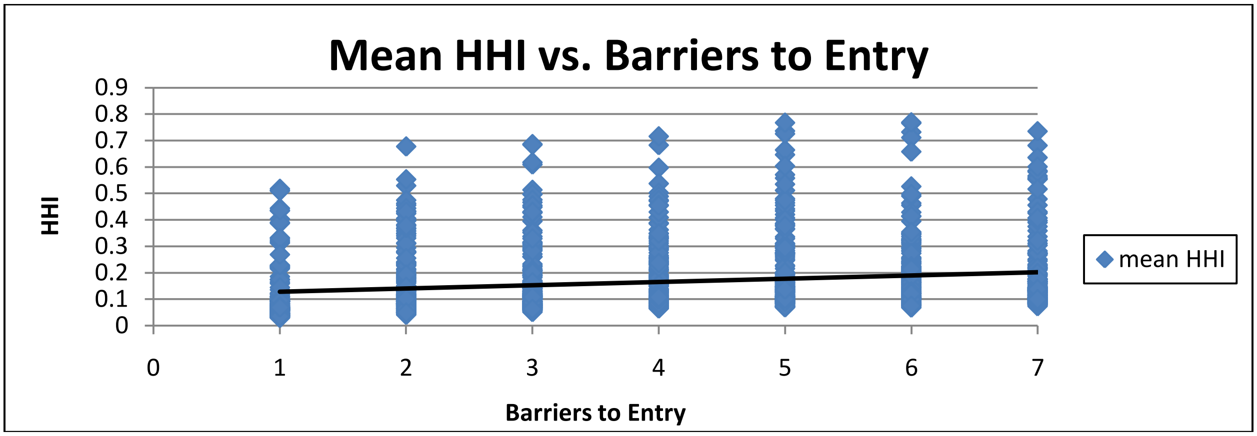

Figure 3: plot of HHI and barrier to entry with regression line.

Table 3: Regression results of HHI on barrier to entry.

					Number of obs =	1275
					F(1, 1273) =	52.11
					Prob > F =	0.0000
					R-Squared =	0.0366
					Root MSE =	.12691
HHI	Coef.	Robust Std. Err.	t	P>|t|	95% Conf. Interval	
Barrier to Entry	.0123558	.0017116	7.22	0.000	.0089979	.0157136
Constant	.1158721	.0072271	16.03	0.000	.1016937	.1300505

4.2 Innovation Specific Parameters

4.2.1 Costs of Innovation

From a Schumpeterian perspective it is the larger firms of concentrated markets that are the driving force of innovation. Other theories state that market concentration and innovation are endogenously determined by other factors. Nelson and Winter [18] suggests that increased rates of innovation would lead to concentrated markets in an evolutionary setting, essentially restating the Schumpeterian hypothesis from the other causal direction. Within the model developed here the higher the cost of innovation the slower firms are to innovate as they will have to accumulate more profit to make such an investment in innovation. The above theory [18] would suggest that decreased rates of innovation (from cost increases) should produce less concentrated markets. That said another study [19] finds that higher innovation costs lead to more concentrated markets, at least within the pharmaceutical industry.

Regression results suggest that cost of innovation has a negative effect on mean HHI which supports the claims of Nelson and Winter [18]. The results are statistically significant yet the total variation in HHI across the parameter space is only 0.07 suggesting this is a parameter of low leverage from a policy perspective. The BP/CW p-value = .0000, which does suggests heterogeneity. As above, a visual inspection of residual plots suggests no functional form for the error term relative to the independent variable. Variance in mean HHI is again quite large producing anywhere from close to monopoly to perfect competition outcomes across the entire parameter space.

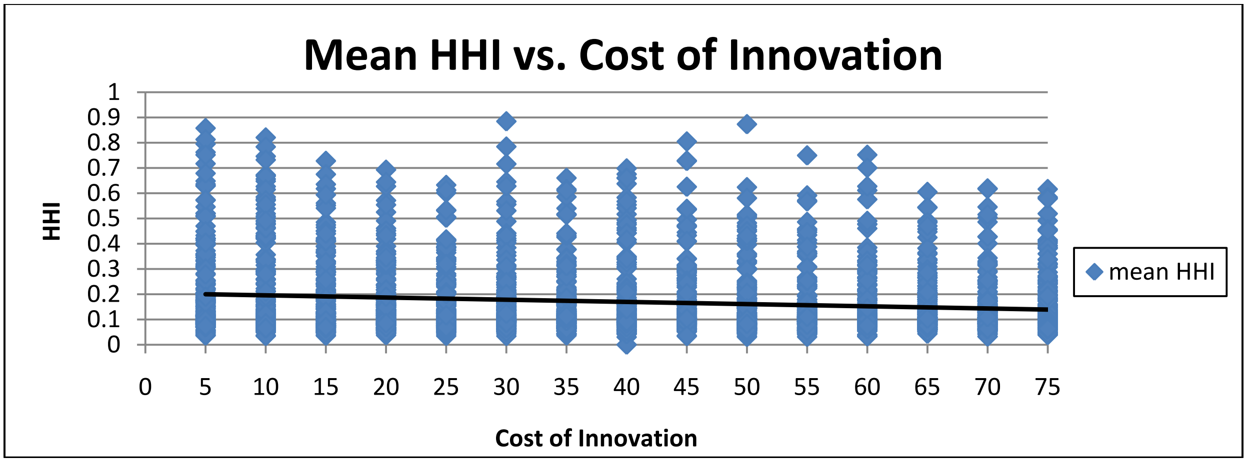

Figure 4: plot of HHI and cost of innovation with regression line.

Table 4: Regression results of HHI on cost of innovation.

					Number of obs =	2698
					F(1, 2696) =	46.85
					Prob > F =	0.0000
					R-Squared =	0.0186
					Root MSE =	.13523
HHI	Coef.	Robust Std. Err.	t	P>|t|	95% Conf. Interval	
Cost of Innovation	-.0008623	.000126	-6.85	0.000	-.0011094	-.0006153
constant	.2042419	.0062602	32.63	0.000	.1919666	.2165171

4.2.2 Returns to Innovation

In the context of the theories provided above increasing the return to innovation implies increasing the rate of innovation, which should result in increased market concentration. Additionally, when firms gain greater returns from innovations they gain a cost advantage over competitors. This increases their market share and profitability providing increased potential for survival in a market as price falls, in essence a first-mover advantage. The result is a more concentrated market. The simulation is consistent with these theories in that the coefficient estimate has the correct sign suggesting returns to innovation have a positive effect on mean HHI though this estimate is not statistically significant and total variation across the parameter space of 0.02 would suggest this to be a parameter of very low leverage. The BP/CW p-value = .1754, which does not suggests heterogeneity. Again, variance in mean HHI is quite large producing anywhere from close to monopoly to perfect competition outcomes across the entire parameter space.

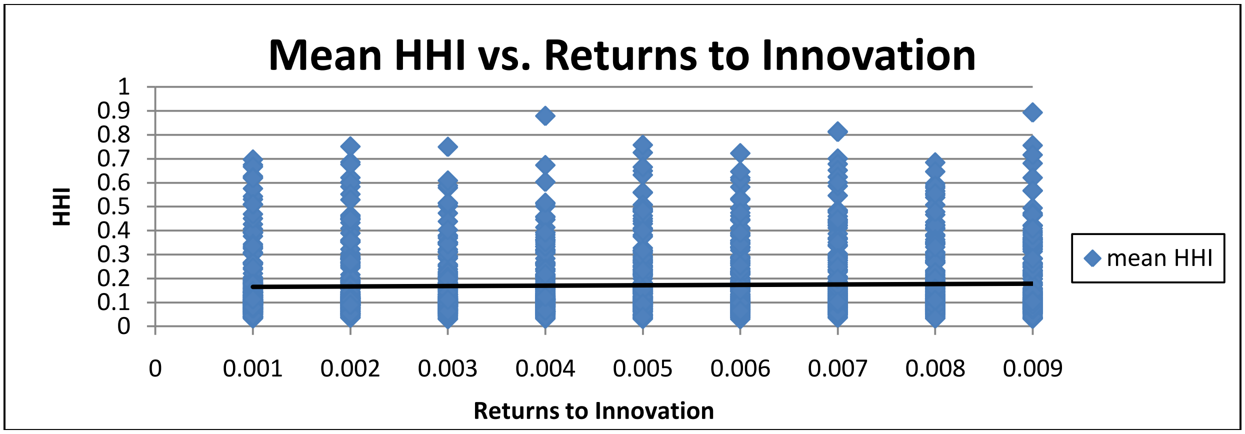

Figure 5: plot of HHI and returns to innovation with regression line.

Table 5: Regression results of HHI on returns to innovation.

					Number of obs =	1628
					F(1,1626) =	1.47
					Prob > F =	.2253
					R-Squared =	.0009
					Root MSE =	.14055
HHI	Coef.	Robust Std. Err.	t	P>|t|	95% Conf. Interval	
Returns to Innovation	1.670415	1.376929	1.21	0.225	-1.030327	4.371156
constant	.1626876	.0076005	21.40	0.000	.1525309	.1775954

4.2.3 Propensity to Innovate

According to Nelson and Winter [18] more innovation should produce more concentrated markets. If firms in a market are more prone to innovation due to a greater access to key resources, human capital, or spillovers from neighboring firms, costs should decrease rapidly placing pressure on non-innovators to exit the market. This would produce a more concentrated market. Simulation results are consistent with this theory; the regression analysis suggests a positive relationship between propensity to innovate and mean HHI and the results are statistically significant. The total variation in HHI across the parameter space is 0.04 suggesting this is a parameter of low leverage. The BP/CW p-value = .0078, which suggests heterogeneity though a visual inspection of the residual plots suggests no specific functional form relative to the independent variable. Variance in mean HHI is quite large producing anywhere from monopoly to perfect competition outcomes across the entire parameter space.

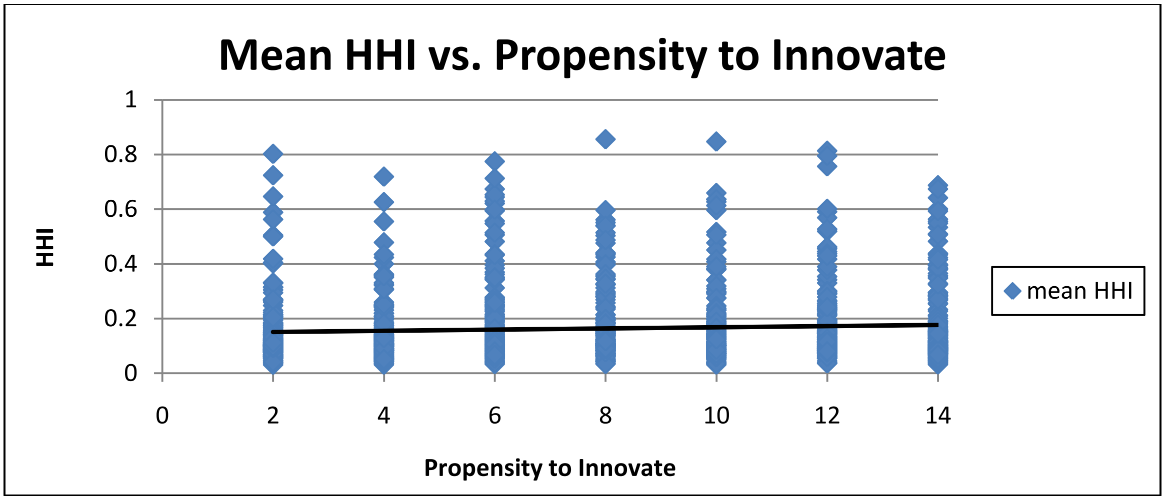

Figure 6: plot of HHI and propensity to innovate with regression line.

Table 6: Regression results of HHI on propensity to innovate.

					Number of obs =	1283
					F(1, 1281) =	5.78
					Prob > F =	0.0164
					R-Squared =	.0042
					Root MSE =	.13391
HHI	Coef.	Robust Std. Err.	t	P>|t|	95% Conf. Interval	
Propensity to Innovate	.0021744	.0009047	2.40	0.016	-.0003995	.0039493
constant	.1462436	.0077849	18.79	0.000	.130971	.1615161

5 Conclusion

The model presented in this paper demonstrates that analytic and agent based models can be implemented together in a simulation setting. Furthermore, it clarifies, and demonstrates the robustness of, the unique sensitivity analysis method developed by Brenner [1]. A method which isolates the effects of individual parameters in a heterogeneous agent based context. Finally, the model itself provides results that are consistent with economic theory. In terms of what mainstream market theory would predict:

- Markets with high costs of production tend to be more concentrated than markets with low costs of production.
- As demand expands all firms become more profitable and more firms enter the market which decreases market concentration.
- As barriers to entry increase the number of firms entering a market decreases and the market becomes more concentrated.

For the parameters specific to the innovation aspect of the model the results were consistent with theories of Nelson and Winter [18] and although the causal direction is reversed Schumpeterian theory as well, in that the results show positive correlation (regardless of causal direction) between innovation and market concentration. These results suggest:

- Higher costs for innovation (slow innovation) and tend to create a less concentrated market.
- Higher propensity toward innovation creates a more concentrated market.

The effect of the last remaining parameter was inconclusive from the sensitivity analysis and within the framework of the model appears to have no significant affect on market concentration outcomes. That said they may be important determinants for one or more of the model's firm and market output variables.

- Changing the cost reductions experienced from innovations for all firms in a market.

The literature on oligopoly theory discusses all of the components explored in the ABS model developed in this paper, as evidenced in the literature review section. An important future development for the model would be to include a learning component for firms. In present form the model consists of firms innovating at random. As stated by Price [7], the benefit of ABS models is that they allow behavioral aspects to be injected into the model. While many analytic models contain no behavioral elements the model presented here would benefit by the inclusion of credible strategies and behaviors for firms pursuing cost reducing innovations. Additionally, it would be useful to expand the behavioral realism of the model by employing the additions discussed throughout the paper.

The great advantage of the ABS approach is that each of these features can be thought of as an independent component and collected together to make a very complex model with many features and possibilities for expansion. The ABS framework does not rely on the closed form solutions that analytic models require. Given the growth in complexity that occurs as these dynamic models are generalized, analytic models tend to be very simple and constrained to focusing on only one particular issue. Though adding components increases model complexity the sensitivity analysis implemented is robust with the only constraint being computational resources which with models including many parameters can be an issue.

Finally, although I do not currently have an appropriate data set, a future goal would be to see if market data could be used to validate the model. With an understanding of the parameter space provided by the sensitivity analysis realistic setting for the parameters could be obtained to generate patterns consistent with real world market performance as previous studies have done based on data in the computer market [2] and energy markets [20], [21], [22]. Parameters of high leverage could then be identified as targets for effective policy.

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