Strategic Vertical Separation

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Abstract

The paper explores incentives for strategic vertical separation of firms in a framework of a simple duopoly model. Each firm chooses either to be a retailer of its own good (vertical integration) or to sell its good through an independent exclusive retailer (vertical separation). In the latter case a two-part tariff is applied. Retailers compete in quantities, goods are perfect substitutes and firms' cost functions are quadratic. I show that the equilibrium outcome crucially depends on the degree of (dis)economies of scale and asymmetry of costs. Two asymmetric equilibria arise, in which one firm separates while another integrates, under conditions that both firms' cost functions exhibit a sufficiently high diseconomies of scale, or extreme asymmetry of costs. Under a moderate asymmetry of costs a unique equilibrium exists in which the firm with the lower degree of diseconomies of scale separates, while its rival integrates. With the degree of diseconomies of scale low for both firms in the unique equilibrium both firms separate.

JEL classification: L22; L42

Keywords: Vertical oligopoly; Vertical Separation; Vertical Integration, Delegation

1 Introduction

The paper examines incentives for strategic vertical separation of firms in Cournot duopoly settings. Vertical separation is defined as selling through an independent exclusive retailer, while vertical integration as selling directly to final consumers. It follows the traditional approach in assuming that, in the case of separation, a firm chooses both the wholesale price, at which he will supply to his retailer, and the franchise fee. So, in this context, separation means that the firm delegates the decision of the quantity to be sold to his retailer, in which case it controls the retailer's objective (retailer's profit function). Using the franchise fee the firm extracts the entire retailer's profit, with the wholesale price being used to set the optimal incentive scheme offered to the retailer. In the case of integration the firm is a retailer of its own good and, as such, the firm's objective is maximization of own profit.

The subject of the possible advantages of vertical separation in strategic duopoly games has been receiving growing attention in the recent economic literature on firm behavior. Bonanno and Vickers (1988) consider a duopoly model with linear costs in which each manufacturer makes the decision of whether to separate or integrate. Assuming price competition among retailers, these authors show that vertical separation is both in the collective, as well as individual, interests of the firms, so that in the equilibrium both firms sell their products through independent retailers. Thus, in the unique equilibrium both firms separate. Lin (1988) considers a model in which the consumers have the discrete choice of buying either one unit of good or not at all and the retailers compete in prices. The author shows that the Nash equilibria of the game are such that either both firms separate, or both integrate. Festmann and Judd (1987) consider separation under Cournot competition with linear demand and constant marginal costs. They show that both manufacturers have an incentive to separate and the resulting equilibrium generates greater output, lower prices and more efficient allocation of production than the Cournot equilibrium. If the firms' cost functions are symmetric, both firms receive lower profits compared to the ones in the Cournot equilibrium of the game. Under asymmetry of the costs, the more efficient firm's profit may be higher than in the Cournot equilibrium of the game. These authors also show that, in the case of Bertrand competition with differentiated products, the owners want their managers to set higher prices, for eliciting higher prices from the competing managers too, with the result that the unique equilibrium, in which both firms separate, generates higher profits and lower output compared to the Bertrand equilibrium. Sklivas (1987) studies a delegation problem in which the owners set the objective functions for their managers at a first stage and, then, managers make a decision. His findings are close to Festmann and Judd (1987). The owners always take advantage of the separation and, in the case that duopolists compete in quantity (prices), both firms earn lower (higher) profits. Gal-Or (1991) considers a model of Bertrand competition between retailers and finds that, in the presence of low product differentiation, the producers may benefit from the double marginalization arising with linear pricing. Such double marginalization results in higher retail prices, with the effect of relieving competitive pressure. Although the author does not make explicit consideration of the separation decision of firms, her findings imply that manufacturers gain from separation, if they can extract all the retailers' profits. Basu (1995) considers a model of managerial delegation in a duopoly with a linear demand, constant marginal costs and competition in quantities. Under the assumption of fixed costs associated with separation, the author shows that asymmetric equilibria arise, even in the symmetric-cost case. In this model, in the absence of fixed costs, if only one firm separates, the profit of the separated (integrated) firm is higher (lower) compared to the Cournot equilibrium. If both firms separate, each firm's profit is strictly lower than in the Cournot equilibrium. The author, further, shows that there exists a level of the fixed cost such that, with only one firm separating, the final profit of the separated firm is still higher than its Cournot profit, moreover, the profit of the integrated firm is higher than in the case of both firms separating. Thus, if separation is associated with additional costs, asymmetric equilibria arise and the outputs levels are as in a Stackelberg equilibrium. Janson (2003) considers a Cournot oligopoly game with a linear demand and constant marginal costs, in which he assumes that separation implies a fixed contracting cost. In this case, asymmetric equilibria emerge, when the Cournot oligopolists supply close substitutes. So, a summary of the literature would be as follows: when goods are imperfect substitutes, under Bertrand competition with constant marginal costs, it is both in the private and the collective interests of the firms to separate. In the case of Cournot competition with a linear demand and constant marginal costs, it is in private interest, but not in the collective interests of each firm to separate. If the case of symmetry of firms, their profits are strictly lower than in the Cournot equilibrium. In the presence of a fixed cost associated with separation, asymmetric equilibria may arise.

In this paper, I extend the earlier analysis by allowing for increasing marginal costs of production and, in particular, for quadratic cost functions. I show that the equilibrium structure critically depends on the slopes of the marginal cost functions (in other words, on the degrees of diseconomies of scale) and the asymetry of costs . If the slopes of both marginal cost functions are sufficiently low in the unique equilibrium of the game both firms separate. Under a moderate asymmetry of costs, in the unique equilibrium of the game, the more efficient firm separates, whereas the less efficient one integrates. Asymmetric equilibria (one firm separating, the other delegating) arise in two cases: firstly, if the slope of each manufacturer's marginal cost is sufficiently high, secondly, if the cost asymmetry is extremely high. The model shows that the optimal distribution policy of a firm depends on both its own and its rival's cost structures. This provides a possible explanation for the widely observed asymmetry in the sales strategies among firms[1]. A strong prediction of the model is that with symmetry of firms, the equilibrium is determined by the degree of diseconomies of scale: if this degree is low, then both firms separate, whereas if it is high, two asymetric equilibria exist. The intuition for these results is as follows. If firm 1 separates and the firm 2 integrates, the firms get the same profits as in a Stackelberg game, with the separated firm being a Stackelberg leader[2]. Suppose two symmetric firms separate. Each firm has an incentive to set a low enough wholesale price in order to increase its retailer's output and its final profit. This results in higher output and lower profits comparing to the Cournot outcome. A central question in this case is whether the firms' profits are lower than the Stackelberg follower's profit. The key difference in the case of linear costs is the strength of competition in the wholesale prices. A decrease in, say, the firm 2's wholesale price results in a decrease in the firm 1's output, and hence in the firm 1's marginal cost[3]. The firm 2's output, as well as the total output, increases, thus the firm 1's marginal revenue declines. The firm 1's best reply, in this case, is to restore a balance between its marginal cost and revenue. If the slope of the firm 1's marginal cost function is high enough, the decline in its marginal cost is higher than

[1] See Buehler and Schmutzler (2005) and Janson (2003) for a detailed discussion of the empirical observations over the asymmetry in vertical structures.

[2] Separation serves as a commitment mechanism in this case: the separated firm commits its retailer's high output by setting a low wholesale price.

[3] This obviously cannot occur if the firms' marginal costs are constant.

that in marginal revenue, and the firm 1 prefers to increase its output. That is, the firm 1 should decrease its wholesale price as a response to a decrease in its rival's wholesale price. In this case, under symmetry of the firms, the firms' wholesale prices are strategic complements. In this case, competition between manufacturers in wholesale prices may be tough, and it results in high output levels, and therefore in low firms' profits, so these may be lower than a Stackelberg follower's profit. Hence, each firm prefers to integrate (and to obtain the Stakelberg follower profit), given that its competitor separates and asymmetric equilibria arise in a completely symmetric game. Suppose now there is a cost asymmetry, that the firm 1 separates, the firm 2 integrates, and let's consider the incentive of the firm 2 to deviate to separation. Separation of the firm 2 has a twofold effect: firstly, in the absence of the firm 1's reaction, the firm 2 could increase its profit by setting its wholesale price at an appropriate level. The increase in firm 2's profit depends on its own cost structure: the lesser efficient the firm 2 is, the lesser the gain obtained. Secondly, if the firm 1's wholesale price is a complement to the firm 2's wholesale price, the firm 1's reaction may imply a significant decrease in its wholesale price, therefore a significant increase in the total output, which decreases the firm 2's profit. The latter effect may dominate the former in two cases: firstly, if the firm 2's marginal cost curve is steep enough. In this case the possible gain from an increase in its retailer's output is small. Secondly, if the firm 1's marginal curve is very steep, hence the firm 1's wholesale price is a strong complement to the firm 2's wholesale price. In this case, an increase in the retailer 1's output is high. Thus, if a firm's, or its rival's, marginal curve is very steep, the firm prefers to integrate, given its rival separates, implying the existence of two asymmetric equilibria. In contrast, if a firm's marginal curve is flat, without its rival's marginal cost curve being very steep, the firm separates, given that its rival separates. In this case there is a unique asymmetric equilibrium in which the more efficient firm separates, while the less efficient firm integrates. Finally, if both marginal curves are sufficiently flat, the first effect dominates the second for both firms and in the unique equilibrium both firms separate.

The organization of the paper is as follows: Section 2 describes the model and provides the characterization of equilibrium under general assumptions on demand and cost functions. Section 3 provides an analysis for the case of quadratic cost and liner demand functions and discusses the robustness of results. Finally, Section 4 concludes. The proofs of the propositions are relegated to the Appendix A.

2 The general model and characterization of equilibrium

The firms F_1, F_2 produce homogeneous good. Let $C_i(q_i)$, $i = 1, 2$ be the cost functions. Let $P(q_1+q_2)$ denote an inverse demand function. In the general case, with the following assumptions on the demand and the costs function[4].

[4] Altough this paper provides final results for quadratic costs and linear demand functions only an analysis in this section allows to discuss robustness of results and also it highlights driving forces in the model.

Assumption 1. $C_i'(q_i) > 0; C_i''(q_i) \geq 0$, for any $q_i \geq 0$.

Assumption 2. $P' < 0; P' + P''q_i < 0; \exists Q > 0 : P(Q) = 0$.

At the first stage of the game, each firm decides whether to sell the good through an independent exclusive retailer, or directly to final consumers, being a retailer of its own good. Following Bonanno and Vickers (1988), I refer to the former case as vertical separation and to the latter case as vertical integration. Thus, at the first stage each firm chooses from the action $m \in M, M = \{S, I\}$, where S and I are interpreted as the choice of the firm to separate and integrate, respectively. If the firm F_i's choice is S, it further sets the two-part tariff (w_i, A_i) on his retailer, where w_i is the wholesale price of the good and A_i is a franchise fee. At the second stage of the game, the decisions of the first stage are observed[5] and the retailers compete choosing their quantities simultaneously and independently. The profit of the integrated firm i is $P(q_1+q_2)q_i - C_i(q_i)$. If the firm i separates,its own and its retailer's profits are $w_iq_i + A_i - C_i(q_i)$ and $P(q_1 + q_2)q_i - w_iq_i - A_i$, respectively.

2.1 The downstream equilibria

The game has four subgames corresponding to the choice $m \in M = \{I, S\}$ taken by each firm at the first stage. If each firm is vertically integrated, the firm i's maximization problem is

$$\max_{q_i} \pi_i = P(q_1 + q_2)q_i - C_i(q_i), i = 1, 2. \tag{1}$$

Given Assumptions 1 ansd 2, the game (1) has a unique Nash-Cournot equilibrium.

Let $\{q_1^C, q_2^C\}$ denote equilibrium quantities and $\pi_i^C = P(q_i^C + q_j^C)q_i^C - C_i(q_i^C), i, j = 1, 2, i \neq j$ denote the equilibrium profits in this subgame[6]. The equilibrium is characterized by the first-order conditions: $P'q_i^C + P - C_i' = 0, i = 1, 2$.

If both firms are vertically separated, the firm i chooses $\{w_i, A_i\}$ to solve its maximization problem:

$$\max_{\{w_i, A_i\}} \pi_i = w_iq_i^* + A_i - C_i(q_i^*), i = 1, 2,$$

where $q_i^* = \arg\max_{q_i}\{P(q_i + q_j^*)q_i - w_iq_i - A_i\}, i, j = 1, 2, i \neq j$ are the output levels resulting in the retailers' competition. By choosing an appropriate level of a franchise fee, each firm extracts

[5]It is assumed that decissions of the first stage are irreversible and therefore there is no commitment problem.
[6]Upper index C indicates the outcome of the subgame $(Integrate, Intergate)$ is a Cournot outcome.

all its retailer's surplus. Thus, the firm i' problems may be written as:

$$\begin{aligned} \max_{w_i} \pi_i &= P(q_1^* + q_2^*)q_i^* - C_i(q_i^*), i = 1, 2, \\ s.t.\ q_i^* &= \arg\max_{q_i}\{P(q_i + q_j^*)q_i - w_i q_i\}. \end{aligned}$$

Hence, for any given $\{w_1, w_2\}$ the retailers' choice of $\{q_1^*, q_2^*\}$ satisfies the first order conditions:

$$P' q_i^* + P - w_i = 0, i = 1, 2. \tag{2}$$

and the firms' choice of $\{w_1, w_2\}$ is a solution of the system:

$$\begin{aligned} \frac{\partial \pi_i}{\partial w_i} &= P' q_i^* \left(\frac{\partial q_i^*}{\partial w_i} + \frac{\partial q_j^*}{\partial w_i} \right) + P \frac{\partial q_i^*}{\partial w_i} - C_i' \frac{\partial q_i^*}{\partial w_i} = \\ &= (P' q_i^* + P - C_i') \frac{\partial q_i^*}{\partial w_i} + P' q_i^* \frac{\partial q_j^*}{\partial w_i} = 0, \\ i, j &= 1, 2. \end{aligned} \tag{3}$$

Plugging $P' q_i^* + P = w_i$ into (3) we obtain $(w_i - C_i') \frac{\partial q_i^*}{\partial w_i} + P' q_i^* \frac{\partial q_j^*}{\partial w_i} = 0$ and the equilibrium values $\{w_1, w_2\}$ and $\{q_1^*, q_2^*\}$ satisfy:

$$w_i = C_i' - P' q_i^* \frac{\partial q_j^*}{\partial w_i} / \frac{\partial q_i^*}{\partial w_i}, i, j = 1, 2, i \neq j. \tag{4}$$

Under Assumptions 1 and 2, $q_i^*(w_1, w_2)$ and $q_j^*(w_1, w_2)$ satisfy $\frac{\partial q_i}{\partial w_i} < 0 < \frac{\partial q_j}{\partial w_i} < \left| \frac{\partial q_i}{\partial w_i} \right|, i, j = 1, 2,$ therefore $w_i < C_i'$. Thus, if both firms separate, in the equilibrium each firm sets its wholesale price lower then its marginal cost. Considering the effect of change in w_j on w_i, an application of the implicit function theorem to (4) gives:

$$\begin{aligned} \frac{dw_i}{dw_j} &= \frac{\frac{d}{dw_j} C_i' - \frac{d}{dw_j} (P' q_i^* \varphi_i)}{1 - (\frac{d}{dw_i} C_i' - \frac{d}{dw_i} (P' q_i^* \varphi_i))} = \\ &= \frac{(C_i'' - P' \varphi_i) \frac{\partial q_i^*}{\partial w_j} - P'' \frac{\partial Q^*}{\partial w_j} q_i^* \varphi_i - P' q_i^* \frac{\partial \varphi_i}{\partial w_j}}{1 - (C_i'' - P' \varphi_i) \frac{\partial q_i^*}{\partial w_i} + P'' \frac{\partial Q^*}{\partial w_i} q_i^* \varphi_i + P' q_i^* \frac{\partial \varphi_i}{\partial w_i}}, \\ i, j &= 1, 2. \end{aligned} \tag{5}$$

where $Q^* = q_i^* + q_j^*$ and $\varphi_i = \frac{\partial q_j^*}{\partial w_i} / \frac{\partial q_i^*}{\partial w_i}$. Note that under a linear demand function: $P'' = 0$, $\varphi_i = const \in [-1, 0]$, $\frac{\partial \varphi_i}{\partial w_j} = \frac{\partial \varphi_i}{\partial w_i} = 0$, $i, j = 1, 2, i \neq j$. Then $\frac{dw_i}{dw_j} = \frac{(C_i'' - P' \varphi_i) \frac{\partial q_i^*}{\partial w_j}}{1 - (C_i'' - P' \varphi_i) \frac{\partial q_i^*}{\partial w_i}}$ and $sign\left(\frac{dw_i}{dw_j}\right) = sign\,(C_i'' - P' \varphi_i)$. Clearly, if the cost functions are also linear, w_i and w_j are always strategic substitutes. In contrast, if C_i'' is sufficiently high, $\frac{dw_i}{dw_j} > 0$, resulting in w_i w_j.being strategic complements. The intuition of this result is as follows: A decrease in w_j results in an decrease in q_i^* and an increase in q_j^*. Moreover, the total output increases ($\frac{\partial Q^*}{\partial w_i} > 0$). If $C_i^{''} > 0$, both the firm i's marginal cost and its marginal revenue decrease ($\frac{d}{dw_j}C_i' = C_i'' \frac{\partial q_i^*}{dw_j} < 0$ and $\frac{d}{dw_j}(P'q_i^*\varphi_i) < 0$). The firm i's best response depends on the relative magnitudes of the changes in the marginal revenue and marginal cost. If $\left|\frac{d}{dw_j}C_i'\right| > \left|\frac{d}{dw_j}(P'q_i^*\varphi_i)\right|$, the best response implies increasing q_i, and decreasing w_i, hence $\frac{dw_i}{dw_j} > 0$. If $\left|\frac{d}{dw_j}C_i'\right| < \left|\frac{d}{dw_j}(P'q_i^*\varphi_i)\right|$, w_i and w_2 are substitutes. For the analysis it is crucial that higher slopes of the firm's marginal cost curves result in a lower degree of substitution. With the slopes steep enough, w_1 and w_2 are complements. If both firms separate, both set the wholesale price bellow marginal cost. If the firms are symmetric ($C_1(q) = C_2(q)$), each firm has a higher output and lower profit than in the Nash-Cournot equilibrium. Moreover, the greater $\frac{dw_i}{dw_j}$ is, the greater is also the difference between the Cournot and equilibrium outcomes in the subgame with the two firms separating. Let $\{q_1^S, q_2^S\}$ be the equilibrium quantities and $\pi_i^S = P(q_1^S + q_2^S)q_i^S - C_i(q_i^S), (i = 1,2)$ the equilibrium payoff of this subgame.

If the firm i integrates and the firm j separates, the firm j chooses w_j to solve its maximization problem:

$$\max_{w_j} \pi_j = P(q_i^* + q_j^*)q_i^* - C_j(q_j^*)$$

where:

$$\begin{cases} q_i^* = \arg\max_{q_i} P(q_i + q_j^*)q_i - C_i(q_i) \\ q_j^* = \arg\max_{q_j} P(q_i^* + q_j)q_j - w_j q_j \end{cases}.$$

The equilibrium values of $\{q_i^*, q_j^*\}$ satisfy the first order conditions:

$$\begin{cases} P'q_i + P - C_i'(q_i) = 0 \\ P'q_j + P - w_j = 0 \end{cases}. \tag{6}$$

Note that both reaction functions determined by (6) are decreasing, the firm i's best response function does not depend on w_j and it is the same as in the subgamewith both firms integrating. The retailer j's best response function is determined by the firm j's choice of w_j at the first stage. Thus the firm j varying w_j (i.e. shifting its retailer's best response curve) may yield any point on the firm i's best response curve as an equilibrium outcome. Clearly, the equilibrium output is the same as under Stackelberg competition with the firm j being the leader. Let $\{q_i^F, q_j^L\}$ and $\{\pi_i^F, \pi_j^L\}$ denote the equilibrium quantities and profits, with the upper indices F and L referring to the integrated and separated firms respectively[7].

[7] The upper index F (L) indicates that the integrated (separated) firm obtains the Stackelbergs follower's (leader's) profit.

2.2 The upstream equilibrium

We summarize the previous results in the following table:

		$M\ 2$	
		Separate	*Integrate*
$M\ 1$	*Separate*	π_1^S, π_2^S	π_1^L, π_2^F
	Integrate	π_1^F, π_2^L	π_1^C, π_2^C

Under Assumptions 1 and 2 we have that $\pi_i^L > \pi_i^C > \pi_i^F$ and $q_i^L > q_i^C > q_i^F, i = 1, 2$. Therefore the subgame $\{I, I\}$ is never an equilibrium outcome. Finally, the equilibrium is determined by a relation of manufacturers' profits π_i^L, π_i^F and $\pi_i^S, i = 1, 2$ and $\{S, I\}$ is equilibrium if $\pi_2^F \geq \pi_2^S$; $\{I, S\}$ is equilibrium if $\pi_1^F \geq \pi_1^S$; $\{S, S\}$ is equilibrium if $\pi_1^F \leq \pi_1^S$ and $\pi_2^F \leq \pi_2^S$. Note that if $\pi_i^F > \pi_i^S, i = 1, 2$, there are two asymmetric strict equilibria $\{S,\ I\}$ and $\{I,\ S\}$, whereas if $\pi_i^F > \pi_i^S$ and $\pi_j^F < \pi_j^S$ there is a unique asymmetric strict equilibrium, in which the firm i integrates and the firm j separates.

3 Linear demand and quadratic costs

Consider the case of a linear demand $P(q_1 + q_2) = 1 - q_1 - q_2$ and quadratic cost functions $C_i(q_i) = \frac{1}{2} d_i q_i^2$ with $d_i \geq 0, i = 1, 2$. Since $P(Q)$ and $C_i(q_i)$ satisfy Assumptions 1 and 2, there exist a unique Nash equilibrium in the Cournot game.

If both firms integrate, they play a standard Cournot game:

$$\max_{q_i} \pi_i = (1 - q_i - q_j) q_i - \frac{1}{2} c_i q_i^2, i, = 1, 2$$

The Nash equilibrium outcome of the game satisfies the first order conditions and yields the solution:

$$1 - (2 + d_i) q_i - q_j = 0.$$

$$q_i^C = \frac{1 + d_j}{(3 + 2d_i + d_j d_i + 2d_j)} \text{ and } \pi_i^C = \frac{(2 + d_i)(1 + d_j)^2}{2(3 + 2d_j + d_j d_i + 2d_i)^2}, i, j = 1, 2.$$

If both firm separate, the first order conditions of the retailers' profit maximization problems

$$1 - 2q_i - q_j - w_i = 0, i, j = 1, 2$$

gives the optimal outputs as functions of $(w1, w2)$, with total output and price:

$$q_i = \frac{1 - 2w_i + w_j}{3}, i, j = 1, 2$$

$$Q = \frac{2 - w_1 - w_2}{3}, P = \frac{1 + w_1 + w_2}{3}$$

The firms' maximization problems are given by:

$$\max_{w_i} \pi_i = \frac{1 + w_i + w_j}{3} \frac{1 - 2w_i + w_j}{3} - \frac{d_i}{2} \left(\frac{1 - 2w_i + w_j}{3} \right)^2, i, j = 1, 2, i \neq j.$$

with first order conditions giving the firms' reaction curves in the space $\{w_1, w_2\}$:

$$w_i = \frac{(-1 + 2d_i)(1 + w_j)}{4(1 + d_i)}, i, j = 1, 2 \tag{7}$$

It holds that: $\frac{dw_i}{dw_j} = \frac{(-1+2d_i)}{4(1+d_i)}$ is strictly increasing in d_i, $\left.\frac{dw_i}{dw_j}\right|_{d_i=0} = -\frac{1}{4}$, $\left.\frac{dw_i}{dw_j}\right|_{d_i=1/2} = 0$, $\frac{dw_i}{dw_j} \underset{d_i \to +\infty}{\to} \frac{1}{2}$. Hence, the degree of substitution between w_i and w_j decreases in d_i, when $d_i < 1/2$, and w_i is complement to w_2, when $d_i > \frac{1}{2}$. Note that if $d_i > \frac{1}{2}$ and $d_i < \frac{1}{2}$ then $\frac{dw_i}{dw_j} > 0$ and $\frac{dw_j}{dw_i} < 0$, That is, w_i is a complement to w_j whereas w_j is a substitute for w_i.The system (7) has a solution $\{w_i = \frac{2d_i - 2d_j + 4d_i d_j - 1}{5 + 6d_i + 6d_j + 4d_i d_j}, i, j = 1, 2\}$. The equilibrium quantities and profits are given by:

$$q_i^S = \frac{2 + 4d_j}{(5 + 6d_i + 4d_i d_j + 6d_j)},$$
$$\pi_i^S = \frac{2(1 + d_i)(1 + 2d_j)^2}{(5 + 6d_i + 4d_i d_j + 6d_j)^2},$$
$$i, j = 1, 2, i \neq j.$$

If the firm i separates and the firm j integrates, the retailers' profit maximization problems:

$$\begin{cases} \max_{q_i} \pi_i = (1 - q_i - q_j)q_i - w_i q_i \\ \max_{q_j} \pi_j = (1 - q_j - q_i)q_j - \frac{1}{2} d_j q_j^2 \end{cases}$$

with the first order conditions and solution:

$$\begin{cases} 1 - 2q_i - q_j - w_i = 0 \\ 1 - (2 + d_j)\, q_j - q_i = 0 \end{cases}$$

$$\begin{cases} q_i = \frac{1+d_j-w_i(2+d_j)}{3+2d_j} \\ q_j = \frac{1+w_i}{3+2d_j} \end{cases}. \tag{8}$$

Plugging (8) into $\pi_i = P(q_i + q_j)q_i - \frac{1}{2}d_i q_i^2$ and optimizing with respect to w_i, we obtain $w_i = \frac{(1+d_j)(2d_i+d_id_j-1)}{(2+d_j)(2+2d_j+d_id_j+2d_i)}$. The equilibrium quantities and profits are given by:

$$\begin{cases} q_i^L = \frac{1+d_j}{(2+2d_j+d_id_j+2d_i)} \\ q_j^F = \frac{1+d_j+2d_i+d_id_j}{(2+d_j)(2+2d_j+d_id_j+2d_i)} \end{cases}$$

$$\begin{cases} \pi_i^L = \frac{(1+d_j)^2}{2(2+d_j)(2+2d_j+d_id_j+2d_i)^2} \\ \pi_j^F = \frac{(1+d_j+2d_i+d_id_j)^2}{2(2+d_j)(2+2d_j+d_id_j+2d_i)^2} \end{cases}$$

By a symmetry argument, if the firm j separates and the firm i integrates then retailers' profits are:

$$\begin{cases} \pi_i^F = \frac{(1+d_i+2d_j+d_id_j)^2}{2(2+d_i)(2+2d_j+d_id_j+2d_i)^2} \\ \pi_j^L = \frac{(1+d_i)^2}{2(2+d_i)(2+2d_i+d_id_j+2d_j)^2} \end{cases}.$$

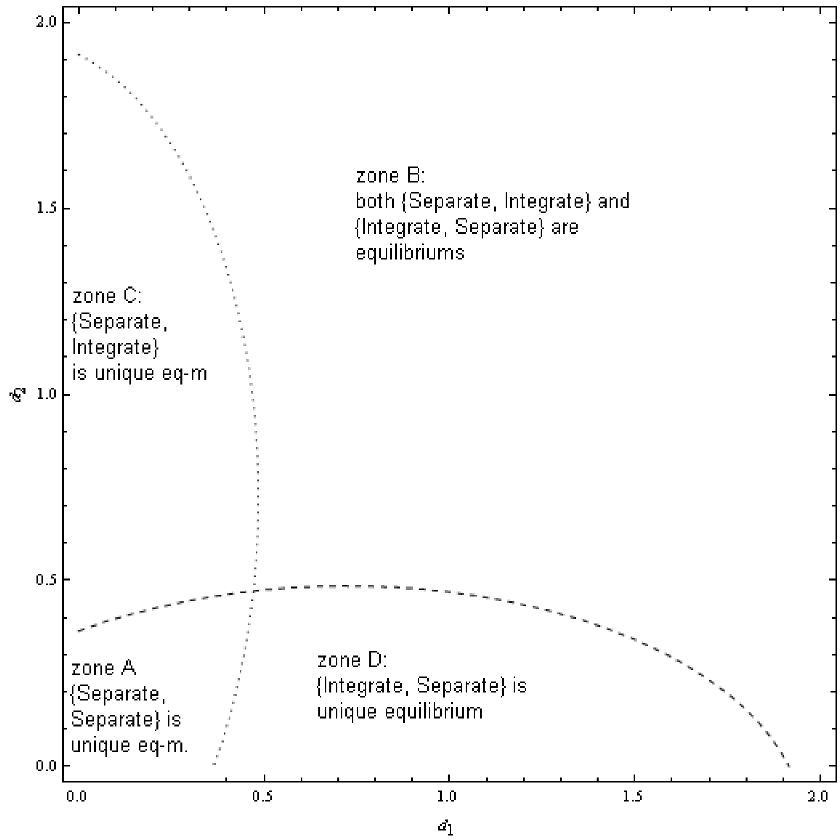
The following proposition says that each firm has an incentive to separate, given that its rival integrates, hence $\{I, I\}$ cannot be an equilibrium outcome.

Proposition 1 *$\pi_i^L > \pi_i^C > \pi_i^F$ for any $d_1 \geq 0, d_2 \geq 0$.*
Proof. *This is obvious given that π_i^L is the Stackerberg leader's profit and π_i^F the Stackerberg follower's profit.* ■

Consider the set (d_1, d_2), $d_i = \theta_i(d_j)$, such that the firm i is indifferent between separating and integrating, given that the firm j separates:

$$\pi_i^F = \frac{(1 + d_i + 2d_j + d_id_j)^2}{2(2 + d_i)(2 + 2d_j + d_id_j + 2d_i)^2} = \frac{2(1 + d_i)(1 + 2d_j)^2}{(5 + 6d_i + 4d_id_j + 6d_j)^2} = \pi_i^S. \tag{9}$$

It can be shown that $\theta_i(d_j)$, determined by (9), is strictly concave and has a unique maximum, $\theta_i(0) > 0$, $\exists \overline{d_j} : \theta_i(\overline{d_j}) = 0$. The Figure 1 gives a graphical representation of $\theta_i(d_j)$ and $\theta_j(d_i)$.



$\pi_2^F > \pi_2^S$ above the dashed line, $\pi_1^F > \pi_1^S$ right at the dotted line.

In the zone A (low d_1 and low d_2), both firms have relatively flat marginal cost curves. Each firm prefers to separate given that its rival separates, hence the unique equilibrium is $\{S, S\}$. The equilibrium profit of each firm is lower than in the Cournot equilibrium, yet higher than the Stackelberg follower's profit: $\pi_i^F < \pi_i^S < \pi_i^C$. Although inside the zone A the firms may differ in efficiency, this difference is sufficient. In the zone C (low d_1 and moderate d_2), the firm 1 is more efficient than firm 2, but the difference in efficiency is not too high. Then the strategy S is dominant for the firm 1, while the firm 2 chooses I if the firm 1 chooses $S : \pi_1^F < \pi_1^S < \pi_i^C; \pi_2^C > \pi_2^F > \pi_2^S$. In the zone D (low d_1 and moderate d_2), the situation is the opposite to that of zone C: the firm 2 is more efficient then the firm 2, but the difference in efficiency is not too high. Then the strategy S is dominant strategy for the firm 1 while the firm 2 chooses I, if the firm 1 chooses $S : \pi_1^F < \pi_1^S < \pi_i^C; \pi_2^C > \pi_2^F > \pi_2^S$. Finally, the zone B is such that (either d_1 or d_2 or both are sufficiently high). Either both firms are less efficient than in the zone A, or the asymmetry in firms' cost is very high. In this case each firm chooses I, if its rival chooses S. Therefore, two asymmetric equilibria, $\{I, S\}$, $\{S, I\}$, exist.

The following proposition summurizes the results:

Proposition 2 *Given $d_1 \geq 0, d_2 \geq 0$, $\{S, S\}$ is an equilibrium, if $(d_1, d_2) \in A = \{(d_1, d_2)|\, d_1 \leq \theta_1(d_2), d_2 \leq \theta_2(d_1)\}$;*

both $\{S, I\}$ and $\{I, S\}$ are equilibria, if $(d_1, d_2) \in B = \{(d_1, d_2)|\, d_1 \geq \theta_1(d_2), d_2 \geq \theta_2(d_1)\}$;

$\{, I\}$ is an equilibrium, if $(d_1, d_2) \in C = \{(d_1, d_2)|\, d_1 \leq \theta_1(d_2), d_2 \geq \theta_2(d_1)\}$;

$\{I, S\}$ is an equilibrium, if $(d_1, d_2) \in D = \{(d_1, d_2)|\, d_1 \geq \theta_1(d_2), d_2 \geq \theta_2(d_1)\}$.

Corollary 3 *In the symmetric game with $C_i(q) = C_j(q) = dq^2$ there exists a unique $\widehat{d}$ such that, if $d < \widehat{d}$, then $\{S, S\}$is the unique equilibrium and, if $d > \widehat{d}$, there are two asymmetric equilibria: $\{I, S\}$ and $\{S, I\}$.*

In particular, $\widehat{d} \approx 0.47 < \frac{1}{2}$ and $\frac{dw_i}{dw_j} = \frac{dw_i}{dw_j} \approx -0.01 < 0$ for $d_1 = d_2 = \widehat{d}$. Thus, asymmetric equilibria in the symmetric game arise even if the wholesale prices are not strategic complements, given that the degree of substitution between them is sufficiently low.

3.1 Robustness

As the analysis in the Section 2 suggests, increasing marginal costs and cost asymmetry are the driving forces for the existence of asymmetric equilibria. According to (5), the degree of substitutability between w_1 and w_2 crucially depends on the slope of marginal cost function (that is, on $C_i''(q)$). Even in the case that the demand function is non-linear, but $C''(q)$ is sufficiently high, w_1 and w_2 are weak substitutes ($\frac{dw_i}{dw_j}$ negative and close to zero), or even complements ($\frac{dw_i}{dw_j} > 0$). This results in strong competition between the manufacturers in the wholesale prices and results in high output levels and low profits. It can be shown that if the goods are imperfect substitutes, the effect of a change in w_i on q_j^* is that the firm j's marginal revenue and its marginal cost are smaller compared to the case of perfect substitutes. Still the substitutability or complementarity between w_j and w_i depends on the slope of the firm j's marginal curve. Hence, given that the degree of goods substitution is sufficiently high, the same qualitative results hold. Thus, the results of the model are robust with respect to the linearity of the demand function and homogeneity of the goods.

4 Conclusion

In this paper I have analyzed the incentives of firms to separate (sell through an independent exclusive retailers), or integrate (be a retailer of one's own good) under quantity competition. The main result is that the equilibrium critically depends on firms' cost structures. For the case of quadratic cost functions, I have shown the following: if the cost asymmetry is small and marginal curves functions are sufficiently flat, in the unique equilibrium of the game both firms separate. If the asymmetry in cost is extremely high, or both firms' marginal cost curves are sufficiently steep, then each firm prefers to integrate, given that it rivals separates, and therefore two asymmetric equilibria arise. Finally, under moderate asymmetry in costs and one firm's marginal curve being steep, with the second firm' marginal curve being flat, there is a unique equilibrium in which the first firm integrates, whereas the second separates. The following intuition for these results is as follows. Each firms prefers to separate, given that its rival integrates. In this case, a separated firm gets the Stackelberg leader's profit and an integrated firm gets the Stackelberg's follower profit. If two symmetric firms separate, their profits depend on strength of competition in the wholesale prices among producers. The strength of this competition is in turn determined by the slope of the marginal cost functions, higher slopes implying stronger competition. A strong competition results in low wholesale prices, high output levels and profits lower than the Stackelberg follower's profit. In this case, there are two asymmetric equilibrium in which one firm separates and the other integrates. If the competition in wholesale prices is weak, each firm prefers to separate given that its rival separates. Thus, in the unique symmetric equilibrium both firms separate.If firms differ in the costs, the degree of asymmetry plays a crucial role. If the asymmetry is not very high, the firm 1's marginal curve is flat and the firm 2's marginal curve steep, the firm 1's dominant strategy is to separate, while the firm 2 prefers to integrate, given that the firm 1 separates. There is a unique equilibrium, in which one firm separates and the other integrates. If the cost asymmetry is very high (e.g., the firm 1 has a flat marginal-cost curve, whereas the firm 2 a very steep one), it is profitable for the firm 1 to integrate, given that the firm 2 separates. This occurs when the increase in firm 1's profit from the increase in its own quantity is lower than the decrease in its profit resulting from the increase in its competitor's quantity. Thus, if the wholesale price of the firm 2 is a strong complement to the one of the firm 1, there are two asymmetric equilibria. Moreover, in one of them the more efficient firm (the firm 1) integrates and the less efficient firm (the firm 2) separates.

My analysis provides a possible explanation for the widely observed asymmetry in firms' sales strategies, based on decreasing economies of scales and cost asymmetries. It is worth to note that in the model separation neither implies a change in the production function, nor is associated with additional costs. In this sense, I have shown the existence of asymmetric equilibria in a "pure" separation game.Moreover, in the paper, I have analyzed the case of quadratic costs and a linear demand function. Further research is needed to be don on the sufficient and necessary conditions for the existence of different equilibria under more general assumptions about cost and demand functions. I am delegating this task to the future.

5 Reference

Basu, Kaushik (1995), Stackelberg equilibrium in oligopoly: An explanation based on managerial incentives", Economic Letters, Vol. 49, pp. 459-464

Bonanno, Giacomo and Vickers, John (1988), "Vertical Separation", The Journal of Industrial Economics, Vol. 36, No. 3, pp. 257-265

Buehler, Stefan and Schmutzler, Armin (2005), "Asymmetric Vertical Integration", Advances in Theoretical Economics, Vol. 5, pp. 1164-1164

Fershtman, Chaim and Judd, Kenneth L. (1987), "Equilibrium Incentives in Oligopoly", The American Economic Review, Vol. 77, No. 5 , pp. 927-940

Gal-Or, Esther (1991), "Duopolistic Vertical Restraints", European Economic Review, Vol. 35, pp. 1237-1253

Janson, Jos (2003), "Coexistance of Strategic Vertical Searation and Integration", International Journal of Industrial Organization, Vol. 21, pp. 699-716

Lin, Y. Joseph (1988), "Oligopoly and Vertical Integration: Note", American Economic Review, Vol. 78, pp. 251-254

Sklivas, Steven D (1987), "The strategic Choice of Managerial Incentives", The RAND Journal of Economics, Vol. 18, No. 3, pp.452-458

Vickers, John (1985), "Delegation and the Theory of the Firm", The Economic Journal, Vol. 95, Supplement: Conference Papers , pp. 138-147