Monetary Expansion and Converging Speed in a Growing Economy

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Abstract

This paper explores the effect of monetary policy on the speed of convergence. Using a neoclassical monetary growth model with a cash-in-advance constraint, we conduct numerical evaluation of the effect of changes in the growth rate of money supply on the converging speed of the economy. We find that, in contrast to fiscal actions, a change in monetary policy may produce little impact on the converging speed. This result indicates that the growth effect of inflation established in the theoretical models of money and growth would be extremely small, if we evaluated it quantitatively.

1 Introduction

In his seminal contribution, Sato (1963) presented the first study on quantitative evaluation of the neoclassical growth model. He found that the calibrated Solow model with empirically plausible parameter values converges to the steady state very slowly: transition from one steady state to another needs almost 100 years to complete 90% of its adjustment process. As pointed out by Sato (1963), this result can be interpreted in two ways. For one thing, the slow adjustment means that fiscal policies have little impact on per capita income in the short run, even though they would have large effect in the very long run. On the other hand, fiscal policies may have lasting and substantial effect on the growth rate of income. For example, if a fiscal action increases the saving rate, growth rate of income rises on the spot and diminishes slowly during the long transition.[1] In the 1960s many authors reconfirmed Sato's (1963) finding in a variety of the neoclassical growth models.

The last decade has witnessed a revival of research interest in speed of convergence in growth models.[2] The recent investigations on this issue have been mostly concerned with the income convergence hypothesis claimed by the empirical studies based on the cross-country regressions (e.g. Barro and Sala-i-Martin 1992). This line of studies intends to explore converging speeds of per-capita income among countries with different development levels. Therefore, the purpose of the recent studies on converging speed in growth models is more ambitious than that of the earlier researches in the 1960s that discussed the adjustment process of a single economy.

In this paper, we follow the original research concern of Sato (1963) and examine the policy impact on adjustment speed in the context of a neoclassical growth model. Departing from the existing studies both in the 60s and the 90s which ignore the monetary side of the economy, we explore the effect of monetary policy on the speed of convergence. More specifically, using a neoclassical monetary growth model with a cash-in-advance constraint, we conduct numerical evaluation of the effect of changes in the growth rate of money supply on the converging speed of the economy. We find that, in con-

[1] See also Sato (1964) for further discussion. In the foreword to his collected works (Sato 1997), he shows an interesting episode as to how he reached idea of Sato (1963).

[2] King and Rebelo (1993) rekindled the research interest in numerical evaluation of transitional dynamics of growth models. See Sato and Mino (1998) for a survey over the recent literature on speed of convergence both in the neoclassical and endogenous growth models.

trast to fiscal actions, a change in monetary policy may produce little impact on the converging speed. Therefore, the growth effect of inflation established in the theoretical models of money and growth would be extremely small, if we evaluated it quantitatively. This conclusion does not fit well with the empirical findings on inflation and growth, because many studies have claimed that inflation may generate a substantial negative impact on long-term economic growth. We suggest alternative formulations that may reconcile the divergence between the calibrated models and empirical reality.

The next section sets up the analytical framework of the paper. Section 3 analyzes the dynamical system and calculates converging speeds. Section 4 considers possible extensions of our formulation that may produce larger growth effect of monetary policy than that obtained in the base model.

2 A Monetary Growth Model

2.1. Production

Following Mankiw, Romer and Weil (1992), we assume that production technology exhibits constant returns to scale with respect to physical capital, K, human capital, H, and effective labor, AL. The production function is specified as

$$Y = K^a H^b (AL)^{1-a-b}, \quad 0 < a, b < 1, \ \ 0 < a + b < 1,$$

The labor efficiency, A, and physical labor, L, grow at constant rates, γ and n, respectively. Hence, the long-term growth rate of income is $\gamma + n$. Since the rates of return to physical and human capital should be the same in a competitive, one good economy, it holds that $\partial Y/\partial K = \partial Y/\partial H$. This condition yields $aY/K = bY/H$, so that the optimal choice between physical and human capital should satisfy $H = (b/a) K$. As a result, the reduced form of the production function is as follows:[3]

$$Y = \phi K^\alpha (AL)^{1-\alpha}, \tag{1}$$

[3] If we use the standard neoclassical formulation in which production function is $Y = K^\alpha (AL)^{1-\alpha}$ with $\alpha = 1/3$, the converging speed of the model is too fast to fit with Barro and Sala-i-Martin (1992) who claimed that the speed of conditional convergence is at most 2.5% per year. Introducing human capital can increase income share of capital up to 0.8 and substantially lowers the converging speed .

where $\phi = (b/a)^b$ and $\alpha = a + b$. The rate of return to physical capital is given by

$$r = a\phi K^{\alpha-1} (AL). \tag{2}$$

Similarly, the real wage rate is $w = (1 - a - b) \phi K^{\alpha} (AL)^{-\alpha}$ and the rate of return to human capital is $r = b\phi K^{\alpha-1} (AL)^{1-\alpha}$.

2.2. Households

There are L households and each supplies one unite of labor in each moment. The representative household maximizes a discounted sum of utilities

$$\int_0^\infty \frac{c^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt, \quad \sigma > 0, \quad \sigma \neq 1,$$

subject to the flow budget constraint

$$\dot{\omega} = (r - n) \omega + w - \tau - c - im - \delta k,$$

and a cash-in-advanced constraint

$$\eta c \leq m, \quad 0 \leq \eta \leq 1. \tag{3}$$

In the above, c is consumption per capita, ω is the real wealth per capita, m is real money balances per capital, $k\,(= K/L)$ is capital-labor ratio, i is nominal interest rate, and τ is per capita tax (transfer from the government if it has a negative value). By definition, $\omega = k + m$.The parameter η medicates how much of consumption good should be purchased by using cash. When η = 1, the cash-in-advance constraint is fully applied to consumption spending.

The Hamiltonian for the household's optimization problem is

$$H = \frac{c^{1-\sigma} - 1}{1-\sigma} + q\left[(r - n)\omega + w - \tau - c - im - \delta k\right] + \lambda(m - \eta c) + \xi\left(\omega - k - m\right).$$

The necessary conditions for an optimum involves:

$$c^{-\sigma} = q + \lambda \eta, \tag{4}$$

$$-iq - \xi + \lambda = 0, \tag{5}$$

$$-\delta q - \xi = 0, \tag{6}$$

$$\lambda\,(m - c\eta) = 0, \;\; \text{with } \lambda \geq 0, \tag{7}$$

$$\dot{q} = q\,(\rho - n) - \xi, \tag{8}$$

together with the transversailty condition, $\lim_{t\to\infty} e^{-\rho t} q\omega = 0$, and the initial condition on ω. Arranging the conditions displayed above, we find that the following equations hold:

$$c^{-\sigma} = q\,[1 + \eta\,(i - \delta)]\,, \tag{9}$$

$$\dot{q} = q\,(\rho + \delta - r)\,. \tag{10}$$

2.3. The Monetary Competitive Equilibrium

In what follows, we assume that the cash-in-advance constraint is always binding, so that $\eta c = m$ for all $t \geq 0$. According to the standard assumption in the money and growth literature, we assume that nominal money supply grows at a constant rate, μ, and newly created money is distributed to each household as a lump-sum transfer. This means that $\tau = -\mu m$. Since m changes according to $\dot{m} = m\,(\mu - \pi - n)\,$, in the monetary equilibrium consumption demand follows

$$\dot{c}/c = \mu - \pi - n. \tag{11}$$

Note that the nominal interest rate satisfies $i = r + \pi$. Thus from (9) and (11), we obtain:

$$\frac{\dot{c}}{c} = \mu + r - \delta + \frac{1}{\eta} - \frac{c^{-\sigma}}{\eta q}. \tag{12}$$

Finally, the market equilibrium condition for the product market gives

$$\dot{K} = Y - cL - \delta K. \tag{13}$$

3 Speed of Convergence

3.1. The Dynamical System

In order to derive a complete dynamical system, let us denote

$$x = Y/K, \;\; z = cL/K, \quad v = \, c^{-\sigma}/q.$$

Using these notations, the rate of return to capital is expressed as $r = ax$ and (13) becomes $\dot{K}/K = x - z - \delta$. In view of those equations, together with (1), (10) and (12), it is easy to show that dynamic behaviors of x, z and v are respectively given by the following differential equations:

$$\frac{\dot{x}}{x} = (1 - \alpha)(\gamma + n + \delta + z - x), \tag{14}$$

$$\frac{\dot{z}}{z} = n + \mu - (1 - a)x + z + \frac{1}{\eta}(1 - v), \tag{15}$$

$$\frac{\dot{v}}{v} = (1 - \sigma)(ax - \delta) - \frac{\sigma}{\eta}(1 - v) - \sigma\mu - \rho. \tag{16}$$

In the balanced growth equilibrium, Y and K grow at the rate of $\gamma + n$, while c, a, m and k grow at γ. Thus the balanced-growth holds when $\dot{x} = \dot{z} = \dot{u} = 0$ in (14), (15) and (16). The steady-state values of x, z and v are:

$$\bar{x} = \frac{1}{a}(\sigma\gamma + \delta + \rho), \tag{17}$$

$$\bar{z} = \frac{1}{a}[\rho + (\sigma - a)\gamma + (1 - a)\delta] - n, \tag{18}$$

$$\bar{v} = \eta[\rho + \mu - (1 - \sigma)\gamma] + 1 \tag{19}$$

By inspecting the coefficient matrix of the linearized system of (14), (15) and (16) evaluated at the steady state, we find that in general the matrix has one negative eigenvalue and two eigenvalues with positive real parts. Since only x is the predetermined variable in the system, the economy may converge monotonically to the steady-state equilibrium on the stable saddle path. Thus the absolute value of the negative eigenvalue represents the speed of convergence of the approximated system.[4]

Notice that if there is no cash-in-advance constraint on consumption, then $v = 1$ for all $t \geq 0$. In this case, the dynamic system consists of (14) and

$$\frac{\dot{z}}{z} = \left(\frac{a}{\sigma} - 1\right)x + z + n + \left(1 - \frac{1}{\sigma}\right)\sigma - \frac{\rho}{\sigma} \tag{20}$$

Since in our model money is superneutral in the balanced-growth equilibrium, the steady-state values of x and z in the non-monetary equilibrium are the same as (17) and (18). The negative eigenvalue involved in the linearized system of (14) and (20) presents the speed of convergence in the non-monetary

[4]See Rebelo and Xie (1999) for further analysis of the transitional dynamics of the cash-in-advance model.

economy.

3.2. Numerical Examples

To conduct numerical experiments, we set

$$a = 0.35, \ \alpha = 0.7, \ \ \gamma = 0.02, \ n = 0.01, \ \delta = 0.05, \ \rho = 0.03, \ \ \sigma = 2.$$

According to the finding by Mankiw, Romer and Weil (1992), we assume that income shares of physical and human capital are 0.35, so that the income share of the aggregated capital, α, is 0.7. We also assume that population grows at 1% per year and the annual rate of technical progress is 2%. The values of δ, ρ and σ, are as the same as those that been commonly used in the real business cycle literature. In this setting, the steady state levels of income capital ratio, x, and consumption capital ratio, z, are $\bar{x} = 0.34286$ and $\bar{z} = 0.27286$, respectively. (Thus consumption income ratio is $cL/Y = 0.785$). Using those values, we calculate the negative eigenvalue of the dynamic system derived above. We inspect two cases: $\eta = 1$ and $\eta = 0.5$. The table shown below displays stable eigenvalue for different levels of money growth rate.

	$\mu = 0.03$	$\mu = 0.1$	$\mu = 0.25$	$\mu = 0.5$	$\mu = 1.0$
$\eta = 1$	-0.02567	-0.02566	-0.02563	-0.02559	-0.02555
$\eta = 0.5$	-0.02555	-0.02554	-0.02553	-0.02552	-0.02515

We see that the stable eigenvalue of the dynamic system for the non-monetary economy is -0.02514. Therefore, the figures shown in the table demonstrate that the monetary economy subject to the cash-in-advance constraint exhibits faster convergence than the corresponding real economy. The figures also show that a rise in the growth rate of money supply lowers the speed of convergence. This confirms the results in the theoretical analyses by Asako (1982) and Abel (1985).[5] Additionally, observe that if $\mu = 0.03$, the steady state value of $v \, (= c^{-\sigma}/q)$ is 1.07, while it is 2.04 if $\mu = 1.0$. Thus a change in money growth rate has a large impact on the divergence between

[5] Abel (1985) revealed that in a cash-in-advance model a rise in money growth depresses capital accumulation in the transition. By using a money-in-the-utility-function model, Asako (1982) found that if real balance and consumption is perfectly complement each other, an increase in money growth rate has negative effect on investment during the transition. As pointed out by Feenstra (1987), the Sidrauski model with a Leontief-type utility function is equivalent to the model with a cash-in-advance constraint on consumption.

the marginal utility of consumption and the shadow value of asset. However, as the table shows, the negative effect of a higher money growth on the speed of convergence is extremely small. For example, if the cash in advance constraint is fully applied ($\eta = 1$) and money growth rate is 0.03 (so that the inflation rate in the steady state is zero), the economy converges to the steady state at 0.02567% per year. Even though money growth rate increases up to 100% per year (the long-run rate of inflation is 97%), the converging speed decreases only by 0.0012% per year. If 50% of consumption spending is subject to the cash-in-advance constraint $(\eta = 0.5)$, the economy converges faster than the case with $\eta = 1.0$. However, the difference in converging speed is again very small.

Since money growth has little effect on the converging speed, the length of transition adjustment is not sensitive to the rate of inflation either. In fact, if $\eta = 1$ and $\mu = 0.03$, the adjustment time to accomplish 90% of transition is about 89.7 years, while 50% adjustment needs 27 years[6]. Those figures in the case of $\eta = 1$ and $\mu = 1.0$ are 90.3 years and 27.2 years, respectively. On the other hand, the corresponding non-monetary economy accomplishes 90% adjustment in 91.6 years and 50% adjustment in 27.5 years. Again, even though a higher money growth makes the transition period longer, in practice the magnitude of the effect is negligibly small.

To sum up, the monetary economy with the neoclassical technology involves adjustment speed that is essentially the same as that in the corresponding non-monetary economy: both economies converge to the steady state slowly. This implies that a change in monetary policy would have a lasting effect on the growth rate of income during the transition. However, the magnitude of such an impact may be very small under a set of realistic parameter values. This conclusion is in contrast to the fact that fiscal actions generally have large effect on income growth at least during the transitional process.

4 Discussion

It is worth emphasizing that the little impact of monetary policy on income growth can be confirmed in the money-in-the-utility-function model as well.

[6] On the converging path, x satisfies $x(t) - x(0) = e^{\lambda t}(x(t) - \bar{x})$, where λ is the stable eigenvalue. The time length to attain $\beta \times 100\%$ is given by $T_\beta = (\log \beta)/\lambda$.

Suppose that the objective functional of the household is

$$\int_0^\infty \frac{\left(c^\psi m^{1-\psi}\right)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt, \quad 0 < \psi < 1, \quad \sigma > 0, \quad \sigma \neq 1.$$

In the Sdrauski model with the above utility function, Fischer (1979) shows that an increase in money growth accelerates capital accumulation during the transition, while money is superneutral in the steady state. It is shown that in this setting the speed of convergence is lower than that of the non-monetary economy and that a higher money growth increases the converging speed. However, we again find that the positive effect of money supply on growth is very small under the parameter values used in the above: see, Gokan and Mino (1999). In this sense, the growth effects of monetary policy in the alternative formulations of money and growth are almost the same from the view point of quantitative evaluation.

In contrast to the quantitative results in theoretical models, many empirical studies on inflation and growth have claimed that there exists a statistically significant negative relation between inflation and growth. For example, Barro (1996 and 1998) concluded that on average a ten present increase in inflation per year depresses growth rate of real income by 0.3% per year. This figure is obviously much larger than those found by our cash-in-advanced model.

One of the natural extension of the base model to reconcile the divergence between theory and empirics is to modify the model to make money affect capital formation directly. A simple formulation is to apply the cash-in-advance constraint to investment as well as to consumption. If (3) is replaced with

$$\theta \dot{k} + c \leq m, \quad 0 \leq \theta \leq 1,$$

we find that a rise in money growth depresses capital accumulation not only in the transition but also in the steady state. It is easy to see that this additional constraint enhances the negative effect of monetary expansion on growth. The cash constraint on investment spending seems to be negligible in advanced countries. However, it would be an appropriate assumption for developing countries where underdevelopment of financial markets would depress investment.

Another consideration is to reformulate money supply behavior. In this paper we have assumed the helicopter drop of money with constant money growth. While this assumption has been standard in money and growth literature, it may not describe well the money supply policies in practice. If we assume that money supply is enodogenously determined rather than exogenously specified, then inflation may be a result of low growth rather than other way around. For example, suppose that unproductive government consumption is fully financed by new money creation. Letting ε be the income share of government consumption, the government budget constraint is $\dot{M} = \varepsilon pY$. Thus, using the previous notations, the growth rate of money supply is given by

$$\frac{\dot{M}}{M} = \mu = \frac{\varepsilon Y}{mL} = \frac{\varepsilon Y}{\eta cL} = \frac{\varepsilon x}{\eta z}.$$

On the other hand, capital formation follows $\dot{K} = (1 - \varepsilon) Y - cL - \delta K$. Consequently, the dynamic system in money finance regime consists of the following dynamic equations:

$$\frac{\dot{x}}{x} = (1 - \alpha)(n + \gamma + \delta + z - (1 - \varepsilon) x)$$

$$\frac{\dot{z}}{z} = n + \frac{\varepsilon x}{\eta z} - (1 - \varepsilon - a) x + z + \frac{1}{\eta}(1 - v)$$

$$\frac{\dot{v}}{v} = (1 - \sigma)(ax - \delta) - \frac{\sigma}{\eta}(1 - v) - \frac{\sigma \varepsilon x}{\eta z} - \rho$$

In this formulation a rise in ε lowers capital formation and raises inflation due to monetary expansion. Therefore, there may exist a substantial negative relation between growth and inflation in the transition as well as in the steady state. Since countries suffered from high inflation and low growth performance (such as several Latin American nations during the 1980s) usually expand money supply to finance their fiscal deficits. Hence, low growth with high inflation is the consequence of too much dependence of the fiscal authority on inflation tax. Our numerical experiments in Section 3 suggest that we should analyze money supply behavior more carefully when we evaluate theoretical frameworks of money and growth in the light of empirical experiences.

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