Monetary Business Cycle Accounting

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Abstract

This paper investigates the quantitative importance of various types of frictions for inflation and nominal interest rate dynamics by extending business cycle accounting to monetary models. Representing a variety of real and nominal frictions as 'wedges' to standard equilibrium conditions allows a quantitative assessment of those frictions. Decomposing the data into movements due to these wedges shows that frictions that are equivalent to wedges in TFP and equilibrium conditions for asset markets are essential. In contrast, wedges in equilibrium conditions for capital accumulation and the resource constraint, and wedges capturing distortionary effects of sticky prices, play only a secondary role.

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1 Introduction

Chari, Kehoe and McGrattan (2007a) develop a data analysis method to investigate the quantitative importance of various classes of frictions for aggregate fluctuations. This method, which they label 'business cycle accounting', is intended to help researchers make decisions about where to introduce frictions in their models so that the models generate fluctuations like those in the data. Chari et al. (2007a), henceforth CKM, focus on fluctuations in four key real variables: output, hours, investment, and consumption. Often, however, economists are also interested in the behavior of the nominal side of the economy, especially in relation to monetary policy, and how it co-moves with real economic activity. This paper therefore extends the method to a class of monetary models in order to assess the quantitative importance of various types of frictions for the dynamics of two key nominal variables, inflation and the short-term nominal interest rate.

Business cycle accounting rests on the insight that models with various frictions can be mapped into a prototype model with a number of time-varying 'wedges'. These wedges distort the equilibrium decisions of agents operating in otherwise competitive markets in the same way as the underlying frictions. Such interpretation of wedges in standard equilibrium conditions has previously been proposed by, for example, Hall (1997) and Mulligan (2002).

Using the equilibrium conditions of the prototype model and data on the model's endogenous variables, the wedges can be uncovered from the data and fed back into the model, one at a time and in various combinations, in order to determine their contribution to the movements in the data. By establishing mappings between different classes of frictions and the wedges, the method can be used to assess the contribution of various types of frictions to fluctuations in the data. By construction, all wedges together account for all of the movements in the data.[1]

CKM provide mappings between a number of detailed models and a prototype stochastic

[1]Papers besides CKM that discuss the method include Christiano and Davis (2006) and Chari, Kehoe and McGrattan (2007b). Studies that apply the method to various episodes in different countries include, among others, Crucini and Kahn (2003), Ahearne, Kydland and Wynne (2005), Chakraborty (2005), Kobayashi and Inaba (2006), and Kersting (2008). All of these studies, however, focus only on the four real variables studied by CKM.

growth model with four time-varying wedges, henceforth referred to as the CKM economy. At face value, fluctuations in these wedges look like fluctuations in total factor productivity, taxes on labor income, taxes on investment, and government consumption. CKM label these wedges *efficiency*, *labor*, *investment*, and *government consumption wedges*, respectively. They demonstrate that input-financing frictions are equivalent to efficiency wedges, labor market distortions, such as labor unions or sticky wages, to labor wedges, investment-financing frictions, such as those in financial accelerator models, to investment wedges, and frictions in international borrowing and lending to government consumption wedges. Applying the method to the Great Depression and the postwar period in the United States, they show that efficiency and labor wedges account for most of the movements in output, hours, investment, and consumption.

In the same spirit, this paper constructs a prototype monetary economy. Specifically, a stochastic growth model with nominal bonds and a monetary authority. This model underlies a large class of monetary business cycle models, such as those of McGrattan (1999), Ireland (2004), and Smets and Wouters (2007). In line with this literature, a monetary authority in the prototype economy follows a simple feedback rule, like that proposed by Taylor (1993). Besides the four wedges of the CKM economy, the prototype monetary economy has two additional wedges: an *asset market wedge*, which acts like a tax on nominal bonds and distorts a standard Euler equation for bonds, and a *monetary policy wedge*, which captures the deviations of the observed nominal interest rate form the rate prescribed by the Taylor rule.

The prototype economy is general enough to capture the distortionary effects of the key frictions considered in the literature. To show this the paper provides three examples of such mappings between detailed models with frictions and the prototype. In particular, it shows that sticky prices are equivalent to equal investment and labor wedges, limited participation in asset markets to asset market wedges, and sticky wages to labor wedges.[2] Furthermore, the paper shows that some detailed monetary policy rules, such as those with

[2] In a working-paper version of this paper (Sustek, 2009) we also show that a model of inflation dynamics based on capacity utilization and energy price shocks studied by Finn (1996) is equivalent to the prototype model with efficiency wedges.

regime changes, can be mapped into a prototype Taylor rule with monetary policy wedges.[3] We also discuss how the effects of these frictions on inflation and the nominal interest rate dynamics in detailed models can be understood through the lenses of a pricing function in the prototype economy.

We apply the method to the dynamics of inflation and the nominal interest rate in the postwar U.S. business cycle with the aim to shed light on two well-known anomalies: the correlations at various leads and lags between output and the nominal interest rate, and between output and inflation. Specifically, both inflation and the nominal interest rate are negatively correlated with future output and positively correlated with past output. This 'inverted leading indicator' property of the nominal interest rate has been pointed out by, among others, King and Watson (1996), and more recently Backus, Routledge and Zin (2007), while the lagging characteristic of inflation has been highlighted by, among others, Fuhrer and Moore (1995), Galí and Gertler (1999), and more recently Wang and Wen (2007). Although we focus on U.S. data, our results are likely to apply to other countries as well. Wang and Wen (2007) and Henriksen, Kydland and Šustek (2008) document that the lead-lag pattern of the two nominal variables observed in the U.S. data is also present business cycles of other developed economies.

Using data on output, hours, investment, consumption, the GDP deflator, and the yield on 3-month Treasury bills, we uncover the realized wedges from the equilibrium conditions of the prototype economy and characterize their behavior over the business cycle. We then feed the wedges back into the model, one at a time and in various combinations, in order to determine how much of the observed movements in the data can be attributed to each wedge.

Previously, the lead-lag pattern of inflation has been usually studied separately from the lead-lag pattern of the nominal interest rate. For example Wang and Wen (2007) only study inflation dynamics, while Backus et al. (2007) only focus on the dynamics of the nominal interest rate (and other yields and asset prices). Our decomposition, however, shows that

[3] The equivalence results established in this paper do not provide an exhaustive list of frictions that can be mapped into our prototype model. We only focus on the most common frictions in the business cycle literature.

the observed dynamics of the two variables over the business cycle are largely driven by the same factors. Specifically, the efficiency and asset market wedges are both necessary, and to some extent also sufficient, for generating the observed lead-lag pattern of the two nominal variables. The other four wedges are substantially less important. Especially the effects of the investment and government consumption wedges on the lead-lag pattern are very small. Interestingly, these findings suggest that sticky prices, a friction often invoked in the study of inflation dynamics, are of second-order importance for the lead-lag pattern. This is because distortions due to sticky prices manifest themselves as movements in investment and labor wedges.

We hope that our findings will provide useful information to researchers constructing detailed models with explicit frictions to analyze the nominal business cycle and monetary policy. Our findings suggest that such models should, first and foremost, include frictions that manifest themselves as efficiency and asset market wedges. Furthermore, such frictions need to generate movements in these two wedges as in the data. Namely, in the data the two wedges are strongly mutually positively correlated but have slightly different phase shift with respect to output. While the efficiency wedge somewhat leads output, the asset market wedge lags output.

Our results are related to two recent papers. Canzoneri, Cumby and Diba (2007) show that the error in estimated Euler equations for bonds systematically moves with the stance of monetary policy for a wide range of utility functions. Atkeson and Kehoe (2008) show that yield curve and exchange rate data imply cyclical movements in the conditional variance of a pricing kernel for government bonds, something that most existing models used for monetary policy analysis omit. Both of these findings are consistent with our result that the asset market wedge is crucial in accounting for the observed inflation and nominal interest rate dynamics over the business cycle.

The rest of the paper proceeds as follows. Section 2 describes the prototype monetary economy, Section 3 provides mappings between frictions and wedges, Section 4 describes the procedure for uncovering the wedges from the data and characterizes the cyclical behavior of the wedges, and Section 5 carries out the decomposition. Section 6 concludes and makes suggestions for future research.

2 The Prototype Economy

2.1 The Economic Environment

The prototype economy is inhabited by an infinitely lived representative consumer and a representative producer. Both are price takers in all markets. In addition, there is a government that taxes the consumer and sets the nominal rate of return on a bond. In period t the economy experiences one of finitely many events z_t. Let $z^t = (z_0, ..., z_t)$ denote the history of events up through and including period t, Z^t the set of all possible histories z^t, $\mathcal{Z}^t$ the appropriate σ-algebra, and $\mu_t(z^t)$ the probability measure associated with this σ-algebra. The initial event z_0 is given. The probability space up through and including period t is thus given by $(Z^t, \mathcal{Z}^t, \mu_t(z^t))$. Furthermore, let $\mu_t(z^{t+1}|z^t)$ denote the conditional probability $\mu_{t+1}(z^{t+1})/\mu_t(z^t)$.

The economy has six exogenous random variables, all of which are measurable functions of the history of events z^t: an *efficiency wedge* $A_t(z^t)$, a *labor wedge* $\tau_{lt}(z^t)$, an *investment wedge* $\tau_{xt}(z^t)$, a *government consumption wedge* $g_t(z^t)$, an *asset market wedge* $\tau_{bt}(z^t)$, and a *monetary policy wedge* $\widetilde{R}_t(z^t)$. The first four wedges are the same as those in the CKM economy (in a sense made precise below) and will therefore be sometimes referred to as the *CKM wedges.*

The consumer maximizes expected utility over stochastic paths of per capita consumption $c_t(z^t)$ and leisure $h_t(z^t)$[4]

$$\sum_{t=0}^{\infty} \sum_{z^t} \beta^t \mu_t(z^t) u\left(c_t(z^t), h_t(z^t)\right)(1+\gamma_n)^t, \tag{1}$$

where β is a discount factor, $u(.,.)$ has the standard properties, and γ_n is a population growth rate. The consumer has to satisfy three constraints: a time constraint, a budget

[4] All quantities in the model are in per capita terms.

constraint, and a law of motion for capital. The time constraint states that

$$h_t(z^t) + l_t(z^t) = 1, \tag{2}$$

where $l_t(z^t)$ is time spent working. The budget constraint requires that

$$\begin{aligned} & c_t(z^t) + \left[1 + \tau_{xt}(z^t)\right] x_t(z^t) \\ & \quad + \left[1 + \tau_{bt}(z^t)\right] \left[(1+\gamma_n)\frac{b_t(z^t)}{(1+R_t(z^t))p_t(z^t)} - \frac{b_{t-1}(z^{t-1})}{p_t(z^t)}\right] \\ & \qquad = \left[1 - \tau_{lt}(z^t)\right] w_t(z^t) l_t(z^t) + r_t(z^t) k_t(z^{t-1}) + T_t(z^t). \end{aligned} \tag{3}$$

Here, $x_t(z^t)$ is investment, $p_t(z^t)$ is a nominal price of goods in terms of a unit of account, $b_t(z^t)$ is holdings of a bond that pays a net nominal rate of return $R_t(z^t)$ in all states of the world z_{t+1} (and is in net zero supply), $w_t(z^t)$ is the real wage rate, $r_t(z^t)$ is the real rental rate for capital, $k_t(z^{t-1})$ is capital held at the start of period t, and $T_t(z^t)$ is a lump-sum transfer from the government. Finally, the law of motion for capital states that

$$(1+\gamma_n)k_{t+1}(z^t) = (1-\delta)k_t(z^{t-1}) + x_t(z^t), \tag{4}$$

where δ is a depreciation rate. Notice that the investment and asset market wedges act like taxes on capital and nominal bond accumulation, respectively, while the labor wedge acts like a tax on labor income.

The producer operates an aggregate constant-returns-to-scale production function

$$y_t(z^t) = A_t(z^t) F\left(k_t(z^{t-1}), (1+\gamma_A)^t l_t(z^t)\right), \tag{5}$$

where γ_A is the growth rate of labor-augmenting technological progress and $F(.,.)$ has the standard properties. The producer maximizes profits $y_t(z^t) - w_t(z^t)l_t(z^t) - r_t(z^t)k_t(z^{t-1})$ by setting the marginal products of capital and labor equal to $r_t(z^t)$ and $w_t(z^t)$, respectively.

The aggregate resource constraint requires that

$$c_t(z^t) + x_t(z^t) + g_t(z^t) = y_t(z^t). \tag{6}$$

Following Taylor (1993), and a large empirical literature on monetary policy (surveyed, for instance, by Woodford, 2003, Chapter 1), most existing monetary business cycle models (e.g. McGrattan, 1999; Ireland, 2004; Primiceri, Schaumburg and Tambalotti, 2006; Smets and Wouters, 2007) describe monetary policy as following a feedback rule like that proposed by Taylor. According to this rule, the monetary authority sets the nominal interest rate in response to movements in output and inflation, while also putting some weight on past interest rates. In order to preserve the structure of this class of models, the government in the prototype economy also follows such a rule

$$\begin{aligned} R_t(z^t) = (1-\rho_R)\left[R + \omega_y\left(\ln y_t(z^t) - \ln y\right) + \omega_\pi\left(\pi_t(z^t) - \pi\right)\right] \\ + \rho_R R_{t-1}(z^{t-1}) + \widetilde{R}_t(z^t), \end{aligned} \tag{7}$$

where $\rho_R \in [0, 1)$, $\pi_t(z^t) \equiv \ln p_t(z^t) - \ln p_{t-1}(z^{t-1})$ is the inflation rate, and a variable's symbol without a time subscript denotes the variable's steady-state (or balanced growth path) value. In addition, in line with much of the literature, we assume that $\omega_\pi > 1$, thus excluding explosive paths for inflation.[5]

2.2 Equilibrium and the Distortionary Effects of the Wedges

A *competitive equilibrium of the prototype economy* is a set of allocations $(c_t(z^t), x_t(z^t), y_t(z^t), l_t(z^t), k_{t+1}(z^t), b_t(z^t))$ and a set of prices $(p_t(z^t), R_t(z^t), r_t(z^t), w_t(z^t))$ such that the alloca-

[5]Most current monetary business cycle models with centralized markets and an interest rate policy rule abstract from money. This is because under an interest rate rule *nominal* money balances are determined residually, after all other equilibrium allocations and prices have been determined (see, for instance, Woodford, 2003). In a working-paper version of this paper (Sustek, 2009) we allow for money in the prototype and detailed economies. It enters the models through a 'shopping time' constraint, which, as Feenstra (1986) shows, is equivalent to a 'money in the utility function' specification. In this setup *real* money balances generally affect equilibrium allocations and prices. However, for U.S. calibration these effects are small and thus do not change our main quantitative results. In addition, the theoretical equivalence results are unaffected by the explicit presence of money, but their exposition becomes somewhat cumbersome.

tions are optimal for the consumer and the producer, the nominal interest rate is set according to the monetary policy rule (7), $b_t(z^t)$ is equal to zero, and the resource constraint (6) is satisfied. Given the producer's optimality conditions $w_t = A_t(z^t)(1+\gamma_A)^t F_{lt}(z^t)$ and $r_t = A_t(z^t)F_{kt}(z^t)$, the consumer's optimal plans have to satisfy the following first-order conditions for labor, capital, and bonds, respectively,

$$\left[1-\tau_{lt}(z^t)\right] A_t(z^t)(1+\gamma_A)^t F_{lt}(z^t) u_{ct}(z^t) = u_{ht}(z^t), \tag{8}$$

$$\sum_{z_{t+1}} Q_t(z^{t+1}|z^t) \frac{\left[1+\tau_{x,t+1}(z^{t+1})\right](1-\delta) + A_{t+1}(z^{t+1})F_{k,t+1}(z^{t+1})}{1+\tau_{xt}(z^t)} = 1, \tag{9}$$

$$\sum_{z_{t+1}} Q_t(z^{t+1}|z^t) \frac{1+\tau_{b,t+1}(z^{t+1})}{1+\tau_{bt}(z^t)} \frac{p_t(z^t)}{p_{t+1}(z^{t+1})} \left[1+R_t(z^t)\right] = 1, \tag{10}$$

where

$$Q_t(z^{t+1}|z^t) \equiv \beta^* \mu_t(z^{t+1}|z^t) \frac{u_{c,t+1}(z^{t+1})}{u_{ct}(z^t)} \tag{11}$$

is a stochastic discount factor, with $\beta^* \equiv (1+\gamma_n)^{-1}\beta$. Here, and throughout the paper, u_{ct}, u_{ht}, F_{kt}, and F_{lt} denote the derivatives of the utility and production functions with respect to their arguments.

The equilibrium conditions (4), (5), (6), (8), and (9) are exactly the same as those in the CKM economy. The equilibrium conditions (7) and (10) are new. As in the CKM economy, the labor wedge distorts the intratemporal optimality condition for labor (8), while the investment wedge distorts the intertemporal optimality condition for investment (9). In addition, the asset market wedge distorts the intertemporal optimality condition for bonds (10). Furthermore, as in the CKM economy, the efficiency wedge determines the amount of output produced by a given amount of inputs (see equation 5), while the government consumption wedge determines the amount of output available for consumption and investment (see equation 6). Finally, the monetary policy wedge captures all deviations of the nominal interest rate from the rate prescribed by the Taylor rule.

Notice that the economy is block recursive. First, equations (4), (5), (6), (8), and (9) determine the equilibrium allocations $(c_t(z^t), x_t(z^t), y_t(z^t), l_t(z^t), k_{t+1}(z^t))$. Then the monetary policy rule (7) and the optimality condition for bonds (10) together determine equilibrium $p_t(z^t)$ and $R_t(z^t)$. As a result of this recursive structure, the CKM wedges affect both real and nominal variables, whereas the asset market and monetary policy wedges affect only the two nominal variables.

The usefulness of this setup is its generality. As we show below, various frictions can be mapped into this prototype economy. The propagation of shocks due to underlying frictions in specific economic environments, including those with sticky nominal prices and wages, will manifest itself as movements in various wedges in the prototype economy. If these frictions manifest themselves as CKM wedges, the propagation of shocks through them affects all variables. If instead they are represented as asset market or monetary policy wedges, the frictions only affect the dynamics of the two nominal variables. Indeed, as a shock can be propagated through a host of frictions, thus generating movements in a number of wedges at the same time, no orthogonality conditions are placed on the joint stochastic process for the wedges.

2.3 The Effects of Frictions on Inflation and the Nominal Interest Rate through the Lenses of the Prototype Economy

It is instructive for the purposes of the current discussion to combine equations (9) and (10) and linearize the resulting equation in the neighborhood of the model's non-stochastic steady state

$$\text{(12)} \quad a_1 E_t \hat{\tau}_{x,t+1} - a_2 \hat{\tau}_{xt} + a_3 E_t \hat{A}_{t+1} + a_4 E_t \hat{l}_{t+1} - a_5 E_t \hat{k}_{t+1} = a_6 E_t \hat{\tau}_{b,t+1} - a_7 \hat{\tau}_{bt} + a_8 \hat{R}_t - a_9 E_t \hat{\pi}_{t+1}.$$

Similarly, we linearize the Taylor rule (7)

$$\hat{R}_t = (1-\rho_R)\omega_y \hat{y}_t + (1-\rho_R)\omega_\pi \hat{\pi}_t + \rho_R \hat{R}_{t-1} + \hat{\hat{R}}_t. \tag{13}$$

In these two equations variables with a 'hat' denote percentage deviations from steady state.[6] In equation (12), the coefficients are defined as follows: $a_1 \equiv (1-\delta)/(1+\tau_x)$, $a_2 \equiv [(1-\delta)(1+\tau_x) + AF_k]/(1+\tau_x)^2$, $a_3 \equiv F_k A/(1+\tau_x)$, $a_4 \equiv AF_{kl} l/(1+\tau_x)$, $a_5 \equiv -AF_{kk}k/(1+\tau_x)$, $a_6 \equiv (1+R)/[(1+\pi)(1+\tau_b)]$, $a_7 \equiv (1+R)/[(1+\pi)(1+\tau_b)]$, $a_8 = 1$, $a_9 \equiv (1+R)/(1+\pi)^2$. Notice that for $\tau_x, \tau_b > -1$, which we assume here (and which is the case in the actual application of the method to the postwar period), all these coefficients are positive.

Assuming, for illustration, that each wedge follows an AR(1) process (in the actual application the wedges will follow a joint VAR process), and combining equations (12) and (13), inflation in period t can be expressed as

$$\begin{aligned}\hat{\pi}_t = \frac{1}{(1-\rho_R)\omega_\pi}[&-(a_2 - a_1\rho_x)\hat{\tau}_{xt} + a_3\rho_A \hat{A}_t + a_4 E_t \hat{l}_{t+1} - a_5 E_t \hat{k}_{t+1} \\ &+(a_7 - a_6\rho_b)\hat{\tau}_{bt} - (1-\rho_R)\omega_y \hat{y}_t - \rho_R \hat{R}_{t-1} - \hat{\hat{R}}_t + a_9 E_t \hat{\pi}_{t+1}],\end{aligned} \tag{14}$$

where ρ_x, ρ_A, and ρ_b are the autocorrelation coefficients of the AR(1) processes for the investment, efficiency, and asset market wedges, respectively. It can be shown that the terms $(a_2 - a_1\rho_x)$ and $(a_7 - a_6\rho_b)$ are positive, for $\rho_x, \rho_b \in (0,1)$.

The difference equation (14) can be solved forward to obtain a particular solution for inflation. Notice, that by appearing in the difference equation, investment, efficiency, asset market, and monetary policy wedges have a *direct* effect on inflation. In addition, investment and efficiency wedges, together with labor and government consumption wedges, have also an *indirect* effect on inflation by affecting equilibrium output, labor, and capital.[7] In a

[6] In the case of the investment, asset market, and monetary policy wedges, the inflation rate, and the nominal interest rate, the variables are expressed as percentage point deviations from steady state.

[7] Although equation (14) is not a particular solution for inflation, we can still use it to discuss the *qualitative* effects of the wedges on inflation in an equilibrium that excludes explosive paths of inflation. In such an equilibrium, the term $\{a_9/[(1-\rho_R)\omega_\pi]\}^i E_t \hat{\pi}_{t+i}$ drops out from the particular solution as $i \to \infty$, while (since $a_9 > 0$) all the other variables have the same qualitative effects on inflation as in the difference

similar way, by substituting (14) into (13), we can characterize the nominal interest rate as a function of the wedges.

In all detailed models with frictions that can be mapped (in a sense made precise in the next section) into our prototype model with wedges, inflation satisfies equation (14). As a result, we can understand the qualitative effects of the underlying frictions on inflation and the nominal interest rate in such detailed models through the lenses of this equation.

As an example, consider an economy in which sticky prices are the only friction (e.g., Ireland, 2004). As the next section shows, sticky prices are equivalent to equal investment and labor wedges in the prototype economy. In a sticky-price economy a negative 'demand' shock (due to, for instance, a positive shock to the nominal interest rate in the Taylor rule) typically leads to a fall in both output and inflation; see Ireland (2004), Figure 1. The propagation of such a shock through sticky prices shows up in our prototype economy as an equal increase in labor and investment wedges. An increase in these two wedges has two effects on inflation. First, there is an indirect effect working through allocations: when the substitution effect in the choice between current and future leisure is sufficiently strong (as is usually the case with standard utility functions and parameter values), an increase in the labor wedge (a tax on labor) in the current period causes a decline in labor supply and a fall in output.[8] A fall in output working through equation (14) increases inflation. Second, there is a direct effect of price stickiness on inflation because τ_x appears in equation (14). In particular, since $a_2 - a_1\rho_x > 0$, an increase in $\hat{\tau}_{xt}$ reduces inflation. When ω_y is not "too large", the direct effect dominates and inflation in sticky-price models falls, following a negative demand shock.

3 Equivalence Results

This section provides three examples of mappings between detailed economies with frictions and the prototype economy. Throughout this section we retain the notation of Section 2. For

equation (14).

[8]In order to make this discussion easier, we abstract from the effects of the increase in the two wedges on $E_t\hat{l}_{t+1}$ and $E_t\hat{k}_{t+1}$, and thus on the expected marginal product of capital.

new variables, notation will be introduced as we go. For brevity, we abstract from population and technology growth. In each example, we assume that the underlying probability space of the detailed economy is the same as that of the prototype. This ensures consistency of expectations across the different economies. In addition, we assume that $u(.,.)$ and $F(.,.)$ are the same across the economies.

3.1 Sticky Prices

3.1.1 A Detailed Economy

Consider an economy with monopolistic competition in product markets and nominal price rigidities. There are two types of producers: identical final good producers and intermediate good producers indexed by $j \in [0,1]$. Final good producers take all prices as given and solve

$$\max_{y_t(z^t),\{y_t(j,z^t)\},j\in[0,1]} p_t(z^t)y_t(z^t) - \int p_t(j,z^t)y_t(j,z^t)dj$$

subject to a production function

$$y_t(z^t) = \left[\int y_t(j,z^t)^{\varepsilon_t(z^t)}dj\right]^{1/\varepsilon_t(z^t)}.$$

Here, $y_t(z^t)$ is aggregate output, $y_t(j,z^t)$ is input of an intermediate good j, $p_t(j,z^t)$ is its price, and $\varepsilon_t(z^t)$ is an exogenous shock that affects the degree of substitutability between intermediate goods (in Subsection 3.1.3 we also consider other shocks). The solution to this problem is characterized by a demand function for an intermediate good j

$$y_t(j,z^t) = \left(\frac{p_t(z^t)}{p_t(j,z^t)}\right)^{\frac{1}{1-\varepsilon_t(z^t)}} y_t(z^t) \qquad j \in [0,1] \tag{15}$$

and a price aggregator

$$p_t(z^t) = \left[\int p_t(j,z^t)^{\frac{\varepsilon_t(z^t)}{\varepsilon_t(z^t)-1}}dj\right]^{\frac{\varepsilon_t(z^t)-1}{\varepsilon_t(z^t)}}.$$

The problem of a producer of an intermediate good j can be split into two sub-problems.

First, for a given level of output $y_t(j, z^t)$ the producer solves

$$\min_{l_t(j,z^t), k_t(j,z^t)} w_t(z^t) l_t(j, z^t) + r_t(z^t) k_t(j, z^t)$$

subject to $F(k_t(j, z^t), l_t(j, z^t)) = y_t(j, z^t)$. Denoting the value function of this cost minimization problem by $\vartheta(y_t(j, z^t), w_t(z^t), r_t(z^t))$, in the second step the producer chooses its price $p_t(j, z^t)$ in order to maximize the present value of profits

$$\sum_{t=0}^{\infty} \sum_{z^t} Q_t(z^t) \left[\frac{p_t(j, z^t)}{p_t(z^t)} y_t(j, z^t) - \vartheta(y_t(j, z^t), w_t(z^t), r_t(z^t)) - \frac{\phi}{2} \left(\frac{p_t(j, z^t)}{\pi p_{t-1}(j, z^{t-1})} - 1 \right)^2 \right]$$

subject to the demand function (15). Here, $Q_t(z^t)$ is an appropriate discount factor and the last term in the square brackets is a price adjustment cost as in Rotemberg (1982). Given a symmetry across producers, all of them choose the same price, capital, and labor.

We use Rotemberg's specification of price stickiness for its ease of exposition. However, as we discuss in Appendix 2, the main result of this section also holds for Calvo-style price setting (Calvo, 1983).

The consumer maximizes the utility function (1), subject to the time constraint (2), the law of motion for capital (4), and the budget constraint

$$\begin{aligned} c_t(z^t) + x_t(z^t) + \frac{b_t(z^t)}{p_t(z^t)(1 + R_t(z^t))} &= w_t(z^t) l_t(z^t) + r_t(z^t) k_t(z^{t-1}) \\ &\quad + \frac{b_{t-1}(z^{t-1})}{p_t(z^t)} + T_t(z^t) + \psi_t(z^t), \end{aligned}$$

where $\psi_t(z^t)$ is profits from intermediate good producers.

The government follows the monetary policy feedback rule (7), but without the monetary policy wedge. Its budget constraint is $T_t(z^t) = 0.5\phi[p_t(z^t)/\pi p_{t-1}(z^{t-1}) - 1]^2$; i.e., we assume that the price adjustment cost acts like a tax that is rebated to the consumer.

An *equilibrium of this sticky-price economy* is a set of allocations $(c_t(z^t), x_t(z^t), y_t(z^t), l_t(z^t), k_{t+1}(z^t)), b_t(z^t))$ and a set of prices $(p_t(z^t), R_t(z^t), r_t(z^t), w_t(z^t))$ that satisfy: (i) a set of consumer's first-order conditions for labor, capital, and bonds, respectively,

$$u_{ct}(z^t)w_t(z^t) = u_{ht}(z^t), \tag{16}$$

$$\sum_{z_{t+1}} Q_t(z^{t+1}|z^t)\left[1 + r_{t+1}(z^{t+1}) - \delta\right] = 1, \tag{17}$$

$$\sum_{z_{t+1}} Q_t(z^{t+1}|z^t)\left[1 + R_t(z^t)\right]\frac{p_t(z^t)}{p_{t+1}(z^{t+1})} = 1, \tag{18}$$

where $Q_t(z^{t+1}|z^t)$ is given by (11); (ii) a set of optimality conditions for the cost minimization problem of intermediate good producers

$$\frac{F_{kt}(z^t)}{F_{lt}(z^t)} = \frac{r_t(z^t)}{w_t(z^t)}, \tag{19}$$

$$y_t(z^t) = F\left(k_t(z^{t-1}), l_t(z^t)\right); \tag{20}$$

(iii) a first-order condition for the profit maximization problem of intermediate good producers (the so-called 'New-Keynesian Phillips Curve')

$$\begin{aligned}&\Phi\left(p_t(z^t)/p_{t-1}(z^{t-1}), \eta_t(z^t), y_t(z^t), \varepsilon_t(z^t)\right)\\ &\quad + \sum_{z_{t+1}} Q_t(z^{t+1}|z^t)\Psi\left(p_{t+1}(z^{t+1})/p_t(z^t), y_{t+1}(z^{t+1}), \varepsilon_{t+1}(z^{t+1})\right) = 0,\end{aligned} \tag{21}$$

where $\eta_t(z^t) \equiv \partial\vartheta_t(z^t)/\partial y_t(z^t)$ is a marginal cost and $\Phi(.,.,.,.)$ and $\Psi(.,.,.)$ are smooth functions; (iv) the resource constraint $c_t(z^t) + x_t(z^t) = y_t(z^t)$; (v) the capital accumulation law (4); (vi) the monetary policy rule (7) without the monetary policy wedge; and (vii) the bond market clearing condition $b_t(z^t) = 0$.

Notice that in this economy equilibrium $r_t(z^t)$ and $w_t(z^t)$ are not equal to the marginal products of capital and labor.

3.1.2 Equivalence Result

Consider now a version of the prototype economy of Section 2 that has an investment wedge that acts like a tax on capital income rather than a tax on investment.[9] The equilibrium condition (9) then becomes

$$\sum_{z_{t+1}} Q_t(z^{t+1}|z^t)\left\{\left[1-\tau_{k,t+1}(z^{t+1})\right]A_{t+1}(z^{t+1})F_{k,t+1}(z^{t+1})+(1-\delta)\right\}=1, \tag{22}$$

where $\tau_{kt}(z^t)$ is the capital income tax and $Q_t(z^{t+1}|z^t)$ is given as before by (11).

Proposition 1: *Consider equilibrium allocations of the economy with sticky prices* $(c_t^*(z^t), x_t^*(z^t), y_t^*(z^t), l_t^*(z^t), k_{t+1}^*(z^t))$ *and prices* $(p_t^*(z^t), R_t^*(z^t), r_t^*(z^t), w_t^*(z^t))$ *that support these allocations. Let the wedges in the prototype economy satisfy:* $A_t(z^t) = 1$, $\tau_{bt}(z^t) = g_t(z^t) = \widetilde{R}_t(z^t) = 0$, *and*

$$\tau_{kt}(z^t) = \tau_{lt}(z^t) = 1 - \frac{r_t^*(z^t)}{F_{kt}^*(z^t)} \tag{23}$$

for all z^t, *where* $F_{kt}^*(z^t)$ *is evaluated at the equilibrium of the sticky-price economy. Then* $(c_t^*(z^t), x_t^*(z^t), y_t^*(z^t), l_t^*(z^t), k_{t+1}^*(z^t))$ *and* $(p_t^*(z^t), R_t^*(z^t))$ *are also equilibrium allocations and prices of the prototype economy.*

Proof See Appendix 1.

This proposition shows that sticky prices act like equal labor and investment wedges. Intuitively, imperfect competition in *product* markets leads to mark-ups that create a distortion in *factor* markets as marginal products of capital and labor are no longer set equal to factor prices. These distortions can be replicated in the prototype economy by choosing τ_{kt} and τ_{lt} according to (23). Based on this result, the effects of sticky prices on inflation and the nominal interest rate can be understood through the pricing function (14) along the lines discussed in Subsection 2.3.

[9] Both types of taxes distort the optimality condition for capital accumulation, but the proof is more straightforward in the case of a capital income tax.

3.1.3 Alternative Specifications of the Detailed Economy

We assumed that the price adjustment cost in the sticky-price economy acts like an implicit tax on firms that is rebated in a lump-sum way to the consumer. If instead we assume that the cost is a pure resource loss, this loss shows up in the prototype economy as a government consumption wedge.

Notice also that various extensions of the price-setting behavior, such as backward indexation, that *only* show up in the New-Keynesian Phillips curve (21) will not change Proposition 1. This is because they do not generate any additional distortions above and beyond preventing r_t to be equal to F_{kt}.

Often, Calvo-style price setting is used instead of Rotemberg's cost of adjustment. In such a case, aggregation issues lead to an efficiency wedge, in addition to the wedges given by equation (23). This is discussed in more detail in Appendix 2.

Finally, the assumption that the exogenous shocks causing fluctuations in the detailed economy are shocks to the elasticity of substitution between intermediate goods is not crucial for equation (23) to hold. If, for example, we instead assume that the source of impulses are monetary policy shocks $\xi_t(z^t)$ in the Taylor rule, we only change Proposition 1 by setting $\tilde{R}_t(z^t)$ equal to $\xi_t(z^t)$.[10]

To summarize, regardless of whether we use Rotemberg- or Calvo-style price setting, or the sources of impulses, a common feature of sticky prices is that the propagation of shocks through this friction manifests itself as equal movements in investment and labor wedges.

3.2 Limited Participation in Asset Markets

3.2.1 A Detailed Economy

Consider now a simple example of a limited participation economy due to Christiano and Eichenbaum (1992), which builds on Lucas (1990) and Fuerst (1992). In their economy, some agents are excluded from the money market at the time of a central bank's money injection.

At the beginning of a period consumers have a stock of nominal wealth, which, before

[10] A more general interpretation of the monetary policy wedge is provided in Subsection 3.4.

observing z_t, they split between deposits with financial intermediaries and 'cash'. After this, they cannot change the composition of their nominal wealth within the period. The amount of their nominal consumption spending within the period is constrained by the sum of cash, labor income, and dividends from firms. Consumers choose plans for consumption, leisure, labor, deposits $q_t(z^{t-1})$, and nominal wealth in the next period $m_t(z^t)$ in order to maximize the utility function (1) subject to the time constraint (2), a 'transaction' constraint

$$p_t(z^t)c_t(z^t) = \left[m_{t-1}(z^{t-1}) - q_t(z^{t-1})\right] + p_t(z^t)w_t(z^t)l_t(z^t) + \varphi_t(z^t), \tag{24}$$

where $\varphi_t(z^t)$ is dividends from firms, and a law of motion for nominal wealth

$$m_t(z^t) = \left[1 + R_t(z^t)\right] q_t(z^{t-1}) + \psi_t(z^t), \tag{25}$$

where $\psi_t(z^t)$ is profits from financial intermediaries.[11]

The intermediaries take deposits from consumers and make loans to firms. They operate in a perfectly competitive market so that the interest rate on deposits is equal to the interest rate on loans.

Firms have access to an aggregate production function $y_t(z^t) = F\left(k_t(z^{t-1}), l_t(z^t)\right)$. They need to finance a fraction ϕ_t of their wage bill $w_t(z^t)l_t(z^t)$ through loans from financial intermediaries, which they repay at the end of the period. Using the consumers' stochastic discount factor, firms maximize a discounted sum of per-period dividends $F(k_t(z^{t-1}), l_t(z^t)) + (1-\delta)k_t(z^{t-1}) - k_{t+1}(z^t) - [1 + \phi_t(z^t)R_t(z^t)]w_t(z^t)l_t(z^t)$ by choosing $k_{t+1}(z^t)$ and $l_t(z^t)$. Notice that the marginal cost of labor to the firms is $[1 + \phi_t(z^t)R_t(z^t)]w_t(z^t)$.

The central bank sets the nominal interest rate according to the feedback rule

$$R_t(z^t) = (1-\rho_R)\left[R + \omega_y\left(\ln y_t(z^t) - \ln y\right) + \omega_\pi\left(\pi_t(z^t) - \pi\right)\right] + \rho_R R_{t-1}(z^{t-1}) + \xi_t(z^t), \tag{26}$$

where $\xi_t(z^t)$ is a monetary policy shock. The central bank implements $R_t(z^t)$ through

[11] Constraint (24) holds with equality when inflation between periods t and $t+1$ is expected to be positive, as firms, acting on behalf of consumers, can pay dividends in an amount that is just enough to satisfy the consumers' consumption, given their labor income and cash balances.

money transfers η_t to the financial intermediaries. Notice that as R_t is a function of z^t, this happens only after consumers made their deposits.

Total funds at the disposal of the intermediaries are thus $q_t(z^{t-1}) + \eta_t(z^t)$ and they are lent to firms at the rate $R_t(z^t)$. Clearing the money market therefore requires

$$q_t(z^{t-1}) + \eta_t(z^t) = \phi_t(z^t)p_t(z^t)w_t(z^t)l_t(z^t) \tag{27}$$

and the money injection $\eta_t(z^t)$ is such that this condition holds at $R_t(z^t)$. Profits of the intermediaries $\psi_t(z^t)$ are given by the interest they earn on the extra money balances injected by the central bank, i.e., $\psi_t(z^t) = R_t(z^t)\eta_t(z^t)$.

An *equilibrium of this economy with limited participation* is a set of allocations ($c_t(z^t)$, $x_t(z^t), y_t(z^t), l_t(z^t), k_{t+1}(z^t), m_t(z^t), q_t(z^{t-1}), \eta_t(z^t)$) and a set of prices ($p_t(z^t), R_t(z^t), w_t(z^t)$) that satisfy: (i) the consumers' first-order conditions for deposits and labor, respectively,

$$\sum_{z_t} \mu_{t-1}(z^t|z^{t-1})\frac{u_{ct}(z^t)}{p_t(z^t)} = \beta \sum_{z_t} \mu_{t-1}(z^t|z^{t-1})\frac{u_{c,t+1}(z^{t+1})}{p_{t+1}(z^{t+1})}(1 + R_t(z^t)), \tag{28}$$

$$u_{ht}(z^t)\frac{1 + \phi_t(z^t)R_t(z^t)}{F_{lt}(z^t)} = u_{ct}(z^t); \tag{29}$$

(ii) the firms' first-order condition for capital

$$u_{ct}(z^t) = \beta \sum_{z_{t+1}} \mu_t(z^{t+1}|z^t)u_{c,t+1}(z^{t+1})\left[1 + F_{k,t+1}(z^{t+1}) - \delta\right], \tag{30}$$

and labor $w_t(z^t) = F_{lt}(z^t)/[1 + \phi_t(z^t)R_t(z^t)]$; (iii) the transaction constraint (24); (iv) the law of motion for nominal wealth (25); (v) the money market clearing condition (27); (vi) the aggregate resource constraint $c_t(z^t)+x_t(z^t) = y_t(z^t)$, where $y_t(z^t) = F\left(k_t(z^{t-1}), l_t(z^t)\right)$; (vii) the capital accumulation law (4); and (viii) the monetary policy rule (26).

3.2.2 Equivalence Result

PROPOSITION 2: *Consider equilibrium allocations of the economy with limited participation* $(c_t^*(z^t), x_t^*(z^t), y_t^*(z^t), l_t^*(z^t), k_{t+1}^*(z^t), m_t^*(z^t), q_t^*(z^{t-1}))$ *and prices* $(p_t^*(z^t), R_t^*(z^t), w_t^*(z^t))$ *that support these allocations. Let the wedges in the prototype economy satisfy:* $A_t(z^t) = 1$, $\tau_{xt}(z^t) = g_t(z^t) = 0$, $\widetilde{R}_t(z^t) = \xi_t(z^t)$, *and*

$$
\begin{aligned}
(31) \qquad \left[1 - \tau_{lt}(z^t)\right] &= \frac{1}{1 + \phi_t(z^t) R_t^*(z^t)}, \\
(32) \qquad \left[1 + \tau_{bt}(z^t)\right] \frac{u_{ct}^*(z^t)}{p_t^*(z^t)} &= \sum_{z_{t+1}} \mu_t(z^{t+1}|z^t) \frac{\beta u_{c,t+1}^*(z^{t+1})}{p_{t+1}^*(z^{t+1})} \left[1 + \tau_{b,t+1}(z^{t+1})\right] \left[1 + R_t^*(z^t)\right]
\end{aligned}
$$

for all z^t, *where* u_{ct}^* *is evaluated at the equilibrium of the detailed economy. Then* $(c_t^*(z^t), x_t^*(z^t), y_t^*(z^t), l_t^*(z^t), k_{t+1}^*(z^t))$ *and* $(p_t^*(z^t), R_t^*(z^t))$ *are also equilibrium allocations and prices of the prototype economy.*

PROOF: See Appendix 1.

Consider now a special case of Proposition 2. Suppose that the fraction of the wage bill financed through loans ϕ_t fluctuates in response to changes in the interest rate so as to leave the effective wage rate $(1+\phi_t R_t)w_t$ unchanged. In this case, monetary policy shocks lead to movements in τ_{bt} but not τ_{lt}. The main point of Proposition 2 is that limited participation in the money market acts like a tax on nominal bonds that distorts the standard equilibrium condition for bonds. Fuerst (1992) labels this distortion a 'liquidity effect'. Liquidity effects thus manifest themselves in the prototype economy as fluctuations in the asset market wedge.

3.3 Sticky Wages

3.3.1 A Detailed Economy

As a final example, consider a sticky-wage economy, similar to the one studied by Chari et al. (2007a). In this economy nominal wages are set before z_t is realized.

The economy is populated by a continuum of infinitely lived consumers differentiated by a labor type $j \in [0, 1]$. Consumers of type j are organized in a labor union j. A representative producer has access to an aggregate production function $y_t(z^t) = F(k_t(z^{t-1}), l_t(z^t))$, where

$$l_t(z^t) = \left[\int l_t(j, z^t)^{\varepsilon_t(z^t)} dj\right]^{1/\varepsilon_t(z^t)} \tag{33}$$

is a labor aggregate and $\varepsilon_t(z^t)$ is a shock to the degree of substitutability between labor types (for reasons similar to those in Subsection 3.1.3, the main result of this section does not depend on the specific source of impulses considered).

The representative producer's problem can be described in two steps. First, for a given $l_t(z^t)$, the producer solves

$$\min_{\{l_t(j,z^t)\}_{j\in[0,1]}} \int W_t(j, z^{t-1}) l_t(j, z^t) dj$$

subject to (33), where $W_t(j, z^{t-1})$ is a nominal wage rate for labor type j, set by union j before the realization of z_t. The solution to this problem gives the producer's demand function for each labor type

$$l_t(j, z^t) = \left[\frac{W_t(j, z^{t-1})}{W_t(z^{t-1})}\right]^{\frac{1}{\varepsilon_t(z^t)-1}} l_t(z^t), \tag{34}$$

where

$$W_t(z^{t-1}) = \left[\int W_t(j, z^{t-1})^{\frac{\varepsilon_t(z^t)}{\varepsilon_t(z^t)-1}} dj\right]^{\frac{\varepsilon_t(z^t)-1}{\varepsilon_t(z^t)}}$$

is the aggregate nominal wage rate. In the second step, the producer chooses $k_t(z^{t-1})$ and $l_t(z^t)$ to maximize profits $F\left(k_t(z^{t-1}), l_t(z^t)\right) - r_t(z^t)k_t(z^{t-1}) - [W_t(z^{t-1})/p_t(z^t)]l_t(z^t)$ by setting the marginal products of capital and labor equal to factor prices.

When setting the wage rate, the union agrees to supply in period t whatever amount of labor is demanded at the real wage rate $W_t(j, z^{t-1})/p_t(z^t)$. The preferences of consumer j are characterized by the utility function (1), where c and h are indexed by j. The consumer/union's problem is to choose plans for $c_t(j, z^t)$, $x_t(j, z^t)$, $k_{t+1}(j, z^t)$, $l_t(j, z^t)$, $b_t(j, z^t)$, and $W_{t+1}(j, z^t)$ to maximize the utility function (1), subject to the labor demand function

(34), the budget constraint (3), and the capital accumulation law (4), where the appropriate quantities are indexed by j. Assuming that k_0 and b_0 are the same for all types, the solution to this problem is symmetric across all j's. The government sets the nominal interest rate according to the policy rule (7) with no monetary policy wedge.

An *equilibrium of this economy with sticky nominal wages* is a set of allocations ($c_t(z^t)$, $x_t(z^t)$,$y_t(z^t)$,$l_t(z^t)$,$k_{t+1}(z^t)$, $b_t(z^t)$) and a set of prices ($p_t(z^t)$,$R_t(z^t)$,$r_t(z^t)$,$W_{t+1}(z^t)$) that satisfy: (i) the consumer's first-order conditions for wages, capital, and bonds, respectively,

$$\frac{\sum_{z_{t+1}} \mu_t(z^{t+1}|z^t) u_{h,t+1}(z^{t+1}) l_{t+1}(z^{t+1})}{\sum_{z_{t+1}} \mu_t(z^{t+1}|z^t)\varepsilon_{t+1}(z^{t+1}) \left[\frac{l_{t+1}(z^{t+1})}{p_{t+1}(z^{t+1})} u_{c,t+1}(z^{t+1})\right]} = W_{t+1}(z^t),$$

$$\sum_{z_{t+1}} Q_t(z^{t+1}|z^t) \left[1 + r_{t+1}(z^{t+1}) - \delta\right] = 1,$$

$$\sum_{z_{t+1}} Q_t(z^{t+1}|z^t)(1 + R_t(z^t))\frac{p_t(z^t)}{p_{t+1}(z^{t+1})} = 1,$$

where $Q_t(z^{t+1}|z^t)$ is given by (11); (ii) the producer's first-order conditions $r_t(z^t) = F_{kt}(z^t)$ and $W_t(z^{t-1})/p_t(z^t) = F_{lt}(z^t)$; (iii) the resource constraint $c_t(z^t) + x_t(z^t) = y_t(z^t)$, where $y_t(z^t) = F(k_t(z^{t-1}), l_t(z^t))$; (iv) the capital accumulation law (4); (v) the monetary policy rule; and (vi) the bond market clearing condition $b_t(z^t) = 0$.

3.3.2 Equivalence Result

CKM show that when

$$\tau_{lt}(z^t) = 1 - \frac{u^*_{ht}(z^t)}{u^*_{ct}(z^t) F^*_{lt}(z^t)}, \tag{35}$$

their real prototype economy is equivalent, in terms of allocations, to the sticky-wage economy just described (see their Proposition 2). Condition (35) also implies equivalence between the detailed economy and our prototype economy. Notice that setting $\tau_{bt}(z^t)$ and $\tilde{R}_t(z^t)$ equal to zero ensures that the optimality condition for bonds and the monetary policy rule in our prototype economy are the same as their counterparts in the sticky-wage economy. Since the other equilibrium conditions in our prototype economy are the same as those in CKM's prototype economy, by setting τ_{lt} according to (35), our prototype economy reproduces the equilibrium allocations, as well as p_t^* and R_t^*, of the sticky-wage economy. Thus, unlike sticky prices, sticky wages only show up in our prototype economy as labor wedges. Notice also that, for reasons similar to those in Section 3.1.3, various extensions of the simple wage-setting behavior assumed here that only affect the optimality condition for labor will not change this result.

A consequence of this result is that, viewed through the lenses of the pricing function (14), the propagation of shocks through sticky wages has only an indirect effect on inflation and the nominal interest rate by affecting equilibrium allocations. This is because the labor wedge does not enter directly the pricing function.

3.4 The Monetary Policy Wedge

The monetary policy wedge represents all aspects of monetary policy above and beyond the responses of the monetary authority to output and inflation as summarized by a standard Taylor rule. As an example, consider a Taylor-type rule with fluctuations in an inflation target, such as that considered by Gavin, Kydland and Pakko (2007). Their monetary policy rule has the form

$$R_t(z^t) = R + \omega_y \left(\ln y_t(z^t) - \ln y\right) + \omega_\pi \left(\pi_t(z^t) - \bar{\pi}_t(z^t)\right) + \rho_R R_{t-1}(z^{t-1}), \tag{36}$$

where $\bar{\pi}_t(z^t)$ is an inflation target, which can change randomly over time, perhaps due to appointments of central bankers with different stance on inflation (e.g., Burns vs. Volcker). This policy rule is equivalent to the prototype rule (7), where the inflation target is constant and the monetary policy wedge is given by $\widetilde{R}_t(z^t) = -\omega_\pi(\bar{\pi}_t(z^t) - \pi)$. In a similar fashion, responses of the monetary authority to variables other than inflation and output also show up as fluctuations in the monetary policy wedge.

4 The Realized Wedges

Our procedure for uncovering the realized values of the wedges from the data follows closely that of CKM. In particular, we assume that the events are governed by a stationary first-order Markov process $\mu(z^{t+1}|z^t)$ and that there is a one-to-one and onto mapping between the events and the wedges. The latter assumption implies that the wedges uniquely identify the underlying events. We can therefore replace in the prototype economy the probability space for the events with a probability space for the wedges without altering the agent's expectations about future events. Since the stochastic process for the events is Markov, the stochastic process for the wedges is also Markov. In particular, following CKM we assume that the process has a VAR(1) form

$$\omega_{t+1} = P_0 + P\omega_t + \varepsilon_{t+1}, \tag{37}$$

where $\omega_t = (\log A_t, \tau_{lt}, \tau_{xt}, \log g_t, \tau_{bt}, \widetilde{R}_t)$ and the shock ε_{t+1} is iid over time and distributed normally with mean zero and a covariance matrix $V = BB'$. There are no restrictions imposed on this stochastic process besides stationarity. In particular, the off-diagonal elements of P and V are allowed to be non-zero, in line with our discussion in Section 2.2.

Uncovering the realized wedges from the data involves three steps. First, we choose functional forms for the utility and production functions, and calibrate their parameter values, as well as the parameter values of the monetary policy rule. Second, we estimate the parameters of the stochastic process (37). And third, we use the equilibrium decision rules and pricing functions of the prototype economy to back out the realized wedges from the data.

In the second and third steps we need to compute the equilibrium decision rules and pricing functions of the prototype economy. Since the state space is large (there are nine state variables in the model, ω_t ,p_{t-1}, R_{t-1}, and k_t), the prototype economy is approximated by a linear-quadratic economy and the equilibrium is computed using the method described by Hansen and Prescott (1995).[12] The outcome is a set of decision rules and pricing func-

[12]Before computing the equilibrium, the model is transformed so that the price level is stationary.

tions that express the deviations of $(\log y_t, \log l_t, \log x_t, \log c_t, \log p_t, R_t)$ from steady state as linear functions of the deviations of the state vector $(\omega_t, \log p_{t-1}, R_{t-1}, \log k_t)$ from steady state. This solution method is also used in the experiments in Section 5. The rest of this section describes the three steps in more detail and characterizes the cyclical behavior of the realized wedges.

4.1 Calibration

We use the same functional forms and parameter values for the utility and production functions as CKM. Namely, the utility function has the functional form $u(.,.) = \lambda \log c_t + (1-\lambda) \log h_t$, while the production function has the form $F(.,.) = k_t^{\alpha}((1+\gamma_A)^t l_t)^{1-\alpha}$. These functional forms are standard in the business cycle literature. The population growth rate γ_n is set equal to 0.0037, the discount rate β to 0.99, the weight on consumption λ to 0.31, the technology growth rate γ_A to 0.004, the depreciation rate δ to 0.0118, and the capital share of output α to 0.35.

The parameters of the monetary policy rule are set equal to fairly standard values in the literature (see Woodford, 2003, Chapter 1, for a survey). The weight on output ω_y is set equal to 0.125 (which corresponds to 0.5 when inflation and the nominal interest rate are expressed at annualized rates), the weight on inflation ω_π to 1.5, and the smoothing coefficient ρ_R to 0.75. Nevertheless, we also study the sensitivity of our findings to alternative parameterizations of the monetary policy rule.

4.2 Estimation of the Stochastic Process

As in CKM, the parameters P_0, P, and B of the stochastic process for the wedges are estimated using a maximum likelihood method. The number of the parameters that need to be estimated is 61. The search for the maximum of the likelihood function is implemented using simulated annealing (see Goffe, Ferrier and Rogers, 1994) in order to thoroughly explore the surface of the objective function. The resulting estimates are contained in Table 1. The likelihood function is based on a state-space representation consisting of the stochastic process for the wedges (37) and the linear approximations of the equilibrium decision rules pricing functions. The estimation is carried out for the period 1959.Q1-2004.Q4 using quarterly data on output (the sum of GDP and imputed services from consumer durables), investment (which includes consumer durables), consumption (the sum of nondurables, services, and imputed flow of services from durables), hours from the Establishment Survey, the GDP deflator, and the yield on 3-month Treasury bills.[13] Capital is computed recursively using the law of motion (4), data on investment, and an initial capital stock.

4.3 Uncovering the Realized Wedges

The estimated stochastic process for the wedges is then used to compute the equilibrium of the model and to uncover the realized wedges from the data. We denote the vector of the realized wedges by $\omega_t^d = (\log A_t^d, \tau_{lt}^d, \tau_{xt}^d, \log g_t^d, \tau_{bt}^d, \widetilde{R}_t^d)$. Notice that $\log g_t^d$ is observed directly from the data as the sum of government consumption and net exports. The realized values of the remaining wedges are obtained from the linear approximations to the following equilibrium decision rules and pricing functions: $\log y_t = y(\omega_t, \log p_{t-1}, R_{t-1}, \log k_t)$, $\log x_t = x(\omega_t, \log p_{t-1}, R_{t-1}, \log k_t)$, $\log l_t = l(\omega_t, \log p_{t-1}, R_{t-1}, \log k_t)$, $\log p_t = p(\omega_t, \log p_{t-1}, R_{t-1}, \log k_t)$, and $R_t = R(\omega_t, \log p_{t-1}, R_{t-1}, \log k_t)$. These approximations constitute a linear system of five equations that in each period can be solved for $(\log A_t^d, \tau_{lt}^d, \tau_{xt}^d, \tau_{bt}^d, \widetilde{R}_t^d)$, using data on $(y_t, x_t, l_t, g_t, p_t, p_{t-1}, R_t, R_{t-1}, k_t)$. Using these equilibrium decision rules and pricing functions to uncover the wedges from the data means that $\log A_t^d$ is essentially obtained from the production function (5), and thus is the standard Solow residual, τ_{lt}^d, τ_{xt}^d, and τ_{bt}^d from the first-order conditions (8), (9), and (10), respectively, and $\widetilde{R}_t^d$ from the monetary policy rule (7).[14] As a result of this procedure, putting all six wedges back into the model at the same time exactly reproduces the data.

[13] The data on output, investment, consumption, and hours are in per capita terms. In addition, a common trend of 0.4% (at a quarterly rate) is removed from per-capita output, investment, and consumption, and a trend of 0.91% (the average postwar quarterly inflation rate) is removed from the price level.

[14] Notice that only τ_{xt}^d and τ_{bt}^d depend on expectations, and thus on the stochastic process for the wedges. Since the model is block recursive, as discussed in Section 2.2, the two new wedges have no direct effect on the CKM wedges. However, as the investment wedge depends on expectations, to the extent that the two new wedges are informative about future values of the investment wedge, the realized values of the investment wedge are somewhat different from those obtained by CKM. This, however, does not change CKM's main result that efficiency and labor wedges account for most of the movements in real quantities. Feeding the wedges back into our model, we can broadly reproduce CKM's results for the postwar period.

4.4 Business Cycle Properties of the Realized Wedges

Tables 2 and 3 provide key business cycle statistics for the realized wedges. In order to focus on their business cycle movements, the wedges are filtered with the HP-filter (see Hodrick and Prescott, 1997), which filters out low frequency movements in these variables. Table 2 shows the standard deviations of the wedges and their correlations with HP-filtered output (in logs) at various leads and lags. In order to have a point of reference for assessing their volatility, the standard deviations of the wedges are expressed as ratios to the standard deviation of HP-filtered output, which is 1.58.

Focusing on the CKM wedges first, we see that the efficiency and investment wedges are, respectively, only 63% and 50% as volatile as output, while the government consumption wedge is 1.5 times as volatile as output, and the labor wedge is about as volatile as output. In addition, both A_t and τ_{xt} are procyclical, while τ_{lt} and g_t are countercyclical. Notice also that the efficiency wedge is more strongly correlated with output at leads than at lags. These findings are broadly in line with those of CKM, except that τ_x is procyclical here, whereas CKM find that it is somewhat countercyclical. This difference is due to the reasons discussed in footnote 14. As CKM provide an interpretation of these findings, we focus on the behavior of the two new wedges.

Clearly, the asset market wedge is the most volatile, moving 2.59 times as much as output. It is also highly procyclical, having a contemporaneous correlation with output of 0.82. Notice also that it is more strongly correlated with past output than with future output.

The high volatility of the asset market wedge reflects the well-known failure of Euler equations with power utility functions to match asset prices (e.g., Hansen and Singleton, 1983). Canzoneri et al. (2007) show that pricing kernels based on other utility functions, including those with habits and Epstein-Zin preferences, also fail to fit the time series for short-term risk-free rate. The strong positive correlation of the asset market wedge with output documented here, however, reveals *systematic* failure of the standard Euler equation for bonds to account for the movements in the risk-free rate over the business cycle.[15]

[15]Canzoneri et al. (2007) find that the errors in their Euler equations are correlated with the stance of

The reason for the strong procyclical movement in the asset market wedge can be understood by inspecting the equilibrium condition (10). When log-linearized, for a CRRA utility function (separable in leisure) the equation becomes

$$(38) \qquad -\log\beta + \gamma E_t \Delta c_{t+1} \approx -(1-\rho_b)\tau_{bt} + R_t - E_t \pi_{t+1},$$

where γ is the inverse of the intertemporal elasticity of substitution, Δc_{t+1} is a consumption growth rate, and where, as in Section 2.3, we assume (for easy exposition) that the asset market wedge follows an AR(1) process with an autocorrelation coefficient $\rho_b \in [0, 1)$.

Figure 1 plots the movements in the level of consumption, inflation, the nominal interest rate, and the asset market wedge during the 1960 and 1990 recessions, which we use as examples to facilitate our discussion. As we can see, a persistent fall in consumption during these downturns implies a negative Δc_{t+1} for the first couple of periods of the recessions. In addition, R_t falls generally by more than π_{t+1}, leading to a fall in the real interest rate. These movements are representative of other postwar recessions, with the exception of the 1982 recession during which the real interest rate increased.[16] The fall in the real rate, however, is larger than the fall in $\gamma\Delta c_{t+1}$ for reasonable values of γ (we use $\gamma = 1$ but this is true also for other plausible values of γ). Essentially, demand for the short-term risk-free bond is greater in downturns than can be justified by standard preferences, driving the real return on these bonds below that predicted by a standard Euler equation. In order to compensate for this 'excessive' fall in the real return, the asset market wedge has to decline, if equation (38) is to hold.

Turning to the monetary policy wedge, as we can see in Table 2, in contrast to the asset market wedge, it is little volatile and only slightly positively correlated with future output, and slightly negatively correlated with past output. This reflects the fact that the Taylor rule captures the movements in the nominal interest rate, for given movements in output

monetary policy.

[16]Although Figure 1 plots the realized values of Δc_{t+1} and π_{t+1}, rather than the agents' expectations of these variables in our model, the agents in the model expect such movements. This is because in most postwar recessions Δc_{t+1} declines during the downturn, and R_t declines by more than π_{t+1}. As the stochastic process for the wedges is estimated on postwar data, agents have expectations about Δc_{t+1} and π_{t+1} that are consistent with the average behavior of these variables in the postwar period.

and inflation, relatively well.

Table 3 displays contemporaneous correlations of HP-filtered wedges with each other. We see that in most cases the wedges are mutually correlated. As the next section shows, the efficiency and asset market wedges are crucial for generating the observed dynamics of the nominal interest rate and inflation over the business cycle. Here we just want to point out that these two wedges are strongly positively correlated with each other, having a coefficient of correlation of 0.53.

5 Accounting for the Nominal Business Cycle

In this section we decompose the observed movements in output, inflation, and the nominal interest rate into movements due to each wedge. Our goal is to use business cycle accounting to shed light on what types of frictions are behind the observed comovement between output and inflation, and between output and the nominal interest rate, over the business cycle.

5.1 The Procedure

The decomposition follows the procedure proposed by CKM.[17] First, we describe the general idea of the decomposition and then explain how we implement it.

Let us assume for now that we observe the events z_t and know the mapping between the events and the wedges. Suppose that we are interested in the movements in the data due to, for example, the distortionary effects of the labor wedge alone. In this case, we feed the realized values of the events into a version of the prototype economy in which only the labor wedge responds to the events, while all other wedges are held constant; i.e., the vector of wedges is $(A, \tau_{lt}(z^t), \tau_x, g, \tau_b, \widetilde{R})$. This procedure isolates the movements in the endogenous variables of the model (and thus also in the data, as movements in all six wedges exactly reproduce the data) due to the distortionary effects of the labor wedge alone. Notice that it also preserves the logic of our equivalence results. Specifically, recall that the detailed economies of Section 3 differ from each other only in terms of their distortions (which show

[17]See CKM for details and a discussion on how this decomposition differs from decompositions carried out with structural VARs.

up as movements in different wedges in the prototype), not in terms of the underlying probability space $(Z^t, \mathcal{Z}^t, \mu_t(z^t))$ or the realization of the events.

In practice, however, we neither observe the events nor know the mapping between events and wedges. Therefore, as in Section 4 in the actual implementation of this procedure, we replace the probability space for the events with the probability space for the wedges implied by the stochastic process (37). This, under our assumption that the wedges uniquely identify the events, preserves the agents expectations about future events. We then solve a version of the prototype economy in which the agents face the stochastic process (37), with the parameter values reported in Table 1, but in which, in the budget and resource constraints, as well as in the monetary policy rule, all wedges except the labor wedge are kept constant at their steady-state values. Thus, only the labor wedge distorts the equilibrium. Let $y^L(\omega_t, p_{t-1}, R_{t-1}, k_t)$, $x^L(\omega_t, p_{t-1}, R_{t-1}, k_t)$, $c^L(\omega_t, p_{t-1}, R_{t-1}, k_t)$, $l^L(\omega_t, p_{t-1}, R_{t-1}, k_t)$, $p^L(\omega_t, p_{t-1}, R_{t-1}, k_t)$, and $R^L(\omega_t, p_{t-1}, R_{t-1}, k_t)$ be the equilibrium decision rules and pricing functions of this modified economy. Starting from p_{-1}, R_{-1}, and k_0 for 1959.Q1, these decision rules and pricing functions are used, together with ω_t^d (the vector of realized wedges obtained using the procedure of Section 4.3), to compute the labor wedge component of the data. Notice that although the other wedges do not distort the equilibrium, they are used to forecast, according to the stochastic process (37), the future values of the labor wedge.

In a similar way we also obtain the components of the data due to the fluctuations in the other wedges, and in their various combinations. Indeed, when we carry out this procedure for all six wedges at the same time, we exactly reproduce the data.

5.2 The Anomalies

The literature has identified two important anomalies in the nominal business cycle. One anomaly concerns the empirical lead-lag relationship between output and the short-term nominal interest rate, pointed out by, among others, King and Watson (1996). They note that the nominal interest rate in the United States is an 'inverted leading indicator', meaning that it is strongly negatively correlated with future output. Accounting this feature of the business cycle within a structural model is important for understanding the mechanism by which asset prices contain information about future economic activity and the effects of monetary policy on real activity (Backus et al., 2007, represents a recent attempt to account for this anomaly).

The second anomaly concerns the empirical lead-lag relationship between output and inflation. As highlighted by, among others, Fuhrer and Moore (1995) and Galí and Gertler (1999), in the U.S. data inflation is positively correlated with past output. Accounting for this feature of the data is crucial for understanding the causes of inflation persistence and the transmission mechanism of monetary policy. Fuhrer and Moore (1995), and recently Wang and Wen (2007), note that models with nominal rigidities have a difficulty accounting for such dynamics. Henriksen et al. (2008) show that a business cycle model in which the central bank follows a Taylor rule, and in which the only impulses are TFP shocks, also cannot account for such a feature of the data.[18]

Table 4 contains the correlations between output and the nominal interest rate, and between output and inflation, at various leads and lags for the postwar period in the United States (all three variables are filtered with the HP-filter). As mentioned above, in the case of the nominal interest rate, the focus of the literature has usually been on the negative lead, whereas in the case of inflation it has been on the positive lag. However, in Table 4 we can see that the two nominal variables have broadly similar dynamics. Both are negatively correlated with future output and positively correlated with past output. The same dynamic pattern is also observed when the data are filtered with Christiano and Fitzgerald (2003) filter, or when inflation and the nominal interest rate are demeaned (and output is detrended). Furthermore, Wang and Wen (2007) and Henriksen et al. (2008) document that this empirical regularity also characterizes the behavior of inflation and short-term nominal interest rates in other major industrialized economies.[19]

[18] Smets and Wouters (2007) show that a particular combination of shocks in their model can generate the observed lead-lag pattern of inflation.

[19] The lead-lag pattern in these two variables is also present both before and after the 1979 monetary policy change, although the actual correlations somewhat changed (see, for instance, Gavin and Kydland, 2000).

5.3 Results of the Decomposition

The results of our decomposition are presented in Figures 2-5. Figures 2 and 3 plot the correlations of HP-filtered output in period t with HP-filtered nominal interest rate in period $t + j$, where $j = \{-5, ..., 0, ..., 5\}$, for various combinations of the wedges in the model (remember that in each case we need to recompute the equilibrium of the model). Figures 4 and 5 then do the same for HP-filtered inflation.

We first focus on the nominal interest rate. We start by leaving out one wedge at a time. As feeding all six wedges back into the model at the same time exactly reproduces the data, this exercise conveys information about the necessity of a particular wedge for reproducing the observed dynamics. We see in the top-left panel that leaving out the efficiency wedge generates the opposite dynamics to that in the data: the nominal interest rate becomes strongly *positively* correlated with future output and only little correlated with past output. Leaving out the labor wedge also deteriorates the observed dynamics, but somewhat preserves its general pattern, leaving the interest rate negatively correlated at leads and positively correlated at most lags. In contrast, leaving out the investment wedge has hardly any effect at all and leaving out the government consumption wedge has only a small effect on the shape of the dynamics. However, when we leave out the asset market wedge, the nominal interest rate becomes strongly countercyclical with no apparent lead-lag structure. Leaving out the monetary policy wedge in contrast does not change the overall pattern of the observed dynamics, although it makes the lead-lag pattern less pronounced than in the data.

These results show that without the efficiency or the asset market wedge the model is unable to produce a lead-lag relationship between output and the nominal interest rate similar to that in the data. We now investigate how far it can go with these two wedges *alone*. This exercise conveys information about the sufficiency of these two wedges for generating the observed dynamics. In the left panel of Figure 3 we see that although these two wedges do not reproduce the correlations in the data, they generate the main qualitative feature of the data: the nominal interest rate is negatively correlated with future output and positively correlated with past output. In the right panel of the figure we see that adding the monetary policy wedge makes this lead-lag pattern in the model more pronounced and closer to that in the data, especially at the lags.

The results are essentially the same for inflation. As we can see in Figure 4, leaving out the efficiency wedge again turns the dynamics around, making inflation positively correlated with future output and negatively correlated with past output. And leaving out the asset market wedge makes inflation strongly negatively correlated with output contemporaneously. In contrast, without any of the other four wedges, the model still produces the right dynamics with respect to output, at least qualitatively. Furthermore, as Figure 5 shows, the efficiency and asset market wedges alone generate a lead-lag pattern similar to that in the data, although less pronounced. Adding the monetary policy wedge somewhat increases the positive correlation of inflation with past output.

The reason for why the efficiency and asset market wedges together generate the right lead-lag pattern of inflation and the nominal interest rate is the following. Since the prototype economy is block recursive, as in a standard real business cycle model the movements in the efficiency wedge account relatively well for the movements in the model's real variables, and output especially. As the asset market wedge is highly positively correlated with the efficiency wedge, it helps the model account for the cyclical movements in the real risk-free rate not captured by standard Euler equations. And because the monetary policy wedge is relatively small (and largely uncorrelated with output over the business cycle) the efficiency and asset market wedges together account for most of the movements in both real and nominal variables at business cycle frequencies. The fact that they have the phase shift with respect to output characterized in Section 4.4 then produces the lead-lag pattern of inflation and the nominal interest rate similar to that in the data.

To summarize, the main finding of the decomposition is that from the perspective of our prototype economy, the key frictions behind the observed dynamics of inflation and the nominal interest rate over the business cycle are those that are equivalent to efficiency and asset market wedges.

5.4 Alternative Parameterizations of the Monetary Policy Rule

The values of ω_π and ω_y in the Taylor rule (7) somewhat differ across empirical studies (see Woodford, 2003, Chapter 1). We therefore study the sensitivity of our findings to these parameters. For space constraints, we only focus on sufficiency of the efficiency and asset market wedges. Figure 6 shows the results for two alternative values of ω_y, 0.05 and 0.175 (which correspond to 0.2 and 0.7 at annualized rates) and two alternative values of ω_π, 1.3 and 1.8. In each case we re-estimate the stochastic process for the wedges and back out the wedges from the re-estimated model. The figure plots the results of this experiment together with the correlations in the data and those for our baseline calibration ($\omega_y = 0.125$ and $\omega_\pi = 1.5$). As we can see, the model produces similar lead-lag patterns under these alternative parameter values to that in the baseline case.

6 Conclusions

As CKM argue, business cycle accounting should guide researchers in making decisions about what types of frictions to introduce into models so that the models exhibit fluctuations like those in the data. This paper has extended the method to two key nominal variables, inflation and the nominal interest rate. The purpose of this extension is to investigate what types of frictions are behind the observed dynamics of these two nominal variables over the business cycle.

Our analysis is based on a prototype economy that underlies a large class of models used to study the business cycle and the effects of monetary policy. The propagation of shocks through various frictions in such models is captured in our prototype economy as variation in wedges that distort the optimality conditions of agents operating in otherwise competitive markets. We have provided examples of such mappings between fictions and wedges for some of the most common frictions in the literature. We have also discussed how the effects of these frictions on inflation and the nominal interest rate can be understood through a pricing function in our prototype economy.

We have then applied the method to two well-known anomalies in the nominal business cycle: the dynamics of inflation and the nominal interest rate with respect to output, as summarized by their correlations with output at various leads and lags. Our decomposition shows that efficiency and asset market wedges are necessary, and to some extent also sufficient, for generating the observed dynamics. In contrast, labor, investment, and government consumption wedges, as well as the monetary policy wedge, play only a secondary role. This finding is particularly interesting as price stickiness, a friction often invoked in the study of inflation dynamics, manifests itself as movements in labor and investment wedges.

We need to stress that our findings do not imply that sticky prices play no role in propagating shocks over the business cycle or in the monetary transmission mechanism. The findings, nevertheless, suggest that frictions and propagation mechanisms that manifest themselves as efficiency and asset market wedges are much more important in accounting for the observed movements in inflation and the nominal interest rate over the business cycle. Our findings also do not mean that the labor wedge is unimportant for the movements in real variables, in particular hours worked, as found by CKM. The block recursive structure of our prototype economy implies that the labor wedge is as important for hours worked as in the CKM economy. It, however, is not as important as the efficiency and asset market wedges in accounting for the two anomalies in the nominal business cycle.

We hope that our findings will provide useful information to researchers constructing detailed models with explicit frictions to analyze the business cycle and monetary policy. To the extent that models used for monetary policy analysis should to be consistent with basic business cycle facts for nominal variables, our findings suggest that such models should, first and foremost, include frictions that manifest themselves as efficiency and asset market wedges. We have documented that these two wedges are strongly positively correlated with each other and with output. The efficiency wedge, however, tends to somewhat lead output, whereas the asset market wedge tends to lag output.

We have provided one possible interpretation of the asset market wedge, based on a simple model of limited participation in the money market. It is of course possible that other asset market frictions might prove promising in generating the observed movements in the asset market wedge. It is also possible that models that can generate countercyclical risk aversion at the aggregate level, or 'flight to quality', which in recessions drives the rate of return on safe assets below that predicted by standard Euler equations, might generate the observed movements in that wedge. We leave, however, such investigation for future research.

Appendix

A.1 Proofs of Propositions 1 and 2

A.1.1 Proof of Proposition 1

The proof proceeds by comparing the equilibrium conditions of the detailed economy with those of the prototype. Notice that when in the prototype $A_t(z^t) = 1$ and $\tau_{lt}(z^t) = \tau_{kt}(z^t) = \tau_{bt}(z^t) = g_t(z^t) = \widetilde{R}_t(z^t) = 0$, the equilibrium conditions in the two economies are the same except that in the prototype the capital rental rate is set equal to the marginal product of capital, whereas in the detailed economy this equilibrium condition is replaced the by optimal price-setting condition (21). Since in the detailed economy $r_t^*(z^t) \neq F_{kt}^*(z^t)$, it follows from the equilibrium condition (19) that also $w_t^*(z^t) \neq F_{lt}^*(z^t)$. The two economies thus differ only in terms of the prices of capital and labor that consumers face. We can, however, eliminate these differences by appropriately choosing $\tau_{kt}(z^t)$ and $\tau_{lt}(z^t)$ in the prototype. In particular, let $\tau_{kt}(z^t)$ satisfy $r_t^*(z^t) = (1 - \tau_{kt}(z^t))F_{kt}^*(z^t)$ and let $\tau_{lt}(z^t)$ satisfy $w_t^*(z^t) = (1 - \tau_{lt}(z^t))F_{lt}^*(z^t)$ for every history z^t, where F_{kt}^* and F_{lt}^* are evaluated at the equilibrium allocations of the detailed economy. Then the first-order conditions for capital and labor in the two economies are the same and the equilibrium allocations $(c_t^*(z^t), x_t^*(z^t), y_t^*(z^t), l_t^*(z^t), k_{t+1}^*(z^t))$ and prices $(p_t^*(z^t), R_t^*(z^t))$ of the detailed economy are also equilibrium allocations and prices of the prototype economy. In addition, since in the detailed economy $w_t^*(z^t) = [F_{lt}^*(z^t)/F_{kt}^*(z^t)]r_t^*(z^t)$, the labor income tax satisfies $r_t^*(z^t) = (1 - \tau_{lt}(z^t))F_{kt}^*(z^t)$ and therefore $\tau_{lt}(z^t) = \tau_{kt}(z^t)$. *Q.E.D*

A.1.2 Proof of Proposition 2

The proof of (31) is based on a similar argument as that of Proposition 1. We therefore concentrate on the proof of (32). The proof again proceeds by comparing the equilibrium conditions of the detailed economy with those of the prototype. Notice that by using the law of iterated expectations, equation (28) can be written as $E_{t-1}[\Lambda_t] = 0$, where

$$\Lambda_t \equiv u_{ct}/p_t - (1 + R_t)\beta E_t[u_{c,t+1}/p_{t+1}], \tag{39}$$

which is generally non-zero. By setting τ_{bt} in the equilibrium condition for bonds (10) in the prototype economy equal to zero in all states of the world, the condition becomes

$$0 = u_{ct}/p_t - (1 + R_t)\beta E_t[u_{c,t+1}/p_{t+1}]. \tag{40}$$

Notice that (40) differs from (39) only in terms of the left-hand side. Choosing τ_{bt} according to (32) implies that the right-hand side of (40), when evaluated at the equilibrium allocations and prices of the detailed economy, is equal to Λ_t. *Q.E.D*

A.2 Calvo-Style Price Setting

An economy with Calvo-style price setting differs from the one of Secton 3.1 only in the price-setting behavior of intermediate good producers. In order to simplify the exposition, we assume that the production function $F(.,.)$ is Cobb-Douglas.

With probability φ an intermediate good producer j is allowed to set its price optimally in period t. Otherwise it has to charge the price chosen last time it was allowed to change it. The shock that determines whether a producer can change its price is iid across producers and time. Producers that are allowed to change price choose $p_t(j)$ to maximize

$$E_t \sum_{i=0}^{\infty} (1-\varphi)^i Q_{t+i} \left[\frac{p_t(j)}{p_{t+i}} y_{t+i}(j) - \kappa_{t+i} y_{t+i}(j) \right],$$

where $\kappa_t = (r_t/\alpha)^{\alpha} [w_t/(1-\alpha)]^{1-\alpha}$ is a marginal cost obtained from the solution of the cost minimization problem, subject to the demand function (15). This profit maximization problem replaces the profit maximization problem in Section 3.1 and its solution is given by

$$(41) \qquad p_t^* = \frac{1}{\epsilon_t} \frac{E_t \sum_{i=0}^{\infty} (1-\varphi)^i Q_{t+i} \kappa_{t+i} p_{t+i}^{1/(1-\epsilon_t)} y_{t+i}}{E_t \sum_{i=0}^{\infty} (1-\varphi)^i Q_{t+i} p_{t+i}^{\epsilon_t/(\epsilon_t-1)} y_{t+i}}.$$

After substituting for $y_t(j)$ from (15) into the profit function of final good producers, a zero-profit condition implies that the aggregate price level has to satisfy

$$p_t = \left[\int p_t(j)^{\epsilon_t/(\epsilon_t-1)} dj \right]^{(\epsilon_t-1)/\epsilon_t},$$

which can be written as

$$(42) \qquad p_t = \left[\varphi (p_t^*)^{\epsilon_t/(\epsilon_t-1)} + (1-\varphi) p_{t-1}^{\epsilon_t/(\epsilon_t-1)} \right]^{(\epsilon_t-1)/\epsilon_t}.$$

Notice that in Rotemberg's model, $p_t(k) = p_t(j)$ for all $j \neq k$ as all producers only differ by their index. Here, however, $p_t(k) = p_t(j)$ only for those k and j that are allowed to change price in period t. Equation (42), together with (41), constitutes a New-Keynesian Phillips curve in a model with Calvo-style price setting. It replaces the New-Keynesian Phillips curve (21) in the model of Section 3.1 and therefore creates the same distortion as (21), namely that $r_t \neq F_{kt}$.

Finally, aggregating output across intermediate good producers by integrating (15) leads to

$$\begin{aligned} y_t &= k_t^{\alpha} l_t^{1-\alpha} \int \left(\frac{p_t}{p_t(j)} \right)^{1/(\epsilon_t-1)} dj, \\ &= p_t^{1/(\epsilon_t-1)} k_t^{\alpha} l_t^{1-\alpha} \int p_t(j)^{1/(1-\epsilon_t)} dj, \\ &= \left(\frac{p_t}{\bar{p}_t} \right)^{1/(\epsilon_t-1)} k_t^{\alpha} l_t^{1-\alpha}, \end{aligned}$$

where

$$\bar{p}_t = \left[\varphi(p_t^*)^{1/(1-\epsilon_t)} + (1-\varphi)p_{t-1}^{1/(1-\epsilon_t)}\right]^{1-\epsilon_t}.$$

The aggregation bias $p_t/\bar{p}_t$ in the aggregate production function shows up in our prototype economy as an efficiency wedge. It is not present with Rotember price setting as $p_t(k) = p_t(j) = p_t$ for all $j \neq k$.

References

Ahearne, A., F. E. Kydland and M. A. Wynne, "Ireland's Great Depression," Working Paper No. 0510, Federal Reserve Bank of Dallas, 2005.

Atkeson, A. and P. J. Kehoe, "On the Need for a New Approach to Analyzing Monetary Policy," NBER Working Paper No. 14260, 2008.

Backus, D. K., B. R. Routledge and S. Zin, "Asset Prices in Business Cycle Analysis," Mimeo, New York University, 2007.

Calvo, G., "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics* 12 (1983), 38398.

Canzoneri, M., R. E. Cumby and B. T. Diba, "Euler Equations and Money Market Interest Rates: A Challenge for Monetary Policy Models," *Journal of Monetary Economics* 54 (2007), 1863–81.

Chakraborty, S., "Business Cycles in a Neoclassical Growth Model: How Important are Technology Shocks as a Propagation Mechanism?," Mimeo, 2005.

Chari, V. V., P. J. Kehoe and E. R. McGrattan, "Business Cycle Accounting," *Econometrica* 75 (2007a), 781–836.

———, "Comparing Alternative Representations, Methodologies, and Decompositions in Business Cycle Accounting," Staff Report No. 384, Federal Reserve Bank of Minneapolis, 2007b.

Christiano, L. J. and J. Davis, "Two Flaws in Business Cycle Accounting," Mimeo, Northwestern University, 2006.

Christiano, L. J. and M. Eichenbaum, "Liquidity Effects and the Monetary Transmission Mechanism," *American Economic Review* 82 (1992), 346–53.

Christiano, L. J. and T. J. Fitzgerald, "The Band Pass Filter," *International Economic Review* 44 (2003), 435–65.

CRUCINI, M. AND J. KAHN, "Tariffs and the Great Depression Revisited," Staff Report No. 172, Federal Reserve Bank of New York, 2003.

FEENSTRA, R. C., "Functional Equivalence between Liquidity Costs and the Utility of Money," *Journal of Monetary Economics* 17 (1986), 271–91.

FINN, M. G., "A Theory of the Capacity Utilization/Inflation Relationship," *Federal Reserve Bank of Richmond Economic Quarterly* 82 (1996), 67–85.

FUERST, T. S., "Liquidity, Loanable Funds, and Real Activity," *Journal of Money, Credit, and Banking* 29 (1992), 3–24.

FUHRER, J. AND G. MOORE, "Inflation Persistence," *Quarterly Journal of Economics* 110 (1995), 127–159.

GALÍ, J. AND M. GERTLER, "Inflation Dynamics: A Structural Econometric Analysis," *Journal of Monetary Economics* 44 (1999), 195–222.

GAVIN, W. T. AND F. E. KYDLAND, "The Nominal Facts and the October 1979 Policy Change," *Reserve Bank of St. Louis Quarterly Review* November/December (2000), 39–61.

GAVIN, W. T., F. E. KYDLAND AND M. R. PAKKO, "Monetary Policy, Taxes, and the Business Cycle," *Journal of Monetary Economics* 54 (2007), 1587–1611.

GOFFE, W. L., G. D. FERRIER AND J. ROGERS, "Global Optimization of Statistical Functions with Simulated Annealing," *Journal of Econometrics* 60 (1994), 65–99.

HALL, R. E., "Macroeconomic Fluctuations and the Allocation of Time," *Journal of Labor Economics* 15 (1997), 223–50.

HANSEN, G. D. AND E. C. PRESCOTT, "Recursive Methods for Computing Equilibria of Business Cycle Models," in T. F. Cooley, ed., *Frontiers of Business Cycle Research* (Princeton University Press, 1995).

Hansen, L. P. and K. J. Singleton, "Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Asset Returns," *Journal of Political Economy* 91 (1983), 249–65.

Henriksen, E., F. E. Kydland and R. Šustek, "The High Cross-Country Correlations of Prices and Interest Rates," Mimeo, University of Oslo, 2008.

Hodrick, R. J. and E. C. Prescott, "Postwar U.S. Business Cycles: An Empirical Investigation," *Journal of Money, Credit, and Banking* 29 (1997), 1–16.

Ireland, P. N., "Technology Shocks in the New-Keynesian Model," *Review of Economics and Statistics* 86 (2004), 923–36.

Kersting, E., "The 1980s Recession in the UK: A Business Cycle Accounting Perspective," *Review of Economic Dynamics* 11 (2008), 179–91.

King, R. G. and M. W. Watson, "Money, Prices, Interest Rates and the Business Cycle," *Review of Economics and Statistics* 78 (1996), 35–53.

Kobayashi, K. and M. Inaba, "Business Cycle Accounting for the Japanese Economy," RIETI Discussion Paper No. 05-E-023, 2006.

Lucas, R. E. J., "Liquidity and Interest Rates," *Journal of Economic Theory* 50 (1990), 237–264.

McGrattan, E., "Predicting the Effects of Federal Reserve Policy in a Sticky-Price Model: An Analytical Approach," Working paper No. 598, Federal Reserve Bank of Minneapolis, 1999.

Mulligan, C. B., "A Dual Method of Empirically Evaluating Dynamic Competitive Equilibrium Models with Market Distortions, Applied to the Great Depression and World War II," NBER Working paper No. 8775, 2002.

Primiceri, G. E., E. Schaumburg and A. Tambalotti, "Intertemporal Disturbances," NBER Working Paper No. 12243, 2006.

Rotemberg, J., "Monopolistic Price Adjustment and Aggregate Output," *Review of Economic Studies* 49 (1982), 517–31.

Smets, F. and R. Wouters, "Shocks and Frictions in U.S. Business Cycles: A Bayesian DSGE Approach," *American Economic Review* 97 (2007), 586–606.

Sustek, R., "Monetary Business Cycle Accounting," Working Paper , Forthcoming, Bank of England, 2009.

Taylor, J. B., "Discretion Versus Policy Rules in Practice," *Carnegie-Rochester Conference Series on Public Policy* 39 (1993), 195–204.

Wang, P. and Y. Wen, "Inflation Dynamics: A Cross-Country Investigation," *Journal of Monetary Economics* 54 (2007), 2004–31.

Woodford, M., *Interest and Prices: Foundations of a Theory of Monetary Policy* (Princeton University Press, 2003).

Table 1: Stochastic process for the wedges[a]

$$P_0 = \begin{bmatrix} -0.0811 & 0.0074 & -0.0336 & 0.0476 & -0.012 & -0.012 \end{bmatrix}$$

$$P = \begin{bmatrix} 0.8581 & -0.0965 & 0.1732 & -0.0064 & -0.0435 & 0.5188 \\ -0.0675 & 1.0610 & 0.0019 & 0.0110 & 0.0467 & -0.7241 \\ -0.0860 & -0.0220 & 1.0890 & 0.0026 & -0.0121 & 0.4020 \\ 0.0821 & 0.0587 & -0.0987 & 1.0061 & 0.0254 & 0.367 \\ 0.0985 & -0.3110 & 0.0870 & -0.0101 & 0.8260 & 0.1200 \\ -0.0217 & 0.0167 & -0.0008 & 0.0009 & 0.0085 & 0.4330 \end{bmatrix}$$

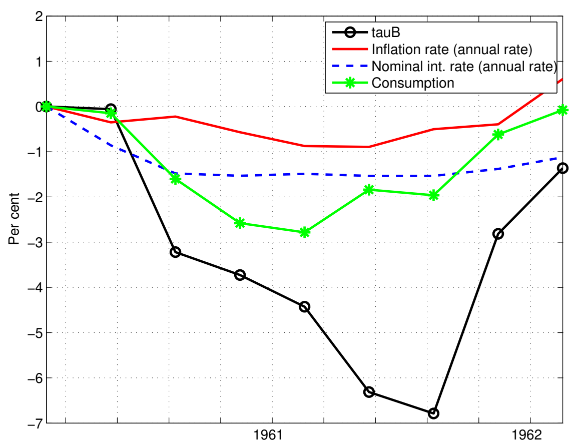
$$B = \begin{bmatrix} 0.0073 & 0 & 0 & 0 & 0 & 0 \\ 0.0038 & 0.0091 & 0 & 0 & 0 & 0 \\ 0.0058 & -0.0009 & 0.0031 & 0 & 0 & 0 \\ 0.0009 & 0.0051 & 0.0119 & 0.0087 & 0 & 0 \\ 0.0005 & -0.0175 & -0.0014 & 0.0015 & 0.0219 & 0 \\ 0.0003 & 9.5e-6 & 0.0001 & -0.0004 & 0.0038 & 0.0011 \end{bmatrix}$$

[a] The equilibrium conditions of the prototype economy imply that in steady state the values of τ_b and $\widetilde{R}$ are zero. This restriction is imposed in the estimation of P_0, P, and B.

Table 2: Business cycle properties of the wedges, 1959.Q1-2004.Q4[a]

Wedges	Rel. std.[b]	Correlations of output in period t with wedges in $t+j$: j = -4	-3	-2	-1	0	1	2	3	4
$\log A_{t+j}$	0.63	0.33	0.49	0.67	0.77	**0.85**	0.62	0.38	0.13	-0.05
$\tau_{l,t+j}$	0.92	-0.17	-0.33	-0.50	-0.67	**-0.74**	-0.78	-0.74	-0.63	-0.43
$\tau_{x,t+j}$	0.50	0.16	0.35	0.54	0.68	**0.79**	0.62	0.44	0.26	0.13
$\log g_{t+j}$	1.51	-0.40	-0.42	-0.45	-0.44	**-0.35**	-0.24	-0.10	0.04	0.20
$\tau_{b,t+j}$	2.59	0.06	0.27	0.48	0.70	**0.82**	0.81	0.72	0.58	0.41
$\widetilde{R}_{t+j}$	0.12	0.11	0.15	0.13	0.15	**0.11**	0.01	-0.09	-0.16	-0.17

[a] The statistics are for wedges and per-capita output filtered with the HP-filter.
[b] The standard deviations are measured relative to that of per-capita output, which is 1.58.



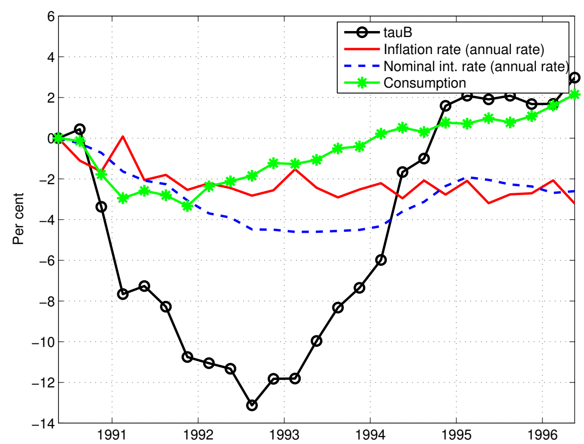


Figure 1: The asset market wedge in the 1960 and 1990 recessions. The plots are for deviations of log per-capita consumption from a linear trend, and of inflation, the nominal interest rate, and the asset market wedge from their mean values. The deviations are normalized to be zero at the start of each recession.

Table 3: Contemporaneous correlations of the wedges with each other: 1959.Q1-2004.Q4[a]

	$\log A$	τ_l	τ_x	$\log g$	τ_b	$\widetilde{R}$
$\log A$	1.00					
τ_l	-0.31	1.00				
τ_x	0.90	-0.28	1.00			
$\log g$	-0.34	0.45	0.01	1.00		
τ_b	0.53	-0.88	0.54	-0.40	1.00	
$\widetilde{R}$	0.19	-0.02	0.17	-0.19	0.35	1.00

[a] The statistics are for wedges filtered with the HP-filter.

Table 4: Comovement in the data between output and the two nominal variables, 1959.Q1-2004.Q4[a]

	Correlations of output in period t with a variable in $t+j$:										
$j =$	-5	-4	-3	-2	-1	0	1	2	3	4	5
R_{t+j}	-0.61	-0.50	-0.34	-0.15	0.10	**0.32**	0.42	0.47	0.46	0.44	0.41
π_{t+j}	-0.42	-0.39	-0.26	-0.11	0.02	**0.12**	0.27	0.38	0.46	0.48	0.45

[a] The statistics are for HP-filtered series.

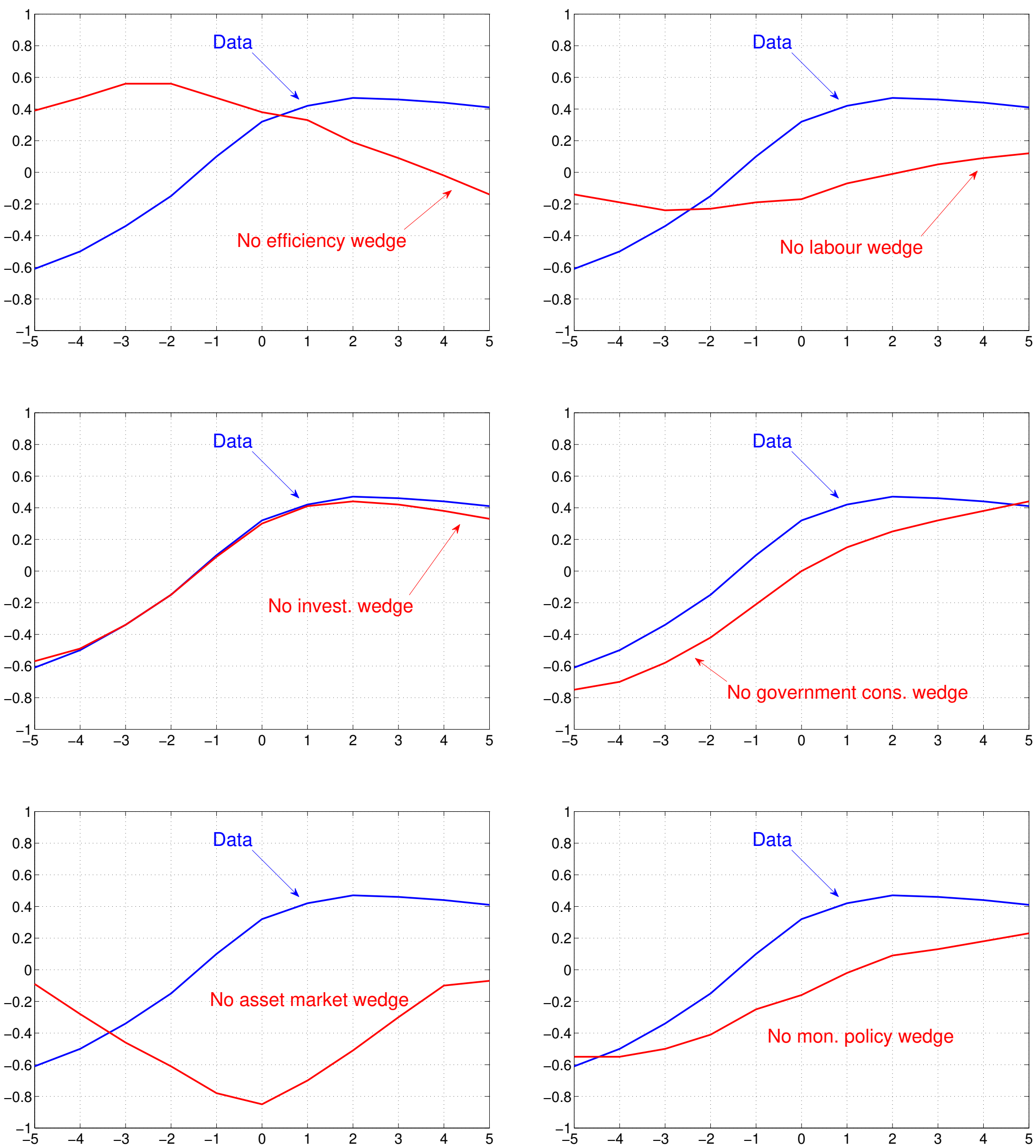


Figure 2: Necessity of wedges for nominal interest rate dynamics.

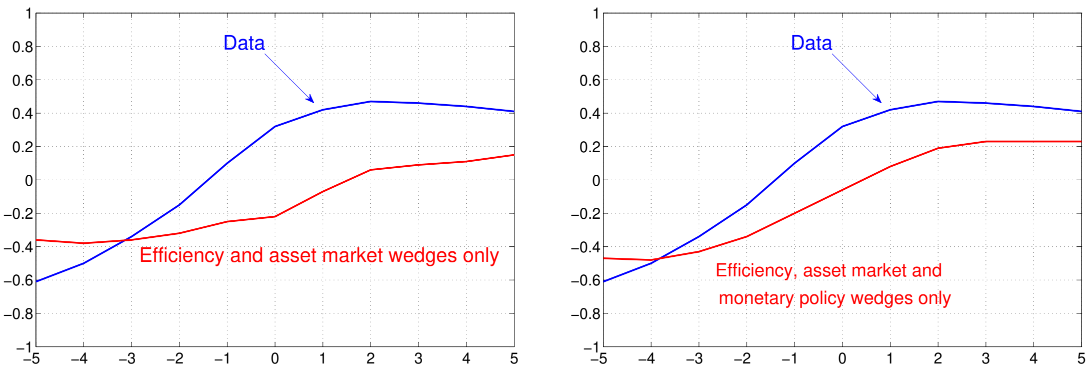


Figure 3: Sufficiency of wedges for nominal interest rate dynamics.

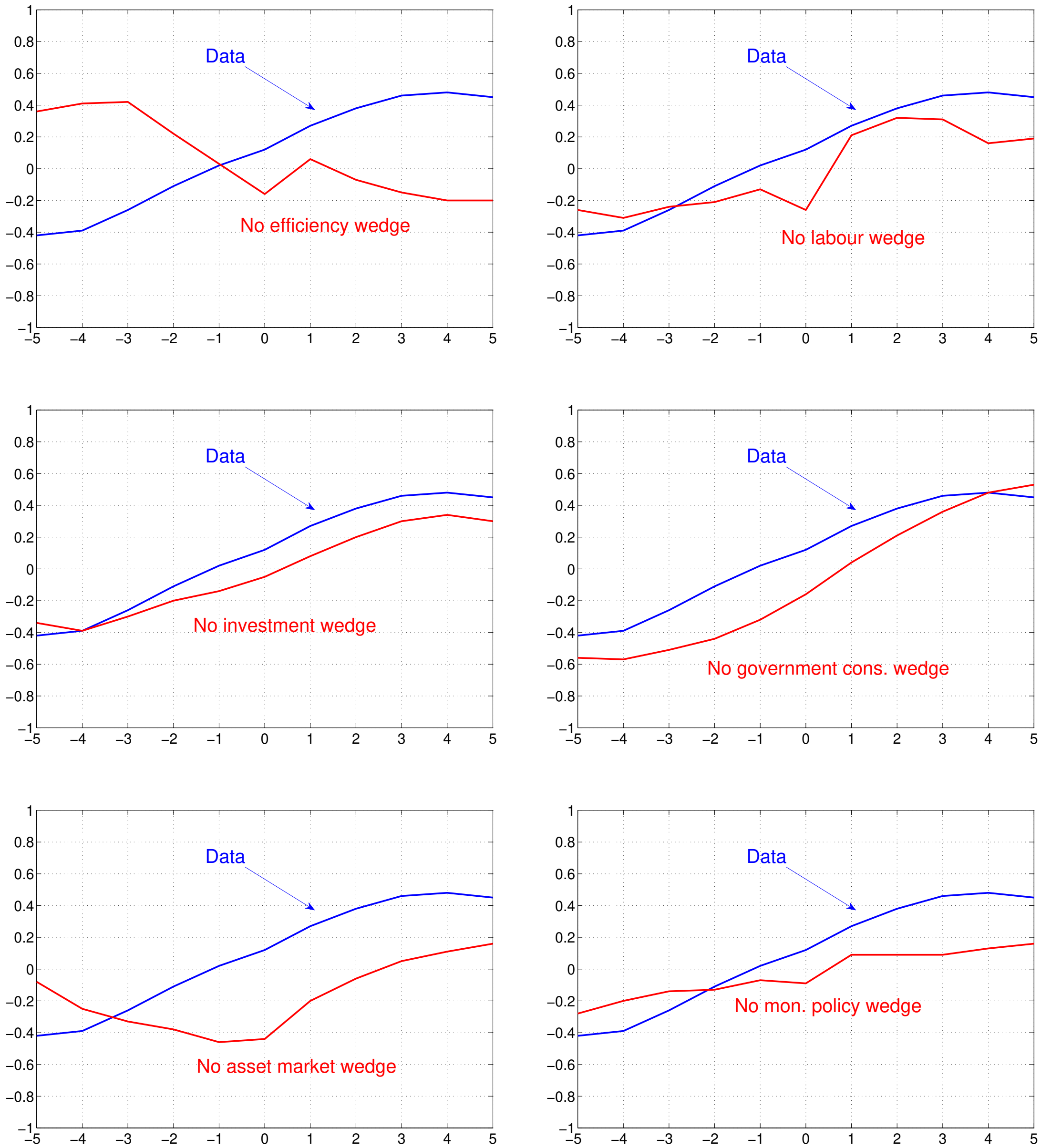


Figure 4: Necessity of wedges for inflation dynamics.

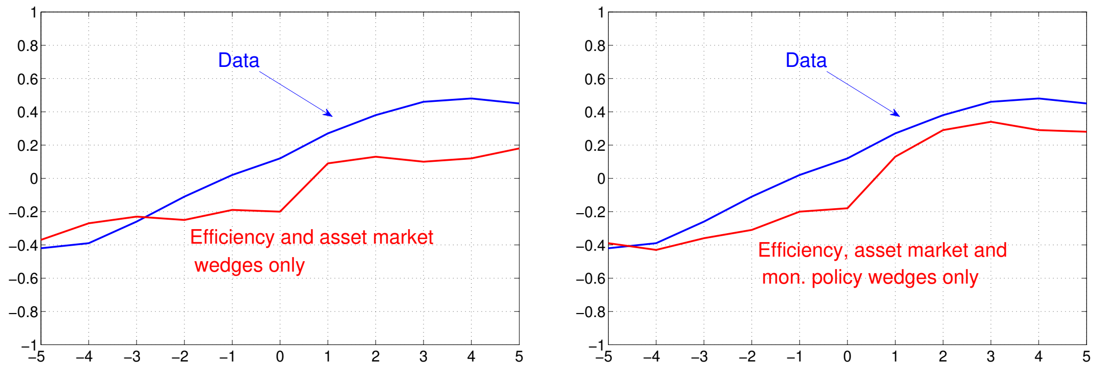


Figure 5: Sufficiency of wedges for inflation dynamics.

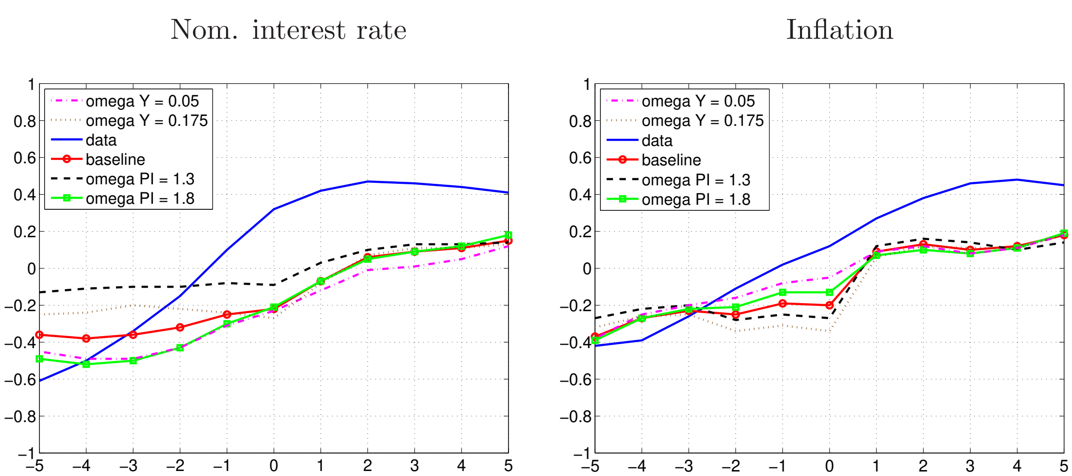


Figure 6: Sensitivity analysis; sufficiency of efficiency and asset market wedges