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Given a commercial banking firm facing credit risk we develop a dynamic hedging model where the bank management can use credit derivatives. In a continuous-time framework optimal hedging strategies, deposit and loan decisions and consumption are studied. It is shown that the optimal hedge ratio consists of two elements: a speculative term which is controlled by the risk premium and the bank's risk aversion; and a pure hedge term which depends on the preferences of bank owners. Primarily the purpose of hedging is to stabilize the consumption path through a reduction in the variability of the dynamics of the wealth accumulation. Furthermore, we demonstrate that the asset/liability management is optimal if marginal cost equal marginal revenue for loans and deposits at each instant.

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1. Introduction

Asset/liability management and especially credit risk management has received increasing attention by intermediaries and financial institutions of all sizes and in all economies. Indeed, as documented in Froot et al. (1993), it is ranked by financial managers as one of their primary objectives.[1] The growth of the global credit derivatives market has been estimated at the end of 2001 to have transactions volume of approximately $1 189 billion or over $1 trillion. The annual surveys of The British Bankers Association forecast dramatic increases reaching a vast volume of $4.8 trillion by 2004. If asset swaps are also included in the market size estimates, the global credit derivation market will exceed $5 trillion by the of 2004. Accounting for the real world importance of loan portfolio and credit risk management there have been many studies concerning the financial and hedging decisions of risk-averse banking firms facing default risk.[2]

Given risk aversion of the owner of the bank and default risk, within a dynamic hedging model we derive the bank's optimal hedge position and we demonstrate that the loan and deposit policy is optimal if marginal costs equal marginal revenue at each instant. In the optimum the marginal utility of consumption equals the marginal utility of wealth at each instant.

The framework we use for our study is the so called industrial organization approach of banking (see Freixas and Rochet (1997)). The approach is focussing on the bank's role as intermediary, but abstracts from informational aspects of banking, adverse selection and moral hazard. Our aim is to study how a risky loan portfolio affects optimal bank asset and liablility management, when credit derivatives to hedge credit risk are available. In the sequel we will use the term credit derivatives for advanced financial instruments such as credit forwards/futures and options.[3] Our objective is to examine how the possibility to sell (or buy) partially or fully of a bank's risky loan portfolio at a market price affects the banks optimal asset and liability management.

What is credit risk and what are credit derivatives? In general credit or default risk is the risk that the borrower does not repay part or the entire financial obligation. Credit derivatives are financial instruments designed to transfer credit risk from one party to another. The legal ownership of the obligation is usually not transferred. Credit derivatives can have the form of forwards, options and swaps which may be imbedded in financial assets such as loans and bonds. Credit derivatives allow an investor to reduce or eliminate default risk or to buy credit risk, expecting to benefit from it. The number of credit derivatives transactions has increased largely worldwide in recent years. The main reasons for the rise of credit derivatives are an increase in corporate and sovereign bankruptcies such as Enron

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[1]See also Bessis (2002), British Bankers' Association (2002).

[2]See, for example, Wong (1997), Louberge and Schlesinger (2002), Broll et al. (2003), Wahl and Broll (2993) and others.

[3]See, for example, Battermann et al. (2000), Broll et al. (2001).

2002, and the Asian Financial Crises in 1997, Russian Financial Crises 1998, and Argentina 2001. There are many ways in which financial managers can utilize credit derivatives. The main applications are hedging, arbitrage and speculation. We are focusing on hedging aspect, i.e., the desire of an investor to reduce risks in order to stabilize the stream of consumption.

The basic motivation of our study can be interpreted as follows. Banks are facing corporate and sovereign credit risks. If a banking firm does not hedge, there will be some stochastic variability in the cash flows. Random fluctuations in cash flows due to credit risk result in variability in the amount of the bank's owner consumption. Variability in consumption will generally be undesirable, to the extent that there is risk aversion. Credit derivatives can reduce this variability in cash flows and increase the expected utility of the owner of the bank.

The plan of the paper is as follows. In section 2 we present an intertemporal model of a competitive commercial banking firm under credit risk, when a credit derivative is available. Section 3 examines optimal loan, deposit and hedging positions of the banking firm. Section 4 concludes the paper.

2. Wealth, Credit Risk and Hedging

Consider a straightforward model of a competitive banking firm in an intertemporal framework. The bank is a classical intermediary, taking deposits D and making loans L. By competitive we mean that the bank is a price taker. The effective rate of loan return, $\tilde{r}$, is uncertain because the credit repayment is uncertain. Equity capital, K_0, of the bank is taken as given. The operational cost of the bank are determined by $G(L, D)$, which is increasing and convex: $G_L > 0$, $G_{LL} > 0$, $G_D > 0, G_{DD} > 0$. The balance sheet constraint of the bank can be written as follows:

$$L + M = K_0 + D, \tag{1}$$

where M is the positive or negative balance of capital invested in or financed from an interbank market at a given deterministic interest rate r_M.

The bank faces credit risk in the sense that repayments are uncertain. The evolution of the effective loan rate, r, is assumed to be stochastic. We assume that the effective loan rate r follows a geometric Brownian motion with a fixed drift rate μ and a fixed variance parameter σ, which is a measure of the diffusion. Mathematically, we write this process as follows:

$$dr/r = \mu dt + \sigma dz \tag{2}$$

where dz is a standard Wiener process.

The banking firm can use the credit derivatives to hedge the default risk. Futures contracts cost nothing to enter but pay off df continuously thereafter due to marking to market. We assume that the time to maturity of the futures contract perfectly matches the investor's hedging horizon. Let r_M denote the risk-free rates of interest in the market. We assume that this interest rates is nonstochastic and constant over time. Hence, in order to prevent arbitrage opportunities, the following relationship must hold between the forward price, which is also the futures price, $f(t)$, and the price of the underlying asset, the effective loan rate, $r(t)$:

$$f(t) = r(t)e^{r_M(T-t)}. \tag{3}$$

Applying Itô's Lemma, we can derive the stochastic process followed by the futures price:

$$df/f = [\mu - r_M]dt + \sigma dz. \tag{4}$$

It is assumed that dr/r and df/f are perfectly correlated. The futures price also obeys a geometric Brownian motion process, with a drift rate $\mu_{RP} = [\mu - r_M]$ that differs from that of the underlying effective loan rate by an amount equal to the constant risk-free rate r_M. μ_{RP} represents the risk premium of the underlying asset.

Now we turn to the risk averse bank owner's wealth accumulation and its utility maximization problem, that is, to the best the bank manager can do in the interest of the owner, who cares about consumption. Consider an investor with a fixed planning horizon T, who has total wealth W, from which she has already invested an amount I in a competitive banking firm. The difference, namely $W - I$, is invested at the riskless market interest rate r_M and generates a cash flow of $r_M(W - I)dt$ per unit of time.

A second cash flow comes from the investment in the banking firm, namely the profits from financial intermediation:

$$\Pi dt = (rL + r_M M - r_D D - G(L, D))dt,$$

where r_D is the given deposit rate. The banking firm can use the risk sharing market to hedge the default risk. The purpose of hedging is to stabilize the consumption path through a reduction in the variability of the wealth accumulation path. Credit derivatives pay off df continuously thereafter due to marking to market. The consumption expenditure of the shareholder over time is denoted by Cdt.

Since loan portfolio revenues over time are uncertain due to the stochastic evolution of the credit risk, the banking firm can hedge against this risk by taking a position in the credit derivative market. The banking firm takes a hedge position with volume H. Hence, the wealth accumulation equation is given by:

$$dW = [r_M(W - I) + \Pi - C]dt - Hdf. \tag{5}$$

As the futures price changes over time, the bank must adjust its margin account. If the futures price rises, $df > 0$, she has to pay an amount Hdf; if the futures price falls, she receives cash payments Hdf.

With the definition of the hedge ratio with respect to wealth as $h = H/W$, and substituting the stochastic process for df in (5) gives the wealth accumulation equation:

$$dW = (r_M W - hW\mu f + \Pi - C - r_M I)dt - hW\sigma f dz. \tag{6}$$

The term $r_M Idt$ represents the opportunity cost of the investment in the banking firm, i.e., the deterministic interest revenue forgone over time.

The investor cares about consumption. $u(C)$ is a von Neumann-Morgenstern utility function which exhibits risk aversion, i.e., $u'(C) > 0, u''(C) < 0$. The parameter β is a subjective discount rate. The risk averse bank manager solves the following maximization problem:

$$\max_{C,L,D,h} E\int_0^T u(C)e^{-\beta t}dt, \tag{7}$$

subject to the stochastic wealth flow constraint (6). Now we turn to the optimal asset/liability management and optimal consumption rules of the banking firm.

3. The Bank's Optimal Asset/Liability Management

Solving the maximization problem (7) subject to the wealth constraint (6) leads to the following

Proposition: *When credit risks are tradeable on a credit derivative market, the optimal loan and deposit decisions, the optimal hedge ratio and the consumption rule are described by:*

$$u'(C^*) = V_W(W,r,t), \tag{8}$$

$$r - r_M = G_L(L^*,D^*), \tag{9}$$

$$r_M - r_D = G_D(L^*,D^*), \tag{10}$$

$$h^* = \frac{\mu_{RP}}{\sigma^2 f V_{WW} W / V_W} + \frac{r V_{Wr}}{f V_{WW} W}. \tag{11}$$

The **proof** is given in the appendix.

Equation (8) determines the optimal consumption decision. It is the familiar condition for optimal intertemporal consumption: the marginal utility of consumption $u'(C)$ has to be equal to the marginal utility of wealth $V_W(\cdot)$, where $V(W,r,t)$ can be identified as the indirect utility function of the bank's owner.

The best the bank management can do is taking deposits and giving loans at each instant of time according to the static optimization rule, which requires that net marginal revenue equals marginal cost, i.e., see (9) and (10). The intermediation margin, $r > r_M > r_D$, is fullfilled.

Equation (11) gives the optimal hedge ratio. The first term in (11) is the speculative component of the optimal hedge policy. The futures market may turn out to be in a backwardation situation, i.e. $\mu_{RP} > 0$, which means that there is a risk premium on the credit derivative. A risk averse investor will take this into account and deviate from the variance minimizing hedge ratio.

As can be seen, the degree of the speculative component depends on the investor's degree of relative risk aversion, which is measured by the expression $-V_{WW}W/V_W$, and the variance of the evolution of the futures price, σ_f^2. The bigger the risk aversion or the bigger the variance of the evolution of the futures price, the smaller becomes the speculative component. The second term of (11) captures the pure hedging motive, i.e. the desire to stabilize consumption over time. We see that the optimal hedge ratio with respect to this motive again depends on preferences.

The economic purpose of hedging with derivatives is to stabilize the intertemporal consumption of the bank's owner. The stabilizing effect on optimal consumption can be seen best when we assume that the risk premium is zero. In this case the change in consumption of time is: $du'(C)/u'(C) = (\beta - r_M)dt$. This means consumption is fully stabilized. When there are no credit derivatives or any other correlated financial asset traded on financial markets optimal consumption over time is

$$du'(C)/u'(C) = (\beta - r_M)dt + \frac{V_{Wr}}{V_W} r\sigma dz.$$

Without hedging consumption is always stochastic which implies a loss in expected utility.

4. Conclusions

This paper presents a banking firm model of dynamic risk management where the underlying source of the risky wealth is an unanticipated change in the default risk. A position in the credit derivative market is used to hedge, ex ante, the uncertain loan revenues. The purpose of hedging is to stabilize the consumption path through a reduction in the variability of the wealth accumulation path. The magnitude and the direction of hedging depends on the preferences, the risk premium and the variance.

Credit derivatives are an important financial instrument in which banks and financial intermediaries, without transferring their portfolio or reducing their portfolio concentration, can buy into their risk of each other. The bartering of risks in such bilateral transactions is enforced through marketable contracts. The credit risk inherent in a portfolio can be securitised and sold in the capital market like any other capital market security. The important underlying economic insight hereby is that the concept of credit derivatives and securitization have joined together to make risk a tradeable commodity enhancing the usage of viable hedging strategies.

However, our analysis is not limited to credit derivatives. Any tradeable financial asset will do. In particular, a so-called macro derivative could serve as a substitute for a missing credit derivate (Wilson (1998), Broll et al. (2003)). The effectiveness of the financial instrument to hedge against credit risk crucially depends on the correlation structure between credit risk and the hedge instrument.

Appendix

The value function $V(W,r,t)$ is defined as follows:

$$V(W,r,t) = \max_{C,L,D,h} E_t \int_t^T U(C)e^{-\beta\tau} d\tau,$$

subject to the dynamic wealth accumulation equation (6) and some initial and terminal conditions. The Bellman equation is[4]

$$\beta V(W,r,t) = \max_{C,L,D,h} \left\{ u(C) + \frac{E(dV)}{dt} \right\},$$

where

$$dV = V_t dt + V_W(dW) + \tfrac{1}{2}V_{WW}(dW^2) + V_r(dr)$$
$$+ \tfrac{1}{2}V_{rr}(dr^2) + V_{Wr}(dW)(dr),$$

and

$$E(dr) = \mu_r r dt,$$
$$E(dr^2) = \sigma_r^2 r^2 dt,$$
$$E(dW) = (r_M W - hWf[\mu_r - r_M] - r_M I + \Pi - C)dt,$$
$$E(dW^2) = h^2 W^2 \sigma_f^2 dt,$$
$$E[(dr)(dW)] = -hW\sigma_f f \sigma_r r \rho_{rf} dt,$$

Substituting these expressions into the Bellman equation yields:

$$\beta V(W,S,t) = \max_{C,L,D,h}\{V_t + \alpha W V_W + \gamma V_W + \delta W^2 V_{WW}$$
$$+ \mu_r r V_r + \tfrac{1}{2}\sigma_r^2 r^2 V_{rr} - \varepsilon W r V_{Wr} + u(C)\}$$

where

$\alpha = (r_m - hf\mu_{RP})$, $\gamma = (-r_M I + \Pi - C)$, $\delta = \tfrac{1}{2}h^2\sigma_f^2 f^2$, $\varepsilon = h\sigma_f \sigma_r f \rho_{rf}$.

Maximization on the right hand side of the equation above and the assumption that df/f and ds/s are perfectly correlated leads to the optimality conditions in the claim.

[4]See, for example, Dixit and Pindyck (1994), Turnovsky (1995).

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