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Bubbles and contagion in English house prices

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Abstract

Using methods originating from statistical physics we model bubbles in English house prices. It is found that there was a nationwide housing bubble 2002-2007. Typically prices were 30-40% over-valued and fell around 20%. London is atypical in that the level of over-pricing was lower, only around 20%, and experienced a drop in prices of only around 15%. There is some suggestion of contagious effects, with the bubble in London affecting prices in Yorkshire and the North.

Keywords: financial crashes, super-exponential growth, illusion of certainty, contagion, housing-bubble, English house prices.

1 Introduction

Housing bubbles are of obvious topical interest given the credit crunch of 2007-8 (*Parkinson et al. (2009)*). The simple truth is that *housing matters*. Housing is typically the major asset in household portfolios and can have major implications for the economy as a whole. As we have seen, the banking sector is particularly exposed to to the housing market (*Hott and Monnin (2008)*), and house price crashes tend to have a stronger impact on the economy than stock market crashes such as the 1987 stock market crash (*Black*

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et al. (2006), Helbling and Terrones (2003)). Symmetrically, however, housing offers potentially greater rewards in that the wealth effects for housing assets are typically greater than those for financial assets (*Case et al. (2005)*). As well as economy-wide issues there are also a number of pertinent housing renewal/social policy implications of the recent crisis in the UK (*Ferrari (2007), Parkinson et al. (2009)*).

Housing markets are particularly susceptible to bubbles. A succession of booms and slumps have been documented in worldwide housing markets; in the UK (*Parkinson et al. (2009), Hott and Monnin (2008), Black et al. (2006)*) and the rest of the world including the USA, Japan and Switzerland (*Hott and Monnin (2008)*). Housing bubbles have been widely studied, see e.g. *Black et al. (2006), Hott and Monnin (2008)* and a host of references therein. The comment is made in *Black et al. (2006)* that since much of the housing market is based on consumption rather than investment, subsequent market inefficiencies mean that housing markets are prone to bubbles and speculative behaviour.

In this paper we apply our theoretical model to English house prices – a subject with obvious socio-economic implications. Our approach allows for univariate and multivariate models and leads to simple answers to questions such as “Are there bubbles?” and “What is the apparent level of over-pricing?”. In particular, we apply the model in *Fry (2009)* to English house prices over the years 2002-2007. For additional background on the modelling work see the papers by Feigenbaum and Sornette and co-workers cited in the references, especially *Johansen et al. (2000)*.

The layout of this document is as follows. Section 2 provides the main analysis. Section 3 concludes. Section 4 provides a self-contained mathematical appendix.

2 Analysis

The analysis in this paper is as follows. We model quarterly house price data for ten English regions obtained from the nationwide website¹ over the years 2002-2007. Our aim in this section is purely data-analytic and a self-contained mathematical Appendix can be found at the end of this paper. Our analysis splits into two parts. Firstly, we use a univariate model where we model data for each region individually. We test for bubbles, and provide estimates of fundamental values and for the level of over-pricing. Further, we are able to demonstrate that prices eventually converge towards fundamental levels out of sample. Secondly, we use a bivariate model which allows us to model data for two regions simultaneously. This gives us an alternative test for bubbles and in addition allows us to

¹<http://www.nationwide.co.uk/hpi/historical.htm>

model contagion. See Section 2.2.

2.1 Univariate analysis

The results obtained using the univariate bubble model (equation (2) in the Appendix) are shown in Table 1. We have strong evidence for bubbles in each of the individual price series. The results for London appear outlying, with the estimated bubble component (equation (7)) comprising only 20% of prices compared to 30-40% for the rest of England and a fall in prices (maximum-to-minimum before subsequent price rises) of 15% compared to 20% for much of the rest of England. These estimates of over-pricing compare with similar estimates of between 12-25% in *Black et al. (2006)* and 28-53% in *Hott and Monnin (2008)*.

This picture of speculative bubbles is reinforced once we take into account of the estimates of fundamental value in equation (6). A graph of observed prices and estimated fundamental price in the years 2002-2007 by region is shown in Figure 1. In all cases prices appear way in excess of fundamental levels. In Table 2 we compare estimates of fundamental value, calculated only using data from 2002-2007, with historically observed prices over the years 2008-2009. The suggestion is that prices have converged towards fundamental values by the first quarter of 2009.

Region	p -value	Estimated speculative component	% Drop
North	0.001	0.375	0.164
Yorkshire	0.009	0.358	0.185
North West	0.002	0.276	0.179
East Midlands	0.000	0.374	0.193
West Midlands	0.000	0.381	0.175
East Anglia	0.000	0.300	0.216
South East	0.000	0.337	0.200
Outer Met	0.001	0.247	0.191
London	0.007	0.203	0.156
South West	0.000	0.362	0.181

Table 1: p -values for null hypothesis of no bubble, estimated speculative component and percentage drop in prices by English region.

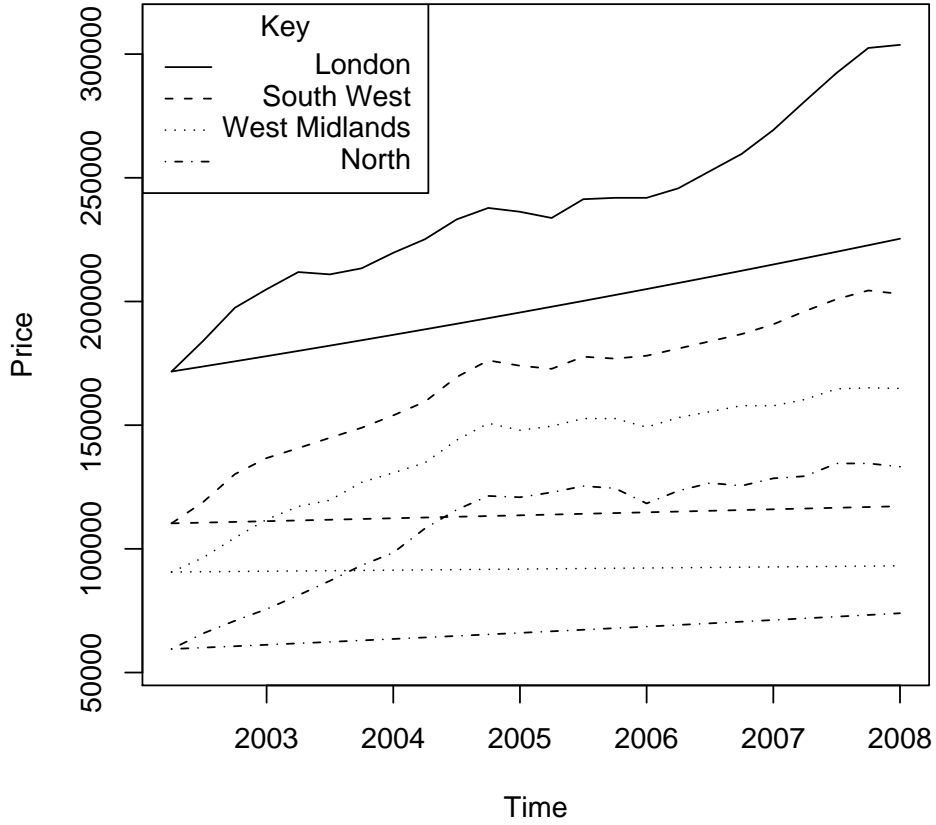


Figure 1: Observed historical prices (wavy lines) and estimated fundamental price (straight lines) by region

Date	Actual price	Estimated fundamental price	95% C. I
2008 Q1	179,363	122,326	100,583-144,068
2008 Q2	174,514	123,602	100,478-146,725
2008 Q3	165,188	124,891	100,338-149,444
2008 Q4	156,828	126,194	100,162-152,226
2009 Q1	149,709	127,511	99,948-155,073
2009 Q2	154,066	128,841	99,695-157,986
2009 Q3		130,185	99,402-160,968
2009 Q4		131,543	99,067-164,018

Table 2: Uk fundamental house prices estimated out-of-sample using data from 2002-2007 only.

2.2 Bivariate bubbles and contagion

Assessing contagion is a delicate theoretical and empirical issue in economics. A distinction needs to be made between genuine contagion and simple co-dependence, with

much of the literature failing to make an adequate distinction between the two (*Forbes and Rigobon (2002)*). Asset prices are assumed to exhibit non-zero correlations in normal times. Contagion occurs when there is a genuine change in correlation structure brought about by specific events or crises. Anything else is simply co-dependence.

In this paper we model contagion as occurring if, under the bubble model given by equation (9), region X is more informative about prices in region Y than Y is about X in a sense to be made precise in the Appendix in Section 4.3. Our analysis in this subsection splits into two parts. Firstly, our aim is to investigate a putative north-south divide. We examine the effect of London upon prices in the Northern and Midlands regions, see e.g. *Parkinson et al. (2009)* Chapter 3. We use equations (8-10) to test for bubbles. Further, we test the additional null hypothesis of no contagion (see equation (11) in Section 4.3). Secondly, we repeat the analysis along geographical lines for each of the Northern, Midlands and South Eastern regions of England.

North-South divide. The results are shown in Table 3 and are suggestive of bubbles in each case, although the p -value of 0.07 for Yorkshire-London is inconclusive in isolation. We test for contagion using a nonlinear t-test based on the delta method, *Bingham and Fry (2010)* Chapter 7, and restrict to a one-sided test since we know apriori that London is much more likely to exert a causal influence on the Northern and Midlands regions than vice-versa. The results give no evidence of contagion in the majority of cases but some contagious effects are apparent with London prices influencing those in the North and Yorkshire.

Geographical contagion. The results are shown in Table 4 and give conclusive evidence of a speculative bubble in each of the pairwise comparisons made. No evidence of contagion is found, suggesting that in each case the neighbouring regions have a roughly equal impact upon each other.

Region	No bubble p -value (One-sided)	No contagion p -value (One-sided)
London-North	0.006	0.058 (·)
London-North West	0.023	0.210
London-Yorkshire	0.071 (·)	0.065 (·)
London-East Mids	0.000	0.242
London-West Mids	0.001	0.281
London-East Anglia	0.001	0.443

Table 3: p -values for null hypotheses of no bubble and of no contagion London versus Northern and Midlands regions.

Area	Regions	No bubble <i>p</i> -value (One-sided)	No contagion <i>p</i> -value (Two-sided)
“North”	North-Yorkshire	0.005	0.899
	North-North West	0.002	0.269
	Yorkshire-North West	0.006	0.408
“Midlands”	East Mids-West Mids	0.000	0.906
	East Mids-East Anglia	0.000	0.573
	West Mids-East Anglia	0.000	0.633
“South East”	London-South East	0.000	0.612
	London-Outer Met	0.001	0.922
	South East-Outer Met	0.000	0.617

Table 4: *p*-values for null hypotheses of no bubble and of no contagion by geographical location.

3 Conclusions

We analysed English house prices over the period 2002-2007 based on the model for asset price bubbles in *Fry (2009)*. Using both univariate and bivariate models strong evidence for bubbles was found, with bubbles estimated to contribute 30-40% of observed prices. This figure compares reasonably to estimates of 12-25% in *Black et al. (2006)* and of 28-53% in *Hott and Monnin (2008)*. Historical price falls were slightly lower than our estimates of over-pricing and were typically in the range of 20-30%. Out of sample, prices appear to converge towards estimated fundamental prices and away from the previous speculative highs. The results for London were slightly atypical in that both the estimated bubble component and the historical fall in prices were less than those experienced in the rest of England. There was some suggestion that prices in London had a contagious effect, causing more rapid price rises in Yorkshire and the North of England than would ordinarily be expected.

4 Mathematical Appendix

4.1 Univariate bubble model (Table 1)

In this subsection we give a brief overview of the model in *Fry (2009)* which forms the basis of the analysis here. Let X_t denote the house price at time t , $t = 1, \dots, n$. We model purely random or non-bubble behaviour as

$$dX_t = rdt + \sigma dW_t. \tag{1}$$

In contrast, our model for a bubble is

$$dX_t = \left(r + \frac{\kappa\beta t^{\beta-1}}{\alpha^\beta + t^\beta} \right) dt + \sqrt{\sigma^2 - \frac{\kappa^2\beta t^{\beta-1}}{\alpha^\beta + t^\beta}} dW_t. \quad (2)$$

The interpretation of (2) is that a representative investor is compensated by the risk of a crash associated with the bubble by an increase in growth $(\kappa\beta t^{\beta-1})/(\alpha^\beta + t^\beta)dt$ term and a decrease in volatility $(-\kappa^2\beta t^{\beta-1})/(\alpha^\beta + t^\beta)$ term. If we define $\Delta X_t = X_{t+1} - X_t$ the log-likelihood function under equation (1) is

$$l(\theta) = -\frac{1}{2} \sum_{t=1}^{n-1} \log(2\pi\sigma^2) - \frac{1}{2\sigma^2} \sum_{t=1}^{n-1} (\Delta X_t - r)^2. \quad (3)$$

Under (2) $l(\theta)$ is given by

$$-\frac{1}{2} \sum_{t=1}^{n-1} \log \left(2\pi \left(\sigma^2 - \kappa^2 \ln \left(\frac{\alpha^\beta + (t+1)^\beta}{\alpha^\beta + t^\beta} \right) \right) \right) - \frac{\left(\Delta X_t - r - \kappa \ln \left(\frac{\alpha^\beta + (t+1)^\beta}{\alpha^\beta + t^\beta} \right) \right)^2}{\sigma^2 - \kappa^2 \ln \left(\frac{\alpha^\beta + (t+1)^\beta}{\alpha^\beta + t^\beta} \right)}. \quad (4)$$

We test for the presence of bubbles by calculating the likelihood ratio statistic (find a maximising set of parameters then subtract (3) from (4) and double) and perform a one-sided test against the mixture distribution

$$\frac{1}{2}\chi_2^2 + \frac{1}{2}\chi_3^2, \quad (5)$$

high values indicating a bubble. The distribution in (5) is obtained by randomly sampling from χ_2^2 with probability 0.5 and from χ_3^2 with probability 0.5. As discussed in *Fry (2009)* under model (2) the mean price is

$$P_B(t) = E(P(t)) = E(e^{X(t)}) = P(1)e^{\left(r + \frac{\sigma^2}{2}\right)(t-1)} \left(\frac{\alpha^\beta + t^\beta}{\alpha^\beta + 1} \right)^{\kappa - \frac{1}{2}\kappa^2}.$$

Under the fundamental model (1) the mean price is $P(1)\exp\{(r + \sigma^2/2)(t-1)\}$, and we estimate fundamental values by

$$P_F(t) = P(1)e^{\left(r + \frac{\sigma^2}{2}\right)(t-1)}. \quad (6)$$

The estimated speculative bubble component is given by

$$1 - \left(\frac{n-1}{\int_1^n \left(\frac{P_B(t)}{P_F(t)} \right) dt} \right) = 1 - \frac{(n-1) (\alpha^\beta + 1)^{\kappa - \frac{1}{2}\kappa^2}}{\int_1^n (\alpha^\beta + t^\beta)^{\kappa - \frac{1}{2}\kappa^2} dt}, \quad (7)$$

which can be calculated numerically.

4.2 Bivariate bubble model (Tables 3-4)

In the previous subsection we described the price of *one* asset. Here, we model the joint behaviour of two assets $Z_t = (X_t, Y_t)^T$. Our fundamental model is

$$dZ_t = rdt + \Sigma^{\frac{1}{2}}dW_t, \quad (8)$$

where r is a 2×1 vector and Σ is a 2×2 covariance matrix (see *Fry (2009)*) for further details. Our model for bubbles becomes

$$dZ_t = \left(r + \frac{\kappa\beta t^{\beta-1}}{\alpha^\beta + t^\beta} \right) dt + \sqrt{\sigma^2 - \frac{\kappa\kappa^T\beta t^{\beta-1}}{\alpha^\beta + t^\beta}} dW_t. \quad (9)$$

For the model (8) the likelihood equation is

$$\begin{aligned} l(\theta) = & -(n-1)\log(2\pi) - \frac{(n-1)}{2}\log(\sigma_X^2\sigma_Y^2 - \sigma_{XY}^2) - \frac{1}{2}\sum_{t=1}^{n-1} \frac{(\Delta X_t - r_X)^2}{\left(\sigma_X^2 - \frac{\sigma_{XY}^2}{\sigma_Y^2}\right)} \\ & + \sum_{t=1}^{n-1} (\Delta X_t - r_X)(\Delta Y_t - r_Y) \left(\frac{\sigma_{XY}}{\sigma_X^2\sigma_Y^2 - \sigma_{XY}^2} \right) - \frac{1}{2}\sum_{t=1}^{n-1} \frac{(\Delta X_t - r_X)^2}{\left(\sigma_Y^2 - \frac{\sigma_{XY}^2}{\sigma_X^2}\right)}. \end{aligned}$$

Under the bubble model (9) the likelihood equation is

$$\begin{aligned} l(\theta) = & -(n-1)\log(2\pi) - \frac{(n-1)}{2}\log(\sigma_{X,t}^2\sigma_{Y,t}^2 - \sigma_{XY,t}^2) - \frac{1}{2}\sum_{t=1}^{n-1} \frac{(\Delta X_t - \mu_{X,t})^2}{\left(\sigma_{X,t}^2 - \frac{\sigma_{XY,t}^2}{\sigma_{Y,t}^2}\right)} \\ & + \sum_{t=1}^{n-1} (\Delta X_t - \mu_{X,t})(\Delta Y_t - \mu_{Y,t}) \left(\frac{\sigma_{XY,t}}{\sigma_{X,t}^2\sigma_{Y,t}^2 - \sigma_{XY,t}^2} \right) - \frac{1}{2}\sum_{t=1}^{n-1} \frac{(\Delta Y_t - \mu_{Y,t})^2}{\left(\sigma_{Y,t}^2 - \frac{\sigma_{XY,t}^2}{\sigma_{X,t}^2}\right)}, \end{aligned}$$

where

$$\begin{aligned} \sigma_{X,t}^2 &= \sigma_X^2 - \kappa_X^2 \ln \left(\frac{\alpha^\beta + (t+1)^\beta}{\alpha^\beta + t^\beta} \right) \\ \sigma_{Y,t}^2 &= \sigma_Y^2 - \kappa_Y^2 \ln \left(\frac{\alpha^\beta + (t+1)^\beta}{\alpha^\beta + t^\beta} \right) \\ \sigma_{XY,t}^2 &= \sigma_{XY}^2 - \kappa_X\kappa_Y \ln \left(\frac{\alpha^\beta + (t+1)^\beta}{\alpha^\beta + t^\beta} \right) \\ \mu_{X,t} &= r + \kappa_X \ln \left(\frac{\alpha^\beta + (t+1)^\beta}{\alpha^\beta + t^\beta} \right) \end{aligned}$$

$$\mu_{Y,t} = r + \kappa_Y \ln \left(\frac{\alpha^\beta + (t+1)^\beta}{\alpha^\beta + t^\beta} \right).$$

We can again use the difference in the log-likelihood to test for bubbles. As before, calculate the likelihood ratio statistic and perform a one-sided test against the mixture distribution

$$\frac{1}{4}\chi_2^2 + \frac{1}{2}\chi_3^2 + \frac{1}{4}\chi_4^2, \quad (10)$$

with high values indicating a bubble. The distribution in (10) is obtained by randomly sampling from χ_2^2 with probability 0.25, from χ_3^2 with probability 0.5, and from χ_4^2 with probability 0.25.

4.3 Contagion from X_t to Y_t

The condition for contagion from X_t to Y_t is

$$\kappa_X < \kappa_Y. \quad (11)$$

We give an explanation for this interpretation below. Under the bubble model (9) the conditional variance of $Y_t|X_t$ is

$$(Var)(Y_t|X_t) = (1 - \rho_t^2) \sigma_{Y,t}^2$$

where

$$\rho_t^2 = \frac{\sigma_{XY}(t-1) - \kappa_X \kappa_Y \ln \left(\frac{\alpha^\beta + t^\beta}{\alpha^\beta + 1} \right)}{\sqrt{\left(\sigma_X^2(t-1) - \kappa_X^2 \ln \left(\frac{\alpha^\beta + t^\beta}{\alpha^\beta + 1} \right) \right) \left(\sigma_Y^2(t-1) - \kappa_Y^2 \ln \left(\frac{\alpha^\beta + t^\beta}{\alpha^\beta + 1} \right) \right)}},$$

$$\sigma_{Y,t}^2 = \sigma_Y^2(t-1) - \kappa_Y^2 \ln \left(\frac{\alpha^\beta + t^\beta}{\alpha^\beta + 1} \right).$$

Symmetrically,

$$(Var)(X_t|Y_t) = (1 - \rho_t^2) \sigma_{X,t}^2,$$

$$\sigma_{X,t}^2 = \sigma_X^2(t-1) - \kappa_X^2 \ln \left(\frac{\alpha^\beta + t^\beta}{\alpha^\beta + 1} \right).$$

Knowing X_t reduces the amount of uncertainty in Y_t by the amount

$$\sigma_{Y,t}^2 - (1 - \rho_t^2) \sigma_{Y,t}^2 = \rho_t^2 \sigma_{Y,t}^2.$$

Similarly, knowledge of Y_t reduces uncertainty in X_t by the amount

$$\sigma_{X,t}^2 - (1 - \rho_t^2)\sigma_{X,t}^2 = \rho_t^2\sigma_{X,t}^2.$$

We say that *contagion* occurs from X_t to Y_t if $\rho_t^2\sigma_{Y,t}^2 > \rho_t^2\sigma_{X,t}^2$, i.e. if X_t is *more informative* about Y_t than Y_t is about X_t , since knowledge of X_t produces the greater reduction in the conditional variance.

In the bubble models (2) and (9), crashes occur when there is a phase transition from stochastic to deterministic behaviour in prices (*Yeomans (1992), Sornette (2004)*), i.e. by a decrease in the volatility function (see equation (2) and *Fry (2009)*). Moreover, in (2) we must have that $\sigma^2 - \kappa^2\beta t^{\beta-1}/(\alpha^\beta + t^\beta) \geq 0$ with $\sigma^2 - \kappa^2\beta t^{\beta-1}/(\alpha^\beta + t^\beta) = 0$ corresponding to the situation where randomness completely disappears and prices are purely deterministic. These considerations imply the constraints

$$\begin{aligned}\sigma_X^2 &= \frac{\kappa_X^2(\beta - 1)^{1-\frac{1}{\beta}}}{\alpha}, \\ \sigma_Y^2 &= \frac{\kappa_Y^2(\beta - 1)^{1-\frac{1}{\beta}}}{\alpha}.\end{aligned}$$

Contagion from X_t to Y_t occurs if

$$\begin{aligned}\sigma_{X,t}^2 &< \sigma_{Y,t}^2, \\ \sigma_X^2(t-1) - \kappa_X^2 \ln\left(\frac{\alpha^\beta + t^\beta}{\alpha^\beta + 1}\right) &< \sigma_Y^2(t-1) - \kappa_Y^2 \ln\left(\frac{\alpha^\beta + t^\beta}{\alpha^\beta + 1}\right), \\ \kappa_X^2 \left[\frac{(\beta - 1)^{1-\frac{1}{\beta}}(t-1)}{\alpha} - \ln\left(\frac{\alpha^\beta + t^\beta}{\alpha^\beta + 1}\right) \right] &< \kappa_Y^2 \left[\frac{(\beta - 1)^{1-\frac{1}{\beta}}(t-1)}{\alpha} - \ln\left(\frac{\alpha^\beta + t^\beta}{\alpha^\beta + 1}\right) \right],\end{aligned}$$

i.e. $\kappa_X < \kappa_Y$.

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