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Abstract

This paper studies the dynamics of bargaining in an intrahousehold context. To explore long-term partner relationships, we analyse bilateral bargaining by considering that spouses take decisions sequentially. We conclude that, for the spouse who takes the second decision, a greater discount factor increases the set of possible sustainable agreements, as well as the proportion of time that this agent devotes to a family good.

Keywords: Family Bargaining, Stackelberg Game, Family Good.
JEL: C71, C72, C62, J12.

Introduction

The theoretical study of family decision-making, and its relationship with consumption and labor supply, has produced a significant body of literature. Some of this work takes as reference the theory of bilateral bargaining, in which it is recognized that families are composed of individuals with separate preferences and objectives. This literature has focused on the household allocation problem in a static setting, in which the results of the bargaining depend on the threat point that is fixed, that is to say, the status quo.

Family bargaining models have mainly identified this threat point with divorce (Manser and Brown, 1980; McElroy and Horney, 1981). In this case, it is assumed that the agents can communicate freely and that the fulfillment of agreements is guaranteed by an external contract or institution. Nevertheless, divorce does not necessarily constitute the only possible threat point in a bargaining process. Lundberg and Pollak (1993) and Chen and Woolley (2001) consider a non-cooperative equilibrium, the Cournot-Nash equilibrium. In this situation, the repeated interaction between agents over time can tacitly lead to cooperative results. More specifically, and in accordance with the folk theorem, a cooperative solution and, therefore, a Pareto-efficient solution, can be derived as a Nash equilibrium in a repeated game, always provided that there is some strategy which penalizes all deviations from the cooperative solution.

However, two questions arise in the intrahousehold bargaining framework. How do individuals arrive at an agreement about independent actions, and how are these agreements monitored and enforced? Households endure for more than a single period, and the multi-period aspect of the game can potentially substantially affect the solution. Thus, it is necessary to view bargaining in a multi-period setting, and to assume that the household decision-making process is a repeated game.

The multi-period analysis only makes sense when we assume that households are formed by individuals with separate preferences and objectives, who are involved in long-term relationships. Recent work on bargaining models of family decision-making has extended the one-period approach to a dynamic setting, focusing on the study of the implications of a couple's inability to make binding, legally-enforceable commitments about future behaviour.[1] A consumption-smoothing problem arises in that setting, which produces the inability of spouses to engage in intertemporal agreements, and that can lead to an inefficient allocation of household resources. The problem here is that a credible promise to compensate public good production with consumption in the future cannot be made, and this reduces incentives to specialize in such production. Even when the analysis focuses on the effect the bargaining power, defined as the amount of money a person earns relative to the partner's income, has on the possibility of implementing efficient outcomes at the household level, the question of making punishment credible has also been limited to the consumption-smoothing problem (see Rainer, 2008).

An alternative dynamic approach, focused on accommodating forward-looking agents in a dynamic environment, has been developed by Andaluz and Molina (2007). They use a repeated non-cooperative game in which both members of a family can contribute voluntarily to the provision of a family public good, and in which the maintenance of cooperative behavior in repeated games requires the threat of punishment through the return to the non-cooperative solution, thus guaranteeing the sustainability of solutions that are more efficient than the Cournot-Nash equilibrium. In this setting, their main finding is that the spouse with the greater bargaining power has a greater incentive to reach an agreement, and that neither of the agents have incentives to deviate from the bargaining solution.

Both the one-period approach and the multi-period non-unitary analysis predict that women's share of home time decreases. In the first case, this is due to an increase in female human capital, whereas in the second case it is due to the impossibility of making agreements in a dynamic setting. However, empirical evidence seems to contradict this implication. At the longitudinal level, Aguiar and Hurst (2007) show that, despite the increase in female labor force participation, specialization within the household has remained relatively unchanged. At the cross-sectional level, Akerlof and Kranton (2000) use PSID data to show certain evidence pointing to the fact that, when a wife works more hours than her husband outside the home, she still undertakes a larger share of housework. Thus, it appears that women's share of home time does not decrease, despite increases in women's relative earnings.

Our paper contributes to the dynamic aspects of partner relationship literature, by analysing the influence of the valuation of the current situation on both the time that each individual devotes to the provision of a family good, and the gains of well-being derived from cooperation. We extend the analysis of the dynamic aspects of the family bargaining process, developing a supergame in which the status quo is not only defined as non-cooperative, but also as sequential. The timing of the game is such that, at every period, the spouses take their decisions sequentially (the Stackelberg equilibrium). The developed model explains situations in which one of the spouses takes a decision, knowing the choice already taken by the other. Then, we address the question of how a household may succeed in using its resources efficiently through informal agreements, enforced through repeated interaction to determine the contribution to the provision of a family good whose consumption is non-rival. To the best of our knowledge, this approach has not previously been considered in the literature of family bargaining.[2]

As regards the main results, a greater discount factor implies an increase in the set of sustainable agreements derived from the bargaining, as well as an increase in the proportion of time devoted to the family good, for the spouse who decides second. These findings, which seem to contradict the one-period non-unitary household approach, confirm previous empirical analysis, in the sense that, if the woman decides second, then her contribution to housework may increase, despite the absence of differences in the opportunity cost of work for both spouses.

I Framework

Our approach captures an environment in which marital partners have separate preferences and objectives, and are involved in a long-term relationship. We develop an infinitely repeated game, in which the two members of a family, spouse 1 and spouse 2, may contribute voluntarily to the provision of a family good whose consumption is non-rival. We suppose that the agents do not know the moment of the dissolution of the marriage, and that the objective of each is to maximize the discounted value of their current utilities:

$$\sum_{t=1}^{\infty} \delta^{t-1} u_j(x_j, Q); (j = 1, 2)$$

where $\delta \in [0, 1]$ denotes the discount factor, common to both agents, x_j indicates the private consumption of agent j, $j = 1, 2$; Q represents the family good, $Q = q_1 + q_2$, with q_j being the proportion of hours that agent j devotes to the provision of this good.

The family good, Q, can include any situation which requires the joint performance of the spouses, e.g., the quality of the children or the maintenance of the home. We assume that the provision of the family good cannot be obtained in the market, that is to say, there are no private substitutes for the family good. Therefore, our model is not applicable to situations where family goods are purchased in the market and shared at home. This assumption is reasonable if we assume that, in certain cases, the private provision of family goods can produce losses of utility to couples, with the study of the trade-off between the time devoted to family goods, and the time devoted to the labour market, being necessary.

We suppose that the utility of each agent takes the following functional form (see Konrad and Lommerud, 2000):

$$u_1 = x_1 + Q - q_1^{\beta}; u_2 = x_2 + Q - q_2^{\alpha} \tag{1}$$

where $x_1 = w_1(1 - q_1)$ and $x_2 = w_2(1 - q_2)$, $w_j \in [0, 1]$ represents the wage rate for agent j and the maximum time available for each spouse is normalized to one.

We assume that the contribution to the family good not only reduces the time available to the labor market, but also has a psychological cost, represented by an increasing and convex function in each of these arguments $\left(q_1^{\beta}, q_2^{\alpha}\right)$, being $\alpha, \beta > 1$. In line with Konrad and Lommerud (2000), we suppose that individuals increasingly dislike spending more time on the production of the family good.

To address the issue of how the bargaining process works over a number of periods, we first solve the one-shot game, a Stackelberg game, and we then use this as the state game of an infinitely repeated game, using reversion to this non-cooperative Stackelberg equilibrium as the punishment for deviators. We then determine the optimum levels of consumption and contribution to the family good among the multiple stationary paths, using the symmetric Nash bargaining solution.

II The one-shot game

As we have mentioned above, in each period t, the non-cooperative equilibrium is the outcome of a Stackelberg game, in which the leader (spouse 1) commits to a certain quantity of provision of family good, while anticipating the optimal contribution of the follower (spouse 2).

As an example of a Stackelberg game, we can consider a situation in which the household division of labor may be affected by social norms, which are to a large extent enforced through non-market interactions.[3] In a less egalitarian social norms framework, the prescription that women should do the work at home may produce an alteration in the decision-making process.[4] As the consumption of the family good is non-rival, following Buchholz et al. (1997), its voluntary provision may be the result of a Stackelberg game, given that one agent can make a credible commitment that she/he will be able to contribute no more than a possibly very small amount of household good. Thus, one spouse becomes a Stackelberg leader, and will take the reactions of the follower into account when deciding the contribution to the family good.

Applying the backward induction procedure, we begin by obtaining the equilibrium corresponding to spouse 2 (the follower). Formally:

$$\begin{aligned} &\underset{x_2,q_2}{Max}\ u_2 = x_2 + Q - q_2^{\alpha} \\ &s.to\ x_2 = w_2(1-q_2) \\ &\qquad q_1 = \bar{q}_1 \end{aligned} \tag{2}$$

From here, we deduce the levels of consumption and the provision of the family good:

$$q_2^* = \left(\frac{1-w_2}{\alpha}\right)^{1/\alpha-1};\, x_2^* = w_2\left[1-\left(\frac{1-w_2}{\alpha}\right)^{1/\alpha-1}\right] \tag{3}$$

and the utility level:

$$u_2^d = w_2 + \bar{q}_1 + (\alpha-1)\left(\frac{1-w_2}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} \tag{4}$$

For spouse 1 (the leader) we formulate the following maximization problem:

$$\underset{x_1,q1}{Max}\ u_1 = x_1 + Q - q_1^{\beta}$$
$$s.to\ x_1 = w_1(1 - q_1)$$
$$q_2 = q_2^*$$

$$\tag{5}$$

and we obtain the level of private consumption, and the provision of the household good made by spouse 1:

$$q_1^* = \left(\frac{1 - w_1}{\beta}\right)^{1/\beta - 1};\ x_1^* = w_1\left[1 - \left(\frac{1 - w_1}{\beta}\right)^{1/\beta - 1}\right] \tag{6}$$

Therefore, the levels of utility in the non-cooperative solution for both spouses are:

$$u_1^*(q_1^*, q_2^*) = w_1 + (\beta - 1)\left(\frac{1 - w_1}{\beta}\right)^{\beta/\beta - 1} + \left(\frac{1 - w_2}{\alpha}\right)^{1/\alpha - 1} \tag{7}$$

$$u_2^*(q_1^*, q_2^*) = w_2 + (\alpha - 1)\left(\frac{1 - w_2}{\alpha}\right)^{\alpha/\alpha - 1} + \left(\frac{1 - w_1}{\beta}\right)^{1/\beta - 1} \tag{8}$$

In addition to the solution obtained in (3) and (6), which is an interior solution, it is possible to analyse the situations in which one of the spouses is the contributor to the family good (see Bucholz et al., 1997).[5] We can distinguish the following types of Stackelberg equilibria:

Case 1 ***The leader is the only contributor.***

In this case, $\bar{Q}$, the family good, is only supplied by spouse 1, from (5) and with $q_2 = 0$, we can obtain the total family good:

$\bar{Q} = \left(\frac{1-w_1}{\beta}\right)^{\frac{1}{\beta-1}}$.

and the levels of utility of both spouses:

$\overline{u}_1(\bar{Q},0) = w_1 + (\beta - 1)\left(\frac{1-w_1}{\beta}\right)^{\beta/\beta-1}$; $\overline{u}_2(\bar{Q},0) = w_2 + \left(\frac{1-w_1}{\beta}\right)^{1/\beta-1}$

Case 2 ***The follower is the only contributor.***

From the maximization problem of spouse 2, (2), and with $q_1 = 0$, *we may determine the levels of provision of family good which are only provided by spouse 2:*

$\widetilde{Q}= \left(\frac{1-w_2}{\alpha}\right)^{\frac{1}{\alpha-1}}$.

with the levels of utility for both spouses in this case being:

$\widetilde{u}_1(0,\widetilde{Q}) = w_1 + \left(\frac{1-w_2}{\alpha}\right)^{\frac{1}{\alpha-1}}; \widetilde{u}_2(0,\widetilde{Q}) = w_2 + (\alpha-1)\left(\frac{1-w_2}{\alpha}\right)^{\alpha/\alpha-1}$

It is straightforward to deduce that $u_i^*(q_1^*, q_2^*) > \overline{u}_i(\bar{Q}, 0)$, and that $u_i^*(q_1^*, q_2^*) > \widetilde{u}_i(0, \widetilde{Q})$, $\forall\ 0 < w_i < 1; (\alpha, \beta) > 1$, $(i = 1, 2)$. Thus, under the structure of preferences defined above, for all values of the parameters, the interior solution constitutes a dominant strategy for both agents in the the Stackelberg equilibrium.[6] This is a reasonable result, since we do not consider the possibility that one spouse compensates the other.

In Figure 1, we represent the curves of indifference of the spouses in the non-cooperative solution. For spouse 1, the slope of the curve of indifference in the non-cooperative equilibrium is zero in (q_1^*, q_2^*), and is increasing and convex if $q_1 > q_1^*$ and $q_2 > q_2^*$, $\left.\frac{dq_2}{dq_1}\right|_{u_1^*} > 0$, $\left.\frac{d^2q_2}{dq_1^2}\right|_{u_1^*} > 0$.

Analogously, for spouse 2, the slope of the curve of indifference that contains the solution of the one shot game is equal to minus infinity in the combination (q_1^*, q_2^*), and is increasing and concave when $q_1 > q_1^*$ and $q_2 > q_2^*$, $\left.\frac{dq_2}{dq_1}\right|_{u_2^*} > 0$; $\left.\frac{d^2q_2}{dq_1^2}\right|_{u_2^*} < 0$.

This situation is clearly inefficient. As Kapteyn and Kooreman (1990) show, all the points located inside the area formed by both curves of indifference are Pareto superior to the equilibrium of the one shot game. They graphically represent several allocations that have been used in the literature. For our purpose, we focus on those points located in the contract curve CC' which are efficient solutions, since, when the decisions are taken in a multiperiod framework, the loss from non-cooperation accumulates, and there appear strong incentives to reach a Pareto Superior agreement.

However, repetition alone is not enough to eliminate the non-cooperative static equilibria. The one-shot Stackelberg equilibrium is another possible outcome of the repeated game. Therefore, it is necessary that both spouses can implicitly create a strategy that deters deviations from a cooperative solution, and reaches a combination that is Pareto superior to the one-shot non-cooperative equilibrium.

We adopt the so-called trigger strategy (Friedman, 1971), so that, when there is a deviation from the cooperative solution, the levels of private consumption and the provision of the family good revert to those of non-cooperative equilibrium. The threat of retaliation, through reversion to this punishment path, is credible, since it is not in the best interest of either agent to deviate unilaterally from this non-cooperative equilibrium, and it sustains Pareto Superior outcomes.

For the sake of simplicity, we only consider the case of stationary paths for all t. Obviously, we have not included all the factors that affect cooperation in a family framework, but we have included certain relevant factors, which have not previously been used in the family bargaining literature.

A stationary path is sustainable in a subgame perfect equilibrium, if it satisfies the following conditions:

$$u_i(x_i, Q) - u_i^* \geq 0; i = 1, 2 \tag{9}$$

$$\frac{u_2(x_2, Q)}{1-\delta} \geq u_2^d(q_1) + \frac{\delta u_2^*}{1-\delta} \tag{10}$$

Condition (9) establishes that both spouses have incentives to cooperate, since the well-being these agents obtain in the cooperative solution is greater than or equal to the well-being obtained in the non-cooperative solution. Condition (10) determines that the spouse who decides second has no incentive to deviate from the efficient solution. However, the follower, given q_1, could react by deviation to maximize his/her own utility. In order to tackle this problem, it is necessary to introduce inequality (10), which states that the discounted value of the well-being of the follower, conforming to the specified path, the left-hand side of the inequality, is greater than the well-being from the optimal one-shot deviation and then reversion from the following period onwards to the punishment path, the right-hand side of the inequality. In this setting, since the one-shot game is sequential, the leader's discount rate plays no role. If spouse 1 deviates from a cooperative agreement, this is immediately observed by spouse 2, thus eliminating any possible short-term well-being gains for spouse 1. Therefore, the maintenance of the cooperative equilibrium depends on the agent who decides second.[7]

Unless δ is very high, constraint (10) is always binding in equilibrium, whereas (9) is not. Denote $\widetilde{\delta}$ the minimum value of discount factor for which (10) is not binding, and let the function $g = g(q_1, q_2, \delta)$represent the long-term gain from the follower's cooperation. Formally:

$$g(q_1, q_2, \delta) = (1 - w_2)q_2 - q_2^{\alpha} + \delta q_1 - (\alpha - 1)\left(\frac{1 - w_2}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \delta\left(\frac{1 - w_1}{\beta}\right)^{\frac{1}{\beta-1}} \tag{11}$$

From (11), we deduce that the set of Pareto-superior solutions to the equilibrium of the one-shot game, sustainable by way of the repeated interaction, is greater when the discount factor is higher. We observe that when $q_1 > q_1^*$ and $q_2 > q_2^*$, the function g is increasing and concave, with the value of its slope being minus infinity in the non-cooperative solution (q_1^*, q_2^*) and increasing in δ.

As shown in Figure 1, among the Pareto-superior combinations (q_1, q_2), we can identify a subset of sustainable solutions which can be achieved by way of repeated interaction. In particular, all the combinations of (q_1, q_2) located to the right of the broken line.

Figure 1. Set of Pareto-Superior Solutions.

(Figure 1 here)

III Bargaining solution

We address the question of how a household may succeed in using its resources efficiently in a self-enforcing manner, that is to say, through informal agreements, enforced through repeated interaction. In order to determine an equilibrium among the multiple stationary paths, it is necessary to specify how the levels of consumption and contribution to the family good are chosen among all possible solutions. Focusing on the study of sustainable solutions, we suppose that there exists a bargaining process, not modeled explicitly here and, as a result, both spouses take their decisions by way of the symmetric Nash bargaining solution.[8] That is to say, they choose the stationary paths of private consumption and family good provision that maximize the product of the utilities, after being normalized by the utility levels of the non-cooperative solution, and within the set of sustainable equilibria. Formally, the problem becomes, for $\delta < \widetilde{\delta}$:

$$\underset{x_1,x_2,q_1,q_2}{Max} J(x_1, x_2, q_1, q_2) = (u_1 - u_1^*)(u_2 - u_2^*)$$

$$s.to\ g(q_1, q_2, \delta) = 0 \tag{12}$$

When δ takes value zero, the non-cooperative solution satisfies restrictions (9) and (10). Alternatively, if this factor takes value one, all the Pareto-superior solutions are indeed sustainable and, consequently, the bargaining agreement constitutes an efficient solution. Between these two extremes, the solution of the previous problem depends on the discount factor through constraint $g(q_1, q_2, \delta) = 0$, in the above maximization problem.[9] In fact, the bargaining solution is determined by way of the tangency between an Iso-J line and an Iso-g line, as shown in Figure 2.

Figure 2. Set of possible sustainable bargaining solutions

(Figure 2 here)

Under some regularity assumptions (see Appendix) we obtain the following proposition:

Proposition 1

For $\delta < \widetilde{\delta}$:

The contribution to the family good of the spouse who decides second (follower) is increasing with respect to the discount factor: $\frac{dq_2}{d\delta} > 0$.

The influence of the discount factor on the contribution to the family good of the spouse who decides first (leader) is ambiguous: $\frac{dq_1}{d\delta} \lesseqgtr 0$.

Proof.(See appendix).

From this result, it is possible to deduce that the agent who decides second, will devote more time to the provision of the family good when the discount factor is greater. However, the path of the contribution to the family good made by the spouse who decides first can be increased or decreased, depending on the discount factor. An increasing evolution implies that the difference between the hours that this agent devotes to the family good in the cooperative solution, and the hours determined in the non-cooperative equilibrium, is not very significant.

When the discount factor is greater, cooperation is more easily sustained, leading to a higher provision of the family good by the follower. Note that we have found that the discount factor can increase the provision of the family good without imposing any restriction on the follower's relative earnings, or on the psychological cost produced by devoting time to the family good. Thus, if the woman is the follower, the higher the discount factor, the higher the provision of the family good, even when the opportunity cost of work is the same for both spouses.

For the leader, spouse 1, there exists a level of provision of the household good, $\hat{q}_1$, with $\hat{q}_1 > q_1^*$, which represents the minimum value from which the relationship between the discount factor and the level of provision of the family good made by this agent becomes negative. By contrast, for $q_1^* < q_1 \leq \widehat{q}_1$, when the discount factor increases, the provision of the family good by the leader increases. There exists a set of sustainable solutions where the spouse who decides first increases or decreases his/her provision of the family good, depending on the discount factor. Thus, situations in which the husband devotes much less time to the family good than does the wife can be possible sustainable agreements, even without differences in the salaries of the spouses, as a result of a high discount factor.

The discount factor also reflects the subjective probability that the game will end. The higher the discount factor, the lower the probability that the game will end in the near future. Even when there is a possibility that the game will end sometime in the future, as in the case of intertemporal agreements within the family, subject to renegotiation resulting from increases in potential earnings, our optimum provision of the family good can support a near-efficient outcome, as long as each spouse believes, with a high enough probability, that the game will continue. Thus, it is possible to make agreements in a dynamic setting with a finite horizon (see Espinosa and Rhee, 1989).

Knowing the evolution of the paths of the provision of the family good, we can deduce the influence of the discount factor on the level of utility derived from the cooperation.

Corollary:

For $\delta < \widetilde{\delta}$:

In the bargaining solution, a variation of the discount factor can increase or reduce the levels of utility for both spouses: $\frac{dU_1}{d\delta} \lesseqgtr 0; \frac{dU_2}{d\delta} \lesseqgtr 0.$

Proof.(See appendix).

Specifically, we observe that, for $q_1 > \widehat{q}_1$,when the discount factor increases, the provision of the family good by the leader decreases, generating opposite effects on the level of utility of the spouses. The level of utility increases for the leader and decreases for the follower. By contrast, for $q_1^* < q_1 \leq \widehat{q}_1$,when the discount factor increases, the provision of the family good by the leader increases, and the levels of utility can increase or decrease for both spouses.

IV Conclusions

Family bargaining models have usually presented the household allocation problem in a static setting. However, households endure for more than a single period, and it is necessary to view bargaining in a multi-period context which can potentially substantially affect the solution.

In this context, we have set up a supergame in an intrahousehold framework, in which both spouses may contribute voluntarily to the provision of a family good. Assuming that the status quo is not only defined as non-cooperative, but also as sequential (equilibrium of Stackelberg), and that the efficient allocation is given by way of the symmetric Nash bargaining solution, we deduce the influence of the valuation of the present on the time that each individual devotes to the provision of the household good, and its effects on the gains of well-being derived from bargaining.

The following conclusions are obtained. Firstly, the set of possible sustainable agreements derived from bargaining is greater when the discount factor of the spouse who decides second is higher.

Secondly, the contribution of the follower to the family good is increasing with respect to the discount factor, whereas the relationship between the discount factor and the contribution made by the leader is ambiguous.

Thirdly, the gains of well-being derived from the bargaining show an ambiguous relationship to the discount factor. The effect of the discount factor will be positive or negative for both spouses, depending on the increase in the time devoted by the leader to the production of the family good in the bargaining solution.

These findings seem to contradict the one-period non-unitary household approach, but they do confirm previous empirical analyses, in the sense that the woman's contributions to housework may increase, if she decides second, despite the absence of differences in the opportunity cost of work for both spouses.

Notes

[1] Some examples are Aura (2002), Basu (2006), Browning (2000), Lundberg and Pollak (2003), Lundberg et al. (2003), Mazzocco (2007), Wells and Maher (1998).

[2] Using methodologies very similar to ours, some studies have examined the relationship between a union and a firm by developing a supergame with a threat to return to a non-cooperative Stackelberg equilibrium (Espinosa and Rhee, 1989; De la Rica and Espinosa, 1997).

[3] Following Vagstad (2001), an average woman about to be married is much better skilled to keep and maintain a house than her coming husband, and this difference is found for a broad range of housekeeping activities. There are skills that rarely are acquired through formal educational, but rather passed on from parents to children or acquired by self studies. Often, skill acquisition, choice of education and many other decisions in life do not reflect rational decision-making but can be seen as responses to some social norms.

[4] This argument is similar to the doing gender hypothesis proposed by the sociological literature to explain the same empirical regularity in a variety of countries (see, among

others, Bittman et al. (2001) for the Australian case and Brines (1994) for the U.S).

[5] It is interesting to note that, under the preferences specified above, the reaction functions for both spouses have null slope, and thus, the provision of the family good by both agents is strategically independent. As a consequence, the levels of consumption and the provision of the family good obtained in the Stackelberg equilibrium coincide with those obtained in a Nash equilibrium.

[6] This result differs from that obtained by Bucholz et al. (1997). However, it is important to note that, in our model, the type of Stackelberg equilibrium does not depend on the aggregate income of the spouses nor on the redistribution of income between spouses. This is due to the fact that the incomes of the agents are endogenously determined, and that one spouse cannot compensate the other spouse.

[7] This is not applicable to the situation of the standard case of Nash reversion, since, in this situation, decisions are taken simultaneously, and to obtain stationary paths it is necessary to include an additional inequality similar to that introduced for spouse 2, (10), but in this case it must also be established for spouse 1 .

[8] This solution implicitly assumes a bargaining process which results in the generalised bargaining solution (see Binmore et al. 1986; Harsanyi, 1977).

[9] It is straightforward deduce that under the structure of preferences used in this analysis, the provision of the family good is independent of the discount factor when we use the Cournot-Nash equilibrium as the threat point in the analysis.

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Appendix

Proof of Proposition

To be able to characterize the solution of the maximization problem propose in (12), we introduce the following assumptions:

We suppose that $J(x_1, x_2, q_1, q_2)$ is strictly concave.

The level curves of $J(x_1, x_2, q_1, q_2)$ are monotone, in that way:

$$\frac{\partial \left(J_1/J_2\right)}{\partial q_1} < 0 \tag{13}$$

For $q_1 > 0, q_2 > 0$, the first order conditions are:

$J_1 + \lambda g_1 = 0$

$J_2 + \lambda g_2 = 0$

$g(q_1, q_2, \delta) = 0$

Being $\lambda > 0$ the multiplier of the problem of maximization. From that it is possible to deduce the following equation:

$$\frac{J_1}{J_2} = \frac{g_1}{g_2} \tag{14}$$

Differentiating with respect to δ, we obtain that:

$(J_{11} + \lambda g_{11})\frac{dq_1}{d\delta} + (J_{12} + \lambda g_{12})\frac{dq_2}{d\delta} + g_1\frac{d\lambda}{d\delta} = -\lambda g_{1\delta}$

$(J_{21} + \lambda g_{21})\frac{dq_1}{d\delta} + (J_{22} + \lambda g_{22})\frac{dq_2}{d\delta} + g_2\frac{d\lambda}{d\delta} = -\lambda g_{2\delta}$

$g_1\frac{dq_1}{d\delta} + g_2\frac{dq_2}{d\delta} = -g_\delta$

These equations can be written in matrix form as:

$$\begin{pmatrix} (J_{11} + \lambda g_{11}) & (J_{12} + \lambda g_{12}) & g_1 \\ (J_{21} + \lambda g_{21}) & (J_{22} + \lambda g_{22}) & g_2 \\ g_1 & g_2 & 0 \end{pmatrix} \begin{pmatrix} \frac{dq_1}{d\delta} \\ \frac{dq_2}{d\delta} \\ \frac{d\lambda}{d\delta} \end{pmatrix} = \begin{pmatrix} -\lambda g_{1\delta} \\ -\lambda g_{2\delta} \\ -g_\delta \end{pmatrix}$$

The matrix on the left hand side is the bordered Hessian. Applying the Cramer's rule it is possible to obtain the changes in q_2 when δ change:

$$\frac{dq_2}{d\delta} = \frac{1}{|D|}\begin{vmatrix} (J_{11} + \lambda g_{11}) & -\lambda g_{1\delta} & g_1 \\ (J_{21} + \lambda g_{21}) & -\lambda g_{2\delta} & g_2 \\ g_1 & -g_\delta & 0 \end{vmatrix} \tag{15}$$

Where $|D|$ is the determinant of the bordered Hessian. The second order conditions of the maximization problem require that $|D|$ be positive.

Therefore, the sigh of $\frac{dq_2}{d\delta}$ is determined by the sign of (15):

$$sign\left(\frac{dq_2}{d\delta}\right) = sign\left[-\lambda\delta(1 - w_2 - \alpha q_2^{\alpha-1}) + (q_1 - q_1^*)(J_{11}\left(1 - w_2 - \alpha q_2^{\alpha-1}\right) - \delta J_{12})\right] \tag{16}$$

Given that $(1 - w_2 - \alpha q_2^{\alpha-1}) < 0 \wedge q_2 > q_2^*$, and under (13) and (14), we deduce that (16) is positive: $\frac{dq_2}{d\delta} > 0$.

Differentiating the restriction with respect to δ, we have:

$$\frac{dq_1}{d\delta} = -\frac{1}{\delta}\left[(q_1 - q_1^*) + (1 - w_2 - \alpha q_2^{\alpha-1})\frac{dq_2}{d\delta}\right] \tag{17}$$

Thus, the sign of $\frac{dq_1}{d\delta}$ is the sign of the numerator:

$$sign\left(\frac{dq_1}{d\delta}\right) = sign\left[-(q_1 - q_1^*) - (1 - w_2 - \alpha q_2^{\alpha-1})\frac{dq_2}{d\delta}\right]$$

Given that $(q_1 - q_1^*) > 0, (1 - w_2 - \alpha q_2^{\alpha-1}) < 0 \wedge \frac{dq_2}{d\delta} > 0$,we deduce that $\frac{dq_1}{d\delta} \lesseqgtr 0$.

From (15) and (17), we can determine a value $\hat{q}_1$, with $\hat{q}_1 > q_1^*$, which represents the minimum value from which the relationship between the discount factor and the level of provision of the family good made by this agent becomes negative. So, when $q_1 > \hat{q}_1$, we obtain that $\frac{dq_1}{d\delta} < 0$ and when $\hat{q}_1 \geq q_1 > q_1^*$ what we obtain is that $\frac{dq_1}{d\delta} > 0$.$\square$

Proof of Corollary

Applying the envelope theorem, we derive the utility function of both spouses (1) with respect to δ :

For the spouse 1, we obtain:

$$\frac{dU_1}{d\delta} = \frac{\partial U_1}{\partial q_1}\frac{dq_1}{d\delta} + \frac{\partial U_1}{\partial q_2}\frac{dq_2}{d\delta} \tag{18}$$

Taking into account (17), this expression takes the following form:

$$\frac{dU_1}{d\delta} = \frac{-(1 - w_1 - \beta q_1^{\beta-1})\,(q_1 - q_1^*)}{\delta} + \left[1 - \frac{(1 - w_1 - \beta q_1^{\beta-1})(1 - w_2 - \alpha q_2^{\alpha-1})}{\delta}\right]\frac{dq_2}{d\delta} \tag{19}$$

Given that $(1 - w_1 - \beta q_1^{\beta-1}) < 0$, $(1 - w_2 - \alpha q_2^{\alpha-1}) < 0$, $\frac{dq_2}{d\delta} > 0$, $(q_1 - q_1^*) > 0, 0 < \delta < 1$, $0 < q_i < 1$ $(i = 1, 2)$,we deduce that:

$\frac{dU_1}{d\delta} \lesseqgtr 0$.

Analogously, we have that:

$\frac{dU_1}{d\delta} = (1 - w_1 - \beta q_1^{\beta-1})\frac{dq_1}{d\delta} + \frac{dq_2}{d\delta} \lesseqgtr 0$.

Given that $\frac{dq_2}{d\delta} > 0$ and with $\frac{dq_1}{d\delta} < 0$, we obtain that $\frac{dU_1}{d\delta} > 0$ with $q_1 > \hat{q}_1$. When $\frac{dq_1}{d\delta} > 0$, we obtain that $\frac{dU_1}{d\delta} < 0$ with $\hat{q}_1 \geq q_1 > q_1^*$.

For the spouse 2,

$\frac{dU_2}{d\delta} = \frac{\partial U_2}{\partial q_1}\frac{dq_1}{d\delta} + \frac{\partial U_2}{\partial q_2}\frac{dq_2}{d\delta}$

Introducing (17), we deduce that:

$\frac{dU_2}{d\delta} = \left(1 - \frac{1}{\delta}\right)\left(1 - w_2 - \alpha q_2^{\alpha-1}\right)\frac{dq_2}{d\delta} - \frac{\left(q_1 - q_1^*\right)}{\delta} \lesseqgtr 0.$

Analogously, we obtain that

$\frac{dU_2}{d\delta} = \left(1 - w_2 - \alpha q_2^{\alpha-1}\right)\frac{dq_2}{d\delta} + \frac{dq_1}{d\delta} \lesseqgtr 0.$

Given that $\frac{dq_2}{d\delta} > 0$ and with $\frac{dq_1}{d\delta} < 0$, we obtain that $\frac{dU_2}{d\delta} < 0$. Remember that it is necessary that $q_1 > \hat{q}_1$ to obtain a negative relationship between the discount factor and the level of provision of the family good made by the spouse 1. When $\frac{dq_1}{d\delta} > 0$, we obtain that $\frac{dU_2}{d\delta} > 0$, with $\hat{q}_1 \geq q_1 > q_1^*$.□

Figure 1. Set of Pareto-Superior Solutions

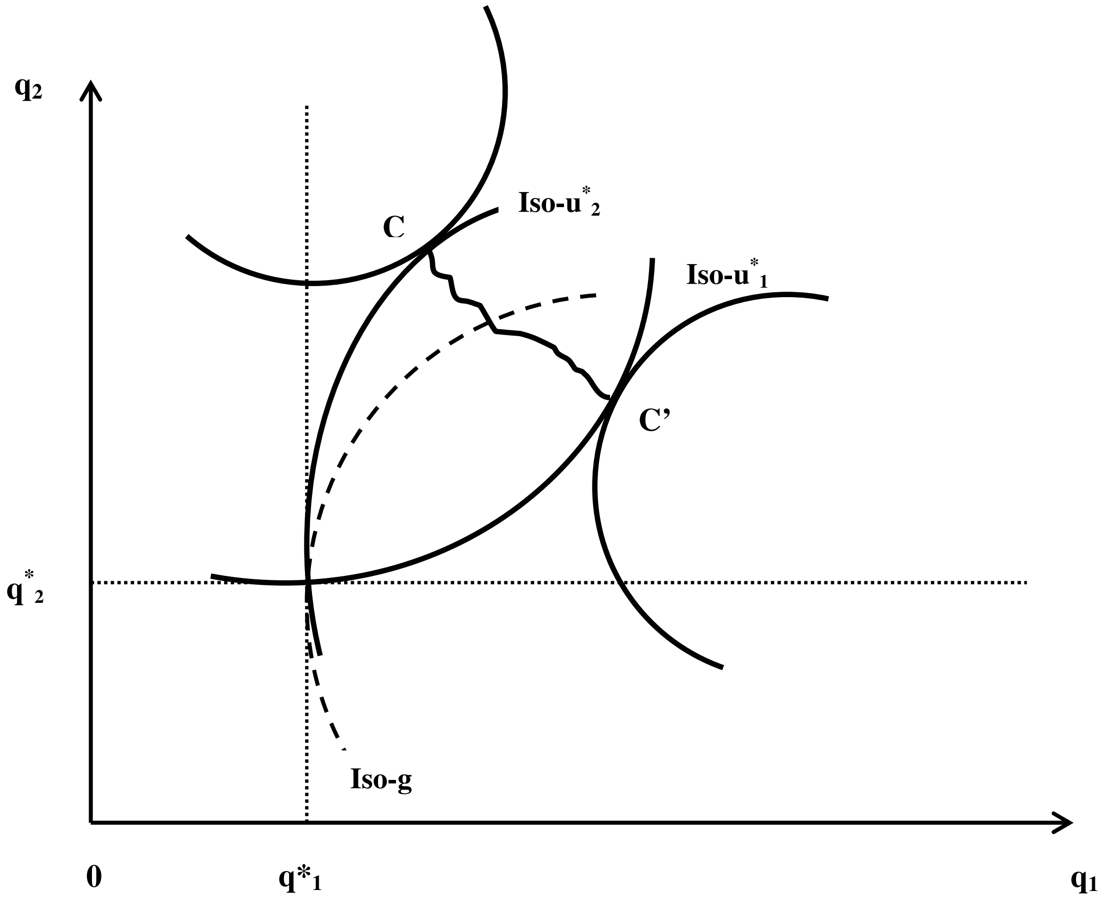

Figure 2. Set of possible sustainable bargaining solutions