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15 October 2009

Online at <https://mpra.ub.uni-muenchen.de/17821/>  
MPRA Paper No. 17821, posted 12 Oct 2009 14:15 UTC

**The Theorem of Proportionality in Mainstream Capital Theory:  
An Assessment of its Applicability**

By

George C. Bitros

**Abstract**

This paper surveys and assesses the empirical literature that bears on the applicability of the theorem of proportionality, which asserts that depreciation is proportional to the outstanding capital stock. All available evidence shows that: a) the rates of depreciation and retirements vary from year to year in response to changes in conventional economic forces like utilization, maintenance and repair, the prices of new capital goods, etc., and b) while the approximation of the distribution of depreciation rates by a single parameter may be characterized by simplicity and ease of use, at the same time it thwarts the advances that can be achieved by returning to a general equilibrium model centered on the time structure of capital and the useful lives of its components. For this reason, it is concluded that, the sooner this theorem is replaced by an endogenous theory of depreciation and replacement, the better for economic theory and policy.

JEL Classification: E220

Keywords: Capital longevity, replacement, depreciation, scrappage, maintenance, utilization, obsolescence.

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## 1. Introduction

Over fifty years ago, when [Schumpeter \(1954\)](#) was writing his monumental history of economic analysis, he characterized the method by which mainstream economic theorists approach the study of economic phenomena as follows:

“Economic theory... cannot indeed, any more than can theoretical physics, do without simplifying schemata or models that are intended to portray certain aspects of reality and take some things for granted in order to establish others according to certain rules of procedure. So far as our argument is concerned, the things (propositions) that we take for granted may be called indiscriminately either hypotheses or axioms or postulates or assumptions or even principles, and the things (propositions) that we think we have established by admissible procedure are called theorems” (p. 15).

By implication, since theorems are deduced from certain axioms by the laws of logic, they may be considered as absolutely true. But then there arises the question: Are the theorems of economics “true” in the same sense as those of algebra and geometry, or are they merely probable, like the propositions of the literary economists? The answer given by [Papandreou \(1958\)](#), who introduced into economics the logician’s distinction between “models” and “theories”, is that there is a fundamental difference. This amounts to the qualification that, while the theorem(s) deduced from an economic model may be considered absolutely true, at the same time though the model itself and its axioms may not be applicable to any real world phenomena, and this in turn may render the theorem(s) in question vacuous. Therefore, to make sure that an economic theorem is useful, we must supplement it with a proposition specifying the real world situation(s) to which it applies. This proposition may be called an applicability or empirical accountability theorem and as such it is highly probable but never absolutely certain. In economics a theorem derived from a model (mathematical, logical or what have you) with one or more applicability theorems appended to it constitutes an economic theory.!

Viewed in the context of these methodological remarks, the theory of depreciation and replacement<sup>2</sup> that dominates mainstream economics was established in three phases. In the first phase, drawing on the claim that the theorem of proportionality could be derived from a capital renewal model of rational entrepreneurial behavior, [Jorgenson \(1963; 1965\)](#) tested it empirically using data from the two-digit standard industrial classification system of manufacturing industries in the United States. In particular, he estimated an equation with gross investment as dependent variable, in which he included the lagged capital stock among the regressors and interpreted its coefficient as an estimate of the replacement rate,  $\hat{\delta}$ , on the understanding that, when capital depreciates at a constant rate, depreciation is dual to replacement. Then he tested

the significance of  $\hat{\alpha}$  and found that it was statistically different from zero; and, finally, on account of his findings he concluded that the theorem of proportionality offered a good explanation of the processes of depreciation and replacement of capital. After this initial phase there followed a second one in which Jorgenson along with several associates contributed a barrage of empirical papers where they demonstrated the wide applicability of the theorem in the framework of the neoclassical theory of investment. Lastly, in the third phase, [Jorgenson \(1974\)](#) derived the theorem formally from renewal theory and assessed the relevant evidence that had emerged in the meantime. As a result of these decade long efforts, the proportional theory of depreciation and replacement gained wide acceptance among mainstream researchers and most previous interest in the time structure of capital nearly eclipsed.

However, soon after the theorem of proportionality was invoked and applied, several economists began to raise doubts about its underpinnings. Some of the doubts emanated from theoretical considerations. Some other derived from empirical studies; and still some other sprung from the nature of the theories and practices adopted in neighboring scientific fields. But all shared a common feature. Namely, they refuted the conceptual and empirical foundations on which it rested. So the questions that come naturally to mind are: Why have all arguments against it failed to attract a significant following among researchers? What inferences might we draw in this regard from the programs of research in micro and macroeconomics? Where else outside the confines of mainstream economics might we look for insights regarding the processes of depreciation and replacement of durable goods? Are there alternative grounds on which to judge its applicability? To shed some light on these questions, in [Bitros \(2009\)](#) I assessed the standing of the theorem of proportionality from a conceptual and methodological standpoint; and, having found that it is too simplistic, to say the least, I concluded that it should be abandoned or at least reconsidered, if not for any other reason, because it retards the progress of theory, particularly in the critical area of research efforts to merge short term with long term macroeconomic analyses. My objective here is to assess its applicability by reference to the results of the empirical tests to which it has been subjected in the last sixty years.

More specifically, based on the approaches adopted by researchers, the present survey covers four categories of tests. The first of them comprises tests that focus on replacement investment. The tests in the second category center on the price-age profiles of various capital goods on the presumption that, if they follow the geometric distribution and remain fairly stable from one year to the next, this would confirm that the rate of depreciation is independent of age and hence that its distribution can be characterized by a single number, i.e.

exactly as the theorem of proportionality maintains. The third category of tests addresses the behavior of retirements on the expectation that, if they remain invariant with respect to such conventional economic forces as utilization, maintenance, technological change, etc., this would constitute evidence in favor of the theorem of proportionality; and, finally, the last category consists of empirically oriented models that test for the microeconomic and macroeconomic implications of replacement, depreciation, and scrappage. From this survey it emerges that the evidence is overwhelmingly against the theorem of proportionality. Thus my overall assessment is that the neoclassical theory of depreciation and replacement lacks theoretical and empirical foundations and that it is time to supplant it with a theory where the distinct processes of depreciation, scrappage and replacement are explained endogenously.

The paper is structured as follows. Section 2 reviews the evidence from empirical studies of replacement investment. Sections 3 and 4 do the same by turning to investigations of economic depreciation and scrappage, respectively. Section 5 presents and comments on the results from miscellaneous tests in the areas of production and productivity, business cycles, and economic growth and development, and, finally, Section 6 concludes with a summary of the findings and the conclusions.

## **2. The evidence from replacement investment**

Suppose that a firm in the transport business purchases a new truck of certain trucking capacity. Can we say from this observation alone by how much the trucking capacity of the firm will change in the period when the new truck is incorporated into its fleet? No, we cannot. Because, even when an old truck is simultaneously retired, it is impossible to observe directly the loss of trucking efficiency that the fleet will suffer during the same period due to usage, technological obsolescence, lapses in maintenance and repairing, etc. From this simple example it follows that depreciation is unobservable. Thus, due to the inherent lack of data, over the years researchers who have attempted to explain replacement investment have followed three approaches. In the first and older of them the analysis centers on the useful life of capital as an endogenous variable.<sup>3</sup> In the second approach, the variable of interest is an estimate of replacement investment obtained through questionnaires or other survey methods; and in the third approach the focus is on the loss-of-efficiency patterns of capital goods. This section reviews the evidence from these three strands of literature.

### **2.1 Tests based on the endogeneity of useful lives**

Before the advent of the neoclassical theory of depreciation and replacement in the early

1960s, not much empirical work of the modern bent took place in this area. If one conducted a search, as one could do easily by looking at the references in [Smith \(1961\)](#) or [Dean \(1962\)](#), one would find that the relevant literature consisted predominantly of models that expanded on various aspects of the classical theory of replacement, which aimed at deriving conditions for optimal decision making in the replacement and in the choice of equipment. In short there were only a few applications and those that existed originated mainly in the field of operations research. However, if one wishes to understand from an economist's point of view where the state of the empirical research stood in the 1950s in this field, no study is more suitable than that by [Smith \(1957\)](#). Why is this so it follows from the remarkable degree to which this study achieved its key objectives. So, let me dwell briefly on the structure of the test and the main findings and conclusions from this study.

The test consisted of two parts. The first of them was designed to compute the optimal average useful life of the truck-tractors in the sample, whereas the second part provided for a comparison between the optimal replacement policy thus computed with the one that was being applied by the truck-tractor owning firm. To compute the optimal useful life, the author estimated with meticulous attention to detail all parameters and variables that enter into the economic replacement model in the form he had advanced it in his 1955 PhD dissertation. In particular, on the revenue side he allowed for the influence of such age-related variables as load factors and technological obsolescence, which was due to the higher efficiency of newer diesel engines; on the cost side he included fuel consumption and maintenance and repair expenditures, which varied also with age; and he treated all other costs as constant. As for the resale price of truck-tractors, he was able to estimate a price-age profile showing an abrupt 30% drop in the price of one-year old units, with their prices falling linearly thereafter. Then, introducing the revenue, cost and resale price functions into the model, he calculated that the optimal useful life of a new truck-tractor lied between 3.2 and 4.2 years, depending inversely on the estimate of the rate of technological obsolescence.

In the sequel he set out to compare the optimal with the actual replacement policy. To do so he calculated from the sample that the actual policy was about 5 years. In turn this implied that the optimal policy was from 1 to 2 years shorter than the actual policy and this was a significant difference that had to be explained. Thus, with the help of the model he calculated the cost that the firm would incur in terms of forgone net profits if it postponed the replacement of the average truck tractor by 1 year or more. From these calculations it emerged that the penalty the firm would pay would be less than 2% of profits,

which indicated that the profitability of the truck owning firm was insensitive to substantial deviations from the optimal replacement policy.

As for the conclusions, it is particularly important to stress the following. According to the results replacement delays are not expected to be costly in terms of forgone profits. Hence, in markets where the firm is faced with capital rationing or rising supply curve of capital, it pays to postpone replacement during a period of expansion and accelerate it during a period of contraction. This implies in turn that: a) the usefulness of an economic replacement theory cannot be discounted unless all items in the current and capital accounts of the modeled firm are non age-related, or if there are some that are age-related, but are negligible relative to the non age-related ones, and b) de-coupling of expansionary from replacement investment is impossible if capital markets are imperfect.

The above evidence in favor of the economic approach to replacement was strongly corroborated by [Rust \(1987\)](#) who studied the behavior of the superintendent of maintenance at the Madison (Wisconsin) Metropolitan Bus Company. Using ten years of monthly data on bus mileage and engine replacements for a subsample of 104 buses in the company fleet, he computed by stochastic dynamic programming procedures a stopping rule as a solution to the trade-off problem between minimizing maintenance costs and minimizing unexpected engine failures. From the results it turned out that the replacement decision was significantly influenced by the expectations of the superintendent regarding the cost of replacements, the cost of regular bus maintenance, his perceptions about the customer goodwill losses due to unexpected engine failures, and other economic considerations. Clearly then, as in [Smith \(1957\)](#) and [Eilon, King and Hutchinson \(1966\)](#) who investigated the replacement of fork lifts, what this study found was that replacement policy matters and that it is a common business practice.<sup>4</sup>

Moreover, similar results have been obtained in more recent decades by investigations in an extensive variety of fixed assets, including nuclear plants, railroad engines and freight cars, oil tankers, cargo ships, etc. In the case of ships for example [Jin and Kite-Powell \(1999\)](#) offer an indication of how large this literature is. Thus, at the level of a business unit operating a fairly well specified durable good, all available evidence refutes the applicability of the theorem of proportionality.

## 2.2 Tests based on estimates of replacement investment

[Feldstein and Foot \(1971\)](#) were among the first to raise serious doubts about the relevance of the neoclassical theory of replacement, i.e. the theorem of proportionality, from an empirical point of view. Using annual data over the 1948-1968 period from the McGraw-Hill *Survey of*

*Business Plans for New Plants and Equipment*, in conjunction with the U.S. Department of Commerce series of planned gross investment, they found that the aggregate replacement/capital stock ratio varied significantly under the influence of conventional economic forces. But two years later [Jorgenson \(1974\)](#) evaluated the consistency with which they had constructed the critical variables in their model and concluded that:

“Feldstein and Foot have not successfully avoided the necessity for direct observation of both replacement investment and capital stock in studying the validity of the geometric approximation to the replacement distribution...”(p. 123).

As it would be expected, this verdict undermined the credibility of their results. However, Jorgenson’s criticism did not apply to [Eisner \(1972\)](#), where the variables of both replacement investment and capital stock had been constructed consistently using data at the firm level from the same survey. Hence his findings that expenditures planned for replacement and modernization: a) were not a constant proportion of capital, and b) related to changes in past and expected sales and previous depreciation charges and profits, corroborated strongly those obtained by [Feldstein and Foot \(1971\)](#). But [Jorgenson \(1974\)](#) failed to make even a passing reference to Eisner’s results.

The same remark holds also for the rebuttal that [Feldstein \(1972\)](#) offered regarding the consistency with which they had constructed the variable of the stock of capital. In particular, here is how he defended the validity of their results:

“[Jorgenson \(1974\)](#) has argued that there is no adequate capital stock series for the estimates of equation 5 in [Feldstein and Foot \(1971\)](#). His criticism is unwarranted. Feldstein and Foot did use the perpetual inventory method to develop a capital stock measure that is consistent with the historical variations in the replacement ratio (1971, p. 53). Jorgenson apparently overlooked this in his criticism of the use of the official Department of Commerce series. Moreover, the estimates in [Feldstein and Foot \(1971\)](#) are quite insensitive to the choice of a capital stock. Nevertheless, it is useful to have a method of testing some implications of the Proportionality Replacement Hypothesis that does not require any capital stock.” (p. 6, ft.)

Following the latter suggestion, [Feldstein \(1972/1974\)](#) conducted a barrage of four tests using only the data on gross investment and planned replacement investment expenditures from the above source and on the basis of the results he concluded that:

“Each of the four tests outlined above provides strong evidence that the proportional replacement hypothesis is inconsistent with actual experience. This incompatibility is so clear that it could not possibly be due to statistical chance or measurement errors. The only reasonable inference would seem to be to reject the hy-



pothesis that expansion investment induces an immediate increase in replacement investment as described by the proportional replacement hypothesis.”(p. 10)

Moreover, the evidence reported by [Smith \(1974\)](#) regarding the replacement of automobiles, using data from the University of Michigan Annual Survey of Consumer Finances, corroborated fully the view of the dissenters that conventional economic forces determine replacement investment. Consequently, the results from this segment of the literature contradicted the theorem of proportionality conclusively.

### 2.3 Tests based on the loss-of-efficiency schedules

Departing from past conventions, [Coen \(1975\)](#) introduced two distinct definitions of depreciation. These are economic depreciation and capacity depreciation. When referring now to economic depreciation economists focus on the losses of market value that durable goods suffer as they grow old, whereas when they refer to capacity depreciation their attention centers on the losses of productive efficiency that durable goods suffer with age. To measure economic depreciation, research efforts have been directed at tracing the relationship between market prices and age, i.e. to identify and estimate price-age profiles. On the other hand, to measure capacity depreciation, economists have attempted to estimate loss-of-efficiency schedules, which are based on the shape of the mortality distribution of investments over the services lives of the capital goods involved. Below I will review the approaches and the results regarding the loss-of-efficiency schedules and relegate the same task for the price-age profiles to the next section.

An old approach, which has shown some revival in recent years, has been to search for replacement “echoes” in gross investment. [Meyer and Kuh \(1957\)](#) adopted it in the context of a sample of firm data over the period 1946-1950 on the rationale that:

“... since replacement investment is included in gross investment the net impact of the echo effect should be ascertainable even when using gross investment as the dependent variable-although perhaps not as precisely as would be desirable.” (p. 93)

Their test was structured as follows: a) the data were grouped into 15 industries, corresponding roughly to the two-digit industries of the standard industrial classification system; b) the dependent variable was defined and measured as gross investment divided by gross fixed assets; c) the age of capital was computed as accumulated depreciation reserves divided by gross fixed assets, and d) the authors estimated two models, i.e. a profit model and a sales model. From the estimations it turned out that the coefficient of the age variable had the right sign and it was statistically significant in both models only in one industry, whereas it was

statistically significant only in three out of thirty regressions. Thus, reflecting on these results, [Jorgenson \(1974\)](#) concluded that:

“The proportion of significant results—three out of thirty regressions—is not out of line with the null hypothesis that the echo effect plays no role in the determination of investment for individual firms.”(p. 211)

However, aside from the problem of the ratio variables that plagued this study, which in the light of more recent analysis by [Kronmal \(1993\)](#) raises the possibility that the estimated coefficients suffered from significant biases, the results might have turned out the way they did because of another reason. This is that during the above period the size of replacement investment in gross investment in the sample might have been relatively small to show any traceable influences of the average age of capital on gross investment. Thus, a more appropriate interpretation would have been that the results were inconclusive.

The next group of researchers addressed the loss-of-efficiency as an engineering process. This was the group of Jorgenson and his associates. As indicated already in the introduction, for them a sufficient test for the theorem of proportionality was to introduce in a gross investment equation the previous period capital stock,  $K_{t-1}$ , and test the sign and statistical significance of its coefficient,  $\delta$ . [Jorgenson \(1963; 1965\)](#) came up with this test as follows. Initially he obtained an expression for the stock of capital that a net worth maximizing firm would desire to have,  $K_t^*$ , assuming that replacement investment was predetermined,  $\delta K_{t-1}$ . Then, with the desired capital stock in hand and the additional assumption that the completion of new investment projects takes time, he obtained the demand for net investment as a distributed lag function of the gap that the firm experienced in its desired capital between the present and the previous period. Finally, adding to net investment the replacement investment, he arrived at the equation of gross investment that he tested. From the above, in conjunction with the conclusions reached in the preceding subsection, it follows that Jorgenson’s test was based on three key assumptions. Namely that: a) the firm operated in perfect capital markets; b) replacement investment was invariant with respect to such variables as utilization, technological obsolescence, maintenance and repair expenditures, etc., and c) net investment evolved independently of replacement investment because the lags that the firm faced in the completion of new projects did not exert any influences on the latter. However, despite the additional issues that these assumptions raised, as well as the strong dissenting view expressed in the following excerpt from [Feldstein \(1972/1974\)](#):

“Although [Jorgenson and Stephenson \(1967; 1969\)](#) claim to have tested the proportionality replacement hypothesis, their calculations actually test a quite different proposition. In estimates of two-digit investment behavior with gross investment as dependent variable, they included the lagged capital stock among the regressors and interpreted its coefficient as an estimate of the replacement rate,  $\hat{\lambda}$ . They then tested and confirmed that  $\hat{\lambda}$  was different from zero but not different from the average annual rate used by the Office of Business Economics to construct the capital stock series. Neither of these tests refers to the stability and constancy of the annual replacement ratio. They show only that on average the amount of replacement is related to the capital stock, a very much weaker proposition than the proportionality replacement hypothesis used in investment studies.”(1972, Footnote 2, p. 3)

all reservations went unheeded and the tide of the neoclassical theory of depreciation and replacement continued to take hold.

To be sure, at some point certain researchers turned their attention to the new project completion lags that [Jorgenson \(1963; 1965\)](#) had introduced in order to derive the investment equation that he tested. Since the lags had been imposed exogenously, their objective was to rationalize the gradual adjustment of the actual to the desired capital stock by grafting into the model internal and/or external adjustment costs. In view of the latter, the neoclassical firm had an incentive to postpone or accelerate replacement so as to allow for a faster or slower adjustment to the desired level of capital. Yet in these endeavors researchers continued to model replacement investment as a constant proportion of the previous period capital stock and the opportunity to switch to a consistent economic theory of replacement was lost. Perhaps this more than anything else explains why ever since mainstream economic theorists have ignored the interdependence of net and replacement investment under adjustment costs.

Still another approach was that of [Coen \(1975; 1980\)](#), which was based on how well gross investment might be predicted by allowing for different combinations of service lives and mortality distributions in a model where the constraints of the theorem of proportionality and the independence of net and replacement investment were relaxed. In his 1975 paper he tested his model by considering 40 such combinations for equipment and 35 for structures, using data from 1949 to 1966 for 21 two-digit standard industrial classification industries from the sector of U.S Manufacturing. Drawing on the results he concluded that:

“Geometric decay of productive capacity—a commonly employed assumption in recent studies of investment behavior—does not appear to underlie actual capital spending decisions. Equipment generally evidences losses in productive capacity as it ages, though not necessarily at a geometric rate, but structures in the major-

ity of industries suffer no loss in productive capacity over their service lives (they resemble one-hoss shays).”(p. 73)

But when he re-estimated the model five years later, he obtained significantly different results. In particular, in his 1980 paper he concluded that:

“The predominant loss-of-efficiency pattern in Table 3.1 is the one denoted by GD-FIN, which is characterized by geometrically decaying weights truncated at the end of service life, the rate of decay being twice the reciprocal of the service life. The straight-line (SL) loss-of-efficiency pattern did, however, yield superior results in many instances. Although there is of course, no way of aggregating the loss-of-efficiency patterns, it seems fair to say that something approximating geometric decay rather than straight line loss-of-efficiency is typical of capital used in manufacturing.” (p. 124)

To explain the striking differences in the two sets of results, and hence in the conclusions, the author offered two explanations. First, that in his 1980 study he used somewhat revised data and, second, that while his earlier results were based solely on the goodness of fit within the 1949-1966 period, his more recent results reflected also the accuracy in postsample predictions for 1967-1971. But the discrepancies raised serious questions about the robustness of his results and rendered highly uncertain the quality of evidence that could be obtained by the procedure that he suggested as well as the theory of investment underlying it.

Also noteworthy is the approach adopted by [Pakes and Griliches \(1984\)](#). To demonstrate the way in which their method for estimating distributed lags in short panels of data may be applied, these researchers estimated a profit equation in which they included among the predictors several lags of past investment expenditures. Then, by interpreting the coefficients of the lagged terms of investment expenditures as estimates of the loss-of-efficiency function they concluded that:

“What is clear is that the usual depreciation schemes which assume that the contribution of past investments declines and immediately with age are wrong. If anything, there may be an ‘appreciation’ in the early years as investments are completed, shaken down, and adjusted to.”(p. 259)

At the same time though they noted that their results derived from a model in which one of the maintained hypotheses was that the time shape of the estimated relationship, i.e. the depreciation pattern, was independent from the circumstances and factor prices that prevailed at the time each vintage of investment was purchased, as well as from their subsequent changes. This qualification is important because it applies with equal force to the evidence from all studies that define and measure capital stock as a distributed lag process of past investments.

Finally, consider the study by [Doms \(1996\)](#). The approach that this author adopted was to introduce the capital stock in the form of a distributed lag function of past investments into a production function and estimate loss-of-efficiency schedules along with all other parameters. A panel data set consisting of annual observations for inputs and outputs at the level of individual raw-steel-producing plants employing the same technology was used in the estimations and the production function took the translog form. The loss-of-efficiency schedules that were computed from the estimated parameters of the production function were such that the author was able to conclude that:

“The geometric decline in efficiency found in this paper supports the geometric pattern of economic depreciation found by [Hulten and Wykoff \(1981a\)](#), since physical deterioration and economic depreciation coincide when deterioration occurs geometrically (see [Jorgenson \(1974\)](#)).” (p. 85)

However, perhaps because: a) both coefficients of the translog production function corresponding to the lagged investment terms were not statistically significant, and b) the model suffered from the limitations stressed in the preceding paragraph, at the end the author surmised that his results were tentative.

In conclusion, the evidence from studies that focus on the patterns by which capital goods lose productive efficiency as they grow old is mixed. Various approaches using various sets of data lead to results that lack satisfactory consistency and stability. Thus, even if one were ready to admit that on balance the evidence from this bibliography tends to favor the geometric distribution, and hence the applicability of the theorem of proportionality, one would be advised not lose sight of the serious shortcomings of the models employed in carrying out the tests. [Pakes and Griliches \(1984\)](#) have spelled out clearly these limitations to warn about the precarious nature of the results from such research endeavors.

### **3. The evidence from studies of economic depreciation**

After it became fairly clear that they tested a weaker proposition rather than the theorem of proportionality, Jorgenson and his associates searched for evidence in the patterns of economic depreciation. Thus, for reasons that will become obvious shortly, it is pertinent to begin with the study by [Wykoff \(1970\)](#), which investigated the case of used automobiles in USA over the 1950-1969 interval. The results from the tests that he performed showed that: a) in the first year depreciation was twice the rate in the succeeding years; b) after the first year depreciation appeared to be exponential, and c) over the said time interval the various auto models depreciated at quite different rates. Reviewing these results four years later [Jorgenson \(1974\)](#)

concluded that they, along with the results from [Griliches \(1960\)](#) for used farm tractors, [Cagan \(1965\)](#) for used automobiles, and [Hall \(1971\)](#) for pickup trucks, supported his contention that depreciation is exponential. But here is how [Wykoff \(1970\)](#) himself interpreted his results:

“This study indicates that the assumptions economists have been making in studies of consumer durables and capital equipment are very strong, and in some cases probably sufficiently far from the mark as to render the results questionable. Depreciation rates for automobiles are not exponential; different types of automobiles display individualistic characteristics as they age, and the assumption that depreciation patterns remain fixed over time is in doubt. Whether or not these results can be generalized to other durables and capital equipment remains to be seen, but certainly more work is needed in automobile studies.”(p. 172)

From these, but also from the results that [O’Herlihy \(1965\)](#) had obtained a few years earlier for automobiles in the United Kingdom, it was fairly clear that at least in the case of automobiles the patterns of economic depreciation rendered the theorem of proportionality inapplicable.

The next major contribution came in the form of a series of highly influential papers by [Hulten and Wykoff \(1981a, b, c\)](#) who investigated the evolution of price-age profiles for vintages from eight classes of commercial and industrial equipment and structures using actual transaction prices. In doing so they regressed first the second hand prices of these assets on their ages and time by applying the Box-Box power transformation. From this operation they found that the depreciation rates varied significantly, particularly in the early years. Then, they regressed the logarithm of the fitted used asset prices from the above regressions on the ages and time and found an average economic depreciation rate, which they called Best Geometric Approximation (BGA). Finally, by drawing on these results, they concluded that:

*“...a constant rate of depreciation can serve as a reasonable statistical approximation to the underlying Box-Cox rates even though the latter are not geometric (their italics). This result, in turn, supports those who use the single parameter depreciation approach in calculating capital stocks using the perpetual inventory method”*([1981a](#), p. 387).

This conclusion was open to the same criticisms that [Feldstein \(1972, p. 3, ft. 2\)](#) addressed to [Jorgenson and Stephenson \(1967; 1969\)](#) almost ten years earlier and implied, in his words, that [Hulten and Wykoff \(1981a, b, c\)](#):

“...tested and confirmed that  $\hat{\delta}$  was different from zero but not different from the average annual rate used by the Office of Business Economics to construct the capital stock series. Neither of these tests refers to the stability and constancy of the annual replacement ratio. They show only that on average the amount of re-

placement is related to the capital stock, a very much weaker proposition than the proportionality replacement hypothesis used in investment studies.”

Perhaps this explains why, when [Hulten, Robertson, and Wykoff \(1989\)](#) revisited this issue several years later, not only did they pay attention to the issue of stationarity, but also they felt obliged, on the one hand, to stress that depreciation rates varied from one year to the next in response to various factors, and on the other, to switch emphasis from the statistical properties of the BGA to the advantages of simplification that it helps achieve. In particular, here is how they interpreted their new findings:

“While depreciation almost certainly varies from year to year in response to a variety of factors, we have found that a major event like energy crises, which had the potential of significantly increasing the rate of obsolescence, did not in fact result in a systematic change in age-price profiles. This lends confidence to procedures that assume stationarity in order to achieve a major degree of simplification (and because of non-stationarity is so difficult to deal with empirically). Or, put simply, the use of a single number to characterize the process of depreciation (of a given type of capital asset) seems justified in light of the results...” (p. 255)

Still, even though the model they and [Wykoff \(1989\)](#) considered was expanded to allow for the quality improvement in the newly purchased capital goods, several problematic aspects in their specification and estimation of price-age profiles came under criticism from three perspectives.

To begin with the most fundamental, consider the research by [Biorn \(1998\)](#).<sup>5</sup> Drawing on the intuition that the price-age profile of a stock of homogeneous capital goods is related to the survival and efficiency patterns of its units, the task undertaken by this author was to set up a neoclassical vintage capital model so as to: a) derive the relationship between the price-age profile and the survival and efficiency curves endogenously; b) highlight the conditions under which the latter curves might be identified and estimated solely from observed second hand prices, and c) conduct an experiment with the help of actual data to check whether the curvatures of these curves were compatible with the observed convexity of the price-age profiles of second hand prices. Reflecting on his results, this author concluded as follows:

“Using U.S. data for one specific category of machines, previously analyzed by [Hulten, Robertson, and Wykoff \(1989\)](#), we find, for our three sets of parametric survival and efficiency curves, evidence of concavity of the survival curve. But we are unable to draw a firm conclusion regarding convexity versus concavity of the efficiency curve. Our conclusion in this respect is far less obvious than those of Hulten and Wykoff and most of their followers, based on similar data, but different price-age profiles. In particular, we cannot exclude that both the survival of capital units and the efficiency of each remaining unit follow concave functions of age, as researchers assuming exponential decay *a priori* do.”(p. 632)

These conclusions undermined seriously the confidence in the evidence regarding the rates of economic depreciation from price-age profiles. But none of the supporters of this approach showed any interest to respond.

The second criticism emanated from the study by [Nelson and Caputo \(1997\)](#). These authors adapted the model presented by [Parks \(1979\)](#) in the light of the flexible functional form introduced by [Hulten and Wykoff \(1981a, b, c\)](#) to explain the depreciation rates from the price-age profiles of two types of aircraft over four five year periods from 1971 to 1991. From the meticulous tests they run it emerged that, even though the rates of depreciation implied by the Box-Cox estimates were not geometric, they could be approximated reasonably well by a constant rate of depreciation. This finding confirmed the results obtained by [Hulten and Wykoff \(1981a, b, c\)](#) and [Hulten, Robertson, and Wykoff \(1989\)](#). But at the same time it turned out that maintenance expenditures related negatively to depreciation rates, which implied that the price-age profiles shifted upwards (downwards) as maintenance expenditures increased (declined). Thus this evidence indicated that in the presence of maintenance expenditures the test of stationarity that the latter authors had performed most likely would have failed. Considering this finding in conjunction with the shift that has taken place in modeling maintenance and repair expenditures in microeconomic and macroeconomic applications, a re-examination in this light of the approach adopted by Hulten et al. in computing rates of economic depreciation from price-age profiles would seem to be overdue.

Lastly, the third criticism stems from the realization that so far all research efforts in the front of price-age profiles consider the retirements of capital to be a mechanistic process, totally invariant with respect to the intensity of utilization, technological change, uncertainty about the rate of technological change,<sup>6</sup> and other market forces. However, if in the coming years empirical research advances in the direction of the recent theoretical contributions cited in [Bitros \(2009\)](#), where retirements of capital along with utilization and maintenance and repair expenditures are treated as an endogenous variables, identification and estimation of price-age profiles from second hand prices alone may become econometrically difficult, if at all possible. For this reason the development of the theory in this area must proceed in parallel with the development of new data sets from second hand markets of structures and equipment.

Why it is very crucial to model retirements explicitly while estimating price-age profiles has been highlighted clearly by the striking differences that [Oliner \(1996\)](#) found in the annual depreciation rates of machine tools industry in the U.S. Carrying out the estimation with and without adjustment of used prices for retirements, he found that:



“ Using the adjusted prices substantially alters the results, with the changes due entirely to the hazard rate for retirement now embedded in the price variable. First column 3 shows that  $\beta_3$  is negative and significant at the 1 percent level, which provides strong evidence that the rate of depreciation becomes more rapid with age. The estimated annual depreciation rate for a forty-year-old machine (shown at the bottom of column 3) is 18.1 percent, more than six times the 2.9 percent rate for a ten-year-old machine. I should stress, however, that the depreciation rate reaches double-digits only after the investment cohort has lost about three-quarters of its initial value. As a result, the cohort depreciation function largely reflects the slower depreciation rates mainly affecting the shape of the right-hand tail. Column 4 shows that the best geometric approximation to the age-varying depreciation rate is 9.5 percent, well above the 3.5 percent rate estimated from observed prices; the difference is the annual average hazard rate.”(p. 74)

Hence, in view of the lack of robustness in such estimates, [Jorgenson \(1994/1996\)](#) ought to have considered the possibility that economic depreciation might not be geometric after all. On the contrary, he chose to side with the results obtained by Hulten and his associates.

In short, the price-age profiles of various classes of durable goods, from which rates of economic depreciation are computed and used in the compilation of capital stock series by means of the perpetual inventory method, shift systematically under the influence of conventional economic forces. This implies that the approximation suggested by Hulten et al. to obtain constant rates of economic depreciation is untenable, thus undercutting the empirical support that it lent to the theorem of proportionality. Therefore, the insistence on an unfounded conceptualization for the sake of achieving a major degree of simplification is accompanied by significant costs in terms of measurement errors and the time is ripe for the practice to be abandoned or at least reconsidered.

#### **4. The evidence from studies of scrappage**

Under the theorem of proportionality producer durables are predicted to remain in production ad infinitum without scrappage ever taking place. This prediction is patently counterfactual and the hypothesis survives only because those who support it argue that it holds as an approximation and usually truncate the process of decay by imposing an arbitrary end of useful life constraint. But if, as found by [Bitros \(1976a;1976b\)](#), [Everson \(1982\)](#), [Kim \(1988\)](#), [Gylfanson and Zoega \(2001\)](#), and many others, the loss-of efficiency of durable goods varies with the intensity of utilization, maintenance, technological obsolescence, and other economic variables, then not only would retirements take place but also the owners of durables may have significant discretion in determining their useful lives. This prospect, in conjunction with the realization that scrappage constitutes a sizeable component of replacement investment, led

empirically minded researchers to inquire whether the scrappage rate related to such variables in a statistically significant way. For, if it did, in all probability the same would be true with respect to the rate of depreciation and replacement.

To this effect [Walker \(1968\)](#) investigated the determinants of auto scrappage. He found that the deviations from an age determined trend in the auto scrappage rate were very well explained by the rate of turnover in automobile ownership and the level of used car prices relative to the costs of representative car repair prices. However, because of various data limitations this evidence was viewed as tentative. For this reason in [Bitros and Kelejian \(1974\)](#) we run a test based on high quality data from the electricity generating capacity in the United States. The results enabled us to conclude that:

“A component of the replacement ratio, namely, the scrappage ratio, is significantly related to such economic variables as gross investment, maintenance expenditures, and the interest rate. Therefore the replacement rate should be also related to these variables” (p. 277).

Since then the literature on scrappage has been enriched significantly. For example, [Cowing and Smith \(1977\)](#) refined further on our data from the electric utilities in the United States and with their estimates reiterated the above conclusion. [Parks \(1977; 1979\)](#) revisited the determinants of scraping rates for postwar vintages of automobiles and found that the probability of a car to be scrapped related significantly to such variables as its age, relative repair costs, and various characteristics of durability. [Lioukas \(1982\)](#) extended further the results on scrappage in the case of electric utilities in the United Kingdom by including in the estimations retirement backlogs. [Cockburn and Frank \(1992\)](#) investigated the retirement of oil tankers and established that it is driven by markets conditions; [Goolsbee \(1998\)](#) researched the retirement of airplanes from the fleet of Boeing 707s and found that fuel costs, the business cycle, the cost of capital and firm financial performance are very important factors for capital retirement decisions; and, last but not least, as evidenced by [Esteban \(2007\)](#), the sprawling literature in the area of accelerated vehicle replacement leaves no uncertainty regarding the economic calculus used by owners of automobiles when deciding whether to participate or not in such government sponsored programs.

To summarize the evidence from this segment of the literature, it suffices to state that there is not a single study having looked into the determinants of scrappage that has not shown that this major component of replacement investment is related systematically to certain key economic variables.

## 5. Other evidence

Indications about the applicability of the theorem of proportionality may be obtained also from research in various other fields of economics that employ capital as a factor of production. For example, production and productivity analyses, growth accounting, factor shares, business cycles, and studies of market structure and profitability are some of the areas where the approach for reckoning depreciation and replacement has paramount implications. Hence, by necessity the survey below is selective.

### 5.1 The evidence from production and productivity analyses

Suppose that the price-age profile of a particular class of productive assets is stationary over the sample period. Also, assume that the Best Geometric Approximation (BGA) to the average depreciation rate does not differ statistically from the depreciation rates of the underlying individual assets at various ages. According to Hulten et al., under these circumstances the BGA could be employed to compute the corresponding capital stock series by means of the perpetual inventory method. Would it then be ok to use this capital stock series, along with output, labor, and other inputs and their prices to estimate the parameters of a production function?

The answer is that it would not be ok and it is based on the following argument, which is due to [Miller \(1990\)](#). The deviations of the BGA from the depreciation rates of the underlying individual assets in each year of the sample would lead to mismeasurements of the capital stock from the one year to the next and the errors in the latter would be correlated with the other variables in the model, thus leading to errors in the covariances from which the conventional econometric techniques estimate the coefficients that enter into the calculation of such standard parameters as the elasticities of substitution between productive inputs, the rates of productivity growth, the biases in technical change, etc. Even worse is the finding by [Barnhart and Miller \(1990\)](#) that small errors in the perpetual inventory series of the capital stock lead to large errors in the covariances of the estimated models. As a result, they concluded the following:

“The constant replacement rate assumption produces systematic non-random errors in the measured capital variable which are correlated with the other independent variables. This causes an obvious problem. The ordinary least squares (OLS) estimators are biased and inconsistent. The main contribution of this paper is to show that this problem is very difficult, if not impossible, to overcome, even with the use of the instrumental variables estimation techniques, the standard solution.” (p. 638).

In the light of this evidence, one would have expected that researchers working in the areas of

production and productivity analysis would have become aware of the difficulties that accompany all capital stock series derived through perpetual inventory procedures and that they would have tried to change course. However, in their great majority applied economic theorists have continued working as if the above papers were never published.<sup>7</sup>

An exception has been [Prucha and Nadiri \(1996\)](#) and [Nadiri and Prucha \(1996\)](#) who started on a new path. To highlight their innovation, let us consider for example their former paper. In it they estimate a model with the help of data from the U.S. Electrical Machinery Industry to analyze the structure of production, the demand for productive inputs, the growth of productivity and the capacity utilization. The model they estimate comprises two variable inputs, labor and materials, two quasi-fixed inputs, physical and R&D capital, and output. Finally, the innovation is that the specification of their model allows for the representative firm in this industry to determine the depreciation rate of the physical capital endogenously, whereas the depreciation rate of the R&D capital is kept fixed. Reflecting on their results, they conclude the following:<sup>8</sup>

“Based on our tests we accept model 2 corresponding to a constant (GB: but endogenously computed) depreciation rate of capital for the U.S. electrical machinery industry. However, for purposes of illustration and comparison we not only report price and output elasticities, estimates of technical change, etc., for model 2 but also for models 1 and 3 (GB: corresponding to an exogenously determined constant depreciation rate and an endogenously determined variable depreciation rate). On the whole the price and output elasticities are similar across models. However, some interesting differences can be observed...in particular...when the depreciation rate is permitted to be endogenously determined... ”

For both models 2 and 3 the depreciation rate is estimated on average to be 0.038 as compared to 0.055 for the OBA capital stock series (GB: which is computed by perpetual inventory methods). This translates into a sizable difference of 16 percent in the level of the capital stock at the end of sample period. Also the ratio of net to gross investment implied by models 2 and 3 is much higher than the ratio implied by the OBA data. All these ratios show sensitivity to the growth in output.”(p. 371)

Consequently, researchers using the perpetual inventory method for obtaining estimates of capital stocks do not have the excuse any more that there are no alternatives. No doubt some alternatives for direct estimates of capital stocks from accounting and engineering data, insurance valuations, etc., always existed. But usually they were rejected on the grounds that they were less accurate than perpetual inventory methods. However now, in the light of the methodology suggested by the above authors, the uncritical insistence on this practice should be discontinued.<sup>9</sup>

## 5.2 The evidence from studies of the business cycle

As capital ages it may yield less output for two reasons. The first of them is sheer wear and tear due to the intensity of its utilization. This is usually referred to in the literature as *output decay*. The second reason is that, in order to produce the same quantity of output, it may absorb more inputs of materials, labor, maintenance, etc. In turn, this is usually referred to as *input decay*. Together these two sources determine the loss-of-efficiency due to usage or *depreciation-in-use*. [Epstein and Denny \(1980\)](#) were the first to introduce depreciation-in-use as a function of the rate of utilization in a short run model of production, thus turning depreciation into an endogenous variable determined by rational entrepreneurial choice. Then, by generalizing the model to the case where the schedule of the marginal efficiency of capital shifts, due to shocks from technological change, [Greenwood, Hercowitz and Huffman \(1988\)](#) obtained strong evidence according to which:

“A variable capacity utilization rate [GB: and hence depreciation-in-use] may be important for the understanding of business cycles. It provides a channel through which investment shocks via their impact on capacity utilization can affect labor productivity and hence equilibrium employment. Such a mechanism may allow for a smaller burden to be placed on intertemporal substitution in generating observed patterns of aggregate fluctuations”(p. 415).

Subsequently, [Burnside and Eichenbaum \(1993;1996\)](#) analyzed the model under the conceptualization that the endogeneity of depreciation derives from the theory of *factor hoarding* and their results confirmed the conclusions in the above passage; and, lastly, [Licandro and Puch \(2000\)](#), [Dueker and Fischer \(2003\)](#), and others, analyzed the implications when depreciation-in-use depends on utilization as well as maintenance and improvement expenditures. Thus, on account of this literature, more and more empirical applications in the areas of real business cycles and dynamic stochastic general equilibrium started to model depreciation-in-use as an economic process.

Consistent with this trend are also the views adopted in empirical applications that address the aggregate implications of investment decisions at the firm level. For an example, consider the study by [Caballero, Engel, Haltiwanger, Woodford and Hall \(1995\)](#), which is representative of a whole class of research efforts to explain the behavior of aggregate investment over the business cycle by introducing various types of convex and non-convex adjustment costs at the representative firm level. These researchers modeled depreciation-in-use as a fixed proportion of the outstanding capital stock and at the same time they introduced retirements explicitly into the equation that traces the time evolution of capital. They

opted for this specification on several grounds. One of them was that ignoring retirements:

“ can yield potentially large measurement errors in the evolution of the capital stock at the plant level, because the average service life distributions are applied to all plants in the same industry” (p. 12).

Thus, echoing perhaps the criticisms mentioned earlier by [Barnhart and Miller \(1990\)](#), five of the most renowned exponents of mainstream economics came full circle around and conceded that capital cannot be measured without direct observations of retirements. But by doing so they abandoned the theorem of proportionality, because, if retirements are modeled explicitly, replacement cannot be invariant with respect to time and its fundamental property of duality with depreciation is lost.

Revisiting the behavior of retirements, a few years later [Goolsbee \(1998\)](#) refuted the applicability of the theorem of proportionality even more conclusively. Let us see why. [Jorgenson \(1974\)](#) had assumed that the loss-of-efficiency of capital goods emanates exclusively from *output decay*. This meant that he abstracted from the presence of technological change, which renders older capital goods obsolete. On the contrary, the view taken by this author was that the vintages effects, i.e. the advanced technical characteristics that are embodied in the newer capital goods, are important and impact differently the service lives of producer durables over the business cycle. To test this so-called embodiment hypothesis he constructed a model of the retirement decision in the airline industry and estimated it with data covering all Boeing 707 planes that were operated by major air carriers in the U.S. from 1972 to 1984. He found that:

“ Capital vintages matter a great deal in the regressions-older planes are much more likely to be retired than younger planes. Further, the business cycle, the costs of capital, the cost of funds, fuel costs, and noise regulation have important effects and the conclusions are highly robust to alternative specifications and functional forms.”(p. 493)

Overall therefore the evidence from studies of the cyclical behavior of retirements contrasts sharply with the neoclassical view that capital: a) is homogeneous across plants, industries, and vintages, and b) depreciates at a constant uniform rate from the one period to the next.

Some support in favor of modeling depreciation and replacement as distinct economic processes has come also from studies that emphasize the Schumpeterian process of creative destruction. [Caballero and Hammour \(1994; 1996\)](#) jumpstarted the research in this direction by proposing and testing a model in which cyclical variations in demand influence

the creation of productive units that embody the latest technology and the retirement of older ones that are obsolete. Among other important results they found that, while during expansions creating new productive units exerts an “insulating” effect in the sense that it prolongs the useful lives of the units already in place, in recessions the same process acts in a “cleansing” fashion, because it precipitates the removal of unprofitable units from production. However, research efforts in more recent years to find “cleansing” effects in episodes of industry “shakeouts”, which coincide with the exit of a sizeable percentage of productive units while the output of the industry is increasing, have been met only with mixed success. For example, [Salvanes and Tveteras \(2004\)](#) found no evidence of such effects in a panel of firm data from the Norwegian manufacturing sector, whereas the estimates by [Jovanovic and Tse \(2007\)](#) pertaining to the exit of firms from the airline industry in the U.S showed that air carriers with older aircraft have higher probability to shut down in recessions. On balance then, and subject to further research, the indications are that creative destruction works procyclically by postponing scrappage in expansions and accelerating it in contractions. For this reason, models that gloss over retirements by adopting the theorem of proportionality may miss an important source of fluctuations.

Lastly, looking for evidence outside mainstream economics, one cannot fail to notice the sharp conflict of the theorem of proportionality with the findings by empirically oriented researchers working along lines suggested by the capital-based Austrian approach to the business cycles. For an example, consider the assessment of the main macroeconomic models of business investment by [Kopcke and Brauman \(2001\)](#). After careful examination of their ability to predict investment expenditures in the 1990s, he concluded that:

“The composition of investment in recent decades has been shifting towards equipment that decays comparatively rapidly, and the relative prices of capital goods have been shifting significantly. In the past, traditional models separated the investment in equipment from that in structures, partly because their characteristics were too distinct to reconcile in one equation. As investors shifted expenditures from structures to equipment, for example, the rate of depreciation of capital that appears as a parameter in a universal investment equation would need to change. Similarly, the shift of investment in equipment towards rapidly depreciating information-processing equipment causes models of gross investment in equipment to err by an increasing margin as they underestimate the need to replace seasoned capital...Ultimately, however, stable models of investment might rest on still finer divisions of investment spending, by separating manufacturing machinery, computers and software, and other types of equipment ([Tevlin and Whelan \(2000\)](#)), or by distinguishing manufacturing plants, office buildings, or wells and mines from other nonresidential structures. Indeed, substantial changes in the characteristics and relative prices of various capital goods might

make this approach more compelling for other, more fundamental reasons. Not only do such changes influence the measurement of aggregates for capital, but they also alter the correlations among measures of capital and other variables...”(pp. 31-32)

From these findings, in conjunction with the ones reported above, it follows that new mainstream and Austrian researchers of business cycles have come closer than anytime before to agreeing that the heterogeneity of capital goods with respect to types and vintages is too important to be ignored in building successful models of cyclical fluctuations. Consequently, it is not surprising that specialists in this area are switching increasingly to models that allow for variable depreciation rates

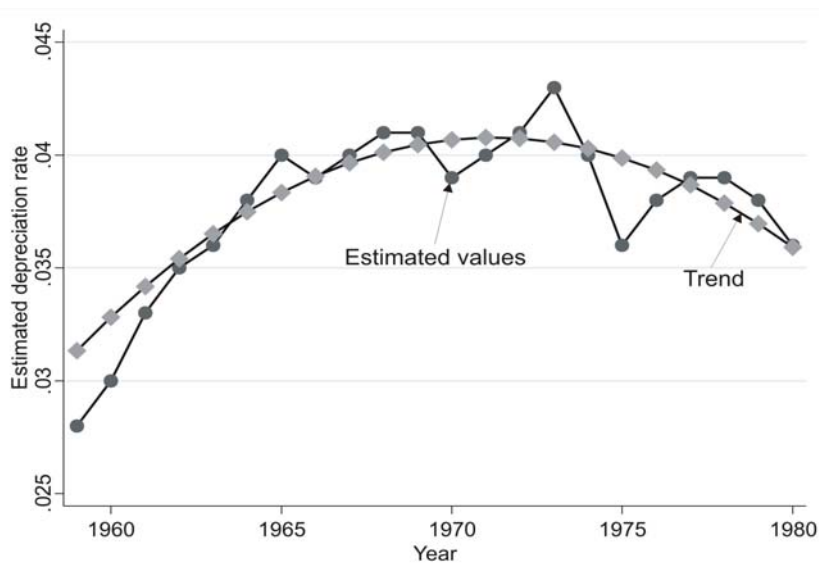
### **5.3 The evidence from studies of economic growth and development**

Jorgenson (1974) obtained the theorem of proportionality in the limit of an infinite series of replacement investments, with constant and growing capital stock. Thus, if there is an area of research where one would expect to find a proper test of its applicability, this is economic growth and development. Hence in closing the presentation it is pertinent to look in these directions.

Turning first to economic growth, recall from the preceding that the loss-of-efficiency of capital: a) almost certainly varies from year to year in response to various economic influences, and b) most likely it varies also over the business cycle because it is closely related to such cyclical variables as utilization, maintenance and repair expenditures, the rate of adoption of technological change, etc. The obvious reason for starting from these findings is that they lead naturally to the following question: Does the variation in the loss-of-efficiency of capital show any time trend, and if so, what are its main characteristics? For, if there is such a trend, even if interrupted from one steady state to another, the rate of depreciation of capital could not possibly be a constant and the theorem of proportionality would not be applicable even in the long run.

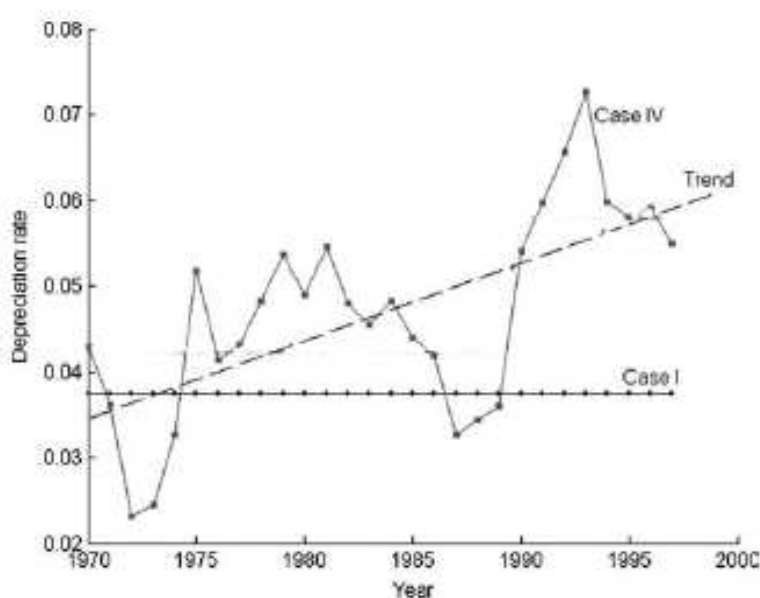
[Prucha and Nadiri \(1996\)](#) estimated consistently annual rates of depreciation in the U.S. Electrical Machinery Industry from 1959 to 1980. Therefore, an interesting test would be to find out whether they showed some time trend and if so what were its general characteristics. The following graph depicts the series of the estimated annual depreciation rates together with a corresponding trend line. From the shape of latter curve it is seen that during the sample period embedded in these estimates was a strong quadratic trend, which on the eve of the energy crisis turned downward. To be sure, in the related literature, authors have





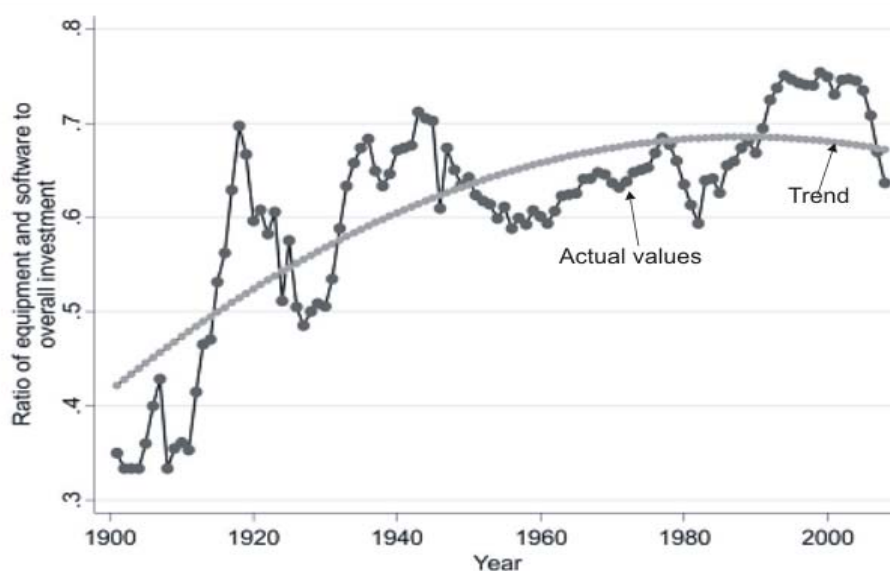
advanced various reasons to explain why this reversal in the trend might reflect what happened actually. But the point of interest here is that the estimated annual depreciation rates for this industry during the sample period displayed a discernible time trend.

[Hernandez and Mauleon \(2005\)](#) obtained equally strong evidence to the same effect for the Spanish economy during the period 1970-1997. The following graph displays the basic results from their study. The horizontal line that corresponds to Case I shows the results when the annual depreciation rates are constrained to be equal to 0.037 throughout, whereas



the line corresponding to Case IV depicts their estimates by applying [Prucha's \(1997\)](#) method as adapted by the authors. From this graph it follows clearly that the estimates exhibited considerable variability around a linear upward trend.

To corroborate further these findings, consider also the following test. According to the conclusions reached in the previous subsection, the variability in the annual depreciation rates at the firm, the sector or the economy level must be dominated by shifts in the time structure of gross investment. Traditionally such expenditures are disaggregated into software, equipment and structures. Even though there may be several reasons that justify this partitioning, perhaps the most fundamental is that the useful lives of the corresponding capital goods differ significantly, with software lasting the least and structures the most. At any level of aggregation, therefore, the changes in the overall depreciation rate from one period to the next must be closely correlated with the changes in the time composition of investment as between short-lived and long-lived capital goods. Consequently, if the ratio of expenditures in software and equipment in total investment is regressed against time, we may be able to test for the existence or not of a trend in the aggregate depreciation rate over the long run. The graph below depicts the actual data and the trend of this ratio for the private nonresidential fixed



depicts the actual data and the trend of this ratio for the private nonresidential fixed investment in the U.S from 1901 to 2008. From this it observed that there is a discernible quadratic trend, which parallels the trend found in the Prucha-Nadiri estimates of the depreciation rate in the U.S. Electrical Manufacturing Industry. In particular, while before the energy crises in the 1970s the depreciation rates were increasing along an upward trend, due mainly to the increase in the share of equipment and software in overall investment, after these crises the trend reversed following a downward direction. Consequently, these findings leave little doubt that the theorem of proportionality does not apply even in the long run.

With respect finally to the evidence from the area of economic development, it is con-

venient to start by noting two crucial differences between the less developed (LDCs) and developed (DCs) countries. These are, first, that the real interest rate in the LDCs is relatively higher than in the DCs, because savings in the former are much lower than in the latter, and, second, that the rate of technological progress in the LDCs is much lower than in the DCs. Thus, if economic agents in the LDCs act in an economizing manner, the optimal useful life of their capital would be expected to be lower than in the DCs, because normally building capital goods that last longer is more costly in terms of savings. But casual observations suggest that in actuality the case is exactly the opposite, because capital in the LDCs is used much longer than in the DCs. The question that arises then is: Do LDCs behave irrationally or are we looking at them through blurred and imperfect lenses? The fault is in the mainstream models of economic development, which are erected generally on the presumption that depreciation is proportional to the outstanding capital stock that they employ.

Direct evidence that this is the case comes from the study by [Bu \(2006\)](#), which investigates the determinants of the rates of economic depreciation in six countries from this group. Using firm level data from surveys of the World Bank, what this researcher finds is that the purchases of new capital goods, on the one hand, and the outlays for their maintenance, repair and improvement, on the other, are substitutes. By implication, economic agents in these countries prolong the useful lives of their fixed capital stock through maintenance and repair, instead of renewing it faster through investment. Is this behavior rational? It is most rational for at least three reasons. First, because these are capital importing countries and such imports are seriously constrained by the lack of foreign exchange and adequate savings. Second, because the technology of maintenance and repair is intensive in the productive factor in which these countries are relatively abundant, i.e. labor; and third, because maintenance and repair is less prone to costly errors due to the relative lack of knowledge in purchasing and installing new capital goods. Consequently, going backwards from experience to theory, models of economic development should allow for a trade-off between maintenance and repair expenditures and purchases of new capital goods. If they do, not surprisingly but expectedly, the optimal policies in LDCs would favor using fixed capital longer than in DCs and doing so by means of investment in maintenance, repair and improvements.

Indirect evidence comes from studies that focus on the relationship between maintenance, investment and scrappage in the DCs. For example, employing fairly good data from the Class-I railroads in the U.S over the period from 1944 to 1970, in [Bitros \(1976a;1976b\)](#) I found that maintenance and repair expenditures related positively to purchases of new loco-

motives and negatively to scrappage, and moreover the same was true for freight cars. These findings implied that investments in maintenance and new rolling stock are complementary, whereas more recently [Mullen and Williams \(2004\)](#) found similar evidence with the help of data from the Canadian manufacturing extending down to four-digit industry level. From this literature it follows that in capital producing countries with ample amounts of savings and efficient money and capital markets investments in maintenance and new capital goods are rather complements. On the contrary, this relationship reverses and maintenance and investment in new capital goods become substitutes in capital importing countries with scarcity of savings and suppressed money and capital markets. The view then that there is no relation between maintenance and investment conflicts sharply with the available evidence.

Having reviewed all the strands of literature that bear on the applicability of the theorem of proportionality, it is time now to summarize the findings and the conclusions.

## 6. Summary of findings and conclusions

The classical theory of replacement, as developed by [Hotelling \(1923\)](#), [Preinreich \(1940\)](#), [Terborgh \(1949\)](#) and [Smith \(1961\)](#), provided for the determination of the optimal replacement rate as a function of market and engineering parameters. The studies by [Smith \(1957\)](#), [Rust \(1987\)](#) and many others have documented the relevance of replacement policies by reference to numerous well-specified durable goods, like truck-tractors, bus-engines, locomotives and freight cars, forklifts, ships, airplanes, etc. So at least at the individual fixed asset level all evidence shows that the theorem of proportionality is inapplicable.

Moving from individual to more aggregate classes of fixed assets, [Feldstein and Foot \(1971\)](#), [Eisner \(1972\)](#) and [Feldstein \(1972/1974\)](#) reported that replacement investment at the level of U.S Manufacturing from 1949 to 1968 was determined by conventional economic forces such as the availability of funds, the level of expansionary investment, the utilization rate, etc. In his rebuttal [Jorgenson \(1974\)](#) took issue with the structure of the tests and the results in the former study. But he remained silent on the latter two. Perhaps he did so on the perception that these studies suffered from the same inconsistencies with those he thought he had discovered in the former study. However, the issues that [Feldstein \(1972/1974\)](#) raised were fundamental and, had the debate continued, most likely the controversy would have been resolved in favor of his arguments and the development of economic theory, as well as econometric and policy applications in various fields, would have switched back to the classical theory of replacement.

Admittedly the evidence from aggregate replacement studies was somewhat soft because

the firm level data used were based on the perceptions of company managers who were asked to indicate how much of next year's planned investment expenditures were for replacement. But in the years that followed that debate researchers sought harder evidence in other directions and their findings ascertained overwhelmingly that the theorem of proportionality applies neither in the short run nor in the long run. More specifically, all studies without exception have shown that depreciation and scrapping vary from year to year in response to changes in conventional economic forces like utilization, maintenance and repair, the prices of new capital goods, etc. The results of research in the areas of production and productivity analyses have shown quite conclusively that under the theorem of proportionality it is extremely difficult, if at all possible, to identify and estimate the main parameters of interest on which policy applications are based. From the research in the area of business cycles, particularly in recent years, it has emerged that vintage models linking depreciation-in-use to utilization and maintenance, on the one hand, and technological obsolescence, on the other, gain significantly in explanatory and predictive power; and lastly, but most importantly, the evidence from studies in the areas of economic growth and development suggest that the theorem of proportionality fails exactly where it ought to apply, i.e. in analyses of the long-run.

However, despite all the voluminous evidence against it, the theorem of proportionality has come to dominate mainstream capital theory and its associated econometric and policy applications. Initially weighing in its favor was the argument by [Hulten and Wykoff \(1981a\)](#) that the distribution of the depreciation rates, as estimated from the price-age profiles of several classes of producer durables, could be represented adequately by a single parameter. But under the piling evidence that the price-age profiles shifted from year to year due to various endogenous and exogenous influences, [Hulten, Robertson and Wykoff \(1989\)](#) and [Hulten and Wykoff \(1996\)](#) fell back to the position that the approximation they suggested was justified because: a) it helped achieve a major degree of simplification, and b) non-stationarity is very difficult to deal with empirically. In turn this concession opened the way for a group of researchers particularly in the areas of business cycles and economic growth to demonstrate that switching to an endogenous theory of depreciation, scrapping and replacement is technically feasible and can be very rewarding in terms of theoretical and empirical implications.

In conclusion, simplicity is not any more a good reason for the continued dominance of the theorem of proportionality. A justification for this assessment is that its employment thwarts the advances that can be achieved in economic theory and econometric applications by returning to a general equilibrium model centered on the time structure of capital and the useful lives of its

components. Indicative of how technically feasible and substantively fruitful such advances might turn out to be are the achievements in recent years in the fronts of economic growth and business cycles, where the adoption of the theorem of proportionality has retreated. Moreover, the returns in terms of precision and robustness of models with endogenous depreciation, scrap-page and replacement can be expected to be even higher in growth accounting, productivity studies, and various other applications, where presently researchers employ estimates of capital stocks based on the perpetual inventory method. On these grounds, the sooner the theorem of proportionality is abandoned or at least revised, the better for economic theory and policy.

## Endnotes

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<sup>1</sup> Unfortunately, too frequently the need for the model and its premises to be empirically applicable or accountable is not emphasized in contemporary economics. To ascertain that this is the case here is a passage from the presidential address of [Leontief \(1971\)](#) to the American Economic Association:

“In the presentation of a new model, attention nowadays is usually centered on a step-by-step derivation of its formal properties. But if the author-or at least the referee who recommended the manuscript for publication-is technically competent, such mathematical manipulations, however long and intricate, can even without further checking be accepted as correct. Nevertheless, they are usually spelled out at great length. By the time it comes to interpretation of the substantive *conclusions*, the assumptions are easily forgotten. But it is precisely the empirical validity of these *assumptions* on which the usefulness of the entire exercise depends”(p. 2).

<sup>2</sup> A comment is in order regarding the distinction of the terms “depreciation,” “retirement” or “scrappage,” and “replacement”. Depreciation is the loss in the earning power of durable goods, which is reflected in the reduction of their resale price. On the other hand, “retirement” or “scrappage” is the amount of productive capacity lost through removal of old and technologically obsolete capital; and, finally, replacement is the productive capacity added through investment in order to replenish the losses in productive efficiency that durable goods suffer through usage and technological progress. Consequently, in general, the processes of “depreciation,” “retirement,” and “replacement” are distinct and this explains why an extensive literature on depreciation has developed independently from the literature that focuses on the issues of “retirement” and “replacement”. But [Jorgenson \(1974\)](#) has shown that, under the theorem of proportionality, depreciation is dual to replacement, which implies that in this and only this case: a) the losses in productive efficiency through scrappage is zero, and, b) the two terms can be used interchangeably. For this reason, when employing them below, I shall exercise extra caution so that their meaning is clear from the context of the discussion.

<sup>3</sup> Researchers in the field of replacement employ various terms to denote the length of time over which a piece of equipment remains economically viable. Some of these terms are useful life, economic life, service life, longevity and durability. Unless noted specifically otherwise, below I will use the term useful life.

<sup>4</sup> Moreover, recently [Cho and Rust \(2008\)](#) confirmed the conclusions reached by [Rust \(1987\)](#) in the case of a company renting automobiles.

<sup>5</sup> In fairness to historical accuracy it should be stressed that criticism of the estimates of economic depreciation derived from price-age profiles along the following lines started to appear much earlier. For an example, see [Biom \(1992\)](#).

<sup>6</sup> In [Bitros \(2008\)](#) I presented and analyzed a model in which the uncertainty about the rate of embodied technological change influences in traceable ways the useful lives of capital goods that one obtains under certainty from the policies of replacement and scrapping. Hence, ceteris paribus, the price-age profiles would be expected to shift with shifts in the degree of uncertainty and cause even switches from replacement to scrapping and vice-versa.

<sup>7</sup> The aversion of researchers in mainstream economics to the issues raised by these papers has been so profound that one rarely finds a single reference to them, not even in passing. This may indicate an unexplained intellectual bias, which retards the progress of economics as a science.

<sup>8</sup> It is very interesting to compare the conclusions below with those reached in Nadiri and Prucha (1996), where the authors restrict the depreciation rates of both physical and R&D capital to a constant. In this paper, they conclude:

“We have specified a model of factor demand that allows for estimating jointly the depreciation rates of both physical and R&D capital for the U.S. Manufacturing sector. The main result of our study is that the depreciation rate for plant and equipment capital is 0.059 and for R&D is 0.12. Our estimate for the depreciation rate of physical capital is generally much lower than those reported by [Epstein and Denny \(1980\)](#), [Bischoff and Kokkenlenberg \(1987\)](#) and [Kollintzas and Choi \(1985\)](#). However, our estimate of the depreciation rate for plant and equipment gross capital is higher than the Bureau of Economic Analysis’s estimate of (on average) 0.036.”(p 54)

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What this evidence shows is that the estimated depreciation rates for physical capital differ significantly across researchers, methodologies of estimation, restrictions imposed on the models, data, and sample periods. In turn though these differences influence the net to gross investment ratio, and hence the margins for errors in the measurement of production and productivity are very wide. For this reason, the returns to more intensive empirical research in this area may be exceedingly high.

<sup>9</sup> In the above papers the authors obtained estimates depreciation rates of physical and R&D capital by self-programmed estimation algorithms. However, since not many researchers have the time or the capability to program their own estimation algorithms, the method these authors devised to estimate constant and variable depreciation rates were of limited appeal. However, [Prucha \(1997\)](#) introduced an approach that permits the same computations using standard econometric packages.



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