Free Entry Bertrand Competition

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October 2009

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Abstract: This paper examines Bertrand competition under free entry, when firm size vis-a-vis market size is exogenously given. A free entry Bertrand Nash equilibrium (FEBE) exists if and only if relative market size is sufficiently large. Further, there is a unique coalition-proof Nash equilibrium price that corresponds to the minimum FEBE price, leads to average cost pricing for all active firms and is decreasing in market size.

Key words: Bertrand competition, free entry, coalition-proof, contestability.

JEL Classification No.: D4, L1.

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1 Introduction

In this paper we study Bertrand competition (i.e. price competition with firms supplying all demand)[1] under free entry, when firm size vis-a-vis market size is exogenously given. The literature on Bertrand competition generally does not allow for free entry, and even when it does, equilibrium properties when firm size can be large relative to market size remain unexplored (more detailed discussion follows). The present paper makes a beginning in this direction. Further, we examine equilibrium prices both when firms behave non-cooperatively, as well as when there is limited collusion among them.

We consider a model of Bertrand competition with free entry, where, following Novshek and Roy Chowdhury (2003), free entry is formalized as the presence of inactive firms in equilibrium. To begin with we examine the case where the firms behave non-cooperatively, i.e. solve for Nash equilibria. We demonstrate that whether a free entry Bertrand Nash equilibrium (henceforth FEBE) exists or not, depends on the size of the market. A FEBE exists if

[1] The assumption that firms supply all demand dates back to Chamberlin (1933) and is appropriate when the costs of turning away customers are very high (see, for example, Vives (1999)). Such costs are routinely assumed in the operations research literature (see, Taha (1982)). Such costs may arise because of either reputational reasons, or governmental regulations. Vives (1999) argues that such regulations are operative in U.S. industries like electricity and telephone. Spulber (1989) argues that the common carrier regulation can lead to a scenario where firms supply all demand. Similarly, under certain sealed bid auctions, winning firms have to supply all demand that comes to them.

and only if the market size is sufficiently large. Further, the set of FEBE prices constitutes an interval.

We next allow for limited collusion among the firms, formalized through the notion of coalition-proof Nash equilibrium developed in Bernheim et al. (1987). We find that a coalition-proof Nash equilibrium exists whenever a FEBE exists. In that case there is a 'unique' coalition-proof Nash price that corresponds to the minimum FEBE price. Further, this equilibrium involves average cost pricing by active firms, leading to zero profits. We also find that for a sufficiently large increase in market size, the equilibrium price in fact decreases.

We then relate our paper to the literature. The early literature focuses on Bertrand competition without free entry, e.g. Vives (1999), Dastidar (1995), Baye and Kovenock (2006), Hoernig (2002, 2007), and Roy Chowdhury and Sengupta (2004). While Vives (1999) and Dastidar (1995) examine the existence of pure strategy Nash equilibria, Hoernig (2002) examines Nash equilibria in mixed strategies, while Baye and Kovenock (2006) and Hoernig (2007) demonstrate that in the presence of fixed costs Nash equilibria may fail to exist. Roy Chowdhury and Sengupta (2004) solves for the coalition-proof Nash equilibrium under Bertrand competition, but does not allow for free entry.

While Novshek and Roy Chowdhury (2004) does allow for free entry, they focus on the limiting case when firm size is vanishingly small. Further, they do not examine the coalition-proof equilibrium. The present paper thus extends the literature by allowing for (a) free entry, (b) relatively large firm size, and (c) both non-cooperative behaviour, as well as limited collusion among the firms.

The rest of the paper is organized as follows. Section 2 sets up the basic model. Non-cooperative Nash equilibria are analyzed in Section 3, while coalition-proof equilibrium is analyzed in Section 4.

2 The Model

The market demand is $rf(p)$, where r is a parameter for market size. There are n firms, where n is large, all producing a single homogeneous good, and having the same cost function, $c(q)$, and the average cost function, $AC(q)$. The case of interest is when r is sufficiently large so that there exists p such that $p > AC(rf(p))$.

The demand and the cost functions satisfy the following two assumptions.

Assumption 1. $f : [0, \infty) \to [0, \infty)$ is continuous and strictly decreasing on $[0, \hat{p})$ where $\hat{p}$ (> 0) is a cutoff price such that, $f(p) > 0$ if and only if $p < \hat{p}$.

Assumption 2. (a) $c : [0, \infty) \to [0, \infty)$ is continuous, except possibly at the origin. Moreover, $c(0) = 0$ and $c(q) > 0$, $\forall q > 0$.

(b) The average cost function $AC : (0, \infty) \to (0, \infty)$, where $b = \lim_{q \to 0} AC(q)$ is well defined (with infinity as a possible limit). There exists $q^* > 0$ such that $AC(q)$ is decreasing (respectively increasing) for $q < q^*$ (respectively $q > q^*$). Further, $\hat{p} > c^* = AC(q^*)$.

We examine a game of free entry Bertrand competition where the firms simultaneously announce their prices, the Chamberlin (1933) assumption holds (so that firms supply all demand) and there is free entry.

Let $D_i(p_1, \cdots, p_i, \cdots, p_n)$ denote the demand facing firm i when the announced price vector is $(p_1, \cdots, p_i, \cdots, p_n)$. Then

$$D_i(p_1, \cdots, p_i, \cdots, p_n) = \begin{cases} 0, & \text{if } p_i > p_j, \text{ for some } j, \\ \frac{rf(p_i)}{m}, & \text{if } p_i \leq p_j, \ \forall j, \text{ and } \#(l : p_l = p_i) = m. \end{cases} \tag{1}$$

The profit of the i-th firm

$$\pi_i(p_1, \cdots, p_n) = (p_i - AC(D_i(p_1, \cdots, p_n)))D_i(p_1, \cdots, p_n). \tag{2}$$

We next consider a situation where m $(\leq n)$ firms all charge the price p, and all other firms charge higher prices. Then the profit of all active firms is given by

$$\pi(p, m) = \frac{rpf(p)}{m} - c(\frac{rf(p)}{m}). \tag{3}$$

Assumption 3. $\pi(p, m)$ is strictly quasi-concave for all m.

Observe that if $\pi(p', m) \geq 0$ for some $p' < \hat{p}$, then for all p'' such that $p' < p'' < \hat{p}$, from strict quasi-concavity it follows that $\pi(p'', m) > \min\{\pi(p', m), \pi(\hat{p}, m)\} = \min\{\pi(p', m), 0\} \geq 0$.

Following Novshek and Roy Chowdhury (2003), free entry is formalized as there being inactive firms in equilibrium. Thus we focus on equilibria where m $(< n)$ (active) firms set the lowest price, and $n - m$ (inactive) firms charge higher prices and have no demand.

Given a price vector $\mathbf{p}$, and a coalition $T \subset N$, we let $\mathbf{p_T}$ denote the price vector corresponding to the coalition T.

Given $\mathbf{p'_T}$ and $\mathbf{p_S}$, $T \subset S$, $(\mathbf{p'_T}, \mathbf{p_{S/T}})$ denotes the s-vector where the prices correspond to $\mathbf{p'_T}$ for firms in T, and to $\mathbf{p_S}$ for firms in S/T.

For a coalition $T \subset N$, the price vector $\mathbf{p'_T}$ constitutes a *profitable deviation* from $\mathbf{p_N}$ if

$$\pi_i(\mathbf{p'_T}, \mathbf{p_{N/T}}) > \pi_i(\mathbf{p_N}),\ \forall i \in T.$$

Definition. A *Bertrand-Nash equilibrium* consists of a price vector $\mathbf{p}$ such that no firm has a profitable deviation.

Definition. A Bertrand-Nash equilibrium price vector, $\mathbf{p}$, is said to be a *free entry Bertrand equilibrium* (FEBE) if some of the firms are not active, i.e. have zero demand.

3 Bertrand Nash Equilibria

In this section we solve for the set of FEBE prices. We begin by introducing some notations.

In case $b \geq \hat{p}$, let $\underline{r}$ be the minimum r such that there exists p satisfying $AC(rf(p)/2) = p$. Otherwise, $\underline{r} = 0$.

$d(r)$ is the minimum p such that $AC(rf(p)) = p$.[2] We assume that at $p = d(r)$, the average cost curve intersects the demand curve from below in

[2] $d(r)$ is well defined given that (a) there exists p such that $p > AC(rf(p))$, and (b) that $\hat{p} > c^* = AC(q^*)$.

the $p - q$ plane.[3]

$d(r,2)$ is the minimum p such that $AC(rf(p)/2) = p$.[4]

Note that for r large, $AC(q)$ is negatively sloped at both $rf(d(r))$ and $rf(d(r,2))/2$. Hence if $d(\underline{r},2) \geq d(\underline{r})$ (so that $d(r,2)$ is well defined for $r \geq \underline{r}$), then an r satisfying $d(r) = d(r,2)$ exists from the intermediate value theorem. Thus let

$$\tilde{r} = \begin{cases} \underline{r}, & \text{if } d(\underline{r},2) < d(\underline{r}), \\ \text{satisfies } d(r) = d(r,2), & \text{otherwise.} \end{cases} \tag{4}$$

The following lemmas will be useful later on.

Lemma 1 *If $\tilde{r}$ satisfies $d(r) = d(r,2)$, then $d(r) \geq d(r,2)$ if and only if $r \geq \tilde{r}$.*

Proof. For r greater than, but close to $\tilde{r}$, $AC(q)$ is positively sloped at $rf(d(r))$ and negatively sloped at $rf(d(r,2))/2$. Hence $d(r) > d(r,2)$. Similarly for r smaller than, but close to $\tilde{r}$, $d(r) < d(r,2)$. Whereas for r sufficiently large (respectively small) $AC(q)$ is positively (respectively negatively) sloped at both $rf(d(r))$ and $rf(d(r,2))/2$. ■

Lemma 2 *There exists some minimal price p' ($\leq d(r)$) and some maximal m' (> 1) corresponding to p', such that $AC(\frac{rf(p')}{m'}) \leq p'$.*

Proof. For $b < \hat{p}$, $\tilde{r}$ solves $d(r) = d(r,2)$ (since in this case $\underline{r} = 0$). Thus for any $r \geq \tilde{r}$, there exists $m' \geq 2$ and $p' < b$ such that $AC(rf(p')/m') = p'$

[3] The set of r for which this assumption is violated has measure zero, and hence is without loss of generality (in the measure theoretic sense).

[4] $d(r,2)$ is well defined for $r \geq \underline{r}$.

and $AC(q)$ is negatively sloped at $rf(p')/m'$. This follows since for m large enough, $rf(c^*)/m < q^*$. Hence $AC(rf(p')/m') = p' > AC(rf(p')/(m'+1)$ (the inequality holds as $AC(q)$ is negatively sloped at $rf(p')/m'$).

So let $b > \hat{p}$. To begin with note that for any $p < \hat{p} < b$, $AC(rf(p)/m) > p$ for m large. It remains to show that there exists $p \leq d(r)$ and $m \geq 2$ such that $AC(rf(p)/m) \leq p$. Let $\hat{r}$ solve $d(r,2) = c^*$. For $\tilde{r} \leq r \leq \hat{r}$, $AC(rf(p)/2) = p$, for $p = d(r,2) \leq d(r)$. Next consider $r > \hat{r}$, and $p = d(r)$. In case $d(r) \geq AC(rf(d(r))/2)$, $\pi(d(r),2) \geq 0$. So let $d(r) < AC(rf(d(r))/2)$. Next let p' be the maximum $p < d(r)$ such that $AC(rf(p')/2) = p'$ (such a p' exists since $r \geq \tilde{r}$). Then, for ϵ (> 0) small, $d(r) > p' - \epsilon > AC(rf(p' - \epsilon)/2)$. ■

We next define the set of FEBE prices
$Q(r) = \{p|$ there exists a FEBE where all active firms charge $p\}$.

We begin by showing that whether an FEBE exists or not depends on the size of the market. In particular, an FEBE exists if and only if the market size exceeds $\tilde{r}$.

Proposition 1 *(i) An FEBE exists if and only if* $r \geq \tilde{r}$*.*

(ii) For any $p \in Q(r)$*,* $p \leq \min\{\hat{p}, d(r), b\}$*.*

Proof. (i) *Step 1.* $r < \tilde{r}$: Suppose to the contrary there exists some $p \in Q(r)$. In case the equilibrium p involves $m > 1$ active firms (so that $\tilde{r}$ solves $d(r) = d(r,2)$), it follows that $p > d(r)$. Otherwise, since $r < \tilde{r}$, from Lemma 1, $d(r,2) > d(r) \geq p$, so that $AC(rf(p)/2) > p$. Further, since $AC(q)$ is negatively sloped at $rf(d(r,2))/2$, $AC(rf(p)/m) \geq AC(rf(p)/2) > p$ $\forall m \geq 2$, so that the active firms make losses. But then, with $p > d(r)$, an inactive firm can charge p', where $d(r) < p' < p$, and make a positive profit. Hence next suppose that p involves a single active firm (for $\tilde{r} = \underline{r}$, this is the only possibility). Then $p = d(r)$ (otherwise for $p > d(r)$ there will be undercutting by some inactive firm, whereas for $p < d(r)$ the single active firm makes losses). But then this firm has zero profits, whereas it can charge a slightly higher price and get a strictly positive profit.

Step 2. $r \geq \tilde{r}$: Given Lemma 2, in the outcome where m' firms each set the price p', and the other firms set higher prices, the active firms all earn a non-negative profit and matching p' by an inactive firm is not profitable. Since $AC(\frac{rf(p')}{2}) \leq p'$, $p' \leq d(r)$, so that undercutting is not profitable either. Consequently, p' can be sustained as an equilibrium.

(ii) Let $p \in Q(r)$. In case $p > d(r)$, then in the equilibrium sustaining p, an inactive firm can charge p', where $d(r) < p' < p$, and make a profit. Whereas if $p > b$, then whenever $\pi(p, m) \geq 0$, $\pi(p, m+1) > 0$ (from the observation following Assumption 3), so that matching this p is profitable for an inactive firm. $\blacksquare$

It is well known that if firms are free to supply less than demand, then with convex costs an equilibrium in pure strategies may not exist - the Edgeworth paradox. This problem is resolved under Bertrand competition without free entry (see, among others, Vives (1999))[5]. Proposition 1(i) shows that depending on market size, the existence result under Bertrand competition without free entry may, or may not extend when entry is free.

[5] As Baye and Kovenock (2006) and Hoernig (2007) demonstrate however, in the presence of fixed costs a Nash equilibrium may not exist.

Remark 1 *The non-existence result for $r < \underline{r}$ however depends on the assumption that prices can vary continuously. Suppose to the contrary that prices can only vary along a grid of size ϵ (> 0), and let $d(r, \epsilon)$ be the smallest price p in the grid such that $p \geq d(r)$. A straightforward modification of the argument in Ray Chaudhuri (1996) shows that for $r < \underline{r}$ (and ϵ sufficiently small), there is a unique equilibrium where there is a single active firm charging the price $d(r, \epsilon)$.*[6]

Remark 2 *It can be shown that $Q(r)$ is empty in case $AC(q)$ is either strictly increasing, or strictly decreasing in q. First consider the case where $AC(q)$ is strictly decreasing in q. Clearly, the only possible FEBE price is $d(r)$. But then there must be at most one active firm charging this price (otherwise the active firms make losses). Hence this firm can increase its price slightly and make a positive profit. Next let $AC(q)$ be strictly increasing. But in any candidate equilibrium involving m active firms, if m firms make non-negative profits in this equilibrium, then matching this price leads to a positive profit for any inactive firm. In case $AC(q) = c$, it is easy to see that $Q(r) = \{c\}$.*

Next turning to the properties of the set of FEBE prices, we show that $Q(r)$ is an interval, bounded and closed from below. While the set of pure strategy Nash equilibrium prices constitutes an interval under Bertrand competition without free entry as well (see, e.g. Vives (1999)), it is interesting that this property goes through with free entry even though in this case different equilibrium prices can correspond to different number of active firms.

[6] The proof is available on request.

Proposition 2 *Let $r \geq \tilde{r}$. The set of FEBE prices, $Q(r)$, is bounded, closed from below and is an interval.*

Proof. Interval. Suppose to the contrary there exists p, p', p'' such that $p < p' < p''$, $p, p'' \in Q(r)$, but $p' \notin Q(r)$. Further, let the FEBE price p involve at most m active firms. Since $p \in Q(r)$, $\pi(p, m) \geq 0$. Hence from the observation following Assumption 3, $\pi(p', m) \geq 0$ (since $p < p' < p'' \leq \hat{p}$). Further, since $p' < p'' \leq b$, for $\tilde{m}$ sufficiently large, $\pi(p', \tilde{m}) < 0$. Thus there exists some m' such that $\pi(p', m') \geq 0 > \pi(p', m' + 1)$. Moreover since $p'' \in Q(r)$, $p' < p'' \leq d(r)$. Consequently, the outcome where m' firms charge p' and all other firms charge higher prices, can be sustained as an equilibrium.

Closed from below. Let $\underline{p}(r)$ denote the infimum of $Q(r)$. If $Q(r)$ is a singleton set, then $\underline{p}(r) \in Q(r)$. So suppose that $Q(r)$ is not a singleton, but $\underline{p}(r) \notin Q(r)$. Since $\underline{p}(r)$ is a boundary point of $Q(r)$, and $p \leq d(r)$ for any $p \in Q(r)$, $\underline{p}(r) < d(r)$ (strict inequality holds since $Q(r)$ is not a singleton). Since $\underline{p}(r) < b$, there exists m' such that $AC(rf(\underline{p}(r))/m') \leq \underline{p}(r) < AC(rf(\underline{p}(r))/m'+1)$. Otherwise there exists $p'' \in Q(r)$ and arbitrarily close to $\underline{p}(r)$ such that $AC(rf(p'')/m) > p''$ $\forall m \geq 2$, which is a contradiction. We can then sustain an equilibrium where m' of the firms charge $\underline{p}(r)$ and all other firms charge higher prices. Thus $\underline{p}(r) \in Q(r)$.[7] ■

Remark 3 *It is easy to show that $Q(r)$ is compact in case $b > \hat{p}$. Given that $b > \hat{p}$, any boundary point of $Q(r)$ must be strictly less than b. We can thus mimic the preceding argument to show that such a boundary point must be in $Q(r)$.*

[7]Given Proposition 1(ii), boundedness is trivial.

Given that $Q(r)$ is closed from below, we can further define

$$\underline{p}(r) = min\{p : p \in Q(r)\}.$$

4 Coalition-proof Nash Equilibrium

In this section we allow for the possibility of limited collusion among the agents. We thus look for equilibria that are immune to 'credible' group deviations. Formally, we solve for coalition-proof free entry Bertrand equilibria (henceforth CPFEBE).

As usual the notion of a CPFEBE is defined recursively.

We first define a '*self-enforcing*' profitable deviation by a coalition. This will be done inductively, using the size of the coalition as the basis for our induction.

We say that a coalition T, with $|T| = 1$, has a *self-enforcing* profitable deviation from $\mathbf{p}$ if T has a profitable deviation from $\mathbf{p}$.

Suppose now we have defined *self-enforcing* profitable deviations $\forall S \subset N$, with $|S| \leq m \leq n-1$, and for all price vectors $\mathbf{p}$. Now consider $T \subseteq N$ such that $|T| = m + 1$. We say that T has a *self-enforcing* profitable deviation from $\mathbf{p}$ if,

i) $\mathbf{p'_T}$ constitutes a profitable deviation for coalition T from $\mathbf{p}$, and

ii) For any $S \subset T, S \neq T$, the coalition S has no *self-enforcing* profitable deviation from $(\mathbf{p'_T}, \mathbf{p_{N/T}})$.

We have thus defined *self-enforcing* profitable deviations from any given price vector $\mathbf{p}$.

Let the FEBE sustaining $\underline{p}(r)$ involve at most $m(\underline{p}(r))$ active firms. We next turn to the task of defining a CPFEBE.

Definition. A vector of prices $\mathbf{p}^*$ is said to be a *Coalition-proof Free Entry Bertrand Equilibrium (CPFEBE)* if no coalition $T, T \subseteq N$, has a *self-enforcing* profitable deviation from $\mathbf{p}^*$, and there are at least $m(\underline{p}(r))$ inactive firms in equilibrium.

Note that the proviso that 'there are at least $m(\underline{p}(r))$ inactive firms in equilibrium' captures the fact that we are focusing on free entry equilibria. In order to simplify the exposition we further make the tie-breaking assumption that a group of inactive firms all prefer to undercut and get zero profits, rather than remain inactive.[8]

Interestingly, a CPFEBE exists whenever a FEBE exists. Further, it is 'unique' and involves all active firms charging $\underline{p}(r)$ (prices charged by the inactive firms only has to satisfy the condition that they are greater than $\underline{p}(r)$, hence the qualification on uniqueness).

Proposition 3 *(i) There is a 'unique' CPFEBE that involves $m(\underline{p}(r))$ active firms charging $\underline{p}(r)$.*

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[8] Otherwise, given that conceptually free entry allows for an infinite number of inactive firms, defining self-enforcing deviations from some vector p^0 becomes notationally cumbersome. In that case one would need to allow for an infinite sequence of deviating price vectors $< p^1, p^2, \cdots >$, $p^i > p^{i+1}$, where p^{i+1} is a deviation from p^i by a group of inactive firms and all deviating firms make positive profits.

(ii) At this CPFEBE all active firms make zero profits.

Proof. (i) Step 1. To show that $\underline{p}(r)$ can be sustained as a CPFEBE.

(1a). To show that there is no 'self-enforcing profitable' deviation by undercutting from $\underline{p}(r)$, it is sufficient to show that no group of m inactive firms have a 'profitable' undercutting deviation from $\underline{p}(r)$. Suppose to the contrary such a profitable undercutting price p' exists. Let m' be the maximum m such that $AC(rf(p')/m') < p'$ (such an m' exists since $p' < \underline{p}(r) \leq b$). Thus $AC(\frac{rf(p')}{m'+1}) \geq p'$. Further, $p' < \underline{p}(r) \leq d(r)$. But then p' can be sustained as a FEBE (with m' firms charging p' and all other firms charging a higher price), which is a contradiction (since $p' < \underline{p}(r)$).

(1b). Next suppose that all the firms charging $\underline{p}(r)$, possibly along with some of the other firms, can deviate to some higher price $\tilde{p}$ and make a gain (any deviation to a price less than $\underline{p}(r)$ is not self-enforcing). But then $m(\underline{p}(r))$ inactive firms can undercut by charging $\underline{p}(r)$. Further, by mimicing the argument in (1a), it can be shown that this deviation is self-enforcing.

(1c) Finally, since $AC(q)$ must be negatively sloped at $\frac{rf(\underline{p}(r))}{m(\underline{p}(r))+1}$, matching $\underline{p}(r)$ by any group of inactive firms is not profitable.

Step 2. Turning to 'uniqueness', suppose to the contrary there is some other CPFEBE, where the active firms charge $p' \neq \underline{p}(r)$. So let $p' < \underline{p}(r)$. But then from 1(a), p' must be a FEBE, a contradiction. If $p' > \underline{p}(r)$, then we can use the argument in step 1(b) to arrive at a contradiction.

(ii) Suppose to the contrary $\pi(\underline{p}(r), m(\underline{p}(r))) > 0$. Since $m(\underline{p}(r))$ is the maximal number of active firms sustaining this price as a FEBE, $\pi(\underline{p}(r), m(\underline{p}(r))+1) < 0$. Thus from continuity there exists $p' < \underline{p}(r)$ such that $\pi(p', m(\underline{p}(r))) >$

$0 > \pi(p', m(\underline{p}(r)) + 1)$. Further, $p' < d(r)$. Thus p' can be sustained as a FEBE, which contradicts the definition of $\underline{p}(r)$. ■

The existence result is interesting given that under coalition-proofness, existence is generally known to be an issue (Bernheim et al. (1987))[9]. While Roy Chowdhury and Sengupta (2004) do prove existence for coalition-proof Bertrand equilibrium, their framework is without free entry. Further, somewhat surprisingly, it turns out that limited collusion in fact leads to the *smallest* FEBE price. In addition, this characterization provides an algorithm for identifying the CPFEBE price that only involves solving for the FEBE set, without explicitly checking for coalition proofness.

The fact that all active firms make zero profits is also of interest since zero profits is not ensured either when there is free entry but no collusion (see Novshek and Roy Chowdhury (2003)), or when there is limited collusion, but no free entry (see Roy Chowdhury and Sengupta (2004)).

Further, the contestability theory developed by Baumol et al. (1977, 1982) (among others) argues that with increasing returns and free entry, the outcome involves a single firm with average cost pricing, i.e. zero profits. Ray Chaudhuri (1996) interprets the contestable outcome as a Bertrand equilibrium when there is increasing returns and pricing is discrete. The present paper extends this literature by showing that even in the absence of increasing returns, free entry, coupled with limited collusion and quasi-convex average costs, can lead to average cost pricing (though, in contrast

[9]Bernheim and Whinston (1987) however show existence for certain games, including Cournot competition

to the contestability literature, in equilibrium there is more than one firm).

We finally examine the impact of an increase in market size on the CPFEBE price, $\underline{p}(r)$. We find that for any sufficiently large increase in demand, i.e. r, the minimal equilibrium price falls. The intuition is that with an increase in demand, the maximal number of active firms that can be sustained in any equilibrium increases. This leads to greater competition, pushing down prices.

Proposition 4 *Given any r', and any p' such that $c^* < p' < \underline{p}(r')$, there exists $\overline{r} > r'$ such that $\forall r \geq \overline{r}$, $\underline{p}(r) \leq p'$.*

Proof. For any p, let $q(p)$ denote the minimal q such that $AC(q) = p$. Fix p'. Let $N(r)$ be the largest integer such that $N(r) < \frac{rf(p')}{q(p')}$. For r sufficiently large, $AC(\frac{rf(p')}{N(r)}) < p' \leq AC(\frac{rf(p')}{N(r)+1})$. Then in the outcome where $N(r)$ firms each set the price p, and the other firms set a higher price, the active firms all earn a non-negative profit and matching p' by an inactive firm is not profitable. In case any firm undercuts the price, then that firm must supply at least $rf(p')$. But for r large, $AC(rf(p')) > p'$. Thus $p' \in Q(r')$ ■

Remark 4 *In case $AC(q)$ is negatively sloped at $rf(\underline{p}(r))/m(\underline{p}(r))$, it can be shown that $\underline{p}(r)$ decreases for any small increase in r. Note that in that case $AC(rf(\underline{p}(r))/m(\underline{p}(r))) = \underline{p}(r) < AC(rf(\underline{p}(r))/m(\underline{p}(r)) + 1)$. For a small increase in r to r', $AC(r'f(\underline{p}(r))/m(\underline{p}(r))) < \underline{p}(r)$ since $AC(q)$ is negatively sloped at $rf(\underline{p}(r))/m(\underline{p}(r))$. Thus from continuity, $AC(r'f(\underline{p}(r))/m(\underline{p}(r))) < \underline{p}(r) < AC(r'f(\underline{p}(r))/m(\underline{p}(r)) + 1)$, so that $\underline{p}(r) \in Q(r')$. Thus from Proposition 3(ii), $\underline{p}(r') < \underline{p}(r)$.*

5 References

Baye, M.R. and D. Kovenock, 2008, Bertrand Competition, in The New Palgrave Dictionary of Economics. Second Edition. Eds. S.N. Durlauf and L.E. Blume. (Palgrave Macmillan).

Baumol, W., E. Bailey and R. Willig, 1977, Weak indivisible hand theorems on the sustainability of prices in a multiproduct monopoly, American Economic Review 67, 350-367.

Baumol, W., J. Panzar and R. Willig, 1982, Contestable Markets and the Theory of Industrial Structure. (Harcourt Brace Jovanovich, New York).

Bernheim, B.D., B. Peleg and M.D. Whinston, 1987, Coalition-proof Nash equilibria. I: Applications, Journal of Economic Theory 42, 13-22.

bernheim, W. and M.D. Whinston, 1987, Coalition-proof Nash equilibria. II: Concepts, Journal of Economic Theory 42, 1-12.

Chamberlin, E.H., 1993, The Theory of Monopolistic Competition. (Harvard University Press: Cambridge, Massachusetts).

Dastidar, K.G., 1995, On the existence of pure strategy Bertrand equilibrium, Economic Theory 5, 19-32.

Hoernig, S.H., 2002, Mixed Bertrand equilibria under decreasing returns to scale: an embarrassment of riches, Economics Letters 74, 359-362.

Hoernig, S.H., 2007, Bertrand games and sharing rule, Economic Theory 31, 573-585.

Novshek, W. and P. Roy Chowdhury, 2003, Bertrand equilibria with entry: limit results, International Journal of Industrial Organization 21, 795-808.

Ray Chaudhuri, P., 1996, The contestable outcome as a Bertrand equilibrium, Economics Letters 50, 237-242.

Roy Chowdhury, P. and K. Sengupta, 2004, Coalition-proof Bertrand equlibria, Economic Theory 24, 307-324.

Spulber, D., 1989, Regulation and Markets. (MIT Press: Cambridge, Mass.).

Taha, H., 1982, Operations Research (Collier Macmillan: New York).

Vives, X., 1999, Oligopoly pricing: Old ideas and new tools. (MIT Press: Cambridge, Massachusetts, London, England).