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Abstract

This paper studies the long-run welfare effect of the extra volatility of country spread due to the possibility of sudden stops. Both analytical and numerical results show that sudden stops have weaker output impact when the small open economy is more open to trade. However, welfare consequences and policy implication of sudden stops depend on the financial friction faced by the small open economy. When it is free to adjust foreign debt, the cost of sudden stops is decreasing in trade openness, which implies the optimality of open trade policy. In this case, external shocks may be welfare improving. In addition, the economy will gain from counter-cyclical tariff rate policies. On the other hand, when it is costly to adjust foreign debt, the cost of sudden stops may be increasing in trade openness, which implies the optimality of a closed trade policy. In this case, the nature of the policy and how the government implements the policy matter. The results hold in economies with and without the working capital constraint, and in economies with GHH preferences and Cobb-Douglas preferences.

Keywords: Trade openness; Welfare cost; Sudden stops; Small open economy; Second order approximation.
JEL classification: E32; E61; F41.

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1 Introduction

In this paper, we analyze an important question, whether a small open economy should adopt an open trade policy or a closed trade policy when external shocks they face become more volatile perhaps due to the possibility of sudden stops, based on recent development in the small open economy literature. The situation is further complicated with competing empirical results about the relationship between the output impact of external shocks and trade openness, which imply opposite recommendations with respect to trade policy. Calvo et al. (2004) and Calvo and Talvi (2005) show that economies more open to trade will adjust their output less when they are hit by sudden stops. However, several other studies show that greater trade openness increases output growth volatility when the economy is exposed to external shocks; see Rodrik (1997), Easterly et al. (2001), and Loayza and Raddatz (2006).

We address the question thus raises dynamic stochastic general equilibrium model by evaluating the relationship between the cost and trade openness in a small open economy. The cost analyzed refers to the welfare cost associated with the extra volatility of country spread arising from the possibility of sudden stops.[1] Country spread refers to the premium the small open economy has to pay to borrow in the international capital market, and trade openness is measured by the ratio of trade turnover to GDP. The results show that (1) with the same external shocks, less open economies will have larger induced endogenous responses of output. (2) The relationship between the cost and trade openness depends on whether it is costly to adjust foreign debt. (3) Exogenous shocks may be welfare improving. And (4) the sensitivity analysis indicates that these results are indeed robust.

The intuition behind the first result is that less open economies tend to have more volatile capital. The reason is that the marginal cost of production, in terms of the price of the imported intermediate input, is higher due to the higher tariff rate, which is also the reason why these economies are less open to trade. As a result, the demand for capital in a less open economy is more elastic than that in a more open economy. Thus, capital will adjust more in a less open economy. When the external shocks become more volatile due to the possibility of sudden stops, the induced additional volatility of capital is larger in a less open economy for the same reason, as is that of output. This result provides a theoretical foundation for the finding of Calvo et al. (2004), even though we do not consider "dollarization" here as Calvo et al. (2004) do.

It turns out that the larger additional volatility of output does not necessarily lead to a higher cost. When the economy incurs no additional expenses in adjusting its foreign debt, the cost of sudden stops decreases with trade openness. In this case, it is optimal to make the

[1] Mendoza (2001), Aguiar and Gopinath (2007) and Loayza et al. (2007) show that a positive possibility of sudden stops increases the volatility of country spread.

trade door open wider when external shocks become more volatile. By doing so, the marginal cost of production can be lowered (because the tariff rate is lowered), and production becomes more efficient. This policy recommendation provides a new perspective by emphasizing the connection, through the tariff rate, between trade openness and production efficiency. As argued in this paper, when trade openness is positively correlated with production efficiency and there are no costs in adjusting foreign debt, the small open economy should adopt open trade policy to deal with the more volatile external shocks. This policy consideration is in line with Loayza et al. (2007). They argue that the self-protection method as addressed in Ehrlich and Becker (1972) may later lead to large domestic shocks.

However, if it is costly to adjust foreign debt, the larger additional volatility of output usually results in a smaller cost, which implies the optimality of a closed trade policy. The main reason is because the ability of households to smooth consumption is compromised due to the debt adjustment costs. The extent to which the ability to smooth consumption is weakened depends on the level of tariff rate. In this case, there may be two offsetting forces: one is the gain from the improvement of production efficiency and the other one is the cost when the ability to smooth consumption is weakened due to financial friction. Consequently, the cost of sudden stops also depends on the level of tariff rate. The numerical results show the later force dominates, which leads to the optimality of a closed trade policy.

The third result is that external shocks may be welfare improving. When the representative household is a net borrower in the international capital market, its consumption decreases at an accelerated rate as world interest rates increase. Thus, the indirect utility function becomes convex in interest rates. As a result, when interest rates are more volatile, the representative household's utility will be higher.[2] This result extends the discussion about the relationship between welfare and economic shocks. The literature has shown that a risk averse household may prefer a volatile economy. For example, Obstfeld and Rogoff (2000) give a closed form solution to an open economy driven by productivity and monetary shocks and illustrate the possible positive welfare effect of economic shocks. Bacchetta and Wincoop (2000) show that depending on the economic structure, high exchange rate volatility may lead to high welfare of risk averse households. Cho and Cooley (2005) discuss the positive welfare effect of productivity shocks extensively in a closed economy. Here we show how the borrowing and lending position affects such a relationship in a dynamic stochastic general equilibrium model.

This paper extends the discussion to evaluate the welfare effect of time-varying tariff rate policy. When the tariff rates are counter-cyclical and constitute a stationary process around a fixed mean, they may improve welfare if there is no financial friction. However, if it is

[2]This result is disturbing because it is in deep contrast to what people generally believe: With the given ability to smooth consumption, economic shocks should always be detrimental to welfare.

costly to adjust the foreign debt position, the nature of the policy and the way how the government implements the policy matter.

The rest of the paper is organized as follows: Section 2 sets up the competitive equilibrium. Section 3 presents some qualitative results. Section 4 carries out numerical analysis. Section 5 discusses time-varying tariff rate policy. And Section 6 concludes.

2 The benchmark economy

The model used in this paper is based on Mendoza (1991) with three modifications: intermediate imported inputs, a working capital constraint, and debt adjustment costs. With the model, there is a role for trade policy when we use Greenwood et al. (1988, GHH hereafter) preferences augmented with the endogenous subjective discount factor. There are three types of agents, domestic households, firms, and the government. There are also three real frictions: capital adjustment costs, debt adjustment costs, and a working capital constraint.

The economy is driven by a joint process of the productivity shocks, the world interest rate shocks, and the country spread shocks. The external shocks of interest in this paper are country spread shocks, whose importance has been documented in Neumeyer and Perri (2005) and Uribe and Yue (2006). The other two shocks are productivity shocks and world interest rate shocks. In this paper, the possibility of sudden stops is represented by an increase in the volatility of country spread. This definition is different from others in the literature. For example, Chari et al. (2005) define sudden stops as exogenous capital inflow reversal. This definition comes from the fact that the possibility of sudden stops not only increases the average country spread that a small open economy has to pay, but also makes the country spread more volatile; see Mendoza (2001). In addition, Aguiar and Gopinath (2007) argues that business cycles in emerging economies are characterized by sudden stops and more volatile output, which contrast with those in developed small open economies.

We focus on this model for several reasons. First, the model provides a simple framework that allows a role for trade policy. Second, the Mendoza (1991) model is widely used in the small open economy literature. Third, recent studies emphasize the use of this benchmark economy in explaining countercyclical real interest rates [Neumeyer and Perri (2005)], stationary trade-balance to GDP ratio [García et al. (2009)], and asset pricing [Jahan-Parvar et al. (2009)].

2.1 The representative household

The representative household chooses hours and consumption to maximize expected lifetime utility:

$$\max_{\{c_t,h_t\}_{t=0}^{\infty}} \qquad \mathbb{E}_0 \sum_{t=0}^{\infty} \theta_t U(c_t, h_t),$$

where $\mathbb{E}_0$ denotes the mathematical expectation operator conditional on information available at time 0. The variables θ_t, c_t, and h_t denote, respectively, the subjective discount factor from period t to period 0, consumption, and hours.

In this paper, we consider two types of preferences: GHH utility in the benchmark economy, and Cobb-Douglas utility as a robustness check.[3] The momentum utility function for GHH utility and the subjective discount factor take the following functional forms

$$U(c_t, h_t) = \left[(c_t - h_t^{\omega}/\omega)^{1-\gamma} - 1\right] / (1-\gamma)\,, \tag{2.1}$$

$$\theta_{t+1} = \beta(\tilde{c}_t, \tilde{h}_t)\theta_t, t \geq 0, \theta_0 = 1, \tag{2.2}$$

$$\beta(c_t, h_t) = (1 + c_t - h_t^{\omega}/\omega)^{-\beta_1}\,. \tag{2.3}$$

The function β represents the subjective discount factor between period t and period $t+1$. The variables $\tilde{c}_t$ and $\tilde{h}_t$ denote the cross-sectional averages of consumption and hours, respectively. They are taken as given by individual households.

The use of the endogenous subjective discount factor is one way to assure stationary behavior of consumption in the small open economy; see Mendoza (1991), Schmitt-Grohé and Uribe (2003), and Mulraine (2004). As long as $\beta_1 < \gamma$, this preference guarantees a unique limiting distribution of state variables; that the stationary cardinal utility is suitable for dynamic programming, and that the consumption good in every period is a normal good; see Mendoza (1991).

The representative household receives the profit, the capital rent, the labor income, and income from intermediate input sale to firms.[4] The household's period budget constraint is given by:

$$d_t + r_t k_t + w_t h_t + r_t^{\mathrm{m}} m_t + \Gamma_t \geq R_{t-1}d_{t-1} + c_t + i_t + (1+\tau)m_t + \Phi(k_{t+1} - k_t) + \Psi(d_t), \tag{2.4}$$

[3]For clarity, we discuss the Cobb-Douglas utility in a separate section.

[4]Even though households receive profit from firms, we do not include profit in the budget constraint because it is well known that the profit is zero with the assumed constant return to scale technology.

where choices i_t, d_t, and m_t denote investment, foreign debt position, and the imported intermediate input, respectively.[5] r_t, w_t, and r_t^{m} denote the rate of return on capital, the wage rate, and the firm-paid price of the imported intermediate input. τ denotes the tariff rate levied on the imported intermediate input. Γ_t denotes the government transfer.

The economy has five state variables. Eq. (2.4) is related to four of them: R^{us}, CR, d_{t-1}, and k_t, which denote, respectively, world interest rates, country spread, debt position from the last period, and physical capital today. In addition, R_{t-1} denotes the country risk free rate faced by individual households. The country interest rate, R, is the product of the world interest rate, R^{us}, and the country spread, CR. The law of motion of capital is given by:

$$k_{t+1} = (1-\delta)k_t + i_t, \tag{2.5}$$

which is standard. $\Phi(k_{t+1} - k_t)$ denotes the capital adjustment cost where $\Phi(k_{t+1} - k_t) = \phi(k_{t+1} - k_t)^2/2$ for computational simplicity. The cost is included because of its empirical relevance. Once we introduce the cost, the model can match the behavior of investment. $\Psi(d_t)$ denotes the debt adjustment costs, where $\Psi(d_t) = \psi\left[\exp(d_t - d) - 1\right]$ and d denotes the non-stochastic steady state of net foreign debt. The inclusion of debt adjustment costs is another way to assure the stationary behavior of debt; see Schmitt-Grohé and Uribe (2003). This allows us to discuss other different preferences when we set the one-period subjective discount factor constant, and this directly follows the exercise in Neumeyer and Perri (2005).

The representative household is subject to the non-Ponzi-game condition

$$\lim_{j\to\infty} \mathbb{E}_t \frac{d_{t+j+1}}{\prod_{s=0}^{j} R_{t+s}} \geq 0. \tag{2.6}$$

The condition rules out the possibility that the representative household borrows to finance its consumption without limit.

Let $\beta^t\mu_t$ and $\beta^t q_t$ be the Lagrangian multipliers associated with Eqs. (2.4) and (2.5). The optimality conditions include the non-Ponzi game condition (2.6), period budget constraints holding with equality (2.4), the law of motion of capital (2.5), and the first order conditions

[5] The price of m_t is normalized to unity. By assuming the relative price of m_t to be unity, we close the door through which the dynamics of terms of trade can affect the economy here.

as follows:

$$
\begin{aligned}
c_t: &\quad 0 = \mu_t - u_{ct}, && (2.7)\\
h_t: &\quad 0 = w_t + u_{ht}/u_{ct}, && (2.8)\\
d_t: &\quad 0 = \mu_t \left[1 - \Psi'(d_t)\right] - \theta_{t+1}/\theta_t \times R_t \mathbb{E}_t \mu_{t+1}, && (2.9)\\
m_t: &\quad 0 = 1 + \tau - r_t^{\mathrm{m}}, && (2.10)\\
i_t: &\quad 0 = 1 - q_t, && (2.11)\\
k_{t+1}: &\quad 0 = \mu_t \left(1 + \phi(k_{t+1} - k_t)\right) - && (2.12)\\
&\quad \theta_{t+1}/\theta_t \times \mathbb{E}_t \mu_{t+1} \left[1 - \delta + \phi(k_{t+2} - k_{t+1}) + r_{t+1}\right].
\end{aligned}
$$

All the first order conditions have their usual interpretations.

2.2 The firms

There are many identical final-good production firms. They (100% owned by domestic households) produce the final good by hiring labor, renting capital, and buying the imported intermediate input from households. Firms use constant return to scale technology to produce:

$$y_t = z_t k_t^{\alpha_k} h_t^{\alpha_h} m_t^{\alpha_m},$$

where $0 < \alpha_k < 1$, $0 < \alpha_h < 1$, $0 < \alpha_m < 1$, and $\alpha_k + \alpha_h + \alpha_m = 1$. The variables y_t, z_t, k_t, h_t, and m_t denote the output of the final good, the total productivity factor, capital, hours and the intermediate input, respectively. In this economy, the total productivity factor is assumed to follow the process[6]

$$\ln(z_{t+1}) \quad = \quad \rho \ln(z_t) + \varepsilon_{t+1}^z, \varepsilon_{t+1}^z \sim \text{IIND}(0, \sigma_z^2), \tag{2.13}$$

where ρ denotes the first-order serial autocorrelation of z and $0 < \rho < 1$, ε_{t+1}^z denotes the technology shocks, IIND denotes identical and independent normal distribution, and σ_z^2 denotes the variance of technology shocks.

Firms are subject to a working capital constraint, so that output will drop in the presence of a positive country spread shock; see Chari et al (2005). In addition, Neumeyer and Perri (2005) show that a working capital constraint helps amplify the effect of fundamental shocks on business cycles. For simplicity, we adapt the same constraint as that in Uribe and Yue

[6]The structural parameters, ρ and σ_z, are calibrated in the Section 4.2.

(2006). In particular, the working capital constraint takes the following form

$$WK_t \;\geq\; \varphi w_t h_t, \tag{2.14}$$

where the variable WK_t denotes the amount of working capital. The parameter $\varphi \geq 0$ denotes the number of quarter wage bills the representative firm needs to pay. If $\varphi = 0$, the working capital constraint is dropped from the economy. The representative firm's debt position evolves as

$$d_t^{\mathrm{f}} \;=\; R_{t-1}d_{t-1}^{\mathrm{f}} - y_t + w_t h_t + r_t k_t + r_t^{\mathrm{m}} m_t + \pi_t - WK_{t-1} + WK_t,$$

where d_t^{f} denotes the debt position of the firms. Defining the net liability of the representative firm as $a_t = R_t d_t^{\mathrm{f}} - WK_t$, we can rewrite the budget constraint of the representative firm as

$$\frac{a_t}{R_t} \;=\; a_{t-1} - y_t + w_t h_t + r_t k_t + r_t^{\mathrm{m}} m_t + \pi_t + \left(\frac{R_t - 1}{R_t}\right) WK_t. \tag{2.15}$$

Since the representative firm is owned by the representative household, the objective function of firms is defined by

$$\max \qquad \mathbb{E}_0 \sum_{t=0}^{\infty} \theta_t \frac{\mu_t}{\mu_0} \pi_t,$$

where μ_t denotes the marginal wealth utility of the representative household. The objective function is the same as that in Uribe and Yue (2006). The representative firm is also subject to the following non Ponzi-game constraint

$$\lim_{j\to\infty} \mathbb{E}_t \frac{a_{t+j}}{\pi_{s=0}^{j} R_{t+s}} \;\leq\; 0. \tag{2.16}$$

Since firms do not make the investment decision, their problem reduces to a static problem to maximize its period profit by choosing k_t, h_t, and m_t, and taking z_t, r_t, w_t, and r_t^{m} as given. The first order conditions for firms are standard:

$$k_t : \qquad r_t = \alpha_k z_t k_t^{\alpha_k - 1} h_t^{\alpha_h} m_t^{\alpha_m}, \tag{2.17}$$

$$h_t : \qquad w_t \left[1 + \varphi \left(R_t - 1\right)/R_t\right] = \alpha_h z_t k_t^{\alpha_k} h_t^{\alpha_h - 1} m_t^{\alpha_m}, \tag{2.18}$$

$$m_t : \qquad r_t^{\mathrm{m}} = \alpha_m z_t k_t^{\alpha_k} h_t^{\alpha_h} m_t^{\alpha_m - 1}. \tag{2.19}$$

These optimality conditions have their usual interpretations. In addition, since any process a_t that satisfies Eqs. (2.15) and (2.16) will be optimal for the representative firm, we follow Uribe and Yue (2006) and set a_t at 0.

2.3 The government

The government collects a stream of tariff tax income, denoted by τm_t. These incomes are rebated back to the domestic households in a lump sum tax, Γ_t. The government's sequential budget constraint is then given by

$$\tau m_t \quad = \quad \Gamma_t, \ t \geq 0. \tag{2.20}$$

We do not consider the government expenditure shocks in order to simplify the discussion.

2.4 Competitive equilibrium

In equilibrium, the capital market, the labor market, and the intermediate input market all clear. The aggregates equal to the counterparts of the representative household's because households are assumed to be identical:

$$\tilde{c}_t = c_t; \tag{2.21}$$

$$\tilde{h}_t = h_t. \tag{2.22}$$

Proposition 2.1 *The competitive equilibrium is defined as a sequence of real allocations* $\{c_t, h_t, d_t, m_t, i_t, k_{t+1}, \tilde{c}_t, \tilde{h}_t, \Pi_t, \Gamma_t\}_{t=0}^{\infty}$*, and prices* $\{\mu_t, q_t, r_t, w_t, r_t^m\}_{t=0}^{\infty}$*, given* d_{-1}, k_0*, the law of motion of the interest rate (4.3), and the law of motion of the total productivity factor (2.13), satisfying the conditions (2.4) with equality, (2.5), (2.6)-(2.12), and (2.17)-(2.22).*

3 Some qualitative analysis

In general, there is no analytical solution to this dynamic stochastic general equilibrium model, and hence policy analysis must depend on numerical results. However, the GHH utility function with the endogenous discount factor allows us to analyze some important qualitative results of this economy without resorting to numerical calculation . Hence, we discuss the role of the tariff rate, output impact of external shocks, and welfare effect of exogenous shocks before we proceed to the numerical analysis.

3.1 The role of the tariff rate

In the optimal tariff literature, a country that has market power on imported goods may gain from protection by setting tariffs on its imports; see Bagwell and Staiger (1999) for theoretical analysis and Broda et al. (2008) for empirical evidence. However, the role of the tariff rate in this paper is different because there is neither any externality nor market failure in our economy. We impose a tariff on the intermediate imported inputs for the following two reasons.

First, the introduction of the tariff rate serves the purpose of controlling the trade openness of the economy. Formally, it is straightforward to show that

$$TO = \frac{\text{Exports} + \text{Imports}}{y-m} = \frac{TB + 2m}{y-m} = s_{tb} + \frac{2}{(1+\tau)/\alpha_m - 1}, \tag{3.1}$$

where TO denotes trade openness, TB denotes trade balance, y denotes output, m denotes the intermediate imported inputs, s_{tb} denotes the trade-balance to GDP ratio, τ denotes the tariff rate, and α_m is a structural parameter. The value added (or GDP) to this small open economy is given $y - m$. Equation (3.1) makes it clear that trade openness is decreasing in the tariff rate. The government can adjust the trade openness by changing the value of the tariff rate.

Second, the imposition of a tariff does not cause loss in the long-run welfare of households with GHH utility augmented by the endogenous subjective discount factor. In other words, the non-stochastic steady state of period utility is independent of the tariff rate. Formally, we have the following non-stochastic steady state of lifetime utility function

$$V = \frac{\left[(c - h^\omega/\omega)^{1-\gamma} - 1\right]/(1-\gamma)}{1-\beta(c,h)} = \frac{\left\{[\log(R)/\beta_1 - 1]^{1-\gamma} - 1\right\}/(1-\gamma)}{1-1/R}, \tag{3.2}$$

where $\beta(c,h)$ denotes the non-stochastic steady state one-period endogenous subjective discount factor. ω, γ, and β_1 are structural parameters and R is the non-stochastic steady state of country interest rate. All of them are fixed when we analyze the welfare cost of sudden stops. The last equality comes from the following Euler equation in the non-stochastic steady state:

$$\beta(c,h)R = (1 + c - h^\omega/\omega)^{-\beta_1} R = 1.$$

It is thus clear from Eq. (3.2) that the non-stochastic steady state lifetime utility is independent of tariff rates (trade openness). In addition, any change of the non-stochastic steady state lifetime utility due to the change in R is independent of the tariff rate (trade openness).

This property justifies our approach of focusing on the cost associated with the volatility of country spread. In principle, the possibility of sudden stops increases not only the volatility of country spread but also the level of country spread. Thus, the cost associated with the possibility of sudden stops could be decomposed into two parts: the cost associated with the change of volatility of country spread (cost of second-order importance), and the cost associated with the change of level of country spread (cost of first-order importance). However, in our economy, the change of non-stochastic steady state lifetime utility due to the change in the level of country spread is independent of trade openness. Thus, when we discuss the cost associated with the possibility of sudden stop, we focus on the cost of second-order importance because the cost of first-order importance is the same across trade openness when we use GHH preferences.

We plot the non-stochastic steady states of some variables against the tariff rate in Figure 1. It is clear that trade openness is decreasing in the tariff rate while the lifetime utility is constant across tariff rates. The results are summarized in the following proposition.

Proposition 3.1 *In a small open economy, if the preferences are given by Eq. (2.1), the endogenous subjective discount factor is given by Eqs. (2.2) and (2.3), and the competitive equilibrium is given by Proposition 2.1, then the non-stochastic steady state lifetime utility is independent of the tariff rate and the trade openness is decreasing in the tariff rate.*

3.2 Why the response of output is larger in a less open economy?

Without debt adjustment costs, we find (as shown in Section 4.4) that the cost (of extra volatility of country spread) is decreasing in trade openness, which implies that open trade policy should be preferred. To understand this result, it is crucial to understand why the less open economy becomes more volatile with the given external shocks.

The higher volatility of output in a less open economy is mainly because capital is more volatile. The reason is that the marginal cost of production, in terms of the price of the imported intermediate input, is higher due to the higher tariff rate. Thus, the demand for capital in a less open economy is more elastic than that in a more open economy. This difference in demand for capital brings in the difference in the volatilities of capital. For any realized external shock, capital in the equilibrium will adjust more in a less open economy. This is because the demand curve is flatter in a less open economy while the supply curve of capital is the same. When the external shocks become more volatile due to the possibility of sudden stops, the additional volatility on capital is larger in a less open economy for the same reason. With the typical calibration of a standard small open economy, this larger additional volatility of capital is transformed into a higher cost, when it is free to adjust foreign debt.

Formally, given the model specification, the rate of return on capital is inversely related to the tariff rate. To see this relationship, we derive the following equation about the rate of return on capital from Eqs. (2.8), (2.10), (2.17), (2.18), and (2.19):

$$r_t = \alpha_k \left(\frac{\alpha_h}{X_t}\right)^{(1-\alpha_m)\varpi_1(\alpha_h+\omega\alpha_m)-1} \left(\frac{\alpha_m}{1+\tau}\right)^{\alpha_m\varpi_1(\alpha_h+\omega\alpha_m)+1} z_t^{\varpi_1\omega} k_t^{-(\omega-1)\alpha_h\varpi_1}, \quad (3.3)$$

where $\varpi_1 = 1/(\omega - \alpha_h - \omega\alpha_m) > 0$ with the calibration. $X_t = [1 + \varphi(R_t - 1)/R_t]$ and when there is no working capital constraint, $X_t = 1$.

It is clear from Eq. (3.3) that the negative relationship holds: for a given state of capital stock and productivity, the rate of return on capital is inversely related to the tariff rate. Or, the rate is positively related to trade openness.

To see how this relationship leads to more volatile capital in a less open economy, we start with a simple case by assuming no adjustment cost, no working capital constraint, and constant productivity. With those assumptions, we combine (2.9) and (2.12) and obtain the following:

$$\begin{aligned} R_t &= \frac{\mathbb{E}_t \mu_{t+1} \left(1 - \delta + r_{t+1}\right)}{\mathbb{E}_t \mu_{t+1}} \\ &= 1 - \delta + \alpha_k \left(\alpha_h\right)^{(1-\alpha_m)\varpi_1(\alpha_h+\omega\alpha_m)-1} \left(\frac{\alpha_m}{1+\tau}\right)^{\alpha_m\varpi_1(\alpha_h+\omega\alpha_m)+1} k_{t+1}^{-(\omega-1)\alpha_h\varpi_1}. \end{aligned}$$

The last equality comes from the fact that k_{t+1} is known at the time t; and productivity is assumed to be constant, so r_{t+1} is known at the time t.

When there is a positive country spread shock, i.e., R_t goes up, the representative household will accumulate less capital for the next period, $\Delta k_{t+1} < 0$. However, the representative household in the less open economy will accumulate even less capital:

$$\Delta k_{t+1}(\text{high tariff}) \quad < \quad \Delta k_{t+1}(\text{low tariff}) < 0.$$

This result holds because the less open economy has to reduce more capital to equalize the rate of return on bond and that on capital, i.e., the demand for capital in less open economies is more elastic. When there is a negative country spread shock, i.e., R_t goes down, the representative household will accumulate more capital for the next period, $\Delta k_{t+1} > 0$. However, the representative household in the less open economy will accumulate even more for the same reason. Since for any country spread shock, the less open economy has a bigger adjustment in capital, it is thus of no doubt that capital is more volatile.

This negative relationship holds even if we bring back the working capital constraint. To see this, we further assume that R_t follows a perfect foresight process, therefore X_{t+1} is known in period t and we obtain the following

$$\frac{\left(R_t + \delta - 1\right)\left(X_{t+1}\right)^{(1-\alpha_m)\varpi_1(\alpha_h+\omega\alpha_m)-1}}{\alpha_k \left(\alpha_h\right)^{(1-\alpha_m)\varpi_1(\alpha_h+\omega\alpha_m)-1}} = \left(\frac{\alpha_m}{1+\tau}\right)^{\alpha_m\varpi_1(\alpha_h+\omega\alpha_m)+1} k_{t+1}^{-(\omega-1)\alpha_h\varpi_1}.$$

It is clear that for any change happens to the left-hand side of the above equation, capital is going to adjust but more in a less open economy, as in the previous case. The difference is that capital may increase, instead of decrease, when R_t increases. This is because when R_t

goes up, it is usually true that R_{t+1} will go up and this will increase X_{t+1} as well. However, if α_m is sufficiently large, the term $(X_{t+1})^{(1-\alpha_m)\varpi_1(\alpha_h+\omega\alpha_m)}$ may go down and bring down the left-hand side of the equation. As a result, k_{t+1} may increase and move in the same direction as R_t. One thing worth mentioning is that this possibility is not in line with empirical facts, and it is never realized in all the simulations conducted in the numerical experiments.

Proposition 3.2 *In a small open economy solely driven by country spread shocks, if there are no capital and debt adjustment costs, productivity is constant, and the competitive equilibrium is given by Proposition 2.1, then capital is more volatile in a less open economy.*

There is no close-form answer to whether capital is more volatile in a less open economy when we include productivity shocks, country spread shocks, and those adjustment costs. We answer that question and the policy implication with numerical results in Section 4.3. It turns out that the same feature, more volatile capital in a less open economy, shows up after we introduce capital adjustment costs, debt adjustment costs, productivity shocks, and even with different preferences. However, the implication for trade policy in economies without debt adjustment costs is different from that in economies with debt adjustment costs; see Section 4.4).

3.3 Why country spread shocks may be welfare improving?

One result of this paper is that country spread shocks may enhance utility. This result seems counter-intuitive. To understand why the risk averse household may like economic uncertainty associated with country spread, we use the following simple two-period model to illustrate the welfare effect of country spread shocks. Suppose the representative household in a small open economy lives for two periods: period 1 and period 2. It has the following endowment flows: 0 in period 1, and y_2 in the period 2. This endowment distribution makes sure that the household will be a borrower in period 1. In period 1, the household solves a perfect foresight problem. Formally, the household chooses consumption and the borrowing and lending position to maximize his utility function:

$$V = \log(c_1) + \log(c_2).$$

The subjective discount factor is assumed to be 1 in order to simplify the discussion. His period budget constraints are $c_1 + b_1 = 0$ and $c_2 = Rb_1 + y_2$. It can be shown that the solutions are $c_1 = y_2/(2R)$ and $c_2 = y_2/2$. The variables c_1, c_2, b_1, and R denote consumption in period 1, consumption in period 2, the borrowing and lending position, and the interest rate, respectively.

Ex ante, the value of R is unknown. Thus, the expected indirect utility is given by

$$\mathbb{E}V = 2\log(y_2/2) - \mathbb{E}\log(R). \tag{3.4}$$

Equation (3.4) clearly shows that the expected utility is convex in R. This implies that when R becomes more volatile, the $\mathbb{E}V$ will be higher. The intuition is: When the representative household is a net borrower in the international capital market, its consumption will decrease at an accelerated rate with country spread. As a result, when the country spread is more volatile, the ex ante expected indirect utility is higher.

A relevant case is where the next period output is a decreasing function of today's interest rate. Suppose $Y_{t+1} = R_t^{-1/\Lambda}$, where $\Lambda > 0$, then the choices of consumption are $c_1 = R_1^{-1-1/\Lambda}/2$ and $c_2 = R_1^{-1/\Lambda}/2$. The ex ante expected indirect utility of the household is given by:

$$\mathbb{E}V = -(1 + 2/\Lambda)\,\mathbb{E}\log R_1 - 2\log 2. \tag{3.5}$$

Compared to (3.4), it is clear that endogenizing output will re-enforce the positive welfare effect of interest rate volatility.

An example in which country spread shocks are detrimental to welfare can be the case in which the small open economy is a net lender in the international capital market. To see this, assume $y_1 > 0$ and $y_2 = 0$ instead of $y_1 = 0$ and $y_2 > 0$ as in the above example. In this case, the representative household is a net lender in the international capital market. It is straightforward to show that, ex ante, we have

$$\mathbb{E}V = 2\log(y_1/2) + \mathbb{E}\log(R).$$

The expected indirect utility is concave in R and it decreases when the volatility of country spread increases.

Combining the two examples together, we have the following proposition.

Proposition 3.3 *In a two-period small open economy, country spread shocks are welfare improving if the representative household does not have endowment income today. Country spread shocks are welfare deleterious if the representative household does not have endowment income tomorrow.*

Ericson and Liu (2009) discuss the welfare effect of interest rate shocks in more detail.

Since the unconditional welfare may increase in the country spread volatility, it is not a surprise to find out that country spread shocks may actually be welfare improving. In Section 4.4, we show with the numerical analysis that whether the cost in the benchmark economy is positive or negative largely depends on the net borrowing and lending position of this small open economy and foreign debt adjustment costs. If, on the other hand, the small open economy is a net borrower in the international capital market or there are costs in adjusting foreign debt, the increase of volatility of country spread may decrease the welfare of the representative household.

4 Quantitative analysis

We apply the second order perturbation method discussed in Schmitt-Grohé and Uribe (2004) to obtain the numerical solution to the competitive equilibrium defined in Proposition 2.1. The perturbation method has been widely used in the literature.[7] We use the second order approximation algorithm because the first order approximation method could not differentiate welfare in two different economies which have the same non-stochastic steady state but different volatilities. Formally, the first order approximation of utility is given by:

$$\mathbb{E}U_t = U + U_x(\mathbb{E}x_t - x) + U_\sigma \sigma,$$

where $\mathbb{E}$ denotes the mathematical unconditional expectation operator. A variable without time-subscript denotes its non-stochastic steady state. The vector x denotes the logarithm of state variables. The parameter σ controls the volatility of the cycles. The row vector U_x denotes the first order derivative of utility with respect to x. The variable U_σ denotes the first order derivative with respect to σ. The first order condition requires that $U_\sigma = 0$; see Schmitt-Groh$\acute{e}$ and Uribe (2004). By assumption, the unconditional expectation of x is zero, i.e., $\mathbb{E}x_t = 0$. For example, $\mathbb{E}\log(z_t) = 0$ if we assume the standard AR(1) process as in the literature. It is also true that $x = 0$. It then comes true that $\mathbb{E}U_t = U$. As a result, there is no way to differentiate policies or economies with the same U.

The second order approximation of utility is given by:

$$\mathbb{E}U_t = U + 1/2\mathbb{E}(x_t - x)' U_{xx}(x_t - x) + 1/2 U_{\sigma\sigma}\sigma^2.$$

The square matrix U_{xx} denotes the Hessian matrix with respect to x. The variable $U_{\sigma\sigma}$ denotes the second order derivatives with respect to σ. With second order approximation, it is clear that both $\mathbb{E}(x_t - x)' U_{xx}(x_t - x)$ and $U_{\sigma\sigma}$ are not necessarily zero. As a result, the second order approximation allows to evaluate different policies or economies with the same non-stochastic steady state, in other words, it allows us to obtain the non-zero welfare cost of the additional volatility of country spread.

[7] We briefly show how to solve the dynamic stochastic general equilibrium in the appendix. Similar descriptions can be found in many other places.

4.1 Welfare cost

We focus on the unconditional cost instead of a conditional cost because the ranking of the conditional cost will depend upon the assumed initial state of the economy; see Schmitt-Grohé and Uribe (2006a,b). Here the cost refers to welfare cost of the additional volatility of country spread due to the possibility of sudden stops. Volatilities of country spread may be different because some economies face the possibility of sudden stops while others do not; see Aguiar and Gopinath (2007).

In our numerical exercise, an economy with highly volatile country spread can be regarded as emerging economies. We set the standard deviation of country spread at 0.0196, a value from Neumayer and Perri (2001), for this type of economy. An economy with less volatile country spread can be regarded as developed small open economies. The standard deviation of country spread is set at 0.0096, an artificial number. We assume this smaller number to serve the purpose to differentiate emerging economies and developed small open economies; see Mendoza (2001). Making that artificial number smaller or larger (as long as smaller than 0.0196) does not change the qualitative results.

Given the tariff rate and the joint stochastic process of total factor productivity, world interest rates, and country spread, the unconditional lifetime welfare, $\mathbb{E}V$, can be written as

$$\mathbb{E}V(\tau, \sigma),$$

where σ denotes the uncertainty of the economy. The above definition means that the unconditional lifetime utility are a function of economic uncertainty and the tariff rate.

The cost is defined as a lump sum consumption, $\lambda(\tau, \sigma)$, by which the representative household is willing to give up to be as well off to avoid uncertainty. This definition can be regarded as equivalent variation in the sense that the change in consumption is equivalent to economic uncertainty in terms of its welfare impact; see Mas-Colell et al. (1995, page 82). Mathematically, the welfare cost of volatility is indirectly defined by

$$\mathbb{E}V(\tau, \sigma) = \frac{\left\{ [c(\tau) - \lambda(\tau, \sigma) - h(\tau)^{\omega}/\omega]^{1-\gamma} - 1 \right\} / (1-\gamma)}{1 - [1 + c(\tau) - \lambda(\tau, \sigma) - h(\tau)^{\omega}/\omega]^{-\beta_1}}. \tag{4.1}$$

The non-stochastic steady consumption and hours are functions of τ and $\mathbb{E}V$ is a function of τ and σ. As a result, the cost is a function of both τ and σ as well. If we remove λ, then the righthand side of Eq. (4.1) denotes the non-stochastic steady state lifetime welfare of the representative household at the tariff rate. One thing to note that we can write down the cost in Eq. (4.1) without changing hours because with GHH momentum utility function, there is no wealth effect on the labor supply.

Once we solve the model, we obtain the numerical value for $\mathbb{E}V(\tau, \sigma)$, $c(\tau)$, and $h(\tau)$.

There is only one unknown in Eqs. (4.1), $\lambda(\tau, \sigma)$. The equation is solved using MATLAB command, fsolve.m.[8] After $\lambda(\tau, \sigma)$ is obtained, the cost of sudden stops is given by

$$\lambda(\tau, \text{sudden stops}) = \lambda(\tau, \sigma = 1.96\%) - \lambda(\tau, \sigma = 0.96\%). \tag{4.2}$$

The difference is used to denote the cost we focus on the cost associated with the additional volatility of country spread due to the possibility of sudden stops.

4.2 Data and calibration

For the benchmark economy, we select the Argentina economy as a representative because it is well known that Argentina has been suffered a lot from sudden stops. We use three different data sources. The first one is the International Financial Statistics of International Monetary Fund, from which we obtain data about GDP, investment (fixed capital formation), total consumption, exports of goods and services, and imports of goods and services.[9] All data are deseasonalized using the X-12 ARIMA procedure provided by the Bureau of Census and deflated by the GDP deflator. We apply HP filter to obtain the cyclical components of each time series and consequently obtain the standard deviations of output, investment, trade openness and consumption, and the first-order serial autocorrelation of output. They are listed in Table 3. With the same data source, we set the non-stochastic steady state trade openness, TO, at 0.31, which is the average of trade openness of Argentina from the first quarter of 1993 to the first quarter of 2009. The non-stochastic steady state trade balance to GDP ratio, s_{tb}, is set at 0.025.[10]

The second data source is the World Trade Organization from which we obtain the data on tariff rates that governments actually charge on imports and the value of imports for products.[11] Products are identified by 6-digit codes under the World Customs Organizations internationally agreed "Harmonized System" for defining product categories. The average of value-weighted ad valorem tariff rates of Argentina for years 1999-2001 is 14%. Thus, we set the non-stochastic steady state tariff rate at 14%.

[8]Similarly, we use the same command to solve (4.4) to obtain welfare cost in the Cobb-Douglas utility case.

[9]Here total consumption is defined in the same way as in Neumayer and Perri (2005): the sum of private consumption, government spending, change in the inventories, and statistical errors and discrepancy.

[10]Note that several papers set $s_{tb} = 0.5\%$ for Argentina. The difference is that we use the national accounts measured in domestic currency while those papers use data measured in the U.S. dollars. For the same IMF data source, if we use the data measured in the U.S. dollars, the average of s_{tb} is around 0.55%. In the sensitivity analysis, we show that they are robust with respect to the values of non-stochastic steady state trade balance.

[11]The data are available at *www.wto.org/english/thewto_e/whatis_e/tif_e/org6_e.htm*.

The third data source is the literature. In particular, the non-stochastic steady state interest rate, R, is set at 1.0275, a value from Uribe and Yue (2006). It is consistent with the average 11% annual real interest rate faced by a small open economy in the international capital market. The non-stochastic steady state world interest rate R^{us} is set at 1.01625, a value from Mendoza and Uribe (2000). The non-stochastic steady state of net foreign debt is given by $d = TB/(R-1)$, where TB denotes the non-stochastic steady state trade balance. In addition, the law of motion of interest rate is assumed to follow the estimated process in Neumeyer and Perri (2001):

$$\begin{pmatrix} \hat{R}_t^{\text{us}} \\ \hat{CR}_t \end{pmatrix} = \begin{pmatrix} 0.73 & 0.04 \\ 0.70 & 0.58 \end{pmatrix} \begin{pmatrix} \hat{R}_{t-1}^{us} \\ \hat{CR}_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{t,R^{\text{us}}} \\ \varepsilon_{t,CR} \end{pmatrix}, \tag{4.3}$$

where the variables with hat denote the percentage deviations from the trend. The variance and covariance of innovations are given by $\sigma_{\varepsilon_{R^{us}}} = 0.42\%$, $\sigma_{\varepsilon_{CR}} = 1.96\%$, and $\rho_{\varepsilon_R, \varepsilon_{CR}} = 0.30$.

We calibrate the economy to match the quarterly data of Argentina. The risk aversion coefficient, γ, is set at 2, a common value used in the business cycle literature. The capital depreciation rate, δ, is set at 0.025, which has also been widely used in the literature. The exponent of labor supply in utility, ω, is set at 1.455, a value from Mendoza (1991).[12] The share of labor income in value added, s_h, and the share of capital income in value added, s_k, are set at 0.62 and 0.38, respectively, the same as in Neumayer and Perri (2005). The parameter φ in Eq. (2.14) is set at 1.2, a value from Uribe and Yue (2006). The value means that the representative firm needs to save money to be able to pay at least 1.2 quarter wage bills.

The parameter α_m is chosen to make sure that in the non-stochastic steady state, trade openness is 31%. The parameters, α_k and α_h, are determined by two conditions: first, in the value added, capital income share is $s_k = 0.38$ and labor income share is $s_h = 0.62$; second, the production is homogeneous of degree one, so $\alpha_k + \alpha_h = 1 - \alpha_m$. The share of investment in value added, s_i, is calculated by the following equation:

$$s_i = \frac{i}{y-m} = \frac{\delta r k}{r(y-m)} = \frac{\delta s_k}{r}.$$

The non-stochastic steady state rate of return on capital, r, is calculated from the non-stochastic steady state optimal condition $r = R - 1 + \delta$. The share of consumption is derived by using the accounting identity in the non-stochastic steady state, $s_c = 1 - s_i - s_{tb}$. From the calibration so far, the determination of the non-stochastic steady state of c and h is independent of β_1. Thus, the parameter β_1 can be calibrated by the following non-stochastic

[12] As argued in Neumayer and Perri (2005), there is no independent estimate of ω. They set it at 1.6.

steady state optimal condition:

$$1 = (1 + c - h^{\omega}/\omega)^{-\beta_1} R.$$

The implied value for β_1 is 0.0659, which is less than $\gamma = 2$. This guarantees the GHH utility function is suitable for dynamic programming when we close the debt adjustment cost channel; see Schmitt-Grohé and Uribe (2003).

Once the above structural parameters are calibrated, their values will be kept constant. One thing worth mentioning is that the non-stochastic steady state trade balance to GDP share will be changed to accommodate the change of the tariff rate in order to have balanced non-stochastic steady state. The values for the structural parameters and some long-run moments are summarized in Table 1.

The last four parameters are the serial correlation of productivity shock, ρ, the standard deviation of the innovation to productivity shocks, σ_z, the capital adjustment cost parameter, ϕ, and debt adjustment cost parameter, ψ. They are model-specific. For each model, to calibrate these parameters, we choose values for them, simulate the model, and repeat this process until the simulated volatilities of output, investment, and trade openness, and the first order autocorrelation coefficient of output match the data as close as possible.

In the numerical analysis, Five different models are considered with GHH preferences. Model (a) is the benchmark economy. When we drop the working constraint from the benchmark economy, we get model (b). If the one-period subjective discount factor in the benchmark economy is fixed, that is model (c). Model (d) is the benchmark economy dropping out debt adjustment costs and model (e) is the benchmark model dropping both debt adjustment costs and the working capital constraint. In Cobb-Douglas preferences case, we consider two different models: one with a working capital constraint, model (f), and the other without, model (g). For each model, we re-search for the values of these four parameters. They are displayed in Table 2.

4.3 Numerical results: Business cycles

The benchmark model - model (a) can explain the business cycles of small open economy well, as shown by the impulse responses and the second moments. Figure 2 shows the impulse responses of some key variables to a positive country spread shock. The solid line represents the economy with a low tariff rate. The dotted line represents the economy with a high tariff rate. When there is a positive country spread shock, the representative household is willing to borrow less because the cost of borrowing rises, and a sudden stop of the type addressed in Chari et al. (2005) emerges. With the working capital constraint, the labor demand decreases even though the labor supply does not move because of the GHH preferences. As a result, the positive country spread shock decreases both hours and output in equilibrium. Consumption drops because of the negative welfare effect. Investment drops dramatically because the opportunity cost of investing is high. Trade balance and current account are thus improved.

Panel (B) and panel (C) in Table 3 display some second moments with different models associated with GHH preferences. The benchmark economy, a model with a 14% tariff rate and high country spread volatility, replicates the selected business cycle moments of Argentina economy. In particular, the generated standard deviation of output is 4.17%, only 0.01 percentage points lower than the observed counterpart. The generated relative standard deviation of investment to that of output is 3.00, only 0.08 percentage points higher than the observed one. The generated standard deviation of trade openness is 2.45, very close to that observed in the data. Finally, the generated first-order serial autocorrelation coefficient is smaller than the data only by 4 percentage points.

Table 3 also lists the generated second moments of consumption and hours. The generated ratio of σ_h/σ_y, 0.75, seems reasonable compared to 0.57 reported in Neumayer and Perri (2005), where σ_h and σ_y denote the standard deviation of hours and that of output, respectively. However, the benchmark model produces a very low ratio of σ_c/σ_y, where σ_c denotes the standard deviation of consumption. In the data, the ratio is about 1.36, while the generated ratio is only 0.80, about 41% lower. This is true with all the models we consider in this paper. The reason is the inclusion of the intermediate imported inputs, m, which allows households smooth consumption. Jahan-Parvar et al. (2009) consider a similar model without m in which consumption is more volatile than output. To reconcile this discrepancy, we suspect that by fixing the supply of m one-period ahead, as Boldrin et al. (2001) do with respect to the quantity supplied of hours, the modified model will generate more volatile consumption. However, given our goal is not to match the moment, we defer that to the future research.

When $\tau = 14\%$, other models are also able to replicate the standard deviations of output and investment, and the first serial autocorrelation of output. However, they clearly miss in generating the standard deviation of trade openness. The benchmark model turns out to be the best model in replicating business cycles moments of Argentina economy.

The most noticeable feature of Table 3 is that, for the same calibration and the same process of driving force, economies less open to trade (when $\tau = 14\%$) have more volatile output than economies more open to trade (when $\tau = 0$). To see this, simply compare models in panel (B) to the corresponding models in panel (C) in the GHH preferences case in Table 3. This result is in line with Calvo et al. (2004), Calvo and Talvi (2005), and Edwards (2004a, 2004b) but without assuming liability-dollarization. This directly follows the intuition we have shown in Section 3.2, even after we introduce capital adjustment costs, debt adjustment costs, and productivity shocks. The same pattern can also be seen from the impulse responses of capital to country spread shocks; see Figure 2.

4.4 Numerical results: Welfare cost

To facilitate the discussion, we start with model (d) and model (e) first, both of which do not have debt adjustment costs. The difference is that model (d) has a working capital constraint while model (e) does not. With model (d), the welfare cost of the additional volatility of country spread is -0.0020419 unit of consumption good in the benchmark economy when the tariff rate is set at 14%, which is about 0.00204% of its non-stochastic steady state consumption. This low value of cost is as expected. With the standard real business cycles models, the welfare cost measured in percentage of consumption is typically small.

The negative welfare cost implies that risk averse households are willing to pay extra to be able to live in the more volatile economy. In other words, when country spread becomes more volatile, households will achieve higher welfare. Since Argentina is a net borrower in the international capital market, this result is not a surprise given the discussion in Section 3.3. In models with GHH preferences, there are at least two ways to obtain a positive welfare cost: if the small open economy is a lender in the international capital market, or if the ability to borrow and lend is compromised. The first condition is confirmed by extra numerical exercises whose results are not reported here. The second condition is evident by the cost associated with model (b) and model (c).

When the tariff rate is set at 0 in model (d), the welfare cost turns out to be -0.0028634 unit of consumption good, which becomes smaller than that in the $\tau = 14\%$ case. Thus, the cost is lower when the economy is more open, which implies that open trade policy is preferred. The same trade policy recommendation is implied in model (e): when $\tau = 14\%$, the welfare cost of the extra volatility of country spread is -0.0024921 unit of consumption good (0.2349% of its non-stochastic state consumption). This is higher than the welfare cost when $\tau = 0$, which is -0.0033563 unit of consumption good.

In this economy, an increase in trade openness always means an improvement in production efficiency as we have shown in Section 3.2. When it is free to adjust foreign debt, there will have no forces against the desire to increase trade openness in order to improve production efficiency. As a result, open trade policy is always preferred. Since both model (d) and model (e) recommend open trade policy, it is clear that a working capital constraint is an irrelevant factor with respect to optimal trade policy here.

Now we turn to models (a) - (c), all of which have debt adjustment costs. With model (a), the welfare cost is -2.2679e-4 unit of consumption good in the benchmark economy when $\tau = 14\%$. It is smaller than the cost of -1.808e-5 units of consumption good when $\tau = 0$. To check whether the result is due to the assumed endogenous subjective discount factor or a working capital constraint, model (b) and model (c) are also considered. The same pattern shows up in model (b) and (c): the cost is increasing in trade openness. Thus, it is optimal to have the closed trade policy when it is costly to adjust foreign debt. Since the result is obtained in all three models, (a), (b), and (c), it is independent of the endogenous subjective discount factor and the working capital constraint.

The numerical results show when it is costly to adjust foreign debt, it is not optimal to have free trade. The underlying reason is because households' ability to smooth consumption by borrowing and lending is compromised. In our numerical examples, the ability in the $\tau = 14\%$ case is only mildly weakened while it is severely weakened in the free trade case. The different impact of debt adjustment costs on that ability outweighs the desire to open the trade to improve the production efficiency. The net effect leads to optimality of the closed trade policy.

We further consider different values of key parameters. In particular, we consider the case when households become more risk averse, i.e., increasing the value of γ from 2 to 5. We also consider the case when labor supply becomes less sensitive to wage rates, i.e., increasing the value of ω from 1.455 to 1.6.[13] Note that Neumayer and Perri (2005) set γ at 5 and ω at 1.6. The numerical results are displayed in Table 4. Once again, we obtain the same results: no debt adjustment costs, open trade policy is preferred; otherwise the closed trade policy is preferred.[14]

Given the above numerical results, we conclude the policy implications are robust. When there are no debt adjustment costs, the desire to improve production efficient leads to the optimality of open trade policy. This is independent of the working capital constraint, the borrowing and lending position, and the key parameter values. When there are debt adjustment costs, the ability to smooth consumption is severely weakened in the $\tau = 0$ case and the induced loss outweighs the improvement from production efficiency, which leads to the optimality of the closed trade policy. This result is independent of the working capital constraint, the key parameter values, and the endogenous subjective discount factor.

Next, we check whether the policy recommendations are robust to a more general class of preferences: Cobb-Douglas utility.

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[13]In this case of $\omega = 1.6$, we set $\beta_1 = 0.589$ to have well-defined non-stochastic steady state.

[14]In the extra numerical exercises, we find that the borrowing and lending position does not affect the negative relationship between the cost and trade openness in model (d) and model (e). We also extend model (e) to two sector economies with homogenous capital and heterogenous capital, and obtain the same policy implication. The results are not reported here but available upon request.

4.5 Numerical Results: Cobb-Douglas preferences

We consider the following Cobb-Douglas utility function:

$$U(c_t, h_t) = \left\{\left[c^{\chi}(1-h)^{1-\chi}\right]^{1-\gamma} - 1\right\} / (1-\gamma).$$

In this case, we set $\beta(c_t, h_t) \equiv \beta^*$ and thus $\theta_{t+1}/\theta_t = \beta^*$. With Cobb-Douglas utility, the inclusion of debt adjustment costs assures stationary behavior of state variables; see Schmitt-Grohé and Uribe (2003). With Cobb-Douglas utility, there is wealth effect on labor supply. Thus, the welfare cost of volatility is defined as

$$\mathbb{E}V(\tau,\sigma) = \frac{\left\{[c(\tau)-\lambda(\tau,\sigma)]^{\chi}\left[1-h(\tau)-\frac{1-\chi}{\chi}\frac{\lambda(\tau,\sigma)}{w(\tau)}\right]^{1-\chi}\right\}^{1-\gamma} - 1}{1-\beta^*}, \tag{4.4}$$

where $w(\tau)$ denotes the non-stochastic steady state wage rate. The reason for $\lambda(\tau)$ showing up twice is because in the non-stochastic steady state, we have

$$\frac{(1-\chi)\,c}{\chi\,(1-h)} = w.$$

As a result, when c decreases, h will increase due to the wealth effect. The cost of sudden stops is also defined by Eq. (5.3).

We calibrate two new parameters, χ, which denotes the consumption share in the utility, and β^*. For simplicity, we set $\beta^* = 1/R$ and follow Neumayer and Perri (2005) by setting χ at 0.24. The rest model-invariant parameters are set at the same values as in the GHH preferences case. The model-specific parameters, ρ_z, σ_z, ϕ, and ψ are re-estimated for the two models we consider, model (f) and model (g). The results are shown in Panel (D) and panel (E) in Table 3. Once again, the following findings with GHH preferences remain true with Cobb-Douglas preferences: (1) output is more volatile when economy is more open, and (2) the welfare cost of extra volatility of country spread is higher in the $\tau = 14\%$ case than that in the $\tau = 0$ case, when it is costly to adjust foreign debt. These are independent of the working capital constraint. Note that here we cannot drop debt adjustment costs because they are required to assure the stationary behavior of the economy after we drop the endogenous subjective discount factor assumption.

However, the policy implication is ambiguous because the non-stochastic steady state lifetime utility depends on the tariff rate. In particular, with Cobb-Douglas preferences, a decrease of the tariff rate will induce households to work more on one hand. The resulted higher output will make households better off. On the other hand, the increased work effort will make households unhappy. These two forces work against each other. With our Cobb-

Douglas utility, households' non-stochastic steady state lifetime utility is maximized when τ is set at a value less than zero; see Figure 3. Thus, if we start at $\tau = 14\%$, it is automatically true that free trade policy is preferred when there is no uncertainty. Given that our goal is the welfare cost of additional volatility, we defer the analysis on the justification of a tariff in a more complicated economic environment in future research.

5 Time-varying tariff rate policy

So far, we have obtained robust results about optimal trade policy: either stay at $\tau = 14\%$ when it is costly to adjust foreign debt; or pursue free trade policy when it is free to adjust foreign debt. One relevant question to ask is whether the government can do better. In other words, can the economy gain from some kind of time-varying tariff rate policy over the constant tariff rate policy of 14%?

To answer this question, we extend models (a), (b), (d), and (e) with $\tau = 14\%$ and high volatility of country spread by introducing a time-varying tariff rate policy.[15] In particular, we consider the following time-varying policies:

$$\hat{\tau}_t = \vartheta * \hat{y}_t, \tag{5.1}$$

$$\hat{\tau}_t = -\vartheta * \hat{CR}_t, \tag{5.2}$$

where a variable with hat denotes the percentage deviation from its non-stochastic steady state. The welfare cost of time-varying tariff rate policy is given by

$$\lambda(\tau_t, \tau) = \lambda(\tau_t, \sigma) - \lambda(\tau, \sigma),$$

where $\lambda(\tau_t, \sigma)$ and $\lambda(\tau, \sigma)$ are obtained by solving the corresponding Eq. (4.1). When the cost is positive, it implies that countercyclical policy is detrimental to welfare compared to the fixed tariff rate policy. When the cost is negative, it implies that countercyclical policy is welfare-improving.

We consider both countercyclical policy and procyclical policy. In the numerical exercise, ϑ varies from -2 to 2. The tariff rate is countercyclical when $\theta > 0$ and procyclical when $\theta < 0$.[16] Countercyclical policy is considered because it is natural for the government to implement certain countercyclical policy to stabilize the economy. Procyclical policy

[15] We do not include model (c) with the intention to simplify the computational task.

[16] Note that the term, procyclical (countercyclical), in this paper means that the tariff rate is high when output is low (high). Our definition is based on whether the policy tends to stabilize output or not. If yes, then we say the policy is countercyclical, otherwise procyclical. Sometimes this relationship is labeled as countercyclical (procyclical) in some literature discussion, for example, Bagwell and Staiger (1995).

is considered because there is bountiful empirical evidence. The most famous example is the Smoot-Hawley Tariff Act passed in June of 1930, which increased the tariff rate to 50%. In the early 1980s, the tariff rate in Chile rose in the face of the debt crisis. After the December 1994 Peso crisis, the general tariff rate in Mexico rose from 8.7% in 1994 to a peak of 12.5% in 1995; see Haltiwanger et al. (2004). The theoretical explanation provided is that political factors affect decision makers in such a way that the procyclical trade policy is the equilibrium outcome; see Bagwell and Staiger (1995).

We plot the welfare cost of countercyclical policy against ϑ in Figure 4. The first column shows the welfare costs associated with the policy (5.1). The second column shows the welfare costs associated with the policy (5.2). When the cost is negative, it means that the economy gains from the countercyclical policy over the constant policy; and vice verse. Two results immediately show up by examining Figure 4. First, when there are no debt adjustment costs (the working capital constraint does not matter), both policies improve the welfare of the economy; see the results associated with model (d) and model (c). Second, when there are debt adjustment costs, the way how the government implements policy matters. In general, the government should set the tariff rate not as a function of country spread gap, but as a function of output gap. This is a surprising result. Because both policy (5.1) and policy (5.2) are countercyclical, therefore we expect both should have increase the welfare. Note that both results are robust with respect to different parameter values.

The welfare cost of procyclical policy is plotted against ϑ in Figure 5. The first and the second columns show the welfare costs associated with the policy (5.1) and the policy (5.2), respectively. When there are no debt adjustment costs (does not matter whether there is a working capital constraint), both policies reduce the welfare of the economy; see the results associated with model (d) and model (c). On the contrary, when there are debt adjustment costs, if the government sets tariff rates as a function of country spread gap, the economy will gain over the constant tariff rate policy. This may provide an economic justification why we observe procyclical tariff rate policy in the real life: because it is not free to adjust foreign debt.

6 Conclusions

We analyze the optimal trade policy when external shocks become more volatile. There are two forces in the economy. One force is the desire to improve production efficiency. This leads to the optimality of open trade policy. The second force is the debt adjustment costs, which compromises the households' ability to smooth consumption through the international capital market. In the models considered, this force dominates and leads to the optimality of a closed trade policy. In addition, countercyclical policy will be preferred if there is no financial friction. However, once the economy faces costs in adjusting its foreign debt, the nature of the policy, countercyclical or procyclical, and the way how the government implement the policy matter. In general, it is optimal to have tariff rates either to positively respond to output gap, or to negatively respond to the country spread gap. These findings are new theoretical results to the literature, which provide economic justifications for two widely observed phenomena: why tariff rates are on average positive and why procyclical policy is implemented. One simple reason is that it is not free to adjust foreign debt.

The results show that it is important for the government to consider the costs they face in the international capital market before it considers to change its trade policy. Given the fact that those economies with a positive possibility of sudden stops usually face debt adjustment costs, it is thus recommended, according to our theoretical results, that they adopt a closed trade policy, even though in this case the economy will be generally more volatile. Thus, even with the findings in Calvo et al. (2004) and Calvo and Talvi (2005), it is still optimal to close the door. Further, our results show that the government needs to pay attention to the anchor to which tariff rates should respond when it tries some forms of countercyclical tariff rate policy.

Barro (2009) claims if a model does not satisfy the "Atkeson-Phelan principle", the welfare analysis based on the model is less meaningful. To satisfy the "Atkeson-Phelan principle," it is necessary for the model to have a good explanation power of the equity prices in emerging economies. Jahan-Parvar et al. (2009) show that debt adjustment costs do make a difference in explaining the equity returns in emerging economies. This paper provides an example confirming the claim in Barro (2009) by showing the difference in optimal trade policy due to debt adjustment costs.

There is, however, scope for improvement in our analysis. For example, in the Cobb-Douglas preferences case, we impose a tariff without an economic reason. To justify the use of the tariff rate, it is sufficient to consider some market failures or externality. We defer this to the future research.

We also assume a reduced form representation of sudden stops simply due to the computational technique concern. The perturbation method we use requires no kinks in policy functions. However, to endogenize sudden stops, it is necessary to introduce kinks to policy functions. Some new algorithm with the penalty function analyzed in den Haan and de Wind (2008) may help solve models with endogenized sudden stops. However, we suspect that our long-run results would be reversed because sudden stops are rare events.

In addition, some business cycle moments generated by our three-input models are quite different from those corresponding moments generated by two-input models. For example, consumption is more volatile than output in Neumayer and Perri (2005). However, it is more smoother than output in our model (c), which is a three-input counterpart of the two-input model in Neumayer and Perri (2005). Some factor inflexibility may help reconcile the difference. We explore this possibility in the future research.

At last, we only consider three types of shocks. Recently, the literature has argued that investment-specific shocks are an important driving force of business cycles. Generalizing the model to allow for investment-specific shocks (and other shocks, such as government expenditure shocks) is an additional extension we plan to consider in later work.

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7 Tables and figures

Table 1: Calibration

Symbol	Definition	Value
γ	Risk aversion coefficient	2
δ	Capital depreciation rate	0.025
ω	Exponent of labor supply in utility	1.5
φ	Working capital constraint parameter	1.2
α_m	Imported input elasticity	0.1422
α_h	Labor elasticity	0.5318
α_k	Capital elasticity	0.3260
β_1	Subjective discount factor parameter	0.0659
s_i	Investment share in value added	0.1773
s_c	Consumption share in value added	0.7977
s_h	Share of labor income in value added	0.62
s_k	Share of capital income in value added	0.38
s_{tb}	Share of trade balance in value added	0.025
R^{US}	Steady state of world interest rate	1.01625
R	Steady state of interest rate	1.0275
TO	Steady state of trade openness	0.31
r	Marginal return to capital	0.0525
τ	Average tariff rate	0.14

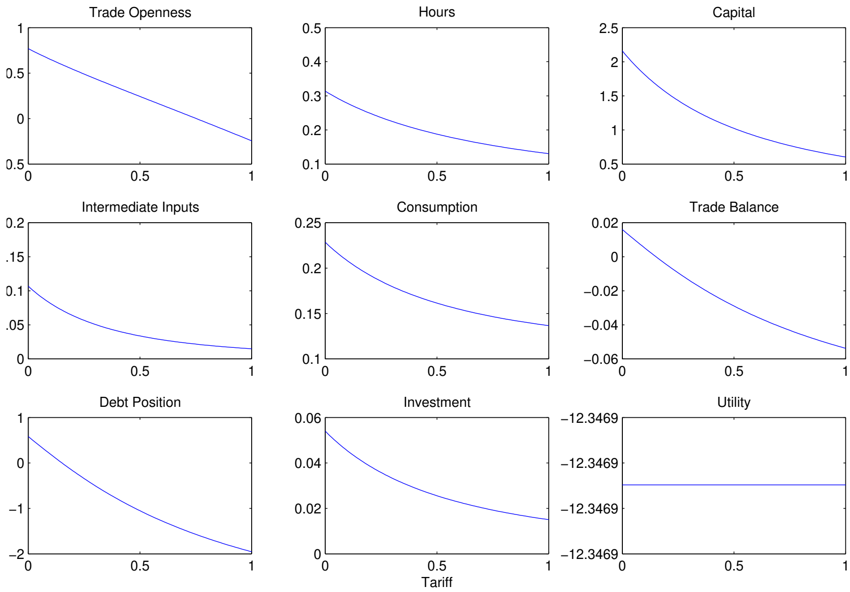
Notes: Table 1 contains the calibration of structural parameters that are invariant to models for the Argentina economy.

Table 2: Calibration of Model Specific Structural Parameters

Models	ρ_z	σ_z	ϕ	ψ	φ
(A) GHH					
(a) Benchmark	0.66	1.25e-2	0.70	0.005	1.2
(b) Without WK	0.68	1.25e-e	0.45	0.005	0
(c) With constant β	0.70	1.20e-2	0.60	0.10	1.2
(d) Without B/L	0.64	1.20e-2	1.20	0	1.2
(e) Without (WK and B/L)	0.67	1.20e-2	1.20	0	0
(B) Cobb-Douglas					
(f) Benchmark	0.72	1.50e-2	5.5	0.05	1.2
(g) Without WK	0.67	1.60e-2	5.0	0.05	0

Notes: Table 2 contains the calibration of structural parameters that are model specific for the Argentina economy. Reported values are calibrated model specific parameters in this study. ρ_z and σ_z are, respectively, the first-order autoregressive parameter and the standard deviation of the productivity process. ϕ and ψ are, respectively, the cost parameters for capital adjustment costs and borrowing and lending costs. When $\phi = 0$, the capital adjustment costs are dropped in the model. When $\psi = 0$, the borrowing and lending costs are dropped in the model. φ is the working capital constraint parameter. When $\varphi > 0$, a working capital constraint is introduced into the model. When $\varphi = 0$, the working capital constraint is dropped. "WK" refers to the model with the imposition of the working-capital constraint. "B/L" denotes debt adjustment costs.

Figure 1: Steady state with GHH utility



Notes: The vertical axis variables represent the aggregate variables of interest. The horizontal variable denotes the tariff rate. The model is the benchmark economy.

Table 3: Business Cycle Moments and Welfare Cost

Data & Models	Business Cycle Moments						Welfare Cost
	Of interest				Implied		
	σ_y	$\frac{\sigma_i}{\sigma_y}$	σ_{TO}	ρ_y	$\frac{\sigma_c}{\sigma_y}$	$\frac{\sigma_h}{\sigma_y}$	(10e-4)
(A) Data	4.18	2.92	2.44	0.82	1.36	–	–
(B) GHH with $\tau = 14\%$							
(a) Benchmark	4.17	3.00	2.45	0.78	0.80	0.75	-2.2679
(b) Without WK	4.21	2.88	1.83	0.79	0.80	0.69	-1.0426
(c) With constant β	4.16	2.92	1.82	0.81	0.63	0.68	0.010
(d) Without B/L	4.25	2.84	3.43	0.81	0.73	0.75	-20.419
(e) Without (WK and B/L)	4.18	2.92	3.37	0.79	0.72	0.69	-24.921
(C) GHH with $\tau = 0\%$							
(a) Benchmark	4.15	2.35	1.20	0.78	0.82	0.73	-0.1808
(b) Without WK	3.70	1.41	1.24	0.79	0.58	0.69	0.0054
(c) With constant β	4.06	2.71	1.73	0.80	0.63	0.68	0.052
(d) Without B/L	4.13	2.66	3.15	0.80	0.84	0.75	-28.634
(e) Without (WK and B/L)	4.08	2.72	3.13	0.78	0.82	0.69	-33.563
(D) Cobb-Douglas with $\tau = 14\%$							
(f) Benchmark	4.22	2.93	3.05	0.76	0.69	0.43	0.066
(g) Without WK	4.19	2.92	3.16	0.67	0.66	0.45	0.049
(E) Cobb-Douglas with $\tau = 0\%$							
(f) Benchmark	4.08	2.81	2.87	0.75	0.71	0.42	0.068
(g) Without WK	4.06	2.80	2.97	0.66	0.68	0.44	0.052

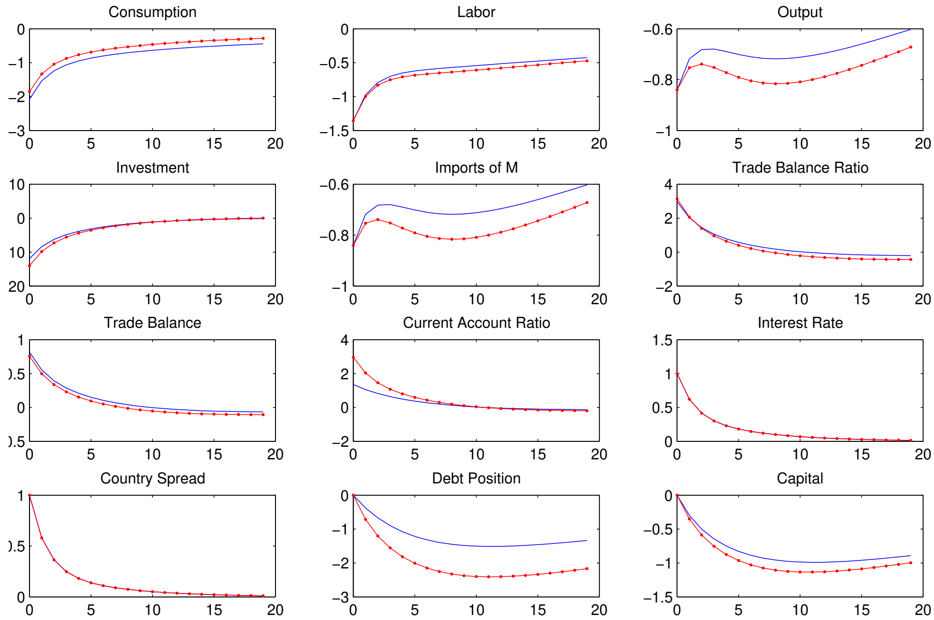
Notes: The moments are calculated based on the standard deviation of country spread shocks being 1.96%. The cost of extra volatility of country spread is defined by Eq. (5.3). All values of standard deviations are in percentages. σ_y, σ_i, σ_{TO}, σ_c, and σ_h are, respectively, the standard deviations of output, investment, trade openness, consumption and hours, and ρ_y denotes the first-order serial autocorrelation of output. The welfare costs are measured in units of consumption good. "Data" report the unconditional sample moments. We do not report the data of σ_h because in general the quality of data for emerging economies is poor; see Aguiar and Gopinath (2007). Neumeyer and Perri (2005) report a value of 0.57 for σ_h/σ_y. "WK" refers to the model with the imposition of the working-capital constraint. "B/L" denotes debt adjustment costs.

Table 4: Sensitivity Analysis – Welfare Cost

	Different Combinations of key parameters			
	$\gamma = 2$	$\gamma = 2$	$\gamma = 5$	$\gamma = 5$
	$\omega = 1.455$	$\omega = 1.6$	$\omega = 1.455$	$\omega = 1.6$
(B) GHH with $\tau = 14\%$				
(a) Benchmark	-2.2679	-2.4701	-1.9306	-2.0940
(b) Without WK	-1.0426	-1.4966	-1.0368	-1.4181
(c) With constant β	0.0100	0.0516	0.0265	0.0516
(d) Without B/L	-20.419	-23.376	-23.745	-26.636
(e) Without (WK and B/L)	-24.921	-26.919	-33.416	-34.198
(C) GHH with $\tau = 0\%$				
(a) Benchmark	-0.1808	-0.3288	-0.1788	-0.3435
(b) Without WK	0.0054	0.0104	0.0028	0.0099
(c) With constant β	0.0518	0.0845	0.0483	0.0677
(d) Without B/L	-28.634	-30.571	-41.633	-42.181
(e) Without (WK and B/L)	-33.563	-34.331	-52.823	-50.579

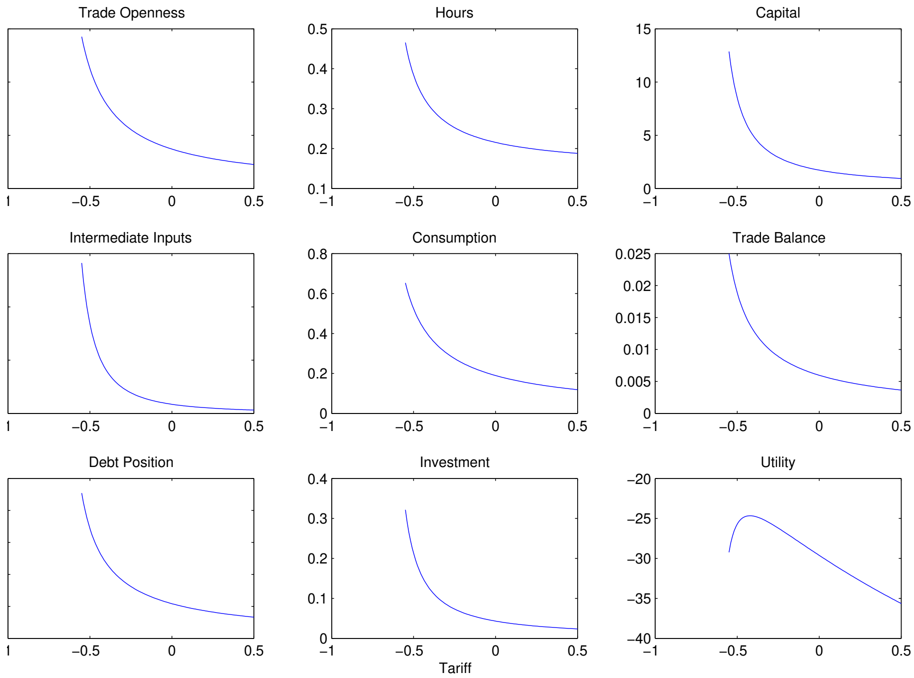
Notes: The cost of extra volatility of country spread is defined by Eq. (5.3) and measured in 10e-4 unit of consumption good. "WK" refers to the model with the imposition of the working-capital constraint. "B/L" denotes debt adjustment costs. When the exponent of labor supply in utility, ω, is set at 1.6, the subjective discount factor parameter, β_1, will be re-calibrated at 0.0589.

Figure 2: Impulse responses to a positive country spread shock



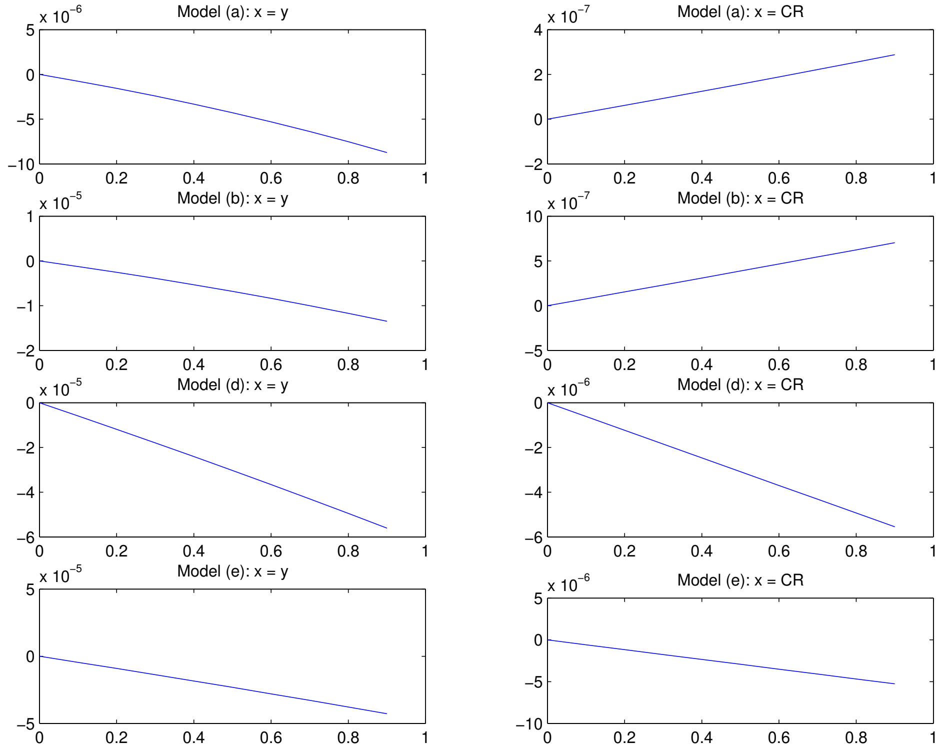
Notes: The benchmark model is the one-sector economy with capital adjustment costs, the working capital, and debt adjustment costs. The solid line corresponds to 0% tariff rate. The dotted Line corresponds to 14% tariff rate.

Figure 3: Steady state with Cobb-Douglas utility



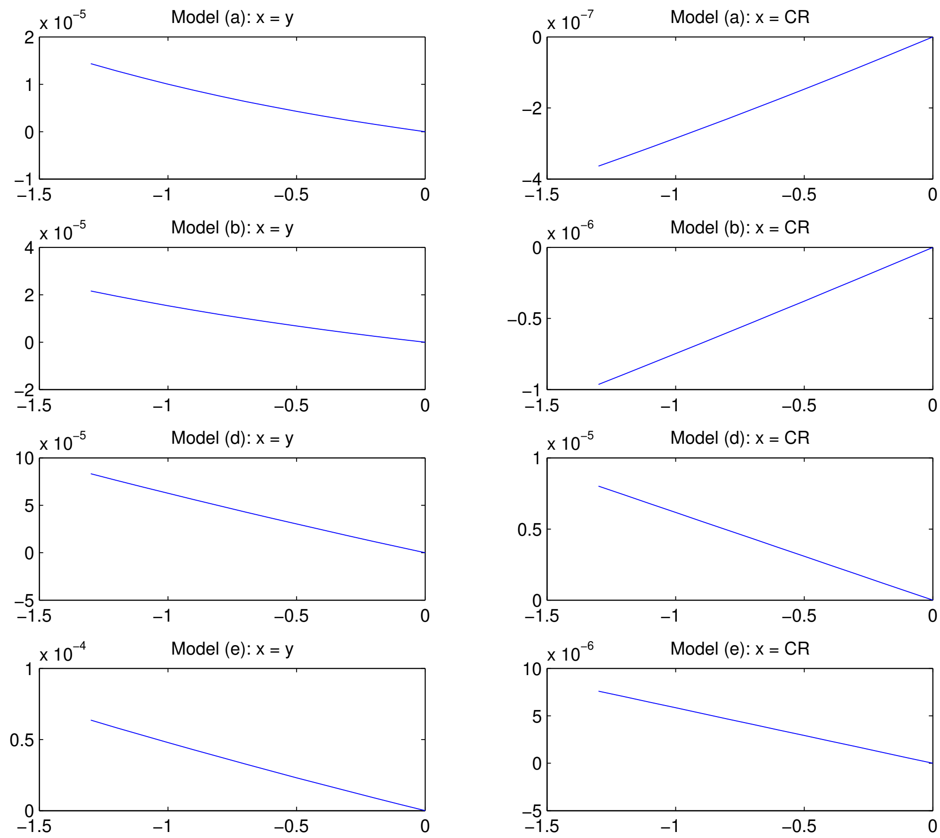
Notes: The vertical axis variables represent the aggregate variables of interest. The horizontal variable denotes the tariff rate.

Figure 4: Welfare cost of countercyclical policies with GHH preferences



Notes: The vertical axis represents the welfare cost of countercyclical tariff rate around $\tau = 14\%$ compared to the fixed tariff rate policy at $\tau = 14\%$. The horizontal variable denotes the response coefficient of tariff rates to the gap of x, where x denotes output in the first column, and country spread in the second column.

Figure 5: Welfare cost of procyclical policies with GHH preferences



Notes: The vertical axis represents the welfare cost of procyclical tariff rate around $\tau = 14\%$ compared to the fixed tariff rate policy at $\tau = 14\%$. The horizontal variable denotes the response coefficient of tariff rates to the gap of x, where x denotes output in the first column, and country spread in the second column.

8 Appendix - Not for Publication

8.1 One-sector model with GHH utility

The Lagrange is

$$\begin{aligned} \mathcal{L} &= E_0 \sum_{t=0}^{\infty} \theta_t \{ U(c_t, h_t) + \mu_t [z_t F(k_t, h_t, m_t) + \Gamma_t + d_t - R_{t-1} d_{t-1} - c_t - i_t \\ & \quad -(1+\tau) m_t - \Phi(k_{t+1} - k_t)] + \mu_t q_t \left[(1-\delta) k_t + i_t - k_{t+1} \right] \}. \end{aligned}$$

The optimality conditions are:

$$\begin{aligned} \mu_t &= \left(c_t - \frac{h_t^{\omega}}{\omega} \right)^{-\gamma}, \\ h_t^{\omega-1} &= w_t, \\ \mu_t \left[1 - \Psi'(d_t) \right] &= \left(1 + \tilde{c}_t - \frac{\tilde{h}_t^{\omega}}{\omega} \right)^{-\beta_1} R_t E_t \mu_{t+1}, \\ 1 + \tau &= (\alpha_m) z_t k_t^{\alpha_k} h_t^{\alpha_h} m_t^{\alpha_m - 1}, \\ q_t &= 1, \\ \mu_t \left[1 + \phi(k_{t+1} - k_t) \right] &= \left(1 + \tilde{c}_t - \frac{\tilde{h}_t^{\omega}}{\omega} \right)^{-\beta_1} E_t \mu_{t+1} \\ & \quad \left[1 - \delta + \phi(k_{t+2} - k_{t+1}) + z_{t+1} \alpha_k k_{t+1}^{\alpha_k - 1} h_{t+1}^{\alpha_h} m_{t+1}^{\alpha_m} \right], \\ 0 &= \alpha_k z_t k_t^{\alpha_k - 1} h_t^{\alpha_h} m_t^{\alpha_m} - r_t, \\ 0 &= \alpha_h z_t k_t^{\alpha_k} h_t^{\alpha_h - 1} m_t^{\alpha_m} - w_t \left[1 + \varphi \left(R_t - 1 \right) / R_t \right], \\ 0 &= \alpha_m z_t k_t^{\alpha_k} h_t^{\alpha_h} m_t^{\alpha_m - 1} r_t^m, \\ y_t + d_t &= R_{t-1} d_{t-1} + c_t + i_t + (1+\tau) m_t + \frac{\phi}{2} (k_{t+1} - k_t)^2 + \Psi(d_t), \\ y_t &= z_t k_t^{\alpha_k} h_t^{\alpha_h} m_t^{\alpha_m}, \\ k_{t+1} &= (1-\delta) k_t + i_t, \\ c_t &= \tilde{c}_t, \\ h_t &= \tilde{h}_t. \end{aligned}$$

We can rearrange and get the following

$$\begin{aligned} h_t &= \varpi_2 \left(z_t k_t^{\alpha_k} \right)^{\varpi_1}, \\ m_t &= \varpi_3 \left(z_t k_t^{\alpha_k} \right)^{\omega \varpi_1}, \end{aligned}$$

where

$$\begin{aligned}
X_t &= \left[1+\varphi\left(R_t-1\right)/R_t\right], \\
\varpi_1 &= \frac{1}{\omega-\alpha_h-\omega\alpha_m}, \\
\varpi_2 &= \left[\left(\frac{\alpha_h}{X_t}\right)^{1-\alpha_m}\left(\frac{\alpha_m}{1+\tau}\right)^{\alpha_m}\right]^{\varpi_1}, \\
\varpi_3 &= \frac{X_t}{\alpha_h}\frac{\alpha_m}{1+\tau}\left[\left(\frac{\alpha_h}{X_t}\right)^{1-\alpha_m}\left(\frac{\alpha_m}{1+\tau}\right)^{\alpha_m}\right]^{\varpi_1\omega}.
\end{aligned}$$

8.1.1 Functional forms and non-stochastic steady state

The functional forms are:

$$\begin{aligned}
F(k,h,m) &= k^{\alpha_k}h^{\alpha_h}m^{\alpha_m}, \\
1 &= \alpha_k+\alpha_h+\alpha_m, \\
F(k^{\mathrm{T}},h^{\mathrm{T}},m) &= k^{\alpha_{k^{\mathrm{T}}}}h^{\alpha_{h^{\mathrm{T}}}}m^{\alpha_m}, \\
F(k^{\mathrm{N}},h^{\mathrm{N}}) &= k^{\alpha_{k^{\mathrm{N}}}}h^{\alpha_{h^{\mathrm{N}}}}, \\
\alpha_k &\geq 0, \alpha_h\geq 0, \alpha_k+\alpha_h\leq 1, \\
U(c,h) &= \frac{\left[c-\omega^{-1}h^{\omega}\right]^{1-\gamma}-1}{1-\gamma}, \\
\Phi(x) &= \frac{\phi}{2}x^2, \\
\Psi(d_t) &= \psi\left[\exp\left(d_t-\bar{d}\right)-1\right], \\
\beta(c,h) &= \left(1+c-\frac{h^{\omega}}{\omega}\right)^{-\beta_1}.
\end{aligned}$$

In the non-stochastic steady state, the optimality conditions are reduced to

$$\begin{aligned}
d &= Rd+c+i+m-k^{\alpha_k}h^{\alpha_h}m^{1-\alpha_k-\alpha_h}, \\
i &= \delta k, \\
\mu &= \left(c-\omega^{-1}h^{\omega}\right)^{-\gamma}, \\
h^{\omega-1} &= \alpha_h k^{\alpha_k}h^{\alpha_h-1}m^{1-\alpha_k-\alpha_h}, \\
1 &= \left(1+c-\frac{h^{\omega}}{\omega}\right)^{-\beta_1}R, \\
1+\tau &= (1-\alpha_k-\alpha_h)k^{\alpha_k}h^{\alpha_h}m^{-\alpha_k-\alpha_h}, \\
q &= 1, \\
R &= \left[1-\delta+\alpha_k k^{\alpha_k-1}h^{\alpha_h}m^{1-\alpha_k-\alpha_h}\right].
\end{aligned}$$

Rearrange and we get three equations of k, h, and m only:

$$\begin{aligned}
\frac{X}{\alpha_h} &= k^{\alpha_k} h^{\alpha_h - \omega} m^{1-\alpha_k-\alpha_h}, \\
\frac{1+\tau}{(1-\alpha_k-\alpha_h)} &= k^{\alpha_k} h^{\alpha_h} m^{-\alpha_k-\alpha_h}, \\
\frac{R+\delta-1}{\alpha_k} &= k^{\alpha_k-1} h^{\alpha_h} m^{1-\alpha_k-\alpha_h},
\end{aligned}$$

where $X = [1+\varphi(R-1)/R]$. We can solve for h as:

$$h = \left[\left(\frac{R+\delta-1}{\alpha_k}\right)^{\alpha_k}\left(\frac{X}{\alpha_h}\right)^{\alpha_h}\left(\frac{1+\tau}{\alpha_m}\right)^{\alpha_m}\right]^{\frac{1}{\alpha_h(1-\omega)}}.$$

Notice without m and without the working capital constraint, the solution to h becomes:

$$h = \left[\left(\frac{R+\delta-1}{\alpha_k}\right)^{\alpha_k}\left(\frac{1}{\alpha_h}\right)^{\alpha_h}\right]^{\frac{1}{\alpha_h(1-\omega)}}.$$

Consider $\alpha_k + \alpha_h = 1$ when there is no m, the solution is the same as the one in SGU (2003), which is:

$$h = \left[\left(\frac{1}{1-\alpha_k}\right)\left(\frac{R+\delta-1}{\alpha_k}\right)^{\frac{\alpha_k}{1-\alpha_k}}\right]^{\frac{1}{1-\omega}}.$$

Thus, the output is

$$\begin{aligned}
Y &= k^{\alpha_k} h^{\alpha_h} m^{\alpha_m} \\
&= \left[\frac{X}{\alpha_h}\frac{\alpha_k}{R+\delta-1}h^{\omega}\right]^{\alpha_k} h^{\alpha_h}\left[\frac{X}{\alpha_h}\frac{\alpha_m}{1+\tau}h^{\omega}\right]^{\alpha_m} \\
&= \left(\frac{\alpha_h}{X}\right)^{\alpha_h}\left(\frac{\alpha_k}{R+\delta-1}\right)^{\alpha_k}\left(\frac{\alpha_m}{1+\tau}\right)^{\alpha_m} \times \\
&\quad \left[\left(\frac{X}{\alpha_h}\right)^{\alpha_h}\left(\frac{R+\delta-1}{\alpha_k}\right)^{\alpha_k}\left(\frac{1+\tau}{\alpha_m}\right)^{\alpha_m}\right]^{1+\frac{\omega}{\alpha_h(1-\omega)}} \\
&= \left[\left(\frac{X}{\alpha_h}\right)^{\alpha_h}\left(\frac{R+\delta-1}{\alpha_k}\right)^{\alpha_k}\left(\frac{1+\tau}{\alpha_m}\right)^{\alpha_m}\right]^{\frac{\omega}{\alpha_h(1-\omega)}}.
\end{aligned}$$

8.1.2 Derivation of Eq. (3.3)

We can rearrange optimality condition with respect to k_t and get the following

$$
\begin{aligned}
r_t &= \alpha_k z_t k_t^{\alpha_k - 1} h_t^{\alpha_h} m_t^{\alpha_m} \\
&= \alpha_k z_t k_t^{\alpha_k - 1} \left[\varpi_2 \left(z_t k_t^{\alpha_k}\right)^{\varpi_1}\right]^{\alpha_h} \left[\varpi_3 \left(z_t k_t^{\alpha_k}\right)^{\omega\varpi_1}\right]^{\alpha_m} \\
&= \alpha_k \varpi_2^{\alpha_h} \varpi_3^{\alpha_m} z_t^{1+\varpi_1\alpha_h+\varpi_1\omega\alpha_m} k_t^{\alpha_k-1+\varpi_1\alpha_k\alpha_h+\varpi_1\alpha_k\omega\alpha_m} \\
&= \alpha_k \varpi_2^{\alpha_h} \varpi_3^{\alpha_m} z_t^{\varpi_1\omega} k_t^{-(\omega-1)\alpha_h\varpi_1},
\end{aligned}
$$

where

$$
\begin{aligned}
\varpi_1 &= \frac{1}{\omega - \alpha_h - \omega\alpha_m}, \\
\varpi_2 &= \left[\left(\frac{\alpha_h}{X_t}\right)^{1-\alpha_m} \left(\frac{\alpha_m}{1+\tau}\right)^{\alpha_m}\right]^{\varpi_1}, \\
\varpi_3 &= \frac{X_t}{\alpha_h}\frac{\alpha_m}{1+\tau} \left[\left(\frac{\alpha_h}{X_t}\right)^{1-\alpha_m} \left(\frac{\alpha_m}{1+\tau}\right)^{\alpha_m}\right]^{\varpi_1\omega}.
\end{aligned}
$$

We can further simplify the expression for r_t by simplifying $\varpi_2^{\alpha_h}\varpi_3^{\alpha_m}$, which is given by

$$
\begin{aligned}
\varpi_2^{\alpha_h}\varpi_3^{\alpha_m} &= \left\{\left[\left(\frac{\alpha_h}{X_t}\right)^{1-\alpha_m} \left(\frac{\alpha_m}{1+\tau}\right)^{\alpha_m}\right]^{\varpi_1}\right\}^{\alpha_h} \\
&\quad \left\{\frac{X_t}{\alpha_h}\frac{\alpha_m}{1+\tau} \left[\left(\frac{\alpha_h}{X_t}\right)^{1-\alpha_m} \left(\frac{\alpha_m}{1+\tau}\right)^{\alpha_m}\right]^{\varpi_1\omega}\right\}^{\alpha_m} \\
&= \left(\frac{\alpha_h}{X_t}\right)^{(1-\alpha_m)\varpi_1(\alpha_h+\omega\alpha_m)-1} \left(\frac{\alpha_m}{1+\tau}\right)^{\alpha_m\varpi_1(\alpha_h+\omega\alpha_m)+1}.
\end{aligned}
$$

And,

$$
\begin{aligned}
\varpi_2^{\alpha_h}\varpi_3^{\alpha_m} &= \left\{\left[\left(\frac{\alpha_h}{X_t}\right)^{1-\alpha_m} \left(\frac{\alpha_m}{1+\tau}\right)^{\alpha_m}\right]^{\varpi_1}\right\}^{\alpha_h} \\
&\quad \left\{\frac{X_t}{\alpha_h}\frac{\alpha_m}{1+\tau} \left[\left(\frac{\alpha_h}{X_t}\right)^{1-\alpha_m} \left(\frac{\alpha_m}{1+\tau}\right)^{\alpha_m}\right]^{\varpi_1\omega}\right\}^{\alpha_m} \\
&= \left(\frac{\alpha_h}{X_t}\right)^{(1-\alpha_m)\varpi_1(\alpha_h+\omega\alpha_m)-1} \left(\frac{\alpha_m}{1+\tau}\right)^{\alpha_m\varpi_1(\alpha_h+\omega\alpha_m)+1}.
\end{aligned}
$$

8.2 One-sector model with Cobb-Douglas utility

If instead we assume Cobb-Douglas utility function

$$U(c,h) = \frac{\left[c^{\chi}(1-h)^{1-\chi}\right]^{1-\gamma}-1}{1-\gamma},$$

we then have the following non-stochastic steady state optimality conditions:

$$\begin{aligned}
0 &= TB + c + i + m - k^{\alpha_k}h^{\alpha_h}m^{1-\alpha_k-\alpha_h}, \\
d + TB &= Rd, \\
i &= \delta k, \\
\mu &= \left[c^{\chi}(1-h)^{1-\chi}\right]^{-\gamma}\chi c^{\chi-1}(1-h)^{1-\chi}, \\
\frac{(1-\chi)\,c}{\chi\,(1-h)}\frac{X}{\alpha_h} &= k^{\alpha_k}h^{\alpha_h-1}m^{1-\alpha_k-\alpha_h}, \\
1 &= \beta R, \\
1+\tau &= (1-\alpha_k-\alpha_h)k^{\alpha_k}h^{\alpha_h}m^{-\alpha_k-\alpha_h}, \\
q &= 1, \\
R+\delta-1 &= \alpha_k k^{\alpha_k-1}h^{\alpha_h}m^{1-\alpha_k-\alpha_h}.
\end{aligned}$$

From the optimality conditions with respect to k and m

$$\begin{aligned}
\frac{1+\tau}{(1-\alpha_k-\alpha_h)} &= k^{\alpha_k}h^{\alpha_h}m^{-\alpha_k-\alpha_h}, \\
\frac{R+\delta-1}{\alpha_k} &= k^{\alpha_k-1}h^{\alpha_h}m^{1-\alpha_k-\alpha_h},
\end{aligned}$$

we have

$$m = \frac{(1-\alpha_k-\alpha_h)}{\alpha_k}\frac{R+\delta-1}{1+\tau}k = F_1 k.$$

Thus,

$$y = k^{\alpha_k}h^{\alpha_h}m^{1-\alpha_k-\alpha_h} = \frac{R+\delta-1}{\alpha_k}k = F_2 k.$$

For the given $s_{tb} = \frac{TB}{k^{\alpha_k}h^{\alpha_h}m^{1-\alpha_k-\alpha_h}-m}$, we can rewrite the period budget constraint as:

$$(1-s_{tb})\left(k^{\alpha_k}h^{\alpha_h}m^{1-\alpha_k-\alpha_h}-m\right) = c + \delta k.$$

Plug the expression of y and m into the period budget constraint, we get

$$(1-s_{tb})(F_2k - F_1k) \quad = \quad c + \delta k.$$

Plug the expression of y and m into the period budget constraint, we get

$$c \quad = \quad (1-s_{tb})(F_2 - F_1)k - \delta k = F_3 k.$$

Rewrite the optimality condition with respect to h:

$$\frac{(1-\chi)c}{\chi(1-h)}\frac{X}{\alpha_h} \quad = \quad k^{\alpha_k}h^{\alpha_h-1}m^{1-\alpha_k-\alpha_h},$$

as

$$\begin{aligned}
\frac{(1-\chi)c}{\chi(1-h)}\frac{X}{\alpha_h} = \frac{(1-\chi)F_3k}{\chi(1-h)}\frac{X}{\alpha_h} &= k^{\alpha_k}h^{\alpha_h-1}m^{1-\alpha_k-\alpha_h} = \frac{y}{h} = \frac{F_2k}{h}, \\
\frac{F_4}{(1-h)} &= \frac{F_2}{h}.
\end{aligned}$$

Thus,

$$h^* \quad = \quad \frac{F_2}{F_2+F_4} = \frac{\frac{R+\delta-1}{\alpha_k}}{\frac{R+\delta-1}{\alpha_k}+\frac{(1-\chi)F_3X}{\chi\alpha_h}} = \frac{\frac{R+\delta-1}{\alpha_k}}{\frac{R+\delta-1}{\alpha_k}+\frac{(1-\chi)[(1-s_{tb})(F_2-F_1)-\delta]X}{\chi\alpha_h}}.$$

Plug h^* and m into the optimality condition with respect to k, we get

$$\begin{aligned}
\frac{R+\delta-1}{\alpha_k} &= k^{\alpha_k-1}(h^*)^{\alpha_h}(F_1k)^{1-\alpha_k-\alpha_h}, \\
k^{\alpha_h} &= (h^*)^{\alpha_h}(F_1)^{1-\alpha_k-\alpha_h}\frac{\alpha_k}{R+\delta-1}, \\
&= (h^*)^{\alpha_h}\left(\frac{(1-\alpha_k-\alpha_h)}{1+\tau}\frac{R+\delta-1}{\alpha_k}\right)^{1-\alpha_k-\alpha_h}\frac{\alpha_k}{R+\delta-1} \\
&= (h^*)^{\alpha_h}\left(\frac{(1-\alpha_k-\alpha_h)}{1+\tau}\right)^{1-\alpha_k-\alpha_h}\left(\frac{R+\delta-1}{\alpha_k}\right)^{-\alpha_k-\alpha_h}.
\end{aligned}$$

8.3 Solving dynamic stochastic general equilibrium

This section shows the general steps to solve dynamic stochastic general equilibrium models by using the perturbation method. There are many other methods that can be used to solve the DSGE models, such as policy function iteration, value function iteration, and etc. The advantage is perturbation method is that it can easily handle a model with many state variables.

1. Set up the dynamic stochastic general equilibrium model. After the first order approximation, list all the equilibrium conditions in the following form:

$$AEx_{t+1} \quad = \quad Bx_t,$$

where

$$x_t \equiv \begin{bmatrix} s_t \\ c_t \end{bmatrix}.$$

The variable s_t denotes state variables. In this paper, state variables include: endogenous but predetermined variables: capital and debt; and exogenous state variables: productivity, world interest rate, and government expenditure. The variable c_t denotes choice variables. In this paper, choice variables include: consumption, hours, and etc. The number of choices is assumed to be n_c.

2. We can apply the Schur decomposition method to the above linear system equation to get the following

$$qAzz'Ex_{t+1} \quad = \quad qBzz'x_t,$$

or

$$aEy_{t+1} \quad = \quad by_t,$$

where $a = qAz$, $b = qBz$, and $y_t = z'x_t$. And a and b are triangle matrix.

3. Partition the system, we get

$$\begin{pmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{pmatrix} Ey_{t+1} \quad = \quad \begin{pmatrix} b_{11} & b_{12} \\ 0 & b_{22} \end{pmatrix} y_t,$$

where y_{2t} is of the order $n_c \times 1$.

4. We thus have the following

$$a_{22}Ey^2_{t+1} \quad = \quad b_{22}y^2_t.$$

Here we put terminal condition on y_{t+1}, such that $\lim_{j\to\infty} E_t y^2_{t+j} < \infty$. Given that $|\frac{a_{22}}{b_{22}}| < 1$, the only solution is $y^2_t = 0$.

5. Since

$$y_t^2 = 0 = z_{12}' s_t + z_{22}' c_t,$$

we get the solution for the choice variables:

$$c_t = -z_{22}^{'-1} z_{12}' s_t.$$

6. Go one step ahead, since $y_t^2 = 0$, we have

$$\begin{aligned} a_{11} E y_{t+1}^1 &= b_{11} y_t^1 \\ \Rightarrow a_{11} E \left(z_{11}' s_{t+1} + z_{12}' c_{t+1} \right) &= b_{11} z_{11}' s_t + z_{12}' c_t. \end{aligned}$$

Given the fact that c_{t+1} is a function of s_{t+1} and s_{t+1} is known at period t, we can solve for s_{t+1} as

$$s_{t+1} = \left[z_{11}' - z_{21}' z_{22}^{'-1} z_{12}' \right] a_{11}^{-1} b_{11} \left[z_{11}' - z_{21}' z_{22}^{'-1} z_{12}' \right] s_t.$$