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Signal Extraction and Forecasting of the UK Tourism Income Time Series. A Singular Spectrum Analysis Approach

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Abstract
We present and apply the Singular Spectrum Analysis (SSA), a relatively new, non-parametric and data-driven method used for signal extraction (trends, seasonal and business cycle components) and forecasting of the UK tourism income. Our results show that SSA outperforms slightly SARIMA and time-varying parameter State Space Models in terms of RMSE, MAE and MAPE forecasting criteria.

Keywords
Singular Spectrum Analysis, Singular Value Decomposition, Business Cycle Decomposition, Tourism Income, United Kingdom, Signal Extraction, Forecasting.

JEL Classification
C14, C53, E32, L83.

1. Introduction

Signal extraction and forecasting are important aspects in tourism policy making in all countries involved in tourism. A meaningful decomposition of an observed time series in signal and noise components leads to a better understanding of the tourism development process, especially in its relation to macroeconomic environment, and more accurate forecasting of tourism demand. Reviewing the relevant literature one can realize that tourism research so far has been dominated by parametric time series methods, both in “atheoretical” (e.g. SARIMA modeling) and theoretical formulations (e.g. cointegration and error correction approaches). It is also true that time domain approaches, as opposed to frequency domain approaches, form the overwhelming majority of these studies. For example, Song and Li (2008) reviewed 121 studies which were published from 2000 onwards and focused on modelling tourism demand: apart from papers on seasonality modelling, only one author (Coshall, 2000) used frequency domain analysis. In his research, Coshall (*op.cit*.) found that cycles of passenger flows from UK to France, Belgium and the Netherlands depend on cycles in exchange rates, not on GDP cycles. From the same review, one can also conclude that non-parametric, data-driven methods are rather the minority in tourism research, while in other scientific and engineering disciplines are quite common.

The Singular Spectrum Analysis (SSA) is a relatively new method of time series analysis and combines elements of various fields of studies such as, among others, statistics and probability theory, dynamical systems and signal processing. It is based on the spectral decomposition of time series (Karhunen, 1946 and Loève, 1945, 1978) and on the Mañé (1981) and Takens (1981) embedding theorem. Although SSA cannot be strictly considered as belonging to statistical spectral methods, it may constitute a stage in the statistical spectral analysis in the sense that the decomposed, by SSA, time series into various components (among them periodic or quasi-periodic ones) may be subject to classical spectral methods since these components are expected to contain, exactly because of the SSA, a “cleaner signal”.

The application of this method can be virtually in any field that requires time series data analysis. Thus, the SSA has been applied so far in hydrology, geophysics, climatology, economics, biology, physics and other quantitatively oriented areas of knowledge. Since the method is still in the early steps of its development, one can expect more methodological advances and practical applications in the near future.

The central concept in SSA is the decomposition of a time series into a group of independent components, which, when they are synthesized, reflect the important characteristics of a time series, that is, trend, periodic / oscillating components (e.g. seasonal and calendar effects, or business cycle in macroeconomics) and uninformative noise. SSA belongs to non-parametric methods, that is, it is a model-free and data-driven method, making no assumptions about data generating processes, combining formal mathematical analysis and visual aid. Its main attractive element is that it depends only on one parameter, the window length, making it simple to understand and relatively “easy” to apply it.

The problems this method aspires to handle are: trends of different resolutions, smoothing, noise reduction, extraction of seasonal components, extraction of cycles with amplitude and frequency modulation, extraction of periodicities with time-varying amplitudes, complex trends and periodicities, discovery of structure in short and noisy time series, change point detection, and, finally forecasting. SSA can decompose one time series (univariate version) or multiple time series (the Multichannel SSA). In this paper we refer only to univariate version of the SSA.

The SSA method was developed independently in the USA and UK under the name SSA in the 1980’s and in Russia (St. Petersburg and Moscow) in the 1990’s under the name Caterpillar-SSA. Since then, hundreds of research papers have been published with the majority of them referring to the natural sciences. For example, Broomhead and King (1986) have applied SSA to the reconstruction of the Lorenz (1963) attractor (meteorology). Examples of other papers are Allen and Smith (1997), Vautard and Ghil (1989), Vautard *et al.* (1992), Ghil *et al.* (2002), Kondrashov and Ghil, (2006), all in climatology, geophysics or atmospheric sciences, Hassani (2007) in a demographic time series and Sella (2008) in macroeconomics. Further, at textbook level, an introduction to SSA is Elsner and Tsonis (1996) and a more advanced treatment is given in Golyandina *et al.* (2001).

In this paper we would like to contribute to the non-parametric literature in tourism economics by presenting and applying the SSA to signal extraction and forecasting of tourism income data of the UK and we compare its forecasting performance with a SARIMA and a State Space Model. To our knowledge, this is the first application of the SSA to tourism economics. In the remainder of the paper we examine methodological aspects of the SSA (Section 2), its application in the UK tourism income data and the empirical results (Section 3) and we conclude the paper in Section 4. The decomposition of the UK tourism income time series by the SSA method has been done by the Caterpillar Software (www.gistatgroup.com).

2. Methodological Aspects of the SSA

This section contains a brief description of SSA-algorithms used for extraction of the signals and their forecasting. SSA comprises of two main stages: decomposition and reconstruction. In turn, the decomposition stage comprises of two steps: embedding and singular value decomposition. The reconstruction stage comprises of two steps, too: grouping and diagonal averaging (or Hankelization). Having completed these steps, one may proceed also to forecasting. The presentation below follows Golyandina *et al.* (2001, Chapters 1 and 2), Hassani (*op.cit.)* and Hassani and Zhigljavsky (2008).

2.1 Stage 1: Decomposition, Step 1: Embedding

Let $N > 2$. Consider a one-dimensional real-valued time series $F = (f_0, f_1, \dots, f_{N-1})$ of length *N*, a positive integer *L* (*window length*) such that $1 < L < N$ and a mapping of the original series into a sequence of *L*-dimensional lagged vectors $\{X_i\}_{i=1}^{K}$, $K = N - L + 1$, by the formula:

$$X_i = (f_{i-1}, \dots, f_{i+L-2})^T, \quad 1 \le i \le K.$$

The Hankel matrix **X=**$[X_1 : \dots : X_K]$ of size $L \times K$ is called the *L-trajectory matrix* (or simply, the *trajectory matrix*) of the series *F*. In linear algebra, a Hankel matrix is a matrix where all the elements along the diagonal *i+j=const.* are equal. In other words, the trajectory matrix is

$$\mathbf{X} = (x_{ij})_{i,j=1}^{L,K} = \begin{bmatrix} f_0 & f_1 & f_2 & \cdots & f_{K-1} \\ f_1 & f_2 & f_3 & \cdots & f_K \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ f_{L-1} & f_L & f_{L+1} & \cdots & f_{N-1} \end{bmatrix}.$$

Note that if *N* and *L* are fixed then there is a one-to–one correspondence between Hankel matrices of size $L \times K$ and time series of length *N*.

2.2 Stage 1: Decomposition, Step 2: Singular Value Decomposition (SVD)

The SSA is based on a particular transformation known in matrix algebra as singular value decomposition (SVD). The SVD of the trajectory matrix **X**=$\left[X_1 : \ldots : X_K\right]$ is a decomposition of **X** in the form **X**=$\sum_{i=1}^{d}\sqrt{\lambda_i}\,U_i\,V_i^T$, where $V_i = \mathbf{X}^{\mathbf{T}} U_i / \sqrt{\lambda_i}$, $d = max(i,\ such\,that\ \lambda_i > 0) = rank$ **X**, $\lambda_1, \ldots \lambda_L$ are the eigenvalues of the $L \times L$ matrix **S** = **XX**$^{\mathrm{T}}$ taken in the decreasing order of magnitude $(\lambda_1 \geq \ldots \geq \lambda_L \geq 0)$ and $U_1, \ldots, U_L$ are the eigenvectors of the matrix **S** corresponding to these eigenvalues**.**

If we define **X**$_i$=$\sqrt{\lambda_i}U_iV_i^T \;\; (i = 1, \ldots, d)$, then the SVD of the trajectory matrix can be written as a sum of rank-one orthogonal matrices:

$$\mathbf{X}=\mathbf{X_1}+\ldots+\mathbf{X_d} \qquad (1)$$

where U_i are the orthonormal eigenvectors of **S** = **XX**$^{\mathrm{T}}$ (in SSA terminology they are called empirical orthogonal functions) and V_i (in SSA terminology they are called principal components) can be regarded as the eigenvectors of the matrix **X**$^{\mathrm{T}}$**X**. The collection $\left(\sqrt{\lambda_i}, U_i, V_i\right)$ is called the *i*-th eigentriple of the matrix **X**, $\sqrt{\lambda_i} \;\; (i = 1, \ldots, d)$ are the singular values of the matrix **X** and U_i, V_i are the left and right singular vectors of **X**, respectively.

SVD is attractive because it ensures optimality. Among all the matrices **X**$^{(r)}$ of rank *r*<*d*, the matrix $\sum_{i=1}^{r}\boldsymbol{X_i}$ provides the best approximation to the trajectory matrix **X**, so that $\left\|\mathbf{X} - \mathbf{X}^{(\mathbf{r})}\right\|$ is minimum. Note that $\left\|\boldsymbol{X}\right\|^2 = \sum_{i=1}^{d}\lambda_i$ and $\left\|\boldsymbol{X_i}\right\|^2 = \lambda_i$ for $i = 1, \ldots, d$. Thus, we can consider the ratio $\lambda_i / \sum_{i=1}^{d}\lambda_i$ as the contribution of the matrix **X**$_i$ in the expansion (1) to the whole trajectory matrix **X**. Consequently, $\sum_{i=1}^{r}\lambda_i / \sum_{i=1}^{d}\lambda_i$, the sum of the first *r* ratios, is the contribution of the optimal approximation of the trajectory matrix by the matrices of rank *r*.

2.3 Stage 2: Reconstruction, Step 1: Grouping

The grouping step corresponds to splitting the elementary matrices $\mathbf{X}_i$ into several groups and summing the matrices within each group. Let $I = \{i_1, \ldots, i_p\}$ be a group of indices. Then the matrix $\mathbf{X}_I$ corresponding to the group I is defined as $\mathbf{X}_I = \mathbf{X}_{i1} + \ldots + \mathbf{X}_{ip}$. These matrices are computed for $I = I_1, \ldots, I_m$ and the expansion (1) leads to the decomposition

$$\mathbf{X} = \mathbf{X}_{I1} + \ldots + \mathbf{X}_{Im} \qquad (2)$$

The procedure of choosing the sets $I_1, \ldots, I_m$ is called the eigentriple grouping.

2.4 Stage 2: Reconstruction, Step 2: Diagonal Averaging

The last step is, in a sense, opposite to the first step and transforms each matrix of the grouped decomposition (2) into a system of new (reconstructed) series of length *N*. This procedure is the so-called Hankelization or diagonal averaging. If z_{ij} stands for an element of a matrix $\mathbf{Z}$, then the *k*-th term of the resulting time series is obtained by averaging z_{ij} over all *i, j* such that $i + j = k + 2$. The result of the Hankelization of a matrix $\mathbf{Z}$ is the Hankel matrix $H\mathbf{Z}$, which is the trajectory matrix corresponding to the time series obtained as a result of the diagonal averaging (see the formal description in Golyandina *et al., op.cit.*). Note that the Hankelization is an optimal procedure in the sense that the matrix $H\mathbf{Z}$ is the closest to $\mathbf{Z}$ (with respect to the matrix norm) among all Hankel matrices of the corresponding size (Golyandina *et al., op.cit.*). In its turn, the Hankel matrix $H\mathbf{Z}$ defines the series uniquely by relating the values in the diagonals to the values in the series. Diagonal averaging applied to a matrix $\mathbf{X}_{Ik}$ produces the series $\widetilde{F}^{(k)} = \left(\widetilde{f}_0^{(k)}, \ldots, \widetilde{f}_{N-1}^{(k)}\right)$. Hence, the original series $f_0, f_1, \ldots, f_{N-1}$ is decomposed into the sum of *m* series: $f_n = \sum_{k=1}^{m} \widetilde{f}_n^{(k)}$.

2.5 Forecasting

Forecasting by SSA can be applied to the time series that approximately satisfies the linear recurrent formula (LRF)

$$f_{i+d} = \sum_{k=1}^{d} a_k f_{i+d-k}\,, \quad 1 \le i \le N-1 \qquad (3)$$

of some dimension *d* with the coefficients $a_1, \ldots a_d$. An important property of the SSA decomposition is that, if the original time series *F* satisfies a LRF (3), then for any *N* and *L* there are at most *d* nonzero singular values in the SVD of the trajectory matrix **X.** Therefore, even if the window length *L* and *K* = *N* - *L* + 1 are larger than *d*, we only need at most *d* matrices **X***i* to reconstruct the series.

Define now the original series $F = (f_0, f_1, \ldots, f_{N-1})$ and the reconstructed series $\tilde{F} = \left(\tilde{f}_0, \tilde{f}_1, \ \ldots \ , \tilde{f}_{N\text{-}1}\right)$. For an eigenvector $U \in R^L$ we denote the vector of the first *L-1* components of the vector *U* as $U^{\nabla} \in R^{L-1}$. Set $v^2 = \pi_1{}^2 + \ldots + \pi_r{}^2 < 1$, where π_i is the last component of the eigenvector U_i $(i = 1, \ldots, r)$. It can be proved that the last component f_L of any vector $F = (f_1, \ldots, f_L)^T$ is a linear combination of the first components $(f_1, \ldots, f_{L-1})$, that is $f_L = a_1 f_{L-1} + \ldots + a_{L-1} f_1$ where the vector of coefficients $A = (a_1, \ldots, a_{L-1})$ can be expressed as $A = \sum_{i=1}^{r} \pi_i \, U_i^{\nabla} / (1 - v^2)$. The forecasts $(f_{N+1}, \ldots, f_{N+M})$ are then obtained as:

$$f_i = \begin{cases} \tilde{f}_i & i = 1, \ldots, N \\ \sum_{j=1}^{L-1} a_j f_{i-j} & i = N+1, \ldots, N+M \end{cases}$$

where *M* is the number of points to forecast for.

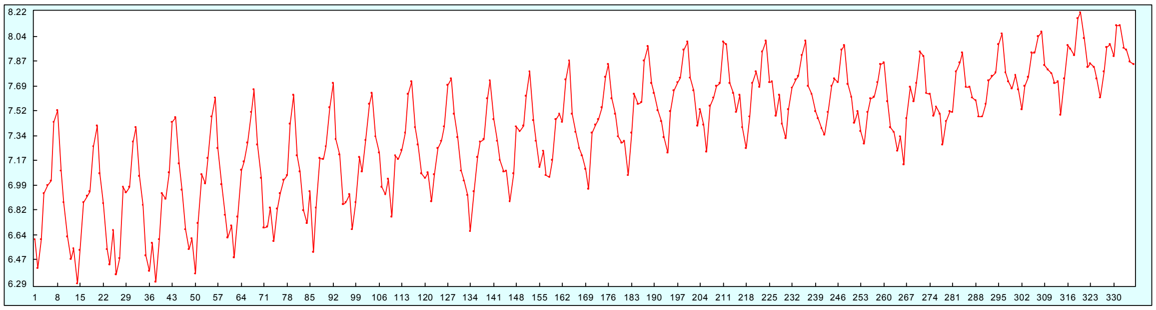
3. Application

3.1 Data

We now apply the SSA to UK tourism income data. They span the time period 1980:I – 2007:12 (monthly data) and refer to the income earned from overseas visitors to UK, in millions of British pounds. The data initially were in current prices and we have converted them into constant prices by means of the retail price index as an approximation of a suitable, in this case, price deflator. The tourism income data are seasonally unadjusted and have been received by the Office of National Statistics of UK, http://www.statistics.gov.uk. The relevant time series is GMAK and the total number of observations is 336. Figure 1 displays the log of the tourism income series. A long-run time-varying trend and strong seasonal components characterize this series. However, the existence of any economic fluctuations at business cycle frequencies (1.5 –

8 or even 10 years) cannot be readily detected visually. We will examine below if such components exist in the tourism income series.

Figure 1: Tourism Income (in logs).



Source: Office of National Statistics of UK and own processing.

3.2 Practical Issues

In practical applications of SSA with observed data (as opposed to simulated data, where the design parameters are already known), some considerations should be taken into account. Since SSA depends on one and unique parameter, the window length L, the issue of its appropriate value is crucial. L should be large enough in order to capture sufficiently the dynamics of the time series but not larger than $N/2$. Further, if any periodic component is known to be present in the time series, then L should be proportional to that period. If more than one periodic components exist in the time series, then L should cover all of them. Therefore, L should be proportional to the highest period we suspect that exists in the time series we analyze. In practice, a length for L equaling approximately 1/4 or 1/5 of the observations is sufficient to capture all the dynamics of the series. At this point, the embedding step in the decomposition stage has been accomplished. Once the window length has been determined, some other information might be valuable in order to proceed to the grouping step of the reconstruction stage. For example, in practice, a periodic component is identified by having two eigentriples with singular values close each other (the exception is at frequency 0.5 which displays one eigentriple with saw-tooth singular vector). Therefore, a plot of the L singular values $\sqrt{\lambda_i}$ against an index $i=1,...,L$, gives important information in the sense that , through this visual aid, one can easily discern the high and the low singular values. Since each singular value $\sqrt{\lambda_i}$ expresses the significance of the corresponding $\mathbf{X}_i$ to the total trajectory matrix $\mathbf{X}$, high singular values imply significance of the corresponding component (trend or periodic). Low singular values imply noise rather than some form of meaningful signal. The decision about which periodic components exist in the time series might also be aided by means of spectral estimation procedures either in the initial time series and/or in the principal components. High spectral densities at particular frequencies imply a periodic component at those frequencies.

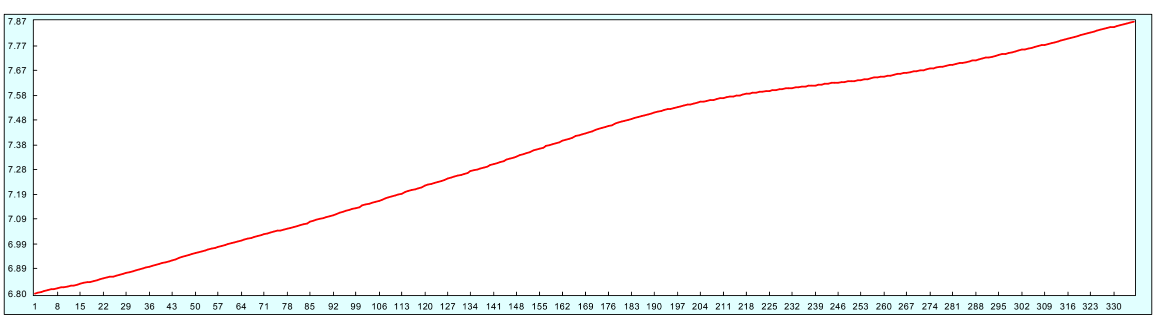
3.3 Estimates

Based on the considerations mentioned above, we have reconstructed the initial series as follows. The window length L has been chosen 84. This is 1/4 of the total observations and, at the same time, covers all known periodicities and, probably, business cycle components. The objective of our analysis is to decompose the initial time series into a meaningful signal, that is, a long-run trend, periodic components of seasonal nature and business cycle movements, and uninformative (white) noise. We emphasize the importance of white noise vis-à-vis coloured noise in the sense that economic time series usually lack very strong signal-to-noise-ratio (as it is the case in many physical applications). For this reason, we think that any informative content, present in a coloured noise, it should be transferred to the systematic part of the model, giving, thereby, a clearer picture of the dynamic properties of the model by separating, as much as possible, the informative signal from the uninformative noise.

The Long-run Trend

Based on the estimates of the singular values, we now proceed to the grouping. First, the long-run trend of the series is estimated by the first eigentriple. See Figure 2 for the long-run trend.

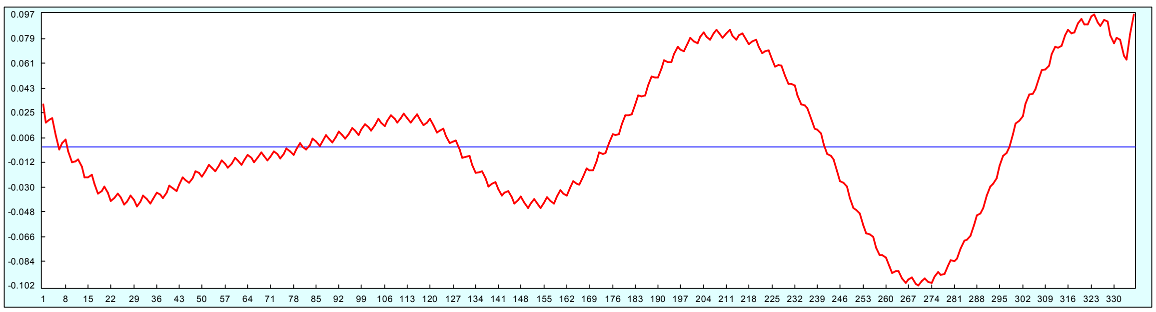
Figure 2: Long-run trend based on eigentriple 1.



Economic Fluctuations

Second, we examine if there are any business cycle components in the tourism income time series. Two other components are obtained by the eigentriples 4 and 7. They are recurrent movements of non-periodic nature, with time-varying amplitude, which could be, probably, characterized as economic fluctuations, lasting approximately 8 years (96 months). Cycles of similar period, of approximately 8 years, have been found in Leon and Eeckels (2009), referring to tourism income of Switzerland. The business cycle components are presented in Figure 3.

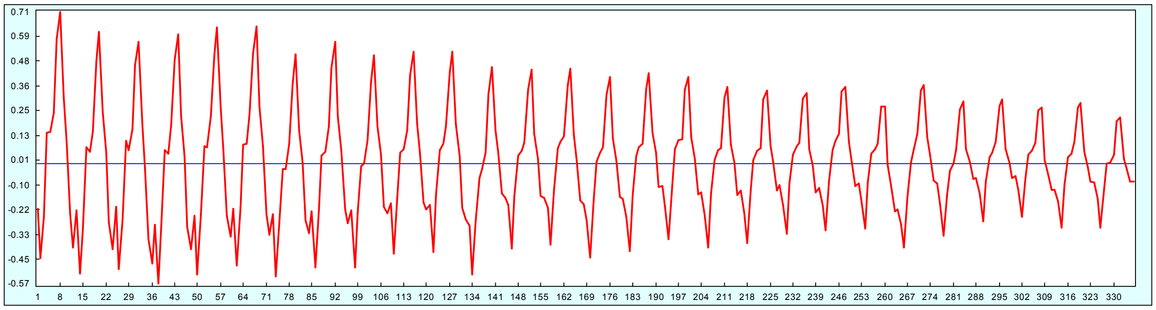
Figure 3: Economic fluctuations based on eigentriples 4 and 7.



Seasonal Components

The next grouping refers to seasonal effects, every 3, 4 6, and 12 months. These seasonal components have been captured by the eigentriples 2-3, 5-6, 8-9, 10-11, 12-13, 14-15, 16-17, 18-19, 20-21, 22-23. It seems that these components are characterized by a dumping oscillator generating mechanism. The inclusion of all these periodicities may seem, at a first glance, somewhat exaggerating due to their relatively low importance of the corresponding singular values. However, they were included on the premise that the remaining components should be unstructured and uninformative, i.e. pure (white) noise. Figure 4 displays the reconstruction of the seasonal components.

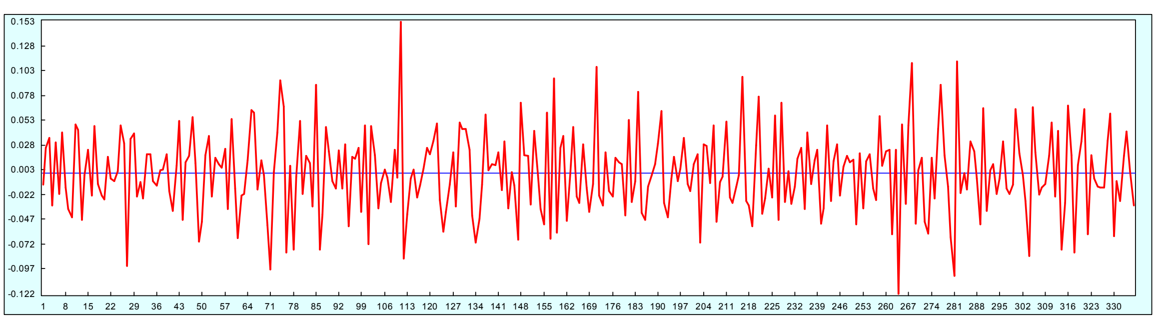
Figure 4: Seasonal components based on eigentriples 2-3, 5-6, 8-9, 10-11, 12-13, 14-15, 16-17, 18-19, 20-21, 22-23.



Noise

The remaining grouping concerns the residuals as estimates of an unobserved noise component. All the remaining eigentriples, from 24-84, are interpreted as noise. Given that the noise component should be fully uninformative, we experimented with spectral estimators and autocorrelation tests on these residuals to ensure that there is no information structure on them. By doing this we are ensured that all information has been transferred to the signal components. Our tests (not shown) confirmed that the residuals were indeed white noise only when all the previous eigentriples 1, 4, 7, 2-3, 5-6, 8-9, 10-11, 12-13, 14-15, 16-17, 18-19, 20-21, 22-23 are interpreted as periodic components, business cycles and long-run trend. Thus, there is evidence that the proposed decomposition in long-run trend, business cycle and seasonal components extracts sufficiently the desired signal since it leaves the noise component uninformative (white noise). Figure 5 presents the reconstruction of the noise component.

Figure 5: Noise component based on eigentriples 24-84.



3.4 Forecasting and Comparison with Other Methods

Having already decomposed the initial time series in a set of separable signal plus noise components, we now want to see how SSA compares with two other times series methods. In these methods we have estimated our parameters from the sample 1980:1 – 2007:12 (336 observations) and we now want to see how they perform out of sample for the period 2008:1 – 2009:5 (17 observations, almost 1.5 year ahead) for which we have actual data from the same statistical sources. We used two models, a SARIMA model and a State Space Model with the time-varying seasonal parameter to follow an AR(1) process. Both these models were selected, after several experimentations with different parameterizations and specifications, on the basis of reaching at a statistically adequate model (white noise residuals).

The comparison is made in terms of RMSE (Root Mean Square Error), MAE (Mean Absolute Error) and MAPE (Mean Absolute Percentage Error) forecasting criteria. The results are displayed in Table 1.

Table 1: Comparison of forecasting performance of the three models.

Model and Forecasting Criterion	SSA	SARIMA	State Space Model
RMSE	0.114	0.115	0.137
MAE	0.095	0.097	0.109
MAPE	1.20%	1.25%	1.40%

The results across these three models are not very different in terms of these forecasting criteria. RMSE is 0.114 for SAA, 0.115 for the SARIMA model and 0.137 for the State Space Model. MAE is 0.095 for SSA, 0.097 for the SARIMA model and 0.109 for the State Model. Finally, MAPE is 1.2% for SSA, 1.25% for the SARIMA model and 1.4% for the State Space Model. Given these estimates, SSA ranks first, the SARIMA model ranks second with slight difference from the SSA model, and the State Space Model ranks third. Hence, SSA is the best forecasting method among these three alternatives, although the difference from the other two models is small. Therefore, a conservative statement that SSA does not seem to be inferior in comparison to the other two very well-established methods, can well be justified.

4. Conclusion

In this paper, we presented and applied the Singular Spectrum Analysis in signal extraction (trends, seasonal and business cycle components) and forecasting of the UK tourism income. Our results show that SSA outperforms slightly the SARIMA and the time-varying parameter State Space Model in terms of RMSE, MAE and MAPE forecasting criteria. Since the method is relatively new, more research is required to show if this method outperforms consistently other time series methods in the area of tourism time series. For example, Ghil *et al*. (*op.cit*.) stress that the use of other spectral methods (e.g. Multi-taper methods or Maximum Entropy) is essential. In fact, in some sense, SSA lacks of statistical rigour, at least in comparison to Fourier methods, for example. Thus, a plurality of spectral methods is necessary to confirm SSA results.

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