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Abstract

We develop a dynamic model that can explain identity switching activities among a stereotyped population, such as passing and selective out-migration, based on the group reputation model developed in Kim and Loury (2008). The more talented members of the population, who gain more by separating themselves from the masses, have a greater incentive to pass for an advantaged group with a higher collective reputation (incurring some cost of switching) or differentiate themselves by adopting the cultural traits of a better-off subgroup to send signals of their higher productivity to employers. We also show how an elite subgroup may grow autonomously out of the stereotyped population, when the most talented members adopt the cultural indices that are not affordable to other members of the population. Those cultural traits or indices are not necessarily relevant for productivity, but should be observable so that they can supplement the imperfect information about the workers' true productivity, as discussed in Fang (2001). We plan to merge this development with our previous work in Kim and Loury (2008) in the future.

KEYWORDS: Endogenous Group Formation, Passing, Partial Passing, Social Elite, Group Reputation, Statistical Discrimination.

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1 Introduction

We develop an identity switching model that can explain social activities such as passing and selective out-migration among a stereotyped group, loosening the assumption of group identity immutability made in Kim and Loury (2008). The more talented members of the group, who gain more by separating themselves from the masses, have a greater incentive to pass for the group with the higher reputation (incurring some cost of switching). They also differentiate themselves by adopting the cultural traits of a better-off subgroup in order to send signals of their higher productivity to employers. Also, we show the dynamic process by which elite subgroups emerge out of disadvantaged populations by adopting unique cultural instruments, as discussed in Fang (2001). The most talented members of the stereotyped population have an incentive to develop distinguished cultural indices for differentiation, which are not affordable to other members of the group. As the most talented adopt these indices, an elite "cultural" subgroup grows autonomously, whose members are preferentially treated by employers.

This paper is closely related to statistical discrimination literature. If a worker's true productivity is not perfectly observable, employers have an incentive to use the collective reputation of the job applicants in the screening process. The individuals who belong to a group with a better collective reputation have a greater incentive to invest in skills because the return for skill investment tends to be greater for them, (and vice versa). With their greater (smaller) skill investment rate, the group maintains a better (worse) collective reputation. Therefore, there are multiple self-confirming equilibria of group reputation (Arrow, 1973; Coate and Loury, 1993). In Kim and Loury (2008), we discuss this externality of group reputation and the stability of multiple equilibria in a dynamic setting. We identify the balanced dynamic paths to the high and low stable reputation equilibria. When the initial reputation of a group is outside the optimistic (pessimistic) path to the high (low) stable reputation equilibrium, the group's reputation deteriorates (improves) over time and ends up in the lower (higher) stable equilibrium. We explain the concept of a reputation trap: if a group's reputation is trapped, the group cannot escape the low skill investment activities without any external interventions such as preferential employers' treatment and/or affirmative action, and offspring of the group consistently suffer from the developed negative stereotype of their ancestors.

In our previous work, there are no implications for multiple social group societies, (except for the policy implication for quota ratio or training subsidy transfer.) An inborn group identity is immutable and each group member is affected only by the collective reputation of his own group. However, when we loosen the immutability assumption, we can explain the relationship between group reputation externality and identity switching between social groups, and the development of an elite group out of a stereotyped population.

The first type of identity switching is "passing." Consider a group in the reputation trap. The talented young members in the group may consider passing for the group with the better collective reputation when the return for passing (such as better treatment in the labor market) outweighs the cost of passing, such as the disconnection from their own ties. A representative historical case is the story of Korean descendants in Japan, who constitute around one percent of the Japanese population. Most of them are the descendants of forced laborers in mines and factories who were brought back by Japan from the peninsula during the period of Japanese imperialism. Their living conditions in Japan were much worse than for Japanese natives, even after the end of World War II. In order to escape negative stereotypes and prejudices, many Korean descendants have passed for native Japanese, changing both surnames and given names at the age when they seek formal employment and marriage. Every year about 10,000 Koreans, out of around 600,000 Korean descendants holding Korean nationality, choose to be naturalized as "official" Japanese, giving up their names and original nationality. Many of the naturalized Koreans conceal their Korean ethnicity, pretending that they have no knowledge about Korean culture and language in order to prevent discrimination in the labor or marriage market (Fukuoka et al., 1998).

Other than the case of Korean descendants in Japan, who share a similar appearance with the Japanese, passing is harder for blacks in the United States who were brought to the country as slaves hundreds of years ago, due to their immutable physical marker. However, a meaningful number of the black population consistently passes for White or other races according to the NLS79 National Longitudinal Survey conducted by the Department of Labor of the US. The survey shows that 1.87 percent of those who had originally answered "Black" in 1979 (when they were 14 to 22 years old), switched to answering the interviewer's race question with either "White," "I don't know," or "other," before 1998 (Sweet, 2004).

The second type of identity switching is "partial passing" or differentiation from others. The term "partial passing" was used first in Loury (2002) to describe the social identity manipulation used by racially marked people to inhibit being stereotyped. When "total passing" for a member of the advantaged group with high reputation is not available due to immutability, the most talented of the stereotyped group are more likely to seek styles of self-presentation that aim to communicate "I'm not one of THEM; I'm one of YOU!" because they are the ones who gain most by separating themselves from the masses (Loury, 2002). That is, they "pass for" the slightly better-off subgroup that maintains a higher reputation than the stereotyped population by adopting the cultural traits of the better-off subgroup. Methods that are known to be used for partial passing among the black population in the US are: affectations of speech, dressing up rather than wearing casual clothes, spending more on conspicuous consumption and so on. For example, blacks earning higher incomes who live in an area where the community income is relatively lower spend more on visible goods to signal their income level and social status, while blacks who live with affluent peers have less need to signal high status (Charles et al., 2007). Also, there is evidence that the more educated (or talented) blacks tend to speak Standard American English rather than African American English (Grogger, 2008).

This selective out-migration to the better-off subgroup may undermine solidarity in the disadvantaged population and cause conflicts among them, such as the accusation of "Acting White" against the ones who practice the partial passing methods (Fryer and Torelli, 2006). The collective reputation of the group with the selective out-migration of the most talented may become worse over time. It would be harder for the stereotyped group to move out of the reputation trap even when an external intervention is made. However, there might be a social gain through this practice. Among many subgroups with the unique cultural traits of the stereotyped population, at least some subgroups would be able to recover their reputation when the talented young members gather around the cultural subgroups. The usage of the observable cultural traits in the screening process can cure to some extent the social inefficiency of the reputation trap, which is caused by imperfect information about the true characteristics of workers.

Also, using the dynamic model developed in this paper, we can explain the emergence of an elite social group out of a stereotyped population. The most talented members of a stereotyped population have an incentive to create a small group with observable distinguished cultural traits so that they can differentiate themselves from the rest in the labor market. The usage of a cultural instrument that is intrinsically irrelevant for productivity to form an elite group is well discussed in Fang (2001) as an explanation for the complexity of elite etiquettes in European (or Confucian) societies and the respect for "Oxford Accent." Skilled and unskilled workers have different incentives to join a group with unique cultural traits that are "expensive" to obtain. Thus, the small group is preferentially treated by employers due to the higher fraction of the skilled workers, even though the cultural traits of the group are not relevant for productivity. Understanding this mechanism, the talented members of the stereotyped population may develop indices for differentiation, which are not affordable to other members of the group. The indices may include the migration of the most talented to affluent residential areas, spending on luxury goods and designer clothing, showing interest in fine arts, and sending children to a private boarding school. Even when there is no a priori difference in cultural traits among the stereotyped population, we may see an autonomously growing elite subgroup with differentiated cultural traits whose members are preferentially treated by employers and considered as distinguished from their peers.

The dynamic model of endogenous group formation in this paper starts with the following basic structure. First, the model is developed based on a dynamic group reputation model in Kim and Loury (2008), using the same notations in the work. We have two identity groups, group A and group B. The groups are identified by cultural traits (and also by physical marker.) Cultural traits may include speaking standard vs. speaking slang, non-smoker vs. smoker, straight sexually vs. gay, fashionable vs. unfashionable, learning etiquette vs. ignoring etiquette, and living in the suburbs vs. living in the inner city. We assume that a worker's preference for those traits is irrelevant to his investment cost for skills: the preference distribution is not correlated with the investment cost distribution among a population. Also, we assume that cultural traits, which are observable by employers, are not associated with productivity, as assumed in Fang (2001). Apart from the immutable group identity, which we have assumed in Kim and Loury (2008), this "cultural" group identity is not determined by nature. Newborn individuals can choose which group they belong to at an early stage of their life. Newborns who "switch" from an inborn identity type to another must incur some cost of switching, which varies across individuals.

This paper is organized into the following sections: Section 2 describes the basic framework of the model; Section 3 examines the dynamic system with no switches and that with switches between two groups, after identifying potential switchers among the population; Section 4 provides an analysis of endogenous group formation including passing, partial passing and the emergence of elite subgroup; and Section 5 contains the conclusion.

2 Framework

In this section, we explain employers' decision making process under the imperfect information about the workers' true productivity, together with workers' decision making process for the skill acquisition and the group identity.

2.1 Employers' Decision

Employers are unable to observe whether a worker is qualified for a task, which is a more demanding and rewarding assignment than other tasks. Employers observe each worker's group identity and a noisy signal $\theta \in [0, \bar{\theta}]$. The distribution of θ depends on whether or not a worker is qualified. The signal might be the result of a test, an interview, or some form of on-the-job training. The signal is uniformly distributed for an unqualified worker in $[0, \theta_u]$, and for a qualified worker in $[\theta_q, \bar{\theta}]$, with $\theta_q < \theta_u$.

In this case, employers will set the hiring standard as either θ_q or θ_u. If the signal is below θ_q, the worker must be unqualified, and, if the signal is above θ_u, the worker must be qualified. If the signal is between θ_q and θ_u, the signal is unable to tell the true characteristic of the worker. Let us denote the probability that, if a worker does invest in skills, his test outcome proves that his is qualified by $P_q(=\frac{\bar{\theta}-\theta_u}{\bar{\theta}-\theta_q})$ and the probability that, if a worker does not invest in skills, his test outcome proves that he is unqualified by $P_u(=\frac{\theta_q}{\theta_u})$.

Assumption 1 (Imperfect Information)**.** *A qualified worker's signal is less informative, compared to an unqualified worker's signal. This is, the payoff uncertainty is greater for qualified workers compared to for unqualified workers: $P_q < P_u$, and equivalently, $\theta_q + \theta_u > \bar{\theta}$.*

The assumption implies that it is relatively harder to confirm qualification for skilled workers, while it is relatively easier to confirm disqualification for unskilled workers. Employers should make a decision to give the benefit of doubt (BOD) if the signal is unclear. If they give BOD to a group, the hiring standard for the group is θ_u, but, if not, the hiring standard for the group is θ_q. Employers' decision to give BOD is determined by the sign of expected payoff, $x_q \cdot Prob[\text{qualified}|\theta] - x_u \cdot Prob[\text{unqualified}|\theta]$, for $\theta_q < \theta < \theta_u$. Using Bayes' rule, the posterior probability that the worker with group identity i and an unclear signal ($\theta_q < \theta < \theta_u$) is qualified is $\frac{\Pi^i(1-P_q)}{\Pi^i(1-P_q)+(1-\Pi^i)(1-P_u)}$. Thus, we can find the threshold level Π^*, above which employers give BOD and below which they do not give BOD, where $\Pi^* \equiv \frac{1-P_u}{\rho(1-P_q)+1-P_u}$ with $\rho = \frac{x_q}{x_u}$.

Lemma 1. *Let us denote ξ_t^i as the indicator of employers' giving BOD to the identity group i at time t:*

$$\xi_t^i = \begin{cases} 0, & \forall\ \Pi_t^i \in [0, \Pi^*) \\ 1, & \forall\ \Pi_t^i \in [\Pi^*, 1]. \end{cases} \tag{1}$$

2.2 Workers' Decision

There are two types of identity groups, A and B. Each individual is born a type A or a type B. Let us denote the population size of the type-A born individuals by L_a and that of the type-B born individuals by L_b. Both L_a and L_b are constant over time and the total population is $L_a + L_b$. Every unit period, λ faction of the total population randomly die and the same fraction are newly born. Thus, λL_a (λL_b) is the size of type-A (type-B) newborns in a unit period.

A newborn can change his inborn identity with incurring some cost k at an early stage of his life. At the same time, he can choose whether to be qualified or not. In order to be qualified, he must incur some cost c. The c and k are nonnegative and distributed with CDF $G(c)$ and $H(k)$ among the newborns, and the two distributions are independent of each other, which means the switching cost is not relevant to the skill investment cost.

Each newborn will choose both identity and qualification at an early stage of his life. Let us denote the lifetime benefits of each choice by W_e^i, where $i \in \{a, b\}$ and $e \in \{q, u\}$. Let us denote the return to skill investment (W_q^i-W_u^i) given the chosen identity i by R^i, and the return to identity switch from i to $-i$ (W_e^{-i}-W_e^i) given the chosen qualification e by Y_e^i: $R^i \equiv W_q^i - W_u^i$ and $Y_e^i \equiv W_e^{-i} - W_e^i$. Note that $R^{-i} - R^i \equiv Y_q^i - Y_u^i$.

Let us denote v_t^i as the "normalized" lifetime BOD expected to be given to a group i member from time t to infinity:

$$v_t^i = (\delta + \lambda) \int_t^{\infty} \xi_\tau^i \cdot e^{-(\delta+\lambda)(\tau-t)} \mathrm{d}\tau. \tag{2}$$

Note that $v_t^i = 1$ when $\xi_\tau^i = 1,\ \forall \tau \in [t, \infty]$. (Let v_t^{-i} denote the normalized lifetime BOD expected to be given to the members of the other group.) By virtue of normalization, the evolution rule of v_t^i is simplified with

$$\dot{v}_t^i = (\delta + \lambda)[v_t^i - \xi_t^i]. \tag{3}$$

Using the notation of v_t^i, the lifetime benefits of each choice (i, e), W_e^i, is expressed as

$$\begin{cases} W_q^i = \int_t^\infty \{w\xi_\tau^i + wP_q(1-\xi_\tau^i)\} \cdot e^{-(\delta+\lambda)(\tau-t)}\mathrm{d}\tau = \frac{wP_q}{\delta+\lambda} + \frac{w(1-P_q)}{\delta+\lambda} \cdot v_t^i \\ W_u^i = \int_t^\infty w(1-P_u)\xi_\tau^i \cdot e^{-(\delta+\lambda)(\tau-t)}\mathrm{d}\tau = \frac{w(1-P_u)}{\delta+\lambda} \cdot v_t^i. \end{cases} \tag{4}$$

Thus, R_t^i and $Y_{e,t}^i$ evaluated by the time t newborn are

$$R_t^i = \frac{wP_q}{\delta+\lambda} + \frac{w(P_u - P_q)}{\delta+\lambda} \cdot v_t^i \tag{5}$$

$$Y_{e,t}^i = \frac{w(1-P_e)}{\delta+\lambda} \cdot (v_t^{-i} - v_t^i). \tag{6}$$

Consider a type-i born individual with the cost set (c, k). The net payoff for each choice (i^*, e^*) denoted by $N_{e^*}^{i^*}$, given $\{i, c, k\}$, is

$$\text{Net Payoff for Choice } (i^*, e^*)_{i,c,k} \begin{cases} N_u^i &= W_u^i \\ N_q^i &= W_q^i - c \\ N_u^{-i} &= W_u^{-i} - k \\ N_q^{-i} &= W_q^{-i} - c - k \end{cases} \tag{7}$$

Comparing the net payoff ($N_{e^*}^{i^*}$) for each choice (i^*, e^*), we can determine the best response, $(i^*, e^*)_{i,c,k}$, for type i newborns with the cost levels of c and k.

Lemma 2. *When $v^{-i} > v^i$, the identity and skill decision for a type-i newborn with the cost set (c, k) is*

$$(i^*, e^*)_{i,c,k} = \begin{cases} (i, u) & \text{if } c > R^i,\ k > Y_u^i \text{ and } k + c > R^{-i} + Y_u^i \\ (i, q) & \text{if } c < R^i \text{ and } k > Y_q^i \\ (-i, u) & \text{if } c > R^{-i} \text{ and } k < Y_u^i \\ (-i, q) & \text{if } c < R^{-i},\ k < Y_q^i \text{ and } k + c < R^{-i} + Y_u^i, \end{cases} \tag{8}$$

and, when $v^{-i} \leq v^i$, no type-i newborn switches his inborn type: $(i^*, e^*)_{i,c,k} = (i, u),\ \forall c \in (0, R^i)$, and $(i^*, e^*)_{i,c,k} = (i, q),\ \forall c \in (R^i, \infty)$.

Proof. When $v^{-i} > v^i$, we know that $R^{-i} > R^i$ and $Y_q^i > Y_u^i$, as described in Panel A of Figure 1. The result is confirmed when comparing $N_{e^*}^{i^*}$ for each range of (c, k). For example, $N_q^i > \max\{N_u^i, N_q^{-i}, N_u^{-i}\}$ if $c < R^i$ and $k > Y_q^i$. When $v^{-i} \leq v^i$, we know that $Y_q^i \leq Y_u^i \leq 0$. Thus, no type-i newborn has a willingness to pay k to switch his inborn type. His choice of qualification depends on R^i. ■

The lemma is described in Panel A of Figure 1 for the case of $v^{-i} > v^i$. The lemma directly proves the following proposition.

Proposition 1. *Under Assumption 1, the more talented the newborn, the more likely that he will switch from his inborn identity type to the other identity type.*

The more talented, the more likely that he will invest in skills. The less talented, the more likely that he will not invest in skills. The return to identity switch for a qualified worker is greater than that for an unqualified worker under Assumption 1: $Y_q^i > Y_e^i$ given $v^{-i} > v^i$. (This is because that the payoff uncertainty is greater for qualified workers than unqualified workers: $1 - P_q > 1 - P_u$. The switch to the group with the better collective reputation can reduce the uncertainty.) Thus, the more talented, the more likely that he will switch to the other type whose members will receive the better treatment by employers.

3 Dynamic Systems

For the purpose of the dynamic analysis, we will simplify both $G(c)$ and $H(k)$. Each cohort of either group is composed of Π_l fraction of low investment cost newborns, $\Pi_h - \Pi_l$ fraction of medium investment cost newborns and $1 - \Pi_h$ fraction of high investment cost newborns. Denoting those cost levels by c_l, c_m and c_h, they satisfy the following condition:

Assumption 2. $c_l < \frac{wP_q}{\delta+\lambda} < c_m < \frac{wP_u}{\delta+\lambda} < c_h$.

With this assumption, we ensure that c_l is small enough that the Π_l faction of low cost newborns always invest in skills, and c_h is big enough that the $1-\Pi_h$ fraction of high cost newborns never invest in skills, regardless of R^i,$R^{-i}$and Y_e^i. Also, each cohort of either group is composed of η fraction of high switching cost newborns and $1-\eta$ fraction of low switching cost newborns. Denoting those cost levels by k_h and k_l, they satisfy the following condition:

Assumption 3. $\frac{w}{\delta+\lambda} - c_m < k_l < \frac{w(1-P_q)}{\delta+\lambda} < k_h$.

With this assumption, we ensure that k_h is big enough that the η fraction of high switching cost never switch their inborn identity types, regardless of their investment cost c. Also, k_l is big enough that the newborns with an investment cost of either c_m or c_h never switch their inborn identity types. However, any newborn with the low investment cost c_l and the low switching cost k_l will switch his identity type and join the other group as long as the return for the identity switch (Y_q^i) is greater than the switching cost k_l. The population distribution that satisfies the two assumptions is depicted in Panel B of Figure 1.

Lemma 3. *Under Assumptions 2 and 3, the newborns with investment cost c_l always invest in skills and the newborns with investment cost c_h never invest in skills. The newborns with switching cost k_h never switch their inborn types. The newborns with investment cost either c_m or c_h never switch their inborn identity types.*

Proof. See the proof in the appendix. ■

The above lemma implies:

Proposition 2 (Potential Switcher)**.** *Under Assumptions 2 and 3, newborns with the cost set (c_l, k_l) are the only potential switchers from their inborn identity types to the other type. Type i born potential switchers switch if and only if Y_q^i is greater than k_l.*

Proof. Lemma 3 implies that newborns with the cost set (c_l, k_l) are the only potential switchers. Also, they will invest in skills whether or not they switch to the other type according to the lemma. Since Y_q^i is the extra benefits of switching for the newborns who will invest in skills $(W_q^{-i} - W_q^i)$, they switch if $Y_q^i > k_l$. Otherwise, the switching cost is greater than (or equal to) the benefits of the switching for the potential switchers. Thus, they do not switch. ■

3.1 Dynamics with Identity Switches Restricted

Before moving to the identity switch dynamics, let us analyze the simplest situation in which no newborn switches his inborn group identity. We can do this by simply imposing a condition that identity switch is prohibited by an authority, or the fraction of newborns with the highest switching cost k_h is one ($\eta = 1$). Each variable in this section is expressed with the superscript "n", symbolizing the condition of identity switch restriction. By equations (3) and (5), we can describe how R_t^n evolves over time:

$$\begin{aligned}
\dot{R}_t^n &= \frac{w(P_u - P_q)}{\delta + \lambda}\dot{v}_t^n \\
&= w(P_u - P_q)(v_t^n - \xi_t^n) \\
&= (\delta + \lambda)\left[R_t^n - \frac{wP_q}{\delta + \lambda} - \frac{w(P_u - P_q)}{\delta + \lambda}\xi_t^n\right].
\end{aligned} \tag{9}$$

Let ϕ_t^n denote the fraction of time t born workers who invest and become qualified:

$$\phi_t^n = \begin{cases} 0, & \forall R_t^n \in [0, c_l) \\ \Pi_l, & \forall R_t^n \in [c_l, c_m) \\ \Pi_h, & \forall R_t^n \in [c_m, c_h) \\ 1, & \forall R_t^n \in [c_h, 1]. \end{cases} \tag{10}$$

Since λ fraction of the total population is replaced with newborns in a unit period, Π_t^n evolves in short time interval Δt in the following way.

$$\Pi_{t+\Delta t}^n \approx \lambda \Delta t \cdot \left(\frac{\phi_t^n + \phi_{t+\Delta t}^n}{2}\right) + (1 - \lambda \Delta t) \cdot \Pi_t^n. \tag{11}$$

By the rearrangement of the equation, we have

$$\frac{\Delta \Pi_t^n}{\Delta t} \equiv \frac{\Pi_{t+\Delta t}^n - \Pi_t^n}{\Delta t} \approx \lambda \left[\frac{\phi_t^n + \phi_{t+\Delta t}^n}{2} - \Pi_t^n\right].$$

Taking $\Delta t \to 0$, we can express how Π_t^n evolves over time:

$$\dot{\Pi}_t^n = \lambda[\phi_t^n - \Pi_t^n]. \tag{12}$$

Therefore, the dynamic system is summarized with

$$\begin{aligned}
\dot{R}_t^n &= (\delta + \lambda)\left[R_t^n - \frac{wP_q}{\delta + \lambda} - \frac{w(P_u - P_q)}{\delta + \lambda}\xi_t^n\right] \\
\dot{\Pi}_t^n &= \lambda[\phi_t^n - \Pi_t^n],
\end{aligned} \tag{13}$$

in which ξ_t^n is a function of Π_t^n and ϕ_t^n is a function of R_t^n, according to equations (1) and (10). Panel A of Figure 2 describes the dynamic paths toward the two stable equilibria, $Q_l\left(\frac{wP_q}{\delta+\lambda},\Pi_l\right)$ and $Q_h\left(\frac{wP_u}{\delta+\lambda},\Pi_h\right)$. Knowing that R_t^n is a linear function of v_t^n in equation (5), we have

$$\phi_t^n = \begin{cases} \Pi_l, & \forall v_t^n \in [0, v^*) \\ \Pi_h, & \forall v_t^n \in [v^*, 1], \end{cases} \quad \text{with } v^* \equiv \frac{(\delta+\lambda)c_m - wP_q}{w(P_u - P_q)}. \tag{14}$$

The usage of v_t^n, instead of R_t^n, can further simplify the dynamic system.

Proposition 3. *The dynamic system with a flow variable* Π_t^n *and a jumping variable* v_t^n *is*

$$\begin{aligned} \dot{v}_t^n &= (\delta+\lambda)[v_t^n - \xi_t^n] \\ \dot{\Pi}_t^n &= \lambda[\phi_t^n - \Pi_t^n], \end{aligned}$$

with demarcation loci of

$$\begin{aligned} \dot{v_t}^n = 0 \ \textit{Locus} &: \ v_t^n = \xi_t^n \\ \dot{\Pi}_t^n = 0 \ \textit{Locus} &: \ \Pi_t^n = \phi_t^n. \end{aligned}$$

Panel B of Figure 2 describes the dynamic paths to two stable equilibria, $Q_l^n\,(0,\Pi_l)$ and $Q_h^n\,(1,\Pi_h)$. Let us denote π^o as the level of reputation at $v^n = v^*$ with which the group at the state (v^*, π^o) can directly reach the upper equilibria Q_h^n along the optimistic path. Also, denote π^p as the level of reputation at $v^n = v^*$ with which the group at the state (v^*, π^p) can directly reach the lower equilibria Q_l^n along the pessimistic path. Using the differential equations in Proposition 3, we can find

$$\pi^o = \Pi_h + (\Pi^* - \Pi_h)v^{*-\frac{\lambda}{\delta+\lambda}}, \tag{15}$$

$$\pi^p = \Pi_l + (\Pi^* - \Pi_l)(1-v^*)^{-\frac{\lambda}{\delta+\lambda}}. \tag{16}$$

In this paper, we assume that δ is big enough that two economically stable states are "separate" from each other. (Refer to Lemma 3 in Kim and Loury (2008) for the definition of separation.) With the separated two equilibria, a group in the lower equilibrium Q_l^n is in a reputation trap, which means the group cannot escape the status of low skill investment activities, owing to the negative influence of the group's bad reputation. A group in the upper equilibrium Q_h^n enjoys the secured BODs given by employers and maintains the high skill investment activities, owing to the positive influence of the group's good reputation. If the two equilibria are separated, the size of overlap L^n is simply the difference between π^p and π^o:

$$L^n = \Pi_l - \Pi_h + (\Pi^* - \Pi_l)(1 - v^*)^{-\frac{\lambda}{\delta+\lambda}} + (\Pi_h - \Pi^*)v^{*-\frac{\lambda}{\delta+\lambda}}. \tag{17}$$

Inside the overlap, the expectation about the future determines the final state, either Q_l^n or Q_h^n. Outside the overlap, the initial reputation is critical: if it is below the overlap, the final state should be the lower equilibrium Q_l^n, and, if it is above the overlap, the final state should be the upper one Q_h^n. (Kim and Loury, 2008)

3.2 Dynamics with Identity Switches from Type j to Type i

Imagine a situation that some fraction of type-j newborns switch to type i consistently since some fixed point of time X. Until the incidence, both group sizes have been constant as L_i and L_j. Under the given assumptions, the exact $(1-\eta)\Pi_l$ fraction of type-j newborns, whose cost set is (c_l, k_l), will switch their inborn identity types to type i, according to Lemma 3 and Proposition 2. Thus, the population sizes of group i and group j eventually arrive $L_i + (1-\eta)\Pi_l L_j$ and $L_j - (1-\eta)\Pi_l L_j$ for each. In the following sections, we will address the dynamic system for group i which benefits from the inflows of skilled workers from type-j newborns, and the dynamic system for group j which loses some of the most talented newborns to group i. Let us denote the size ratio of group j and group i by $\tilde{L}_i(\equiv \frac{L_j}{L_i})$.

3.2.1 Dynamic System of Group i with Inflows from Group j

Let us denote the size of the type-i skilled workers at time t by Z_t^i, and the total size of the type-i workers at time t by M_t^i. Note that M_X^i is L_i, and M_t^i increases consistently over time with the inflows from the type-j newborns since time X. Thus, M_t^i changes in short time interval Δt:

$$M_{t+\Delta t}^i = (1 - \lambda\Delta t)M_t^i + L_i\lambda\Delta t + L_j\lambda\Delta t \cdot \Pi_l'. \tag{18}$$

Taking $\Delta t \to 0$, we have the evolution rule of M_t^i:

$$\dot{M}_t^i = \lambda[L_i + L_j\Pi_l' - M_t^i]. \tag{19}$$

Then, since M_X^i is L_i, M_t^i can be expressed explicitly:

$$M_t^i = L_i + L_j\Pi_l' \cdot [1 - e^{-\lambda(t-X)}]. \tag{20}$$

The Z_t^i changes in short time interval Δt, denoting $(1-\eta)\Pi_l$ by Π_l':

$$Z_{t+\Delta t}^i = (1-\lambda\Delta t)Z_t^i + L_i\lambda\Delta t\cdot\frac{\phi_t^i+\phi_{t+\Delta t}^i}{2} + L_j\lambda\Delta t\cdot\Pi_l'. \tag{21}$$

Taking $\Delta t \to 0$, we have the evolution rule of Z_t^i:

$$\dot{Z}_t^i = \lambda[L_i\phi_t^i + L_j\Pi_l' - Z_t^i]. \tag{22}$$

As far as ϕ_t^i is constant over time ($\phi_t^i = \bar{\phi}^i$), Z_t^i can be expressed explicitly, knowing $Z_X^i = \Pi_X^i \cdot L_i$:

$$Z_t^i = L_i\bar{\phi}^i + L_j\Pi_l' + [L_i\Pi_X^i - L_i\bar{\phi}^i - L_j\Pi_l']e^{-\lambda(t-X)}. \tag{23}$$

Therefore, using equations (20) and (23), we can express the reputation of group i at time t:

$$\Pi_t^i\left(=\frac{Z_t^i}{M_t^i}\right) = \frac{L_i\bar{\phi}^i + L_j\Pi_l' + [L_i\Pi_X^i - L_i\bar{\phi}^i - L_j\Pi_l']e^{-\lambda(t-X)}}{L_i + L_j\Pi_l'\cdot[1-e^{-\lambda(t-X)}]}. \tag{24}$$

Since we already know that $\dot{v}_t^i = (\delta+\lambda)[v_t^i - \xi_t^i]$, as far as ξ_t^i is constant ($\xi_t^i = \bar{\xi}^i$),

$$v_t^i = \bar{\xi}^i + (v_X^i - \bar{\xi}^i)e^{(\delta+\lambda)(t-X)}. \tag{25}$$

After the rearrangement, we have the following useful outcome:

$$e^{-\lambda(t-X)} = \left[\frac{v_X^i - \bar{\xi}^i}{v_t^i - \bar{\xi}^i}\right]^{\frac{\lambda}{\delta+\lambda}}. \tag{26}$$

From equations (24) and (26), we can achieve the following useful lemma:

Lemma 4. *Suppose the Π_l' fraction of type-j newborns consistently switch to type i since $t = X$. Given constant $\bar{\xi}^i$ and $\bar{\phi}^i$, we can express the relationship between the initial state (v_X^i, Π_X^i) and the state at time t (v_t^i, Π_t^i):*

$$\left[\frac{v_X^i - \bar{\xi}^i}{v_t^i - \bar{\xi}^i}\right]^{\frac{\lambda}{\delta+\lambda}} = \frac{L_i\bar{\phi}^i + L_j\Pi_l' - (L_i + L_j\Pi_l')\Pi_t^i}{-L_i\Pi_X^i + L_i\bar{\phi}^i + L_j\Pi_l' - L_j\Pi_l'\Pi_t^i}. \tag{27}$$

Also, we can evaluate the following, using equations (18) and (21),

$$\frac{\Delta\Pi_t^i}{\Delta t} \equiv \frac{\Pi_{t+\Delta t}^i - \Pi_t^i}{\Delta t} \quad = \quad \frac{1}{\Delta t}\cdot\left[\frac{Z_{t+\Delta t}^i}{M_{t+\Delta t}^i} - \Pi_t^i\right].$$

Taking $\Delta t \to 0$, we have the evolution rule of Π_t^i:

$$\dot{\Pi}_t^i = \frac{\lambda[(L_i\phi_t^i + L_j\Pi_l') - (L_i + L_j\Pi_l')\Pi_t^i]}{M_t^i}. \tag{28}$$

Proposition 4. *Suppose that the Π_l' fraction of type-j newborns switch to type i consistently since time X. Then, the dynamic system with a flow variable Π_t^i and a jumping variable v_t^i is*

$$\begin{aligned} \dot{v}_t^i &= (\delta+\lambda)[v_t^i - \xi_t^i] \\ \dot{\Pi}_t^i &= \frac{\lambda[(L_i\phi_t^i + L_j\Pi_l') - (L_i + L_j\Pi_l')\Pi_t^i]}{M_t^i}, \end{aligned}$$

with demarcation loci of

$$\begin{aligned} \dot{v_t}^i = 0 \ \textit{Locus} &: \ v_t^i = \xi_t^i \\ \dot{\Pi}_t^i = 0 \ \textit{Locus} &: \ \Pi_t^i = \frac{L_i\phi_t^i + L_j\Pi_l'}{L_i + L_j\Pi_l'}. \end{aligned}$$

Corollary 1. *In the dynamics of group i which is growing with the inflows of the most talented type-j newborns, the reputation of group i improves faster (or deteriorates slower) compared to that of the no-switches dynamics: $\dot{\Pi}_t^i > \dot{\Pi}_t^n$, $\forall \phi_t^i \in \{\Pi_l, \Pi_h\}$, $\forall t \in (X, \infty)$, except when $\Pi_X^i = 1$. (Note that when $\Pi_X^i = 1$, $\dot{\Pi}_X^i = \dot{\Pi}_X^n$.)*

Proof. See the proof in the appendix. ■

The dynamics generates two stable equilibria: $Q_l'(0, L_i')$ and $Q_h'(1, H_i')$, where $L_i' = \frac{L_i\Pi_l + L_j\Pi_l'}{L_i + L_j\Pi_l'}$ and $H_i' = \frac{L_i\Pi_h + L_j\Pi_l'}{L_i + L_j\Pi_l'}$. Both of them are positioned higher than stable equilibria in no-switches dynamics, $Q_l^n(0, \Pi_l)$ and $Q_h^n(1, \Pi_h)$. Let us denote ${\pi^o}_i'$ as the time-X reputation level Π_X^i at $v^n = v^*$ with which group i at the time-X state $(v^*, {\pi^o}_i')$ can directly reach the upper equilibria Q_h' along the optimistic path. Also, denote ${\pi^p}_i'$ as the level of reputation at $v^n = v^*$ with which group i at the time-X state $(v^*, {\pi^p}_i')$ can directly reach the lower equilibrium Q_l' along the pessimistic path. Using Lemma 4, we can compute both of them. For the first, apply $\bar{\xi}^i = 0$, $\bar{\phi}^i = \Pi_h$, $(v_X^i, \Pi_X^i) = (v^*, {\pi^o}_i')$ and $(v_t^i, \Pi_t^i) = (1, \Pi^*)$:

$${\pi^o}_i' = \Pi_h + \tilde{L}_i\Pi_l'(1-\Pi^*) - [\tilde{L}_i\Pi_l'(1-\Pi^*) + (\Pi_h - \Pi^*)] \cdot v^{*-\frac{\lambda}{\delta+\lambda}}. \tag{29}$$

For the second, apply $\bar{\xi}^i = 1$, $\bar{\phi}^i = \Pi_l$, $(v_X^i, \Pi_X^i) = (v^*, \pi^{p\prime})$ and $(v_t^i, \Pi_t^i) = (0, \Pi^*)$:

$${\pi^p}_i' = \Pi_l + \tilde{L}_i\Pi_l'(1-\Pi^*) - [\tilde{L}_i\Pi_l'(1-\Pi^*) + (\Pi_l - \Pi^*)] \cdot (1-v^*)^{-\frac{\lambda}{\delta+\lambda}}. \tag{30}$$

Comparing ${\pi^o}'_i$ and ${\pi^p}'_i$ with π^o and π^p in equations (15) and (16), we have the following result.

Corollary 2. *Both ${\pi^o}'_i$ and ${\pi^p}'_i$ in group i dynamics with the inflows of the most talented type-j newborns are smaller than π^o and π^p in the dynamics with no identity switches: ${\pi^o}'_i < \pi^o$ and ${\pi^p}'_i < \pi^p$.*

The optimistic path from $(v^*, {\pi^o}'_i)$ and the pessimistic path from $(v^*, {\pi^p}'_i)$ are described in Figure 3.

3.2.2 Dynamic System of Group j with Outflows to Group i

According to Proposition 2, outflows to group i should be among the most talented type-j newborns with the lower switching cost k_l. Note that when the potential switchers start to switch at time X, the reputation level of group j should be lower than π^o, which is the lower boundary of the optimistic path in the no-switches dynamics: $\Pi^j_X < \pi^o$.

Lemma 5. *When type-j potential switchers start to switch at time X, $\Pi^j_X < \pi^o$ and, consequently, $v^j_X = 0$.*

Proof. See the proof in the appendix. ■

As the $\Pi_l(1-\eta)$ fraction of type-j newborns switches to type i since time X, $M^j_X = L_j$ and M^j_t decreases over time. Then, M^j_t changes in the short time interval Δt:

$$M^j_{t+\Delta t} = (1-\lambda\Delta t)M^j_t + L_j\lambda\Delta t[1-\Pi'_l]. \tag{31}$$

Taking $\Delta t \to 0$, we have

$$\dot{M}^j_t = \lambda[L_j(1-\Pi'_l) - M^j_t]. \tag{32}$$

Then, since M^j_X is L_j, M^j_t can be expressed explicitly:

$$M^j_t = L_j(1-\Pi'_l) + L_j\Pi'_l e^{-\lambda(t-X)}. \tag{33}$$

Also, the size of skilled workers among group j changes over time:

$$Z^j_{t+\Delta t} = (1-\lambda\Delta t)Z^j_t + L_j\lambda\Delta t \cdot \Pi_l\eta. \tag{34}$$

Taking $\Delta t \to 0$, we have the evolution rule of Z_t^j:

$$\dot{Z}_t^j = \lambda[L_j \Pi_l \eta - Z_t^j]. \tag{35}$$

Since $Z_X^j = \Pi_X^j \cdot L_j$, the Z_t^i can be expressed explicitly:

$$Z_t^j = L_j \Pi_l \eta + L_j(\Pi_X^j - \Pi_l \eta)e^{-\lambda(t-X)}. \tag{36}$$

Thus, we can evaluate the following, using equations (34) and (31),

$$\frac{\Delta \Pi_t^j}{\Delta t} \equiv \frac{\Pi_{t+\Delta t}^j - \Pi_t^j}{\Delta t} = \frac{1}{\Delta t} \cdot \left[\frac{Z_{t+\Delta t}^j}{M_{t+\Delta t}^j} - \Pi_t^j \right].$$

Taking $\Delta t \to 0$, we have the evolution rule of Π_t^j:

$$\dot{\Pi}_t^j = \frac{\lambda L_j [\Pi_l \eta - (1 - \Pi_l')\Pi_t^j]}{M_t^j}. \tag{37}$$

Therefore, using the above lemma, we can reach the following results:

Proposition 5. *Suppose that the $\Pi_l(1-\eta)$ fraction of type-j newborns switch to type i consistently since time X. Then, the dynamic system with a flow variable Π_t^j and a jumping variable v_t^j is*

$$\begin{aligned} \dot{v}_t^j &= (\delta + \lambda)[v_t^j - \xi_t^j] \\ \dot{\Pi}_t^j &= \frac{\lambda L_j [\Pi_l \eta - (1 - \Pi_l')\Pi_t^j]}{M_t^j}, \end{aligned}$$

in which $v_t^j = \xi_t^j = 0$, $\forall t \in (X, \infty)$, and Π_t^j approaches monotonically $L_j''(\equiv \frac{\Pi_l \eta}{1 - \Pi_l(1-\eta)})$, which is smaller than Π_l.

Proof. Since $\pi^o > \Pi_l$ and $\Pi_l > L_j''(\equiv \frac{\Pi_l \eta}{1-\Pi_l(1-\eta)})$, $\pi^o > L_j''$. For any $\Pi_X^j < \pi^o$, Π_t^j approaches L_j''. Under the no-switches dynamics, the reputation recovery path is not available for any initial reputation level $\Pi_0^n \in (0, \pi^o)$. Therefore, the reputation recovery path should not be available to group j which is losing their most talented newborns to the other group, which implies $v_t^j = \xi_t^j = 0$, $\forall t \in (X, \infty)$, as $v_t^n = \xi_t^n = 0$, $\forall t$, for any $\Pi_0^n \in (0, \pi^o)$. ■

The dynamics of group j is displayed in Figure 4. Note that whenever the most talented type-j newborns switch to type i, group j is positioned on the pessimistic path with $\xi_t^j = 0$, $v_t^j = 0$ and $R_t^j = \frac{wP_q}{\delta+\lambda}$, $\forall t \in (X, \infty)$. The state of group j losing the most talented to group i converges to $Q''(0, L_j'')$, where $L_j'' = \frac{\Pi_l \eta}{1-\Pi_l(1-\eta)}$, which is smaller than Π_l.

Corollary 3. *In the dynamics of group j which is losing some of the most talented newborns to group i, the reputation of group j deteriorates faster (or improves slower) compared to that of the no-switches dynamics:* $\dot{\Pi}_t^j < \dot{\Pi}_t^n, \forall t \in (X, \infty)$.

Proof. See the proof in the appendix. ■

4 Endogenous Group Formation

In order to analyze the endogenous process of group formation, we impose the following reasonable assumptions about the behaviors of group members: 1) Group members can make a consensus for the group state that will be realized in the far future, within a reasonably short period. They can agree quickly with the path to be taken, when multiple equilibrium paths (optimistic and pessimistic) are available. 2) Whenever multiple equilibria are possible for the future group state, group members tend to choose the equilibrium with the higher group reputation. Whenever multiple paths are available, the group tends to choose the (optimistic) path that leads to the higher group reputation. 3) Once group members agree with a future group state, they behave in a way to arrive there as early as possible. Once group members choose the path to take, they determine the level of a jumping variable in a way that the group state reaches the equilibrium as fast as possible. 4) When two groups hold expectations about the future that conflict with each other, they can reach "social consensus" toward the future within a reasonably short period. For example, when it is impossible that both groups take the optimistic path, one group gives up the option to take the optimistic path within a reasonably short period.

We assume that the overlap in the no-switches dynamics is placed within the two stable equilibria: Q_l^n and Q_h^n. Under the constraints that newborns cannot switch, any group in the lower equilibrium Q_l^n is in the reputation trap, and cannot escape the low skill investment activities.

By Lemma 5, we know that $v_X^j = 0$ when the Π_l' fraction of type-j newborns start to join group i. Since they switch their inborn types only when $Y_{q,t}^j > k_l$, we can find the threshold level of v_X^i:

$$Y_{q,X}^j = \frac{w(1-P_q)}{\delta+\lambda}(v_X^i - v_X^j) = \frac{w(1-P_q)}{\delta+\lambda}v_X^i > k_l.$$

Therefore, the threshold level of v_X^i, denoted by $\hat{v}_X^i$, is $\frac{(\delta+\lambda)k_l}{w(1-P_q)}$, given $v_X^j = 0$. We impose the following condition that is not critical in the structure of the given dynamic model, but useful to achieve the main results more effectively:

Condition 1. *The level of* k_l *ensures* $\hat{v}_X^i > v^*$: $k_l > \frac{1-P_q}{P_u-P_q}\left(c_m - \frac{wP_q}{\delta+\lambda}\right)$.

It is notable that there always exists a positive range of k_l that satisfies the condition and Assumption 3 for any c_m satisfying Assumption 2: the range of k_l is $\left(\max\left\{\frac{w}{\delta+\lambda} - c_m, \frac{1-P_q}{P_u-P_q}\left(c_m - \frac{wP_q}{\delta+\lambda}\right)\right\}, \frac{w(1-P_q)}{\delta+\lambda}\right)$ for given $c_m \in \left(\frac{wP_q}{\delta+\lambda}, \frac{wP_u}{\delta+\lambda}\right)$. Readers may try and confirm that the following results in this paper can be replicated for the case of $\hat{v}^i_X \leq v^*$: $k_l \leq \frac{1-P_q}{P_u-P_q}\left(c_m - \frac{wP_q}{\delta+\lambda}\right)$.

4.1 Group i Equilibrium Path with Skill Inflows from Group j

Since k_l is less than $\frac{w(1-P_q)}{\delta+\lambda}$ by Assumption 3, we know that $v^* < \hat{v}^i_X < 1$ under Condition 1. Then, we find the corresponding threshold level of group i reputation, $\hat{\Pi}^i_X$, above which the initial reputation of group i can lead the low cost talented type-j newborns to switch to type i immediately. Using Lemma 4, $\hat{\Pi}^i_X$ is computed applying $\bar{\xi}^i = 0$, $\bar{\phi}^i = \Pi_h$, $(v^i_X, \Pi^i_X) = (\hat{v}^i_X, \hat{\Pi}^i_X)$ and $(v^i_t, \Pi^i_t) = (1, \Pi^*)$:

$$\hat{\Pi}^i_X = \Pi_h + \tilde{L}_i \Pi'_l (1 - \Pi^*) - [\tilde{L}_i \Pi'_l (1 - \Pi^*) + (\Pi_h - \Pi^*)] \cdot \hat{v}^i_X{}^{-\frac{\lambda}{\delta+\lambda}}. \tag{38}$$

However, if group i members can expect the inflows of the talented type-j newborns in the future, they may increase their skill investment rate much earlier even before the incidence of the skill inflows, as described in Panel A of Figure 5. Before the incidence of the skill inflows, the evolution rules for Π^i_t and v^i_t should follow the rules in the no-switches dynamics summarized in Proposition 3. Thus, applying $\xi^n_t = 0$, $\phi^n_t = \Pi_h$, $(v^i_0, \Pi^i_0) = (v^*, \pi^{o\prime\prime}_i)$ and $(v^i_t, \Pi^i_t) = (\hat{v}^i_X, \hat{\Pi}^i_X)$ to the differential equations in the proposition, we can find the group i reputation level $\pi^{o\prime\prime}_i$ at $v^i = v^*$ with which the group state (v^i_t, Π^i_t) reaches $(\hat{v}^i_X, \hat{\Pi}^i_X)$ that initiates the switching of the talented type-j newborns:

$$\pi^{o\prime\prime}_i = \Pi_h - (\Pi_h - \hat{\Pi}^i_X) \cdot [\hat{v}^i_X / v^*]^{\frac{\lambda}{\delta+\lambda}}. \tag{39}$$

The following lemma summarizes the relative size of π^o_i, $\pi^{o\prime}_i$, and $\pi^{o\prime\prime}_i$.

Lemma 6. *Since $v^* < \hat{v}^i_X < 1$, $\pi^{o\prime}_i < \pi^{o\prime\prime}_i < \pi^o$.*

Proof. Compare π^o, $\pi^{o\prime}_i$ and $\pi^{o\prime\prime}_i$, using equations (15), (29) and (39) and applying equation (38):

$$\begin{aligned} \pi^{o\prime\prime}_i - \pi^{o\prime}_i &= \tilde{L}_i \Pi'_l (1 - \Pi^*) \cdot (\hat{v}^i_X{}^{\frac{\lambda}{\delta+\lambda}} - v^*{}^{\frac{\lambda}{\delta+\lambda}}) v^*{}^{-\frac{\lambda}{\delta+\lambda}} > 0 \\ \pi^o - \pi^{o\prime\prime}_i &= \tilde{L}_i \Pi'_l (1 - \Pi^*) \cdot (1 - \hat{v}^i_X{}^{\frac{\lambda}{\delta+\lambda}}) v^*{}^{-\frac{\lambda}{\delta+\lambda}} > 0. \end{aligned} \tag{40}$$

■

If $\pi^{o\prime\prime}_i$ is below Π_l, the optimistic path that can reach $(\hat{v}^i_X, \hat{\Pi}^i_X)$ is extended further up to the $\Pi^i = 0$ horizontal line and, thus, the group i even with zero reputation can take the optimistic path to $Q'_h(1, H'_i)$, as described in Panel B of Figure 5. Therefore, we can find the effective threshold of group i reputation $\tilde{\Pi}^i$, above which the optimistic path to the higher equilibrium $Q'_h(1, H'_i)$ is available to group i members.

Proposition 6. *Given $v_X^j = 0$, the effective threshold of group i reputation $\tilde{\Pi}^i$ is as follows, above which group i can move out of the reputation trap and reach the high reputation equilibrium $Q'_h(1, H'_i)$:*

$$\tilde{\Pi}^i = \begin{cases} \Pi_h - (\Pi_h - \hat{\Pi}_X^i) \cdot [\hat{v}_X^i / v^*]^{\frac{\lambda}{\delta+\lambda}} \ (= \pi^{o\prime\prime}_i) & \text{if } \pi^{o\prime\prime}_i \geq \Pi_l, \\ 0 & \text{if } \pi^{o\prime\prime}_i < \Pi_l. \end{cases} \tag{41}$$

The following corollary shows the important role of relative size between two groups in the determination of the effective reputation threshold for group i ($\tilde{\Pi}^i$):

Corollary 4. *There exists the threshold $\tilde{L}_i^*$ of $\tilde{L}_i$ in $(0, \infty)$, above which $\tilde{\Pi}^i = 0$ and below which $\tilde{\Pi}^i = \pi^{o\prime\prime}_i$:*

$$\tilde{L}_i^* = \frac{\Pi'_l(1 - \Pi^*) \cdot (1 - \hat{v}_X^{i\ \frac{\lambda}{\delta+\lambda}}) v^{*-\frac{\lambda}{\delta+\lambda}}}{\Pi_h - \Pi_l + (\Pi^* - \Pi_h) v^{*-\frac{\lambda}{\delta+\lambda}}}. \tag{42}$$

Proof. Using equations (38) and (39), we can get $\tilde{L}_i^*$ that satisfies $\pi^{o\prime\prime}_i = \Pi_l$. $\tilde{L}_i^*$ is positive because the denominator is positive: $\Pi_h - \Pi_l + (\Pi^* - \Pi_h) v^{*-\frac{\lambda}{\delta+\lambda}} = \pi^o - \Pi_l > 0$.■

4.2 Search For Final State Given Initial State (Π_0^b, Π_0^a)

Now let us get return to the original question, the dynamics of groups A and B. Using the findings in the previous sections, we can search for the final state for each initial state (Π_0^b, Π_0^a) under the imposed assumptions and a condition.

First, check the state evolution under the constraints that type-B newborns can switch to the other type but the switches of type-A newborns to type B are not permitted. In this case, only type-B potential switchers with the cost set (c_l, k_l) may consider switching (Proposition 2). By Lemma 5, there is no switching of type-B newborns when $\Pi_0^b \geq \pi^o$. When $\Pi_0^b < \pi^o$, $v_t^b = 0, \forall t$, whether or not the type-B potential switchers switch, according to the dynamics summarized in Propositions 3 and 5. According to Proposition 6, since $v_X^b = 0$ with $\Pi_0^b < \pi^o$ given, group A with its initial reputation above $\tilde{\Pi}^a$ can reach the high reputation equilibrium Q'_h. Below $\tilde{\Pi}^a$, the group's reputation ends up with Π_l.

Therefore, we can summarize the basin of attraction for each potential attractor in the following way:

$$\text{Basins of Attraction I} \begin{cases} \{(\Pi_0^b, \Pi_0^a) | \pi^o \leq \Pi_0^b \leq 1, \pi^o \leq \Pi_0^a \leq 1\} & \text{for attractor } (\Pi_h, \Pi_h), \\ \{(\Pi_0^b, \Pi_0^a) | \pi^o \leq \Pi_0^b \leq 1, 0 \leq \Pi_0^a < \pi^o\} & \text{for attractor } (\Pi_h, \Pi_l), \\ \{(\Pi_0^b, \Pi_0^a) | 0 \leq \Pi_0^b < \pi^o, \tilde{\Pi}^a \leq \Pi_0^a \leq 1\} & \text{for attractor } (L_b'', H_a'), \\ \{(\Pi_0^b, \Pi_0^a) | 0 \leq \Pi_0^b < \pi^o, 0 \leq \Pi_0^a < \tilde{\Pi}^a\} & \text{for attractor } (\Pi_l, \Pi_l). \end{cases} \tag{43}$$

These basins of attractions are displayed in Panel A of Figure 6 for the case of $\tilde{\Pi}^a = \pi^{o''}_{\ a}$ (that is, $\pi^{o''}_{\ a} > \Pi_l$). Any initial position in the basin of attraction for the attractor (L_b'', H_a') that is colored in yellow in the panel will lead the type-B potential switchers to start to join group A at time X in order to obtain the benefits of superior collective reputation of group A in the labor market. Note that, given $0 \leq \Pi_0^b < \pi^o$, the type-B potential switchers immediately start to switch when the initial reputation Π_0^a is greater than $\hat{\Pi}_X^a$; If Π_0^a is smaller than $\hat{\Pi}_X^a$, though Π_0^a is greater than $\tilde{\Pi}^a$, the type-B potential switchers do not switch right away, but wait until the group A's reputation Π_t^a improves up to $\hat{\Pi}_X^a$.

Second, check the state evolution under the constraints that type-A newborns can switch to the other type but the switching of type-B newborns to type A are not permitted. In this case, only type-A potential switchers with the cost set (c_l, k_l) may consider switching (Proposition 2). With the same logic above, we can summarize the basin of attraction for each potential attractor:

$$\text{Basins of Attraction II} \begin{cases} \{(\Pi_0^b, \Pi_0^a) | \pi^o \leq \Pi_0^a \leq 1, \pi^o \leq \Pi_0^b \leq 1\} & \text{for attractor } (\Pi_h, \Pi_h), \\ \{(\Pi_0^b, \Pi_0^a) | \pi^o \leq \Pi_0^a \leq 1, 0 \leq \Pi_0^b < \pi^o\} & \text{for attractor } (\Pi_l, \Pi_h), \\ \{(\Pi_0^b, \Pi_0^a) | 0 \leq \Pi_0^a < \pi^o, \tilde{\Pi}^b \leq \Pi_0^b \leq 1\} & \text{for attractor } (H_b', L_a''), \\ \{(\Pi_0^b, \Pi_0^a) | 0 \leq \Pi_0^a < \pi^o, 0 \leq \Pi_0^b < \tilde{\Pi}^b\} & \text{for attractor } (\Pi_l, \Pi_l). \end{cases} \tag{44}$$

These basins of attractions are displayed in Panel B of Figure 6 for the case of $\tilde{\Pi}^b = \pi^{o''}_{\ b}$ (that is, $\pi^{o''}_{\ b} > \Pi_l$). Any initial position in the basin of attraction for the attractor (H_b', L_a'') that is colored in orange in the panel will lead the type-A potential switchers to start joining group B at time X. Note that, given $0 \leq \Pi_0^a < \pi^o$, the type-A potential switchers immediately start to switch when Π_0^b is greater than $\hat{\Pi}_X^b$. Otherwise, they do not switch until group B's reputation Π_t^b improves further up to $\hat{\Pi}_X^b$. From Basins of Attraction I and II summarized above, we have the following lemma.

Lemma 7 (Basin of Attraction with Switching)**.** *For any type $i \in \{A, B\}$, the following is true: under the constraints that only type i newborns can switch and the type -i newborns are restricted not* *to switch, any initial position in* $\{(\Pi_0^i, \Pi_0^{-i})|0 \leq \Pi_0^i < \pi^o, \tilde{\Pi}^{-i} \leq \Pi_0^{-i} \leq 1\}$ *will lead the low-cost type* i *potential switchers to start to join group* $-i$ *at some point of time* X. *An initial position in other areas never initiate the switching of the type* i *potential switchers in the future.*

By lifting both constraints above, we can obtain the full dynamic picture: the overlap of Panel A and Panel B of Figure 6 generates Panel A of Figure 7 (for the case that $\tilde{\Pi}^a = \pi^{o\prime\prime}_a$ and $\tilde{\Pi}^b = \pi^{o\prime\prime}_b$). First, when both Π_0^a and Π_0^b are greater than π^o, no potential switcher considers the switching (Lemma 5), and the state (Π_t^a, Π_t^b) approaches the high reputation symmetric equilibrium (Π_h, Π_h). Also when Π_0^a and Π_0^b are smaller than $\tilde{\Pi}^a$ and $\tilde{\Pi}^b$ for each, no potential switcher considers switching (Proposition 6), and the state (Π_t^a, Π_t^b) approaches the low reputation symmetric equilibrium (Π_l, Π_l). For the other initial positions, (Π_0^b, Π_0^a) belongs to either the basin of attraction for (L_b'', H_a'), which is the area satisfying $\Pi_0^b < \pi^o$ and $\Pi_0^a \geq \tilde{\Pi}^a$ or the basin of attraction for (H_b', L_a''), which is the area satisfying $\Pi_0^a < \pi^o$ and $\Pi_0^b \geq \tilde{\Pi}^b$, or to both areas. For the former case, type B potential switchers consistently join group A from the time X and the state (Π_t^b, Π_t^a) approaches (L_b'', H_a'). For the latter case, type A potential switchers consistently join group B from the time X and the state (Π_t^b, Π_t^a) approaches (H_b', L_a''). The basin of attraction for (L_b'', H_a') and that for (H_b', L_a'') are overlapped in the area: $X \equiv \{\tilde{\Pi}^b \leq \Pi_0^b < \pi^o$ and $\tilde{\Pi}^a \leq \Pi_0^a < \pi^o\}$, in which both type-A potential switchers and type-B potential switchers have an incentive to switch to the other type as long as the potential switchers of the other type do not switch. Therefore, in this case, the social consensus about the future among the whole population may determine the future state: if people believe that group A will grow as an elite group with a higher reputation, only type-B potential switchers may switch to the other type and, consequently, the future state approaches (L_b'', H_a'). If people believe that group B will grow as an elite group with a higher reputation, only type-A potential switchers may switch to the other type and, consequently, the future state approaches (H_b', L_a'').

Lemma 8 (Overlap of Basins of Attraction)**.** *In an overlapped area of the basin of attraction for* (L_b'', H_a') *and that for* (H_b', L_a'')*, the expectation about the future among the whole population determines the final state. In the areas other than the overlap, the initial state* (Π_0^b, Π_0^a) *decisively determines the final state.*

Example 1 (Expectation - Point A in Figure 7)**.** *In a point* A*, in which the initial statuses of two groups are identical in terms of the group size and the group reputation, the difference between two groups' reputations grows over time as the potential switchers of one group consistently migrate to the other group. The expectation (social consensus) about the future determines which group grows as an elite group and which one keeps on losing its reputation.*

Using Lemma 7 and Lemma 8, we can summarize the future state for each initial state (Π_0^b, Π_0^a) in the following way:

Theorem 1. *For given initial state* (Π_0^b, Π_0^a)*, the final state* $\lim_{t\to\infty}(\Pi_t^b, \Pi_t^a)$ *is*

$$\lim_{t\to\infty}(\Pi_t^b, \Pi_t^a) = \begin{cases} (\Pi_h, \Pi_h) & \text{if } (\Pi_0^b, \Pi_0^a) \in \{\pi^o \leq \Pi_0^b \leq 1, \pi^o \leq \Pi_0^a \leq 1\} \\ (\Pi_l, \Pi_l) & \text{if } (\Pi_0^b, \Pi_0^a) \in \{0 \leq \Pi_0^a < \tilde{\Pi}^a, 0 \leq \Pi_0^b < \tilde{\Pi}^b\} \\ (L_b'', H_a') & \text{if } (\Pi_0^b, \Pi_0^a) \in \{0 \leq \Pi_0^b < \pi^o, \tilde{\Pi}^a \leq \Pi_0^a \leq 1\} - X \\ (H_b', L_a'') & \text{if } (\Pi_0^b, \Pi_0^a) \in \{0 \leq \Pi_0^a < \pi^o, \tilde{\Pi}^b \leq \Pi_0^b \leq 1\} - X \\ (L_b'', H_a') \text{ or } (H_b', L_a'') & \text{if } (\Pi_0^b, \Pi_0^a) \in X, \end{cases} \tag{45}$$

in which $X \equiv \{\tilde{\Pi}^b \leq \Pi_0^b < \pi^o \text{ and } \tilde{\Pi}^a \leq \Pi_0^a < \pi^o\}$, $L_a'' = L_b'' = \frac{\Pi_l \eta}{1-\Pi_l(1-\eta)}$, $H_a' = \frac{L_a \Pi_h + L_b \Pi_l'}{L_a + L_b \Pi_l'}$ *and* $H_b' = \frac{L_b \Pi_h + L_a \Pi_l'}{L_b + L_a \Pi_l'}$.

First, note that both (Π_l, Π_h) and (Π_h, Π_l) are not stable. The newborn potential switchers in a disadvantaged group may switch their inborn types (thus incurring the cost of switching) and try to join the advantaged group to take the benefits of their superior reputation, which is often called a "passing" behavior.

Corollary 5 (Instability of (Π_l, Π_h) and (Π_h, Π_l))**.** *Both* (Π_l, Π_h) *and* (Π_h, Π_l)*, which are stable in a no-switches dynamics (Proposition 3), are not stable in a dynamic system with switches allowed.*

Example 2 ("Passing" - Points B or B' in Figure 7)**.** *Given the initial points* $B(\Pi_l, \Pi_h)$ *or* $B'(\Pi_h, \Pi_l)$ *in Figure 7, the talented newborns with the lower switching cost among a stereotyped disadvantaged population consistently pass for the advantaged group. Consequently, the reputation of the disadvantaged group becomes even worse and that of the advantaged group becomes even better.*

The above theorem implies the followings: 1) when both groups' reputations are good enough $(\Pi_0^b > \pi^o, \Pi_0^a > \pi^o)$, the two groups' reputations tend to converge to the high reputation level Π_h; 2) when both are very bad $(\Pi_0^b < \tilde{\Pi}^b, \Pi_0^a < \tilde{\Pi}^a)$, they tend to converge to the low reputation level Π_l; and 3) otherwise, the little better-off group's reputation tends to improve over time and approach a reputation even higher than Π_h, while the little worse-off group's reputation tends to deteriorate over time to a reputation level even worse than Π_l, as the potential switchers of the worse-off group consistently differentiate themselves from their own group and join the little better-off group that is expected to grow as an elite group. Let us define this behavior as "partial passing" (Loury 2002). Those who commit partial passing might be blamed by their peer members for the differentiation from them, which is often convicted as "Action White."

Corollary 6 (Divergence among Disadvantaged Population)**.** *Unless the reputations of two disadvantaged groups* ($\Pi_0^b < \pi^o, \Pi_0^a < \pi^o$) *are very bad* ($\Pi_0^b < \tilde{\Pi}^b, \Pi_0^a < \tilde{\Pi}^a$)*, the reputations of the two groups tend to diverge over time, as the potential switchers of the little worse-off group consistently migrate to the little better-off group.*

Example 3 ("Partial Passing" - Points C or C' in Figure 7)**.** *Consider two subgroups with distinguished cultural traits of the stereotyped population. Assume a small difference in their initial reputations. The talented newborns of the worse-off subgroup have an incentive to differentiate themselves from the other members of the group and join the better-off subgroup, by collectively adopting the cultural traits of the better-off subgroup. Owing to the "partial passing" activities of the talented young members of the worse-off subgroup, the reputation of the slightly better-off subgroup may improve significantly over time, as the percentage of the qualified workers among them grows continuously. This partial passing or differentiation activities can help the talented young members in the stereotyped population to be less influenced by the negative stereotype in the labor market.*

Now, let us consider two disadvantaged groups with the different group sizes. When group sizes are different, the low cost talented newborns of the bigger group have greater incentive to switch to the smaller group than the other way around, because the talented newborns' switching of the bigger group can make a significant improvement in the reputation of the smaller group, but the newborns' switching of the smaller group to the bigger group would not make enough difference. Thus, with the others being equal, the smaller group is more likely than the bigger group to become an "elite" group. Panel B of Figure 7 displays this tendency: ${\pi^{o\prime\prime}}_b > {\pi^{o\prime\prime}}_a$ when $L_a < L_b$.

Corollary 7 (Selective Out-Migration from Bigger Group to Smaller Group)**.** *Among the two disadvantaged groups A and B* ($\Pi_0^b < \pi^o, \Pi_0^a < \pi^o$)*, the smaller group is more likely to grow as an "elite" group and the bigger one is more likely to remain as a disadvantaged group in the reputation trap.*

Proof. See the proof in the appendix. ■

Example 4 (Separating from Masses - Point D in Figure 7)**.** *Consider two subgroups A and B in the stereotyped population, with the different group sizes* ($L_a < L_b$)*, which means the cultural traits of B-type are more popular a priori than those of A-type. The talented newborns of the majority subgroup B, who suffer from the group's negative stereotype in the market, may seek a way to differentiate themselves from the masses. One way to do this is to collectively join the less popular cultural group A while incurring the costs of adopting the new cultural traits. Even when the initial reputation of the minority cultural group A is worse than that of the majority group B as noted in Point D in Figure 7*

($\Pi_0^a < \Pi_0^b$), the selective out-migration of the most talented of the larger group B to the smaller group A can improve the smaller group A's reputation fast, and thus the talented newborns of group B can escape the reputation handicap in the labor market.

The feasibility of identity-switching is represented by the parameter η: the greater η is, the more newborns never consider switching due to the very high switching cost. Also, the feasibility is represented by the parameter k_l: the greater k_l is, the less affordable the switching is for the newborns who may consider switching.

Corollary 8 (Switching Feasibility)**.** *The less feasible the switching is, the less likely the divergence of group reputations arises.*

Proof. See the proof in the appendix. ■

If the social identity manipulation such as partial passing is not available, any subgroup in the stereotyped population may not recover its reputation, moving out of the reputation trap. The usage of the identity manipulation can help some disadvantaged subgroups to build up their reputations with the inflows of the most talented from other subgroups, and with their greater skill investment activities. In this sense, the identity manipulation or the usage of the cultural instrument in the labor market (Fang 2001) can improve the social efficiency. However, with the selective out-migration, other disadvantaged subgroups losing the most talented may suffer further from having the worse collective reputation, which may undermine solidarity in the disadvantaged population and cause conflicts between the subgroups (Loury 2002).

Corollary 9 (Social Efficiency)**.** *The behavior of the social identity manipulation such as partial passing may improve the social efficiency, and the usage of the observable cultural traits in the screening process may cure to some extent the social inefficiency caused by the imperfect information in the labor market.*

Proof. Suppose both Π_0^a and Π_0^a are below π^o. Without the usage of the cultural traits in the screening process in the labor market, the total size of skilled workers would be $(L_a + L_b)\Pi_l$. With the usage of the cultural traits, the final state may approach either (L_b'', H_a') or (H_b', L_a''), which means the total size of skilled workers may approach either $L_b\Pi_l + L_a\Pi_h$ or $L_a\Pi_l + L_b\Pi_h$. Both are greater than $(L_a + L_b)\Pi_l$. ■

4.3 Autonomous Emergence of an Elite Group among the Stereotyped

In this section, we show how a small elite subgroup with the unique cultural traits can emerge autonomously among a negatively stereotyped population. First, the emergence is feasible whenever there exists a sufficiently small subgroup with unique cultural traits that are expensive enough to obtain for the other members in the stereotyped population except the most talented (Panel A in Figure 8). Second, the most talented members of a stereotyped population have an incentive to create a small group with the observable distinguished cultural traits so that they can differentiate themselves from the rest in the labor market (Panel B of Figure 8).

4.3.1 Small Cultural Group's Growing as an Elite Group

We know that a symmetric initial position (Π_l, Π_l) is stable if the identity switch between the groups is not allowed in the given model (Proposition 3). Once the switch is feasible, the symmetric initial position (Π_l, Π_l) is not stable any more if the size disparity between the groups is sufficiently big, which is implied in Corollary 4:

Lemma 9. *The* (Π_0^b, Π_0^a) *is not stable at* (Π_l, Π_l)*, when either* $\tilde{L}_a > \tilde{L}_a^*$ *or* $\tilde{L}_b > \tilde{L}_b^*$*.*

Proof. According to Corollary 4, $\tilde{\Pi}^i$ is zero when $\tilde{L}_i > \tilde{L}_i^*$. According to Theorem 1, the basin of attraction for (Π_l, Π_l) is $\{0 \leq \Pi_0^a < \tilde{\Pi}^a, 0 \leq \Pi_0^b < \tilde{\Pi}^b\}$. Thus, with $\tilde{L}_i > \tilde{L}_i^*$, (Π_l, Π_l) cannot belong to the basin of attraction for (Π_l, Π_l). ■

Suppose $(\Pi_0^i, \Pi_0^j) = (\Pi_l, \Pi_l)$ at time zero. If L_i is small enough that $\tilde{L}_i (\equiv L_j/L_i)$ is greater than $\tilde{L}_i^*$, $\tilde{\Pi}^i$ is equal to zero (Corollary 4) and, consequently, $v_0^i > v^*$, which means the medium investment cost newborns with type i immediately invest in skills expecting the better group reputation and the more preferential treatment in the future. Therefore, the group i's reputation improves immediately beyond the low reputation level Π_l. The unequal reputations between two groups emerge autonomously as the talented type-j newborns join group i (Corollary 7), and the final state approaches the asymmetric stable state $(\Pi_\infty^i, \Pi_\infty^j) = (H_i', L_j'')$, as displayed in Panel A of Figure 8. Note that the instability of (Π_l, Π_l) has arisen because the most talented of the disadvantaged population have an incentive to join a small subgroup i expecting the fast reputation improvement of the group in the future with their joining the subgroup and the increased skill investment activities among the type i newborns. Thus, the above lemma implies the following proposition.

Proposition 7 (A Small Group's Growing as Elite Group)**.** *Imagine a negatively stereotyped population at the low reputation level* Π_l*, which is a stable equilibrium in the no-switches dynamics. As far as there exists a sufficiently small subgroup with unique cultural traits for which the switching cost is large* *enough that it satisfies Assumption 3, the most talented young members of the stereotyped population have an incentive to differentiate themselves from the masses joining the small cultural subgroup. The small subgroup emerges as an elite group out of the stereotyped population.*

4.3.2 Endogenous Creation of Elite Group

So far, we have assumed that groups with different cultural traits are exogenously given. Imagine a negatively stereotyped group B. If we allow that the most talented members of the stereotyped group can find proper cultural indices for differentiation and create a distinguished cultural group A by adopting specific indices, the condition in Lemma 9 is immediately satisfied because $\tilde{L}_a \approx \infty > \tilde{L}_a^*$, as far as the chosen indices are rare in nature ($L_a \approx 0$). Therefore, an elite group consisting of the most talented can emerge immediately and the size of the group will grow over time, as described in Panel B of Figure 8: the size of the created cultural group is close to zero in the beginning, but increases up to $\Pi_l' L_b$. From the beginning, the reputation of the created group is one, which means that most members of the group are skilled workers. The reputation of the stereotyped population will become even worse over time as they lose the most talented newborns of the group to the distinguished cultural group A. A real life example would be the migration of talented members of a stereotyped population to specific residential areas that are not affordable to less talented peer members. Spending money on luxury goods and designer clothing that are not affordable to other members of the stereotyped population would be another example of differentiation by the most talented. They might also commit to fine arts or send their children to a private boarding school to signal their higher social status to outsiders.

Corollary 10 (Emergence of Elite Group)**.** *If the talented young members of a stereotyped group can find proper cultural indices for differentiation, which are not affordable to other members of the group, they will form an elite subgroup based on the indices, incurring a cost to obtain them. Through this, they can immediately escape statistical discrimination practices and will be preferentially treated in the labor market.*

5 Conclusion

The externality of group reputation is important to explain the discriminatory practices by employers and the different skill investment activities across social groups. In our previous work in Kim and Loury (2008), we suggest the concept of a reputation trap by developing a dynamic model of group reputation. Once a group's collective reputation enters the range below a threshold, the group cannot escape the lower skill investment activities and the negative stereotype. A group with the reputation above the threshold can build up its reputation through young members' optimism about the future and their collective action for skill achievement.

This paper discusses the identity-switching behaviors of the most talented young members of a stereotyped population in a reputation trap, who have greater incentives to separate themselves from the stereotyped masses. We have presented three different identity-switching activities for the differentiation: passing, partial passing and elite culture development. Passing for an advantaged group would be the most efficient way for differentiation if the identity switching cost is not large. The most talented who succeed in passing can take advantage of the superior collective reputation of the group immediately. We often observe the passing activities among the stereotyped population who "fortunately" share a similar appearance with the advantaged population (eg. Korean descendants in Japan). When passing is not available, the talented members of the stereotyped population may consider the "partial" passing. They "pass" for the better-off subgroup with the unique cultural traits in order to send signals of their higher productivity to employers. The partial passing is a common activity among physically marked stereotyped people (eg. Blacks in the United States). Finally, the most talented individuals may develop distinguished cultural indices that are not affordable to the less talented members of the stereotyped population. The talented young members adopting the indices may form an elite cultural subgroup, whose members are distinguished from the rest in the population and who will be preferentially treated by employers.

Note that, in the given dynamic identity-switching model, we have simplified the composition of a population in the following way. Group members are classified into three categories: the most talented, the medium talented and the least talented. Each talent group is classified again into individuals with higher switching cost and those with lower switching cost. Making some assumptions, we argue that the most talented members with the lower switching cost are identified as the only potential switchers who may consider the identity switching. The identification of the potential switchers was an important starting point for the analysis of endogenous group formation. We were able to find the exact dynamic paths tracing the decision-process of the potential switchers. However, some researchers may generalize our findings using continuous distribution functions of skill investment costs and switching costs, without introducing the potential switchers.

As we have discussed the endogenous group formation under the existence of group reputation externality, we may develop a similar work under the existence of social network externality. Kim (2009) developed a dynamic model of group inequality through the channel of social network externalities. He suggested the concept of a network trap. Once the quality of a group's social network enters some range below a threshold, the group cannot escape the low skill investment activities due to the negative influence of the network effects. The most talented of a disadvantaged social group in the network trap may consider switching to other social groups with a better network quality. There are several practices of this kind that we may observe in the real world, such as people moving to a residential area with more members of advantaged social groups, sending their children to a private school in which descendants of the advantaged group are prevalent, and attending social clubs where they can associate with members of advantaged social groups. If switching to advantaged social groups is not possible, the most talented may build up an elite subgroup with entering barriers, separating themselves from the low quality social network. Because the most talented individuals of a disadvantaged social group have a greater incentive to separate themselves from their peers than the most talented of an advantaged social group, we might observe a more divisive culture among the disadvantaged group than among the advantaged group. Some researchers may develop a dynamic model of endogenous group formation through this channel of social network externality.

6 Appendix: Proofs

6.1 Proof of Lemma 3

The results are driven using Lemma 2. The first argument is obvious because $\frac{wP_u}{\delta+\lambda}$ is $\max\{R^i\}$ and $\frac{wP_q}{\delta+\lambda}$ is $\min\{R^i\}$, which means $c_l < \min\{R^i, R^{-i}\}$ and $c_h > \max\{R^i, R^{-i}\}$. The second argument is obvious as well because $\frac{w(1-P_q)}{\delta+\lambda}$ is $\max\{Y_q^i\}$, which means $k_h > \max\{Y_q^i\}$. By the above argument, we need to check the following two newborn sorts for the third argument: (c_m, k_l) and (c_h, k_l).

[Newborns with Cost Set (c_m, k_l)] First, let us prove that they never switch their inborn types when they have chosen to be unqualified, $(i^*, e^*)|_{i,c_m,k_l} \neq (-i, u)$. Suppose they can switch. Then, by Lemma 2, $k_l < Y_u^i \leq \frac{w(1-P_u)}{\delta+\lambda}$. However, by Assumptions 2 and 3, $c_m + k_l > \frac{w}{\delta+\lambda}$ and $\frac{wP_q}{\delta+\lambda} < c_m < \frac{wP_u}{\delta+\lambda}$, which implies $k_l > \frac{w(1-P_u)}{\delta+\lambda}$. Thus, they contradict each other. Second, let us prove that they never switch their inborn types when they have chosen to be qualified: $(i^*, e^*)|_{i,c_m,k_l} \neq (-i, q)$. Suppose they can switch. Then, by the lemma and equations (5) and (6), c_m and k_l should satisfy $c_m + k_l < R^{-i} + Y_u^i = \frac{wP_q}{\delta+\lambda} + \frac{w(1-P_q)}{\delta+\lambda}v^{-i} - \frac{w(1-P_u)}{\delta+\lambda}v^i$, which implies $c_m + k_l < \frac{w}{\delta+\lambda}$. This contradicts Assumption 3.

[Newborns with Cost Set (c_h, k_l)] First, let us prove that they never switch their inborn types when they have chosen to be unqualified: $(i^*, e^*)|_{i,c_h,k_l} \neq (-i, u)$. Suppose they can switch. Then, by Lemma 2, $k_l < Y_u^i \leq \frac{w(1-P_u)}{\delta+\lambda}$. As shown already, $k_l > \frac{w(1-P_u)}{\delta+\lambda}$ by Assumptions 2 and 3. So, there is a contradiction. Second, let us prove that they never switch their inborn types when they have chosen to be qualified: $(i^*, e^*)|_{i,c_h,k_l} \neq (-i, q)$. Suppose they can switch. Then, by the lemma, $c_h < R^{-i} \leq \frac{wP_u}{\delta+\lambda}$, which contradicts Assumption 2. QED.

6.2 Proof of Corollary 1

Replacing M_t^i with $L_i + \rho L_j \Pi_l'$ in equation (28), where $\rho \in [0, 1]$, we have

$$\begin{aligned}
\dot{\Pi}_t^i &= \frac{\lambda[(L_i\phi_t^i + L_j\Pi_l') - (L_i + L_j\Pi_l')\Pi_t^i]}{L_i + \rho L_j \Pi_l'} \\
&= \frac{\lambda[L_i(\phi_t^i - \Pi_t^i) + L_j\Pi_l'(1 - \Pi_t^i)]}{L_i + \rho L_j \Pi_l'} \\
&= \frac{\lambda[L_i(\phi_t^i - \Pi_t^i) + L_j\Pi_l'(1 - \Pi_t^i) + \rho L_j\Pi_l'(\phi_t^i - \Pi_t^i) - \rho L_j\Pi_l'(\phi_t^i - \Pi_t^i)]}{L_i + \rho L_j \Pi_l'} \\
&= \lambda(\phi_t^i - \Pi_t^i) + \frac{\lambda[L_j\Pi_l'(1 - \Pi_t^i - \rho(\phi_t^i - \Pi_t^i))]}{L_i + \rho L_j \Pi_l'} \\
&= \dot{\Pi}_t^n + \frac{\lambda L_j\Pi_l'[(1-\rho)(1 - \Pi_t^i) + \rho(1 - \phi_t^i)]}{L_i + \rho L_j \Pi_l'}
\end{aligned}$$

Since ϕ_t^i is either Π_l or Π_h, $\dot{\Pi}_t^i > \dot{\Pi}_t^n$ except when $\Pi_t^i = 1$ and $\rho = 0$. Since $\rho = 0$ when time is X, $\dot{\Pi}_t^i > \dot{\Pi}_t^n$ except when $\Pi_X^i = 1$. QED.

6.3 Proof of Lemma 5

Suppose that $\Pi_X^j \geq \pi^o$. Then, $v_X^j \geq v^*$ because the optimistic path to Q_h^n is available to group j. Then, we have the following:

$$
\begin{aligned}
Y_{q,X}^j &= \frac{w(1-P_q)}{\delta+\lambda} \cdot (v_X^i - v_X^j) \\
&\leq \frac{w(1-P_q)}{\delta+\lambda} \cdot (1 - v^*) \\
&= \frac{w}{\delta+\lambda} - c_m - \frac{w(1-P_u)}{\delta+\lambda} \cdot v^* \quad \left(\because v^* \equiv \frac{(\delta+\lambda)c_m - wP_q}{w(P_u - P_q)} \right).
\end{aligned}
$$

Thus, $Y_{q,X}^j + \frac{w(1-P_u)}{\delta+\lambda} \cdot v^* \leq \frac{w}{\delta+\lambda} - c_m$. Since $\frac{w}{\delta+\lambda} - c_m < k_l$ by Assumption 2, $Y_{q,X}^j < k_l$, which contradicts to Proposition 2 that $Y_{q,X}^j < k_l$ when type-j potential switchers switch. QED.

6.4 Proof of Corollary 3

Replacing M_t^j with $L_j - \rho L_j \Pi_l'$ in equation (37), where $\rho \in [0, 1]$, we have

$$
\begin{aligned}
\dot{\Pi}_t^j &= \frac{\lambda L_j[\Pi_l \eta - (1-\Pi_l')\Pi_t^j]}{L_j - \rho L_j \Pi_l'} \\
&= \frac{\lambda L_j[\Pi_l - \Pi_t^j + \Pi_l'(-1+\Pi_t^j)]}{L_j - \rho L_j \Pi_l'} \\
&= \frac{\lambda L_j[\Pi_l - \Pi_t^j + \Pi_l'(-1+\Pi_t^j) - \rho\Pi_l'(\Pi_l - \Pi_t^j) + \rho\Pi_l'(\Pi_l - \Pi_t^j)]}{L_j(1-\rho\Pi_l')} \\
&= \lambda(\Pi_l - \Pi_t^j) + \frac{\lambda L_j \Pi_l'[-1 + \Pi_t^j + \rho(\Pi_l - \Pi_t^j)]}{L_j(1-\rho\Pi_l')} \\
&= \dot{\Pi}_t^n + \frac{\lambda \Pi_l'[-1 + \Pi_t^j + \rho(\Pi_l - \Pi_t^j)]}{1-\rho\Pi_l'}
\end{aligned}
$$

Therefore, as far as $\Pi_t^j < 1$, $\dot{\Pi}_t^j < \dot{\Pi}_t^n$. Since $\Pi_X^j < \pi^o$ (Lemma 5) and Π_t^j monotonically approaches L_j'', $\Pi_t^j < 1$, $\forall t \in (X, \infty)$. QED.

6.5 Proof of Corollary 7

First, you may check the difference between $\pi^{o\prime\prime}_a$ and $\pi^{o\prime\prime}_b$, using equations (38) and (39):

$$
\pi^{o\prime\prime}_a - \pi^{o\prime\prime}_b = \left[\frac{L_b}{L_a} - \frac{L_a}{L_b} \right] \cdot \Pi_l(1-\Pi^*)(1 - \hat{v}_X^{i\,-\frac{\lambda}{\delta+\lambda}}) \cdot [\hat{v}_X^i / v^*]^{\frac{\lambda}{\delta+\lambda}}. \tag{46}
$$

Thus, when $L_a < L_b$, $\pi^{o\prime\prime}_a < \pi^{o\prime\prime}_b$. Also, you may check the partial derivative of $\pi^{o\prime\prime}_i$ with respect to L_i:

$$\begin{aligned}
\frac{\partial \pi^{o\prime\prime}_i}{\partial L_i} &= \frac{\partial \hat{\Pi}^i_X}{\partial \tilde{L}_i} \cdot \frac{\partial \tilde{L}_i}{\partial L_i} \cdot [\hat{v}^i_X / v^*]^{\frac{\lambda}{\delta+\lambda}} \\
&= -\Pi_l'(1-\Pi^*)(1-\hat{v}^i_X{}^{-\frac{\lambda}{\delta+\lambda}}) \cdot \frac{L_j}{L_i^2} \cdot [\hat{v}^i_X / v^*]^{\frac{\lambda}{\delta+\lambda}} > 0
\end{aligned}$$

Thus, the bigger the size of the group L_i, the greater $\pi^{o\prime\prime}_i$, that is, the smaller the basin of attraction for the attractor with H_i'. QED.

6.6 Proof of Corollary 8

The divergence occurs either in the basin of attraction for (L_b'', H_a') or in that for (H_b', L_a''). We can show that the basins tend to shrink with the greater η, or with the greater k_l, using equations (38) and (39):

$$\begin{aligned}
\frac{\partial \pi^{o\prime\prime}_i}{\partial \eta} &= \frac{\partial \hat{\Pi}^i_0}{\partial \eta} \cdot [\hat{v}^i_0 / v^*]^{\frac{\lambda}{\delta+\lambda}} \\
&= -\tilde{L}_i \Pi_l (1-\Pi^*)(1-\hat{v}^i_0{}^{-\frac{\lambda}{\delta+\lambda}}) \cdot [\hat{v}^i_0 / v^*]^{\frac{\lambda}{\delta+\lambda}} > 0 \\
\frac{\partial \pi^{o\prime\prime}_i}{\partial k_l} &= \tilde{L}_i \Pi_l'(1-\Pi^*) \cdot v^{*-\frac{\lambda}{\delta+\lambda}} \cdot \frac{\partial \hat{v}^i_0{}^{\frac{\lambda}{\delta+\lambda}}}{\partial k_l} \\
&= \tilde{L}_i \Pi_l'(1-\Pi^*) \cdot v^{*-\frac{\lambda}{\delta+\lambda}} \cdot \frac{\partial \hat{v}^i_0{}^{\frac{\lambda}{\delta+\lambda}}}{\partial k_l} > 0. \left(\because \hat{v}^i_0 = \frac{(\delta+\lambda)k_l}{w(1-P_q)}\right)
\end{aligned}$$

QED.

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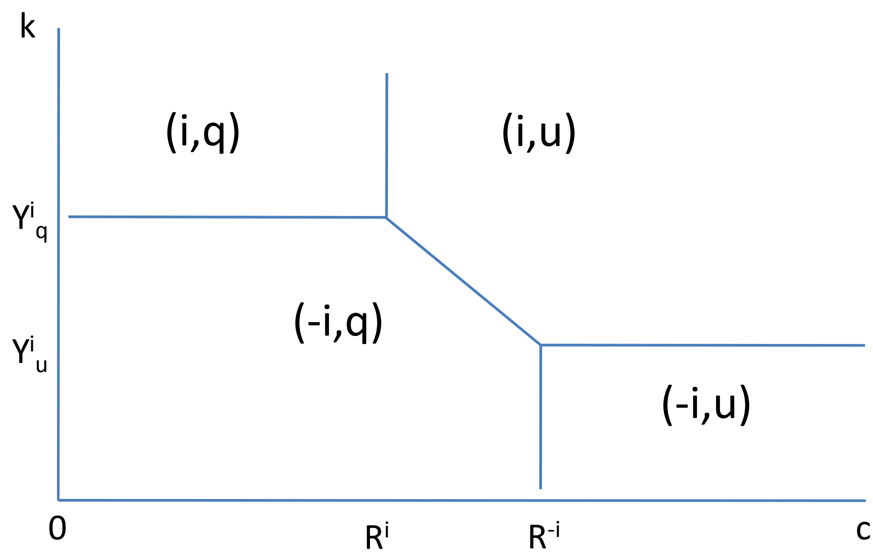
Figure 1. Costs Distribution and Newborns' Decision

Panel A. Identity and Skill Decision of Type i Newborns (given $v^{-i}>v^{i}$)



Panel B. Costs Distribution under Assumptions 2 and 3

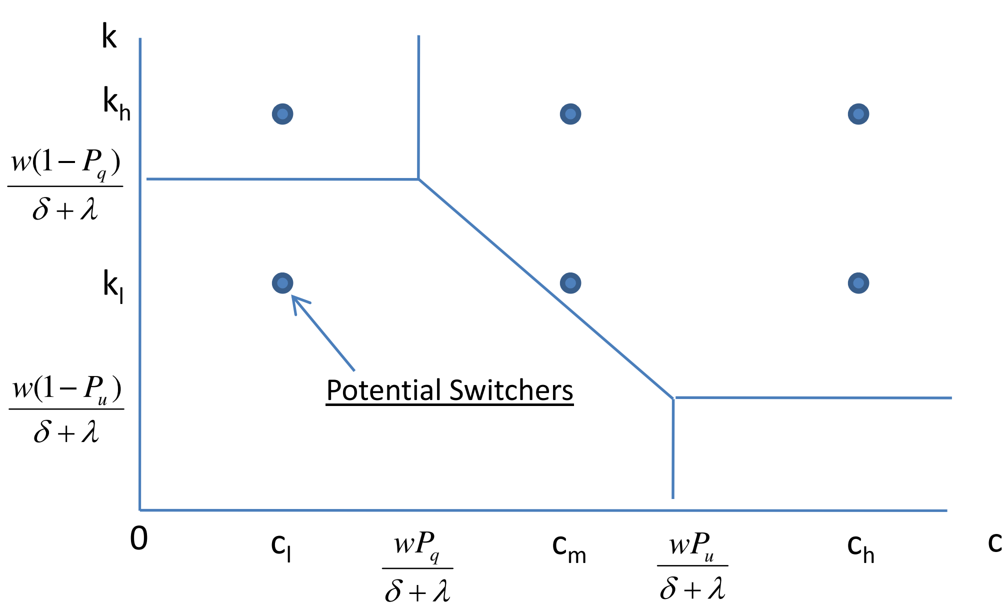
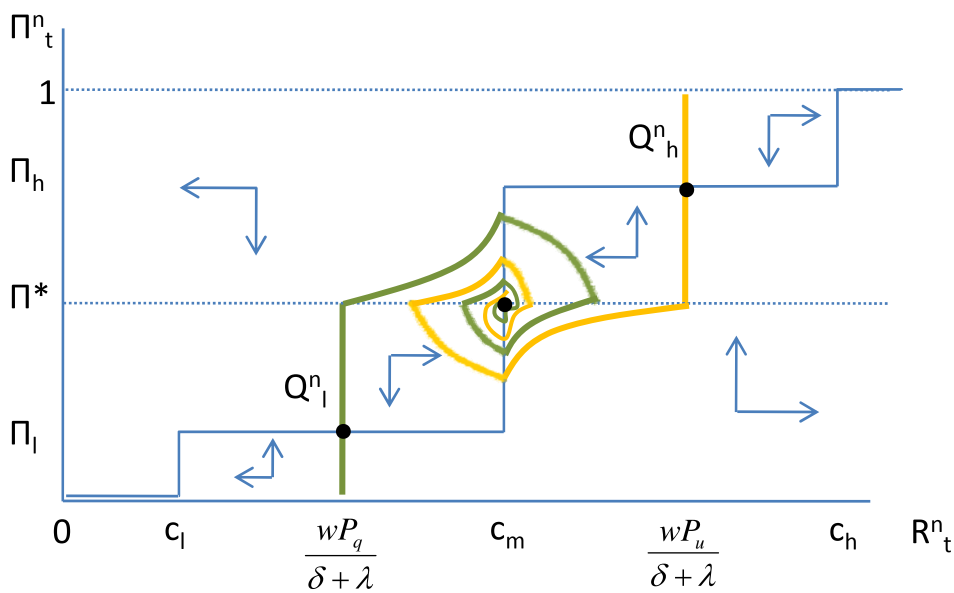


Figure 2. Dynamics with No Switches between Types

Panel A. Equilibrium Paths in (R^n_t, Π^n_t) coordinates



Panel B. Equilibrium Paths in (v^n_t, Π^n_t) coordinates

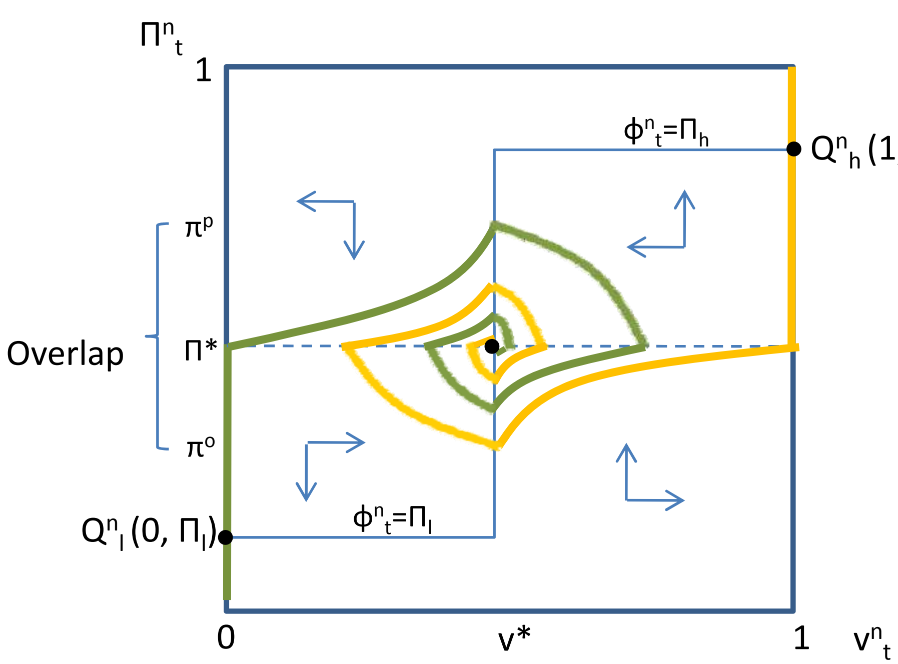
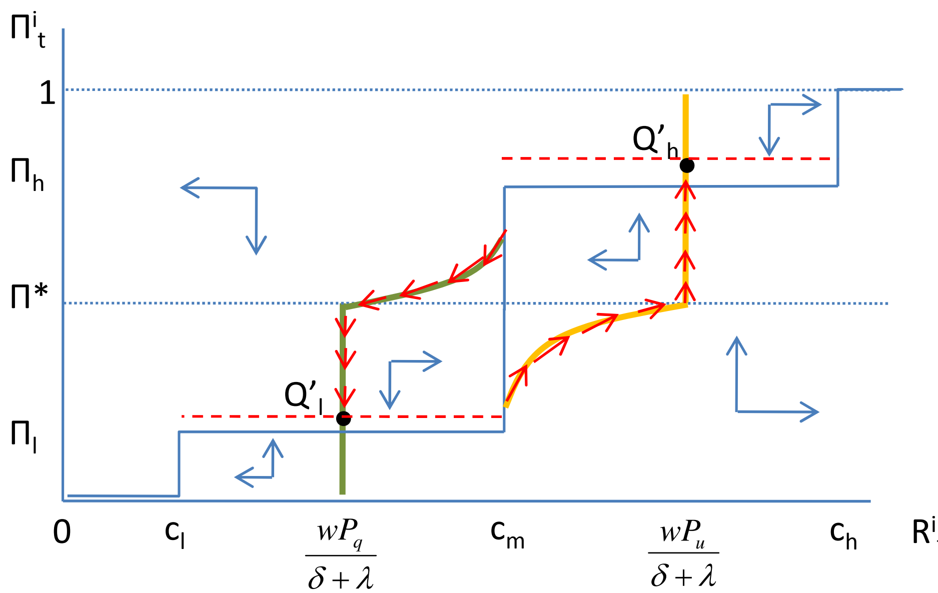


Figure 3. Dynamics of Group i with Inflows from Group j

Panel A. Equilibrium Paths in (R^i_t, Π^i_t) coordinates



Panel B. Equilibrium Paths in (v^i_t, Π^i_t) coordinates

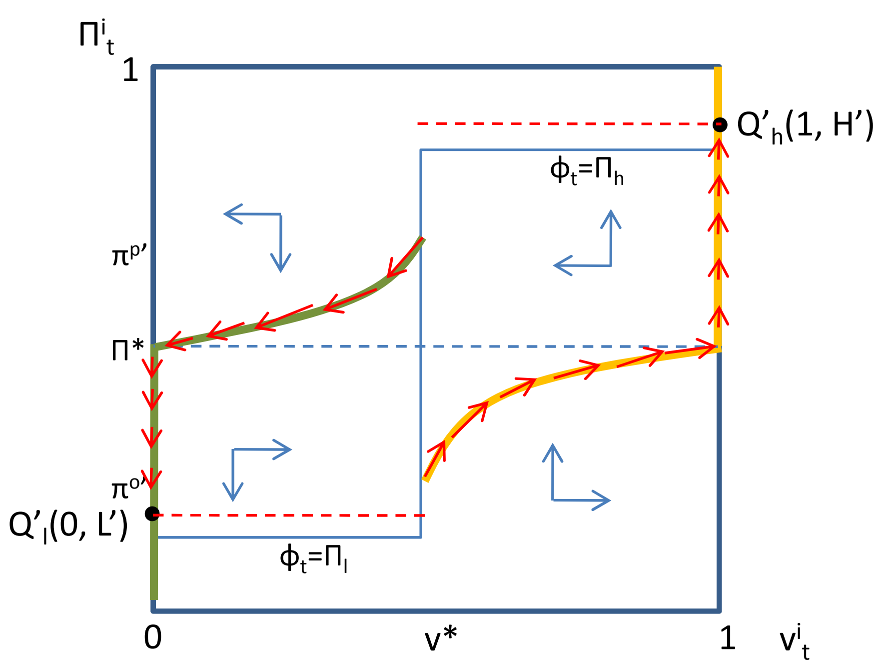
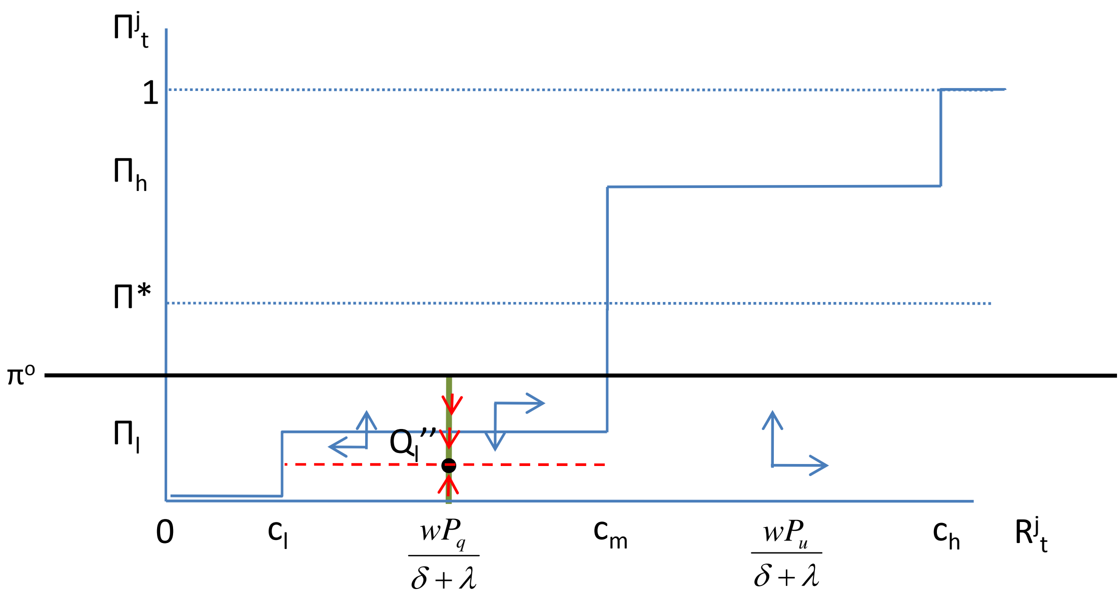


Figure 4. Dynamics of Group j with Outflows to Group i

Panel A. Equilibrium Paths in (R^j_t, Π^j_t) coordinates



Panel B. Equilibrium Paths in (v^j_t, Π^j_t) coordinates

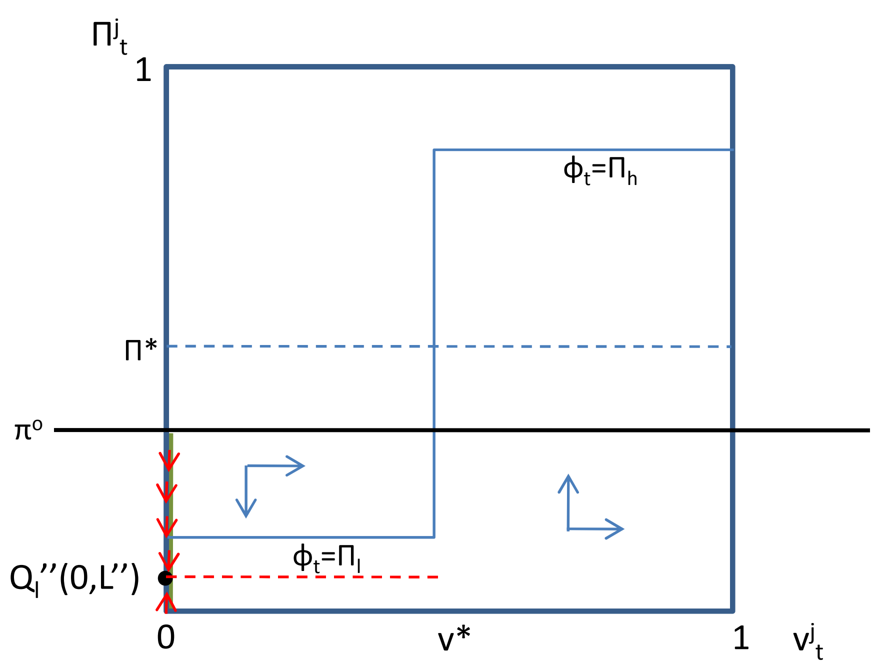


Figure 5. Group i Equilibrium Path with Inflows from Group j

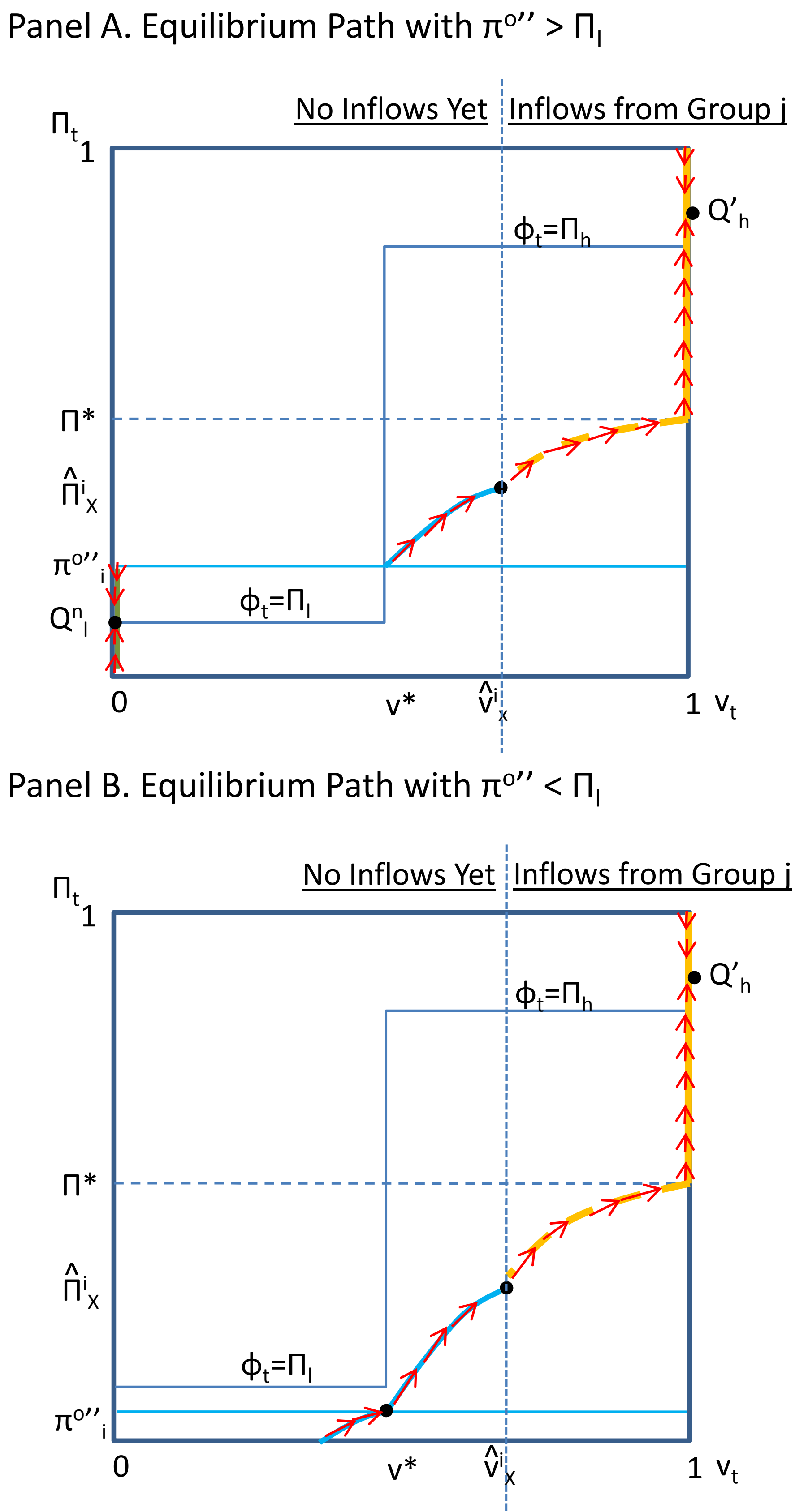
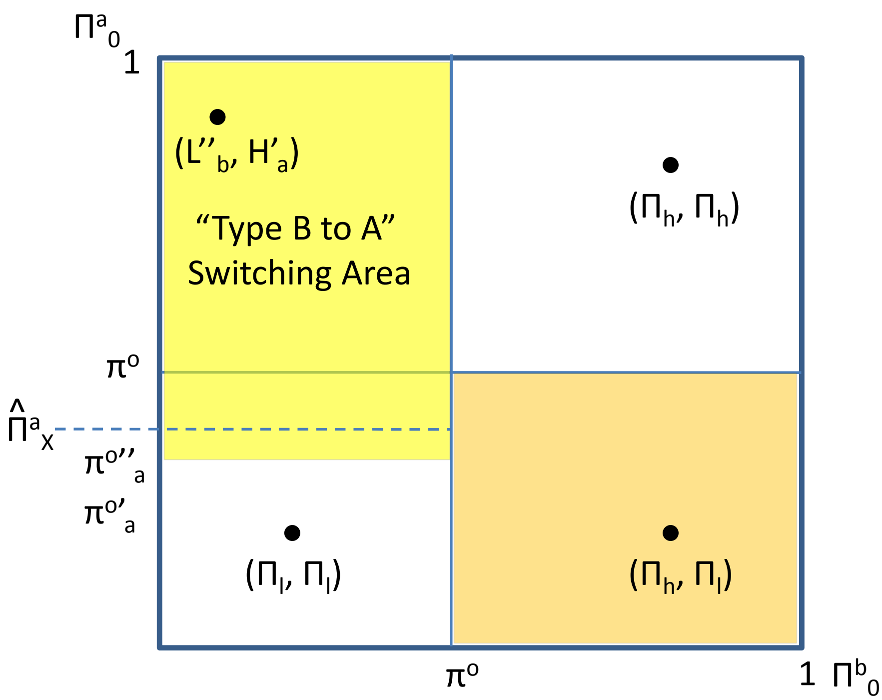


Figure 6. State Evolution From Initial State (Π^b_0, Π^a_0)

Panel A. Under Constraints that Only Type-B Newborns Switch



Panel B. Under Constraints that Only Type-A Newborns Switch

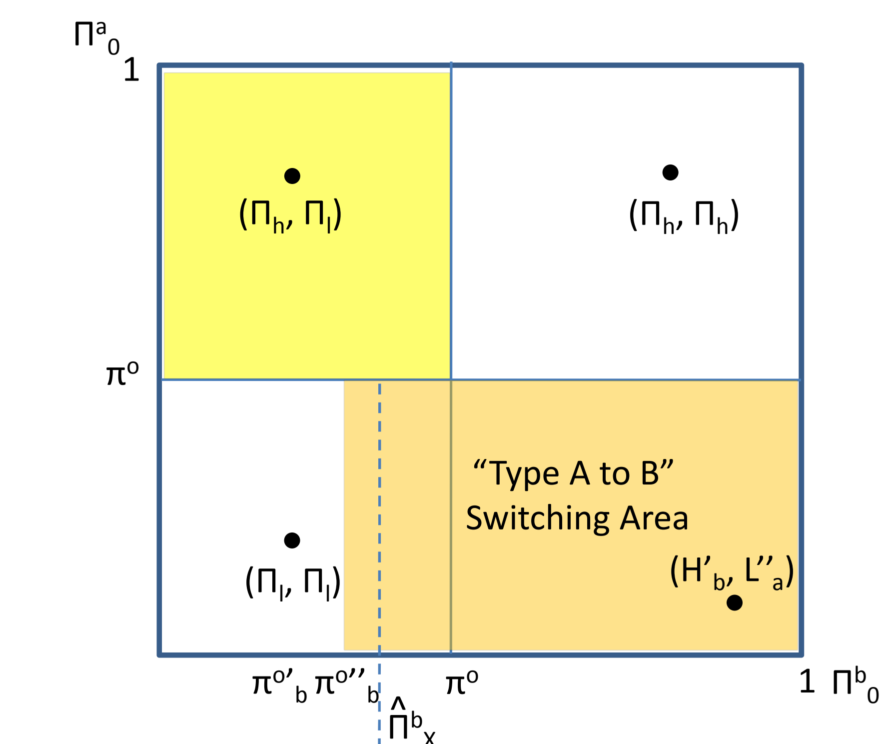
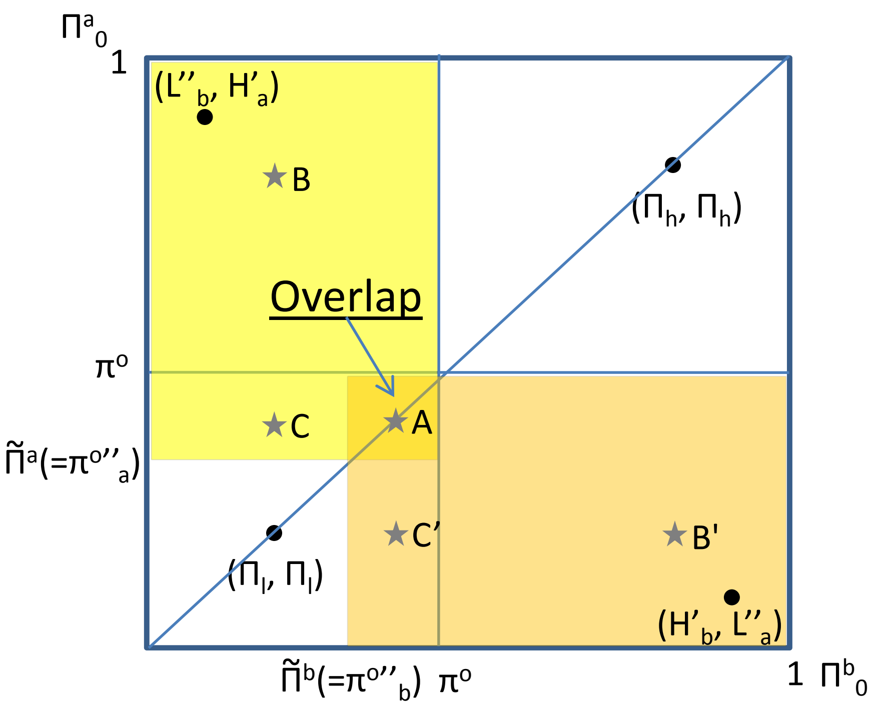


Figure 7. Endogenous Group Formation

Panel A. Skill Divergence with Equal Group Sizes (Given $L_a=L_b$)



Panel B. Skill Divergence with Unequal Group Sizes (Given $L_a<L_b$)

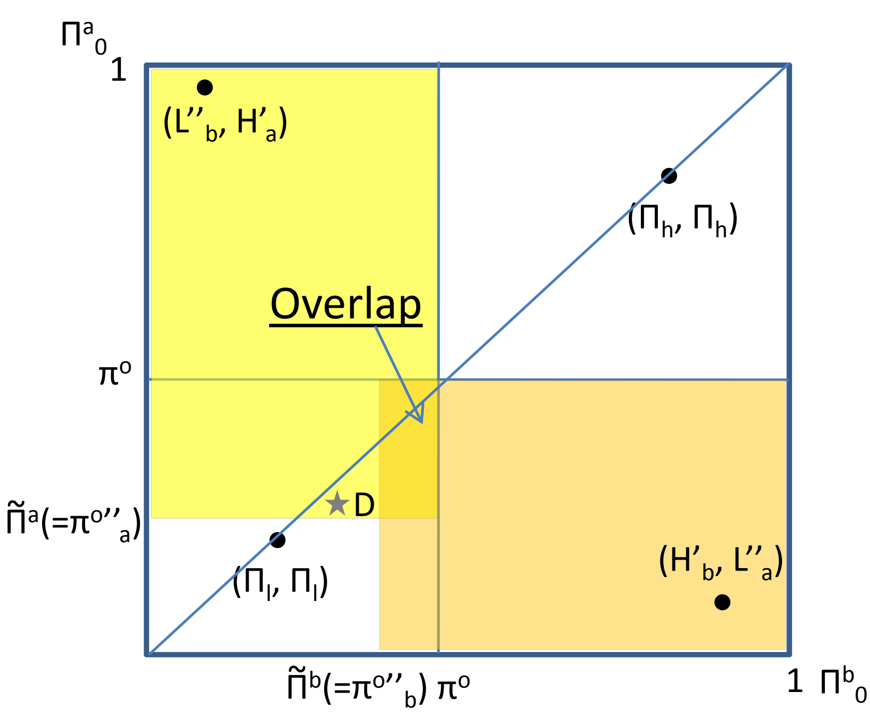
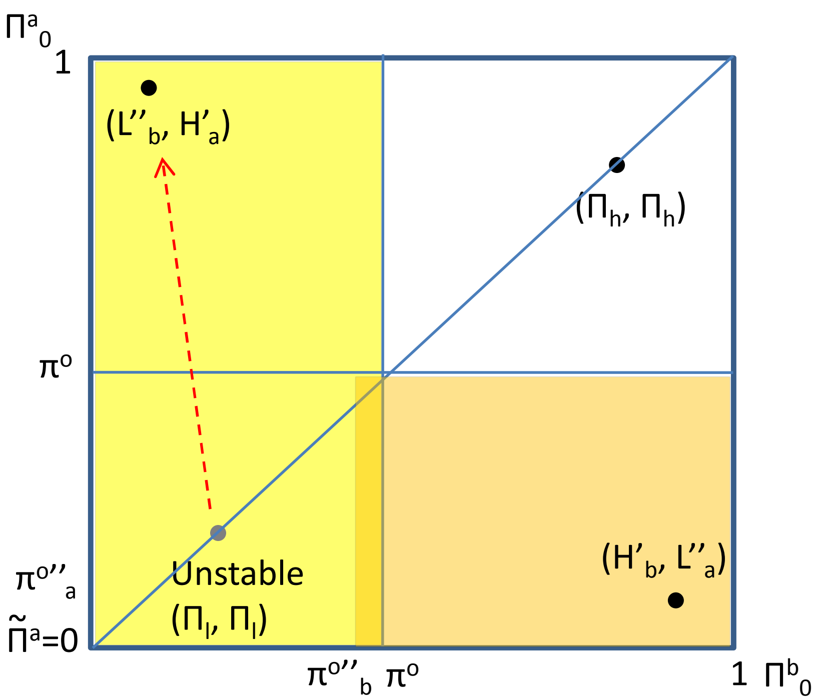


Figure 8. Autonomous Emergence of Elite Group

Panel A. Small Cultural Group's Growing as Elite Group ($L_a << L_b$)



Panel B. Endogenous Creation of Elite Group (Given $L_a \approx 0$)