Sweet Talk: A Theory of Persuasion

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Abstract

This paper introduces a model of *sweet talk* in which a seller may acquire verifiable information and selectively disclose it to a buyer to negotiate a deal. We start by analyzing a model with common priors in which the seller generates information for two reasons: a *trading* motive and a *profit* motive that is, to make trade possible or to increase the gains from it. There exists a negotiation region in which the seller continues to reveal information even if trading is already profitable. We extend the model, allowing for different prior beliefs about the value of the object, arguing that a complementarity between the seller's confidence and the precision of his information endogenously arises. Appointing an optimistic salesman may be costly because he may destroy profitable trading opportunities. We also allow the seller to choose in which market to trade: a matching market with a fixed price or a haggling market. Our model also provides a testable difference between a model of trading with homogenous priors and one with heterogeneous priors and finds application in understanding contracts as reference points.

JEL Classification:D82, D83, D86

Keywords: persuasion games, haggling, heterogeneous priors, overconfidence, consummate and perfunctory performance.

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1 Introduction

"Negotiation is a very, very delicate art. Sometimes you have to be tough, sometimes you have to be as sweet as pie." *D. Trump*

Stock issuers disclose financial statements about their earnings forecasts, and investment banks certify the value of a debt issued to investors; in general, salesmen emphasize the qualities of their product to persuade consumers to buy it. In many contexts, then, buyers rely on seller-supplied information. Rational buyers are aware that the information provided may be manipulated and especially selectively disclosed; in fact, regulators constantly monitor the information released by informed agents. The United States Securities and Exchange Commission, for example, requires public companies to disclose meaningful financial and other information to the public to provide a common pool of knowledge for all investors because "only through the steady flow of timely, comprehensive, and accurate information can people make sound investment decisions." However, the disclosure of information is crucial in many other industries, such as the pharmaceutical industry. Recently, many policy proposals have tried to change the testing and reporting requirements requested by the Food and Drug Administration for the approval of drugs. In all of these cases, after the release of new information, parties update their expectations about the object's value and begin to negotiate to reach an agreement on the terms of the contract between them. An issuer may have to lower the price of its initial public offering if negative news about the firm's future earnings become available. A pharmaceutical company may be forced to withdraw its product from the market, if new research certifies this product has some unexpected side effects.

This paper provides a novel model of negotiations through persuasion, in which the seller acquires verifiable information and strategically discloses it in order to persuade the buyer of the intrinsic value of the object, which in turn affects the trading price. It differs from the common models of bargaining used in the literature[1] for two main reasons. First, bargaining between a seller and a buyer usually takes place after the buyer has already decided to buy the object. Second, in models of bargaining, the buyer and the seller have valuations determined *ex ante* about the object, and the bargaining procedure only determines how the parties will split the surplus through the price.

[1] A review of the relevant literature is presented in the next section.

The model presented here builds instead on the theory of *persuasion games,* games in which a seller provides verifiable information to buyers to influence the actions they take, i.e., buying the object or not, but they do not explain how prices are settled[2]. Information can be "verifiable" either because buyers can directly check its accuracy or because there are institutions in place that effectively deter misleading reports by sellers. However, this theory does not explain why, in reality, we encounter many cases in which negotiations with the exchange of relevant information take place, how the information may be used to strengthen each party position and, most importantly, how this information may endogenously determine prices.

A growing literature has analyzed the transmission of strategic information between an informed party and an uninformed agent, who usually has to choose an action based upon the information revealed by the informed agent. Since Crawford and Sobel (1982), many authors have focused their attention on what is called *cheap talk*, that is, a kind of information transmission that lacks a direct linkage with the players' payoffs. This paper introduces instead what we called *sweet talk;* that is, we allow a party to acquire verifiable information and to use it to persuade an uninformed agent to take a specific action. Examples of this kind of situation are very common in reality: a seller may show to the buyer the salient characteristics of the product, trying to hide its drawbacks with respect to the competitors. A stock issuer may truthfully disclose its financial statements, but may omit to reveal other important information.

I start analyzing a model with common priors, in which the seller generates information for two different reasons. First, if the prior about the object's value is low, the seller discloses information in order to persuade the buyer that the object is worth buying; that is, she has a *trading motive* to acquire information. Second, even if the buyer's belief about the object makes trade possible, the seller may acquire information to increase the price at which she sells the good; that is, she has a *profit motive*. We assume that the seller strategically discloses information to the buyer when she is able to acquire it. The signal is non-falsifiable: she cannot lie, but she can decide to withhold the evidence. Once the buyer observes the signal, she can decide to trade or to wait for more information.

Based on this model, I address the following questions:

(1) What is the seller's optimal disclosure strategy?

[2] This is the reason why, to avoid any confusion with bargaining models, I will use the terms *negotiations, haggling, sweet talk* and *persuasion* interchangeably.

(2) Under what conditions does the seller prefer to negotiate rather than trade as soon as it becomes profitable?

(3) Is overconfidence a valuable asset in the market? That is, is it better to have an optimistic salesman, who has a higher incentive to produce information to persuade the buyer or a salesman with a prior belief about the quality of the good closer to the buyer's?

(4) Under what conditions does the seller decide to sell the good at a fixed price, without allowing for information acquisition?

(5) What are the implications in terms of optimal contracts?

I show that there exists a unique threshold equilibrium in which trading occurs only if the signal is above a cutoff. However, the seller may disclose his information even when the signal she gets is not accurate enough to convince the buyer to trade; that is, she will disclose it if she believes that it is at least high enough to induce the buyer to continue the negotiation. Moreover, on the equilibrium path the seller always tries to acquire information about the good.

I extend the model by allowing the buyer and the seller to have different prior beliefs about the value of the object. Even the simplest model highlights the importance of negotiation and information transmission to reach an agreement. I show that there exists a negotiation region in which the seller provides information to the buyer to increase the object's value in his eyes. Characterizing the equilibrium with heterogeneous priors gives us the ability to analyze the role of overconfidence. An overconfident seller truly believes that the information she is going to acquire will turn out to show that the product has higher value than what the buyer thinks, and as a result, she engages in too much haggling. That is, a seller with beliefs closer to those held by the buyer tends to trade as soon as she is able to show that the joint surplus from trading is positive, while for an overconfident seller, the profit motive leads him to waste possible profitable trading opportunities.

I then turn to a more general setting, in which I allow the seller to choose in which market to sell the object, a matching market, in which she offers the object at a fixed price to a randomly matched buyer, or in a "haggling market," where she might persuade the buyer. I argue that in equilibrium, only very confident sellers are willing to haggle with the buyers. The trade-off is clear; while posting a fixed price without the possibility of sweet talk avoids any pointless negotiation, it may also eliminate some trading opportunities.

The framework and the results presented so far have a variety of applications; I present one. Following the recent analysis by Hart and Moore (2008) and Hart (2009), in which they show that in many cases the buyer has to face an additional "shading" cost to ensure a consummate performance by the seller, I provide an alternative based upon "persuasion costs." I identify a trade-off between a flexible contract and a rigid one, due to the fact that only according to the first one may negotiation take place ex post. It is a trade off similar to those studied by the cited papers but does not employ any behavioral assumption. Moreover, in the case in which the parties' beliefs are far from each other, we argue that the flexibility or the incompleteness of the contract should be reduced.

Finally, the model also provides a testable difference between a model of trading with homogenous prior beliefs and one with heterogeneous priors. In fact, while the bargaining stage of the game ends up yielding the same outcome, the difference relies in the information acquisition stage.

The paper is structured as follows. Section II is devoted to presenting the directions in which we depart from the existing literature. Section III introduces the base model and discusses the main assumptions. Section IV characterizes the equilibrium in the case with heterogeneous priors analyzing the value of being overconfident and identifies the optimal seller's decision between posting a fixed price in a matching market and participating in an haggling market. Section V applies the model to two recent strands of the literature, while Section VI concludes and illustrates avenues for future research.

2 Related Literature

This paper spans and borrows from several literatures. We examine the connections to each of them.

Persuasion games. Since Grossman (1981) and Milgrom (1981), an influential strand of the literature has analyzed settings in which an informed party can manage the disclosure of information, which cannot be misrepresented but only hidden. Subsequent papers, such as Milgrom and Roberts (1986), Farrell (1986), Lipman and Seppi (1995) and Shin (1994), have provided cases in which the receiver is able to discount fully the reports of the senders so as to completely reveal that party's type. This is the so-called unraveling argument, or skeptical equilibrium, as defined by Milgrom and Roberts (1986). Shin (1994) generalizes the previous models, showing that even when the sender is not perfectly informed about the state, she will follow a "sanitization strategy," in which the good states are revealed while the bad realization of the signals is suppressed. More recently, Glazer and Rubinstein (2004, 2006) have characterized the mechanism that minimizes the probability of a mistake by the decision maker. That is, they study a mechanism that maximizes the probability that the listener accepts the sender's request when it is justified and rejects the request when it is unjustified, given that the speaker maximizes the probability that his request is accepted. Milgrom (2008) observes that Glazer and Rubinstein (2006) characterize the equilibrium where the buyer's decision is binary and highlights that "There is, as yet, no extension of that model that endogenizes prices." One of the methodological contributions of the paper is precisely to study price determination in a persuasion game. This paper shares with Shin (2003) the interest on the effects of information on prices; in fact, Shin (2003) provides an interesting treatment of how the selective reporting of information of the sort considered here affects security price dynamics. Milgrom (2008) is an excellent review of the works on persuasion games, and it highlights the main difference between cheap talk and persuasion, namely, the possibility for the seller to reveal information selectively without lying. Our model departs from these models in that the buyers do not have a binary choice, i.e., to buy or not to buy, but the price at which trade occurs is influenced by sweet talk. Notice also that the buyer's skeptical strategy, which is central in the literature supporting an unraveling argument, has little traction here. In our model, the buyer correctly believes that the seller selectively discloses his information, but she does not have any new information with positive probability. This means that in the literature there exists only a trading motive and not a profit motive for disclosing information. Finally, our model is also related to Caillaud and Tirole (2007) for their analysis on building consensus; they adopt a mechanism design approach to explore the strategies that a sponsor of a proposal may employ to convince a group to approve the proposal. Finally, the biggest departure from the persuasion game literature comes from the observation that the negotiation between a seller and a buyer is usually not only about the possibility of trading but on which *price* to trade.

Bargaining. Shavell (1994) consider a seller-buyer relationship where each party may acquire information about the value of the good; however, this information does not always have social value; that is, it does not increase value. In contrast, I suppose that the seller acquires information about the good's value and that both parties revise their valuations based on the information disclosed. In our setting, we also identify the conditions under which haggling with the buyer can result in a more efficient trading relationship than does posting a price with no information communication. The trade-off between different selling mechanisms has already been analyzed by Wang (1993), Peters and Severinov (1997), Kultti (1999), among others, but we stress the persuasion costs as a key element of this decision.

Heterogeneous priors. In Section IV, I drop the common prior assumption to analyze how different types of sellers behave as a function of their priors and to determine whether overoptimistic or impartial sellers have the greater incentive to acquire information. Recently, many papers have questioned the use of the common prior approach, using a different framework (Morris 1994, 1997; Yildiz 2003, 2004; Harris Raviv 1993; Hong and Stein 2007). This paper is also related to the strand of the literature on belief formation, such as Benabou and Tirole (2006) or Benabou (2008). The communication game presented here builds upon the model provided by Che and Kartik (2009). They study a context in which a decision maker and an adviser have different prior beliefs over the state of the world but where the adviser can acquire "hard" evidence. They show that the decision maker chooses an adviser with at least some difference of opinion, in order to motivate him to acquire information. Our model departs from Che and Kartik (2009) since we introduce, for the first time, the possibility of trading in a persuasion game.

Organizational strand. I claim that this model may find applications to common problems in organizations, such as settings in which the agent may decide to adhere to the spirit, if not the letter of the contract, i.e., consummate versus perfunctory performance. For example, in an organization, the principal authority may make it prohibitively costly for the agent to completely renege on the terms of the contract but may allow the agent to perform the assigned task in different ways. Our model is then related to Hart and Moore (2008), which analyzes the optimal contracts when the contract itself provides a reference point for a trading relationship, that is, a flexible contract can be very costly when the parties withhold some part of consummate performance if they feel shortchanged. They say: "If the buyer prefers a and the seller b, the buyer may have to spend time persuading the seller of the reasonableness of the choice a in order to ensure consummate performance by the seller. These persuasion costs are a plausible alternative to the shading costs we have focused on. Modeling persuasion costs is not easy, but it is an interesting topic for future research." Our paper is the first to provide a model for these persuasion costs, allowing me to highlight the drawbacks associated with the seller's persuasion motive. There are cases, in fact in which the seller, in a sense, is held up and is forced to provide much more information, bearing the related cost, than what efficiency requires.

3 The Base Model

A buyer (he) and a seller (she) meet to trade a single, indivisible good. The good's value for the seller is commonly known to be zero, while its value for the buyer is $v \in \{-1, +1\}$. In the base model, the two parties have a common prior about v, namely, $\Pr(v = 1) = q_0$.

There are two periods of negotiations, each involving information acquisition, disclosure, and possibly trade. We analyze a two-period model; otherwise, if the surplus is positive, the parties will be induced to trade at the end of period 1, lacking the ability to conduct any further negotiation.

In each period, the seller can first exert *unobservable* effort to acquire information about the value of the good. If effort is exerted, the seller obtains a verifiable signal $\sigma \in \{-1, +1\}$ with probability $\rho \in (0, 1)$. This signal agrees with the true value of the good with probability $p \in (1/2, 1)$, that is, $\Pr(\sigma = v|v) = p$. Even though the signal's value is verifiable, its existence is not: the seller can conceal the signal (report $\sigma = \emptyset$), and conversely, she cannot prove that she did not obtain a signal.

If the seller obtains a signal, she can decide whether or not to disclose it to the buyer. Following this, the seller makes a *take-it-or-leave-it* price offer to the buyer. If the offer is accepted by the buyer, then the parties trade and the game is over. The buyer's ex-post payoff is $v - x$, while the seller's is x, where x is the price offer accepted by the buyer.

If the offer is rejected in period 1, then play continues in period 2. In period 2, rejecting the offer yields zero payoffs to both parties. We assume that both parties are risk neutral and have infinitesimal discounting; that is, they prefer positive payoffs earlier, but, for simplicity, we do not formally introduce a discount factor. The timing is summarized in Figure 1.

In the base model we also assume that the cost of the seller's effort to acquire information is arbitrarily small and positive (i.e. infinitesimal). As a result, the seller acquires information whenever (in equilibrium) she is strictly better off by doing so, and she does not acquire information in the case that she is indifferent.

The other significant simplification in the base model is that we assume that the seller makes a take-it-or-leave-it offer in the bargaining phase of each period. Since the buyer has no private information, this assumption implies that the buyer does not get any of the social surplus generated by trade. We will argue at the end of this Section why this is a less stronger assumption than what it appears.

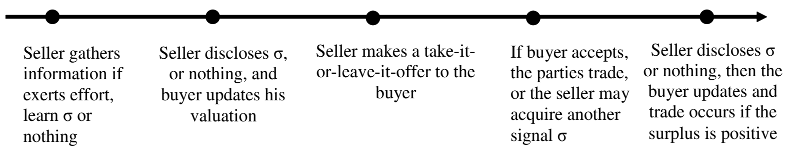

Figure 1: : Timeline

3.1 Analysis of the Base Model

We solve the game backwards.

3.1.1 Continuation equilibrium in the persuasion phase

Assume that we enter the last period so that the buyer believes that the good's value is $v = 1$ with probability q_2. We show that there is a unique continuation equilibrium given q_2 at $t = 2$ irrespective of the values of all the other endogenous variables.

Denote d_2 the seller's belief (probability assessment) at the beginning of period 2 that by exerting effort she gets a positive signal. For example, if the seller and the buyer have symmetric information about the good's value at the beginning of period 2, then $d_2 = \rho\left(pq_2 + (1-p)(1-q_2)\right)$. However, note that the seller may have private information about the true value of v at the beginning of period 2.

If the seller exerts effort and gets a positive signal, then she reveals it, as this is the last period and a positive signal can only help selling the good at a higher price. By an analogous argument, if the seller gets a negative signal, then she conceals it. If she gets no signal (either because she did not exert any effort or because she did but did not get anything), then there is nothing to disclose.

Irrespective of the seller's expected behavior in period 2, if she discloses a positive signal at $t = 2$, then the buyer's updated belief regarding $v = 1$ becomes $q_2^+ = q^+(q_2)$, where

$$q^+(q) = \frac{pq}{pq + (1-p)(1-q)} \tag{1}$$

If the seller is expected to exert effort, then the buyer's belief upon not being shown a signal is $q_2^\emptyset = q^\emptyset(q_2)$, where

$$q^\emptyset(q) = \frac{q(1-\rho) + \rho q(1-p)}{(1-\rho) + \rho\left[(1-q)p + q(1-p)\right]}$$

If the seller is not expected to exert effort and no signal is disclosed, then the buyer's belief that $v = 1$ is $q_2^{\emptyset} = q_2$ After the information acquisition and disclosure phase, at the end of period 2, the seller offers the good at the price $\max\left\{2q_2^{+} - 1, 0\right\}$ in the case that she disclosed a positive signal, and at $\max\left\{2q_2^{\emptyset} - 1, 0\right\}$ in case that she did not. Note that the price she can get in the case of no disclosure depends on whether or not she is expected to exert effort in period 2 (that is, it depends on the continuation equilibrium). In what follows, we write $\max\{A, 0\} = (A)_{+}$.

In period 2, given a continuation equilibrium (that is, given whether or not the seller is expected to exert effort), the seller exerts effort if, and only if,

$$\left(2q_2^{\emptyset} - 1\right)_{+} < d_2\left(2q_2^{+} - 1\right)_{+} + (1 - d_2)\left(2q_2^{\emptyset} - 1\right)_{+}$$

that is, if and only if

$$\left(2q_2^{\emptyset} - 1\right)_{+} < \left(2q_2^{+} - 1\right)_{+} \tag{2}$$

Proposition 1 *For every* $q_2 \in (0, 1)$, *there exists a unique continuation equilibrium in period 2, such that:*

If $q_2 \leq 1 - p$, *then the seller does not exert effort in period 2; there is no disclosure, there is no profitable trade, and the seller's profit (social surplus) is zero.*

If $q_2 > 1 - p$, *then the seller exerts effort in period 2. If she obtains a positive signal, then she discloses it, and trade takes place at a price* $2q_2^{+} - 1 > 0$, *which also equals the expected social surplus and the seller's profit. If she obtains a negative signal or no signal at all, then there is no disclosure; profitable trade takes place at price* $2q_2^{\emptyset} - 1$ *provided that it is positive.*

Profitable trade occurs in period 2 as $q_2 \to 1$, *or* $p \to 1$, *or both.*

Note that the seller's expected profit at the beginning of period 2 is positive and equals $d_2\left(2q_2^{+} - 1\right)_{+} + (1 - d_2)\left(2q_2^{\emptyset} - 1\right)_{+}$ if, and only if, $q_2 > 1 - p$. Otherwise the seller's expected profit is zero.

Proof. Recall that the seller prefers to exert effort in period 2 if and only if condition (2) holds. Suppose that in equilibrium the seller is not expected to exert effort in period 2. Then condition (2) becomes $\left(2q_2^{\emptyset} - 1\right)_{+} < \left(2q_2^{+}(q_2) - 1\right)_{+}$ as $q_2^{\emptyset} = q_2$ and $q_2^{+} = q_2^{+}(q_2)$, which must fail to hold in a no-effort continuation equilibrium. Since $q < q^{+}(q)$ for all $q \in (0, 1)$, condition (2) fails if and only if $q^{+}(q_2) \leq 1/2$. By equation (1), this is equivalent to $q_2 \leq 1 - p$. There is no profitable trade and no surplus, as $2q_2^+ - 1 < 0$ by $p > 1/2$. This proves the existence of an equilibrium with no effort and no profitable trade if, and only if, $q_2 \leq 1 - p$. Suppose that in equilibrium the seller is expected to exert effort in period 2. In such an equilibrium condition (2) must hold with $q_2^{\emptyset} = q^{\emptyset}(q_2)$ and $q_2^+ = q^+(q_2)$. Since $q^{\emptyset}(q) < q < q^+(q_2)$ for all $q \in (0,1)$, condition (2) holds if, and only if, $q_2^+(q_2) > 1/2$. By equation (1), this is equivalent to $q_2 > 1 - p$. Therefore, an equilibrium with effort in period 2 exists if, and only if, $q_2 > 1 - p$. This completes the proof of the Proposition. ■

When the seller possesses very precise information, she tends to be more willing in providing information because the buyer knows she is more reliable. This leads to an upward revision, which may induce parties to trade.

3.2 Equilibrium play in period 1

We look for an equilibrium in period 1 knowing that there is a unique continuation equilibrium (as a function of the buyer's beliefs) in period 2, as described in Proposition 1.

The first useful result is stated in the following lemma:

Lemma 1 *There is no equilibrium in the game where the seller does not exert effort at $t = 1$ but where a trade occurs either at $t = 1$ or $t = 2$ at a positive price with positive probability.*

Proof. The proof is in the Appendix. ■

The previous lemma is very intuitive: if the seller is so pessimistic in the first place such that she does not gather information then surely there will not be any trade at period 2, when the buyer has already negatively updated his information because $\sigma = \emptyset$. Moreover, if the seller is confident enough to exert effort, it is optimal to do it at $t = 1$, with no delay. We now define a region of parameters in which we have no trade and no provision of effort.

Proposition 2 *Define $\underline{q}$ such that $q^+(\underline{q}) = 1 - p$. If $q_1 \leq \underline{q}$, then the unique equilibrium involves no effort and no profitable trade in either period.*

Proof. First note that there is no equilibrium in which the seller only exerts effort in period 2 as that would imply there is a profitable sale in period 2 with positive probability contradicting Lemma 1. Hence, if there is an equilibrium with no effort in period 1 (implying no effort and no trade in either period), then $q^+(q_1) \leq 1 - p$. Conversely, if $q^+(q_1) \leq 1 - p$, then it is not worth it for the seller to exert effort in period 1, because even if she obtains and discloses a positive signal, she cannot make a profitable trade right away (as $1-p<1/2$), nor can she get a positive payoff in the continuation (as no effort and no trade are expected with $q_2 \equiv q^+(q_1) \leq 1-p$). The threshold defined by $q^+\left(\underline{q}\right)=1-p$ is given by $\underline{q} \equiv \frac{(1-p)^2}{p^2+(1-p)^2}$. ■

Now we let $q_1 > \underline{q}$; then, in any equilibrium, the seller exerts effort at $t=1$.

Suppose that the seller gets a positive signal in period 1 and discloses it so that both parties believe that $v=1$ with probability $q^+(q_1)$. The seller can either trade right away at the (fair) price $2q^+(q_1)-1$, or trigger a continuation by asking for a higher price. Clearly, if $q^+(q_1) \in (1-p, 1/2]$, then the seller cannot sell the good at a positive price in period 1, and so she prefers to negotiate further (as negotiation yields a positive expected profit). Interestingly, even when there exists a profitable trading opportunity, i.e., $q^+(q) > 1/2$, the seller may decide to negotiate.

For all $q^+(q_1) = q_1 > 1/2$, define the seller's payoff-difference between trading at $t=1$ and negotiating,

$$\Delta(q) = 2q - 1 - \left[d(q)\left(2q^+ - 1\right)_+ + (1-d(q))\left(2q^{\emptyset} - 1\right)_+\right]$$

where $d(q) \equiv \rho(pq + (1-p)(1-q))$ is the expected probability that the seller obtains a positive signal by exerting effort in period 2. In deciding to negotiate, the seller trades off a sure payoff now for an expected increase in payoff tomorrow. If the seller is very confident of getting a positive signal, then negotiating may be the optimal strategy, even when trading at $t=1$ would yield positive payoff.

Lemma 2 *If $\rho < 1$ and $q^{\emptyset}(q) < 1/2$, there exists a unique threshold $\overline{q} \in (1/2, 1)$ such that the seller who discloses a positive signal in period 1 prefers trading in period 1 rather than continuing negotiation if and only if $q^+(q_1) \geq \overline{q}$.*

Proof. The proof is in the Appendix. ■

The previous lemma has identified the seller's optimal decision rule between trading and negotiating when she gets a positive signal. Intuitively, when it would be profitable to trade even if the seller does not disclose, then immediate trade is always better. If, instead, not disclosing could trigger no trade from the buyer, the seller prefers to negotiate. There exists a trading region, a negotiating region and a no-effort one as depicted in Figure 2. The uniqueness of the equilibrium follows from the fact that effort is unobservable. That is, if the buyer expects the seller to exert effort, then it is optimal for the seller to do so. On the other hand, if the buyer does not expect the seller to acquire any new information, the seller has a profitable deviation strategy in acquiring

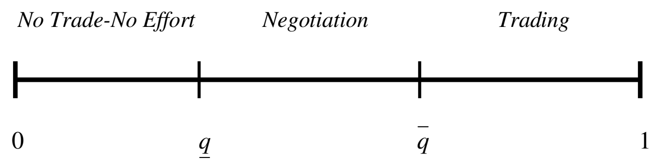

Figure 2: : Threshold Equilibrium

it, as he can always disclose it only if it is positive. This eliminates the other possible equilibria. Next, we should investigate what happens when the seller gets a negative or no signal in period 1.

Suppose the seller exerts effort, that is $q_1 > \underline{q}$, there are two cases. First, if she gets no signal, she will try again in period 2, as trade was not profitable in period 1, and then there will not be any trade after the information acquisition attempt, either. Then, the analysis follows Proposition 1. Second, if the seller gets a negative signal, she will not disclose it and will thus have private information about the object's value. The seller downward revises the probability that by exerting effort, she gets a positive signal to $d'(q_s) \equiv E[\sigma_2 = 1|\sigma_1 = -1]$, which is $d'(q_s) < d(q)$. The expected profit is now $d'(q_s)\left(2q_2^+ - 1\right)_+ + (1 - d'(q_s))\left(2q_2^\emptyset - 1\right)_+$, where the prices at which she could sell the object are computed using the buyer's information (the buyer already takes into account the fact that the seller may have gotten a negative signal when she does not disclose). Since $d'(q_s)$ does not matter in the decision of whether to acquire information, she will behave as in the symmetric information case. The only difference in the private information case is that the expected profit for the seller might be negative, due to a re-weighting, which has decreased the expected profit. However, since it is strictly increasing in q_s, there will exist a threshold for the seller's belief $\widehat{q}_s$ to start with, such that getting a negative signal at $t = 1$ has not negatively impacted his decision of exerting effort at $t = 2$, i.e., $d'(q_s)\left(2q_2^+ - 1\right)_+ + (1 - d'(q_s))\left(2q_2^\emptyset - 1\right)_+ > 0$, as long as $q_s > \widehat{q}_s$. This means that her expected profit will be positive if she does not discount too much the probability of getting a positive signal in the period after observing a negative one in the past. The repeated feature of the model and the possibility that the seller will withhold information, endogenously, generates a difference in priors at the beginning of period 2.

The difference between immediate trade and continuing in the persuasion phase $\Delta(q, q_s)$ is strictly positive when $q_2^{\emptyset} > 1/2$, while when $q_2^{\emptyset} < 1/2$ we have that $\Delta(q, q_s^*) = 0$ identifies as before a cutoff, which is now function of the buyer's valuation. Then, the following proposition:

Proposition 3 *If the seller exerts in the first period, there exists a unique equilibrium of this subgame in which:*

If the seller gets a positive signal in period 1, there is immediate trade if and only if $q^+(q_1) \geq \overline{q}$*; otherwise, she engages in the negotiation.*

If the seller gets no signal in period 1, she will exert effort in period 2 if and only if $q_1 > 1 - p$*.*

If the seller gets a negative signal and $q_s < q_s^*$*, there is no disclosure and no profitable trade, and the seller's profit is zero. If* $q_s > q^*$*, instead, there is immediate trade if and only if* $q^+(q_1) \geq \overline{q}_s$*; otherwise she engages in the negotiation.*

Proof. The proof is in the Appendix. ■

Now the question is if the second threshold $\overline{q}$ will be reached and under what conditions this will be the case. It depends on how informative the signals that the seller shows to the buyer are. When his arguments are not so informative, i.e., when p is low, she is not able to shift the buyer's posterior until $\overline{q}$; that is, she will not be able to sell his good at $t = 1$. However, if she produces incontrovertible evidence, where p is close to 1, she will be able to convince the buyer immediately and then sell the good without further negotiation. Notice that the two cutoffs are endogenous and that they are a function of the accuracy of the seller's information. When p is close to $1/2$, that is, his arguments are not convincing, we have $\underline{q} \to 1/2$; that is she has a lower incentive to acquire information in the first place. On the other hand, when p is close to one, we have $\underline{q} \to 0$; this means that she will almost surely exert effort but that it will be harder to reach the trading region as $\overline{q}$ is closer to 1[3]. This suggests that when the seller's arguments are really informative she has a greater incentive to negotiate due to the higher impact his information has on the buyer's beliefs. Given the previous discussion, note the following:

Remark 1 *Negotiations happen only if* $p >> 1/2$*.*

When the seller's arguments are uninformative, the buyer is not willing to listen, and trading is thus determined only based upon the prior information about the object value, i.e., $\underline{q} \approx \overline{q}$.

[3] Precisely, the limit of $\overline{q}$ as p increases is $\frac{1}{2-\rho}$.

To complete the equilibrium characterization, we need to analyze the seller's incentives to exert effort in the first period as a function of the reliability of his information. That is, we compute the expected gains from gathering information as a function of p.

3.2.1 Vague Evidence: $p \approx \frac{1}{2}$

In the analysis above we have supposed that the seller is willing to exert effort in the first period, but this is true only if the prior is greater than a threshold q_0^v which should be less than $\overline{q}$ where the subscript stands for *vague* evidence. Given the results above, we know that if the seller ever puts in effort she will do that in the first period; moreover, she will never decide to trade the good in the current period without providing evidence, because she will not be able to reach the *trade* region. We can define his continuation payoff:

$$\pi_s(q) = \begin{cases} d(q)(2q^+ - 1)_+ + (1 - d(q))\left(2q^{\emptyset} - 1\right)_+ & if\ q_1^+\left(q^0\right) > \underline{q} \\ 0 & otherwise \end{cases}$$

The intuition is that q_0^v must be such that if the seller gets a positive signal, she updates the posterior until $\underline{q}$; otherwise, his continuation payoff will be zero anyway. In fact, if in the second period $q < \underline{q}$, she will not exert any effort, and this will lead to zero payoff.

The following lemma formalizes this argument:

Lemma 3 *The seller puts in effort in the first period only if* $q > q_0^v \in \left(0, \underline{q}\right)$.

Proof. The proof is in the Appendix. ■

Intuitively, if the seller has only vague arguments to convince the buyer then she has lower upside value from acquiring information; however, if she wants to keep the buyer in the relationship, $q > \underline{q}$, she will exert effort. This is one of the expressions of the *trading motive.*

3.2.2 Persuasive Arguments: $p \approx 1$

In this case the seller has the opportunity to convince the buyer showing him just one piece of evidence, because the buyer knows that her information is reliable. By Proposition 1, we have a unique equilibrium and we can find a new threshold for the prior as:

$$q_+\left(q_0^p\right) = \overline{q}$$

The existence and uniqueness follows directly from the continuity of q_1^+ in q_0 and from the uniqueness of the "*trading*" threshold. We find that $q_0^p \equiv \frac{(1-p)(1-(1-p)\rho)}{2p\rho - 2p^2\rho - \rho} > q_0^v$, this means that if the common initial belief about the object is high enough, the seller may ends up trading it in the first period, while if the prior is low enough, she will haggle with the buyer in order to reach a positive price, but she could also not succeed in persuading him.

Then, the equilibrium is summarized in the following proposition.

Proposition 4
- *The seller prefers to negotiate with the buyer rather than trade at the current valuation if p is low. Then, she puts in effort in the first period only if $q^0 > q_0^v$, while in the second period she will acquire more information if $q > 1 - p$. If she gets a positive signal she will sell the good at price:*

$$x^2 = \begin{cases} \left(2q_2^+ - 1\right)_+ & \text{if } \sigma = 1 \\ \left(2q_2^\emptyset - 1\right)_+ & \text{if } \sigma \in \{\emptyset, -1\} \end{cases}$$

- *The seller will instead prefer to trade at the current price if p is close enough to* 1, *and she will exert effort in the first period if $q^0 > q_0^p$ and she will trade in the second period at prices $x^1\left(q^1\right)$.*

Proof. Follows from the previous Lemmas and the discussion in the text. ■

3.3 Discussion

In this section, we discuss some of the assumptions made above.

Small cost of acquiring or process information. Assuming that the seller may acquire information at a positive but arbitrarily small cost allows us to sharply characterize the equilibrium in the second period with a single cutoff strategy. At the same time, introducing a positive cost $c_s > 0$ opens the question of what is the efficient amount of information that the seller should acquire. This normative observation will drive some of the insights in the case of heterogeneous priors. Since for the buyer, acquiring information is costless, he would induce the seller to gather as much information as possible, as the buyer enjoys its option value without bearing its cost. However, a positive cost c would make the analysis much more obscure. We introduce it when we discuss the benefits of having an overconfident seller.

We could also consider a positive cost c_b for the buyers to process the information that is revealed. This is the case, for example, when they have to verify the accuracy of the information transmitted or to check its congruence. Although the nature of the equilibrium will be preserved, it would change the thresholds, which will now depend on that. If the information revealed is very difficult to process, represented by an increase in c_b, then the buyer will be willing to listen to the seller only if p and ρ are higher than before; otherwise, he will find it optimal to walk away.

Finite horizon. We carried over our analysis in a three-period model with almost no loss of generality. In fact, as it is true that the qualitative results will be robust to this kind of generalization, after T periods of negotiations between the parties, the posterior could approximate 1, which would induce the seller to no longer acquire information. Suppose that after T periods, the seller has disclosed s times a positive signal about the value of the good. Therefore, we have

$$\Pr\left(v=1 \mid \sum_{i=1}^{T} 1\{\sigma_i=1\}=s\right)=\frac{\binom{T}{s}(\rho p)^s(1-\rho p)^{T-s}q}{\binom{T}{s}(\rho p)^s(1-\rho p)^{T-s}q+\binom{T}{s}(\rho(1-p))^s(1-\rho(1-p))^{t-s}(1-q)}$$

which approximates 1 as $s\to\infty$. Even in this case, his optimal strategy would be to exert effort only for intermediate values of the posterior belief and then to trade when it is close enough to 1.

Bargaining. We have restricted our attention to a setting in which the seller makes a *take-it-or-leave-it* offer to the buyer. However, we could extend the model to a different bargaining procedure. Consider the case in which the seller has disclosed his signals in both periods. Here, assuming a common discount factor δ, sequential bargaining would lead the parties to split the surplus equally with a price $x_i(q^+)=(q_i^+-\frac{1}{2})_+$. However, since the seller may withhold his signal in the first period, she could in principle possesses private information if $\sigma_1=-1$. In this case, the seller becomes more *impatient,* as she believes that the next period she will receive with higher probability a negative signal. She then has incentives to close the deal sooner, charging a lower price. In this scenario the parties have different priors, at the beginning of period 2, about the object value, q_s and q_b. Note that for the seller the value of having the object is zero and the buyer's posterior is commonly known between the parties. The bargaining stage of the game is thus not affected. They split the *buyer's* surplus, if it is positive[4]. The reason is that the difference in priors, Δq, matters only in the acquisition stage of the game, due to the seller's assessment of how likely it is to get a positive signal. This has two implications. First, the equilibrium characterized in the previous section is robust to different bargaining procedures. Second, this suggests a testable difference between a model with homogenous priors and one with heterogeneous ones: the information acquisition stage, not the bargaining outcome.

[4] Note that this game is different from the bargaining game with differing priors considered by Yildiz (2004), because he assumes different priors on how likely each party will make an offer and not on the object value.

Homogeneous priors. We have shown that even in a model with homogeneous priors the seller and the buyer engage in a relationship with information transmission in order to generate surplus to split at the bargaining stage of the game. The following section analyzes our framework in the case in which the buyer and the seller have different priors about the object.

4 Persuasion with Heterogeneous Priors

We now extend the model to the case in which the seller and the buyer have different priors. Although the equilibrium takes the same form, this section will allow us to study a number of related issues.

4.1 The Model

We assume that the prior beliefs of each agent are common knowledge and $\Pr(v = 1) = q_i^0$, for $i = s, b$. There is a significant and growing literature that analyzes games with heterogeneous priors. Spector (2000) and Banerjee and Somanathan (2001) do so in communication models with exogenous information; examples in other contexts are Harrington (1993), Yildiz (2003), Van den Steen (2005), and Eliaz and Spiegler (2006). Our model is different from the model of bargaining with heterogeneous priors proposed, for example, by Yildiz (2004). Our model shows, in fact, that the seller is only more impatient than the buyer depending on a sequence of signals about the value of the good; that is, she is more impatient only when the buyer is more excited about the value of the good.

Since the priors are commonly known, it is key to note that the bargaining stage of the game is not changed, as already noted for the common prior setting. At the time of trading the seller knows the buyer's valuation of the good and then the seller can appropriate, as in the homogeneous case, all of the surplus. We can assume that $q_s^0 > q_b^0$, that is, that the seller has a trading incentive to start with to persuade the buyer.

Second period. Let us begin analyzing what happens in the second period. They will both have two different posteriors about the value of the object q_s^1 and q_b^1 coming from the previous period even if they have observed the same signal. Moreover, if the seller did not disclose his negative signal in the first period, she has private information about the value of the object. At this stage, the seller has to decide whether or not to exert effort. The posteriors are commonly known only if she discloses his signal in the first period, then if $q_b^1 < q_s^1$ there is no surplus to appropriate, in fact, due to an *adverse selection effect* the price offered will never be accepted by the buyer. Hence, the seller will certainly exert effort due to the trading motive. Let us define as before:

$$d_s^t\left(q_s^t\right) = \rho\left(pq_s^t + (1-p)\left(1-q_s^t\right)\right)$$

Interestingly, even with heterogeneous priors, the seller may decide to acquire more information in the case in which $q_b^1 > q_s^1$ if the following condition holds:

$$\left(2q_{\emptyset b}^2 - 1\right)_+ < d_s^1\left(q_s^1\right)\left(2q_{+b}^2 - 1\right)_+ + \left(1 - d_s^1\left(q_s^1\right)\right)\left(2q_{\emptyset b}^2 - 1\right)_+ . \tag{3}$$

The intuition is that when q_s^1 is big enough and his signal is accurate enough, high ρ , she truly believes that she will end up getting a positive signal, increasing in this way her expected payoff. Consequently, there now is also a *profit motive.* There is no surplus from trading if the buyer's posterior is lower than one half; the trading price that matters is simply the buyer's posterior. However, the probabilities used by the seller to take the expected value of putting in effort are his own posteriors about the value of the good. In fact, in the second period, the seller's posterior may differ from that of the buyer in the case where she decides not to disclose his negative signal. We have that condition (3) defines a unique threshold as shown by the following lemma:

Lemma 4 *The seller puts in effort if and only if* $q_b^1 > q_{b*}^1$.

Proof. Equation (3) gives us the following condition:

$$\frac{pq_{b*}^1}{pq_{*b}^1 + (1-p)\left(1-q_{*b}^1\right)} > \frac{1}{2}$$

which can be solved for $q_{b*}^1 = (1-p)$ as in Lemma 1. ■

Intuitively, the previous lemma shows that the seller will be more willing to provide evidence to convince the buyer of the value of the good as his signal's informativeness increases.

First period. I can define the seller's payoff-difference between trading at $t = 1$ and negotiating, by

$$\Delta(q) = 2q_b - 1 - \left[d\left(q_s\right)\left(2q_b^+ - 1\right)_+ + \left(1 - d\left(q_s\right)\right)\left(2q_b^\emptyset - 1\right)_+\right] \tag{4}$$

where the only difference from the homogeneous case is given by a different probability of getting a positive signal, $d\left(q_s\right)$, which can be now a function of the seller's private information. In the case in which she discloses his signal in the first period, we are back to the homogeneous priors case, and we can define the same threshold $\overline{q}_s = \overline{q}_b \in (1/2, 1)$ such that the seller who discloses a positive signal in period 1 prefers trading in period 1 rather than continuing negotiation if and only if $q^+(q_1) \geq \overline{q}_i$ for $i = s, b$. Let us suppose that the seller withheld his signal in period 1. In this case, she possesses private information, and the condition $\Delta(q) = 0$ can be rewritten as:

$$d(q_s) = \frac{q_b - q_b^{\emptyset}}{q_b^+ - q_b^{\emptyset}} \tag{5}$$

which belongs to the unit interval, as $q_b^+ > q_b > q_b^{\emptyset}$. We then have the following proposition:

Proposition 5 *There exists a unique threshold equilibrium in which:*

(i) *If the seller gets a positive signal in period 1, she discloses it and there is immediate trade if and only if* $q^+(q_1) \geq \overline{q}$*; otherwise, she engages in the negotiation.*

(ii) *If the seller gets no signal in period 1, she will exert effort in period 2 if and only if* $q_2 > 1-p$.

(iii) *If the seller gets a negative signal, there is immediate trade if and only if* $q^+(q_1) \geq \overline{q}_s$*; otherwise she engages in the negotiation.*

Proof. Points (i) and (ii) are equivalent to what happens in the homogeneous priors case. Condition (5) defines a unique threshold $\overline{q}_s$. It follows from the fact that $d(q_s)$ is a continuous and strictly increasing function of q_s. The right-hand side of (5) belongs to the unit interval as $d(q_s)$. ■

Intuitively, condition (5) sorts the sellers according to their level of confidence and their willingness to negotiate with the buyer. Notice that the RHS is an increasing function of the buyer's posterior. Then, when the buyer is already enthusiastic about the value of the good the seller should sell it without any further negotiation. In the case of heterogeneous beliefs, we are able to write the seller's belief as a function of the buyer's valuation. We can establish that more confident sellers negotiate more and moreover that they will be less inclined to delay trading when they correctly anticipate that not disclosing the signal may trigger a decline from the buyer, i.e., when $q_b^{\emptyset} < 1/2$, fewer types of sellers will negotiate. The analysis of the first period effort choice is analogous to the homogeneous priors case.

4.2 Is Overconfidence Valuable?

Although the equilibrium structure is similar to the case of common priors, the heterogeneity in the prior beliefs gives us a natural way to answer a related and relevant question: does an overconfident seller always perform better?[5] Intuition tells us that a very confident salesman may play a key role in the negotiations, as she will be more willing to acquire information, which will enhance his ability to persuade the buyer. However, if the acquisition of information is costly, it may be optimal to choose a less confident seller, especially, when this information is imprecise. This means that there exists an interesting endogenous *complementarity* between competence and confidence in this model; a higher q_s increases the expected profit when it is associated with higher p, i.e., the higher accuracy of the seller's information.

To address this question, consider a slightly modified setting in which there is a positive cost for acquiring information $c > 0$, which is not paid by the seller but by a principal who owns the good. The owner has to appoint a salesman, using the seller's level of confidence captured by his prior q_s[6]. To focus only on the interesting case in which there is a possible efficiency loss in overinvesting in persuasion, suppose that the parties have already negotiated, such that $q_b > q_s > 1/2$, and let us compute the seller's type that maximizes the principal expected payoff[7].

Define the following cumulative distribution function:

$$\Phi\left(q_s\right) \equiv \Pr\left(\Delta\left(q_s\right) < t\right)$$

we can then denote the probability that q_s belongs to the set A, where it is defined as $A \equiv \{q_s : \Delta\left(q_s\right) > 0\}$, by $1 - \Phi\left(\widetilde{q}_s\right)$ where $\widetilde{q}_s$ is the cutoff at which $\Delta\left(q_s\right)$ becomes positive. The principal expected profits as a function of the seller's belief are given by

$$Max_{q_s}\pi = \left[1 - \Phi\left(\widetilde{q}_s\right)\right] q_b + \Phi\left(\widetilde{q}_s\right)\left[d\left(q_s\right) q_b^{+} + \left(1 - d\left(q_s\right)\right)\left(2q_b^{\emptyset} - 1\right)_{+} - c\right]$$

we take expectation over the set $A \equiv \{q_s : \Delta\left(q_s\right) > 0\}$, that is the set of beliefs such that the seller is not willing to negotiate. The first term is the expected profit in the case of a seller who immediately trades with the buyer, as there is a positive surplus from trading. The second term is the expected payoff when the seller is confident enough to acquire new information and to persuade the seller at cost c for the principal. To simplify notation and make the intuition clearer, let us

[5] For an excellent review on psychological literature on overconfidence see Odean (1998) and references therein. For empirical evidence on overconfidence in financial markets see Barber and Odean (2001), Glaser and Weber (2003), and Statman, Thorley, and Vorkink (2003), among many others.

[6] To make the result sharper, I assume that there is no exogenous cost in appointing a more optimistic salesman. Introducing, for example, a convex cost $c\left(q_s\right)$, with $c' > 0$ and $c'' > 0$, would induce an interior solution for every range of the parameters.

[7] We abstract from the salesman's optimal payoff function. This is equivalent to assuming that he gets paid a fixed share of the generated income. This is a very commonly used contract in the sales industry.

define $\delta \equiv \Pr\left(q_b^{\emptyset} > 1/2\right)$ which is independent of q_s, and the density function of the seller's type by $\phi(q_s)$. The first order condition is given by[8]

$$\phi\left(d(q_s)\, q_b^{+} - q_b\right) + \Phi(\widetilde{q}_s)\left(d'(q_b^{+} - \delta q_b^{\emptyset})\right) = c\Phi(\widetilde{q}_s) - \phi\left[(1 - d(q_s))\,\delta q_b^{\emptyset} - c\right] \tag{6}$$

where on the left-hand side we have the benefits of having a more confident seller and on the right-hand side we have the associated costs. Equation (6) has an intuitive interpretation. The first term represents the increased charged price in the case in which the seller shows a positive signal to the buyer. The second term is the benefit deriving from a *positive* signal, which is conditional on having chosen a confident seller, i.e., $q_s \in A$. On the right-hand side, we have the fixed cost of engaging in negotiation with the buyer and, more interestingly, the second term is the cost associated with selling the good at a lower price $q_b^{\emptyset}$, if no information is revealed, which can well be lower than the initial buyer's belief q_b. The following proposition identifies the condition under which we have an interior q_s^* :

Proposition 6 *In the case of costless information,* $c = 0$, *equation (6) has a corner solution at* $q_s = 1$. *For intermediate values of the parameter, and when c is small enough, we can find an interior solution. Moreover:*

- *For given* ρ, *when* $p \to \frac{1}{2}$ *we might have an interior solution only if*

$$q_b < \frac{\rho}{2} q_b^{+} + \left(1 - \frac{\rho}{2}\right) q_b^{\emptyset} \delta \tag{7}$$

- *For given* p, *when* $\rho \to 1$ *we have that* $\delta \to 0$ *then an interior solution, if any, solves*

$$c(\Phi(q_s) + \phi) = \phi\left(d(q_s)\, q_b^{+} - q_b\right) + \Phi(q_s)\, d' q_b^{+}$$

Proof. The proof is in the Appendix. ■

Before interpreting the condition used in the proposition, we can also state the following comparative statics result

Lemma 5 *The profit function* $\pi^s(q_s, p)$ *has increasing differences in* q_s *and* p, *that is* $\pi^s(q_s', p) - \pi^s(q_s, p)$ *with* $q_s' > q_s$ *is non decreasing in* p.

[8] We have dropped the dependence of $d'(q_s)$ on q_s because $d(q_s)$ is a linear function.

Proof. Since the profit function is differentiable, we can easily derive the first-order condition (6) with respect to p, and show that $\frac{\partial \pi^s}{\partial q_s \partial p} \geq 0$. ■

The supermodularity of the profit function ensures that the value of hiring a seller who is very enthusiastic about the value of the good is increasing in his ability or expertise, in providing accurate information. This result points out that a new dimension, the *reliability* of the information disclosed, rather than the willingness to acquire it, may be key. The proposition also has an interesting interpretation. First, picking a perfectly confident salesman is optimal only when acquiring information has no cost, as in this case, the principal benefits come from the option value of raising the trading price. However, in reality there are many costs associated with gathering and transmitting information, i.e., writing an informative prospectus, running quality tests and spending time in presenting the results. Second, when the signal that the seller may get is not very informative, indicated by a value for p close to one half, it is optimal to have a "buyer-minded" seller if condition (7) fails to hold, i.e., there is a corner solution with q_s equal to its lower bound. The condition ensures that the price that the seller may charge without gathering any information is lower than the expected price in the case of information provision. This is, for example, the case of those salesmen who are able to negotiate better just through sweet talking without really conveying any useful information to the buyers. Third, when the probability of receiving useful information about the quality of the good is very high, the buyer heavily discounts the reservation price when he does not receive any communication from the seller. In many contexts, the buyer is aware that a simple investigation, such as testing the good, would reveal important information about its quality, and this induces him to be suspicious about a seller who claims that the test is not feasible or that it did not reveal anything.

An overconfident salesman will acquire information without internalizing the exogenous cost c and, more importantly, will underestimate the probability of missing some profitable opportunities.

4.3 Matching Market vs Negotiating

We now investigate whether the seller may do better allocating the price in a matching market, without allowing for any persuasion.

Matching market. Suppose that potential buyers arrive according to a Poisson process[9]. The

[9] We have explored other matching functions, which do not affect the qualitative results. Even allowing for auctions if more than one buyer shows up, only marginally increases the expected profits of a seller who enters in a matching market, without changing the main insights. Details are available upon request.

rate of arrival is λ; thus, the probability of exactly k potential buyers arriving within an interval of time of length t is given by

$$P_k(t) = \frac{(\lambda t)^k}{k!} e^{-\lambda t} \; k = 0, 1, 2...$$

The value of the object for the potential buyer i is assumed to be drawn independently according to the differentiable cumulative distribution function F on the support $\left[\underline{q}_b, \overline{q}_b\right]$ with probability density function $f(q_b) > 0$ and $\underline{q}_b > 0$. To post the price, the seller incurs a cost of displaying at rate θ until an arriving buyer agrees to pay the posted price. Since the expected length of time to sell the object is $1/\lambda [1 - F(p)]$, the price charged by the seller is then

$$p^* = \arg\max_p \; \pi^s(p) \equiv p - \frac{\theta}{\lambda [1 - F(p)]}$$

assuming that the second order condition is satisfied, i.e., $\frac{d^2 \pi^s(p)}{dp^2} \leq 0$, the expected profit for the optimal p^* may be rewritten as

$$\pi^s(p^*) = p^* - \frac{1 - F(p^*)}{f(p^*)}$$

then it does not depend on the prior seller's belief q_s.

Haggling market. The seller will decide to negotiate with the buyer depending on the pair (q_s, q_b). Let us now take q_s as given so that we can analyze the optimal seller's strategy. The seller will acquire information only if $q_b < \widehat{q}_b$, where the cutoff is optimally determined and $\widehat{q}_b \geq q_s$, as shown in Proposition 5. The expected profit of the seller is given by

$$\pi^s(q_s) = \int_{\underline{q}_b}^{\overline{q}_b} \{ I_{\{q_b > \widehat{q}_b\}} q_b + I_{\{q_b < \widehat{q}_b\}} [d(q_s)(2q_b^+ - 1)_+ + (1 - d(q_s))(2q_b^\emptyset - 1)_+] \} dF(z) \tag{8}$$

where we can characterize the optimal threshold $\widehat{q}_b$ with the following lemma.

Lemma 6 *The persuasion cutoff $\widehat{q}_b(q_s)$ increases with the seller's prior belief q_s.*

Proof. The cutoff is defined as $\widehat{q}_b(q_s) = \arg\max \pi^s(q_s)$. Then as $\frac{d\pi^s(q_s)}{dq_s} > 0$, we can apply the envelope theorem to prove that $\widehat{q}_b(q_s)$ is an increasing function of q_s. ■

At this point, we can rewrite (8), employing the fact that $I_{\{q_b > \widehat{q}_b\}} = 1 - F(\widehat{q}_b)$. The seller will prefer to post a fixed price only if

$$\pi^s(p^*) - \pi^s(q_s) > 0$$

or, more precisely,

Proposition 7 *There exists a belief q_s^h such that for every $q_s < q_s^h$ the seller posts a fixed price; otherwise, she tries to persuade the buyer. The range of parameters for which haggling is the best option is increasing in p and ρ.*

Proof. The proof is in the Appendix. ■

The previous proposition provides a testable implication of our theory; the more confident the seller is, the more likely it should be to observe him selling the good in bilateral trade rather than in an anonymous random-matching market. The reason is that in the latter, it is difficult to persuade the buyer of the value of the good. This may be one of the reasons why goods whose value cannot be objectively measured, such as antiques, fashion clothing and used cars, are rarely sold through anonymous random-matching mechanisms, in contrast to, for example, books and high-tech products usually sold online.

5 Application

In this section, we explore the role played by persuasion costs in explaining when writing a flexible contract may not be optimal.

5.1 Contracts and Persuasion Costs

A buyer and a seller meet at date 0 and can trade at date 1. We assume that there is symmetric information throughout and that the parties are risk neutral and face no wealth constraints. I consider the case where the buyer wants one unit of an indivisible good from the seller at date 1; however, there is uncertainty about the buyer's valuation v_b and the seller's outside option v_s, while cost c is known at date 0. This uncertainty is resolved at date 1, but v_b and v_s are not verifiable and thus state-contingent contracts are not feasible. We interpret v_i for $i = S, B$, as subjective valuations about the good and normalize them to $v_i \in [0, 1]$. For example, a buyer may order the good but only after the seller has provided it, he can find out and appreciate its true characteristics, and he may even be able to test it before buying it. In other words, the valuations are the subjective probabilities that the good is worth 1, and its objective value is $v \in \{-1, 1\}$. On the other hand, the seller may have incurred some cost in producing it in addition to that expected at date 0, or, more interestingly, she can believe herself to be able to sell the good to some other customer if she breaches the contract. This interpretation of v_i as subjective valuations makes the lack of state-contingent contracts less important for the result. Finally, we suppose that trade ($q = 1$) is voluntary; that is, a party cannot be punished for breaching the contract.

Let us start observing that the first-best trading rule is given by

$$q = 1 \Leftrightarrow \; v_b - v_s \geq c.$$

We distinguish between a rigid contract and a flexible agreement. A rigid contract consists of a no-trade price p_0 and a trade price p_1. Given the voluntary trade assumption, trade will occur if and only if $v_b - p_1 \geq -p_0$ and $p_1 - c \geq v_s + p_0$; that is,

$$q = 1 \;\; \Leftrightarrow \;\; v_b \geq p_1 - p_0 \geq v_s + c.$$

Given the existence of transfers, we can normalize p_0 to zero, as only the difference between p_1 and p_0 matters. It is immediately apparent that trade occurs less often than in the first-best case.

A flexible contract, instead, specifies a no-trade price p_0 and an interval of trading prices $\left[\underline{p}, \overline{p}\right]$ [10]. In this case,

$$q = 1 \;\; \Leftrightarrow \exists \; \overline{p} \leq p_1 \leq \underline{p} \text{ s.t. } v_b \geq p_1 - p_0 \geq v_s + c, \tag{9}$$

That is, trade occurs if and only if parties can find a price in the specified range, such that both parties are willing to trade. Condition (9) simplifies to

$$q = 1 \;\; \Leftrightarrow v_b - v_s \geq c, \;\;\; v_b \geq \underline{p}, \;\; \overline{p} \geq v_s + c$$

as, in this case as well, only the difference between the prices matters.

We now come to the assumption that represents a significant departure from Hart and Moore (2008) and from the relevant literature. We suppose that price is determined as the outcome of a negotiation between the parties, in which each of them may acquire with probability ρ a signal σ, at cost $c_i(\phi)$, where $i = S, B$ and $c^{'} > 0, \;\; c'' > 0,$ about the good's objective value $v \in \{-1, 1\}$, where $\phi_i = \Pr(\sigma = v|v)$. That is, each party may acquire information and may also increase its precision at an increasing cost. After this persuasion phase, the party who has gathered the new evidence offers a price. The assumption of costly information makes sure that in equilibrium only the more confident of the parties, will try to persuade the counterparty. Assume also that

[10] More general contracts than those considered in the text are possible for example, a contract could allow both p_0 and p_1 to vary. However, since the main motivation for this section is to provide an alternative to the model by Hart and Moore (2008), we will focus on these two simple contracts, which are the main object of their analysis.

$$d^{+}(p_1^{+} - v_s^{+}) + \left(1 - d^{+}\right)(p_1^{\emptyset} - v_s^{\emptyset}) \geq c_s\left(\phi_s^*\right) \tag{10}$$

and

$$d^{-}(v_b^{-} - p_1^{-}) + \left(1 - d^{-}\right)(v_b^{\emptyset} - p_1^{\emptyset}) \geq c_b\left(\phi_b^*\right) \tag{11}$$

where d^+ is, as defined in the base model, the probability of receiving a positive signal, i.e., $d^+ = \rho(\phi v_s + (1-\phi)(1-v_s))$, and, analogously, d^- is the probability of receiving a negative signal; $d^- = (\phi(1-v_b) + (1-\phi)v_b)$. The costs are computed at the optimal effort choice ϕ_i^*. Thus, the conditions (10) and (11) ensure that in expectation it is worth it to attempt to persuade the counterparty. It is important to remark that the expectation is taken according to each player's belief. Notice that, an overconfident seller will try to boost the buyer's valuation up to the point at which she will find it optimal to sell him the good. On the other hand, a *picky* buyer may try to show to the seller all of the good's drawbacks, simply to lower the selling price. We now analyze two different cases.

Case 1. Suppose that at date 1, $v_s + c > v_b$. Because, the seller in this case believes that she will be able to sell the good for a higher price to another buyer, if she does not persuade the buyer, she will find it optimal to breach the contract. Given condition (10), she acquires the signal and discloses it only if it is positive. The buyer expects the seller to show him the signal; he will then update his beliefs according to

$$v_b^{+} = \frac{\phi v_b}{\phi v_b + (1-\phi)(1-v_b)}$$

if the seller shows indeed some new evidence of the qualities of the good, while he will update downward to

$$v_b^{\emptyset} = \frac{v_b(1-\rho) + \rho v_b(1-\phi)}{(1-\rho) + \rho\left[(1-v_b)\phi + v_b(1-\phi)\right]}$$

in the case of no disclosure. In this case, the seller may offer at most

$$p_1(\phi_s, \overline{p}) = Min\left(d^{+}v_b^{+} + \left(1 - d^{+}\right)v_b^{\emptyset},\ \overline{p}\right).$$

The seller, then, has complete control over the price but has to stick within the contract; she cannot exceed $\overline{p}$. To complete the analysis, let us clarify the optimal persuasion strategy. The optimal effort level ϕ_s^* solves the seller's first-order condition differentiated with respect to the precision of his information

$$p_1'(\phi_s^*, \overline{p}) = c_s'(\phi_s^*).$$

The seller increases the precision of his information, up to the point at which the expected increase in the trading price is equal to the marginal cost of effort.

Case 2. Suppose, instead, that at date 1, $v_b > v_s + c$. This may capture some colorful applications, such as the case of picky buyers who have already decided to trade but try to achieve the best feasible deal. In this case, the buyer will offer a price

$$p_1\left(\phi_b, \underline{p}\right) = Max\left(d^- v_s^- + \left(1 - d^-\right) v_s^{\emptyset},\ \underline{p}\right)$$

and his precision ϕ_b^* makes the expected reduction in price equal to its marginal cost. Observe, however, that in this case we may have an additional inefficiency. Here, the buyer may boost the seller's valuation up to the point where $v_s^+ > v_b$; this induces him to breach the contract. Define by δ the probability that the buyer destroys this trading opportunity, i.e., $\eta \equiv \Pr\left(v_s^{\emptyset} > v_b - c\right)$.

Thus, an optimal contract solves

$$Max_{\underline{p},\overline{p}}\ V\left(v_s, v_b\right) = \int \begin{bmatrix} v_b - v_s - c - I_{\{v_s + c > v_b\}} c_s\left(\phi_s^*, \overline{p}\right) \\ -I_{\{v_s + c < v_b\}} \{c_b\left(\phi_b^*, \underline{p}\right) + \eta\left(v_b - v_s - c\right)\} \end{bmatrix} dF\left(v_s, v_b\right) \quad (12)$$

$s.t.$

$$\begin{aligned} v_b - v_s &\geq c \\ v_b &\geq \underline{p}, \\ v_s + c &\leq \overline{p} \end{aligned} \quad (13)$$

where F is the distribution of $\left(v_s, v_b\right)$. We derive an interesting trade-off: a large interval $\left[\underline{p}, \overline{p}\right]$ makes it more likely that trade will occur if $v_b - v_s \geq c$, in fact, in the limit in which $\underline{p} \to -\infty$ and $\overline{p} \to \infty$, the trading rule becomes the first-best one. However, it also increases the *persuasion* costs. If $\overline{p}$ increases, the seller will exert more effort in the persuasion phase of the relationship, as the contract allows him to charge a higher price. On the other hand, if for example the buyer asks the seller to be arbitrarily refunded for the time spent or makes her pay a penalty, if the buyer believes that the good has a lower quality than expected, i.e., $\underline{p} < 0$, he will try to demonstrate to the seller the malfunction of the product, exerting more effort.

Even in the presence of persuasion costs, we are able to achieve the first-best with a simple contract $\underline{p} = \overline{p}$, under some conditions.

Proposition 8 *A simple contract achieves the first best if* (i) *only* v_b *varies;*(ii) *only* v_s *varies; or* (iii) *the smallest element of the support of* v_b *is at least as great as the largest element of the support of* $v_s + c$.

If only v_b varies, choose a simple contract with $p = c + v_s$. In case (ii), choose a contract $p = v_b$. If (iii) holds, choose a simple contract with p between the smallest v_b and the largest $c + v_s$.

We now turn to the analysis of the effects that the parameters have on the optimal contract.

Proposition 9 *The optimal $\overline{p}(v_s)$ is a decreasing function of the seller's valuation v_s, while the buyer's valuation increases the optimal $\underline{p}(v_b)$. Then, as the pair (v_b, v_s) increases, the optimal contract becomes less flexible.*

Proof. The proof is in the Appendix. ■

We have introduced persuasion costs as an alternative to shading costs, and it has given us not only the ability to derive the same trade-off between the flexibility and the rigidity of a contract, as in Hart and Moore (2008), without any behavioral assumption but has also allowed us to shed new light on a novel issue. In the case in which the parties' beliefs are far from one another, the incompleteness or flexibility of the contract should be reduced. In our environment, disagreement has a high cost and induces the parties to persuade each other at a cost that is increasing in their valuations. Disagreement also carries a cost in terms of losing trading opportunities.

Although we have restricted our attention to these two types of contracts, leaving the analysis of the optimal contract in presence of persuasion costs as an open question, we believe that the main qualitative result would hold.

When the parties may have a large disagreement in opinions, a rigid contract becomes a better option, as it reduces any attempt to persuade the counterparty.

6 Conclusion

Probably the most important contribution of this paper is to provide a framework in which to analyze the costs related to negotiation through persuasion. This provides, in the spirit of Williamson (1985), a new rationale for transaction costs. The introduction listed the main insights. Rather than restating them, let us conclude with a couple of avenues for future research.

First, we focused on the transaction costs of negotiating deals, but it is certainly worth considering the nature of the relation between persuasion and *ex post* transaction costs. Each party may incur some costs in order to induce the other party to perform according to the spirit of the contract or to *adapt* to contingencies that are not describable ex ante.

Second, as argued by Bernheim and Whinston (1998), the incompleteness of contracts may be a deliberate choice of sophisticated parties. Understanding whether an agent may strategically decide to leave the contract incomplete because he believes himself to be able to negotiate with the other party a better deal ex post, acquiring and disclosing new information, would shed new light on the role played by heterogeneous beliefs regarding the formation of the contract.

Third, the ability of the seller to persuade and influence the buyer's decision is affected by past negotiated deals. A seller may find himself stuck in a trading relationship in which his *perceived* informativeness does not allow him to affect the buyer's beliefs; that is, the trading relationship may display path dependence. A seller may not want to regret having abused his ability to sweet talk.

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7 Appendix

Proof of Lemma 1. First, suppose that no effort is expected in period 1, and, after no disclosure, profitable trade is expected to take place at $t = 1$. This implies $q_1 > 1/2$, otherwise the seller could not sell the good at a positive price at $t = 1$. Consider the seller's deviation to exerting effort and disclosing a positive signal, if obtained. With positive probability the seller gets a positive signal. Both parties' beliefs are updated to $q^+(q_1) > q_1$. The seller can now offer a price slightly below $2q^+(q_1) - 1$ but above $2q_1 - 1$. The offer is immediately accepted by the buyer; the good's expected value exceeds the price, and the buyer knows that if he rejects the offer, in the continuation the seller exerts effort (as $q_2 = q^+(q_1) > q_1 > 1/2 > 1 - p$), and he (the buyer) gets zero surplus. The deviation is profitable for the seller because the cost of effort is arbitrarily small. Therefore, profitable trade and no effort at $t = 1$ is impossible in equilibrium. Second, suppose that no effort is expected in period 1 and that after no disclosure, the parties expect to continue to negotiate in period 2 and then trade at a positive price with positive probability. By Proposition 1, this implies $q_2 \equiv q_1 > 1 - p$ (effort is expected in period 2). Suppose the seller exerts effort in period 1 and does exactly as she would do after exerting effort in period 2 (discloses a positive signal if obtained, and sells the good at the price she would charge in period 2). The seller's expected equilibrium profit in period 2 is positive, which she receives in period 1 instead of period 2 by deviating in this way. Hence the deviation is profitable for the seller. We conclude that there is no equilibrium in which the seller exerts no effort in period 1 but then trades the good at a positive price with positive probability in either period 1 or period 2. ■

Proof of Lemma 2. Direct calculations reveal that $\Delta(1/2) = \rho(1-p) - 1 < 0$ and $\Delta(1) = 0$. Moreover, when $\rho \leq 1$ and $\left(2q^{\emptyset}(q) - 1\right) > 0$, $\Delta(q) = 0$ for all q. This means that she is indifferent between immediate trade and persuading, but the seller will then prefer to trade immediately if there is even an infinitesimal cost of acquiring new information. However, when $\rho < 1$ and $q^{\emptyset}(q) < 1/2$ the payoff-difference becomes

$$\Delta(q) = 2q - 1 + \rho - \rho q - \rho p$$

which is strictly increasing in q. Then, there exists a unique threshold $\overline{q} \equiv \frac{1-\rho+\rho p}{2-\rho}$, such that $\Delta(q) > 0$ iff $q > \overline{q}$. This is exactly the same threshold that determines $q^{\emptyset}(q) > 1/2$. Note also that $\overline{q} \geq \underline{q}$ and $\overline{q} > 1/2$. ■

Proof of Lemma 3. *Step 1.* Let us first find the threshold. From the discussion in the text we know that the threshold must satisfy the following condition:

$$q_+(q_0^v) = \underline{q}$$

but we know that $\underline{q} = \frac{(1-p)^2}{p^2+(1-p)^2}$, so:

$$\frac{pq_0^v}{pq_0^v + (1-p)(1-q_0^v)} = \frac{(1-p)^2}{p^2 + (1-p)^2}$$

Solving for q_0^v, we get $q_0^v = \frac{(1-p)^3}{\left[p^2+(1-p)^3\right]}$.

Step 2. Now, we have to make sure that $q_0^v < \underline{q}$, which is equivalent to: $\frac{(1-p)^3}{\left[p^2+(1-p)^3\right]} < \frac{(1-p)^2}{p^2+(1-p)^2}$, which is always true, given $p \in \left(\frac{1}{2}, 1\right)$. ■

Proof of Proposition 6. To show the existence of an interior solution, we can employ a continuity argument. The first-order condition is continuous in q_s, and we know that if $c = 0$ the optimal q_s is 1. Hence, suppose to rise the cost of acquiring information by ε, we can find a $q_s \approx 1$ that satisfies the principal optimal condition. For the first point it suffices to observe that if condition (7) does not hold, the first-order condition is negative for any value of q_s. The condition $\delta \to 0$ derives directly from the inspection of $q_b^{\emptyset}$ when ρ is close to one. ■

Proof of Proposition 7. The seller enters in the matching market if

$$\pi^s (p^*) - q_b^m \frac{1 - F(\widehat{q}_b)}{F(\widehat{q}_b)} \geq \int_{\underline{q}_b}^{\overline{q}_b} [d(q_s)(2q_b^+ - 1)_+ + (1 - d(q_s))(2q_b^{\emptyset} - 1)_+]\} dF(z)$$

where q_b^m is the mean buyer's valuation. Notice that the left-hand side is constant with respect to q_s : then, the cutoff q_s^h is defined by the previous equation holding with equality. A necessary condition to induce the seller to enter in this market is that the price she expects to charge p^* must be greater than what she can get from the buyer with mean valuation. The comparative statics is a direct application of the envelope theorem. ■

Proof of Proposition 9. First, note that the choice of the optimal $\underline{p}(v_b)$ is not affected by the seller's prior belief. We can show that the value function in (12) has increasing differences in $(v_s, -\overline{p})$:

$$\frac{\partial\ V(v_s, v_b)}{\partial(-\overline{p})} = -\frac{\partial c_s}{\partial \phi_s^*} \frac{\partial \phi_s^*}{\partial(-\overline{p})} \geq 0$$

and then we have that

$$\frac{\partial^2\ V(v_s, v_b)}{\partial(-\overline{p}) \partial(v_s)} \geq 0.$$

The first inequality derives from the observation that increasing $(-\overline{p})$ decreases the effort choice ϕ_s^*, as it poses a limit to the seller's profit and that because the cost is increasing in the information precision, the overall effect is negative. The second inequality derives from the fact that increasing the seller's prior belief increases again the optimal choice of effort because the seller believes that she will get a positive signal with higher probability, i.e., d^+ increases; we can then conclude that the value function $V(v_s, v_b)$ presents increasing differences in $(v_s, -\overline{p})$. The optimizer $\overline{p}(v_s)$ is thus decreasing in v_s. To show that the optimal $\underline{p}(v_b)$ is increasing in v_b, we can simply differentiate $V(v_s, v_b)$ with respect to v_b :

$$\frac{\partial\ V(v_s, v_b)}{\partial v_b} = 1 - [1 - F(c)] \left[\underbrace{\frac{\partial c_s}{\partial \phi_s^*} \frac{\partial \phi_s^*}{\partial v_b}}_{(-)} + \eta + \underbrace{\frac{\partial \eta}{\partial v_b}}_{(-)} v_b \right] \geq 0$$

where the signs of the derivatives come from the observation that increasing the buyer's assessment of the good quality reduces his probability of getting a negative signal and thus induces a lower effort. The effect on η follows immediately from its definition. ■