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Abstract

We analyze how a contest organizer chooses the winner when the contestants.efforts are already exerted and commitment to the use of a given contest success function is not possible. We define the notion of rationalizability in mixed-strategies to capture such a situation. Our approach allows to derive different contest success functions depending on the aims and attitudes of the decider. We derive contest success functions which are closely related to commonly used functions providing new support for them. By taking into account social welfare considerations our approach bridges the contest literature and the recent literature on political economy.

Keywords: Endogenous Contests, Contest Success Function, Mixed-Strategies.

JEL Classification: C72 (Noncooperative Games), D72 (Economic Models of Political Processes: Rent-Seeking, Elections), D74 (Conflict; Conflict Resolution; Alliances).

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1 Introduction

In a contest players exert effort in order to win a certain prize. Contests have been used to analyze a variety of strategic situations ranging from rent-seeking and lobbying to conflict, arms races, warfare, and promotional efforts, as well as to sports.[1]

In many of these situations there is a contest organizer who awards the prize.[2] For example in litigation a court decides on the winner; in lobbying, rent-seeking and rent-defending contests bureaucrats or politicians award a prize; in internal labor market tournaments jobs are allocated by the manager of an organization; or in beauty contests the decision where to locate an event is taken by a committee.

A crucial determinant for the equilibrium predictions of contests is the specification of the so-called contest success function (CSF) which relates the players' efforts and win probabilities (also interpreted as shares of the prize).[3] In this paper we provide a framework in which CSFs are derived as an optimal choice of the contest administrator.[4] See our remarks in the final section regarding the interpretation of our results when the contest is not organized by a contest administrator. The administrator is unable to commit to a CSF so she chooses the probabilities with which the prize is given in order to maximize her utility given the choices of the contestants. Assuming complete information, contestants anticipate the choice made by the contest organizer. We apply this framework by postulating different utility functions for the decider. Of course the plausibility of our results hinges on the plausibility of these utility functions. Thus we decide to use only the most popular utility functions in the theory under risk: expected utility and (a special case of) prospect theory. We also follow the recent political economy literature (see Grossman and Helpman, 2001; Persson and Tabellini, 2000) and let the decider care about social welfare - albeit in contrast to that literature in the form of a generalized utilitarian planner. Somewhat surprisingly, this allows us to derive the three most commonly used types of CSFs: non-deterministic CSFs with an additive structure (Proposition 5), deterministic CSFs (Proposition 4 and Example 1) and difference-form CSFs (Propositions 1, 2 and 3).

Our approach differs from other models in which commitment is possible such as the menu auction approach (see Grossman and Helpman, 1994). Dasgupta and Nti (1998) analyzed the optimal design of a contest when the contestants have identical valuations, the planner might retain the prize and when the contest success function must be of the class axiomatized by Skaperdas (1996) (see equation (2) below). They find that, when the planner has a low valuation

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[1] For a survey see Konrad (2007).

[2] We use the terms contest administrator, contest organizer, decider and planner interchangeably. Participants in the contest are called contestants, rent seekers or contenders.

[3] A prominent example of this interpretation as shares is Wärneryd (1998). He analyzes a contest among jurisdictions for shares of the GNP and compares different types of jurisdictional organization.

[4] Other alternative foundations for CSFs are provided by Blavatskyy (2008), Münster (2009) and Rai and Sarin (2009) who offer axiomatic characterizations of contest success functions following the seminal paper by Skaperdas (1996). The earlier work on foundations for contest success functions has been extensively reviewed by Konrad (2007).

for the prize, the optimal CSF is such that the probability that a contestant obtains the prize is the ratio of her effort with respect to total effort. A closely related paper is Epstein and Nitzan (2006) which with respect to commitment can be seen as the benchmark that is the opposite of our paper. In Epstein and Nitzan the contest organizer decides first whether to have a contest at all, and if a contest takes place, she chooses the CSF among the elements of a fixed set of CSFs maximizing ex-ante utility. In doing so, the organizer anticipates the equilibrium efforts of contestants and needs to be able to commit to employing a given CSF once efforts are exerted. In contrast, in our approach the organizer decides on the winner once efforts are exerted.[5] There are further approaches motivating or endogenizing the CSF and some of them imply that it is not necessary to assume commitment. In one recent approach contestants might be uncertain about a characteristic of the organizer and as a result view the determination of the winner as probabilistic although the organizer chooses in a deterministic way (Corchón and Dahm, 2009; Skaperdas and Vaidya, 2009). In another, efforts are affected by exogenous shocks so that the performance of contestants is different from efforts which generates randomness from the point of view of contestants (Jia 2007).

Our approach cannot be construed as criticism of the commitment assumption. The commitment case is an important benchmark case. It is, however, not always clear how the decider can be trusted to maintain her word. In other cases the planner may prefer a policy of wait and see instead of announcing a certain CSF. Therefore, it is important to know what happens when, after contestants have exerted efforts, the contest administrator is no longer constrained by her word and could choose the winner in a different way. Thus our paper can be viewed as a check on the properties of CSFs in case commitment becomes unlikely. Our main conclusion is that well-known CSFs arise under natural specifications of the preferences of the contest organizer. Thus our approach could be considered as a back up for the use of well-known CSFs.

2 Preliminaries

A contest administrator conducts a contest among n contestants denoted by $i \in N := \{1, ..., n\}$. Each contestant has a valuation for the prize, denoted by $V_i \in \mathbb{R}_+$, and exerts effort $G_i \in \mathbb{R}_+$ in order to affect the probability of winning the prize which is given by the CSF.

Formally, a *contest success function* $\boldsymbol{p}(\boldsymbol{G}) = (p_1(\boldsymbol{G}), p_2(\boldsymbol{G})..., p_n(\boldsymbol{G}))$ associates, to each vector of efforts $\boldsymbol{G}$, a lottery specifying for each agent a probability p_i of getting the prize. That is, $p_i = p_i(\boldsymbol{G})$ is such that, for each contestant $i \in N$, $p_i(\boldsymbol{G}) \geq 0$, and $\sum_{i=1}^n p_i(\boldsymbol{G}) = 1$.

We say that a CSF is *imperfectly discriminating* if $G_i > 0$ implies that $p_i(\boldsymbol{G}) > 0$.[6] An

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[5]There are further differences to our paper. In our approach the contest organizer is completely unconstrained in her choice of the contest success function, rather than choosing among the elements of a fixed set of contest success functions. Also, our approach is not restricted to the case of two contestants.

[6]This is essentially axiom 1 in Skaperdas (1996). The name of this property refers to the fact that a contest can be interpreted as an auction where the prize is auctioned among the agents and efforts are bids. In standard auctions the highest bid obtains the prize with probability one. Here, any positive bid entitles the bidder with a positive probability to obtain the object, so it is as if the bidding mechanism did not discriminate perfectly

example for such a function is the most commonly used CSF introduced by Tullock (1980) which is given by

$$p_i = \frac{G_i^R}{\sum_{j=1}^n G_j^R}, \text{ for } i = 1, ..., n, \tag{1}$$

where R is a positive parameter. A generalization of this form is

$$p_i = \frac{f_i(G_i)}{\sum_{j=1}^n f_j(G_j)}, \text{ for } i = 1, ..., n, \tag{2}$$

where the $f_i(\cdot)$ are positive increasing functions of its argument. For the case in which $f_i(\cdot) = f(\cdot)$ for $i = 1, ..., n$, (2) has been axiomatized by Skaperdas (1996). A different class of CSFs are difference-forms (Hirshleifer, 1989; Baik, 1998; Che and Gale; 2000). The linear difference-form contest in Che and Gale (2000) is defined as

$$p_i = \max\left\{\min\left\{\frac{1}{2} + s(G_i - G_j), 1\right\}, 0\right\} \text{ for } i = 1, 2 \text{ and } j \neq i, \tag{3}$$

where s is a positive scalar. Notice that the linear difference-form is not imperfectly discriminating. Another example for a not imperfectly discriminating CSF is a function that assigns probability one to the contestant exerting the highest effort, like in an all-pay auction.

Contestants are risk-neutral. Defining $a_i(\boldsymbol{G}) = 1$ when the contest is all-pay (effort is irreversible so that all contestants pay their bid) and $a_i(\boldsymbol{G}) = p_i(\boldsymbol{G})$ when the contest is winner-pay (effort is like a promise so that only the winner pays his bid), the expected utility of a contestant is

$$u_i(p_i, \boldsymbol{G}) = p_i(\boldsymbol{G})V_i - a_i(\boldsymbol{G})G_i.$$

While contests are usually analyzed as all-pay, winner-pay contests have been analyzed in Skaperdas and Gan (1995), Wärneryd (2000) and Yates (2007). There is also a large literature on the first-price (sealed bid) auction which constitutes the extreme case of a winner-pay contest in which the highest bidder wins with probability one.

The timing is as follows. In an all-pay contest contenders exert effort in the first stage (simultaneously), while the administrator assigns win probabilities or shares of the prize in the second stage. In a winner-pay contest contenders promise effort in the first stage (simultaneously) but effort is not exerted yet. In the second stage the organizer determines the outcome of the contest. In the third stage the winner exerts the efforts promised.[7] When contestants play mixed strategies, in stage 1 contestants anticipate the CSF and choose a mixed strategy. Then the actual effort levels are realized and the organizer observes them. In stage two the organizer chooses (given the realization of efforts) the CSF which is the one contestants have anticipated in stage 1.

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among bids.

[7] Our approach also works if in the first stage contestants exert effort sequentially (in the case of the all-pay contest) or promise effort sequentially (in the case of the winner-pay contest). We assume that in a winner-pay contest the promise of effort in stage one will be fulfilled in stage three. A reason for this could be that the prize will only be delivered when effort has been exerted.

3   Rationalizability in Mixed-Strategies

In this section we formalize the idea that the contest organizer's choice in the second stage is optimal from her view point. Suppose that, after effort has been exerted or promised in the first stage, the decider assigns win probabilities or shares of the prize maximizing her objective function. Since there is no uncertainty and contestants know the organizer and her incentives, rent-seekers are able to anticipate in the first stage which CSF will be chosen.

We capture the idea that a particular CSF, which is used when contestants make their effort choices, is also optimal from the point of view of the contest organizer with the following definition. Denote by S^n the $n-1$ dimensional simplex.

Definition 1 *The contest success function $\boldsymbol{p}(\boldsymbol{G}) = (p_1(\boldsymbol{G}), p_2(\boldsymbol{G})..., p_n(\boldsymbol{G}))$ is rationalizable in mixed-strategies if there is a function $W(\boldsymbol{p}, \boldsymbol{G})$ such that for all $\boldsymbol{G} \in \mathbb{R}^n$,*

$$\boldsymbol{p}(\boldsymbol{G}) = argmax \ W(\boldsymbol{p}, \boldsymbol{G}), \ \boldsymbol{p} \in S^n.$$

Here the term mixed-strategies refers to the fact that technically a CSF is a probability distribution even though, as we have already mentioned, it might be interpreted as shares of the prize. In our approach, the contest success function is the best reply of a contest organizer with payoff function $W(\boldsymbol{p}, \boldsymbol{G})$. As we will see in the sequel, the payoff function allows to take into account a variety of objectives and attitudes of the decider. The use of mixed-strategies here might be motivated analogously to the classical argument in favor of mixed-strategies, namely that mixed-strategies produce unpredictable choices that cannot be exploited by an opponent. This might translate to enabling the organizer to avoid that contestants forecast perfectly her choice, which might reduce incentives to exert effort. There are, however, differences between our approach and the way mixed-strategies are usually employed. Usually a player chooses a mixed-strategy when she is indifferent between the pure strategies involved. In contrast, we will be able to derive mixed-strategies which are strictly preferred to any other pure or mixed-strategy. This is because our planners are not expected utility maximizers.

4   Generalized Utilitarian Planner

As commonly assumed in the recent political economy literature (see Grossman and Helpman, 2001; Persson and Tabellini, 2000) let the decider's objective function depend on expected social welfare.[8] Consider a generalized utilitarian planner whose payoff function is a constant elasticity of substitution function

$$W(\boldsymbol{p}, \boldsymbol{G}) = \begin{cases} \left(\sum_{i=1}^{n} (u_i(p_i, \boldsymbol{G}))^{1-r} \right)^{1/(1-r)} & \text{if} \quad r \neq 1 \\ \sum_{i=1}^{n} \ln (u_i(p_i, \boldsymbol{G})) & \text{if} \quad r = 1 \end{cases}. \qquad (4)$$

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[8]In that literature the objective function is a weighted average of social welfare and lobbying efforts.

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The positive parameter r represents the degree of inequality aversion of the planner. If the CSF specifies win probabilities, inequality can be interpreted as referring to expected utility. The utilitarian case corresponds to $r = 0$. When $r = 1$, the Bernoulli-Nash case obtains. When r goes to infinity, the Rawlsian case arises and the concern is with the least well-off only.

We consider an all-pay contest.[9] The planner maximizes $W(\boldsymbol{p}, \boldsymbol{G})$ as defined in (4) with respect to $\boldsymbol{p}$. It is instructive to start with the Bernoulli-Nash case in which $r = 1$.

Proposition 1 *Suppose the contest is all-pay and maximizing (4) with respect to $\boldsymbol{p}$ has an interior solution. The contest success function*

$$p_i = \frac{1 - \sum_{j=1}^{n} (G_j/V_j)}{n} + G_i/V_i \ \text{for } i = 1, ..., n \tag{5}$$

is the unique one that can be rationalized in mixed-strategies by a function fulfilling (4) with $r = 1$.

Proof. Notice first that the objective function is strictly concave, as the Hessian matrix is a diagonal matrix which has as its iith element $-(V_i)^2/(p_iV_i - G_i)^2$. Since the solution is interior, the first order conditions imply

$$(p_iV_i - G_i) V_j = (p_jV_j - G_j) V_i, \ \text{for } i, j = 1, ..., n.$$

Rearranging we obtain

$$p_i = \frac{p_nV_n - G_n}{V_n} + \frac{G_i}{V_i} \ \text{for } i = 1, ..., n.$$

Adding up over all contestants yields

$$p_n = \frac{1 - \sum_{j=1}^{n} (G_j/V_j)}{n} + G_n/V_n$$

and replacing this in the previous equation we obtain (5). ∎

Notice that the expression in (5) is a generalized difference-form. To see this consider the following corollary.

Corollary 1 *Suppose the contest is all-pay and maximizing (4) with respect to $\boldsymbol{p}$ has an interior solution.*

1. *If there is a common value V, the contest success function*

$$p_i = \frac{1}{n} + \frac{1}{nV} \left((n - 1) G_i - \sum_{j \neq i} G_j \right) \ \text{for } i = 1, ..., n \tag{6}$$

   is the unique one that can be rationalized in mixed-strategies by a function fulfilling (4) with $r = 1$.

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[9] We omit the discussion of winner-pay contests as the mathematical structure of the problem is closely related to Proposition 5. See the discussion after that proposition.

2. If $n = 2$, the contest success function

$$p_i = \frac{1}{2} + \frac{1}{2} \left(\frac{G_i}{V_i} - \frac{G_j}{V_j} \right) \text{ for } i = 1, 2 \text{ and } j \neq i \tag{7}$$

is the unique one that can be rationalized in mixed-strategies by a function fulfilling (4) with $r = 1$.

Notice that the CSFs in (6) and (7) are very related to the linear difference-form contest in Che and Gale (2000) given in (3). When there are two contestants and when $p_i > 0$ for $i = 1, 2$, then (5) coincides with (3) if there is a common value V and $V = 1/(2s)$. This threshold $V = 1/(2s)$ has the interesting interpretation that a contestant can only guarantee success by exerting at least this amount more than the opponent.

As Baik (1998) and Che and Gale (2000) have shown, in many difference-form contests there are problems concerning the existence of pure strategy equilibria. But, as we already discussed, our approach can also work when contestants play a mixed-strategy.

We consider now the case in which $r \neq 1$ and $r \in (0, \infty)$.

Proposition 2 *Suppose the contest is all-pay and maximizing (4) with respect to $\boldsymbol{p}$ has an interior solution. The contest success function*

$$p_i = \frac{1 - \sum_{j=1}^{n} (G_j/V_j)}{\sum_{j=1}^{n} (V_j/V_i)^{\frac{1-r}{r}}} + G_i/V_i \text{ for } i = 1, ..., n \tag{8}$$

is the unique one that can be rationalized in mixed-strategies by a function fulfilling (4) with $r \neq 1$ and $r \in (0, \infty)$.

Proof. Suppose $r \neq 1$ and $r \in (0, \infty)$ and consider

$$W'(\boldsymbol{p}, \boldsymbol{G}) = \sum_{i=1}^{n} (p_i V_i - G_i)^{1-r}.$$

Notice that for $r < 1$, $W(\boldsymbol{p}, \boldsymbol{G})$ is an increasing transformation of $W'(\boldsymbol{p}, \boldsymbol{G})$, while for $r > 1$, $W(\boldsymbol{p}, \boldsymbol{G})$ is a decreasing transformation of $W'(\boldsymbol{p}, \boldsymbol{G})$. The Hessian matrix of $W'(\boldsymbol{p}, \boldsymbol{G})$ is a diagonal matrix which has as its iith element $-r(1-r)(V_i)^2(p_i V_i - G_i)^{-r-1}$. $W'(\boldsymbol{p}, \boldsymbol{G})$ is, hence, strictly concave for $r < 1$ and strictly convex for $r > 1$. $W'(\boldsymbol{p}, \boldsymbol{G})$ has thus a unique maximizer in the first case and a unique minimizer in the latter case. Both correspond to the unique maximizer of $W(\boldsymbol{p}, \boldsymbol{G})$.

Let us maximize $W'(\boldsymbol{p}, \boldsymbol{G})$ as defined above subject to $p \in S^n$, where S^n is the $n-1$ dimensional simplex. Since the solution is interior, the first order conditions imply

$$(p_i V_i - G_i)^r V_j = (p_j V_j - G_j)^r V_i, \text{ for } i, j = 1, ..., n.$$

Rearranging we obtain

$$p_i = \frac{p_n V_n - G_n}{(V_n)^{1/r}} (V_i)^{\frac{1-r}{r}} + \frac{G_i}{V_i} \text{ for } i = 1, ..., n.$$

Adding up over all contestants yields

$$p_n = \frac{1 - \sum_{j=1}^{n} (G_j/V_j)}{\sum_{j=1}^{n} (V_j)^{\frac{1-r}{r}}} (V_n)^{\frac{1-r}{r}} + G_n/V_n$$

and replacing this in the previous equation we obtain (8). ■

Notice that when r goes to one, (8) becomes (5), while when there is a common value (8) becomes (6). Consider now the Rawlsian case in which r goes to infinity.

Proposition 3 *Suppose the contest is all-pay and maximizing (4) with respect to $\boldsymbol{p}$ has an interior solution. The contest success function*

$$p_i = \frac{1 - \sum_{j=1}^{n} (G_j/V_j)}{\sum_{j=1}^{n} V_i/V_j} + G_i/V_i \ for \ i = 1, ..., n \tag{9}$$

is the unique one that can be rationalized in mixed-strategies by a function fulfilling (4) with $r = \infty$.

Proof. For r equal to infinity the objective function (4) becomes

$$W(\boldsymbol{p}, \boldsymbol{G}) = \min\{p_1 V_1 - G_1, ..., p_n V_n - G_n\}. \tag{10}$$

If the solution is interior, it must hold that

$$p_i V_i - G_i = p_j V_j - G_j, \ for \ i, j = 1, ..., n.$$

This implies that

$$p_i = \frac{p_n V_n - G_n}{V_i} + \frac{G_i}{V_i} \ for \ i = 1, ..., n.$$

Adding up over all contestants yields

$$p_n = \frac{1 - \sum_{j=1}^{n} (G_j/V_j)}{\sum_{j=1}^{n} 1/V_j} (V_n)^{-1} + G_n/V_n$$

and replacing this in the previous equation we obtain (9). ■

Notice that when r goes to infinity, (8) becomes (9). Note also that when there is a common value all cases considered so far yield the same CSF.

Corollary 2 *Suppose the contest is all-pay, maximizing (4) with respect to $\boldsymbol{p}$ has an interior solution and there is a common value. The contest success function (6) is the unique one that can be rationalized in mixed-strategies by a function fulfilling (4) with $r > 0$.*

We may also consider an individual rationality constraint in the planner's problem adding the restrictions that $u_i \geq 0$, $i = 1, 2, ..., n$. Notice that in the cases considered in Propositions 1,

2 and 3 as long as $\sum_{j=1}^{n} G_j/V_j \leq 1$ it can be easily shown that there is a subset of the simplex from which win probabilities can be chosen such that the individual rationality constraint is satisfied. Note also that in the case of a common value the assumption that the sum of individual efforts weighted by the valuation must be smaller than one says that the rent is not completely dissipated. The assumption that $\sum_{j=1}^{n} G_j/V_j \leq 1$ together with $G_i/V_i > 0$, $i = 1, 2, ..., n$ is also sufficient to guarantee that maximizing (4) with respect to $\boldsymbol{p}$ has an interior solution. On the one hand, it is immediate that under these assumptions $p_i > 0$, $i = 1, 2, ..., n$. On the other hand, computing the sum of the win probabilities using (8) shows that $\sum_{j=1}^{n} p_j = 1$. Thus, we have that $p_i \in (0, 1)$, $i = 1, 2, ..., n$. This shows that our interiority assumption can be satisfied under certain conditions.

Finally let us consider the case of $r = 0$ in which the planner is utilitarian. In this case, in general, there will be no interior solution and the assignment of win probabilities depends only on valuations. More precisely, the planner prefers that the contestant with the highest valuation wins the prize. In the common value case the objective function of the planner becomes insensitive to whom wins the contest.

Proposition 4 *Suppose the contest is all-pay and $W(\boldsymbol{p}, \boldsymbol{G})$ follows (4) with $r = 0$. The contest success function is such that the contestant with the highest valuation wins the prize: If $p_i > 0$, then $V_i = \max\{V_1, V_2, ..., V_n\}$.*

5 Other Approaches

Suppose that the prize is the right to supply a certain good (i.e. Olympic Games) and that the quality of this good is positively related with the effort made by the winner. Let $f_i(G_i)$ be the quality of the prize if agent i wins the contest. Then, expected quality, identified with the utility of the planner is $W(\boldsymbol{p}, \boldsymbol{G}) = \sum_{i=i}^{n} p_i f_i(\boldsymbol{G})$. Since expected utility theory has a linear structure similar to the utilitarian planner of the previous section, imperfectly discriminating CSFs cannot be rationalized.[10] This is illustrated by the following example.

Example 1 *Incumbency advantage (Konrad (2002)). Let the contest be winner-pay and $n = 2$. Assume $f_1(G_1) = G_1$ and $f_2(G_2) = bG_2 - a$, where $b \in (0, 1]$ and $a \geq 0$. When $a = 0$ and $b = 1$, this function rationalizes the standard first-price (sealed bid) auction in mixed-strategies. For other parameter values it rationalizes a biased version of it.*

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[10]It is also difficult to derive imperfectly discriminating CSFs using expected utility theory when the contest is all-pay. Suppose the organizer obtains $f_i(\boldsymbol{G}) = \sum_{j=1}^{n} G_j$ no matter which contestant wins, then the organizer's payoffs do not depend on the assignment rule. The organizer is, thus, indifferent between CSFs and any rule including imperfectly discriminating ones can be rationalized in mixed-strategies. But notice that $f_i(\boldsymbol{G})$ does not need to be the same for all i, in which case imperfectly discriminating CSFs cannot be generated. To see this assume that $\boldsymbol{p}(\boldsymbol{G})$ is imperfectly discriminating and $\boldsymbol{p}(\boldsymbol{G})$ can be rationalized in mixed-strategies. Then there exist $\hat{\boldsymbol{G}} \in \mathbb{R}_{++}^n$ and $i, j \in N$ such that $f_i(\hat{\boldsymbol{G}}) > f_j(\hat{\boldsymbol{G}})$ and maximizing $W(\boldsymbol{p}, \hat{\boldsymbol{G}})$ requires $f_j(\hat{\boldsymbol{G}}) = 0$. This contradicts that $\boldsymbol{p}(\boldsymbol{G})$ is imperfectly discriminating.

In order to derive imperfectly discriminating CSFs one needs thus to assume some form of non-expected utility theory.

Consider prospect theory. A prospect is an utility level $f_i(G_i)$, which we identify with the quality of the prize, and a probability p_i. In prospect theory the former is transformed through a value function, while the latter enters the utility through a weighting function. Both functions are assumed to be power functions.[11]

More precisely, consider the following functional form for $W(\boldsymbol{p}, \boldsymbol{G})$ which corresponds to a special case of the class postulated by Kahneman and Tversky (1979, p. 276) for regular prospects, namely[12]

$$W(\boldsymbol{p}, \boldsymbol{G}) = \sum_{i=1}^{n} p_i^{\alpha} f_i(G_i)^{1-\alpha}, 1 > \alpha > 0. \tag{11}$$

Notice that (11) is the sum of n terms—one associated to each contestant. In each such term both components of a prospect are combined in a Cobb-Douglas way under the assumption of constant returns to scale.

Finally, all these contestants' specific Cobb–Douglas functions are aggregated in an additive way. This reflects that contestants are perfect substitutes from the contest organizer's point of view and implies that the marginal product of a contestant's effort does not depend on the effort of others. Notice lastly that except for the contestants' specific $f_i(\cdot)$'s, (11) is symmetric in contestants. For example, the exponents α and $1 - \alpha$, which measure the elasticity of the contest organizer's payoff with respect to effort and win probability, take the same value for every participant. Moreover, there is no contestants' specific scaling parameter. Now we have the following result:

Proposition 5 *The contest success function of the form (2) is the unique one that can be rationalized in mixed-strategies by a function fulfilling (11).*

Proof. Let us maximize $W(\boldsymbol{p}, \boldsymbol{G})$, $\boldsymbol{p} \in S$. Since $W(\boldsymbol{p}, \boldsymbol{G})$ is continuous on $\boldsymbol{p}$ and S is compact, a maximum exists. Since $W(\boldsymbol{p}, \boldsymbol{G})$ is strictly concave on $\boldsymbol{p}$ and S is convex the maximum is unique. Consider the first order conditions of the maximization with respect to $\boldsymbol{p}$

$$\alpha p_i^{\alpha-1} f_i(G_i)^{1-\alpha} - \lambda = 0, \, i = 1, 2, ..., n.$$

Clearly, the maximum is interior because if $p_i \to 0$, the left hand side of the above equation goes to infinity. The above equations imply that

$$p_i^{\alpha-1} f_i(G_i)^{1-\alpha} = p_j^{\alpha-1} f_j(G_j)^{1-\alpha}, \, i, j = 1, 2, ..., n.$$

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[11] Value functions in the form of power functions are often used. For a discussion and axiomatic analysis of the so-called probability weighting functions, including the power function employed in (11), see Prelec (1998). Of course, an important difference between our setting and standard applications of prospect theory is that here win probabilities are an object of choice of the decider.

[12] Although Kahneman and Tversky consider $n = 2$, they consider the extension to more outcomes "straight-forward" (p. 288).

Which yield

$$p_i = \frac{f_i(G_i)p_n}{f_n(G_n)}, \ i = 1, 2, ..., n.$$

Substituting these equations in the simplex we obtain that

$$p_n = \frac{f_n(G_n)}{\displaystyle\sum_{j=1}^{n} f_j(G_j)}.$$

Substituting this equation in the previous one we obtain (2). ∎

Notice that besides prospect theory there might be other interpretations of (11) that depending on the context might be meaningful. For instance, by choosing $f_i(G_i) = (G_i)^{\frac{\alpha}{1-\alpha}}$ and by applying to (11) the increasing transformation $V(\boldsymbol{p}, \boldsymbol{G}) = (W(\boldsymbol{p}, \boldsymbol{G}))^{1/\alpha}$, we obtain the CES utility function with the form

$$V(\boldsymbol{p}, \boldsymbol{G}) = \left(\sum_{i=1}^{n} (p_i G_i)^{\alpha}\right)^{1/\alpha} \text{ with } \alpha = 1 - \frac{1}{\sigma}, \tag{12}$$

where σ is the elasticity of substitution which is usually assumed to be strictly larger than one. Since the maximizer is not affected, we obtain $p_i = (G_i)^{\sigma-1} / \left(\sum_{j=1}^{n} (G_j)^{\sigma-1}\right)$, which allows to explain the exponents in this CSF as the elasticity of substitution of the decider.[13] In another example one might proceed similarly and choose $f_i(G_i) = (V_i - G_i)^{\frac{\alpha}{1-\alpha}}$. This corresponds to (4) under the assumption of a winner pay contest.

6 Concluding Remarks

We have shown that contest success functions can be viewed as optimal choices of a contest organizer, given efforts of contestants. This implies that it is not necessary to assume that a contest organizer is able to commit to employ a given contest success function. This approach does not rely on uncertainty or reputation effects. We have shown that our approach works both for all-pay and winner-pay contests. It can also motivate both interpretations of contest success functions: win probabilities and shares of the prize. Our exercise yields contest success functions that were already 'popular' providing new support for them.

Notice that the our approach is very much related to at least two classic problems.

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[13] Under the assumption that the CSF assigns shares of the prize, the CES function indicates a taste for variety. A politician might have a taste for variety when the lobbying efforts are based on different expertise e.g. when they are used to draft legislation. One interpretation of equation (12) is then that lobbies offer to invest effort G_i in drafting a complete bill and that the decider prefers to have parts of the legislation drafted by different lobbies. Notice that in order to capture a taste for variety the sum in equation (12) cannot be linear in shares, because $\left(\sum_{i=1}^{n} p_i^{\gamma} (G)^{\beta}\right)^{1/\beta} > G$ requires $\gamma < 1$.

The first classic problem is the equivalence between revealed preference and utility maximizing choices. For a long time our profession struggled to find a condition under which these two approaches were equivalent, a final solution being obtained by Kihlstrom, Mas-Colell and Sonnenschein (1976). In our case we show the equivalence between contest success functions whose axiomatic properties were known and those arising from the actions of a benevolent and well-informed planner.

The second classic problem is the equivalence between market equilibrium and welfare optimum, i.e. the so-called two fundamental theorems of welfare economics. These theorems assert the equivalence between market equilibrium and allocations obtained by a benevolent and well-informed planner, see Mas-Colell (1985 and 1986). In our case, we find that some contest success functions maximize expected social welfare. As in the case of the two fundamental theorems of welfare economics we do not mean that the planner actually exists. The planner is just a surrogate of what the system achieves by its own forces. Thus, in our case, contest success functions can be determined by the pure form of conflict, random elements, etc. What our results say is what these contest success functions are like if they were chosen by a planner (with a visible hand) in order to maximize social welfare.

An interesting implication of our analysis is that contest success functions might depend on the valuations of contestants. This is reasonable when the decider takes into account the welfare effects of her decision on contestants which in a contest model depend on valuations. In this sense our analysis bridges the gap between the contest literature and the recent literature on political economy in which the decider takes into account the welfare of rent-seekers (see Grossman and Helpman (2001)).

By postulating reasonable aims for the decider we have derived commonly used contest success functions and new ones with a mathematical structure similar to popular ones. Future research might postulate further payoff functions for the contest organizer and investigate the consequences for the contest success functions which arise.

In contrast, in the contest literature many studies suppose that the contest organizer is only interested in maximizing total expected effort. But as Konrad (2007, p. 69) writes "this is, at best, an approximation to what contest organizers care about in many applications." In the present paper we have postulated alternative aims for the contest organizer and shown how these aims translate into contest success functions. We hope that our approach opens the door to taking into account further variables influencing contest organizers.

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