Lifetime Network Externality and the Dynamics of Group Inequality

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Abstract

The quality of one's social network significantly affects his economic success. Even after the skill acquisition period, the social network influences economic success through various routes such as mentoring, job searching, business connections, or information channeling. In this paper I propose that a social network externality which extends beyond the education period – what I call a Lifetime Network Externality – is important in explaining the evolution of between-group inequality in an economy. When the members of a group believe that the quality of their social network will be better in the future, more young group members invest in skill achievement because they expect higher returns on investment realized over the working period. As this is repeated in the following generations, the quality of the group's network improves over time. Combining the Lifetime Network Externality, which operates during the labor market phase of a worker's career, with the traditional concepts of peer and parental effects, which operate during the educational phase (Loury 1977), I suggest a full dynamic picture of group inequality in an economy with multiple social groups. I define a notion of Network Trap, wherein a disadvantaged group cannot improve the quality of its network without a governmental intervention, and I explore the egalitarian policies to mobilize the group out of this trap. This social capital approach suggests a positive effect of equality on economic growth in later stages of economic development and a positive effect of inequality in the early stage of economic development, consistent with Galor and Zeira (1993). Unlike the previous literature, the conclusion is derived without imposing the standard assumption of credit market imperfections. Therefore, this implies that equality, by helping disadvantaged groups to move out of the network trap, has a positive effect on economic development even in a matured economy without binding credit constraints, or in a society with public provision of schooling.

KEYWORDS: Lifetime Network Externality, Group Inequality, Network Trap, Social Capital, Economic Development.

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1 Introduction

The acquisition of human capital occurs within a social context, and can be facilitated by access to the right social networks. This paper examines one mechanism by which such social network externalities affect the evolution of economic inequality between social groups.[1] The interaction between network externalities during the education period and during the working period produces a unique dynamic structure for the evolution of group inequality. The education period network externalities operate as *a historical force* that restricts a group to be subject to the current network quality, while the working period (or lifetime) network externalities operate as *a mobilization force* that leads a group to enhance (or shrink) the skill investment activities by holding an optimistic (or pessimistic) view about the future network quality. In the model to follow I identify what I will call the Network Trap, in which the human capital development of a social group is trapped by the externality of social networks. Also, I examine possible egalitarian policies to mobilize a disadvantaged group out of the trap and improve its skill investment activities. Considering that human capital is a prime engine of economic growth in the modern economy, I describe the macroeconomic effects of group inequality on economic development (Loury 1981, Galor and Zeria 1993). The model I create finds a positive effect of equality on the economic growth in most developmental stages. Unlike the previous literature, this conclusion is derived without imposing the standard assumption of imperfect credit markets. Therefore, the model implies a positive effect of equality even in an economy with no credit constraints: equality, a more equal distribution of social network capital in this study, has a positive effect on the economic development, by helping the disadvantaged groups to move out the network trap and enhance the skill investment activities, even in the society with public provision of schooling.

Lifetime Network Externality

Socioeconomic disparities between social groups constitute a challenge in many countries around the world. Even though social groups may educate their children within an identical educational system and work in the same market economy, their skill achievement ratios and wage levels can be significantly different. It is hard to conceive of a single root cause of inequality between groups since the manner in which social groups are formed is unique to each society. For instance, groups form along racial line in societies such as the Unites States, South Africa, New Zealand and Australia, but form along religious lines in Turkey, Iraq, Pakistan, Northern Ireland and Israel. While ethnicity is

[1] Another approach to explaining group inequality explores the discrimination story: either taste-based discrimination (Becker 1957) or statistical discrimination through imperfect information (Arrow 1972, Phelps 1972 and Coate and Loury 1993). This paper focuses on the social network externality not because discrimination is a less important issue, but because the purpose of the work is to explore the dynamic structure of group inequality through the network externalities. A companion paper (Kim and Loury 2008) explores the dynamic structure of group inequality due to reputational externalities in the context of statistical discrimination.

important in some countries such as Singapore, Indonesia, and the Balkan states, we often see caste-like social division in India and Gypsies in Europe. In many western countries, the population is divided into immigrants and non-immigrants, while population in the Americas is divided into indigenous peoples and European descendants.[2]

Even though these cases are distinct from one another, a salient feature of the issue is consistent throughout all the cases: divided social interactions between groups occurs over the whole lifetime. The social network externality around the skill acquisition period and the consequent development bias has long been discussed since the pioneering work of Loury (1977). In his theory, a human being is socially situated in that familial and communal resources explicitly influence a person's acquisition of human capital through various routes, including the constraints of training resources, of nutritional and medical provision, of after-school parenting, of peer effects, of role models, and even of the psychological processes that shape one's outlook on life.[3] A number of subsequent theoretical works discussed the development bias, emphasizing the network externalities over the skill acquisition period, including Akerlof (1997), Lundberg and Startz (1998) and Bowles, Loury and Sethi (2007).[4] However, the theoretical work continues to confront some empirical evidence that it cannot fully embrace. Consider a few examples:

1. Over the industrialization process of South Korea in the 1970s and 1980s, the socioeconomic disparity between Youngnam and Honam regional groups increased significantly, even when the educational system was strictly based on the public provision of schooling, and when the familial and communal environment did not carry a big difference between two regional groups: both groups were in an early stage of development, poor and low skilled, and shared a similar cultural base. It is often argued that social connections and mentoring networks played a key role in the emergence of group disparity in South Korea (Ha 2007, Kim 2002).

2. In France, where the public school system is well established, the violence of second generation immigrant youth in 2005 caused nearly 9,000 cars to be torched and dozens of buildings damaged in a riot. Most of the rioters were unemployed youth who arguably suffered from social exclusion

[2] Also, groups have formed along linguistic lines in nations such as Canada, Switzerland, and South Africa (Anglo-African and Afrikaners). Region of family origin influences the social interactions in nations such as Spain, the United Kingdom, and South Korea (Youngnam, Honam).

[3] His theory is supported by numerous empirical work, which includes the peer influence (Anderson 1990), community effects (Cutler and Glaeser 1997, Weinberg et al. 2004), racial network effect (Hoxby 2000, Hanusheck, Kain and Rivkin 2002) and academic peer effect (Kremer and Levy 2003, Zimmerman and Williams 2003).

[4] Akerlof (1997) provides a theoretical argument, which states that concerns for status and conformity are the primary determinant of an individual's educational attainment, child bearing, and law-breaking behavior. Lundberg and Startz (1998) argue that group disparities in earnings can persist indefinitely when the average level of human capital in a community affects the accumulation of human capital of the following generations. The recent work by Bowles, Loury and Sethi (2007) shows how group disparity can persist in a highly segregated society, and how it can disappear as integration is facilitated, in the presence of network externality over the skill acquisition period.

in French society, and from the lack of a job network.

3. In his examination of the jobless black underclass in New York City, Waldinger (1996) concludes that black unemployment originates from the lack of access to the ethnic networks through which workers are recruited for jobs in construction and service industries.[5]

These examples illustrate the importance of social network externalities that operate beyond the education period – what I am calling the Lifetime Network Externalities. This effect has been emphasized in numerous empirical papers in the economics and sociology literature. The sociologist Granovetter (1975) has been one of the pioneers of this line of inquiry. His work sheds light on the role played by interpersonal relationships, such as friends and relatives, in channeling information about jobs and job applications. He and other researchers have found that approximately fifty percent of all workers employed found their jobs on the basis of recommendation and word-of-mouth (Granovetter 1973, Myers and Shultz 1951, Rees and Shultz 1970, Campbell and Marsden 1990).[6] The role of ethnic networks in job search is emphasized in numerous empirical work such as immigrants in Australia (Mahuteau and Junankar 2008), Mexican immigrants in the US (Livingston 2006 and Munshi 2003) and migrants to urban centers in India (Banerjee 1981, 1983).[7] The effects of social network go beyond just finding jobs. Friends and acquaintances of the same occupation may help workers to increase productivity and decrease the psychological stress of maintaining the occupation. Empirical papers show that a worker with richer social networks can be more efficient in contacting business partners (clients and customers) and handling specific work troubles (Fafchamps and Minten 1999, Laband and Lentz 1995, 1999, Falk and Ichino 2005, Khwaja e al. 2008). The mentoring effects of the social network can help to increases job satisfaction, to minimize the turnover rate (Rockoff 2008, Castilla 2005, Cardoso and Winter-Ebmer 2007, Bilimoria et al. 2006), and to heighten the recognition of opportunities in the

[5] In the postwar era of New York, the manufacturing industries where the blacks occupied jobs moved out or eroded while the job opportunities in the service sector continued to grow with whites moving out of the sector. The immigrants who entered the low skilled service sector expanded their economic base through the ethnic networks, while the native blacks left behind jobless. Given employers' preference for hiring through networks, information about job openings rarely penetrated outside the immigrant groups (Waldinger 1996). This empirical evidence brings a very different perspective from the spatial mismatch hypothesis (Kain 1968, Raphael 1998, Ross 1998), which insists that blacks in central cities lost jobs as employment moved to suburbs. The case in New York City reveals that blacks lost jobs even when whites moved out leaving jobs for minorities in the cities.

[6] Other researchers concludes that, among many different job search methods, personal connection of friends or relatives is most widely used among unemployed youth in the US (Holzer 1987,1988, and Blau and Robins 1990), and in the UK (Gregg and Wadsworth 1996) and in Egypt (Assaad 1997, Wahba and Zenou 2005): Holzer (1988) finds 85.2% of jobseekers used friends/relatives ties, 79.6% used direct application without referral, 53.8% used state agency, and 57.8% used newspaper advertisement.In their study, the acceptance rate of job offers obtained through personal connection is highest (eg. about 82 percent in Holzer (1988)), implying that job offers through personal connection generally have higher wages or more appealing nonwage characteristics.

[7] Observing the evidence, Montgomery (1991) constructs a theoretical model that explains why firms hiring through referral might earn higher profits and why workers who are well connected might fare better than poorly connected workers. Montgomery (1992) suggests another interesting model in which the widespread use of employee referrals, combined with a tendency to refer others within their own social network, might generate persistent inequality between groups of workers.

entrepreneurial process (Ozgen and Baron 2007). The empirical work suggests that the better the quality of one's social network, the higher the benefits one can expect, and, consequently, the more incentive one has to invest in the acquisition of skills.

Dynamic Structure of Group Inequality

We conclude that both kinds of externalities – those operating during the education period and those at work over the course of a worker's lifetime – affect a social group's overall skill investment rate.[8] As mentioned, this paper explores the dynamic structure of group inequality generated by the interaction between these two types of network externalities. These two effects operate via different channels. With the education period network externality, change in a group's status tends to be subject to the "past": by altering skill investment cost, the current stock of network human capital directly affects the investment rate in a newborn cohort. By contrast, with the lifetime network externality, change in a group's status tends to be subject to the "future": by altering the future benefits anticipated to accrue from skill acquisition, the expected success of one's network influences skill investment in an entering cohort.

This latter effect implies a unique feature of the dynamic structure: the possibility of workers acting together to improve, or deteriorate, the quality of a group's social network. For instance, suppose that a group's network quality is relatively poor, but that a newborn cohort happens to believe the quality of group's network will be better in the future. If this belief leads more newborn group members to acquire skills, then the next newborn cohort will find the overall network quality has improved because of the enhanced skill investment of the previous cohort. If the next newborn cohort, and the following cohorts, continue to hold the optimistic view of the future, they will keep the enhanced skill investment rate and the quality of group's social network will improve over time thereby justifying the optimistic beliefs of earlier cohorts. However, suppose that the newborn cohort held a pessimistic view that the network quality will be even worse in the future. Fewer members of the newborn cohort will invest in the skill achievement because the expected benefits have declined. As the following cohorts continue to hold the pessimistic view, the network quality will be deteriorated over time. So, this pessimistic belief could also be self-fulfilling.

However, collective action to influence such beliefs may not be feasible for all social groups with unequal network quality. The potential impact of altering beliefs is restricted by the strength of education period network externalities. That is, collective action through optimism or pessimism cannot play any role when the quality of network is too good or too bad.[9] Therefore, the analysis

[8] For example, when a group's social network contains more highly skilled members, then more of its newborns will invest in skills – not only because they have lower costs over the skill acquisition period, but also because they expect greater benefits from a given skill investment to accrue over their lifetimes.

[9] Suppose that a group's network quality is very poor. The newborn group members may consider enhancing the

of the dynamic structure of network externalities focuses on the identification of the following two ranges: (1) the network quality range mainly governed by the *historical force* of the education period network externality, and (2) the network quality range mainly governed by the *mobilization force* of the lifetime network externality. The former is defined as *deterministic range*, and the latter as *overlap*, as Krugman (1991) denotes in his argument for the relative importance of history and expectations. In the dynamic system developed in this paper, there exists a unique equilibrium path in a deterministic range, and there are two equilibrium paths available in an overlap in which a group's expectation toward the future determines the path to be taken.[10] This insight is expanded to the multi-group economy, defining notions of *social consensus* and *folded overlap*. In a folded overlap, two or more equilibrium paths can exist. The path to be taken is determined by the social consensus, which is a combination of groups' expectations toward the future.[11]

An interesting feature of the multi-group economy is the existence of a *network trap*, where a social group maintains a high skill investment rate, and another group is trapped by the "past," that is, the adverse effects of bad-quality education period network externality. To mobilize the disadvantaged group out of the trap, two egalitarian policies are examined: integration and affirmative action. If the disadvantaged group is a minority, the integration policy alone can save the group out of the trap. If it is not, integration may cause both groups to fall down to the lower investment rates, as discussed in Bowles et al. (2007). In this case, a combination of the two policy measures may help to solve the problem.

Macroeconomic effects of Inequality

Finally, I examine the macroeconomic effects of group inequality. Since human capital has been

skill investment rate by holding the expectation that the group's network quality will be improved over time and their skill investment will be paid back in the future. However, they will realize very soon that the scenario would never occur in the real world: the following generations cannot invest enough due to the serious adverse effects of poor quality network externality over the education period, and, consequently, the network quality cannot be improved substantially even in the far future. This is the situation of "the past" that traps the disadvantaged group. The opposite scenario is plausible for the case of network quality that is too good. The newborn group members may consider lowering their skill investment rate by holding a pessimistic expectation toward the future. However, they will soon realize that the scenario would never occur because a sufficient number of following generations would continue to invest, due to the good quality network externality over the education period.

[10] Adsera and Ray (1998) argue that overlap is generated only when agents can have an incentive to choose the option that offers less appealing benefits at the moment of decision. In the example of Krugman (1991) regarding industry specialization, overlap is generated because agents can have an incentive to choose the option that offers even loss at the moving moment, because its cost is lower than the cost of moving in the future. In my model, the incentive is originated by the nature of the overlapping generation structure. Since agents are given only one chance to choose their occupational type at the early stage of their lives, they choose a type that gives less appealing benefits at the moment of skill investment decision, expecting the average lifetime benefits accumulated in the future.

[11] For example, suppose a two-group economy. In a folded overlap with four equilibrium paths available, the economy may evolve to the highest (lowest) level of development with both groups' optimistic (pessimistic) expectations. If one group holds an optimistic expectation and the other holds a pessimistic expectation, the economy will evolve to a mediocre level of development with unequal distribution of wealth between two groups. By identifying folded overlaps and deterministic ranges, we can analyze the dynamic process of the group inequality evolution: conditions under which group disparities grow and conditions under which groups' network qualities converge.

the prime engine for economic growth in the modern economy, aggregate skill investment activities can be directly interpreted as a stage of economic development. Thus far, most of the literature has discussed the topic under the assumption of an imperfect credit market (Loury 1981, Galor and Zeira 1993, Benabou 1996, Durlauf 1996).[12] This paper shows the positive effects of equality on economic growth in most development stages, consistent with Galor and Zeira (1993), even without imposing the assumption of an imperfect credit market. When *social network capital*, the average human capital in one's social network, is more equally distributed, more social groups can be encouraged to develop their skill investment ratios, moving out of the network trap where they had fallen. It is noteworthy that, contrary to the previous theoretical works (Galor and Moav 2004), equality of social network capital can enhance the process of economic development even in the society with a perfect public school system, or in well-developed countries where credit constraints are no longer binding for human capital investment. In addition, this social capital approach demonstrates the positive effects of inequality on economic growth in the early stage of economic development. In the early stage of development, the concentration of social network capital to selective groups may help the economy move out of the low skill investment steady state by giving opportunities for the groups to take the collective action needed to improve their skill investment ratios.

Overall, the model is consistent with the empirical finding that income tends to be more equally distributed in developed countries than in less developed countries, a phenomenon many economists, including Kuznets (1955), tried to explain. By departing from the poor equal society with low skill investment ratios, the economy may move to a more developed stage with some groups in the network trap, where selective groups maintain high investment ratios while others continue the low skill investment activities. Egalitarian policies can move the economy towards the high investment equal society, in which all social groups participate in the high skill investment activities.

As an application of the dynamic network model, I address the regional group inequality issue that emerged in South Korea during its industrialization process. Both Youngnam and Honam groups were in the low investment steady state after the Korean War (1950-53). The Youngnam-based regime in the 1960s through the 1970s helped the regional group to be more successful in an initial state-led industrialization process and, consequently, to move into an overlap area, where the group was given an advantaged position to exercise collective action to increase the group's human capital investment and build a better network quality. After rapid industrialization and urbanization in the 1970s and 1980s, Homan was identified as being in a network trap, where the group's skill investment ratio was significantly less than that of the Youngnam group. As the between-group social interactions

[12] Bowles et al. (2007) have successfully modeled the intergenerational human capital externalities without imposing the imperfect credit market assumption.

proceeded and the political power was transferred to Homan in the 1990s, younger members of the Honam significantly enhanced their skill investment activities. Therefore, both the dominating position of Youngnam group in the early stage of economic development, and the more equal distribution of social network capital in the later stage of economic development, promoted the greater human capital investment of the economy and the faster economic growth.

This paper is organized into the following sections: Section 2 describes the basic structure of the model; Section 3 develops the dynamic model with network externalities and economic players' forward-looking decision making; Section 4 provides an analysis on the homogeneous group economy; Section 5 provides an analysis on the multiple group economy; Section 6 examines the egalitarian policies to mobilize disadvantaged groups out of the network trap; Section 7 examines the macroeconomic effects of inequality; Section 8 presents an application of the dynamic model on the regional group disparity in South Korea; and Section 9 contains the conclusion.

2 Basic Structure of the Model

The analysis in this paper focuses on the two-group economy because the most interesting features of dynamic structure associated with social interactions between groups are contained in the two-group economy. The way to extend to arbitrary n-group economy is discussed in Section 7.1. The two social groups are denoted by group one and group two. Population shares are denoted by β^1 and β^2 respectively with $\beta^1 + \beta^2 \equiv 1$. Suppose that there are two types of occupations, skilled or white color jobs and unskilled or blue color jobs. Each agent decides whether to be a skilled worker or not at his early days of life. Once he becomes a skilled worker, he lives as a skilled worker until he dies. Otherwise, he lives as a unskilled worker until he dies. Let s_t^i denote the fraction of skilled workers in group $i \in \{1, 2\}$ at time t, which is called *group i skill level at time t*. The fraction of skilled workers in the overall population at time t is then $\bar{s}_t \equiv \beta^1 s_t^1 + \beta^2 s_t^2$, which is a proxy of economic development as the economic growth is largely attributed to the human capital accumulations in the modern economy (Abramovitz 1993).

Let σ_t^i denote the fraction of skilled workers in the social network of an individual belonging to group $i \in \{1, 2\}$ at time t, which is called *group i network quality at time t*. This depends on the levels of human capital in each of the two groups as well as the extent of segregation η: $\sigma_t^i \equiv \eta s_t^i + (1 - \eta)\bar{s}_t$. When η=1, σ_t^i is equal to $\bar{s}_t$ for any group i, indicating that there is no difference in the network quality across social groups. When $\eta = 0$, σ_t^i is equal to the skill level of group i (s_t^i), indicating the total segregation across groups. Note that, with this structure, the number of total contacts by group 1 members of group 2 members equals that by group 2 members of group 1 members: $(1 - \eta)\beta^2$ times population share of group 1 (β^1) equals $(1-\eta)\beta^1$ times population share of group 2 (β^2). The σ_t^i is a convex combination of s_t^i and s_t^j with their wights k^i and $1-k^i$,

$$\sigma_t^i \equiv k^i s_t^i + (1-k^i)s_t^j, \text{ where } k^i = \eta + (1-\eta)\beta^i. \tag{1}$$

The k^i represents the degree of influence from its own group skill level and $1-k^i$ represents that from the other group's skill level. It is noteworthy that k^i is an increasing function of the societal segregation level (η), and that of the population share of the group (β^i): as the society is more segregated, its own group skill level influence more on its network quality, and as the population size is bigger, the network quality is less affected by the other group's skill level.

Each newborn individual at time t makes a skill investment decision, comparing the cost of skill acquisition with the expected benefits of investment. The cost to achieve skill at time t depends on the innate ability a and the quality of social network at time t σ_t^i: $C(a, \sigma_t^i)$ is an increasing function in both arguments a and σ_t^i, which satisfies $\lim_{a\to 0} C(a, \sigma_t^i) = \infty$ and $\lim_{a\to\infty} C(a, \sigma_t^i) = 0$ for any $\sigma_t^i \in [0, 1]$. The cost includes both mental and physical costs that are incurred for the skill achievement. The lower one's innate ability or the worse the quality of one's social network, the more mentally stressful the skill acquisition process is or the more materials he must spend for the achievement.

The expected benefits of investment to a newborn individual of group i born at time t, $\Pi_t^i \in (0, \infty)$, is the benefits of skill investment to be realized over the whole lifetime from time t until he dies, which is the difference between the expected lifetime benefits of being skilled ($B_s^i(t)$) and that of being unskilled ($B_u^i(t)$): $\Pi_t^i \equiv B_s^i(t) - B_u^i(t)$. I rule out the high and low skill complementarity for the reasons discussed below. Thus, both $B_s^i(t)$ and $B_u^i(t)$ are functions of the sequence of the expected network quality from time t to infinite: $\{\sigma_\tau^i\}_{\tau=t}^\infty$, implying that $\Pi_t^i \equiv \Pi(\{\sigma_\tau^i\}_{\tau=t}^\infty)$. Let us call Π_t^i *group i benefits of investment at time t*, because it depends on the group specific sequence of network quality. The benefits reflect both psychological and material benefits, about which we will discuss in section 3.2.

With this setup, the education period network externality comes into $C(a, \sigma_t^i)$ and lifetime network externality comes into $\Pi(\{\sigma_\tau^i\}_{\tau=t}^\infty)$. A newborn individual of group i at time t will invest for the skill achievement only when $C(a, \sigma_t^i)$ is less than or equal to $\Pi(\{\sigma_\tau^i\}_{\tau=t}^\infty)$. Suppose that ability $a \in (0, \infty)$ is distributed among newborn cohort in a S-shaped CDF function $G(a)$: there exists $\hat{a}$ such that $G''(a) > 0, \forall a \in (0, \hat{a})$ and $G''(a) < 0, \forall a \in (\hat{a}, \infty)$, implying that its PDF function is bell-shaped. Suppose that $G(a)$ is identical for all groups, consistent with Loury's (2002) axiom of anti-essentialism. We can find the threshold ability level for group i such that newborn individuals of group i whose innate ability is at least the threshold invest in the skill acquisition.

Lemma 1. *Given* $\{\sigma_t^i\}_{t\to\infty}$*, there exists a unique threshold ability level* $\tilde{a}_t^i$ *that satisfies* $C(\tilde{a}_t^i, \sigma_t^i) = \Pi(\{\sigma_\tau^i\}_{\tau=t}^{\infty})$.

Proof. This is derived from the fact that $C(a, \sigma_t)$ is a decreasing function with respect to a that satisfies $\lim_{a\to 0} C(a, \sigma_t) = \infty$ and $\lim_{a\to\infty} C(a, \sigma_t) = 0$ for any $\sigma_t \in [0, 1]$. ■

Let us define a function A which represents the unique threshold ability: $\tilde{a}_t^i \equiv A(\sigma_t^i, \Pi_t^i)$. A is a decreasing function with respect to both arguments σ_t^i and Π_t^i. Thus, the fraction of time t newborn individuals of group i who invest in skill, denoted by x_t^i, is expressed by

$$x_t^i = 1 - G(A(\sigma_t^i, \Pi_t^i)). \tag{2}$$

In developing the basic structure of this model, I am indebted to Bowles et al. (2007) for suggesting the simplest way to encompass both the intergenerational network externality and the extent of segregation. The smart representation of the newborn cohort's decision making helps to reflect the intergenerational network externality without imposing the assumption of credit market imperfections, which most previous theoretical work on the intergenerational dynamics relied on (Loury 1981, Banerjee and Newman 1993, Galor and Zeira 1993). The so called (η, β) structure used in the paper to represent the segregation level in a society enables the model to reflect the integration effects in the simplest and the most effective way.[13] Bowles et al., assuming the presence of education period network externality, prove the instability of an equal society in a highly segregated economy combined with the complementarity of high- and low-skill workers, as well as the instability of an unequal society in a highly integrated economy.

My main departure from their model is the replacement of the two-period overlapping generational structure with an infinite horizon structure. With this adjustment, we let the economic agents at the decision moment of skill investment to consider the benefits of skill investment accrued over the entire lifetime. Another departure is the assumption of exogenous wages: I rule out the high-low skill complementarity, which was an essential part of Bowles et al., in order to concentrate on the net effects of the two types of network externalities, education period and lifetime. The assumption of no complementarity can arguably be accepted for the correct description of the modern economy, where the human capital is considered the prime engine of economic development, due to skill-biased technologies and international capital flow (Goldin and Katz 2001, Galor and Moav 2004). The wage divergence between skilled and unskilled workers caused by trade openness and globalization does support the exogenous wages in the modern world (Wood 1994, Richardson 1995). One important

[13] The η indicates the level of segregation in a society, and β indicates the population share of a disadvantaged group. Chaudhuri and Sethi (2008) is another paper that adopts the structure.

departure is an introduction of the S-shaped $G(a)$ function. With this specific functional form, which is arguably the best way to reflect the innate ability distribution among a newborn cohort, I can explicitly express the dynamic structure with network externalities in a heterogeneous group economy, and explore the macroeconomic effects of inequality.

3 Dynamic Model with Network Externalities

In this section, I construct the dynamic system for the two-group economy, in which two types of network externalities, education period and lifetime, play a crucial role in explaining the evolution of group skill level s_t^i and that of group benefits of investment Π_t^i.

3.1 Education Period Network Externality and Evolution of Group Skill Level

I assume that a worker is subject to the "poisson death process" with parameter α: in a unit period, each individual faces α chances to die. We assume that the total population of each group is constant, implying that the α fraction of a group's population is replaced by newborn group members in a unit period. Since x_t^i is the fraction of newborn group members who invest in skill, the group i's skill level s_t^i evolves in a short time interval Δt in the following way:

$$s_{t+\Delta t}^i \approx \alpha \ \Delta t \cdot \left(\frac{x_t + x_{t+\Delta t}}{2} \right) + (1 - \alpha \Delta t) \cdot s_t.$$

By the rearrangement of the equation, we have

$$\frac{\Delta s_t^i}{\Delta t} \equiv \frac{s_{t+\Delta t}^i - s_t^i}{\Delta t} \approx \alpha \left[\frac{x_t^i + x_{t+\Delta t}^i}{2} - s_t^i \right].$$

Taking $\Delta t \to 0$, we have the evolution rule of group skill level,

$$\dot{s}_t^i = \alpha [x_t^i - s_t^i]. \tag{3}$$

There is a direct way to achieve the same result. We can define s_t^i as $s_t^i \equiv \int_{-\infty}^{t} \alpha x_\tau^i e^{-\alpha(t-\tau)} d\tau$. Taking a derivative with respect to time t, we have $\dot{s}_t = \alpha[x_t - s_t]$. The speed of group skill level change is determined by the difference of the skill level of newborn cohort and the skill level of "old" cohorts. If the fraction of newborn group members who invest in skill acquisition is greater than the fraction of skilled workers in the group population, the group skill level improves. Otherwise, it declines. There is no change in skill level, if the fraction of newborn members who invest in skill is equal to the group's current skill level. Combining this with the determination rule of x_t^i in equation (2), we have the evolution rule with education period network externality reflected,

$$\dot{s}_t^i = \alpha[1 - G(A(\sigma_t^i, \Pi_t^i)) - s_t^i]. \tag{4}$$

3.2 Lifetime Network Externality and Evolution of Group Benefits of Investment

As discussed earlier, we rule out the high and low skill complementarity. The expected benefits of skill investment depends on the expected quality of the network in the future and the exogenous wage levels for each type of worker. Let us assume the base level salary for a skilled worker (white-color worker) is w_s and that for an unskilled worker (blue-color worker) is w_u. A skilled worker obtains extra benefits from his social network, both psychological and material. The more skilled workers he has in his network, the more appropriate job position he may find for his specific skills. He may get more comfort and mentoring out of the informal network, and the cost for maintaining jobs may decline. Information flows along the synapses of the social network. A skilled worker can be more efficient in contracting his customers and handling specific work troubles with more skilled workers in his network. Let $S_s(\sigma_t^i)$ denote the extra benefits of having skilled workers in the social network of a skilled worker from group i. Even an unskilled worker may obtain more benefits from having more skilled workers in his network, but to a lesser degree than a skilled worker would get. Let $S_u(\sigma_t^i)$ denote the extra benefits of having skilled workers in the social network of an unskilled worker from group i. Both $S_s(\sigma_t^i)$ and $S_u(\sigma_t^i)$ are increasing functions of σ_t^i. We assume that $S_j(0) = 0, \forall j \in (s, u)$, and $\frac{\partial S_s}{\partial \sigma_t^i} > \frac{\partial S_u}{\partial \sigma_t^i}$, implying that a skilled worker would obtain higher marginal benefits for having an additional skilled worker in his social network.

In the same way, an unskilled worker would obtain more extra benefits from having more unskilled workers in his network. For example, a car mechanic would find a better car center that fits his speciality when he has more mechanics in his network. He would be more efficient in handling a specific mechanical problem when he can confer with more mechanics in his network. Since $(1 - \sigma_t^i)$ represents the fraction of unskilled workers in the social network of worker from group i, let $U_u(1-\sigma_t^i)$ denote the extra benefits of having unskilled workers in the social network of an unskilled worker from group i. Even a skilled worker would obtain more benefits from having more unskilled workers in his network, but obviously to a lesser degree than an unskilled worker would get. In the previous example, at least, he would find a better car center to fix his broken car when he has more mechanics in his network. Let $U_s(1 - \sigma_t^i)$ denote the extra benefits of having unskilled workers in the social network of a skilled worker from group i. Both $U_u(1 - \sigma_t^i)$ and $U_s(1 - \sigma_t^i)$ are increasing functions of $(1 - \sigma_t^i)$. We assume that $U_j(0) = 0, \forall j \in (s, u)$, and $\frac{\partial U_u}{\partial(1-\sigma_t^i)} > \frac{\partial U_s}{\partial(1-\sigma_t^i)}$, implying that an unskilled worker would obtain higher marginal benefits than a skilled worker from having an additional unskilled worker in his social network.

Suppose a worker discounts the benefits realized in the future with a discounting factor ρ. We have assumed that each worker faces α chances to die in a unit period, under the poisson process. Accordingly, given the sequence of the expected network quality from time t to infinity, $\{\sigma_\tau^i\}_{\tau=t}^{\infty}$, the expected lifetime benefits of being skilled ($B_s^i(t)$) and that of being unskilled ($B_u^i(t)$) are

$$\begin{aligned} B_s^i(t) &= \int_t^\infty [w_s + S_s(\sigma_\tau^i) + U_s(1-\sigma_\tau^i)]e^{-(\rho+\alpha)(\tau-t)}\mathrm{d}\tau, \\ B_u^i(t) &= \int_t^\infty [w_u + S_u(\sigma_\tau^i) + U_u(1-\sigma_\tau^i)]e^{-(\rho+\alpha)(\tau-t)}\mathrm{d}\tau, \end{aligned}$$

where ρ is a discounting factor and α is a poisson death rate. Since the expected benefits of investment to a group i individual born at time t is $\Pi_t^i \equiv B_s^i(t) - B_u^i(t)$,

$$\Pi_t^i = \int_t^\infty [w_s - w_u + S_h(\sigma_\tau^i) - S_u(\sigma_\tau^i) + U_s(1-\sigma_\tau^i) - U_u(1-\sigma_\tau^i)]e^{-(\rho+\alpha)(\tau-t)}\mathrm{d}\tau.$$

Replacing $w_s - w_u$ with $\bar{\delta}$, and $S_h(\sigma_\tau^i) - S_u(\sigma_\tau^i) + U_s(1-\sigma_\tau^i) - U_u(1-\sigma_\tau^i)$ with $f(\sigma_t^i)$, we have

$$\Pi_t^i = \int_t^\infty [\bar{\delta} + f(\sigma_\tau^i)]e^{-(\rho+\alpha)(\tau-t)}\mathrm{d}\tau, \tag{5}$$

where $\bar{\delta}$ indicates the base salary differential and, $f(\sigma_\tau^i)$ the difference between the extra benefits of being skilled and that of being unskilled at time τ. Let us call $\bar{\delta} + f(\sigma_\tau^i)$ *time τ net benefits of being skilled*, which is an increasing function of σ_t^i because $\frac{\partial S_s}{\partial \sigma_t^i} > \frac{\partial S_u}{\partial \sigma_t^i}$ and $\frac{\partial U_u}{\partial(1-\sigma_t^i)} > \frac{\partial U_s}{\partial(1-\sigma_t^i)}$. The more skilled workers at time τ in a worker's social network, the greater the net benefits of being skilled. I assume that $\bar{\delta} + f(0) > 0$, which implies that the base salary differential is big enough that the net benefits of being skilled is always positive.

Taking the derivative with respect to time t, we have the evolution rule of the group i benefits of investment Π_t^i,

$$\dot{\Pi}_t^i = (\rho+\alpha)\left[\Pi_t^i - \frac{\bar{\delta} + f(\sigma_t^i)}{\rho+\alpha}\right]. \tag{6}$$

The change of the lifetime benefits of investment evaluated at time t is determined by the difference between the current level of lifetime benefits of investment and the current level of net benefits of being skilled. If the current level of net benefits of being skilled $\left(\frac{\bar{\delta}+f(\sigma_t^i)}{\rho+\alpha}\right)$ is greater than the current lifetime benefits of investment (Π_t^i), at the next time $t + \Delta t$, lifetime benefits of investment would be smaller than the current level: $\Pi_{t+\Delta t}^i < \Pi_t^i$. If they are equal to each other, there will be no change in the expected lifetime benefits of investment.

3.3 Dynamic System with Network Externalities

Thus far, we have examined how two variables, group skill level s_t^i and group benefits of investment Π_t^i, evolve over time. The first variable indicates the fraction of skilled workers in the population of group i. This is adjusted every minute by the fraction of skilled workers among the group i newborn cohort. Thus, it is a flow variable, which cannot make a sudden jump at a point of time. The second variable indicates the expected benefits of investment for a newborn individual of group i born at time t, which is determined by the sequence of the expected network quality in the future, $\{\sigma_\tau\}_{\tau=t}^{\infty}$. Since it depends on the expected network qualities, it can make a sudden jump at any point of time by changing the expectation of $\{\sigma_\tau\}_{\tau=t}^{\infty}$. Thus, it is a jumping variable.

In the dynamic system that represents a two-group economy, there exist two flow variables, s_t^1 and s_t^2, and two jumping variables, Π_t^1 and Π_t^2. Using equations (4) and (6), we can construct the following dynamic system.[14]

Theorem 1 (Dynamic System)**.** *In a two-group economy, the dynamic system with two flow variables s_t^1 and s_t^2 and two jumping variables Π_t^1 and Π_t^2 is summarized by the following four-variable differential equations:*

$$\begin{aligned}
\dot{s_t}^1 &= \alpha\left[1 - G(A(\sigma_t^1, \Pi_t^1)) - s_t^1\right] \\
\dot{s_t}^2 &= \alpha\left[1 - G(A(\sigma_t^2, \Pi_t^2)) - s_t^2\right] \\
\dot{\Pi}_t^1 &= (\rho+\alpha)\left[\Pi_t^1 - \frac{\bar{\delta} + f(\sigma_t^1)}{\rho+\alpha}\right] \\
\dot{\Pi}_t^2 &= (\rho+\alpha)\left[\Pi_t^2 - \frac{\bar{\delta} + f(\sigma_t^2)}{\rho+\alpha}\right],
\end{aligned}$$

where

$$\begin{aligned}
\sigma_t^1 &= k^1 s_t^1 + (1-k^1)s_t^2, \ \textit{with } k^1 = \eta + (1-\eta)\beta^1 \\
\sigma_t^2 &= k^2 s_t^2 + (1-k^2)s_t^1, \ \textit{with } k^2 = \eta + (1-\eta)\beta^2.
\end{aligned}$$

4 Homogeneous Group Economy

The dynamic system in Theorem 1 is defined in a four-dimensional space of (s^1, s^2, Π^1, Π^2). It is hard to have a clear imaginary view of the dynamic structure through the phase diagram, the direction field, and the stationary points in this complex system. Even after succeeding in clarifying those technical aspects, the intuitive interpretation of the system is a more challenging task. Let us start

[14] Refer to Section 7.1 for the expansion of this dynamic system to the arbitrary n-group economy.

with a simplest structure of the economy, in which two social groups are fully integrated becoming a homogeneous social group, or in which a social group is totally separated from all other social groups. In the middle of the analysis of this homogeneous group economy, we define the essential concepts to interpret the dynamic system with network externalities. In section 5, we will come back to the two-group economy.

4.1 Steady States and Economically Stable States

Suppose a homogeneous social group or social groups in a fully integrated society, in which $s_t = \sigma_t$. The skill level at time t directly represents the network quality at time t. The dynamic system is simply

$$\begin{aligned} \dot{s}_t &= \alpha[1 - G(A(s_t, \Pi_t)) - s_t] \\ \dot{\Pi}_t &= (\rho + \alpha)\left[\Pi_t - \frac{\bar{\delta} + f(s_t)}{\rho + \alpha}\right], \end{aligned} \tag{7}$$

and two demarcation loci (isoclines) of the time dependent variables are represented by

$$\dot{s}_t = 0 \text{ Locus} \quad : \quad s_t = 1 - G(A(s_t, \Pi_t)) \tag{8}$$

$$\dot{\Pi}_t = 0 \text{ Locus} \quad : \quad \Pi_t = \frac{\bar{\delta} + f(s_t)}{\rho + \alpha}. \tag{9}$$

In the demarcation locus for $\dot{\Pi}_t$, Π_t is simply an increasing function of s_t, as denoted by a dotted curve in Panel B of Figure 1. The demarcation locus for $\dot{s}_t$ is represented by (s_t, Π_t) that satisfies the following two equations that are associated with the threshold ability level for the skill achievement:

$$\begin{aligned} s_t &= 1 - G(\tilde{a}) \\ \tilde{a} &= A(s_t, \Pi_t). \end{aligned}$$

The first one is denoted by the solid curve in Panel A of Figure 1 in $(s_t, \tilde{a})$ domain, which is simply a S-shaped curve. The second one is denoted by the dotted curves for each level of Π_t (iso-Π curves), in the same panel. As Π_t increases, the corresponding iso-Π curve moves down. (The curves tend to be convex as the marginal impact of a network quality on the threshold ability level may decline as s_t increases.) The combinations of (s_t, Π_t) that satisfy the above two equations are represented by the solid curve in Panel B of the figure, which is the demarcation locus for $\dot{s}_t$. Since we have achieved two demarcation loci, we can identify steady states in this system.

Lemma 2. *If there exist s' and s'', where $s' < s''$, that satisfy both $s' > 1 - G(A(s', \frac{\bar{\delta}+f(s')}{\rho+\alpha}))$ and*

$s'' < 1 - G(A(s'', \frac{\bar{\delta}+f(s'')}{\rho+\alpha}))$, *there are at least three steady states.*

Proof. See the proof in the appendix. ■

Note that, as long as the the network externality in the skill acquisition period is strong enough ($\frac{\partial A}{\partial s_t}$ is big enough), the dotted curves in Panel A (iso-Π curves) are tangent to the S-shaped curve ($s_t = 1 - G(\tilde{a})$ curve) at two distinct points, $(s_u, \tilde{a}_u)$ with $\Pi_t = \Pi_u$ and $(s_d, \tilde{a}_d)$ with $\Pi_t = \Pi_d$, where $\Pi_u > \Pi_d$. In the specific case with two tangent points, the multiple steady state condition in Lemma 2 is simply satisfied when $\Pi_u > \frac{\bar{\delta}+f(s_u)}{\rho+\alpha}$ and $\Pi_d < \frac{\bar{\delta}+f(s_d)}{\rho+\alpha}$, as you can observe in Panel B of Figure 1.

Without loss of generality, we assume that there exist three steady states when the condition of Lemma 2 is satisfied.[15] When the condition is not satisfied, it is most likely that there exists a unique steady state.[16] For example, if the base salary differential $\bar{\delta}$ is too big or too small, the $\dot{\Pi}_t = 0$ locus will be placed too high or too low, and there is a unique intersection between the two loci. Whatever the initial position s_0 is, the group state will move toward the unique steady state.[17] That is, if the base salary for a skilled job is much greater (smaller) than that for an unskilled job, the group skill level s_t converges to a high (low) skill steady state, regardless of the initial network quality. This is certainly not an interesting case: the network externalities do not generate any difference in the final economic outcome. Therefore, we will focus on the case with three steady states in this paper.

Proposition 1. *Without loss of generality, there is a range of the base salary differential* $\bar{\delta} \in (\bar{\delta}_l, \bar{\delta}_h)$, *with which there exists three distinct steady states in a homogenous group economy.*

Proof. Without loss of generality, assume that the dynamic system has two distinct salary differentials $\bar{\delta}_l$ and $\bar{\delta}_h$, with which the $\dot{\Pi}_t = 0$ locus is tangent to the $\dot{s}_t = 0$ locus in the (s_t, Π_t) plane. Between the two levels, there will be at least three steady states. Without loss of generality, there are three steady states in the interval $(\bar{\delta}_l, \bar{\delta}_h)$. ■

Let us denote the three steady states by $E_l(s_l, \Pi_l)$, $E_m(s_m, \Pi_m)$ and $E_h(s_h, \Pi_h)$, where $s_l < s_m < s_h$. The final economic state (s, Π) will be on either one of them. In order to examine the conversing process to a steady state, we need a phase diagram with direction arrows (laws of motion), which are displayed in Figure 2. The characteristics of the steady states are summarized by the following lemma.

Lemma 3. *Among three steady states,* $E_l(s_l, \Pi_l)$, $E_m(s_m, \Pi_m)$ *and* $E_h(s_h, \Pi_h)$, E_l *and* E_h *are saddle points and* E_m *is a source.*

[15] There exist more than three steady states only for very peculiar functional forms of G, A or f.

[16] It is possible that there are only two steady states, for example, when the $\dot{\Pi}_t = 0$ locus is tangent to the $\dot{s}_t = 0$ locus. We ignore this case because it can occur with a measure zero probability.

[17] The unique steady state is a saddle point, which is easily proven as the same way for the proof of Lemma 3. The economic state with an arbitrary s_0 converges to the unique steady state following the saddle path.

Proof. See the proof in Appendix. ■

We can identify the equilibrium path (saddle path) to each saddle point, E_l and E_h, as described in Figure 2. In the given example of the figure, the equilibrium paths spiral out of a source E_m. Given an initial network quality $s_0 \in (0,1)$, the newborn agents will calculate the expected benefits of investment Π_0. Based on the calculated Π_0, each agent with different innate ability (a) makes his own skill investment decision. In their calculation of Π_0, they will use the evolution rules of s_t and Π_t, summarized in the formula (7). They understand that either E_h or E_l should be the final economic state. If they believe that E_h will be realized in the future, they will choose Π_0 on the equilibrium path to E_h. As the following generations keep the same belief, the group state will be moving along the equilibrium path, and eventually arrive at E_h. If they believe that E_l will be realized in the future, they will choose Π_0 on the equilibrium path to E_l. As the following generations keep the same belief, the group state will move toward E_l along the path. Since there is no equilibrium path that converges at E_m, the newborn agents who understand the evolution rules of s_t and Π_t will never choose E_m as the final group state. Choosing E_h is called sharing an optimistic social consensus, while choosing E_l is called sharing a pessimistic social consensus.

Although all three steady states are mathematically unstable, two saddle points, E_l and E_h, are "economically" stable in a sense that there exists a converging path to these states for any perturbation at the states: rational economic agents who understand the dynamic system can take the saddle path to lead the group back to the original state. However, the source E_m is "economically" unstable, because any small perturbation from the state will lead the group to move away from it: rational economic agents will take either the optimistic path to E_h or the pessimistic path to E_l, because there is no converging path to E_m.

Definition 1 (Economically Stable States)**.** *A state (s', Π') is an economically stable state if there exists a converging phase path to the state for any s in the neighborhood of s'. Otherwise, it is an economically unstable state.*

In the given economy with three steady states, E_l and E_h are economically stable states and E_m is an economically unstable state.

4.2 Overlap and Deterministic Ranges

Let the network quality e_o denote the lower bound of the optimistic path to E_h, and the network quality e_p the upper bound of the pessimistic path to E_l. As shown in the example of Figure 2, there exists a unique optimistic path for an initial network quality in the interval $(e_p, 1)$: if the initial network quality is good enough, there is only one reasonable social consensus about the future, that is E_h, and the group state (s_t, Π_t) will move toward the high skill equilibrium (s_h, Π_h) by self-fulfilling investment activities. Also, there exists a unique pessimistic path for an initial network quality in the interval $(0, e_o)$: if the initial network quality is poor enough, there is only one reasonable social consensus about the future, that is E_l, and the group state (s_t, Π_t) moves toward the low skill equilibrium (s_l, Π_l), by the self-fulfilling investment activities. However, if the initial network quality is mediocre in (e_o, e_p), there exist multiple social consensuses about the future, E_l and E_h, that are available to the group. The final economic state depends on the social consensus that the newborn agents of the group choose. If they and the following generations are optimistic, E_h will be realized in the end. If they all are pessimistic, E_l will be realized in the end. Therefore, in the interval $[e_o, e_p]$, the social consensus determines the future, while the historical position determines the future outside the interval. We denote $[e_0, e_p]$ as *overlap* as Krugman (1991) denotes. We denote the ranges $(0, e_p)$ and $(e_o, 1)$ as deterministic ranges as the macroeconomic literature denotes.

Proposition 2. *In a homogeneous group economy with two economically stable states E_l and E_h, there exists a positive range of overlap, $[e_o, e_p]$, with $e_o < e_p$, where the social consensus about the future determines the final economic state among E_l and E_h.*

Proof. See the proof in Appendix. ■

Corollary 1. *In a homogeneous group economy with two economically stable states E_l and E_h, the deterministic range for E_h is $(e_p, 1)$, and the deterministic range for E_l is $(0, e_o)$, where $e_o < e_p$.*

Proof. Since there exists a unique equilibrium path outside the overlap, both ranges $(e_p, 1)$ and $(0, e_o)$ are deterministic ranges. Since e_p is the upper bound of the pessimistic path toward E_l, $(e_p, 1)$ is a deterministic range for E_h. Since e_o is the lower bound of the optimistic path toward E_h, $(0, e_o)$ is a deterministic range for E_l. ■

4.3 Mobilization Force and Historical Force

There are two kinds of network externalities, education period and lifetime, that affect the structure of the economy. In order to examine the direct effect from each network externality, we compare three distinct cases: 1) lifetime network externality only 2) education period network externality only and 3) both education and lifetime network externalities.

Lifetime Network Externality Only: the Mobilization Force

Panel A of Figure 3 describes the first case: there exists very negligible peer effects or parental effects together with perfect provision of public schooling or no credit constraints in the skill acquisition period. Since there is almost no education period network externality, the iso-Π curves described in

Panel A of Figure 1 is almost flat: $\frac{\partial A(s_t,\Pi_t)}{\partial s_t} \simeq 0$, or $\tilde{a} \equiv A(\Pi_t)$, ignoring the s_t term. Therefore, the $\dot{s}_t = 0$ locus will be like a S-shaped curve that satisfies $s_t = 1 - G(A(\Pi_t))$. As you can observe in Panel A of Figure 3, the overlap may to cover the whole range of network quality $[0,1]$.[18] In this case, the historical position of initial network quality s_0 does not provide any constraint in the determination of the final economic state. The final economic state entirely depends on the social consensus, regardless of s_0. (With the bigger discounting factor ρ, the overlap may not cover the whole range of network quality $[0,1]$.[19] Even in this case, the overlap range will be much greater than that in Panel C of Figure 3.) In other words, skill investment activities of newborn agents tend to be subject to the "future": the expected benefits of skill acquisition that accrues over the lifetime.

Suppose the initial network quality is $s_0 \in (s_l, s_h)$. If the group members believe that the final state is E_h instead of E_l, the future benefits anticipated to accrue from skill acquisition Π_0^{op} are greater than the current level of network benefits ($\frac{\bar{\delta}+f(s_0)}{\rho+\alpha}$), and more newborns invest in skills. The group's network quality improves over time and the group state converges to (s_h, Π_h) along the optimistic path, as displayed in Panel A of Figure 1. If they believe that the final state is E_l instead of E_h, the future benefits anticipated to accrue from skill acquisition Π_0^{pe} is smaller than the current level of network benefits ($\frac{\bar{\delta}+f(s_0)}{\rho+\alpha}$), and less newborns invest in skills. The group's network quality deteriorates and the group state converges to (s_l, Π_l). Therefore, the social consensus toward the future determines the future. Group members can work together to improve the quality of the group's social network by sharing the optimistic social consensus, or to deteriorate the quality of the network by sharing the pessimistic social consensus. This is what I call the *mobilization force* of network externalities.

Education Period Network Externality Only: the Historical Force

Panel B of Figure 3 describes the opposite regime, where there is no network externality over the course of a worker's lifetime. The benefits of skill acquisition is just the wage differential $\bar{\delta}$ at each period, and the consequent lifetime benefits are $\frac{\bar{\delta}}{\rho+\alpha}$. In this case, the expectation toward the future does not play any role because the benefits of skill acquisition is fixed. The skill investment activities of newborns are subject to the "past": the cost level to achieve the skill. If the initial network quality is good, the cost for the skill achievement is low. Consequently, a large fraction of newborns invest in skill. Then, the network quality in the next period is even better because of the enhanced skill investment rate in the previous period. Thus, even more newborns invest in skill in the next period.

[18]In this case, another limit set exists, which is a loop located between the optimistic path and the pessimistic path. If economic agents believe that the network quality will fluctuate forever, the group state will be on this loop. I rule out this unique case in my study.

[19]Note that, the bigger ρ, the bigger $\dot{\Pi}_t$ and the longer the vertical direction arrows in the phase diagram. As the slopes of the saddle paths are steeper, the overlap may not cover the whole range of network quality $[0,1]$, and is limited between $[s_l, s_h]$.

The network quality improves over time. If the initial network quality is poor, a small fraction of newborns invest in skill. The next period network quality is even worse, and even less newborns invest in skills. The network quality deteriorates over time.

The two examples are displayed in the Panel. If the initial network quality is bad, below s_m, the network quality converges to the low skill equilibrium E_l. If it is good, above s_m, the network quality converges to the high skill equilibrium E_h. Therefore, the final economic state entirely depends on the history, an initial network quality of the group. This is what I call the *historical force* of network externalities.

Both Lifetime and Education Period Network Externalities: Two Forces Combined

Panel C of the figure display how the two forces of network externalities are interwoven in the dynamic structure of network quality evolution. The mobilization force of lifetime network externalities is constrained by the historical force of education period network externalities. The overlap is the network quality range mainly governed by the mobilization force, while the deterministic ranges are the network quality ranges mainly governed by the historical force. In the overlap, the group can be mobilized toward the high skill steady state E_h by sharing the optimistic view together, or toward the low skill steady state E_l by sharing the pessimistic view together. Outside the overlap, it is the initial historical position that determines the final state. If it is high (low) enough, the group status moves toward the high (low) skill steady state.

4.4 Size of Overlap

In the previous sections, the importance of overlap has been emphasized. Whether the initial position is in the overlap or outside the overlap determines whether the group members can work together to change the future by sharing optimism (or pessimism). Outside the overlap, the future is determined through a mechanical tatonnement process. The bigger size of overlap indicates the dynamic structure that is more governed by the mobilization force or the power of collective action. It is worthwhile to check how the overlap size is determined, because this is an indication of the relative power of the mobilization force and the historical force. In order to make a comparative statics analysis, let us simplify the given model using the linear functional forms of the cost function $C(a, s_t)$ and the benefits function $f(s_t)$:

$$C(a, s_t) = \psi(a) - k_1 s_t \tag{10}$$

$$f(s_t) = q_0 + q_1 s_t, \tag{11}$$

where k_1 represents the influence of education period network externality, and q_1 the influence of lifetime network externality. We further assume that the innate ability equals across the population: $a \equiv \bar{a}$. The newborn agents with the unique innate ability $\bar{a}$ decide to invest when the benefits is greater than the cost: if $\Pi_t > \psi(\bar{a}) - k_1 s_t$, $x_t = 1$ and $\dot{s} = \alpha[1 - s_t]$, and if $\Pi_t < \psi(\bar{a}) - k_1 s_t$, $x_t = 0$ and $\dot{s} = \alpha[0 - s_t]$. Therefore, the slanting part of the $\dot{s} = 0$ locus in Appendix Figure 1 is $\Pi_t = \psi(\bar{a}) - k_1 s_t$, which is a demarcation line that sharply divides the evolution rule of $\dot{s}_t$. The demarcation locus for Π_t is $\Pi_t = \frac{\bar{\delta}+q_0+q_1 s_t}{\rho+\alpha}$. In this simple system without hurting the essential structure of the economy, we can explicitly find the optimistic and pessimistic paths and the relevant size of overlap.

Lemma 4. *In the simple homogenous economy with equations* (10) *and* (11) *and the unique innate ability level of $\bar{a}$, the optimistic equilibrium path (s^{op}, Π^{op}) above two demarcation loci and the pessimistic equilibrium path (s^{pe}, Π^{pe}) below two demarcation loci are*

$$\begin{aligned} \Pi^{op} &= \frac{q_1}{\rho + 2\alpha} s^{op} + \frac{(\bar{\delta} + q_0)(\rho + 2\alpha) + q_1 \alpha}{(\rho + \alpha)(\rho + 2\alpha)} \\ \Pi^{pe} &= \frac{q_1}{\rho + 2\alpha} s^{pe} + \frac{\bar{\delta} + q_0}{\rho + \alpha}. \end{aligned}$$

Proof. See the proof in Appendix. ■

This linear equilibrium paths are described in Appendix Figure 1 with the corresponding demarcation loci. Using the calculated equilibrium paths and the slanting part of the $\dot{s} = 0$ locus ($\Pi = \psi(\bar{a}) - k_1 s$), we can obtain the overlap size $\bar{L}$:

$$\overline{L} = \frac{\alpha}{(\alpha + \rho)(1 + (k_1/q_1)(\rho + 2\alpha))}. \tag{12}$$

Using the outcomes, we have the following results that have deep economic implications.

Proposition 3. *In a simple economy defined in Lemma 4, the bigger the relative influence of lifetime network externality over education period network externality (the bigger q_1/k_1), the larger the size of overlap. The less the economic agents discount the future (the smaller ρ), the larger the size of overlap.*

Proof. The first derivatives of equation (12) give the results. ■

The first result implies that, when the lifetime network externality is relatively more influential, the mobilization force of network externalities is stronger compared to their historical force; collective action facilitated by the formation of social consensus can play a bigger role in the determination of the final economic state. The lower discounting factor means the greater forward-looking decision making of economic agents, which implies the expectation toward the future can play a bigger role in the determination of the final outcome. This fact is reflected in the second result of the proposition.

5 Heterogeneous Group Economy

Now we come back to the two group economy summarized in the four variable dynamic system of Theorem 1. We assume that the conditions in Lemmas 2 and 3 are applied to this economy, so that there exist three steady states at skill levels s_l, s_m and s_h in the fully integrated economy, or in each group's economy with social interactions fully separated between two groups. Note that there will be no group disparity in the long run if there exists a unique steady state: whatever the initial skill composition (s_0^1, s_0^2) is, the economy state (s_t^1, s_t^2) converges to the unique steady state as time goes by. Therefore, there will no issue for the persistent group disparity through the channel of network externalities in this case, and this is certainly not an interest in this study.

5.1 Heterogeneous Economy with Total Segregation

Let us start with the simplest case of the two group economy: fully separated social interactions between two groups ($\eta = 1$), which can help us to have an intuition about the four dimension dynamic structure of two group economy. The structure of this special case can be directly inferred from the properties of the homogeneous economy because there are no interactions between the two. Using the same definition of economically stable states in the homogeneous economy (Definition 1), a steady state $(s^{1\prime}, s^{2\prime}, \Pi^{1\prime}, \Pi^{2\prime})$ is called an economically stable state if there exists a converging phase path to the state in the neighborhood of $(s^{1\prime}, s^{2\prime})$. Obviously, there are nine steady states in this economy: $Q_{ij}(s_i, s_j, \Pi_i, \Pi_j)$ for $i \in \{l, m, h\}$ and $j \in \{l, m, h\}$. Among them, the following four are economically stable states: $Q_{ll}(s_l, s_l, \Pi_l, \Pi_l)$, $Q_{lh}(s_l, s_h, \Pi_l, \Pi_h)$ $Q_{hl}(s_h, s_l, \Pi_h, \Pi_l)$ and $Q_{hh}(s_h, s_h, \Pi_h, \Pi_h)$. Those are depicted with dark circles in (s^1, s^2) domain in Figure 4. The other five economically unstable states are depicted with gray circles in the domain. Two separated dynamic structures are displayed beside the domain in the figure. In an economically unstable state, any arbitrary shock to the position may lead the economic state $(s_t^1, s_t^2, \Pi_t^1, \Pi_t^2)$ to move away from it, while the economic state can come back to the original steady state after any small shock in an economically stable state.

Since there are four economically stable states, a society with an initial skill composition (s_0^1, s_0^2) will move to either of them in the long run. Once a social consensus about the future is constructed in the society, the economic state $(s_t^1, s_t^2, \Pi_t^1, \Pi_t^2)$ will move to the future state of social consensus following a unique converging path to the state. Let us check available social consensuses and corresponding equilibrium paths for each initial skill composition (s_0^1, s_0^2) in this totally segregated economy.

5.1.1 Stable Manifolds and Manifold Ranges

First, regardless of the position of s_0^2, group 1 with an initial skill level s_0^1 can move toward either the skill level s_l or s_h following the same rule in the homogeneous economy. The optimistic path to s_h is available to the group with an initial skill position $s_0^1 \in [e_o, 1]$ and the pessimistic path is available to the group with an initial skill position $s_0^2 \in [0, e_p]$. Those available equilibrium paths are described in Panel A of Figure 5: the available optimistic path in pink and the pessimistic path in blue. In the interval (e_o, e_p), which is an overlap, both optimistic and pessimistic paths are available. The same is true for group 2, as displayed in Panel B of the figure.

Therefore, when both s_0^1 and s_0^2 are greater or equal to e_o, the equilibrium path to $Q_{hh}(s_h, s_h, \Pi_h, \Pi_h)$ is available to the society. The unique converging path to Q_{hh} is a combination of two optimistic equilibrium paths, $(s_t^1, \Pi_t^1)_{op}$ for group 1 and $(s_t^2, \Pi_t^2)_{op}$ for group 2. In the same way, the equilibrium path to $Q_{ll}(s_l, s_l, \Pi_l, \Pi_l)$ is available when both s_0^1 and s_0^2 are smaller or equal to e_p. The unique converging path to Q_{ll} is the combination of two pessimistic paths, $(s_t^1, \Pi_t^1)_{pe}$ and $(s_t^2, \Pi_t^2)_{pe}$. The set of initial positions (s_0^1, s_0^2) where the converging path to Q_{hh} is available is called Manifold Range for Q_{hh}, and colored in darker green in Panel C of Figure 3. The set of initial positions where the converging path to Q_{ll} is available is called Manifold Range for Q_{ll}, and colored in lighter green in the same panel. In the same way, we define the Manifold Ranges for Q_{hl} and Q_{lh}. The manifold range for Q_{hl} is the set of (s_0^1, s_0^2)s with $s^1 \geq e_o$ and $s^2 \leq e_p$, and is described in lighter orange in Panel D. The Manifold Range for Q_{lh} is the set of (s_0^1, s_0^2)s with $s^1 \leq e_p$ and $s^2 \geq e_o$, and is described in darker orange in the panel.

In geometry, a collection of points on all converging paths to a limit set Q is defined as *a stable manifold to the limit set Q*.[20] Using the concept, we define the stable manifold to an economically stable state Q_{ij}.

Definition 2 (Stable Manifold SM_{ij})**.** *Stable manifold SM_{ij} is a collection of $(s_0^1, s_0^2, \Pi_0^1, \Pi_0^2)$s that converge to an economically stable state Q_{ij} in the dynamic system defined in Theorem 1:*

$$SM_{ij} \equiv \{(s_0^1, s_0^2, \Pi_0^1, \Pi_0^2) \in \mathbb{R}^4 | (s_t^1, s_t^2, \Pi_t^1, \Pi_t^2)|_{(s_0^1, s_0^2, \Pi_0^1, \Pi_0^2)} \to Q_{ij}\}.$$

The manifold range to Q_{ij} is redefined using the concept of stable manifold, which is just a projection of the stable manifold to Q_{ij} to the (s^1, s^2) plane.

[20] A limit set in geometry is the state a dynamic system reaches after an infinite amount of time has passed, by either going forward or backward in time.

Definition 3 (Manifold Range M_{ij})**.** *Manifold range M_{ij} is a collection of (s_0^1, s_0^2)s in SM_{ij}:*

$$M_{ij} \equiv \{(s_0^1, s_0^2) \in [0,1]^2 | (s_t^1, s_t^2, \Pi_t^1, \Pi_t^2)|_{(s_0^1, s_0^2, \Pi_0^1, \Pi_0^2)} \rightarrow Q_{ij}\}.$$

5.1.2 Folded Overlaps and Deterministic Ranges

All manifold ranges are put together in Panel E of Figure 5. There are nine distinct areas: all manifold ranges are folded in the center square, two manifold ranges are folded in the rectangles surrounding the center square, and a unique manifold range exists in each of the corner. In the center square, where four manifold ranges are folded, four social consensuses about the future are available to the members in the society: the consensus of Q_{hh}, that of Q_{lh}, that of Q_{hl} and that of Q_{ll}. Depending on the constructed social consensus, the economic state will move toward one of four economically stable states along a unique converging path. The social consensus is a combination of the expectation to group 1's final state and that of group 2's final state, as summarized in the following table.

Group 2 \ Group 1	Pessimistic Expectation	Optimistic Expectation
Optimistic Expectation	Q_{lh}	Q_{hh}
Pessimistic Expectation	Q_{ll}	Q_{hl}

If the initial skill composition (s_0^1, s_0^2) is in one of four rectangles, where two social consensuses are available, the expectation to one group's final state is critical in the determination of the social consensus about the future. For example, in the rectangle placed in the top middle, two manifold ranges are folded, M_{lh} and M_{hh}, and two social consensus about the future are available: the consensus of Q_{lh} and that of Q_{hh}. Group 2 will move toward s_h regardless of social consensus, because only the optimistic equilibrium path is available to the group: $s_t^2 \rightarrow s_h$. Group 1's expectation toward the future is important in the determination of social consensus and in the consequent equilibrium path. If group 1 holds an optimistic expectation toward the future, the economic state $(s_t^1, s_t^2, \Pi_t^1, \Pi_t^2)$ will converge to Q_{hh}. Otherwise, it will converge to Q_{lh}.

If an initial skill composition (s_0^1, s_0^2) is in one of four corner areas, the economic state will converge to the unique economically stable state (Q_{ij}) in the area. Social consensus about the future is fixed as Q_{ij} among the rational economic agents. Thus, the expectation toward the future does not play any critical role in the determination of the final state, but the location of the initial skill composition, so called *history*, determines the final state. To clarify the distinct areas determined by manifold ranges, I define a folded overlap where multiple manifold ranges are folded, and a deterministic range which is covered by a unique manifold range.

Definition 4 (N-Folded Overlap)**.** *N-folded overlap of M^1, M^2, ... M^n is an overlapped area of those* *n manifold ranges, M^1, M^2, ... M^n.*

Definition 5 (Deterministic Range)**.** *Deterministic range for Q_{ij} is the part of the manifold range for Q_{ij} (M_{ij}) that does not belong to any folded overlaps.*

Therefore, the characteristics of the two group economy with social interactions fully segregated between groups are summarized in the following way:

Proposition 4. *In the two group economy with total segregation, the (s^1, s^2) domain is composed of one four-folded overlap, four two-folded overlaps and four deterministic ranges. If the initial skill composition (s_0^1, s_0^2) is in the four-folded overlap, four final economic states are available, depending on the social consensus about the future. If it is in a two-folded overlap, one group's expectation toward the future determines the final economic state among two possible destinations. If it is in a deterministic range, the economic state converges to the unique economically stable state belonging to the range.*

5.2 Heterogeneous Economy in General

Now we turn to the two group economy with an arbitrary segregation level η. Concepts of stable manifolds, manifold ranges, folded overlaps and deterministic ranges, defined in the previous section, will be useful in the following analysis of the complex dynamic structure with arbitrary η.

5.2.1 Identifying Steady States

In order to proceed with the analysis of the dynamic system in Theorem 1, we first need to identify the steady states that satisfy $\dot{s}_t^1 = \dot{s}_t^2 = \dot{\Pi}_t^1 = \dot{\Pi}_t^2 = 0$. Let $(s^{1**}, s^{2**}, \sigma^{1**}, \sigma^{2**}, \Pi^{1**}, \Pi^{2**})$ denote a steady state, where two sets, (s^1, s^2) and (σ^1, σ^2), are bijective with parameters η, β^1 and β^2.

First, let us identify "partial" steady states $(s^{i*}, \sigma^{i*}, \Pi^{i*})|_{s^j}$ which are $(s_t^i, \sigma_t^i, \Pi_t^i)$s that satisfies both $\dot{s}_t^i = \dot{\Pi}_t^i = 0$ and $\sigma_t = k^i s_t^i + k^j s_t^j$, given s_t^j. The following three equations characterize the set of partial steady states $(s^{i*}, \sigma^{i*}, \Pi^{i*})|_{s^j}$:

$$\dot{s}^i = 0 \text{ Condition} \quad : \quad s^{i*} = 1 - G(A(\sigma^{i*}, \Pi^{i*})) \tag{13}$$

$$\dot{\Pi}^i = 0 \text{ Condition} \quad : \quad \Pi^{i*} = \frac{\bar{\delta} + f(\sigma^{i*})}{\rho + \alpha} \tag{14}$$

$$\text{Clearing Condition} \quad : \quad \sigma^{i*} = k^i s^{i*} + (1 - k^i) s^j. \tag{15}$$

By equation (14), $A(\sigma^{i*}, \Pi^{i*})$ is a function of σ^{i*}. Let us denote $\tilde{A}(\sigma^{i*}) \equiv A(\sigma^{i*}, \frac{\bar{\delta}+f(\sigma^{i*})}{\rho+\alpha})$. Panel A of Figure 6 describes $\tilde{A}(\sigma^{i*})$, which is a decreasing function and steeper than iso-Π curves $A(\sigma, \Pi)$ at each point (σ^{i*}, Π^{i*}).[21] In the panel, I place $s^{i*} = 1 - G(\tilde{a})$ locus together with $\tilde{A}(\sigma^{i*})$ locus, sharing the x-axis together. Note that $\tilde{A}(\sigma^{i*})$ locus must pass through $s^{i*} = 1 - G(\tilde{a})$ locus at three points of $s^{i*}(\sigma^{i*})$, s_l, s_m and s_h, because we have assumed three steady states (E_l, E_m and E_h) in a homogeneous economy. In Panel B, using two curves in Panel A, we can denote the set of (s^{i*}, σ^{i*})s that satisfy both equations (13) and (14), which is represented by $s^{i*} = 1 - G(\tilde{A}(\sigma^{i*}))$ curve in Panel B.[22] This curve must pass through three symmetric points, (s_l, s_l), (s_m, s_m) and (s_h, s_h). Finally, given s^j, (s^{i*}, σ^{i*}) must satisfy the clearing condition in equation (15), which is represented by the slashed dotted line in the panel. Thus, for given s^j, three points colored in blue indicate the corresponding partial steady states, which are intersections of the $s^{i*} = 1 - G(\tilde{A}(\sigma^{i*}))$ curve and the $\sigma^{i*} = k^i s^{i*} + (1 - k^i)s^j$ line.

The second step is to collect all partial steady states $(s^{1*}, \sigma^{1*}, \Pi^{1*})|_{s^2}$, and $(s^{2*}, \sigma^{2*}, \Pi^{2*})|_{s^1}$, in order to identify (global) steady states $(s^{1**}, s^{2**}, \sigma^{1**}, \sigma^{2**}, \Pi^{1**}, \Pi^{2**})$. Panel A of Figure 7 indicates the former and Panel B of the figure indicates the latter. In the top figure of Panel A, the slashed lines with different levels of s^2 help to identify (s^{1*}, σ^{1*}) for each level of s^2. Note that the slope of the slashed line is k^1. Consequently, all partial steady states are denoted by $s^{1*}(s^2)$ locus in the bottom figure. In the same way, in the top figure of Panel B, the slashed lines with different levels of s^1 help to identify (s^{2*}, σ^{2*}) for each level of s^1. The slope of the slashed line is $\frac{1}{k^2}$. All partial steady states are denoted by $s^{2*}(s^1)$ locus in the bottom figure of the panel. As we overlap the two partial steady state curves, $s^{1*}(s^2)$ and $s^{2*}(s^1)$, finally we can identify the (global) steady states in Panel C. Note that each partial steady state locus is characterized with, using equations (13), (14) and (15),

$$s^{i*}(s^j) \text{ Locus} : s^{i*} = 1 - G(\tilde{A}(k^i s^{i*} + (1 - k^i)s^j)), \forall s^j \in [0, 1]. \tag{16}$$

The following proposition characterizes the (global) steady states.

Proposition 5. *The (global) steady states (s^{1**}, s^{2**}) are a set of (s^1, s^2)s that satisfy the following two equations:*

$$\begin{aligned} s^{1**} &= 1 - G(\tilde{A}(k^1 s^{1**} + (1 - k^1)s^{2**})) \\ s^{2**} &= 1 - G(\tilde{A}(k^2 s^{2**} + (1 - k^2)s^{1**})), \end{aligned} \tag{17}$$

where $\tilde{A}(\sigma) \equiv A(\sigma, \frac{\bar{\delta} + f(\sigma)}{\rho + \alpha})$.

[21] $\frac{\partial \tilde{A}}{\partial \sigma}|_{\sigma^{i*}} (= \frac{\partial A}{\partial \sigma}|_{(\sigma^{i*}, \Pi^{i*})} + \frac{\partial A}{\partial \Pi}|_{(\sigma^{i*}, \Pi^{i*})} \cdot \frac{\partial \Pi}{\partial \sigma}|_{\sigma^{i*}}) < \frac{\partial A}{\partial \sigma}|_{(\sigma^{i*}, \Pi^{i*})}$.

[22] Refer to two examples (σ_a, s_a) and (σ_b, s_b) illustrated in Panels A and B.

5.2.2 Characteristics of Steady States

Let us denote four regions in the (s^1, s^2) plane by Regions 1, 2, 3 and 4 in clockwise order, which are divided by one vertical line ($s^1 = s_m$) and one horizontal line ($s^2 = s_m$), and the left and top region is denoted by region 1, as displayed in Panel C of Figure 7. In order to analyze the number of steady states in each region, I impose the following assumption without hurting the essential structure of the model.

Assumption 1 (Smoothing Condition)**.** *The function $G(\tilde{A}(\sigma))$ has one point of inflection as the ability distribution $G(a)$ has one point of inflection.*

There must be at least one inflection point between $\sigma = s_l$ and $\sigma = s_h$, because we have assumed three steady states in a homogeneous economy. As graphics in Figure 7 manifest, the economic structure cannot embed more than two steady states when the function $G(\tilde{A}(\sigma))$ has no inflection point. The assumption imposes the uniqueness of the inflection point. This assumption is achieved when the curvature of the S-shaped $G(a)$ function is strong enough that the curvature of the function $\tilde{A}(\sigma)$ does not distort the overall S-shape of $G(\tilde{A}(\sigma))$. This assumption helps the model to be more tractable than the case of $G(\tilde{A}(\sigma))$ with multiple inflection points. We call this assumption *smoothing condition*, because we rule out unnecessary local fluctuations of $G(\tilde{A}(\sigma))$ with imposing this assumption. The assumption implies that there exists $\hat{\sigma} \in (s_l, s_h)$ such that $G(\tilde{A}(\sigma))'$ is decreasing in $[0, \hat{\sigma}]$ and increasing in $[\hat{\sigma}, 1]$. Let us define a function $D^j(s^{i*})$ as the unique s^j given s^{i*} on the $s^{i*}(s^j)$ locus , which is $D^j(s^{i*}) \equiv \frac{\tilde{A}^{-1}G^{-1}(1-s^{i*})-k^i s^{i*}}{1-k^i}$, according to formula (16). Then, we have the following useful results.

Lemma 5. *Under Assumption 1, the $s^{i*}(s^j)$ locus with $\eta < 1$ has one point of inflection, when the locus is defined over the range of $s^j \in (-\infty, \infty)$: $D^j(s^{i*})$ is concave with $s^{i*} < 1 - G(\tilde{A}(\hat{\sigma}))$, and convex with $s^{i*} < 1 - G(\tilde{A}(\hat{\sigma}))$.*

Proof. See the proof in Appendix. ■

The partial steady state loci are composed of a concave part and a convex part, regardless of η and β^1.

Lemma 6. *As η declines, the partial steady state locus $s^{i*}(s^j)$ tends to be flatter: the distance $|D^j(s^{i*}) - s^{i^*}|$ shrinks as η declines. The partial steady state locus $s^{i*}(s^j)$ with the bigger β^i is steeper than that with the smaller β^i: The bigger β^i, the larger the distance $|D^j(s^{i*}) - s^{i^*}|$ and the steeper the slope $\left|\frac{\partial(D^j(s^{i*})-s^{i*})}{\partial s^{i*}}\right|$.*

Proof. See the proof in Appendix. ■

The $D^j(s^{i*})$ curve gets closer to the diagonal as η declines, which implies the $s^{i*}(s^j)$ locus tends to be flatter as η declines. The second one implies that, the bigger the size of the group, the more distant the curve $D^j(s^{i*})$ is from the diagonal and the steeper the curve is. Suppose group 1 is the minority and group 2 is the majority. Figure 8 shows the two partial steady state loci, $D^2(s^{1*})$ and $D^1(s^{2*})$, for each segregation level η. Note that the locus for group 2 is less sensitive to the change of η because the population size of the group is bigger than group 1 ($\beta^1 < \beta^2$). From the above lemmas, we have the following results in terms of the number of steady states.

Proposition 6. *The total number of steady states decreases from nine to three as η declines from one to zero, regardless of the population composition (β^1, β^2). Regardless of η and (β^1, β^2), there exist three symmetric steady states, (s_l, s_l), (s_m, s_m) and (s_h, s_h). All other steady states are asymmetric.*

Proof. See the proof in Appendix. ■

The network qualities (or skill levels) of two groups at a steady state vary depending on the location of each steady state:

Corollary 2. *Regardless of $\eta < 1$ and (β^1, β^2), there exists a unique steady state (s_h, s_h) in Region 2, and (s_l, s_l) in Region 4. Regardless of $\eta < 1$ and (β^1, β^2), any steady state $(s^{1**}, s^{2**}, \sigma^{1**}, \sigma^{2**})$ in Region 1 satisfy $s_l < \sigma^{1**}(s^{1**}) < s_m$ and $s_m < \sigma^{2**}(s^{2**}) < s_h$, and any steady state in Region 3 satisfy $s_m < \sigma^{1**}(s^{1**}) < s_h$ and $s_l < \sigma^{2**}(s^{2**}) < s_m$.*

Proof. See the proof in Appendix. ■

Therefore, we can conclude that two groups are equally better off if the economy state is on any steady state in Regions 2 and 4. Group 2 is better off than group 1 on any steady state in Region 1 and group 1 is better off than group 2 on any steady state in Region 3. In order to analyze the welfare implications of the model, we need to define the *Pareto* dominant (or inferior) steady state.

Definition 6 (Pareto Dominance)**.** *A steady state (s^1, s^2) is a strictly Pareto dominant steady state if both $s^1 > s^{1\prime}$ and $s^2 > s^{2\prime}$ are satisfied for any other steady state $(s^{1\prime}, s^{2\prime})$.*

Definition 7 (Pareto Inferiority)**.** *A steady state (s^1, s^2) is a strictly Pareto inferior steady state if both $s^1 < s^{1\prime}$ and $s^2 < s^{2\prime}$ are satisfied for any other steady state $(s^{1\prime}, s^{2\prime})$.*

With the definitions, we have the following result.

Corollary 3. *Regardless of $\eta < 1$ and β^1, (s_h, s_h) is a strictly Pareto dominant steady state and (s_l, s_l) is a strictly Pareto inferior steady state.*

Proof. Because $|D^j(s^{i*}) - s^{i*}|$ is monotonically decreasing as η declines (Lemma 6), both groups' skill levels become less than s_h at any steady state with $\eta < 1$, except the fixed steady state (s_h, s_h). Also, both groups' skill levels become greater than s_l at any steady state with $\eta < 1$, except the fixed steady state (s_l, s_l). ■

We adopt the following notation rule for the deeper analysis in the following sections.

Notation 1. *When $\eta = 1$, each steady state (s_i, s_j) is denoted by Q_{ij} for $i, j \in \{l, m, h\}$. As η declines, each steady state is denoted by its original notation at $\eta = 1$.*

Thus, when we have less than nine steady states, we can identify each steady state by following its original name in the economy with $\eta = 1$. Note that the locations of the following three steady states do not change with varying η or β: Q_{ll}, Q_{mm} and Q_{hh}. Other steady states continuously move as η or β changes. Particulary, the locations of Q_{lh} and Q_{hl} are denoted by the following rule.

Notation 2. *The skill levels of group 1 and group 2 at Q_{lh} are denoted by (s'_l, s'_h) and the skill levels of group 1 and group 2 at Q_{hl} are denoted by (s''_h, s''_l).*

When $\eta < 1$, the followings should hold by corollary 2: $s'_l > s_l$, $s'_h < s_h$, $s''_h < s_h$ and $s''_l > s_l$.

5.2.3 Demarcation Surfaces

The vector field $(\dot{s}^1_t, \dot{s}^2_t, \dot{\Pi}^1_t, \dot{\Pi}^2_t)$ is $(0, 0, 0, 0)$ at each steady state identified in the previous section. At any other state $(s^1_t, s^2_t, \Pi^1_t, \Pi^2_t)$, the vector field is identified by the state's location in a four dimensional Euclidian space $\mathbb{E}^4$. It is hard to observe the moving direction at each state because the four dimensional space is not visible. Fortunately, the dynamic system in Theorem 1 implies each component of the vector field can be displayed in a three dimensional Euclidian space $\mathbb{E}^3$, either in the (s^1, s^2, Π^1) coordinates or in the (s^1, s^2, Π^1) coordinates. Therefore, we can identify the demarcation manifolds in each three dimensional Euclidian space, which turn out to be two dimensional manifolds, surfaces:

$$\dot{s}^1_t = 0 \text{ Surface} \quad : \quad 1 - G(A(\sigma^1, \Pi^1)) = s^1 \tag{18}$$

$$\dot{s}^2_t = 0 \text{ Surface} \quad : \quad 1 - G(A(\sigma^2, \Pi^2)) = s^2 \tag{19}$$

$$\dot{\Pi}^1_t = 0 \text{ Surface} \quad : \quad \Pi^1 = \frac{\bar{\delta} + f(\sigma^1)}{\rho + \alpha} \tag{20}$$

$$\dot{\Pi}^2_t = 0 \text{ Surface} \quad : \quad \Pi^2 = \frac{\bar{\delta} + f(\sigma^2)}{\rho + \alpha}. \tag{21}$$

In the space above the $\dot{s}^i_t = 0$ surface, s^i_t increases over time and, in the space below the surface, it decreases over time. In the space above the $\dot{\Pi}^i_t = 0$ surface, Π^i_t increases over time and, in the space below the surface, it decreases over time. Therefore, when an initial steady state $(s^1_0, s^2_0, \Pi^1_0, \Pi^2_0)$

is identified to be in the above (below) $\dot{s}_t^1 = 0$ surface in the (s^1, s^2, Π^1) coordinates, $\dot{s}_0^1$ is positive (negative). When the initial state is identified to be in the above (below) $\dot{\Pi}_t^1 = 0$ surface in the (s^1, s^2, Π^1) coordinates, $\dot{\Pi}_0^1$ is positive (negative). The same is true for $\dot{s}_0^2$ and $\dot{\Pi}_0^2$ in the (s^1, s^2, Π^2) coordinates.

Figure 9 illustrates the $\dot{s}_t^1 = 0$ surface and the $\dot{\Pi}_t^1 = 0$ surface together in one (s^1, s^2, Π^1) coordinates. The second pictures in the five panels show the sliced segments of the two surfaces for each level of s^2. The $\dot{\Pi}_t^1 = 0$ surface is captured with $\Pi^1 = \frac{\bar{\delta}+f(\sigma^1)}{\rho+\alpha}$ in the second picture of each panel. The sliced segment of the surface in each panel is $\Pi^1 = \frac{\bar{\delta}+f(\sigma^1)}{\rho+\alpha}$ with σ^1 ranging over $[(1-k^1)s^2, k^1+(1-k^1)s^2]$. For example, in panel A with $s^2 = 1$, σ^1 ranges over $[1-k^1, 1]$. The segment of the $\dot{s}^1 = 0$ surface should satisfy the following two conditions with s^2 given:

$$\begin{aligned} \tilde{a} &= A(\sigma^1, \Pi^1) \\ 1 - G(\tilde{a}) &= s^1 \end{aligned}$$

Points that satisfy the first condition are depicted as dotted gray curves for different levels of Π^1, named by *iso*-Π *curves*, in the first picture of each panel. Points that satisfy the second condition are depicted as a solid curve with the range of σ^1 in $[(1-k^1)s^2, k^1+(1-k^1)s^2]$ in the same picture. The points that satisfy both conditions together constitute the segment of the surface given s^2. This is displayed in the second picture of each panel as a solid curve. Adding up all these segments for each s^2 in $[0, 1]$, we can construct the $\dot{\Pi}_t^1 = 0$ surface and the $\dot{s}_t^1 = 0$ surface. Note that the intersections of two segments given s^2 are the partial steady states $(s^{1*}, \sigma^{1*}, \Pi^{1*})|_{s^2}$.

5.2.4 Economically Stable States and Stable Manifolds

Since we have identified steady states and the vector field $(\dot{s}_t^1, \dot{s}_t^2, \dot{\Pi}_t^1, \dot{\Pi}_t^2)$, we are ready to identify the economically stable states.

Theorem 2. *Regardless of η and (β^1, β^2), steady states Q_{ll}, Q_{hh}, Q_{lh} and Q_{hl} are economically stable states and all others are economically unstable states.*

Proof. See the proof in Appendix. ■

Those economically stable states are illustrated in Figure 10 with dark circles. Since we do not impose functional forms of G, A and f, we do not have the explicit form of steady states. Consequently, we are not able to calculate eigenvalues for those steady states. However, it is possible to identify eigenvalues at three symmetric steady states because they are fixed points. We can show that there are two positive and two negative eigenvalues at steady states Q_{ll} and Q_{hh}.

Lemma 7. *At the economically stable states Q_{ll} and Q_{hh}, the solutions of the following equation constitute eigenvalues, among which two are positive and two are negative:*

$$[\lambda^2 - \mathbb{R}\lambda + \mathbb{H}] \cdot [\lambda^2 - (\eta\mathbb{R} + (1-\eta)\rho)\lambda + \eta\mathbb{H} - (1-\eta)\alpha(\alpha+\rho)] = 0, \tag{22}$$

where $\mathbb{R} = [-GA'_\sigma + \rho]_{Q_{ii}}$ *and* $\mathbb{H} = [-\alpha(\alpha+\rho)(G'A'_\sigma + 1) - \alpha G' A'_\Pi f']_{Q_{ii}}$.

Proof. See the proof in Appendix. ■

Knowing this result, without loss of generality, we assume that two positive eigenvalues and two negative eigenvalues are at all economically stable states, because the dynamic structure of those economically stable states are locally identical. As far as the four eigenvalues are distinct, we can calculate the exact equilibrium path to each economically stable state in the neighborhood of the state.

Corollary 4. *Suppose four eigenvalues are distinct. The unique equilibrium path, given an initial skill composition $(s^1(t_0), s^2(t_0))$ in the neighborhood of an economically stable state $Q_{ij}(s^{1\prime}, s^{2\prime}, \Pi^{1\prime}, \Pi^{2\prime})$, is*

$$\begin{bmatrix} s_t^1 \\ s_t^2 \\ \Pi_t^1 \\ \Pi_t^2 \end{bmatrix} = \begin{bmatrix} W_{11} e^{\Lambda_1 (t-t_0)} W_{11}^{-1} \\ W_{21} e^{\Lambda_1 (t-t_0)} W_{11}^{-1} \end{bmatrix} \begin{bmatrix} s^1(t_0) - s^{1\prime} \\ s^2(t_0) - s^{2\prime} \\ s^1(t_0) - s^{1\prime} \\ s^2(t_0) - s^{2\prime} \end{bmatrix} + \begin{bmatrix} s^{1\prime} \\ s^{2\prime} \\ \Pi^{1\prime} \\ \Pi^{2\prime} \end{bmatrix} \tag{23}$$

, where Λ_1 is a diagonal matrix containing two negative eigenvalues and B is a 4×4 matrix whose column vectors are eigenvectors corresponding to two negative eigenvalues and two positive eigenvalues, which is composed of four 2×2 matrices $W_{ij}s$, $B = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix}$.

Proof. See the proof in Appendix. ■

The set of these equilibrium paths are stable manifold in the neighborhood of the state. The stable manifold theorem helps us to understand the shape of the stable manifold.

Corollary 5. *By the virtue of the Stable Manifold Theorem, the stable manifold to an economically stable state $Q_{ij}(s^{1\prime}, s^{2\prime}, \Pi^{1\prime}, \Pi^{2\prime})$ is two dimensional and is tangent to the stable subspace $\mathbb{E}^S$ of the linearized differential system at Q_{ij}. Suppose four eigenvalues are distinct. The stable subspace $\mathbb{E}^S$ is represented by the following two planes:*

$$\mathbb{E}^S : \quad \begin{bmatrix} \Pi_t^1 \\ \Pi_t^2 \end{bmatrix} = [W_{21}][W_{11}]^{-1} \begin{bmatrix} s_t^1 - s^{1\prime} \\ s_t^2 - s^{2\prime} \end{bmatrix} - \begin{bmatrix} \Pi^{1\prime} \\ \Pi^{2\prime} \end{bmatrix}, \tag{24}$$

where the first plane is determined in the (s^1, s^2, Π^1) *coordinates and the second one in the* (s^1, s^2, Π^2) *coordinates.*

Proof. The stable manifold theorem manifests that if the linearized dynamic system at an economically stable state Q_{ij} has two eigenvalues with negative real parts and two eigenvalues with positive real parts, a stable manifold SM_{ij} is two dimensional, which is tangent to the stable subspace $\mathbb{E}^S$ of the linearized differential system at Q_{ij}. In Lemma 7, we have presented the characteristics of eigenvalues at any economically stable state. The stable subspace $\mathbb{E}^S$ is derived from Corollary 4. ■

Therefore, we can infer that the stable manifold, which is two dimensional in a four dimensional Euclidian space $\mathbb{E}^4$, is the combination of two surfaces, one which is identified in the (s^1, s^2, Π^1) coordinates, and the other is identified in the (s^1, s^2, Π^2) coordinates. The part of the stable manifold to the stable state Q_{lh} is illustrated with blue and red curves in Figure 10. In the sliced phase spaces in the figure, we can check the relative position of the stable manifold with respect to demarcation surfaces of the time dependent variables (the $\dot{s}^i = 0$ surface and the $\dot{\Pi}^i = 0$ surface). In Panel B of Figure 11, I display the full picture of the stable manifold with the blue and red curves, which is a combination of two surfaces. The blue surface indicates Π^1 on the stable manifold given (s^1, s^2). The red surface indicates Π^2 on the stable manifold given (s^1, s^2). The manifold range (M_{lh}), which is a projection of stable manifold SM_{lh} to the (s^1, s^2) plane, is depicted in orange color in Panel A of the figure, together with the blue and red curves. Panel C of the figure displays manifold ranges of all economically stable states and their overlapped areas. In the middle, we have a four-folded overlap, and there are two-folded overlaps surrounding that. We can observe tiny areas of three-folded overlaps.

The number of stable states decreases from four to two as η declines. The manifold ranges and their overlapped areas are depicted for each level of η in Figure 12. Two manifold ranges M_{lh} and M_{hl} disappear at some level of η, as the two stable states disappear. Two manifold ranges M_{ll} and M_{hh} tend to expand as η declines, while the other two manifold ranges M_{lh} and M_{hl} tend to shrink as it declines. All manifold ranges are greater when the lifetime network externalities are more influential, as the size of overlap is greater with the stronger lifetime network externalities in a homogeneous economy (Proposition 3).

Proposition 7. *As* η *declines, the manifold ranges* M_{ll} *and* M_{hh} *tend to expand, while the manifold ranges* M_{lh} *and* M_{hl} *tend to shrink. All manifold ranges tend to expand with the stronger lifetime network externalities (greater* $f'(\sigma)$*).*

Proof. The first argument is obvious when we look at the case with no lifetime externalities $f(\sigma)' = 0$. In this case, manifold ranges are not overlapped at all, as illustrated in Appendix Figure 2.

Each manifold range is a basin of attraction for an attractor (economically stable state). The basins are separated by separatrices that are connecting saddle points (economically unstable states). As economically unstable states get closer to the diagonal with the declined η (Lemma 6), the basins for economically unstable states should be shrinking. This analysis for the special case of $f'(\sigma) = 0$ is directly applied to the general case, because the only difference is bigger manifold ranges with greater lifetime externalities (positive $f'(\sigma)$). The second argument is true because the size of overlap in a homogeneous economy is analogous to the folded overlaps in a heterogeneous economy. When $f'(\sigma) = 0$, there is no overlap in a homogeneous economy, and no folded overlap in a heterogeneous economy. With greater $f'(\sigma)$, they get larger, as Propostion 3 proves. ■

5.3 Social Consensus and Network Trap

In the given economy, a maximum of four economically stable states exist. In $Q_{hh}\,(Q_{ll})$, both groups' skill levels and network qualities are $s_h\,(s_l)$. In Q_{lh}, group 1's skill level and network quality are below s_m, while group 2's skill level and network quality are above s_m, according to Corollary 2. In Q_{hl}, we have the opposite result. Social consensus plays a critical role in the determination of the final destination of the economy. If optimism prevails and newborn members believe in the better network quality of the future, they are encouraged to invest more, expecting the higher benefits of investment to accrue over their lifetimes. If pessimism prevails and newborn members believe that network quality of the future gets worse, they are discouraged to invest, due to the declined expected benefits of investment. The social consensus about the future should be one of the above four stable states, because any other state is not stable and thus cannot be the final destination. In the analysis of the model, I propose that members in a society can agree to a social consensus within a reasonably short time period.

Suppose the society is located in a four-folded overlap. The society faces four possible destinations. The final destination is determined by the belief of members in the society. If both group members are optimistic about the future, Q_{hh} will be realized. If both are pessimistic about the future, Q_{ll} will be realized. If newborn members of group 1 are pessimistic about the group's network quality and newborn members of group 2 are optimistic about the group's network quality, the social consensus about the future will be formed as Q_{lh}. The economic state (s_t^1, s_t^2) will move toward the final destination Q_{lh} gradually. In the same way, if the social consensus is formed with group 1's optimism and group 2's pessimism, the state will move toward Q_{hl}. Thus, the society can be mobilized to any stable state depending on the chosen social consensus: the *mobilization force* of lifetime network externalities strongly influence in this economy.

Suppose the society is located in a two-folded overlap: for example, an overlap of M_{hh} and M_{lh}, in which group 2's network quality is relatively better than group 1's network quality. Newborns of group 2 will keep the optimistic view toward the group's future because the following generations will maintain the high investment rate, due to the good quality of network externalities over the education period. Thus, there are two social consensuses available to the members in the society: Q_{hh} and $Q_{lh.}$. If newborn members of group 1 share the optimistic view toward the group's future, the social consensus will be formed as Q_{hh} instead of Q_{lh}. The skill investment rate of group 1 will be enhanced significantly and the economic state (s_t^1, s_t^2) will move toward Q_{hh}. If they are pessimistic, the skill investment rate of group 1 will deteriorate over time and the economic state will move toward Q_{lh}. Thus, group 1's expectation toward the future determines the social consensus about the future, and strongly affects the skill investment pattern of group 1 newborns. The mobilization force of the lifetime network externalities influences group'1 future, while the historical force of the education period externalities influences group 2's skill investment.

When an initial economic state is in a deterministic range, "rational" group members understand that there is only one possible future. The social consensus will be quickly formed, and the economic state will move toward the stable state gradually. Among four deterministic ranges, two of them lead the economic state (s_t^1, s_t^2) to an asymmetric steady state. If an initial economic state is in one of those ranges, "rational" newborns of one group will share a pessimistic view toward the group's future, while "rational" newborns of the other group will share an optimistic view toward the group's future. Envision a society in a deterministic range for Q_{lh}. The current network quality of group 1 is so poor that there is no way to recover the skill investment rate of the group, while that of group 2 is so good that newborn members can maintain the high skill investment rate, benefiting from the good quality education period network externalities. Thus, the *historical force* of education period network externalities determines the future.

In many societies around the world, the economic state of the society is at an asymmetric stable state belonging to a deterministic range: for example, either Q_{lh} or Q_{hl} in a two group economy. At this state, a disadvantaged group cannot be mobilized to improve its skill investment rate, because the network externalities over the skill acquisition period have a strong adverse effect on the cost of skill achievement. This is a case where a social group is trapped by the network externalities.

Definition 8 (Network Trap). *Q_{lh} (Q_{hl}) is called a network trap of group 1 (group 2) if it belongs to a deterministic range.*

6 Egalitarian Policies in Network Trap

In this section, we discuss the egalitarian policies in a society placed in a network trap. Suppose that the economic state (s^1, s^2) is at Q_{lh} in a deterministic range, as Panel B of Figure 12 illustrates. Group 1 is disadvantaged in the social network structure. The skill disparity between two groups will persist indefinitely, without any governmental intervention or a structural change of the economy. Since Q_{hh} is a Pareto dominant state, the government has an incentive to pursue egalitarian policies to mobilize the society toward the equality of his skills. We analyze two kinds of egalitarian policies, the integration between groups and the implementation of affirmative actions such as quota and training subsidies. Then, we will discuss the way to implement an effective policy for the different sizes of the disadvantaged group. In the end, we emphasize the importance of good leadership of the disadvantaged group in the mobilization out of the trap: encouragement of optimism and fostering of within-group cooperation.[23]

6.1 Integration Effect

Imagine a society in a network trap of group 1, Q_{lh}. First, note that the asymmetric stable state disappears as integration is facilitated. The threshold level of the segregation level $\hat{\eta}$ depends on the population size of the disadvantaged group: $\hat{\eta} \equiv \hat{\eta}(\beta^1)$. The following Lemma summarizes the shape of the function $\hat{\eta}(\beta^1)$.

Lemma 8. *There exists $\hat{\beta}$ such that $\hat{\eta}(\beta^1)$ is strictly decreasing in $(0, \hat{\beta})$ and strictly increasing in $(\hat{\beta}, 1)$.*

Note that, as integration proceeds, either Q_{lh} and Q_{mh} are merged together or Q_{lh} and Q_{lm} are merged together, before Q_{lh} disappears. Therefore, before the steady state Q_{lh} disappears, the state must get into either M_{hh} or M_{ll} or both, because the manifold range M_{hh} (M_{ll}) always covers the unstable state Q_{mh} (Q_{lm}) as far as $\eta \neq 1$. With this fact integrated with the above Lemma, we have the full picture of the integration effect, which is summarized in Figure 13. In the diagram, $\eta^*(\beta^1)$ indicates the threshold level of η for Q_{lh}'s entering M_{hh}. Note that $\eta^*(\beta^1) > \hat{\eta}(\beta^1)$ with $\beta^1 \in (0, \hat{\beta})$. $\eta^{**}(\beta^1)$ indicates the threshold level of η for Q_{lh}'s entering M_{ll}. Note that $\eta^{**}(\beta^1) > \hat{\eta}(\beta^1)$ with $\beta \in (\hat{\beta}, 1)$. Let us denote β^* with which $\eta^*(\beta^1) = \hat{\eta}(\beta^1)$, and β^{**} with which $\eta^{**}(\beta^1) = \hat{\eta}(\beta^1)$, where $\beta^* > \beta^{**}$, as displayed in the figure. As far as both groups are optimistic rather then pessimistic, the integration can lead the society to the pareto dominant Q_{hh} when β^1 is in $(0, \beta^*)$, because Q_{lh} will

[23] This is what civic leaders, civic organizations, and religious groups can contribute for a more egalitarian society. In US history, many civil rights activists and organizations including Rev. Martin Luther King Jr. and the National Urban League contributed to the improvement of the black community.

move into the manifold range M_{hh} before its disappearance. However, if $\beta^1 > \beta^*$, the integration will lead to the manifold range M_{ll} and the society may fall down to Q_{ll} as Q_{lh} disappears.

Proposition 8. *Suppose members in the society are optimistic rather than pessimistic. As integration proceeds, the economic state moves to the pareto dominant state Q_{hh} with $\beta^1 \in (0, \beta^*)$, and it moves to the pareto inferior state Q_{ll} with $\beta^1 \in (\beta^*, 1)$.*

However, if members in the society are pessimistic rather optimistic, they may tend to choose Q_{ll} rather than Q_{hh} for their social consensus about the future, when the economic state is in the overlap of M_{ll} and M_{hh}. They would tend to stay at Q_{lh} rather than moving toward Q_{hh}, when the equilibrium path to Q_{hh} is feasible. In this pessimistic society, integration will lead both groups to fall down to Q_{ll} as far as $\beta^1 > \beta^{**}$, as Figure 13 illustrates.

Corollary 6. *Suppose members in the society are pessimistic rather than optimistic. As integration proceeds, the economic state moves to the pareto dominant state Q_{hh} with $\beta^1 \in (0, \beta^{**})$, and it moves to the pareto inferior state Q_{ll} with $\beta^1 \in (\beta^{**}, 1)$.*

In either situation, integration has an adverse effect for the welfare improvement when the population size of the disadvantaged group is relatively big. In this case, we should consider other policy tools together with the integration for the effective implementation of egalitarian policies.

6.2 Affirmative Action Policies

We consider two types of affirmative action policies: training subsidies and quota system. First, consider the training subsidy policy. Government can impose some taxes on the advantaged group (or skilled workers in general) and transfer the resources to the disadvantaged group (or unskilled workers in general) for the purpose of the enhanced skill investments. This policy targets decreasing the education cost for the disadvantaged group, while increasing the cost for the advantaged group. For simplicity, suppose that the policy is implemented in a way that all members of the disadvantaged group experience a certain amount of cost decrease for skill acquisition and those of the advantaged group experience a certain amount of cost increase for its acquisition. This effect is well reflected by the shifted-up $\dot{\Pi}_t^1 = 0$ surface and the shifted-down $\dot{\Pi}_t^2 = 0$ surface, because the cost increase (decrease) has the exactly same impact with the increased (decreased) benefits of investment on each individual's decision making process, which is a simple cost-benefits comparison in the model. If then, as Figure 9 shows, the shift-up of the $\dot{\Pi}_t^1 = 0$ surface leads the shift-down of the $D^2(s^{1*})$ curve, and the shift-down of the $\dot{\Pi}_t^2 = 0$ surface leads the shift-up of the $D^2(s^{1*})$ curve (or the shift-to-right in (s^1, s^2) plane). This impact is summarized in Panel A of Figure 14. As the curves shift enough, the steady state Q_{lh}, which was a network trap of group 1, moves into the manifold range of Q_{hh}. Thus, by overturning the social consensus from Q_{lh} to Q_{hh}, the economic state can move toward a high skill symmetric state.

Quota policy places some group 1 members, who are unskilled, into skilled job positions that otherwise would go to skilled members of group 2. Suppose the current economic state is $Q_{lh}(s_l', s_h')$. The skilled job positions are fixed as $\bar{s} = \beta^1 s_1' + \beta^2 s_h'$ in this economy. The government intervenes to mitigate the skilled job disparity between two groups, $|s_2 - s_1|$. The higher fraction of group 1 take the skilled job positions, and the lesser fraction of group 2 takes the skilled job positions under the constraint of $\bar{s} = \beta^1 s_1' + \beta^2 s_h'$. If this external intervention can lead the economic state (s^1, s^2) into the manifold range M_{hh}, the society will start to move toward the high skill symmetric state. More of group 1 newborns will be motivated to invest in skill acquisition, by sharing the optimistic view about the future. This process is displayed in Panel B of Figure 14.

With the imposition of affirmative actions, members of group 2 may suffer temporarily, but the group state will improve in the end: both the group's skill level and the network quality will approach s_h, which is greater than s_h'. The effectiveness of affirmative action is restricted by the size of the disadvantaged group. As it increases in size, the stronger action is required to mobilize the disadvantaged group out of the trap. If it is too big, there might be no way to implement effective affirmative policies to make society equal.

6.3 Policy Implementation

As discussed earlier, the population size of the disadvantaged group is critical for the effective implementation of the egalitarian policies. If it is small enough, any one type of policy may solve the problem (Proposition 8). This is depicted in the Panel As of Figure 15. The state X in Panel A1 of the figure indicates the original economic state, which is a network trap of group 1. As integration proceeds, X moves to X' in Panel A2, which belongs to the manifold range M_{hh}. With the overturn of social consensus, the economic state starts to move toward Q_{hh}. If the integration policy is difficult to implement due to the rigid division of social interactions, other affirmative action policies can handle the problem. A quota way is illustrated in Panel A1: the state (s^1, s^2) is relocated to Z' in M_{hh} by the quota implementation, and moves toward Q_{hh} as social consensus overturns from Q_{lh} to Q_{hh}. (The training subsidy strategy is illustrated in Panel A of Figure 14.)

However, if the population size of group 1 is too big, there is be no way for any type of policy to improve the situation. This is depicted in the Panel Bs of Figure 15. The integration alone will lead the original economic state X in Panel B1 to the pareto inferior state X'' in Panel B3. A quota way alone cannot solve the problem as illustrated in Panel B1: the straight line constraint ($\bar{s} = \beta^1 s_1' + \beta^2 s_h'$) does not go through the manifold range M_{hh}. The same is true for training subsidy policy: if the disadvantaged group is too big, per capita training subsidies would be very small and would not significantly change the dynamic structure of the model (eg. the $\dot{\Pi}^1$ surface does not shift up enough). It is important to know that the mixed policies can be effective to mobilize the society to the pareto dominant state Q_{hh}, even when the majority group is disadvantaged. The mixed policy of integration and quota system is depicted in Panel B2 of Figure 15. With the integration between groups, the group state will moves closer to the center: X in Panel B1 to X' in Panel B2. Then, the straight line from the position X may pass through the manifold range M_{hh}. Thus, this mixed policy helps the majority group to move out of the trap. A bundle of three policy methods - integration, quota and training subsidy - might be more effective in the implementation.

In the implementation of egalitarian policies, one important factor is the social consensus. Even though an effective policy leads the economic state to the overlap of M_{hh} and M_{lh}, the society may stay at Q_{lh} consistently if the pessimism prevails in the society and newborns are not motivated to improve their skill investment rates. Therefore, the effective policy should come with the overturn of social consensus. Forward looking decision making and the optimistic view toward the future are crucial parts of the effective policy implementation. Another factor that can improve the effectiveness of policy implementation is the fostering of within-group cooperation. Even though the social interaction between two social groups cannot be proceeded significantly, it can be easier to improve the quality of social interactions among the disadvantaged group members. If the quality of relationship is improved, the lifetime benefits of investment increase: the slope $f(\sigma^1)$ gets steeper. We can check that the manifold range M_{hh} expands with the steeper $f(\sigma^1)$ in the given model. The fostering of within-group cooperation can help the group move out the trap with the expanded folded overlap, when the proper egalitarian policies are activated together. These two factors, the optimism in the society and the within-group cooperation, can be facilitated by non-governmental institutions such as civic groups and religious institutions, and by civic leaders who can motivate and integrate the disadvantaged group members.

7 Macroeconomic Effects of Inequality

Human capital has been the prime engine of economic growth in the modern economy (Goldin and Katz 2001, Abramovitz and David 2000). The accumulation of intangible capital contributed to growth significantly, replacing the importance of physical capital accumulation in the early stage of the Industrial Revolution (Galor and Moav 2004). Because inequality is greatly associated with overall human capital achievement, it is natural to think about the macroeconomic effects of inequality (Benabu 1996). Along this line, Loury (1981) shows the positive effect of egalitarian policies on overall economic activities, under the intergenerational transfer of skill achievement. Galor and Zeira (1993) show the positive effect of equality on economic development, identifying the multiple equilibria with the assumption of indivisible human capital investment. The credit market constraint is the underlying force of the intergenerational mobility restriction in these studies. Unlike the previous literature, I suggest the positive effect of equality on economic development without imposing the assumption of credit market imperfection. Even in an economy with a perfect credit market, the social network externalities still restrict the skill achievements of the disadvantaged groups. With more equal distribution of social capital across social groups, the society can encourage more newborns from disadvantaged groups to invest in skill acquisition, and reach a more developed stage of an economy.

Of course, social network externality is a broad concept that can include accessibility to physical resources. For example, children of a rich community can afford the higher tuition for private schools and tend to have a better quality of schooling. It is easier for college graduates of the rich community to obtain the seed money for starting a business than those of the poor community. Therefore, in a society with credit constraints, the social network externalities will be stronger, both during education period and over the lifetime. However, the conclusion of the model sharply contrasts to the prediction of the previous literature, which suggests an equal society in the matured economy where credit constraint does not bind for the skill investment (Galor and Moav 2004), or an equal society with the centralized provision of training (perfect public school system) (Loury 1981). Even in the sufficiently developed economy with no binding of credit constraint or in the society with public provision of schooling, the social network externalities over the skill acquisition period (such as peer effects, parental effects, role models, and medical and nutritional provision) and over the working period (such as mentoring, job search and business connections) still influence the incentives for skill acquisition and work as a major force of the intergenerational mobility constraint. Therefore, unlike the conclusions of the previous studies, equality, namely more equal distribution of social network capital in this study, will have a positive effect on the economic development even in the matured economy or in the society with a perfect public school system.

7.1 Multiple Equilibria as Different Development Stages

In the developed model, we have four economically stable states, two symmetric ones and two asymmetric ones. The two symmetric states, Q_{hh} and Q_{ll}, indicate the most developed stage and the least developed stage: $\bar{s} = s_h$ and $\bar{s} = s_l$ for each. Two asymmetric states indicate the mediocre levels of development with group 2's better off (Q_{lh}) and group 1's better off (Q_{hl}), which are defined as network traps in the model: $\bar{s} = \beta^1 s_l' + \beta^2 s_h'$ and $\bar{s} = \beta^1 s_h'' + \beta^2 s_l''$ for each. When the economy is trapped in either Q_{lh} or Q_{hl}, the egalitarian policies discussed in the previous section can help society to be mobilized to the most developed stage of Q_{hh}: integration or affirmative actions such as training subsidies and quotas can be the tool to motivate more newborns of disadvantaged groups to invest in skills. The structural change of the educational system or redistribution policy of income can help the group to move out of the trap by mitigating the adverse effects of poor education period network externalities, such as better public education system or more progressive tax system: with this structural change, the folded overlap may expand covering the unequal steady state. Panel A depicts the threshold level of economic development by the red line, above which egalitarian policies can promote the economic growth helping the economic state move into the manifold range M_{hh}. Below the threshold level, the policies may not be effective in the promotion of growth.

The development stages can be more than four in a multi-group economy. If the number of social groups is n, the maximum number of economically stable state is 2^n. Each of them can serve as a development stage of an economy. The case of three group economy is displayed in Panel C of Figure 16. Maximum eight development stages are identified in a three dimensional Euclidian space with the coordinates (s^1, s^2, s^3), in which h (l) indicates the skill level of a group above (below) the medium skill level s_m. For example, (h, h, l) indicates group 1 and group 2 achieve the higher skill level while group 3 is left behind with the low skill acquisition rate. The following proposition analogous to Theorem 1 summarizes n group economy, denoting the integration level between group i and group j by η_{ij} and the average skill level of the two groups by $\bar{s}_{ij}(t) \equiv \frac{\beta^i s_t^i + \beta^j s_t^j}{\beta^i + \beta^j}$:

Proposition 9 (N-Group Economy)**.** *In a n-group economy, the dynamic system with n flow variables* $(s_t^1, \cdots, s_t^n)$ *and n jumping variables* $(\Pi_t^1, \cdots, \Pi_t^n)$ *is summarized by the following* $2n$*-variable differential equations:*

$$\begin{bmatrix} \dot{s_t}^i & = & \alpha[1 - G(A(\sigma_t^i, \Pi_t^i)) - s_t^i] \\ \dot{\Pi}_t^i & = & (\rho + \alpha)\left[\Pi_t^i - \frac{\bar{\delta} + f(\sigma_t^i)}{\rho + \alpha}\right] \end{bmatrix}_{i \in \{1, 2, \cdots, n\}}, \tag{25}$$

where $\sigma_t^i = (\eta_{i1}, \eta_{i2}, \cdots, \eta_{in}) \cdot (\bar{s}_{i1}(t), \bar{s}_{i2}(t), \cdots, \bar{s}_{in}(t))$*, with* $\eta_{ij} = \eta_{ji}$ *and* $\sum_{k=1}^n \eta_{ik} = 1$*.*

In this expanded (η, β) structure, the quality of social network of group i (σ_t^i) is an inner product of two vectors, a vector of between-group integration levels and a vector of between-group average skill levels. $\eta_{ij} = 1$ indicates the perfect integration between group i and group j and their perfect segregation from all other social groups. Then, the quality of the social network of group i is equal to the average skill level between two groups: $\sigma_t^i = \bar{s}_{ij}(t)$. $\eta_{ij} = 0$ indicates zero contacts between the two social groups. Then, the quality of the social network of group i is not affected by the group j's skill level: $\frac{\partial \sigma_t^i}{\partial s^j(t)} = 0$. In this dynamic system, there are maximum 3^n steady states including maximum 2^n economically stable states. The stable manifold to each economically stable state is an n dimensional manifold defined in $2n$ dimensional Euclidian space $\mathbb{E}^{2n}$. The manifold range of an economically stable state is a projection of n dimensional stable manifold to n dimensional Euclidian space $\mathbb{E}^n$ with the coordinates $(s^1, s^2, \cdots, s^n)$. Using the same notation rule defined in Notation 1, we have the following implication for this economy.

Corollary 7. *In an economy with n social groups, there are maximum 2^n distinct development stages. $Q_{hh\cdots h}$ is the most developed stage and a pareto dominant steady state, and $Q_{ll\cdots l}$ is the least developed stage and a pareto inferior steady state.*

Proof. The proof is analogous to the proofs for the two group economy at Corollary 3 and Theorem 2. ■

This implies countries in the world might be in different development stages due to the different social network structure. In order to understand how equalitarian policies can promote economic development, check the following simple example: Suppose the initial economic state is (h, l, l) in a three group economy. Group 1 is a sufficiently big group and those three social groups are fully segregated. Suppose the initial state is in a deterministic range. Thus, the unequal state persists. If the integration between group 1 and group 2 is facilitated, the economic state will move into the folded overlap area of M_{hll} and M_{hhl}. As members of group 2 are motivated to increase the skill investment rate, the economy will move toward the state (h, h, l). If the integration is between group 1 and group 3, the economic state will move toward (h, h, h), which is the most developed stage of the economy. Thus, an egalitarian policy, integration, will help the economy grow. The case is roughly illustrated in Panel C of Figure 16.

7.2 Positive Effect of Inequality

In most development stages, the egalitarian policies might facilitate the economic growth. However, if the economy is in its early stage of development, the effect is obscure. As Panel A of Figure 16 describes, there is no way to enter M_{hh} if the economic state is positioned below the threshold level of economic development depicted by the red line. Instead, there can be a positive effect of inequality, consistent with Galor and Tsiddon (1997). Suppose the initial economic state is at Q_{ll} in a simple two group economy. Also, suppose the government has resources to invest for human capital development in the society, which might be borrowed from abroad or gained from selling natural resources. Panel B of Figure 16 illustrates the resource constraint for human capital development. As far as two social groups are separated significantly, the unequal distribution of development resources can be the best strategy for growth, because the unequal distribution may lead the skill composition into the manifold range M_{lh} or M_{hl}, while the equal distribution is more likely to lead it into a deterministic range of Q_{ll}. That is, when the resources are limited in the early stage of development, the concentration of social capital to some selective groups can promote the groups to enhance their skill investment rates significantly because they expect the increased return on skill achievement through the network externality channel. This might explain the concentration of education facilities in selective cities in many developing countries, rather than the equal distribution all over the countryside. The current group inequality that exists in many less developed countries can be a byproduct of an initial economic development promotion.

This positive effect of inequality in the early stage of development along with the positive effect of equality in the later stage of development is consistent with the empirical findings that income tends to be more equally distributed in developed countries than less developed countries and the early stage of economic development often comes along with the growing inequality.

8 Application: Regional Group Inequality in South Korea

In this section, I present a historical example of between-group disparity - regional group disparity in South Korea. The example displays how an initially advantaged group enhances its skill acquisition activities by holding an optimistic view about the group's network quality, and reinforces its dominant position. Most social interactions in Korean society had occurred within each region (Youngnam, Honam, Chungcheong, Kangwon, etc.) before the rapid urbanization in the last decades, as displayed in Figure 17. Even after the urbanization, which caused a huge population to migrate to South Korea's main metropolis, Seoul, over the industrialization process, the regional based social interactions have been the strongest in the social interactions among Seoul migrants through hometown gatherings, high school alumni, or extended family reunions. Two regional groups, Youngnam and Honam, are most distinguished due to their rivalry size and geographical separation by the Taebaek Mountains that separate the peninsula.[24] In the 1950s after the Korean war (1950-53), there was a negligible difference between these two regional groups: both were poor and low skilled, as indicated by point A in the skill composition map of Panel A of Figure 18. Over the next decades, the between-group disparity has grown significantly: for example, among leaders in contemporary Korean society, 43.35% were born in Youngnam and 21.88% were born in Honam.[25] As Appendix Figure 3 demonstrates,

[24]According to the 1949 Census, Youngnam constituted 31.43%, Honam 25.24%, Metropolitan Area (Seoul and Gyunggi) 20.69%, Chungcheong 15.73%, Kangwon 5.65% and Cheju 1.26% of the total population.

[25]Source: Chosun Daily Leaders' Database in 2002 (www.dbchosun.com); Eui-Young Yu (2003). Note that these are calculated excluding Seoul born leaders because Seoul natives (about 5% of the population) were exceptionally more successful than migrants from the outskirts.

members of the Youngnam are much more represented in most professions than those of the Honam. The following explanations present the process of the group inequality evolution in the early stage of economic development and the diminishing between-group inequality in the later stage of economic development.

Emergence of Initial Group Disparity in the 1960s and 1970s

In the 1960s and 70s, the industrialization was strongly pushed forward by President Park's administration, whose regional origin was Youngnam. Ministerial officials from his native province, Youngnam, were favored for the stability of the military regime (Ha 2007).[26]Youngnam-created companies and businessmen took advantage of the social connections to the administration, while the rivalry social group Honam, which was least connected to the administration, was most disadvantaged. In the early 80s, about half of the largest conglomerates were Youngnam-based, while only ten percent of them were Honam-based.[27] More Youngnam-born workers were hired by big companies and promoted to the manager level using the social ties and connections. The Youngnam-dominating circumstance led Honam group to be against the Park's political party denoted by "Industrial party", and Youngnam group to be strongly supportive for the party as reflected in the presidential elections since 1971 in Appendix Figure 4. The disparity emerged under the Park's regime is described in Panel B of Figure 18: denoting the skill levels of Honam and Youngnam groups by s^h and s^y respectively, the state (s^h, s^y) moved from a low-skilled equal state A to a unequal state A' in an overlap range of M_{ll} and M_{lh}.

Enhanced Human Capital Investment of the Youngnam Group since the Mid 1970s

Even after the assassination of President Park in 1979, the Youngnam based military regime continued until the early 1990s. Youngnam-created business conglomerates were successful in the global market. Members of the Youngnam maintained the optimistic view about the future that the network quality of the group persistently improves over time. As the dynamic model of this paper predicts, they enhanced human capital investment expecting the higher returns accrued over their lifetimes. The college advancement rate in Figure 19 well reflects the enhanced skill investment activities of young members of the Youngnam: since the late 70s, their college advancement rate started to be significantly higher than other regional groups. It maintained 7 to 13 percent higher rate than the national average in the 80s and 90s. More importantly, the higher college advancement rate

[26] Among ministerial officials between 1962 and 1984, excluding Seoul or North Korea born officials, 48.76% were born in Youngnam while only 16.25% were born in Honam; Among CEOs of major banks, 52.63% were born in Youngnam and 5.26% were born in Honam. (Hankook Daily 1/27/1989)

[27] Among the founders of the largest fifty conglomerates in 1985, 22 were born in Youngnam, four in Honam, twelve in Seoul and Gyunggi, five in North Korea and seven in other regions (MH Kim 1991). The politically connected firms are favored by lenders in the developing countries (Khwaja and Mian 2005).

continued under the strictly equal provision of schooling in those days: since the late 70s, all private secondary schools were merged into the public school system and the salary of the teachers became identical across all secondary schools in the country.[28] The evolution of skill composition (s^h, s^y) over the period is described in Panel C of Figure 18. The members of the Youngnam significantly improved skill investment rates with an optimistic view about the future, while those of the Honam continued the lower skill investment activities with a pessimistic view.

Enhanced Human Capital Investment of the Honam Group since the Early 1990s

The situation started to change in the early 1990s. The first democratic regime took place in the 1992 presidential election and a Honam-born candidate was elected as the President for the first time in the next election of 1997.[29] Social integration between two regional groups proceeded over time. The power transfer from Youngnam to Honam and the progressed social interaction between two regional groups helped young members of the Honam to hold the optimistic view that the network quality of the group will improve over time. They enhanced the skill investment activities expecting the higher lifetime returns on the investment. Figure 19 demonstrates the highest level of college advancement rate of Honam since 1994. The enhanced skill investment activities of Honam is described in Panel D of Figure 18: the skill composition (s^h, s^y) was placed in an overlap range of M_{lh} and M_{hh} in the early 1990s due to the integration effect and the power transfer to Honam. As members of the Honam hold an optimistic view about the future, the skill composition (s^h, s^y) started to move toward the high-skilled equal state (s_h, s_h).[30]

The dominating position of the Youngnam regional group helped the increased human capital investment of the Korean economy in the 70s and 80s. Noting that the human capital accumulation is a driving force of the economic development, the dominating position of a selective group provided a positive effect on economic growth, which is consistent with what the given dynamic model suggests for the early stage of economic development. The power transfer from one group to another group and the more equally distributed social network capital helped another social group Honam to improve its skill investment activities significantly. This promoted the further economic growth by the improved human capital investment in the economy. Thus, as the model predicts, the equality has a positive

[28] Even more, over the same period, any type of private tutoring was prohibited by law. Also, students were randomly assigned to the schools in most cities.

[29] As Appendix Figure 4 displays, a fraction of the original democratic party was merged into the industrial party in 1992, which had been led by President Park in the 70s. The democratic leader YS Kim, the candidate of the "new" industrial party, competed against another democratic leader DJ Kim, the candidate of the democratic party, in the 1992 presidential election.

[30] The underclass of Seoul continue lower skill investment activities. It is plausible that they are trapped in the network structure due to the urban poverty problem and the consequent low quality of education period network externalities. This might persist in the future. It is noteworthy that a considerable percentage of the Seoul underclass are migrants from Honam who moved in the 70s and 80s.

effect on economic growth in the later stage of economic development.

9 Conclusion

This paper explores the dynamic structure of group inequality evolution through the channel of social network externalities. The interaction of two kinds of network externalities, those operating during the skill acquisition period and those at work over the course of a worker's life, provides a unique dynamic picture with folded overlaps and deterministic ranges. The former are the skill composition ranges in which the mobilization force of lifetime network externality is most influential, and the latter are ranges in which the historical force of education period network externality is most influential. Unequal stable states in deterministic ranges are defined as network traps, in which a disadvantaged social group cannot improve its skill investment rate without a governmental intervention or a structural change of the economy. Egalitarian policies to mobilize the group out of the trap are examined. Any type of egalitarian policy, integration, quota or training subsidies, can be effective in an economy with a minority disadvantaged group. If the disadvantaged group is the majority, one policy alone cannot solve the problem, but a combination of different policies may mobilize the group to change its skill investment activities.

The dynamic model of the paper identifies multiple steady states of groups' skill levels that can be interpreted as different development stages, considering that the economic growth is driven by the accumulation of human capital in the modern economy. The positive effect of egalitarian policies on the economic development is discussed. When social network capital, the quality of social network, is more equally distributed between social groups, more disadvantaged group members are motivated to invest in their skills with the increased return on skill acquisition to accrue over their lifetime and, thus, the economy grows. However, if the economy is in early stage of development, the unequal distribution of social capital could be better for the economic growth to take off, because at least selective groups are motivated to develop their skills even under the strong adverse effects of poor quality network externalities over the skill acquisition period. This implies that unequal skill distribution between social groups in many less developed countries might be the byproduct of an initial economic development promotion. It is noteworthy that the macroeconomic effects of equality/inequality have been examined even without imposing the standard assumption of an imperfect credit market. Therefore, unlike the previous studies (Loury 1981, Galor and Zeira 1993), the result implies a positive effect of equality even in an economy where a credit constraint is not binding for the skill achievement, or where public provision of education is well cultivated.

The theoretical framework in this paper is unique in terms of its dynamic structure with multiple overlaps. The folded overlaps and social consensus in the model are innovative ideas to deal with distinct social groups with different economic statuses and expectations toward the future. The concepts can be applied to other research areas dealing with heterogeneous economic groups, such as trade between nations and games between teams. Also, the rational expectation framework, combined with an overlapping generation structure, provides a unique way to analyze the intergenerational social mobility. A similar method is applied to my companion paper (Kim and Loury 2008) for the analysis of the evolution of group reputation. Future research related to intergenerational social mobility may adopt this method. Noting the importance of social networks to one's economic success, the lack of theoretical works along this line is awaiting research in the field of economics. The theoretical framework suggested in this paper could be a good building block for more research on the social networks and social capital.

10 Appendix: Proofs

10.1 Proof of Lemma 2

Let us define a function $s_y(s_x)$: $s_y \equiv 1 - G(A(s_x, \frac{\bar{\delta}+f(s_x)}{\rho+\alpha}))$. Then, s' and s'' satisfy $s' > s_y(s')$ and $s'' < s_y(s'')$, according to the given condition. Because $A(0, \frac{\bar{\delta}+f(0)}{\rho+\alpha}) < \infty$, $s_y(0) > 0$, which implies that at least one steady state exists in $(0, s')$. Because $A(1, \frac{\bar{\delta}+f(1)}{\rho+\alpha}) > 0$, $s_y(1) < 1$, which implies that at least one steady state exists in $(s'', 1)$. By the continuity of $s_y(s_x)$, there must be at least one steady state in (s', s''). QED.

10.2 Proof of Lemma 3

Using an implicit function theorem, we have the following result from equation (8) for any state on the $\dot{s} = 0$ locus:

$$\frac{\partial \Pi}{\partial s}\Big|_{(\dot{s}=0 \text{ locus })} = -\frac{1 + G'A'_s}{G'A'_\Pi}\Big|_{(\dot{s}=0 \text{ locus })}. \tag{26}$$

From equation (9), we have the following result for any state on the $\dot{\Pi} = 0$ locus:

$$\frac{\partial \Pi}{\partial s}\Big|_{(\dot{\Pi}=0 \text{ locus })} = \frac{f'}{\rho + \alpha}\Big|_{(\dot{\Pi}=0 \text{ locus })}. \tag{27}$$

From the demarcation loci described in Panel B of Figure 1, we know that the slope at the $\dot{s} = 0$ locus is greater than that at the $\dot{\Pi} = 0$ locus at the steady states E_l and E_h, and the slope at the $\dot{s} = 0$ locus is smaller than that at the $\dot{\Pi} = 0$ at the steady state E_m:

$$-\frac{1 + G'A'_s}{G'A'_\Pi} > \frac{f'}{\rho + \alpha} \quad (\text{at } E_l \text{ or } E_h). \tag{28}$$

$$-\frac{1 + G'A'_s}{G'A'_\Pi} < \frac{f'}{\rho + \alpha} \quad (\text{at } E_m). \tag{29}$$

Given the dynamic system in equation (7), its linearization around a steady state $(\bar{s}, \bar{\Pi})$ is

$$\begin{aligned} \dot{s}_t &= \alpha[-G'A'_s - 1](s_t - \bar{s}) + \alpha[-G'A'_\Pi](\Pi_t - \bar{\Pi}) \\ \dot{\Pi}_t &= -f'(s_t - \bar{s}) + (\rho + \alpha)(\Pi_t - \bar{\Pi}). \end{aligned}$$

Therefore, the Jacobian matrix J_E evaluated at a steady state is

$$J_E \equiv \begin{bmatrix} -\alpha G'A'_s - \alpha & -\alpha G'A'_\Pi \\ -f' & \rho + \alpha \end{bmatrix}_{(\bar{s}, \bar{\Pi})}.$$

Consequently, its transpose is $trJ_E = -\alpha G'A'_s + \rho$ and the determinant is $|J_E| = -\alpha(\rho + \alpha)(G'A'_s + 1) - \alpha f'G'A'_\Pi$. Since trJ_E is positive, every steady state is unstable. $|J_E|$ is negative at E_l and E_h because of equation (28), which implies that those are saddle points. $|J_E|$ is positive at E_m because of equation (29), which implies that E_m is a source, either an unstable node or an unstable focus. QED.

10.3 Proof of Proposition 2

Suppose $e_o \geq s_m$. This means that the saddle path to E_h intersects the $\dot{\Pi} = 0$ locus between s_m and s_h. Let us denote the intersection point by $C(s^c, \Pi^c)$, where $s_m \leq s^c < s_h$. Because it is on the $\dot{\Pi} = 0$ locus, $\Pi^c|_{\dot{\Pi}=0 \text{ locus}} = \frac{\bar{\delta}+f(s^c)}{\rho+\alpha}$. Because it is on the saddle path to E_h,

$$\Pi^c|_{\text{saddle path}} = \int_t^{\infty} [\bar{\delta} + f(s_\tau)]e^{-(\rho+\alpha)(\tau-t)}\mathrm{d}\tau,$$

where $s_t = s^c$, $s_\tau > s^c, \forall \tau > t$, and $\lim_{\tau\to\infty} s_\tau = s_h$. Therefore, we have

$$\begin{aligned} \Pi^c|_{\text{saddle path}} &= \int_t^{\infty} [\bar{\delta} + f(s^c) + (f(s_\tau) - f(s^c))]e^{-(\rho+\lambda)(\tau-t)}\mathrm{d}\tau \\ &= \frac{\bar{\delta} + f(s^c)}{\rho+\alpha} + \int_t^{\infty} [f(s_\tau) - f(s^c)]e^{-(\rho+\lambda)(\tau-t)}\mathrm{d}\tau. \end{aligned} \tag{30}$$

Then, $\Pi^c|_{\text{saddle path}} > \Pi^c|_{\dot{\Pi}=0 \text{ locus}}$ because $f(s_\tau) - f(s^c) > 0, \forall \tau > t$. This contradicts the assumption that there exists an intersection of the locus and the saddle path between s_m and s_h. Therefore, $e_o < s_m$.

In the same way, we can prove that $e_p > s_m$. Thus, a positive range of overlap $[e_o, e_p]$ exists, where $e_o < e_p$. Since two saddle paths in the overlap exist (one path to E_h, and the other path to E_l), the social consensus determines the one to be taken. QED.

10.4 Proof of Proposition 3

[Optimistic Path] Above the two demarcation loci, the dynamic system is determined by

$$\begin{aligned} \dot{s}_t &= \alpha(1 - s_t) \\ \dot{\Pi}_t &= (\rho + \alpha)\Pi_t - q_1 s_t - \bar{\delta} - q_0. \end{aligned} \tag{31}$$

In this dynamic system, two eigenvalues are $-\alpha$ and $\alpha + \rho$ and the steady state $(\bar{s}, \bar{\Pi})_{op}$ is $(1, \frac{\bar{\delta}+q_0+q_1}{\rho+\alpha})$. Then, we have the explicit functions of s_t and Π_t that satisfy the saddle path condition $lim_{\tau\to\infty}(s_t, \Pi_t) = (\bar{s}, \bar{\Pi})_{op}$:

$$\begin{aligned} s_t &= C^* e^{-\alpha t} + \frac{\bar{\delta} + q_0 + q_1}{\rho + \alpha} \\ \Pi_t &= C^* \frac{\rho + 2\alpha}{q_1} e^{-\alpha t} + 1, \end{aligned} \tag{32}$$

where C^* depends on the initial condition (s_0, Π_0) on the saddle path. Thus, we have the saddle path that converges to $E_h(1, \frac{\bar{\delta}+q_0+q_1}{\rho+\alpha})$: $\Pi^{op} = \frac{q_1}{\rho+2\alpha}s^{op} + \frac{(\bar{\delta}+q_0)(\rho+2\alpha)+q_1\alpha}{(\rho+\alpha)(\rho+2\alpha)}$.

[Pessimistic Path] Under the two demarcation loci, the dynamic system is determined by

$$\begin{aligned} \dot{s}_t &= -\alpha s_t \\ \dot{\Pi}_t &= (\rho+\alpha)\Pi_t - q_1 s_t - \bar{\delta} - q_0. \end{aligned} \tag{33}$$

In this dynamic system, two eigenvalues are $-\alpha$ and $\alpha+\rho$ and the steady state $(\bar{s},\bar{\Pi})_{pe}$ is $(0, \frac{\bar{\delta}+q_0}{\rho+\alpha})$. Then, we have the explicit functions of s_t and Π_t that satisfy the saddle path condition $lim_{\tau\to\infty}(s_t,\Pi_t) = (\bar{s},\bar{\Pi})_{pe}$:

$$\begin{aligned} s_t &= C^* e^{-\alpha t} + \frac{\bar{\delta}+q_0}{\rho+\alpha} \\ \Pi_t &= C^* \frac{\rho+2\alpha}{q_1} e^{-\alpha t}, \end{aligned} \tag{34}$$

where C^* depends on the initial condition (s_0,Π_0) on the saddle path. Thus, we have the saddle path that converges to $E_l(0, \frac{\bar{\delta}+q_0}{\rho+\alpha})$: $\Pi^{pe} = \frac{q_1}{\rho+2\alpha}s^{pe} + \frac{\bar{\delta}+q_0}{\rho+\alpha}$.

10.5 Proof of Lemma 5

By the implicit function theorem imposed at equation (16), we have the following first order derivative:

$$\frac{\mathrm{d}\ s^j}{\mathrm{d}\ s^{i*}} = \frac{1}{1-k^i}\left[\frac{1}{-(G(\tilde{A}(\sigma^{i*}))'} - k^i\right]. \tag{35}$$

By assumption 1, there exists $\hat{\sigma}$ such that $G(\tilde{A}(\sigma))'$ is decreasing in $[0,\hat{\sigma}]$ and increasing in $[\hat{\sigma},1]$. As equation (16) implies, σ^{i*} is monotonically increasing with s^{i*}. Therefore, $\frac{\mathrm{d}\ s^j}{\mathrm{d}\ s^{i*}}$ is decreasing where $s^{i*} < 1-G(\tilde{A}(\hat{\sigma}))$, and increasing where $s^{i*} < 1-G(\tilde{A}(\hat{\sigma}))$. Equivalently, $D^j(s^{i*})$ is concave where $s^{i*} < 1-G(\tilde{A}(\hat{\sigma}))$, and convex where $s^{i*} < 1-G(\tilde{A}(\hat{\sigma}))$. QED.

10.6 Proof of Lemma 6

Note that $|D^j(s^{i*}) - s^{i*}| = \left|\frac{\tilde{A}^{-1}G^{-1}(1-s^{i*})-s^{i*}}{1-k^i}\right| = \frac{|\sigma^{i*}-s^{i*}|}{1-k^i}$, because $s^{i*} = 1-G(\tilde{A}(\sigma^{i*}))$. $|\sigma^{i*}-s^{i*}|$ is fixed as Panel B of Figure 6 describes. The first derivative gives $\frac{\partial|D^j(s^{i*})-s^{i*}|}{\partial\eta} = \frac{|\sigma^{i*}-s^{i*}|}{(1-\beta^i)(1-\eta)^2}$. Thus, $|D^j(s^{i*}) - s^{i*}|$ shrinks as η increases. Also, it becomes larger with the bigger β^i because $\frac{\partial|D^j(s^{i*})-s^{i*}|}{\partial\beta^i} = \frac{|\sigma^{i*}-s^{i*}|}{(1-\beta^i)^2(1-\eta)}$. Finally, let us denote the slope $\left|\frac{\partial(D^j(s^{i*})-s^{i*})}{\partial s^{i*}}\right|$ by Q:

$$\begin{aligned} Q &= \left|\frac{1}{1-k^i}\left[\frac{1}{-(G(\tilde{A}(\sigma^{i*}))'} - k^i\right] - 1\right|, \text{ because of equation (35).} \\ &= \frac{1}{1-k^i}\left|\frac{1}{-(G(\tilde{A}(\sigma^{i*}))'} - 1\right|. \end{aligned} \tag{36}$$

Then, the first derivative with respect to β^i is $\frac{\partial Q}{\partial\beta^i} = \frac{1}{(1-\beta^i)^2(1-\eta)}\left|\frac{1}{-(G(\tilde{A}(\sigma^{i*}))'} - 1\right|$. Thus, the slope is steeper with the bigger β^i. QED.

10.7 Proof of Proposition 6

The total number of steady states is nine with $\eta = 1$, as discussed in section 5.1. By the dynamic system in a homogeneous economy summarized in (7), the three states, (s_l, s_l), (s_m, s_m) and (s_h, s_h), are steady states in a heterogeneous economy regardless of η and β^1. For example, in the case of (s_h, s_h), $\sigma^1 = \sigma^2 = s^1 = s^2 = s_h$. They satisfy $\dot{s}^1 = \dot{s}^2 = \dot{\Pi}^1 = \dot{\Pi}^1 = 0$ in the dynamic system summarized in Theorem 1. First of all, I claim that there are no symmetric steady states other than those three. Suppose that a symmetric steady state $(\hat{s}, \hat{s})$ exists, which is not one of the three. Since $\sigma^1 = \sigma^2 = s^1 = s^2 = \hat{s}$, this implies

$$\begin{aligned}
\dot{s_t}^1 &= \alpha\left[1 - G(A(\hat{s}, \Pi_t^1)) - \hat{s}\right] = 0\\
\dot{s_t}^2 &= \alpha\left[1 - G(A(\hat{s}, \Pi_t^2)) - \hat{s}\right] = 0\\
\dot{\Pi}_t^1 &= (\rho+\alpha)\left[\Pi_t^1 - \frac{\bar{\delta} + f(\hat{s})}{\rho+\alpha}\right] = 0\\
\dot{\Pi}_t^2 &= (\rho+\alpha)\left[\Pi_t^2 - \frac{\bar{\delta} + f(\hat{s})}{\rho+\alpha}\right] = 0.
\end{aligned}$$

This contradicts that there are only three skill levels (s_l, s_m, s_h) that satisfy formula (7). Therefore, there are only three steady states regardless of η and β^1.

Secondly, let us prove that the total number of steady states is three with $\eta = 0$. This is true when there are no asymmetric steady states with $\eta = 0$. Suppose an asymmetric steady state $(\hat{s}^1, \hat{s}^2)$ exists, where $\hat{s}^1 \neq \hat{s}^2$. Since two groups are fully integrated, $\sigma^1 = \sigma^2 = \bar{s}$. Since it is a (global) steady state, it should be a partial steady state. By equations (13) and (14), s^{i*} is uniquely determined by σ^{i*}, which implies that $\hat{s}^1 = \hat{s}^2$ when $\sigma^1 = \sigma^2$. This contradicts that it is an asymmetric steady state. Therefore, there is no asymmetric steady state when $\eta = 0$. Since there are only three symmetric steady states, the number of steady states is three when two groups are fully integrated.

The total number of steady states monotonically decreases from nine to three as η declines, because $|D^j(s^{i*}) - s^{i*}|$ is monotonically decreasing as η declines (Lemma 6) and there is a unique inflection point in the partial steady state loci ($D^2(s^{1*})$ and $D^1(s^{2*})$) (Lemma 5). This implies the number of steady states decreases from three to zero as η declines, in Regions 1 and 3, and there is always a unique steady state in Regions 2 and 4. QED.

10.8 Proof of Corollary 2

The uniqueness of the steady states in Regions 2 and 4 is already proven in the proof of Proposition 6. Let us prove that all steady states satisfy $\sigma^{1**} < s_m$ and $\sigma^{2**} > s_m$ in Region 1. The distance $|D^j(s^{i*}) - s^{i*}|$ is monotonically decreasing as η declines (Lemma 6), which means the partial steady state loci move closer to the diagonal as η declines. This implies that the following should hold: $s^{1**} < s_m$ and $s^{2**} > s_m$ at any steady state (s^{1**}, s^{2**}) with $\eta < 1$. In Region 1, the partial steady state locus $s^{1*}(s^2)$ is below the $s_m = k^1 s^{1*} + (1 - k^1)s^2$ line, because (s^{1*}, σ^{1*}) must satisfy $s^{1*} = 1 - G(\tilde{A}(\sigma^{1*}))$ from equations (13) and (14), and, due to its monotonicity, $\sigma^{1*} < s_m$ when $s^{1*} < s_m$. By the same reasoning, the partial steady state locus $s^{2*}(s^1)$ is above the $s_m = k^2 s^{2*} + (1-k^2)s^1$ line, because $\sigma^{2*} > s_m$ when $s^{2*} > s_m$. Therefore, σ^{1**} is smaller than s_m because any steady state (s^{1**}, s^{2**}) in Region 1 must be below the $s_m = k^1 s^{1*} + (1-k^1)s^2$ line. σ^{2**} is greater than s_m because the steady state must be above the $s_m = k^2 s^{2*} + (1-k^2)s^1$ line.

Now, let us prove that $\sigma^{1**} > s_l$ and $\sigma^{2**} < s_h$. Since the distance $|D^j(s^{i*}) - s^{i*}|$ is monotonically decreasing as η declines (Lemma 6), any steady state in Region 1 should satisfy the following two conditions with $\eta < 1$: $s_l < s^{1**} < s_m$ and $s_m < s^{2**} < s_h$. This implies that $s_l < \sigma^{1**} < s_h$ and $s_l < \sigma^{2**} < s_h$. Therefore, we can conclude that $s_l < \sigma^{1**}(s^{1**}) < s_m$ and $s_m < \sigma^{2**}(s^{2**}) < s_h$ for any steady state in Region 1 with $\eta < 1$ given. In the same way, we can prove that all steady states satisfy $s_m < \sigma^{1**}(s^{1**}) < s_h$ and $s_l < \sigma^{2**}(s^{2**}) < s_m$ in Region 3. QED.

10.9 Proof of Theorem 2

Let us check the local stability at one steady state Q_{hh}. We have the following Jacobian matrix at the steady state $Q_{hh}(s_h, s_h, \Pi_h, \Pi_h)$:

$$\mathbf{J_{Q_{hh}}} = \begin{bmatrix} \alpha[-G'A'_\sigma(\eta + (1-\eta)\beta^1) - 1] & \alpha[-G'A'_\sigma(1-\eta)\beta^2] & \alpha[-G'A'_\Pi] & 0 \\ \alpha[-G'A'_\sigma(1-\eta)\beta^1] & \alpha[-G'A'_\sigma(\eta + (1-\eta)\beta^2) - 1] & 0 & \alpha[-G'A'_\Pi] \\ -f'_\sigma(\eta + (1-\eta)\beta^1) & -f'_\sigma(1-\eta)\beta^2 & \rho + \alpha & 0 \\ -f'_\sigma(1-\eta)\beta^1 & -f'_\sigma(\eta + (1-\eta)\beta^2) & 0 & \rho + \alpha \end{bmatrix}_{Q_{hh}}.$$

Let us denote $\mathbf{J_{Q_{hh}}} - \lambda\mathbf{I}$ using 2×2 matrices J_{ij}s: $\mathbf{J_{Q_{hh}}} - \lambda\mathbf{I} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$. We need to calculate the determinant of $\mathbf{J_{Q_{hh}}} - \lambda\mathbf{I}$ in order to find eigenvalues. Note that $|\mathbf{J_{Q_{hh}}} - \lambda\mathbf{I}| \equiv |J_{22}| \cdot |J_{11} - J_{12}J_{22}^{-1}J_{21}|$. Let us denote the second term by J': $J' \equiv J_{11} - J_{12}J_{22}^{-1}J_{21}$. Using the explicit forms of J_{ij}s, J' is

$$\begin{aligned} J' &= J_{11} - \begin{bmatrix} \alpha[-G'A'_\Pi] & 0 \\ 0 & \alpha[-G'A'_\Pi] \end{bmatrix} \cdot \begin{bmatrix} (\rho + \alpha - \lambda)^{-1} & 0 \\ 0 & (\rho + \alpha - \lambda)^{-1} \end{bmatrix} \\ &\quad \cdot \begin{bmatrix} -f'_\sigma(\eta + (1-\eta)\beta^1) & -f'_\sigma(1-\eta)\beta^2 \\ -f'_\sigma(1-\eta)\beta^1 & -f'_\sigma(\eta + (1-\eta)\beta^2) \end{bmatrix}. \end{aligned} \tag{37}$$

Thus, its determinant is

$$\begin{aligned} |J'| &= \left| J_{11} - \alpha\xi \begin{bmatrix} \eta + (1-\eta)\beta^1 & (1-\eta)\beta^2 \\ (1-\eta)\beta^1 & \eta + (1-\eta)\beta^2 \end{bmatrix} \right|, \text{ where } \xi = \frac{G'A'_\Pi f'_\sigma}{\rho + \alpha - \lambda}. \\ &= [\lambda - \alpha(-G'A'_\sigma\eta - 1) + \alpha\xi\eta].[\lambda - \alpha(-G'A'_\sigma - 1) + \alpha\xi]. \end{aligned} \tag{38}$$

The result is achieved with a bit messy calculation. Therefore, we have the determinant of $\mathbf{J} - \lambda\mathbf{I}$:

$$
\begin{aligned}
|\mathbf{J}_{Q_{hh}} - \lambda\mathbf{I}| &= |J_{22}| \cdot [\lambda - \alpha(-G'A'_\sigma\eta - 1) + \alpha\xi\eta] \cdot [\lambda - \alpha(-G'A'_\sigma - 1) + \alpha\xi] \\
&= [\lambda^2 - \lambda(-\alpha G'A'_\sigma + \rho) - \alpha(\alpha+\rho)(G'A'_\sigma + 1) - \alpha G'A'_\Pi f'_\sigma]_{Q_{hh}} \\
&\quad \cdot[\lambda^2 - \lambda(-\alpha G'A'_\sigma\eta + \rho) - \alpha(\alpha+\rho)(G'A'_\sigma\eta + 1) - \alpha G'A'_\Pi f'_\sigma\eta]_{Q_{hh}}. \qquad (39)
\end{aligned}
$$

Taking $|\mathbf{J}_{Q_{hh}} - \lambda\mathbf{I}| = 0$, we can obtain four eigenvalues at the steady state. First, note that $[-\alpha(\alpha+\rho)(G'A'_\sigma + 1) - \alpha G'A'_\Pi f'_\sigma]_{Q_{hh}} < 0$ by equation (28). Thus, the first term of the determinant has one positive and one negative eigenvalue. That is, the local stability condition at E_{hh} in a homogeneous economy implies one negative and one positive eigenvalue in a heterogeneous economy at Q_{hh}. Also, we have $[-\alpha(\alpha+\rho)(G'A'_\sigma\eta+1) - \alpha G'A'_\Pi f'_\sigma\eta]_{Q_{hh}} < 0$ because $-\alpha(\alpha+\rho)(G'A'_\sigma\eta+1) - \alpha G'A'_\Pi f'_\sigma\eta = \eta(-\alpha(\alpha+\rho)(G'A'_\sigma + 1) - \alpha G'A'_\Pi f'_\sigma) - \alpha(\alpha+\rho)(1-\eta)$. Therefore, there are two positive eigenvalues and two negative eigenvalues.

There exists a unique equilibrium path if the number of jumping variables equals the number of eigenvalues with a positive real part (Buiter, 1984). Since we have two jumping variables, Π^1_t and Π^2_t, we know the existence of the unique equilibrium path in the neighborhood of (s_h, s_h). Therefore, Q_{hh} is an economically stable state. The four steady states Q_{ll}, Q_{hh}, Q_{lh} and Q_{hl} are identical in terms of their local dynamic structures, as manifested by local demarcation surfaces at those states. We can conclude that those four steady states are economically stable states. Without loss of generality, we can infer that two eigenvalues with a positive real part and two with a negative real part exist at those states.

All other steady states, Q_{lm}, Q_{mh}, Q_{ml}, Q_{hm} and Q_{mm}, are economically unstable steady states. For example, check the local stability of Q_{mm}. Using equation (39), we have the determinant $\mathbf{J}_{Q_{mm}} - \lambda\mathbf{I}$:

$$
\begin{aligned}
|\mathbf{J}_{Q_{mm}} - \lambda\mathbf{I}| &= [\lambda^2 - \lambda(-\alpha G'A'_\sigma + \rho) - \alpha(\alpha+\rho)(G'A'_\sigma + 1) - \alpha G'A'_\Pi f'_\sigma]_{Q_{mm}} \\
&\quad \cdot[\lambda^2 - \lambda(-\alpha G'A'_\sigma\eta + \rho) - \alpha(\alpha+\rho)(G'A'_\sigma\eta + 1) - \alpha G'A'_\Pi f'_\sigma\eta]_{Q_{mm}} \qquad (40)
\end{aligned}
$$

We know that $[-\alpha(\alpha+\rho)(G'A'_\sigma+1) - \alpha G'A'_\Pi f'_\sigma]_{Q_{mm}} > 0$, by equation (29), and $-\alpha G'A'_\sigma + \rho > 0$ because of $A'_\sigma < 0$. Thus, the first term of the determinant implies two eigenvalues with positive real parts. The second term implies at least one eigenvalue with positive real part because $-\alpha G'A'_\sigma\eta + \rho > 0$. Therefore, at least three eigenvalues have positive real parts. Since we have only two jumping variables, we cannot always find a unique equilibrium path in the neighborhood of (s_m, s_m). Thus, Q_{mm} is an economically unstable state. Now check other states. Since all other four are identical in terms of their dynamic structures, we need to check only one of them: Q_{mh}. When $\eta = 1$, there must be three eigenvalues with positive real parts and one negative eigenvalue, because group 1 is at an economically unstable state E_m and group 2 is at an economically stable state E_h in the separated dynamic structures of two groups. Thus, Q_{mh} is an economically unstable state since the number of positive eigenvalues exceeds the number of jumping variables: except $s^1 = s_m$, there is no converging path to the state in the neighborhood of (s_m, s_h). We cannot explicitly calculate the signs of eigenvalues with $\eta < 1$. However, the qualitative approach using demarcation surfaces identified in section 5.2.3 helps us to conclude that it cannot be an economically stable state for any η, because we can easily find at least one point (s^1, s^2) in the neighborhood of (s'_m, s'_h), in which a converging equilibrium path to $Q_{mh}(s'_m, s'_h)$ does not exist. QED.

10.10 Proof of Lemma 7

The given determinant equation is obtained in the proof of Theorem 2. Both $\mathbb{R}$ and $\eta\mathbb{R} + (1-\eta)\rho$ are positive because $A'_\sigma < 1$. Both $\mathbb{H}$ and $\eta\mathbb{H} - (1-\eta)\alpha(\alpha+\rho)]$ are negative at economically stable states, Q_{ll} and Q_{hh}, because of E_l and E_h are economically stable states in a homogeneous economy and satisfy condition (28).

10.11 Proof of Corollary 4

At an economically stable state Q_{ij}, the linearized dynamic system is expressed with the Jacobian matrix $\mathbf{J}_{Q_{ij}}$:

$$\begin{bmatrix} \dot{\tilde{s}}(t) \\ \dot{\tilde{\Pi}}(t) \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} \tilde{s}(t) \\ \tilde{\Pi}(t) \end{bmatrix}, \tag{41}$$

where $\mathbf{J}_{Q_{ij}} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$, $\tilde{s}(t) = \begin{bmatrix} s_t^1 - s^{1\prime} \\ s_t^2 - s^{2\prime} \end{bmatrix}$ and $\tilde{\Pi}(t) = \begin{bmatrix} \Pi_t^1 - \Pi^{1\prime} \\ \Pi_t^2 - \Pi^{2\prime} \end{bmatrix}$, in which J_{ij} is 2×2 matrix. Let us define the expectation operator E with $I(t)$, which is the information set conditioning expectations formed at time t: for any vector x, $E_t x(\tau) \equiv E(x(\tau)|I(t))$. This means that $E_t x(\tau)$ is the expected value x at time τ given the information set at time t. Let us define $\dot{x}(t)$ as $\dot{x}(t) \equiv \lim_{u\to t} \frac{x(u)-x(t)}{u-t}$. Then, $E_t\dot{x}(\tau) = E\left(\lim_{u\to\tau} \frac{x(u)-x(\tau)}{u-\tau}\middle| I(t)\right)$. Taking the expectation operator at both sides of the above equation, we have

$$\begin{bmatrix} E_t\dot{\tilde{s}}(t) \\ E_t\dot{\tilde{\Pi}}(t) \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} E_t\tilde{s}(t) \\ E_t\tilde{\Pi}(t) \end{bmatrix}. \tag{42}$$

Note that there are two positive eigenvalues and two negative eigenvalues at an economically stable state according to Theorem 2. Since they are distinct by assumption, there are four linearly independent eigenvectors. Then, we have Jordan form with a diagonal matrix Λ:

$$J_{Qij} = B\Lambda B^{-1}, \tag{43}$$

in which $\Lambda = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix}$ with $\Lambda_1(\Lambda_2)$ containing two negative (positive) eigenvalues, and the column vectors of B are the corresponding eigenvectors. Let us partition B and B^{-1} into four 2×2 matrices: $B = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} V_{11} & V_{21} \\ V_{12} & V_{22} \end{bmatrix}$. Let us define two dimensional vectors $\tilde{p}(t)$ and $\tilde{q}(t)$ as

$\begin{bmatrix} \tilde{p}(t) \\ \tilde{q}(t) \end{bmatrix} = B^{-1} \begin{bmatrix} \tilde{s}(t) \\ \tilde{\Pi}(t) \end{bmatrix}$. Then, using the Jordan form, we have the following result.

$$\begin{bmatrix} E_t\dot{\tilde{s}}(t) \\ E_t\dot{\tilde{\Pi}}(t) \end{bmatrix} = B\Lambda B^{-1} \begin{bmatrix} E_t\tilde{s}(t) \\ E_t\tilde{\Pi}(t) \end{bmatrix} \Rightarrow \begin{bmatrix} E_t\dot{\tilde{p}}(t) \\ E_t\dot{\tilde{q}}(t) \end{bmatrix} = \Lambda \begin{bmatrix} E_t\tilde{p}(t) \\ E_t\tilde{q}(t) \end{bmatrix}. \tag{44}$$

Therefore, we have $E_t\dot{\tilde{q}}(t) = \Lambda_2 E_t\tilde{q}(t)$. This is true for any time $\tau \geq t$: $E_\tau\dot{\tilde{q}}(\tau) = \Lambda_2 E_\tau\tilde{q}(\tau)$. Taking the expectation operator E_t at both sides, we have $E_t E_\tau \dot{\tilde{q}}(\tau) = \Lambda_2 E_t E_\tau \tilde{q}(\tau)$. Note that, for any random vectors u,v and w, $E(E(u|v,w)|w) = E(u|w)$. Since $I(\tau) \supseteq I(t)$, we have the consequent result,

$$E_t\dot{\tilde{q}}(\tau) = \Lambda_2 E_t\tilde{q}(\tau). \tag{45}$$

This means that a forward looking individual's expectation for time τ variation of $\tilde{q}(\tau)$, given information set $I(t)$, follow the above equation. An forward looking individual can expect $\tilde{q}(\tau)$ to be

$$E_t\tilde{q}(\tau) = e^{\Lambda_2 \tau} K, \quad \forall \tau \geq t, \tag{46}$$

in which K is a two dimensional arbitrary constant. The forward looking "rational" individuals who know that $\tilde{q}(\tau)$ should not explode over time will adjust their jumping variables (Π_t^1, Π_t^2) in order to make $E_t\tilde{q}(\infty) \neq \infty$, which implies $K = 0$. This is a typical transversality condition. Therefore, we have $E_t\tilde{q}(\tau) = 0$. This should be true for all $\tau \geq t$. We have

$$E_t\tilde{q}(t) = 0 \Rightarrow V_{21}\tilde{s}(t) + V_{22}\tilde{\Pi}(t) = 0, \tag{47}$$

because $q(t) = V_{21}\tilde{s}(t) + V_{22}\tilde{\Pi}(t)$ and, for any vector $x(t)$, $E_t(x(t)) = x(t)$. Therefore, we have

$$\begin{aligned} \dot{\tilde{s}}(t) &= J_{11}\tilde{s}(t) + J_{12}\tilde{\Pi}(t) \quad (\because \text{ equation (41)}) \\ &= (J_{11} - J_{12}V_{22}^{-1}V_{21})\tilde{s}(t) \quad (\because \text{ equation (47)}) \\ &= (J_{11} + J_{12}W_{21}W_{11}^{-1})\tilde{s}(t) \quad (\because BB^{-1} = I) \\ &= (J_{11}W_{11} + J_{12}W_{21})W_{11}^{-1}\tilde{s}(t) \\ &= W_{11}\Lambda_1 W_{11}^{-1}\tilde{s}(t) \quad (\because \mathbf{J}_{Qij}B = B\Lambda). \end{aligned}$$

Therefore, we know how the skill composition evolves around an economically stable state given (s_0^1, s_0^2):

$$\begin{aligned} \tilde{s}(t) &= e^{W_{11}\Lambda_1 W_{11}^{-1}(t-t_0)}\tilde{s}(t_0) \\ &= W_{11}e^{\Lambda_1(t-t_0)}W_{11}^{-1}\tilde{s}(t_0). \end{aligned} \tag{48}$$

The corresponding benefits of investments are

$$\begin{aligned}\tilde{\Pi}(t) &= W_{21}W_{11}^{-1}\tilde{s}(t) \quad (\because \text{ equation (47) and } BB^{-1}=I) \\ &= W_{21}e^{\Lambda_1(t-t_0)}W_{11}^{-1}\tilde{s}(t_0). \end{aligned} \tag{49}$$

Applying $\tilde{s}(t) = \begin{bmatrix} s_t^1 - s^{1\prime} \\ s_t^2 - s^{2\prime} \end{bmatrix}$ and $\tilde{\Pi}(t) = \begin{bmatrix} \Pi_t^1 - \Pi^{1\prime} \\ \Pi_t^2 - \Pi^{2\prime} \end{bmatrix}$,we have the unique equilibrium path given (s_0^1, s_0^2) in the neighborhood Q_{ij}:

$$\begin{bmatrix} s_t^1 \\ s_t^2 \\ \Pi_t^1 \\ \Pi_t^2 \end{bmatrix} = \begin{bmatrix} W_{11}e^{\Lambda_1(t-t_0)}W_{11}^{-1} \\ W_{21}e^{\Lambda_1(t-t_0)}W_{11}^{-1} \end{bmatrix} \begin{bmatrix} s^1(t_0) - s^{1\prime} \\ s^2(t_0) - s^{2\prime} \\ s^1(t_0) - s^{1\prime} \\ s^2(t_0) - s^{2\prime} \end{bmatrix} + \begin{bmatrix} s^{1\prime} \\ s^{2\prime} \\ \Pi^{1\prime} \\ \Pi^{2\prime} \end{bmatrix}.$$

QED.

10.12 Proof of Lemma 8

Note that, as integration proceeds, either Q_{lh} and Q_{mh} are merged together or Q_{lh} and Q_{lm} are merged together before Q_{lh} disappears. First, envision a threshold segregation level for a sufficiently small $\beta^{1\prime}$: $\hat{\eta}(\beta^{1\prime})$. With the threshold level, the $D^2(s^1*)$ curve will be tangent to the $D^1(s^{2*})$ curve and Q_{lh} will be merged with Q_{mh}, as Panel C of Figure 12 illustrates approximately. Now, let us increase $\beta^{1\prime}$ to $\beta^{1\prime} + \epsilon$ holding $\eta = \hat{\eta}(\beta^{1\prime})$. With this increase, $D^1(s^{2*})$ moves away from a diagonal because of the increased β^1 and $D^2(s^{1*})$ curve moves closer to the the diagonal because of the increased β^2, according to Lemma 6. Thus, two steady states, Q_{lh} and Q_{mh}, get more distant from each other. In order to merge them and to make $D^2(s^1*)$ curve tangent to the $D^1(s^2*)$ curve, the lower segregation level is required. Therefore, $\hat{\eta}(\beta^{1\prime}) > \hat{\eta}(\beta^{1\prime} + \epsilon)$, which implies $\hat{\eta}(\beta^1)$ is a strictly deceasing function with the lower level of β^1.

Now, imagine a threshold segregation level for a sufficiently great $\beta^{1\prime\prime}$: $\hat{\eta}(\beta^{1\prime\prime})$. With this threshold level, the $D^1(s^2*)$ curve will be tangent to the $D^2(s^{1*})$ curve and Q_{lh} will be merged with Q_{lm}, as Panel B-2 of Figure 15 illustrates approximately. Now, let us decrease $\beta^{1\prime\prime}$ to $\beta^{1\prime\prime} - \epsilon$ holding $\eta = \hat{\eta}(\beta^{1\prime\prime})$. With this decrease, $D^2(s^{1*})$ moves away from a diagonal because of the increased β^2 and the $D^1(s^{2*})$ curve moves closer to the the diagonal because of the decreased β^1, according to Lemma 6. Thus, two steady states, Q_{lh} and Q_{lm}, get more distant from each other. In order to merge them and to make the $D^1(s^2*)$ curve tangent to the $D^2(s^1*)$ curve, the lower segregation level is required. Therefore, $\hat{\eta}(\beta^{1\prime}) > \hat{\eta}(\beta^{1\prime} - \epsilon)$, which implies $\hat{\eta}(\beta^1)$ is a strictly increasing function with the higher level of β^1.

Finally, imagine a group 1 population size of $\hat{\beta}$, with which all three steady states, Q_{lh}, Q_{mh} and Q_{lm}, are merged together at some level of segregation: $\hat{\eta}(\hat{\beta})$. With an increase of β^1 to $\hat{\beta} + \epsilon$, $D^1(s^{2*})$ moves away from a diagonal and the $D^2(s^{1*})$ curve moves close to the the diagonal, which means only one steady state Q_{mh} survives and the two others disappear. This implies the threshold level of segregation should be higher with $\hat{\beta}+\epsilon$: $\hat{\eta}(\hat{\beta}) < \hat{\eta}(\hat{\beta}+\epsilon)$. With a decrease of β^1 to $\hat{\beta}-\epsilon$, $D^2(s^{1*})$ moves away from a diagonal and the $D^1(s^{2*})$ curve moves closer to the the diagonal, which means only one steady state Q_{lm} survives and the two others disappear. This implies the threshold level of segregation should be higher with $\hat{\beta}-\epsilon$: $\hat{\eta}(\hat{\beta}) < \hat{\eta}(\hat{\beta}-\epsilon)$. Therefore, $\hat{\eta}(\hat{\beta})$ is a local minima.

Therefore, with $\beta^1 \in (0, \hat{\beta})$, Q_{lh} and Q_{mh} are merged at the threshold segregation level (before their disappearance), and the threshold level $\hat{\eta}(\beta^1)$ is a strictly decreasing function of β^1. With $\beta^1 \in (\hat{\beta}, 1)$, Q_{lh} and Q_{lm} are merged at the threshold segregation level, and the threshold level $\hat{\eta}(\beta^1)$ is a strictly increasing function of β^1. QED.

Reference

Abramovitz, M. (1993). "The Search for the Sources of Growth: Areas of Ignorance, Old and New," *Journal of Economic History*, 53: 217-243.

Adsera, A and D. Ray (1998). "History and Coordination Failure," *Journal of Economic Growth* 3: 267-276.

Anderson, Elijah (1990). *Streetwise: Race, Class, and Change in an Urban Community*, Chicago: University of Chicago Press.

Akerlof, George (1997). "Social Distance and Social Division," *Econometrica* 65: 1005-1027.

Arrow, K.J. (1973). "The Theory of Discrimination," In Ashenfelter and Rees, editions, *Discrimination in Labor Markets.*

Assaad R.(1997). "Kinship Ties, Social Networks and Segmented Labor Markets: Evidence from the Construction Sector in Egypt," Journal of Development Economics, 52: 1-30.

Banerjee, B. (1981). "Ruralurban Migration and Family Ties: an Analysis of Family Considerations in Migration Behavior in India," *Oxford Bulletin of Economics and Statistics*, 43: 321355.

Banerjee, B. (1983). "Social Networks in the Migration Process: Empirical Evidence on Chain Migration in India," *Journal of Developing Areas* 17: 185196.

Banerjee A. and A. Newman (1993). "Occupational Choice and the Economic Development," *Journal of Political Economy*, 101: 274-298.

Becker, G. S. (1957). *The Economics of Discrimination,* Chicago: University of Chicago Press.

Benabou, R. (1996). "Inequality and Growth," *NBER Macroeconomics Annual* 11: 11-74.

Bilimoria1, D., S. Perry, X. Liang, E. Stoller, P. Higgins, and C. Taylor (2006). "How Do Female and Male Faculty Members Construct Job Satisfaction? The Roles of Perceived Institutional Leadership and Mentoring and their Mediating Processes," *The Journal of Technology Transfer*, 31.

Blanchard, O. and C. Kahn (1980). "The Solution of Linear Difference Models under Rational Expectation Models," *Econometrica*, 48: 1305-1311.

Blau, D.M. and P.K. Robins (1990). "Job Search Outcomes for the Employed and Unemployed," *Journal of Political Economy* 98: 637655.

Bowles, S., G.C. Loury and R. Sethi (2007). "Group Inequality," Unpublished Manuscript, Barnard College, Columbia University.

Buiter, Willem (1982). "Predetermined and Non-Predetermined Variables in Rational Expectations Models," *Economics Letters*, 49-54.

Buiter, Willem (1984). “Saddlepoint Problems in Continuous Time Rational Expectations Models: A General Method and Some Macroeconomic Examples,” *Econometrica*, 52: 665-680.

Campbell, Karen E. and Peter V. Marsden (1990). “Recruitment and Selection Processes: The Organizational Side of Job Searches,” in *Social Mobility and Social Structure*, Ronald L. Breiger (ed.), New York: Cambridge University Press.

Cardoso, Rute and Rudolf Winter-Ebmer (2007). “Mentoring and Segregation: Female-Led Firms and Gender Wage Policies,” *IZA Discussion Papers*, 3210.

Casella, Alessandra and James Rauch (2001). *Networks and Markets*, Russell Sage Foundation Publications.

Castilla, Emilio (2005). “Social Networks and Employee Performance in a Call Center,” *American Journal of Sociology* 110: 1243-83.

Chaudhuri, S. and R. Sethi (2008). “Statistical Discrimination with Peer Effects: Can Integration Eliminate Negative Stereotypes?” *Review of Economic Studies* 78: 579 - 596.

Coate, S. and G.C. Loury (1993). “Will Affirmative-Action Policies Eliminate Negative Stereotype?” *American Economic Review* 83: 1220-1240.

Cutler, David and Edward Glaeser (1997). “Are Ghettos Good or Bad?” *Quarterly Journal of Economics* 112: 827-872.

Durlauf, Steven (1996). “A theory of persistent Income Inequality,” *Journal of Economic Growth* 1:75-94.

Durlauf, Steven and Marcel Fafchamps (2004). “Social Capital,” (unpublished ms.).

Fafchamps, M. and B. Minten (1999). “Relationship and Traders in Madagascar,” *Journal of Development Studies* 35: 1-35.

Fernandez, Roberto, Emilio Castilla and Paul Moore (2000). “Social Capital at Work: Networks and Employment at a Phone Center,” *American Journal of Sociology* 105: 1288-356.

Falk, Armin and Andrea Ichino (2005). “Clean Evidence of Peer Effects,” *Journal of Labor Economics* 25.

Fryer, Roland, Glenn C. Loury and Tolga Yuret (2006). “An Economic Analysis of Color-Blind Affirmative Action,” *Journal of Law, Economics and Organization*, forthcoming.

Fryer, Roland and Glenn C. Loury (2005). “Affirmative Action and Its Mythology,” *Journal of Economic Perspectives* 19.

Galor, O. and J. Zeira (1993). “Income Distribution and Macroeconomics,” *Review of Economic Studies,* 60: 35:52.

Galor, O. and D. Tsiddon (1997). “The Distribution of Human Capital and Economic Growth,” *Journal of Economic Growth*, 2: 93-124.

Galor, O. and O. Moav (2002). "From Physical to Human Capital Accumulation: Inequality and the Process of Development," Unpublished Manuscript.

Galor, O. and O. Moav (2006). "Das Human-Kapital: A Theory of the Demise of the Class Structure," *Review of Economic Studies*, 73: 85-117.

Gibbons, Robert (2004). "What is Economic Sociology and Should Any Economists Care?" *Journal of Economic Perspectives* 18.

Granovetter, Mark. S. (1973). "The Strength of Weak Ties," *American Journal of Sociology* 78: 1360-1380.

Granovetter, Mark S. (1975). *Getting A Job: A Study of Contracts and Careers*, Chicago: University of Chicago Press; 2nd Edition 1995.

Granovetter, Mark. S. (1985). "Economic Action and Social Structure: A Theory of Embeddedness," *American Journal of Sociology* 91: 481-510.

Gregg, P. and J. Wadsworth (1996). "How Effective Are State Employment Agencies? Jobcentre Use and Job Matching in Britain," *Oxford Bulletin of Economics and Statistics* 58: 4367.

Hanushek, Eric, John Kain, and Steve Rivkin (2002). "New Evidence about Brown vs Board of Education: The Complex Effects of School Racial Composition on Achievement," Working Paper 8741, National Bureau of Economic Research.

Ha, Yong Chool (2007). "Late Industrialization, the State, and Social Changes: The Emergence of Neofamilism in South Korea," *Comparative Political Studies* 40: 363-382.

Holzer, Harry J. (1987). "Hiring Procedures in the Firm: Their Economic Determinants and Outcomes," NBER Working Papers No. 2185.

Holzer, Harry J. (1988). "Search Method Use by Unemployed Youth," *Journal of Labor Economics* 1-20.

Hoxby, Caroline (2000). "Peer Effects in the Classroom: Learning from Gender and Race Variation," Natioinal Bureau of Economic Research.

Kain, John F. (1968) "Housing Segregation, Negro Employment and Metropolitan Decentralization," *Quarterly Journal of Economics*, 82: 175-197.

Khwaja, Asim, Atif Mian and Abid Qamar (2008). "The Value of Business Networks," Unpublished Manuscript.

Khwaja, Asim and Atif Mian (2005). "Do Lenders Favor Politically Connected Firms? Rent Provision in an Emerging Financial Market," *Quarterly Journal of Economics*, 120.

Kim, Man-Hum (1991). "Sixth Korean Republic and the Deepening of Regional Conflict," in Kim Hak-Min et al. eds. *Study on Regional Conflict in South Korea*, Seoul: Hakmin. (in Korean)

Kim, Yong-Hak (2002). "Social Networks Within Korea: With a Special Reference to Regional Ties," mimeo.

Kim, Young Chul and Glenn Loury (2008). "Group Reputation and Dynamics of Statistical Discrimination," Unpublished Manuscript, Brown University.

Kremer, Michael and Dan Levy (2003). "Peer Effects and Alcohol Use Among College Students," National Bureau of Economic Research.

Krugman, Paul (1991). "History Versus Expectations," *Quarterly Journal of Economics* 106: 651-667.

Laband, David and Bernard Lentz (1995). "Workplace Mentoring in the Legal Profession," *Southern Economic Journal* 61:783-802.

Laband, David and Bernard Lentz (1999). "The Impact of Having a Mentor on Earnings and Promotion: Evidence from a Panel Study of Lawyers," *Applied Economics Letters* 6: 785-787.

Lee, Kun-bok and Ki-young Lee (2007). "Effects of Job Obtaining Method on the Survival Rate," mimeo (in Korean).

Livingston Gretchen (2006). "Gender, Job Searching, and Employment Outcomes among Mexican Immigrants," *Population Research and Policy Review*, 25: 43-66.

Loury, Glenn C. (1977). "A Dynamic Theory of Racial Income Differences," in *Women, Minorities and Employment Discrimination*, Phyllis Wallace and Annette LaMond (Eds).

Loury, Glenn C. (1995). "Economic Discrimination: Getting to the Core of the Problem," in *One By One From the Inside Out: Essays and Reviews on Race and Responsibility in America.* New York: The Free Press.

Loury, Glenn C. (2000). "Social Exclusion and Ethnic Groups: The Challenge to Economics," The Annual World Bank Conference on Development Economics 1999, Boris Pleskovic and Joseph E. Stiglitz (eds.), The World Bank.

Loury, Glenn C. (2002). *The Anatomy of Racial Inequality*, Harvard U Loury, Glenn C. (2002). *The Anatomy of Racial Inequality.* Harvard University Press

Lucas, Robert (1988). "On the Mechanics of Economic Development," Journal of Monetary Economics, 22: 3-42.

Lundberg, Shelly and Richard Startz (1998). "On the Persistence of Racial Inequality," *Journal of Labor Economics* 16: 292-322.

Mahuteau, Stephane and Promod Junankar (2008). "Do Migrants Get Good Jobs in Australia? The Role of Ethnic Networks in Job Search," IZA Discussion Papers, 3489.

Montgomery, James (1991), "Social Networks and Labor Market Outcomes: Toward an Economic Analysis," *American Economic Review* 81(5):1408-18.

Mookherjee, Dilip and Debraj Ray (2006), "Occupational Diversity and Endogenous Inequality," Unpublished Manuscript.

Munshi, K. (2003). "Networks in the Modern Economy: Mexican Migrants in the US Labor Market," *Quarterly Journal of Economics* 118: 549599.

Myers, Charles A. and George P. Shultz (1951). *The Dynamics of a Labor Market,* New York: Prentice-Hall.

Myrdal, Gunnar. (1962), *An American Dilemma: The Negro Problem and Modern Democracy.* New York: Pantheon Books.

Eren Ozgena, Robert A. Baron (2007). "Social sources of information in opportunity recognition: Effects of mentors, industry networks, and professional forums," *Journal of Business Venturing* 22: 174-192.

Phelps, E (1972). "The Statistical Theory of Racism and Sexism," *American Economic Review* 62: 659-661.

Raphael, Steven (1998). "The Spatial Mismatch Hypothesis and Black Youth Joblessness: Evidence from the San Francisco Bay Area," *Journal of Urban Economics*, 43: 79-111.

Rees, Albert (1966). "Information Networks in Labor Markets," *American Economic Review (Papers and Proceedings)* 56: 559-66.

Rees, Albert and George P. Shultz (1970). *Workers in an Urban Labor Market,* Chicago: University of Chicago Press.

Richardson, J. David (1995). "Income Inequality and Trade: How to Think and What to Conclude," *Journal of Economic Perspectives* 9: 33-55.

Rockoff, Jonah E. (2008). "Does Mentoring Reduce Turnover and Improve Skills of New Employees? Evidence from Teachers in New York City." NBER Working Papers No. 13868.

Ross, Stephen L. (1998). "Racial Differences in Residential and Job Mobility: Evidence Concerning the Spatial Mismatch Hypothesis," *Journal of Urban Economics*, 43: 112-135.

Sethi, Rajiv and Rohini Somanathan (2004). "Inequality and Segregation," *Journal of Political Economy* 112.

Wahba, Jackline and Yves Zenou (2005). "Density, social networks and job search methods: Theory and application to Egypt," *Journal of Development Economics* 78: 443-473.

Waldinger, Roger (1996). *Still the Promised City?: New Immigrants and African-Americans in Post-Industrial New York,* Cambridge, MA: Harvard University.

Weinberg, Bruce, Patricia Reagan and Jeffrey Yankow (2004). "Do Neighborhoods Affect Hours Worked?" Journal of Labor Economics, 22: 891-921.

Wood, Adrian (1994). *North-South Trade, Employment and Inequality*, Oxford: Clarendon Press.

Yu, Eui-Young (1990). "Regionalism in the South Korean Job Market: An Analysis of Regional-Origin Inequality among Migrants in Seoul," *Public Affairs* 63: 24-39.

Yu, Eui-Young (2003). "Regionalism in South Korea: Regional Distribution of Power Elites," in Sung-Gook Kim et al. eds. *What the Social Ties Mean to US*, Seoul: Tradition and Modernity (in Korean).

Zimmerman, David and Gordon Williams (2003). "Peer Effects in Higher Education," National Bureau of Economic Research.

Figure 1. Steady States in the Homogeneous Economy

Panel A Finding the $\dot{s}_t=0$ Locus

Panel B Steady States with the $\dot{s}_t=0$ and $\dot{\pi}_t=0$ loci

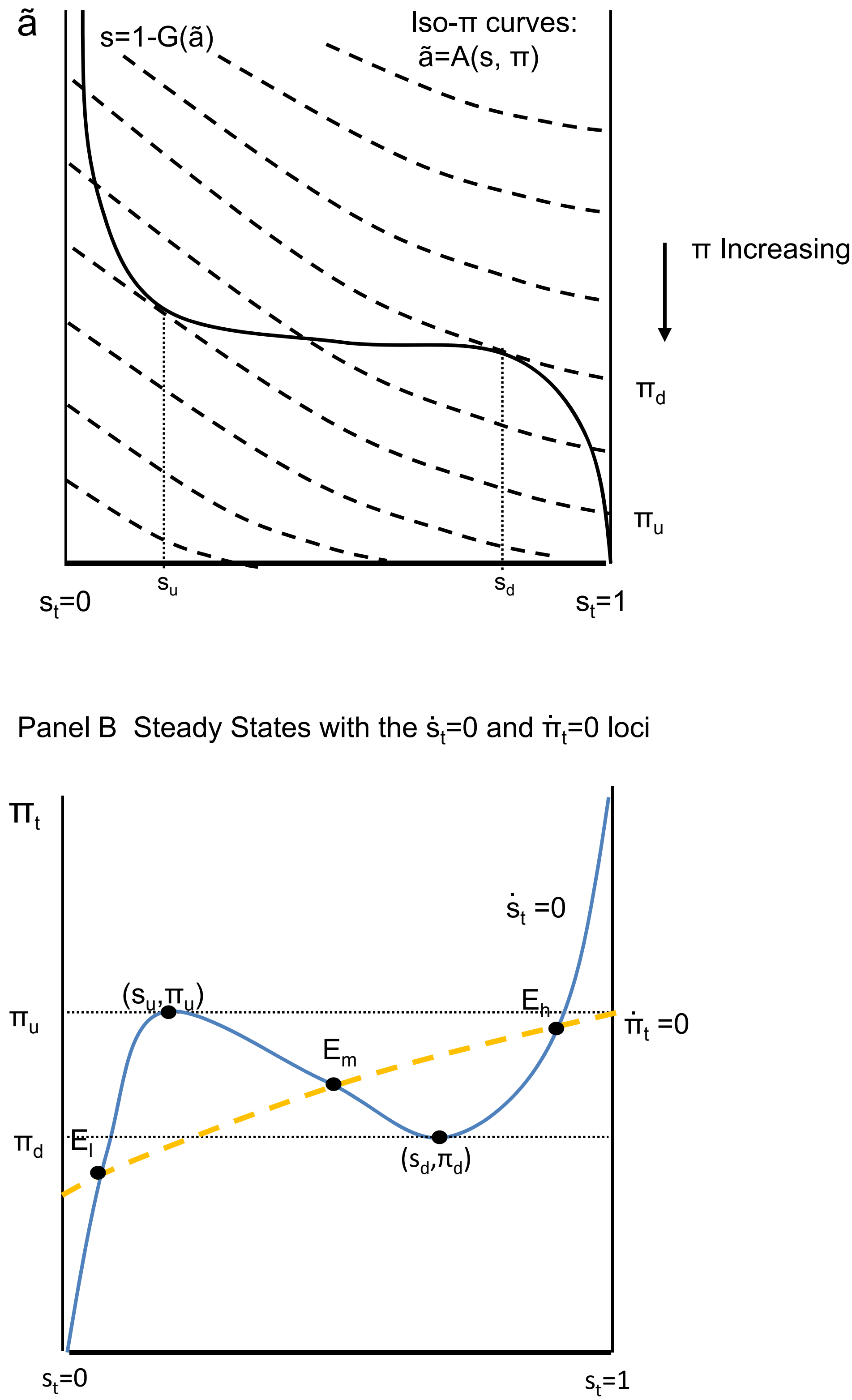


Figure 2. Equilibrium Paths in the Homogenous Economy

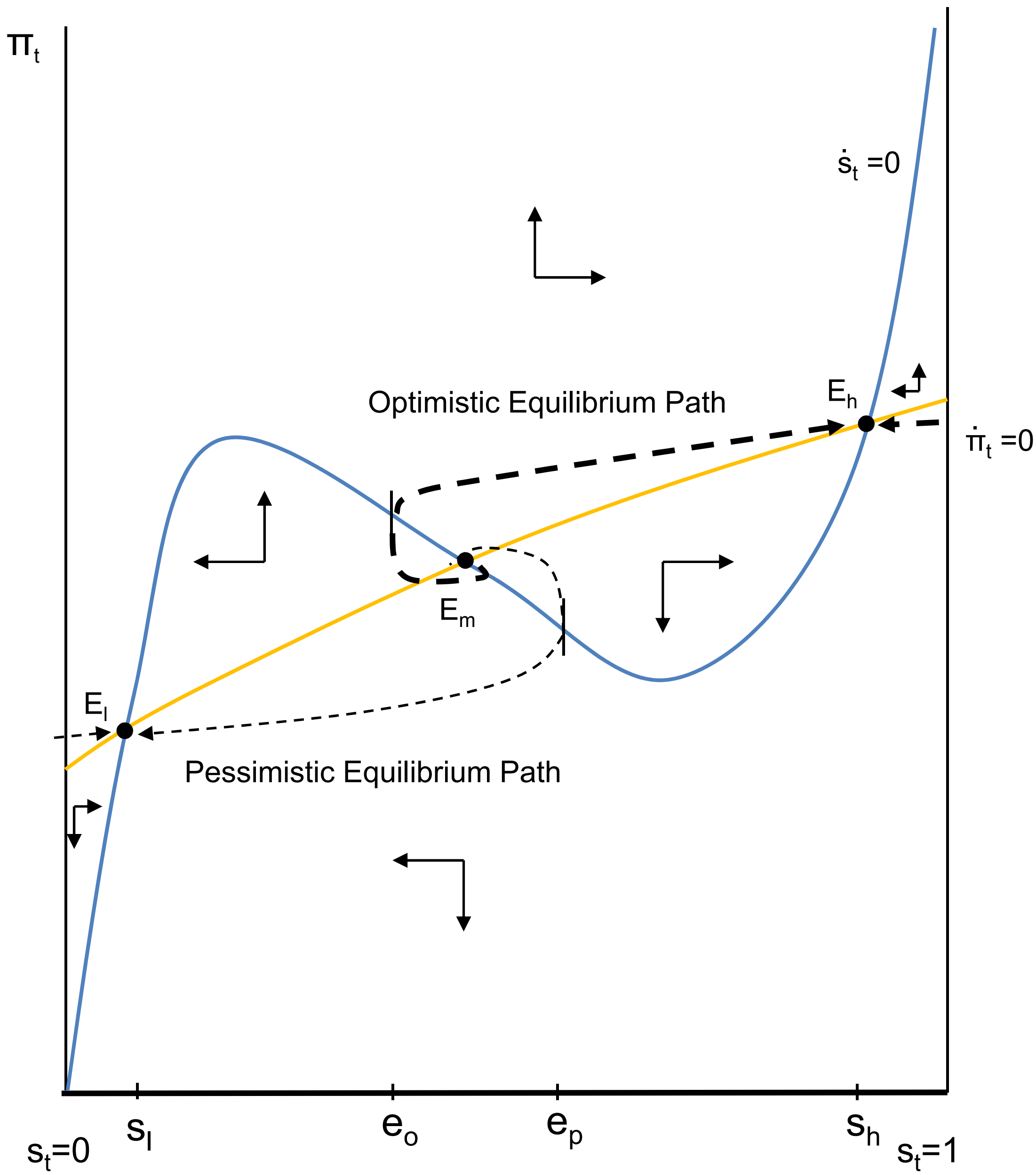


Figure 3. Comparison of Three Distinct Cases

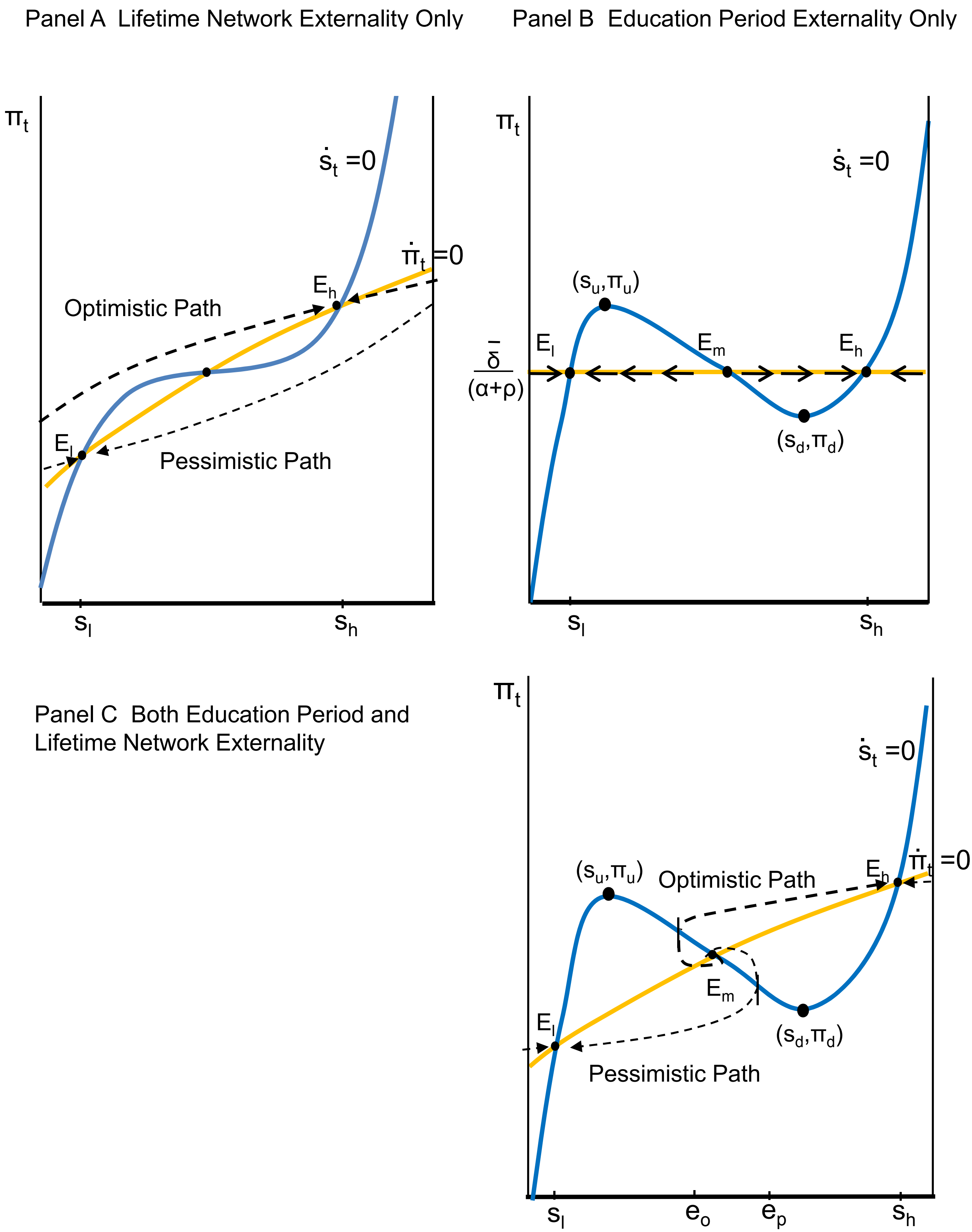


Figure 4. Economically Stable States with Total Segregation

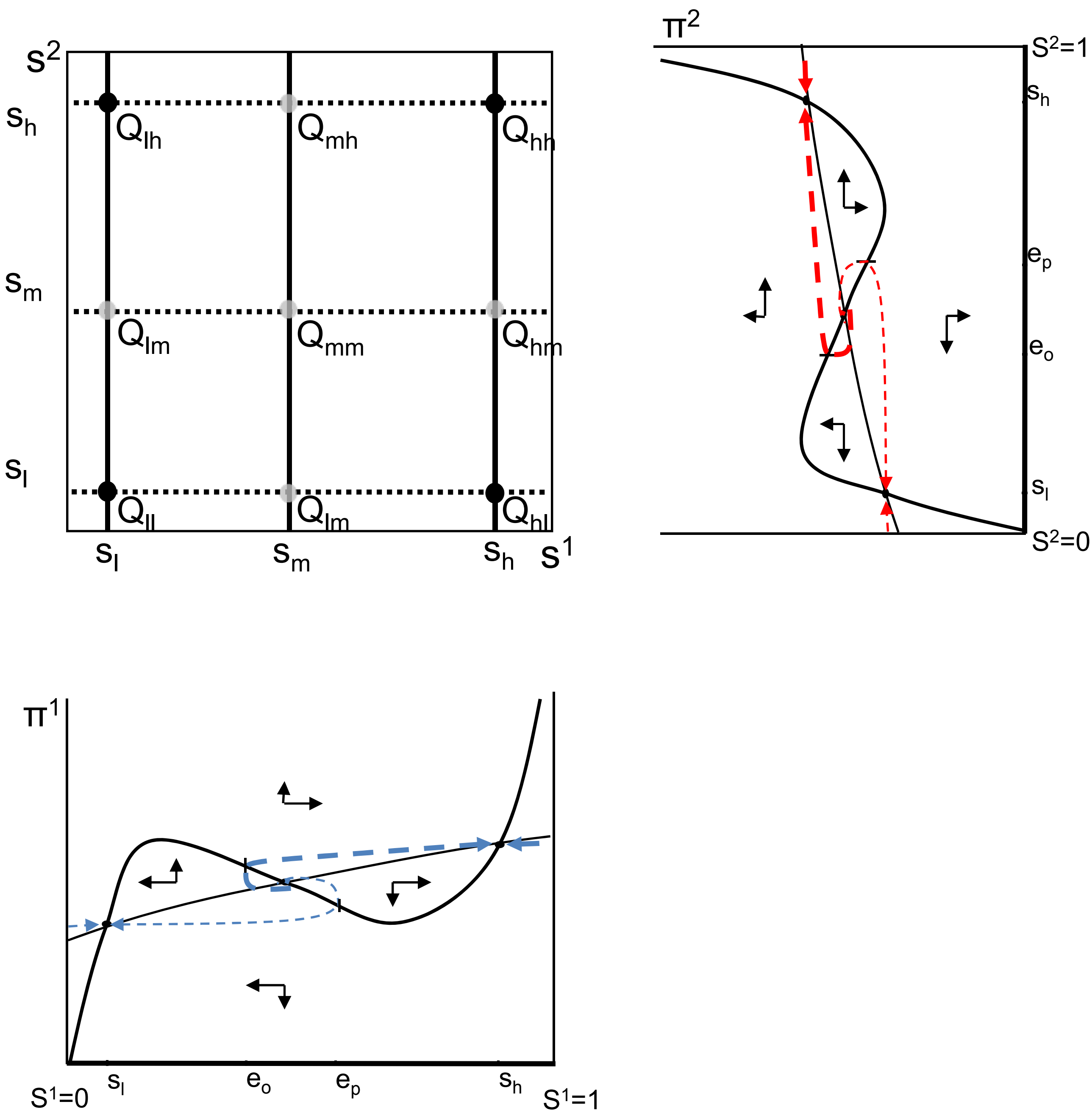


Figure 5. Manifold Ranges and Overlaps with Total Segregation

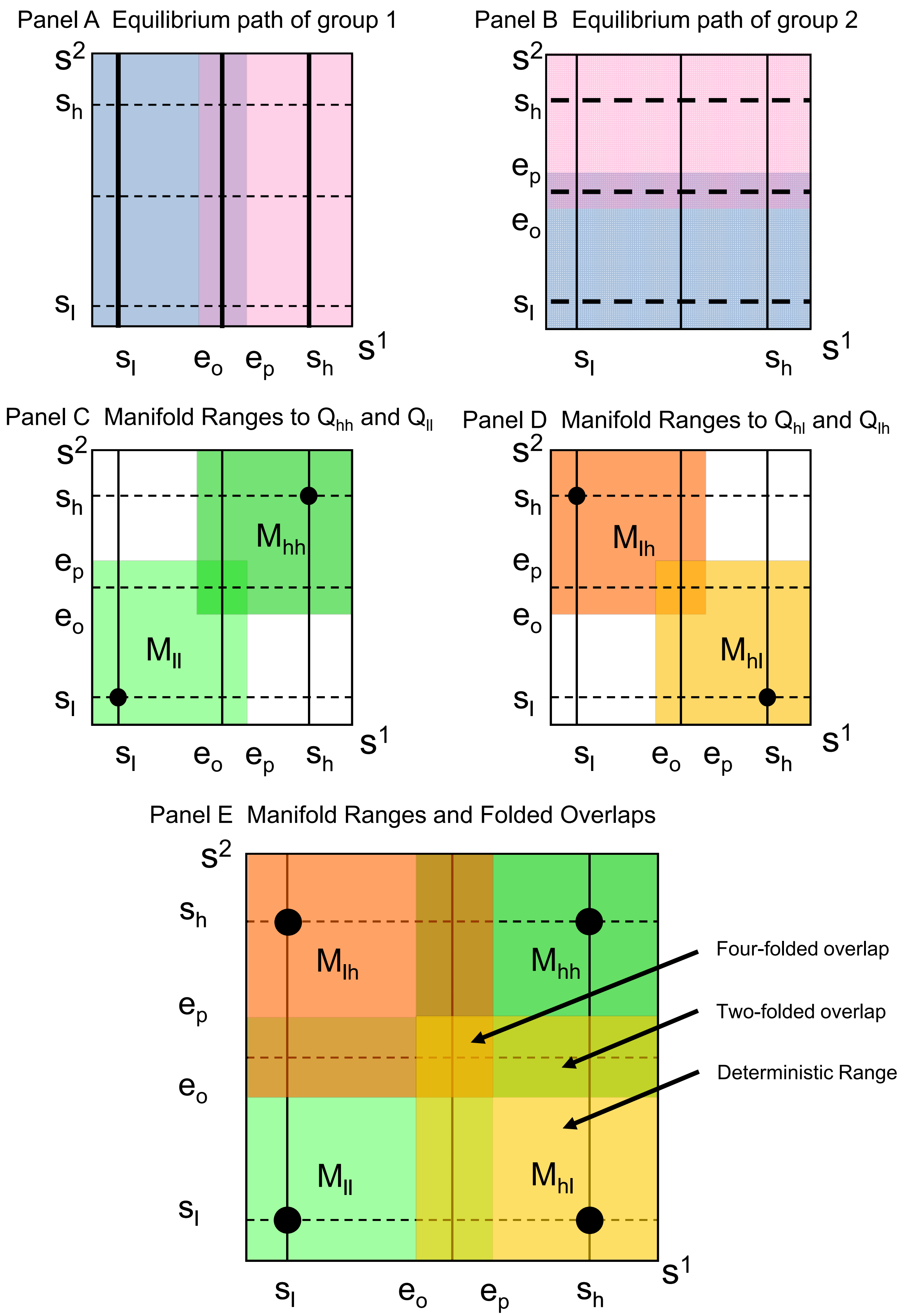


Figure 6. Partial Steady States (s^{i*},σ^{i*}) Given s^j

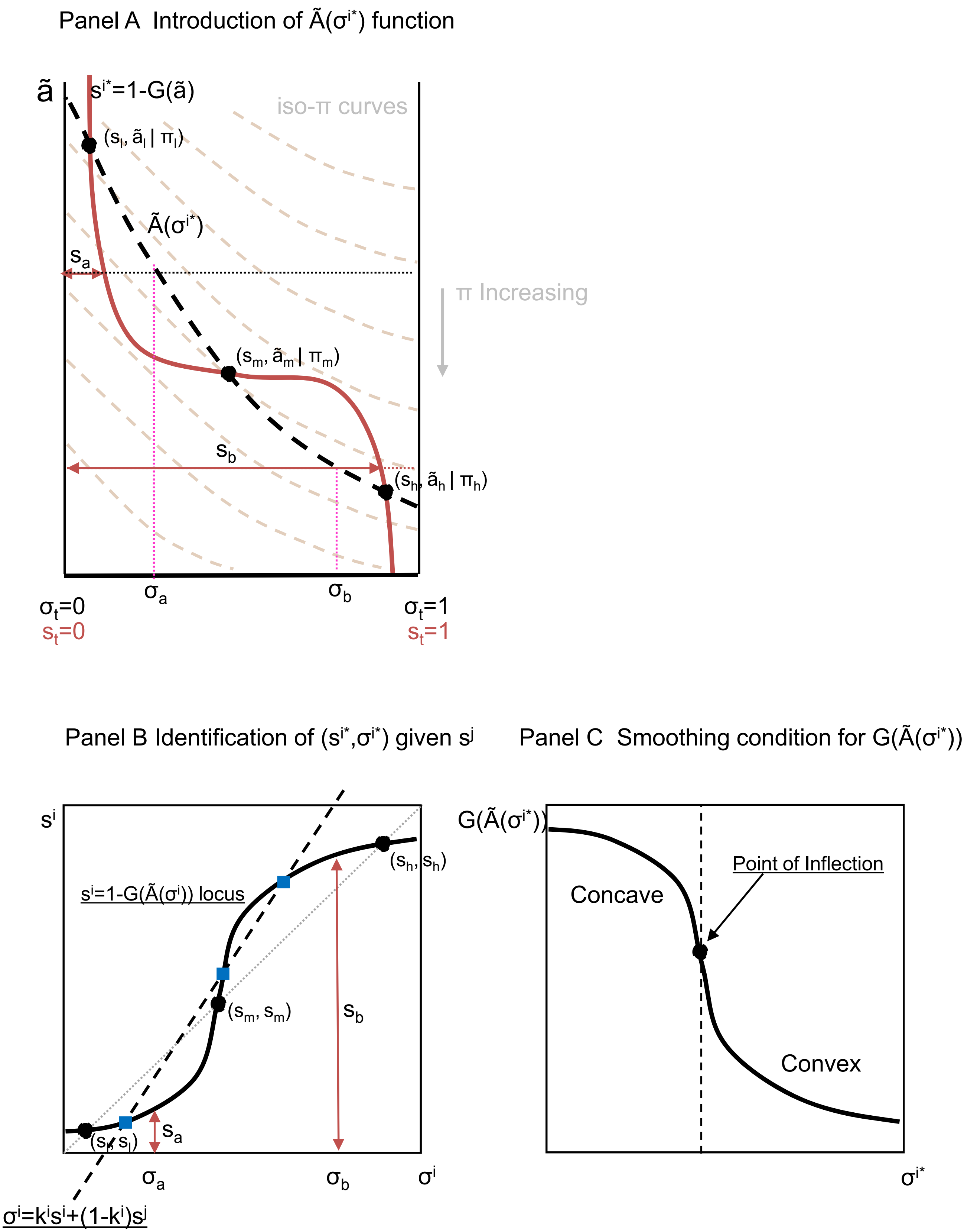


Figure 7. Global Steady States with η and β Given

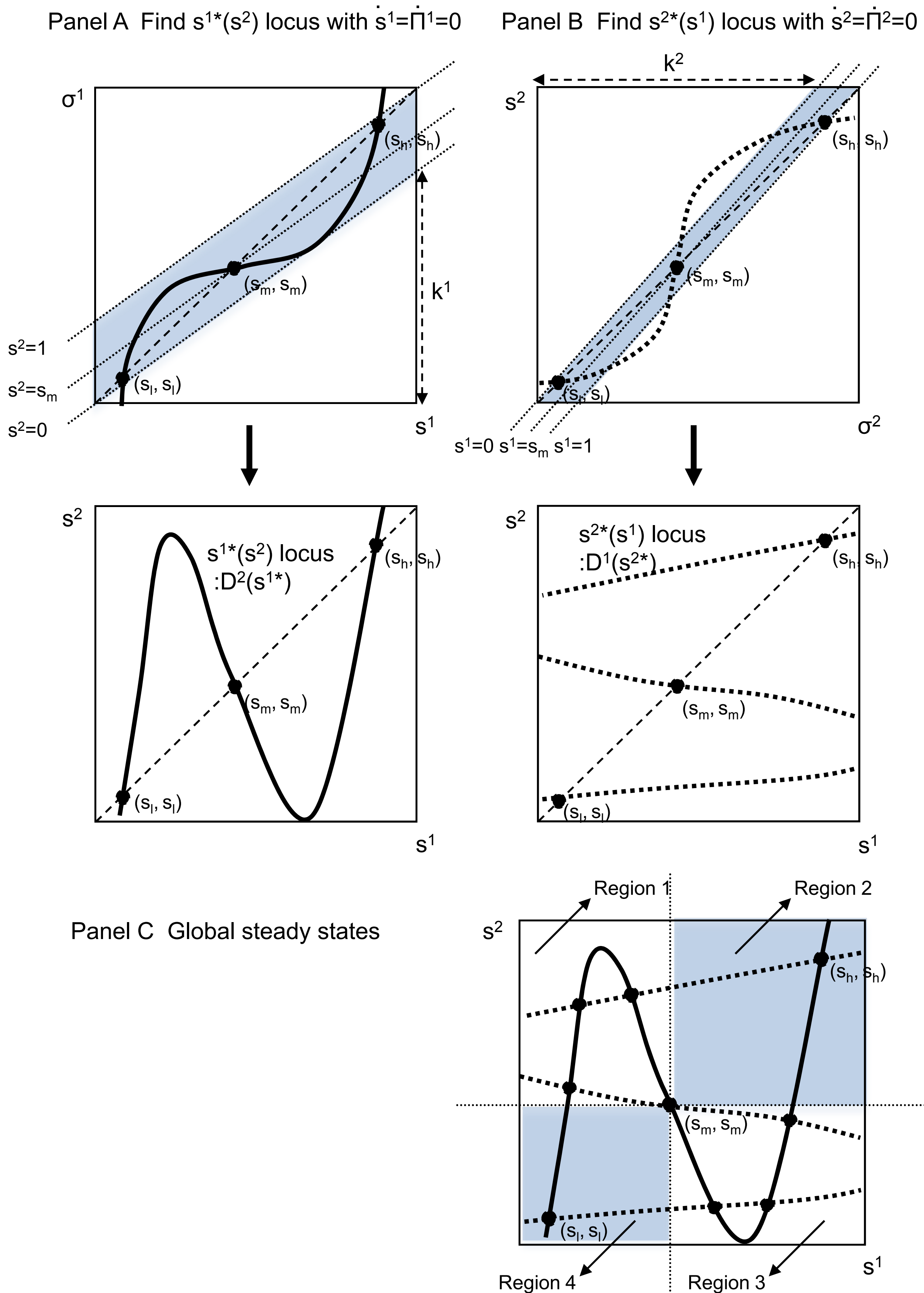


Figure 8. Steady States for Each Level of η (Given Small β^1)

- Segregation level η declines in Panel A, B, C, D, E, F order, with η=1 in Panel A and η=0 in Panel F.

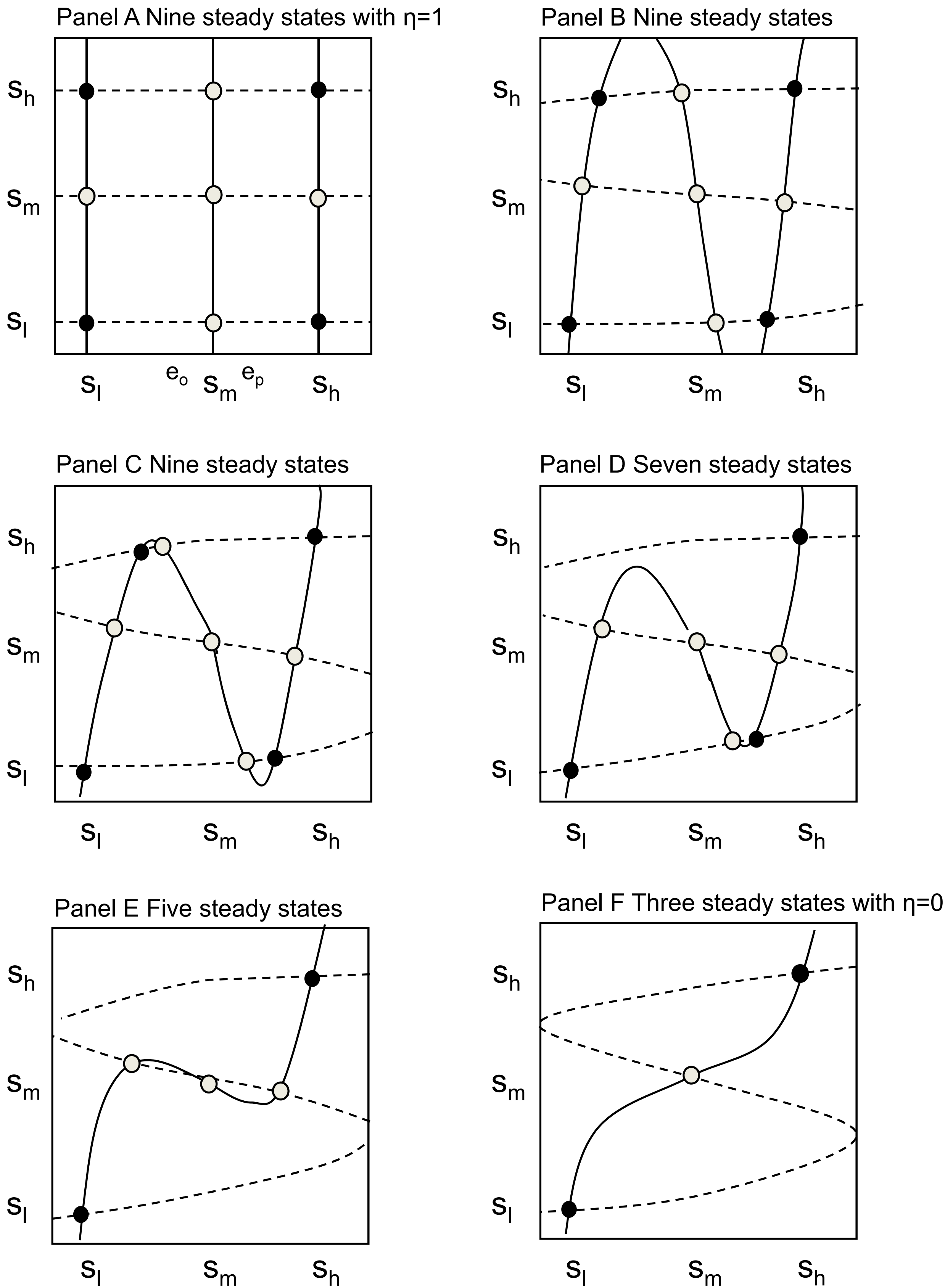


Figure 9. $\dot{s}^1_t$ and $\dot{\Pi}^1_t$ Demarcation Surfaces

-The sliced segments of the surfaces for each level of s^2 are depicted in the second picture of each panel.

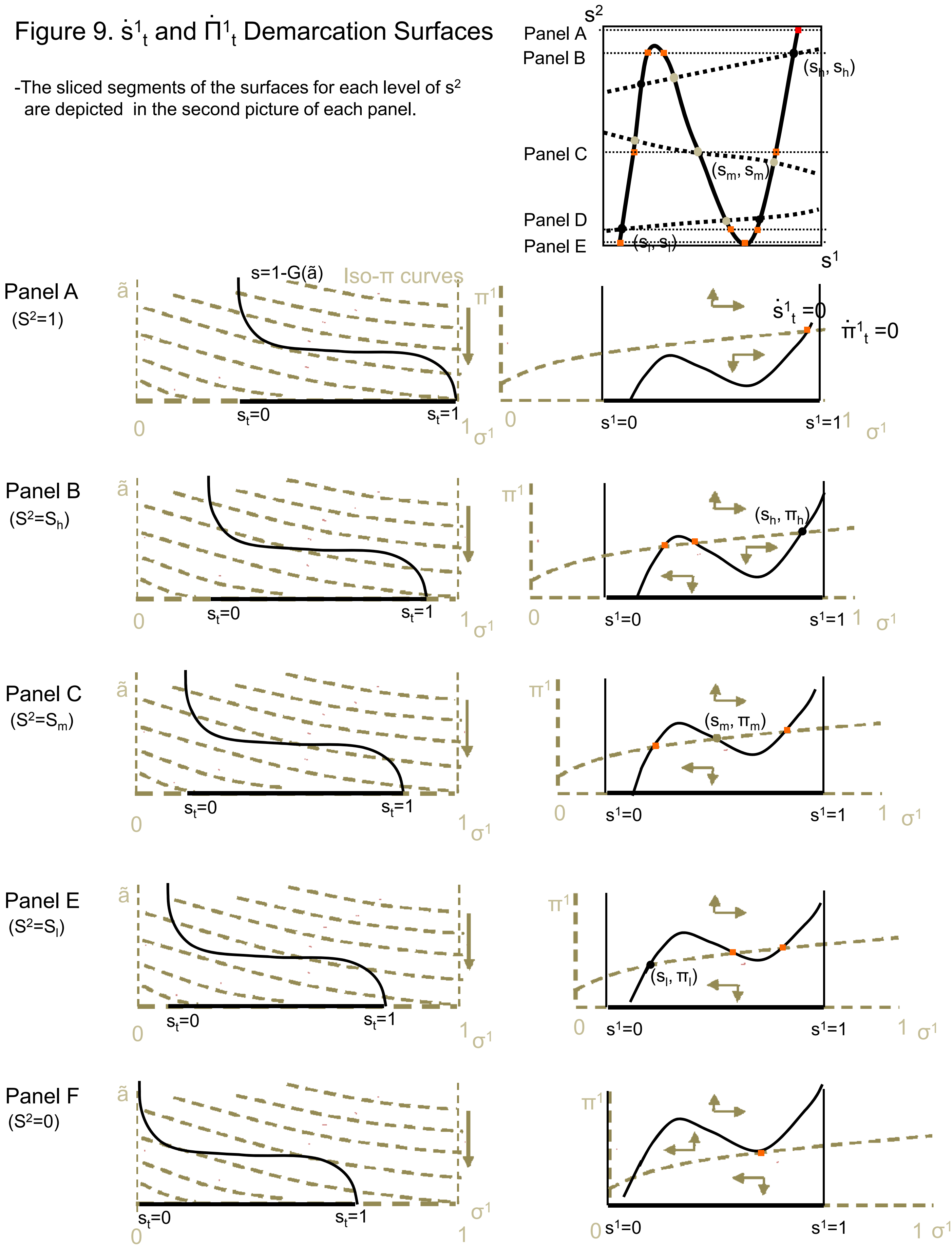


Figure 10. Economically Stable States and Location of Stable Manifold

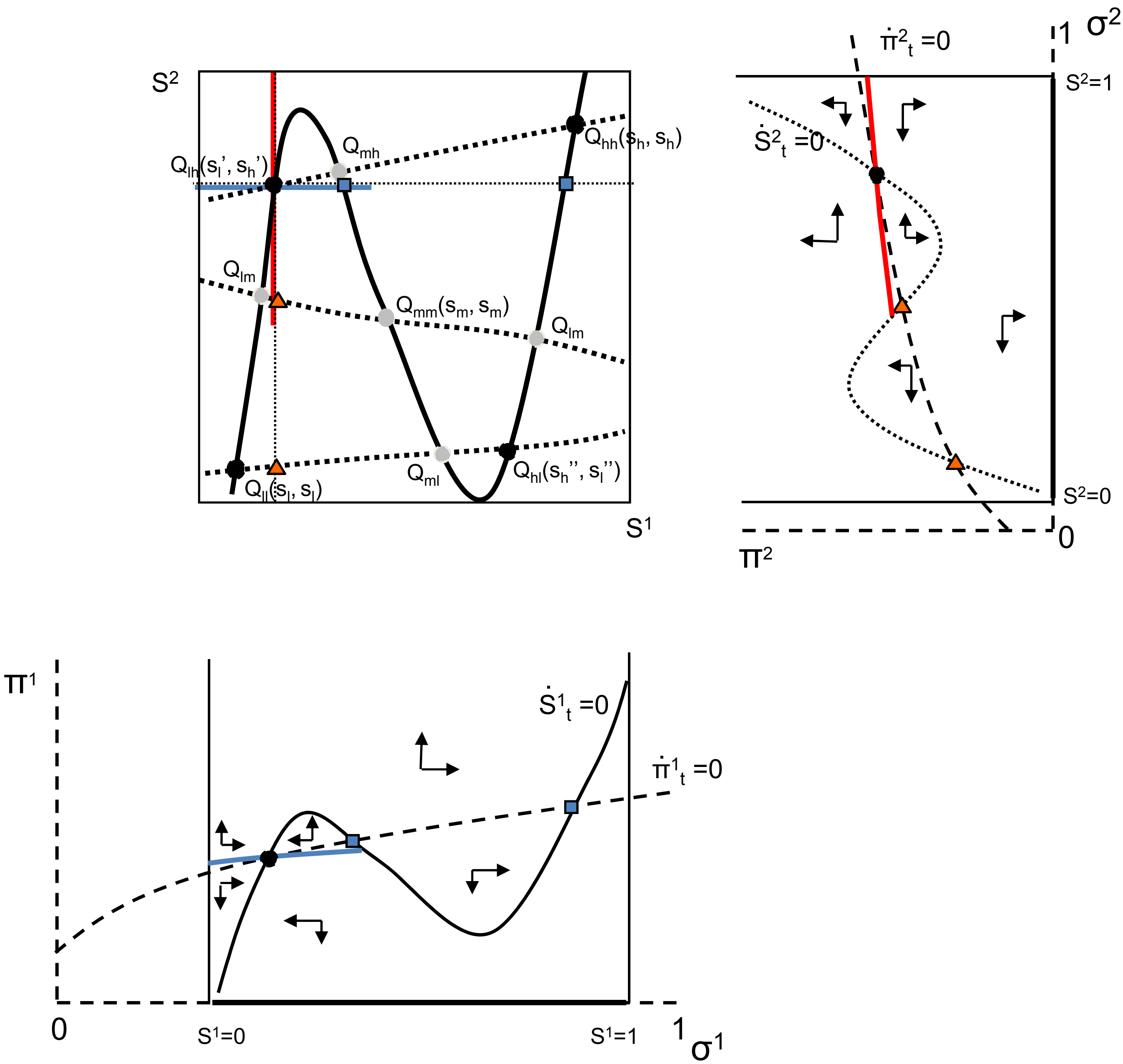


Figure 11. Stable Manifolds and Folded Overlaps

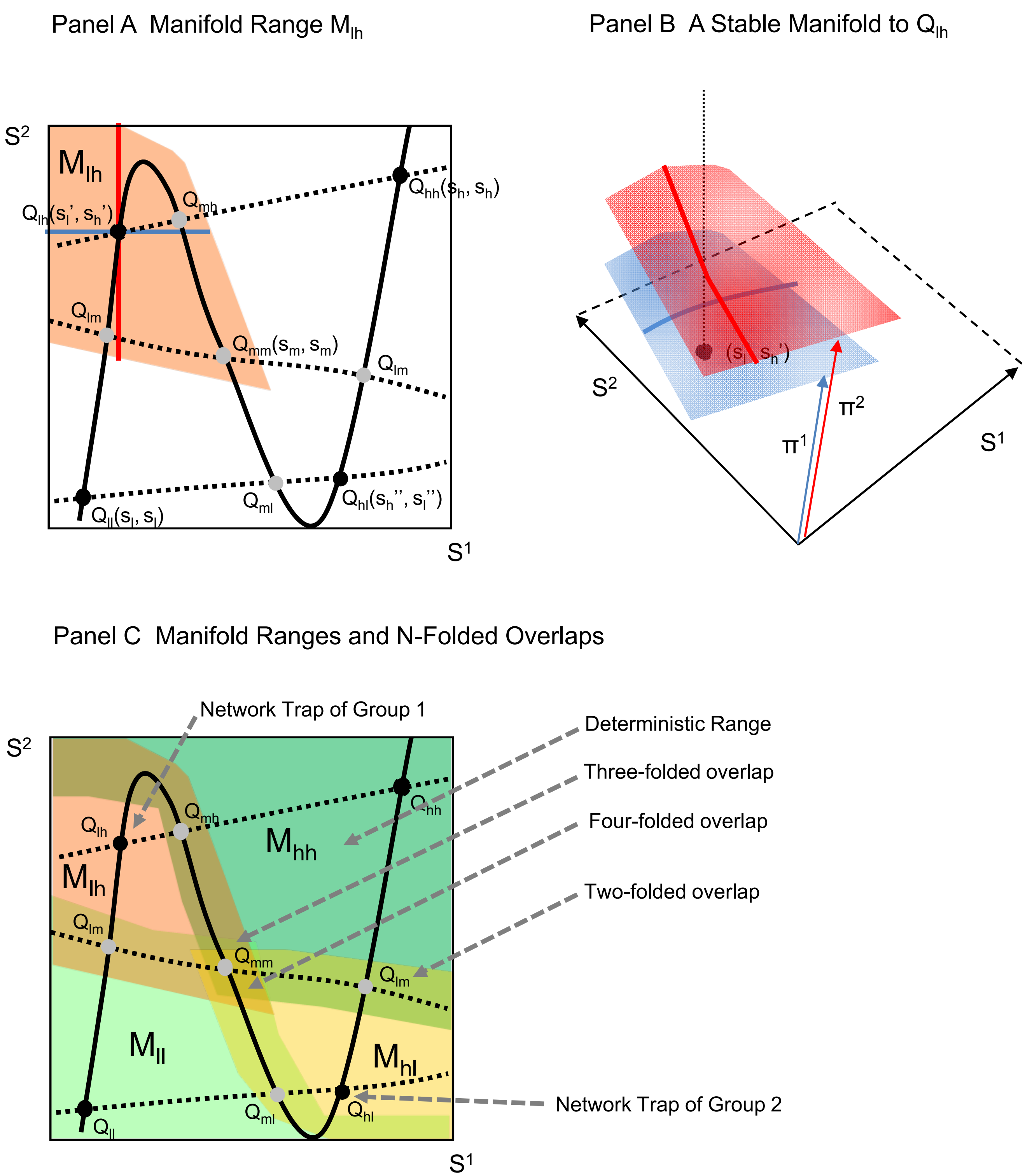


Figure 12. Sable States and Manifold Ranges for Each Level of η (Given Small β[1])

- Segregation level η declines in Panel A, B, C, D, E, F order, with η=1 in Panel A and η=0 in Panel F.

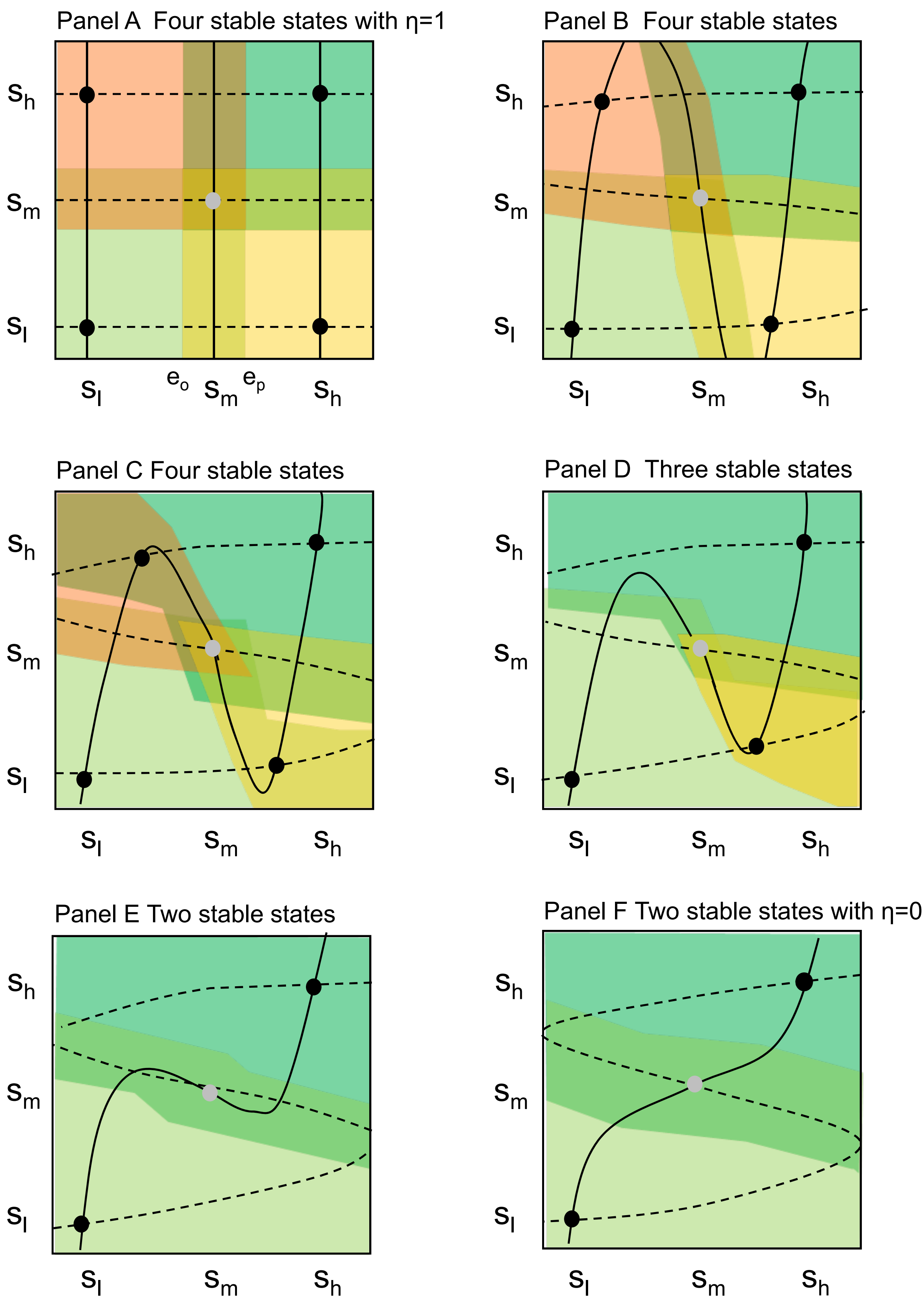


Figure 13. Integration Effect: Economic State Move as η Declines

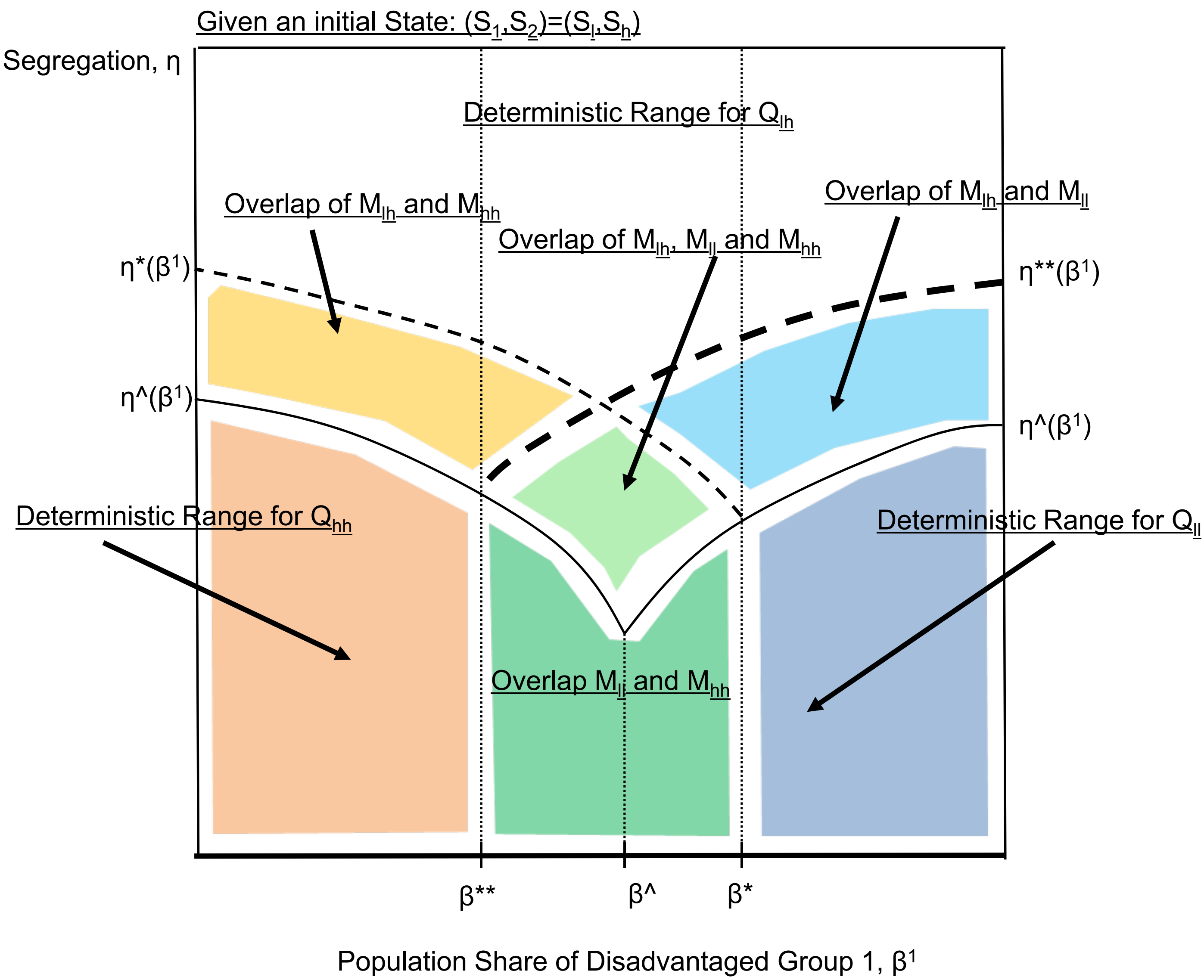
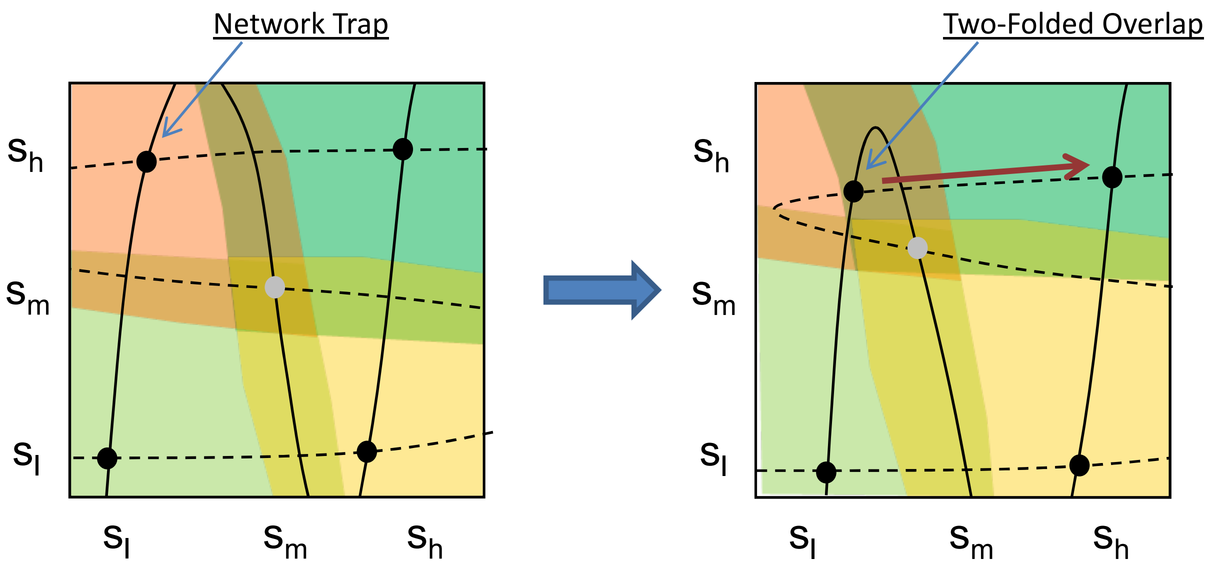


Figure 14. Affirmative Action Policies

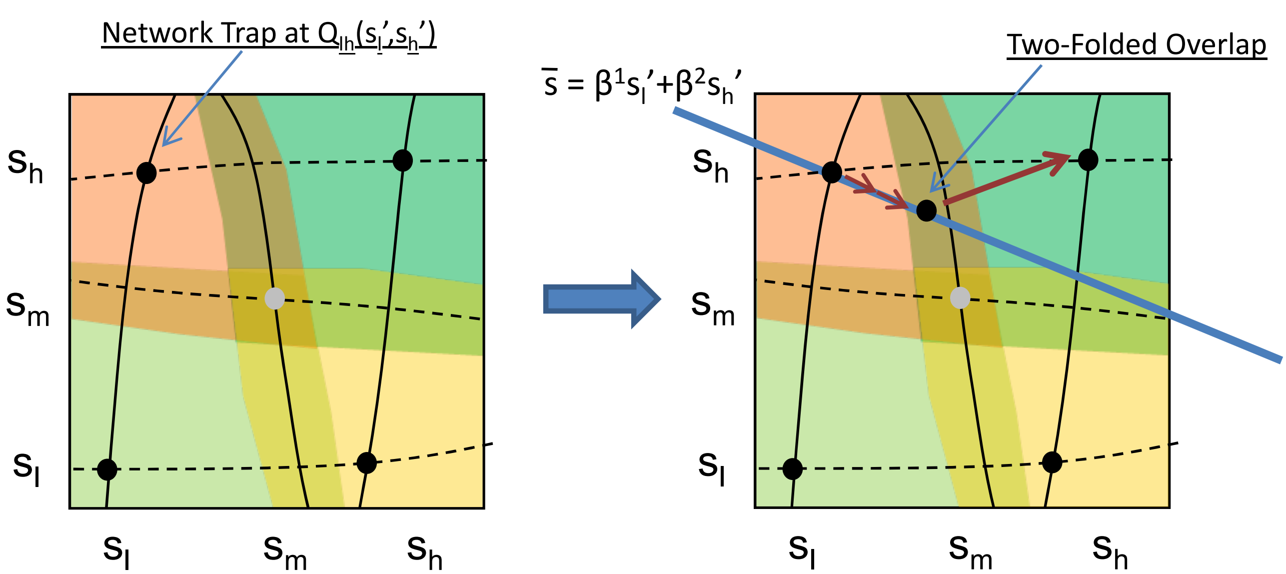
Panel A Asymmetric Training Subsidy



(Group 1 in Network Trap)

(Q_{lh} Moves into Overlap)

Panel B Quota System



(Group 1 in Network Trap)

(Economic State Placed in Overlap)

Figure 15. Equalization Policy Implementation

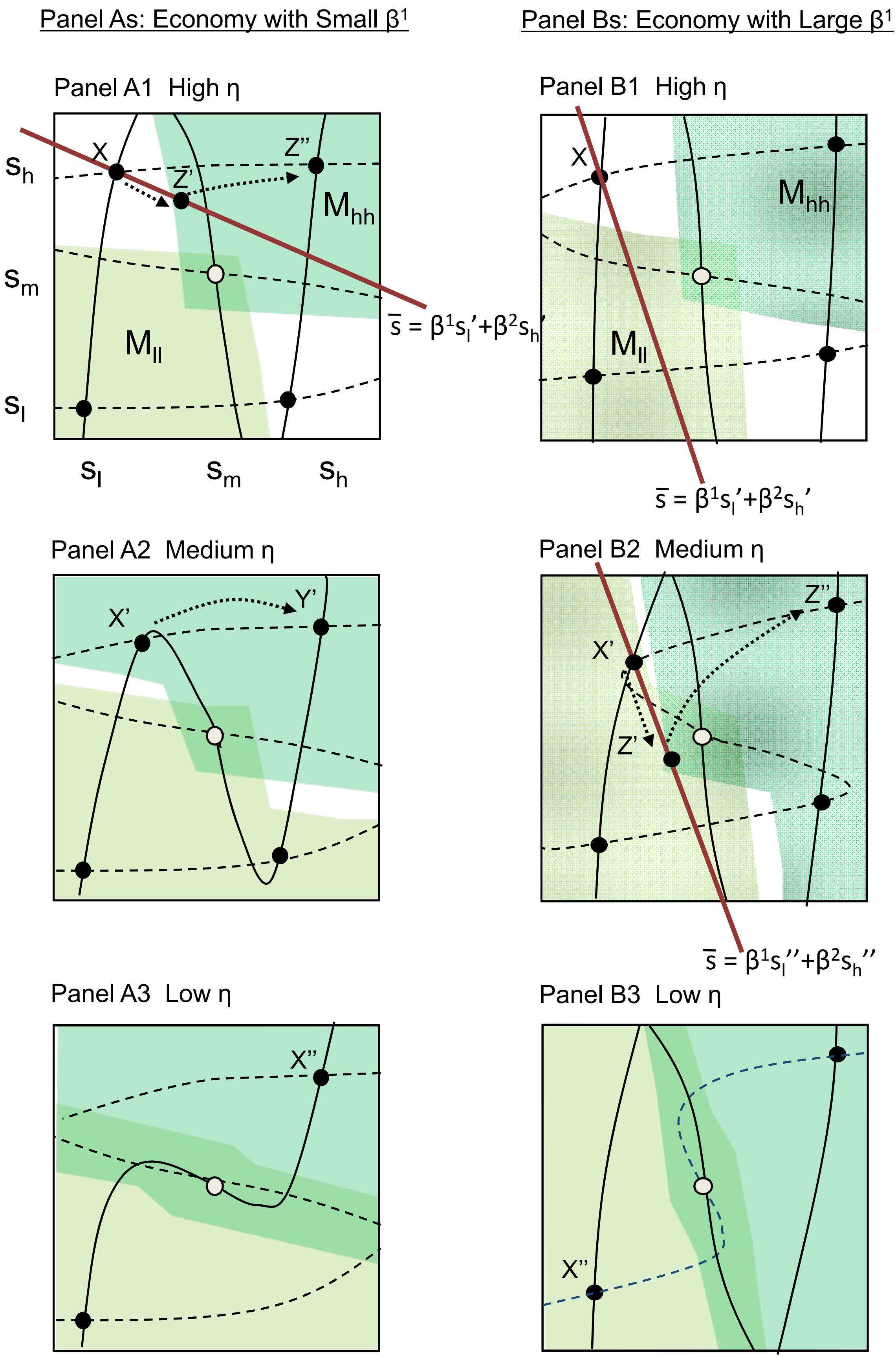
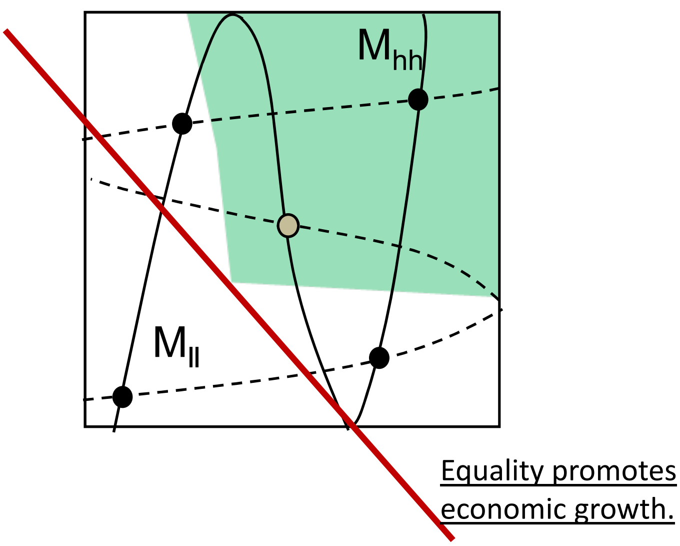
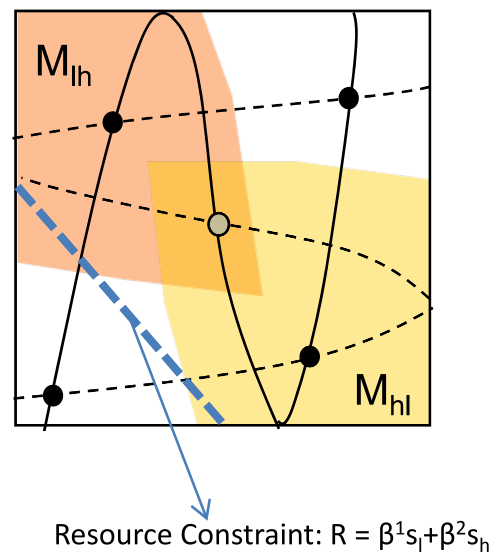


Figure 16. Macroeconomic Effects of Inequality

Panel A Positive Effect of Equality



Panel B Positive Effect of Inequality



Panel C Multiple Stable States in Three Group Economy

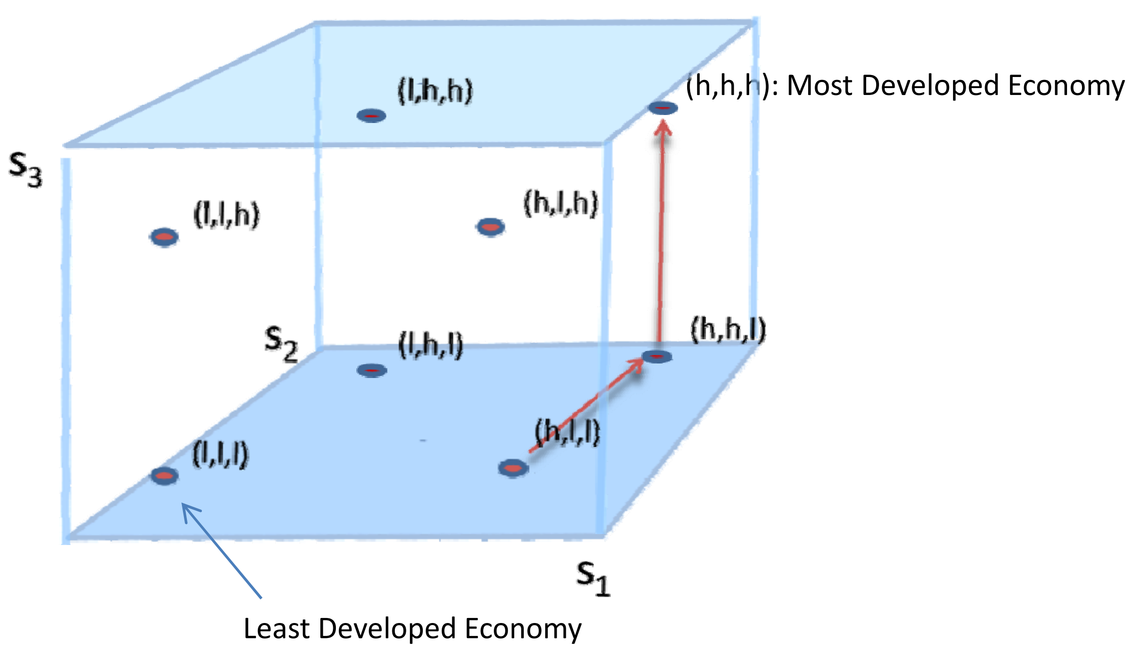


Figure 17. [Application] Urbanization and Regional Groups in S. Korea

Metropolitan Population
- 20.1% (5.0 mil, 1960)
- 35.5% (13.3 mil, 1980)
- 48.3% (23.8 mil, 2007)

Figure 18. [Application] Evolution of Regional Group Disparity in S. Korea

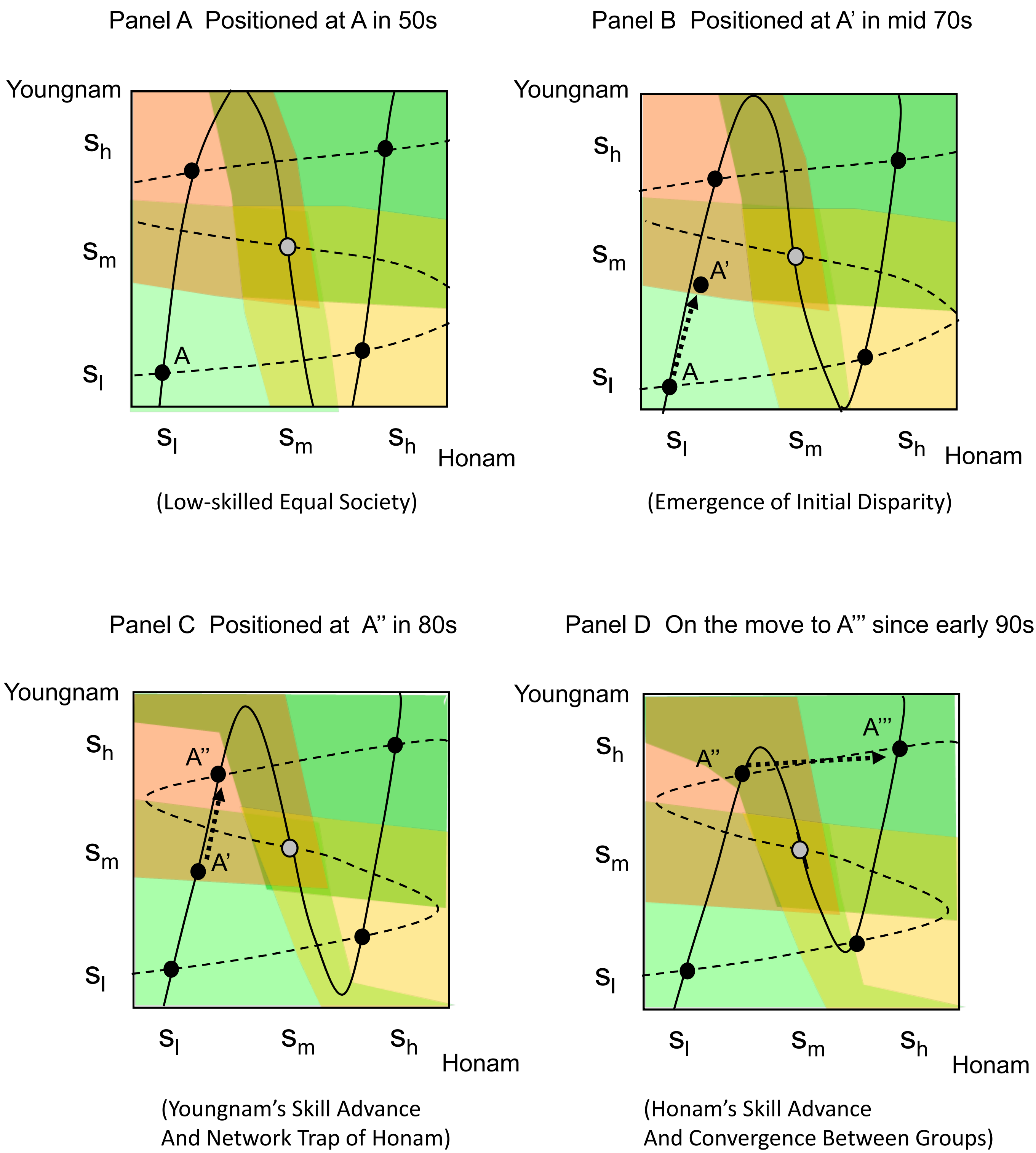
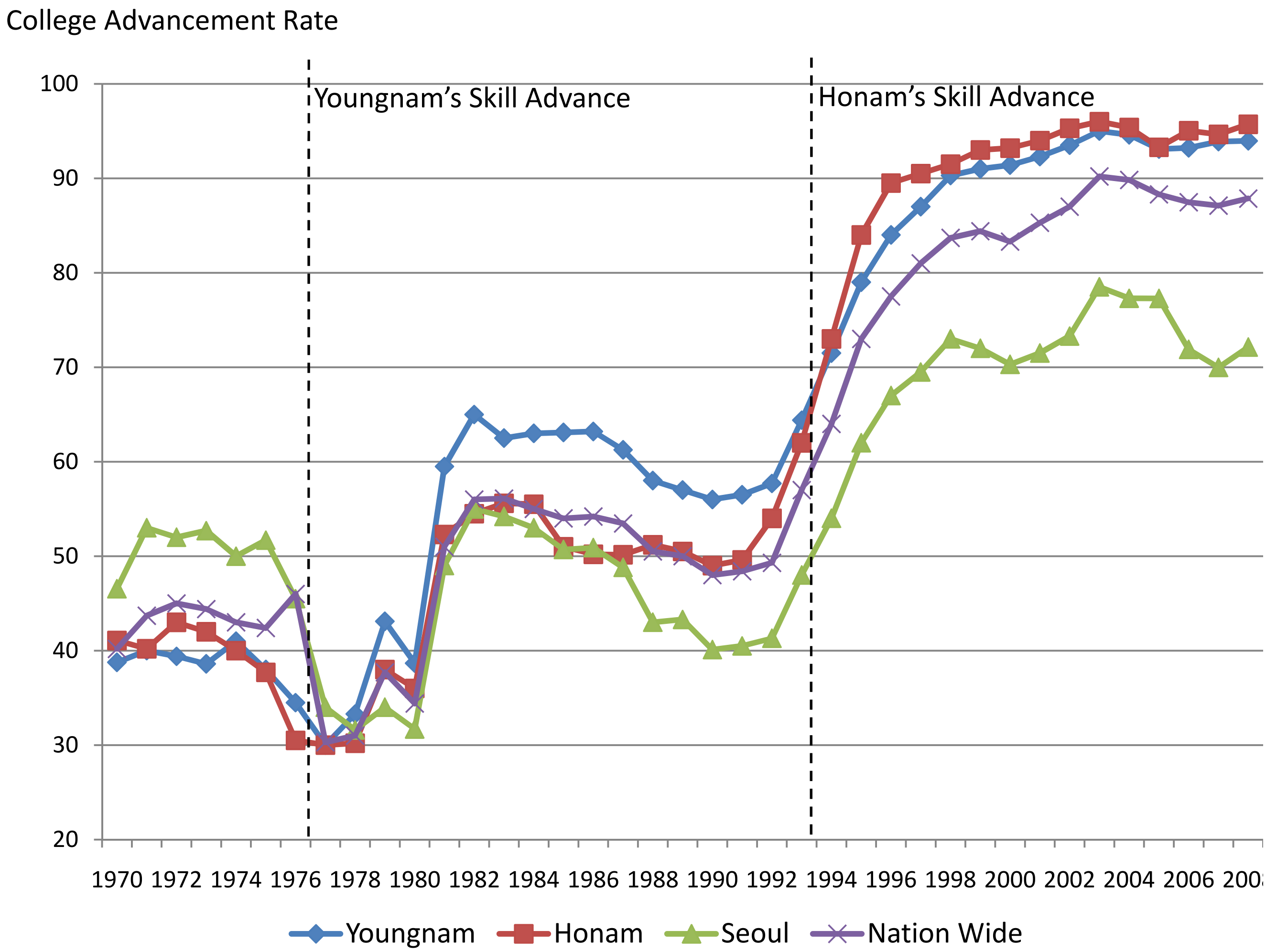


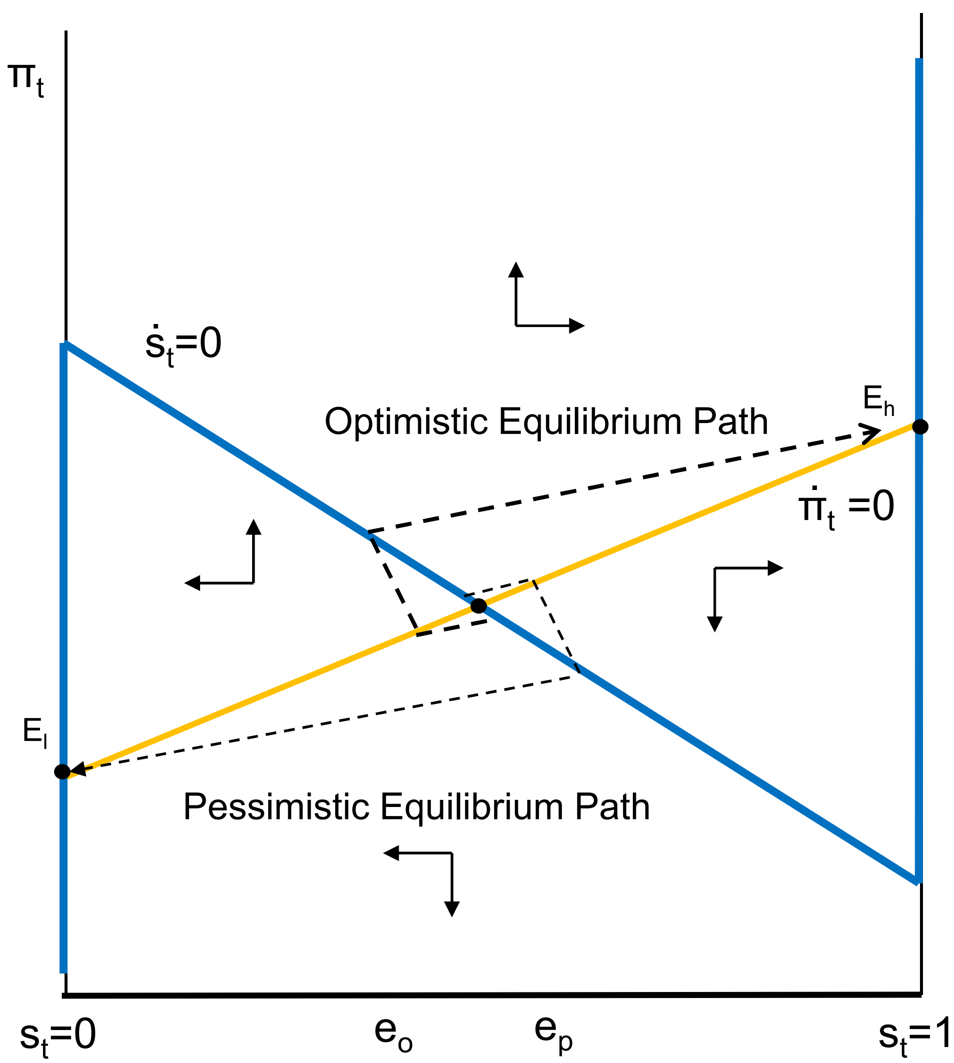
Figure 19. [Application] College Advancement Rate in Each Region



• Source: Statistical Yearbook of Education (South Korea)

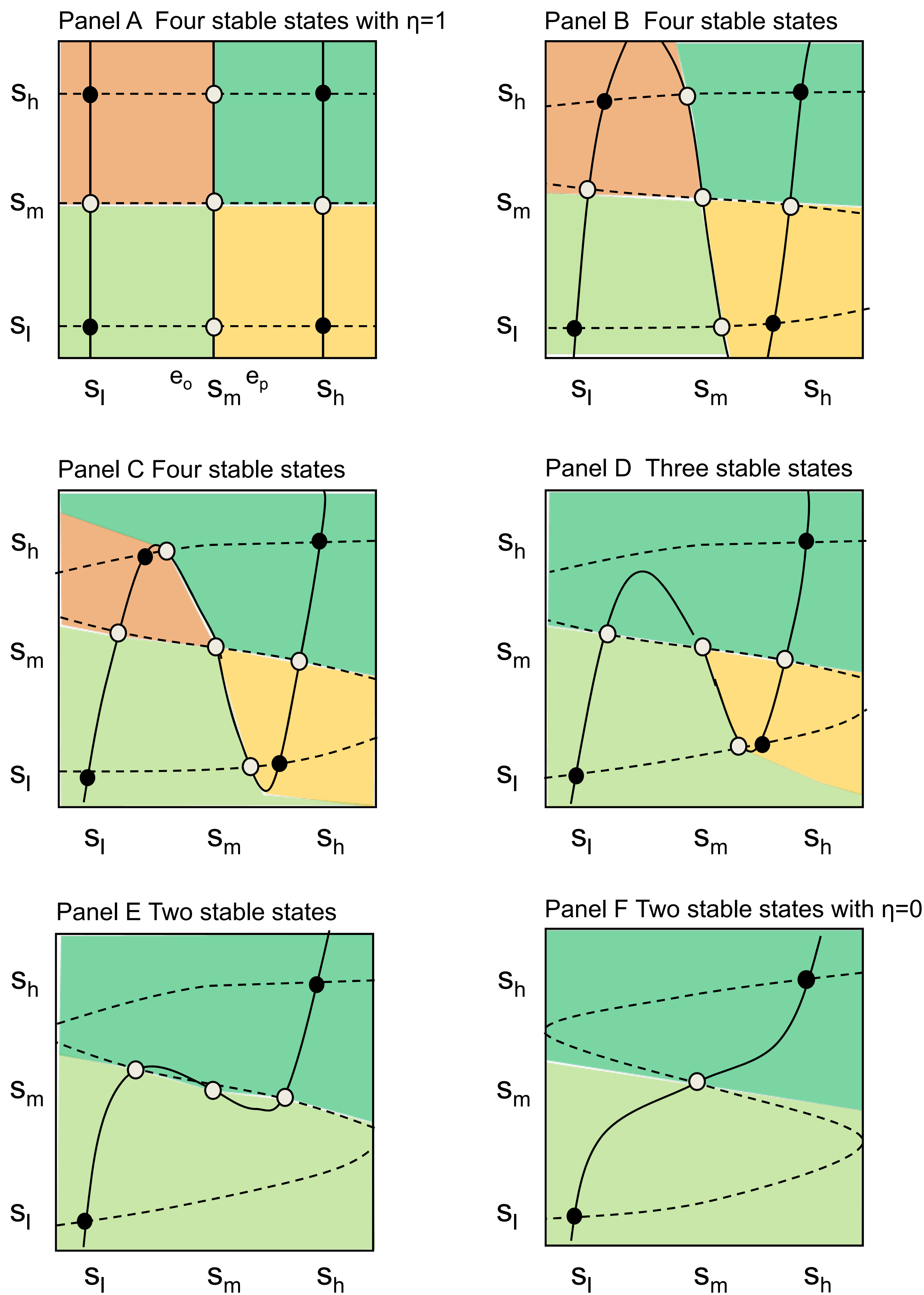
• The statistics rule out the vocational high schools. Note that an equalized public school system was established in the mid 1970s. Before then, the Seoul's college advancement rate was the highest because the major prestigious private schools were located in Seoul.

Appendix Figure 1. Size of Overlap in a Simplified Homogeneous Economy

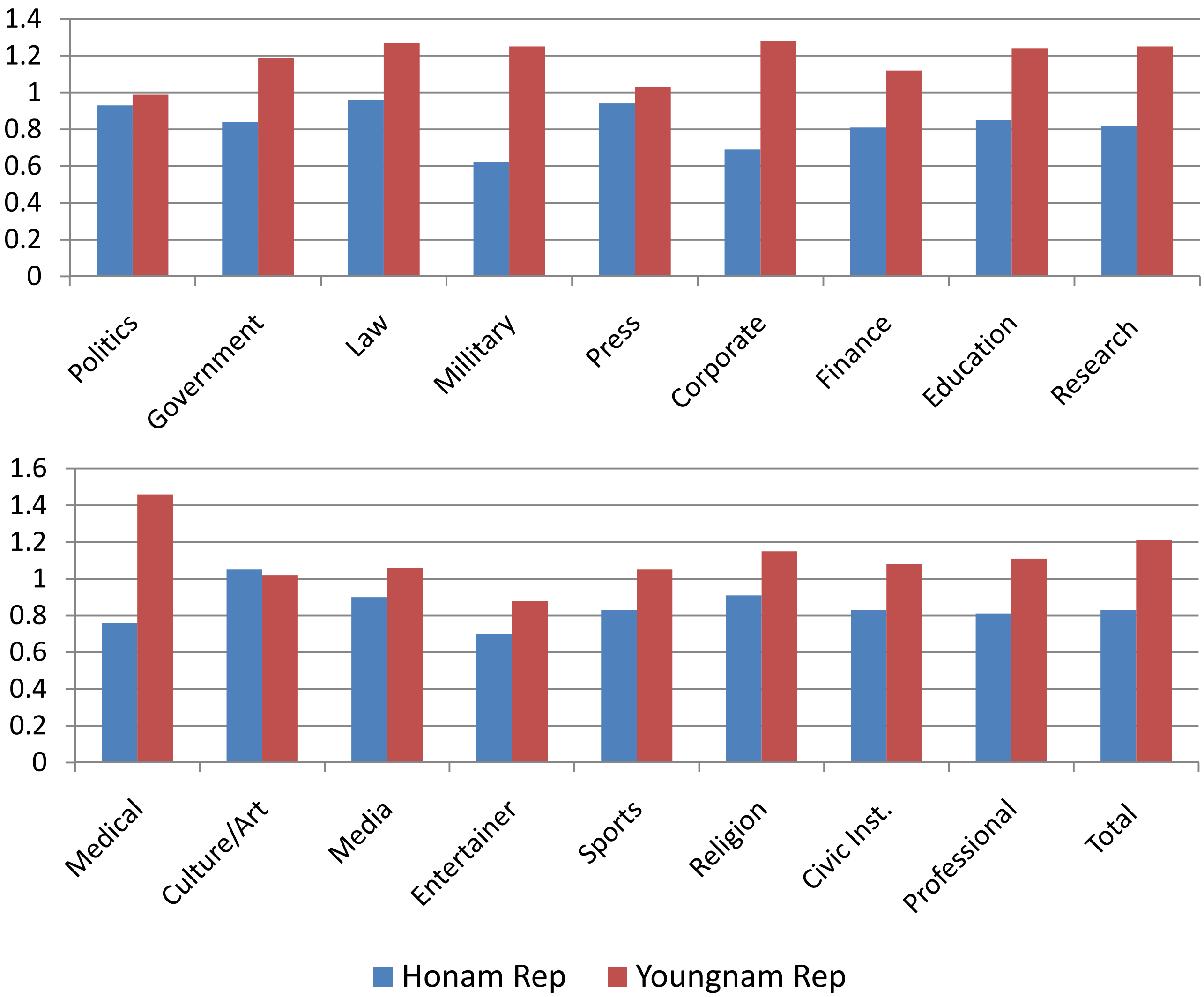


Appendix Figure 2. Manifold Ranges with No Lifetime Externalities

- Segregation level η declines in Panel A, B, C, D, E, F order, with η=1 in Panel A and η=0 in Panel F.



Appendix Figure 3. Regional Group Disparity in S. Korea



• Source: Chosun Daily Leaders' Database www.dbchosun.com; Eui-Young Yu (2003)
• Rep index: Rep index is the ratio between leader's birthplace percentage in 2002 and newborn percentage of the birthplace in 1970. The newborn distribution is a proxy of the regional distribution of young families in 1970. Rep index is calculated excluding Seoul born natives (about 5% of the population).

Appendix Figure 4. Regional Voting in Presidential Elections of S. Korea

Election Year	1963		1967		1971		1987		
Parties	Democratic	Industrial	Democratic	Industrial	Democratic	Industrial	Democrat 1	Democrat 2	Industrial
Candidates	BS Yoon	CH Park	BS Yoon	CH Park	DJ Kim	CH Park	DJ Kim	YS Kim	TW Roh
Honam	41	59	52	48	64	36	88.4	1.2	9.9
Youngnam	36	64	27	73	25	75	5.0	41.6	48.8
Nationwide	49	51	45	55	46	54	27.1	28	38.6
Birthplace of Candidate	Chungcheong	Youngnam	Chungcheong	Youngnam	Honam	Youngnam	Honam	Youngnam	Youngnam

Election Year	1992		1997		2002		2007	
Parties	Democratic	Industrial	Democratic	Industrial	Democratic	Industrial	Democratic	Industrial
Candidates	DJ Kim	YS Kim	DJ Kim	HC Lee	MH Roh	HC Lee	DY Jung	MB Lee
Honam	91	4.2	93.5	3.8	92.5	5.4	79.5	9.0
Youngnam	10	98	12.3	58.4	24.5	70.3	9.1	62
Nationwide	33.8	42	40.3	38.7	48.9	46.6	26.1	48.7
Birthplace of Candidate	Honam	Youngnam	Honam	North Korea	Youngnam	North Korea	Honam	Youngnam

• Source: National Election Commission (South Korea)

• [Anecdote] The Democratic Party has been based on Honam region and the Industrial Party based on Youngnam region since 1971 election. There was no presidential election between 1971 and 1987 due to President Park's dictatorship and the second military coup by President Chon in 1980. In 1987 election, the democratic party was split into two. "Democratic 1" gave the candidacy to DJ Kim born in Honam, and "Democratic 2" to YS Kim born in Youngnam. In 1992 election, the "Democratic 2" party was merged into the Industrial Party and two democratic leaders, DJ Kim and YS Kim, competed for the presidency. In 1997 and 2002, the honam-based Democratic Party won the presidential elections.