Decomposing Federal Funds Rate forecast uncertainty using real-time data

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Abstract

Using real-time data I estimate out-of-sample forecast uncertainty about the Federal Funds Rate. Combining a Taylor rule with a model of economic fundamentals I disentangle economically interpretable components of forecast uncertainty: uncertainty about future economic conditions and uncertainty about future monetary policy. Uncertainty about U.S. monetary policy fell to unprecedented low levels in the 1980s and remained low while uncertainty about future output and inflation declined only temporarily. This points to an important role of increased predictability of monetary policy in explaining the decline in macroeconomic volatility in the U.S. since the mid-1980s.

Keywords: monetary policy reaction function, interest rate uncertainty, state-space model

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1 Introduction

The aim of this paper is to study uncertainty in the U.S. money market by estimating changes in uncertainty about forecasts of the Federal Funds Rate. Using real-time forecasts of U.S. output gaps and inflation rates I estimate a time-varying interest rate rule for the Federal Reserve System (Fed) over the time period from 1966-2007 and decompose the forecast uncertainty implied by the interest rate rule into components representing uncertainty about the future state of the economy, about future monetary policy, and a residual element.

Estimates of interest rate uncertainty are important for a wide range of financial market applications such as portfolio allocation, derivative pricing, risk management etc. The Federal Funds Rate is the indicator of monetary policy in the U.S. Hence, uncertainty about future money market rates reveals information about the credibility and predictability of the central bank's monetary policy. Keeping this uncertainty low is an important goal of central banks' communication policy (for example, European Central Bank (2008), Reinhart (2003)). The level of interest rate uncertainty has also been shown to affect economic stability (e.g. Poole (2005)).[1]

The empirical importance of time-variation in uncertainty about short-term interest rates has been documented in many studies. The most widely used approach is to construct measures of interest rate uncertainty from the time series of historical interest rate changes by estimating ARCH or GARCH models (e.g. Chuderewicz (2002) and Lanne and Saikkonen (2003)), stochastic volatility models (e.g. Caporale and Cipollini (2002)) or regime switching models of volatility (e.g. Sun (2005)). As an alternative, derivative prices can be used to obtain market-based estimates of interest rate uncertainty (e.g. Fornari (2005)). An important drawback of these approaches is however, that changes in the extracted measure of uncertainty are difficult to interpret economically.

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[1]For example, an increase in the volatility of money market rates can be transmitted through the yield curve (Ayuso et al. (1997)) causing the volatility of longer-term interest rates to rise as well which has negative effects on real growth (e.g. Muellbauer and Nunziata (2004)) and investment (e.g. Byrne and Davis (2005)).

The contribution of this paper is to show that economically meaningful estimates of interest-rate uncertainty can be obtained by recognizing that the most important driving force of short-term interest rates is monetary policy. Hence, an interpretation of interest-rate uncertainty must be based on a model that accounts for how financial markets perceive monetary policy to respond to changes in the state of the economy.

The starting point of the analysis is the famous Taylor rule (Taylor (1993)) which is widely accepted as a descriptive model of how the Fed sets the Federal Funds Rate in response to (expected) economic conditions. Even though the Fed certainly does not follow a Taylor rule mechanically, financial market participants often use Taylor-type rules as forecasting tools.

Federal Funds Rate forecasts from a Taylor rule require predictions of the state of the economy the Fed will have to respond to in the future. Thus, uncertainty about the forecasts of the information the central bank is expected to act upon is one source of uncertainty about future interest rates.

The second element of uncertainty is related to imperfect knowledge about the central bank's reaction to given future economic conditions: The reaction coefficients in estimated simple interest rate rules such as the Taylor rule have been shown to change over time (e.g. Mehra (1999), Judd and Rudebusch (1999), Clarida et al. (2000), Tchaidze (2001), Gordon (2005)) and this variation is a second source of uncertainty about the future Federal Funds Rate.

One cause for this is that the coefficients in monetary policy reaction functions derived optimally depend on the central bank's preferences about output stabilization, inflation and possibly other goals, as well as on structural parameters of the model of the economy: Changes in preferences and changes in the structure of the economy will both affect the coefficients in the monetary policy reaction function. Another explanation is that simple interest rate rules are only approximations to the optimal monetary policy reaction function. Central banks base their policy decisions on a much more comprehensive data set than a simple Taylor-type interest rate rule which only accounts for (forecasts of) the output gap and inflation. Situations with identical (forecast) values of the output gap and inflation can be significantly different economically if judged by the much larger optimal information set. Thus, the central bank does not necessarily have to react to (apparently) identical economic situations in the same way and this

will lead to changing reaction coefficients in estimated simple interest rate rules. Finally, changes in the interest-rate rule coefficients can also result from fitting a linear reaction function when the true reaction function is in fact non-linear.

The third source of Federal Funds Rate forecast uncertainty is due to the fact that the estimated reaction function is an approximation. The approximation error of the Taylor rule relative to the actual Federal Funds Rate is represented by the error term in the empirically estimated interest rate rule.

These separate components of interest-rate uncertainty can be related to the discussion of the causes for the observed decline in macroeconomic volatility in the U.S. since the mid-1980s. The results in this paper shed light on changes in the predictability of the output gap, of inflation and of U.S. monetary policy through time. It has been argued whether the decline in output and inflation volatility was caused by a reduction in shocks to the U.S. economy ("good luck"), changes in the structure of the U.S. economy or by improvements in the Fed's monetary policy (e.g. Gordon (2005), Stock and Watson (2003)). The the first two explanations are related to the predictability of macroeconomic fundamentals. This paper shows a trend decline in forecast uncertainty about fundamentals throughout the 1990s followed by an increase after 2001. However, the level of forecast uncertainty about fundamentals in the late 1980s and in the 1990s was, on average, not much different from before. The "good policy" version of the argument is related to an improved predictability of monetary policy, i.e. a decline in monetary policy shocks as represented by the deviations from the Fed's monetary policy reaction function. According to the results presented in this paper an important decline in uncertainty about the unpredictable component of the Fed's interest rate policy occurred in the early 1980s followed by a period of extremely low uncertainty about the Fed's monetary policy reaction function in the 1990s.

Empirical studies of monetary policy reaction functions have shown that the use of ex-post revised data results in distorted estimates of reaction coefficients (e.g. Orphanides (2001), Perez (2001)). The estimation of a monetary policy reaction function using ex-post revised data assumes too much information on part of the monetary policy authority: It contains observations that were not available at the time of the actual monetary policy decision and some observations have undergone revisions relative to the

information that the central bank had to act upon.[2] Hence, the results presented in this paper are derived from recursive estimates using a real-time data set of macroeconomic variables.

This paper offers a new application for the growing empirical literature on time-varying monetary policy rules: the study of uncertainty about future monetary policy. Previous analyses have focused on ex-post descriptions of central bank behavior. For example, Clarida, Galí and Gertler (2000) provide evidence of pronounced changes in Taylor-type interest rate rules for the U.S. using split-sample regressions. They show a strong shift in the Fed's reaction function related to the appointment of Fed Chairman Volcker in 1979. More recently, Boivin (2006) and Kim and Nelson (2006) estimate forward-looking Taylor rules with time-varying parameters and report sizeable but more gradual changes in the coefficients. Trecroci and Vassali (2006) show that time-varying monetary policy reaction functions for the U.S., the U.K., Germany, France and Italy perform superior to constant parameter rules in accounting for observed changes in interest rates.[3] However, most of these studies on time-varying monetary policy reaction functions use ex-post revised data which might bias the results.[4]

The paper is structured as follows: Section 2 outlines the empirical models for the monetary policy reaction function and for the economic fundamentals that enter into it. Section 3 presents the data set and explains how the real-time data are used in the estimation. The estimation results for interest-rate forecast uncertainty are presented in Section 4.

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[2]See also Orphanides (2002, 2003) for a discussion of the importance of using real-time data for the empirical modelling of monetary policy.

[3]Time-varying Taylor rules have also been estimated for the Deutsche Bundesbank by Kuzin (2005) and using a regime-switching model by Assenmacher-Wesche (2008).

[4]An exception is Boivin (2006) who uses the Fed's own forecasts of economic fundamentals.

2 A model of policy and economic fundamentals

2.1 The Taylor rule

The empirical model for the Federal Funds Rate is based on the notion that the Fed adjusts the Federal Funds Rate in response to the current or expected state of the economy. Thus, Federal Funds Rate forecasts are affected by two sources of uncertainty: (i) uncertainty about the future state of the economy and (ii) uncertainty about future policy responses to a given state of the economy. The first type of uncertainty concerns forecasting future values of the variables in the central bank's reaction function while the second type concerns forecasts of future values of the reaction function coefficients.

The standard approach to model the setting of the short-term interest rate by the central bank is the specification of a monetary policy reaction function, i.e. an interest rate rule, that relates the short-term interest rate as the monetary policy instrument, to other economic variables. The most widely used type of interest rate rules is represented by the Taylor rule (Taylor (1993)) which assumes the central bank to react to current or expected inflation and output gaps

$$i_t^* = \bar{r}_t + \bar{\pi}_t + \alpha_{\pi,t}(E_t\pi_{t+k} - \bar{\pi}_t) + \alpha_{z,t}E_t z_{t+j}, \tag{1}$$

where i_t^* is the target short-term interest rate, $\bar{r}_t$ is the time-varying equilibrium real interest rate, π_{t+k} is the inflation rate k periods in the future, $\bar{\pi}_t$ is the inflation target, and z_{t+j} is the output gap j periods ahead. Equation (1) allows for time variation in the reaction coefficients $\alpha_{\pi,t}$ and $\alpha_{z,t}$.

The actual short-term interest rate is adjusted gradually towards the target interest rate given by (1)

$$i_t = (1 - \rho_t)i_t^* + \rho_t i_{t-1} + \epsilon_t, \qquad 0 < \rho_t < 1, \tag{2}$$

where ϵ_t is a random disturbance term which represents the non-systematic element of monetary policy and the approximation error of the Taylor rule relative to the actually observed interest rate.

The time-varying reaction coefficients in (1) are assumed to follow random walks. This assumption together with imposing the restriction $0 < \rho_t <$ leads to the following representation for the time-varying interest rate rule

$$
\begin{aligned}
i_t &= (1 - \rho_t)(\bar{r}_t + \bar{\pi}_t + a_{\pi,t}(E_t\pi_{t+k} - \bar{\pi}_t) + a_{z,t}E_t z_{t+j}) + \rho_t i_{t-1} + \epsilon_t \\
&= \beta_{0,t} + \beta_{\pi,t}E_t\pi_{t+k} + \beta_{z,t}E_t z_{t+j} + \rho_t i_{t-1} + \epsilon_t \tag{3} \\
\rho_t &= \frac{1}{1 + \exp(-\beta_{\rho,t})} \tag{4} \\
\beta_{t+1} &= \beta_t + w_{t+1}, \quad w_t \sim \text{i.i.d} N(0, \Sigma_w), \tag{5}
\end{aligned}
$$

with $\beta_{0,t} = (1 - \rho_t)\left(\bar{r}_t + (1 - a_{\pi,t})\bar{\pi}_t\right), \beta_{\pi,t} = (1 - \rho_t)a_{\pi,t}, \beta_{z,t} = (1 - \rho)a_{z,t},$ $\beta_t = [\beta_{0,t} \quad \beta_{\pi,t} \quad \beta_{z,t} \quad \beta_{\rho_t}]', w_t = [w_{0,t} \quad w_{\pi,t} \quad w_{z,t} \quad w_{\rho_t}]'$ and Σ_w as a diagonal matrix.

Concerning the forecast horizons k and j various assumptions have been used in the literature. In this paper, I assume $k = j = 2$. Due to the high degree of autocorrelation of the forecasts the choice of the forecast horizon has only modest effects on the results (e.g. Boivin (2006)).

Since several studies have documented important variation in the variance of the interest-rate shock ϵ (e.g. Stock and Watson (2003), Cogley and Sargent (2003)) the variance of the disturbance term is approximated by a GARCH(1,1) process.

$$
\begin{aligned}
\epsilon_t | \Psi_{t-1} &\sim N(0, \sigma_{\epsilon,t}^2) \tag{6} \\
\sigma_{\epsilon,t}^2 &= \kappa_0 + \kappa_1 \epsilon_{t-1}^2 + \kappa_2 \sigma_{\epsilon,t-1}^2, \tag{7}
\end{aligned}
$$

where Ψ_{t-1} is the period $t - 1$ information set.

2.2 Output gap and inflation forecasts

The output gap which enters the Taylor rule (3) is an unobservable variable and can only be inferred indirectly from the observed output dynamics. Various empirical decompositions of actual output into a long-run trend component (potential output) and a short-run cyclical component (output gap) have been suggested in the literature.

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These include, among others, the Hodrick-Prescott filter as well as decompositions suggested by Watson (1986) and Clark (1989).

The output gap is related to the inflation rate by a Phillips curve-type relationship. To exploit both sources of information, it is preferable to jointly model the dynamics of inflation and of the output gap using an unobserved components model suggested by Kuttner (1994): The output equation is based on Watson (1986) and decomposes the log of real GDP (y) into a random walk and a stationary AR(2) component

$$
\begin{align}
y_t &= n_t + z_t \tag{8}\\
z_t &= \phi_1 z_{t-1} + \phi_2 z_{t-2} + e_t^z \tag{9}\\
n_t &= \mu_y + n_{t-1} + e_t^n, \tag{10}
\end{align}
$$

where n represents the trend component and follows a random walk with drift μ_y while z is the (log) deviation of real GDP from potential output, i.e. the output gap. After some preliminary estimations, inflation dynamics were modelled as an MA process in which the change in the rate of inflation depends on the lagged output gap[5]

$$
\Delta \pi_t = \gamma z_{t-1} + \delta(L)\nu_t, \tag{11}
$$

where $\delta(L)$ is a lag polynomial of order three and ν is a normally i.i.d error term. The model (8) - (11) can be written in state-space form leading to the observation equation (see Appendix A)

$$
Y_t = \mu + H\tilde{x}_t + e_t, \tag{12}
$$

with $Y_t = (\delta y_t \quad \delta \pi_t)'$ and $\tilde{x}_t$ as a vector of unobserved components including the current and lagged output gaps.

The transition equation for the state variables is given by

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[5]Preliminary tests strongly reject the hypothesis of a stationary inflation rate and suggest a model in first differences.

$$\tilde{x}_{t+1} = F\tilde{x}_t + \zeta_{t+1}. \tag{13}$$

This model generates forecasts for the output gap and for the inflation rate that are used to estimate the Taylor rule (3). Since output and inflation cannot be observed within the current period the forecasts for the output gap and the inflation rate in $t+2$ are based on information up to and including period $t-1$. The forecast for the output gap two periods ahead is

$$z_{t+2|t-1} = 1'_z FFF\tilde{x}_{t|t-1}, \tag{14}$$

where 1_z is a unit vector for the first element of $\tilde{x}$. The forecast of inflation in $t+2$ based on data available in $t-1$ is given by

$$\pi_{t+2|t-1} = \pi_{t-1} + 1'_\pi \left[3\mu + H(I + F + FF)\tilde{x}_{t|t-1} \right]. \tag{15}$$

These forecasts are used as explanatory variables in the estimation of the Taylor rule (3) by replacing $E_t\pi_{t+2}$ and $E_t z_{t+2}$ by $\pi_{t+2|t-1}$ and $z_{t-1|t+2}$.

The two-step estimation approach of using model-generated forecasts in the estimation of a Taylor-type interest rate rule is related to the one advocated in Nikolsko-Rzhevsky (2008): Since the Fed's internal forecasts of future economic conditions (Greenbook forecasts) are available only with a lag of five years, Nikolsko-Rzhevsky (2008) estimates various univariate and multivariate forecasting models to generate out-of-sample forecasts closely tracking the Greenbook forecasts. These time-series of model-generated forecasts are used to estimate a forward-looking Taylor rule for the Fed. Similarly, McCulloch (2007) estimates a forward-looking Taylor rule using an adaptive least squares technique. The forecasts which enter the monetary policy reaction function are generated from structural vector autoregressions. While the two-step procedures employed in these papers is similar to the one presented here, these papers do not consider forecast uncertainty.

3 Data and Estimation

The hyperparameters in equations (12) and (13) were estimated by maximum likelihood using the Kalman-Filter. Quarterly observations of output and inflation for the U.S. were obtained from the Real-Time Data Set for Macroeconomists (RTDSM) at the Federal Reserve Bank of Philadelphia.[6] Output is real GNP (after 1993 real GDP) while the inflation rate is 100 times the quarterly log difference of the GNP/GDP deflator. The output and inflation series are grouped into data vintages containing only time series that would have been available at a specific point in time. In the RTDSM the first real-time vintage is available for 1965Q4 and contains time series from 1947Q1 to 1965Q3. For each of the following quarters new vintage series are available with new observations for the most recent quarter and revised data for some of the previous observations. Since both the price level and real output are observed with a one period lag each vintage ends one quarter before the date it applies to. The four vintages from 1993 contain missing observations for the time period from 1947Q1 to 1959Q1. Hence, all time series used in the estimation were chosen to start in 1959Q1 The policy indicator i_t is the quarterly average of the Federal Funds Rate. In contrast to the data on output and inflation the Federal Funds Rate is not subject to revisions.

Table 1 is a stylized representation of real-time observations on a variable x. The columns contain the data vintages beginning with $\tau_0 = 1965Q4$ and ending in $T = 2007Q3$. $x_{t|\tau}$ is variable x in period t as observed from period τ. For the RTDSM $t_0 = 1947Q1$ and $t < \tau$ because the variables are observed with a lag of one period.

The empirical model of the output gap and the inflation rate (12) - (13) is estimated recursively to generate series of two-period-ahead forecasts of output gap and inflation. At each date only the time series that would have actually been available to the central bank are used to estimate the model parameters and the output gap series and to derive the forecasts. The first vintage used is 1966Q1 with the last observation in 1965Q4. Hence, the first forecasts for the output gap and for the inflation rate are $z_{1966Q3|1965Q4}$ and $\pi_{1966Q3|1965Q4}$. For 1966Q2 the model is re-estimated from the 1966Q2 vintage and new forecasts $z_{1966Q4|1966Q1}$ and $\pi_{1966Q4|1966Q1}$ are constructed etc.

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[6]A detailed description of the data set is given in Croushore and Stark (1999, 2001, 2003).

	τ_0	τ_0+1	$\ldots$	T-1	T
t_0	$x_{t_0\|\tau_0}$	$x_{t_0\|\tau_0+1}$	$\ldots$	$x_{t_0\|T-1}$	$x_{t_0\|T}$
t_0+1	$x_{t_0+1\|\tau_0}$	$x_{t_0+1\|\tau_0+1}$	$\ldots$	$x_{t_0+1\|T-1}$	$x_{t_0+1\|T}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
τ_0-1	$x_{\tau_0-1\|\tau_0}$	$x_{\tau_0-1\|\tau_0+1}$	$\ldots$	$x_{\tau_0-1\|T-1}$	$x_{\tau_0-1\|T}$
τ_0	-	$x_{\tau_0\|\tau_0+1}$	$\ldots$	$x_{\tau_0\|T-1}$	$x_{\tau_0\|T}$
τ_0+1	-	-	$\ldots$	$x_{\tau_0+1\|\tau_0}$	$x_{\tau_0+1\|\tau_0+1}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
$T-2$	-	-	$\ldots$	$x_{T-2\|T-1}$	$x_{T-2\|T}$
$T-1$	-	-	$\ldots$	-	$x_{T-1\|T}$
T	-	-	$\ldots$	-	-

Table 1: Stylized real-time data set

The coefficients of the time-varying Taylor rule are estimated recursively from these model-generated forecasts starting in 1966Q1 since the Federal Funds Rate cannot be viewed as the principal U.S. monetary policy indicator before this date (e.g. Lansing (2003)). For each quarter from 1966Q1 to 2007Q3 the hyperparameters in (3)-(7) are re-estimated using the real-time forecasts for the inflation rate and the output gap extended by the newest available forecasts. Since the reaction coefficients are assumed to follow random-walk processes the Kalman Filter is initialized with a diffuse prior.

Two assumptions are required to actually estimate the monetary policy reaction function from the model-generated forecasts of economic fundamentals: First, the contemporaneous value of $x_t = (1 \quad \pi_{t+2\|t-1} \quad z_{t+2\|t-1} \quad i_{t-1})'$ that underlies the central bank's decision is known to the public. Second, x_t must be exogenous to β_t. For example, the model does not allow for asymmetries in the interest rate response to the output gap or inflation, i.e. for the β parameters to vary systematically with changes in the output gap or inflation forecasts.

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4 Estimation results

Figure 1 presents two time series of one-sided Kalman filter estimates of output gaps. The solid line is the output gap estimated from ex-post revised data (1959Q4 - 2007Q3) while the dashed line represents output gap estimates in real-time, i.e. the estimates that would have been obtained at each point in time using the most recent available data at that specific point in time. The difference between both time series is the real-time measurement error in the terminology of Orphanides and van Norden (2002). The estimated output gap from ex-post revised data is smoother than the real-time output gap and the real-time estimates for particularly negative values of the output gap are much more pronounced.

« insert Figure 1 »

Figure 2 compares one-sided estimates of output gaps over time for three different vintages. While the dashed line in Figure 1 shows the output gap series estimated from the latest available vintage of data. Figure 2 traces estimated output gaps obtained for the time period from 1959Q4 to 1997Q1 from three specific vintages. In contrast to Figure 1 along a specific line only one unchanged time series for inflation and output is used in the estimation. The solid line shows output gap estimates using the data set from 2007Q2, the dashed line from 2002Q2, and the dotted line from 1997Q2. The data sets differ to the extent to which the data has been revised and in the number of observations which is higher for younger vintages. The low points of the business cycle tend to be more pronounced for shorter data sets with less revisions. As we move to the right and approach the final observation of each data set the estimates diverge more strongly since data revisions are more pronounced close to the release date of the data.[7]

« insert Figure 2 »

Figure 3 presents the inflation forecast which is used together with the forecast for the output gap in the estimation of the forward-looking Taylor rule. The top panel shows actual inflation together with the inflation forecast. Forecast errors are presented in the bottom panel. The RMSE of the inflation forecast is 0.039.

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[7]For similar results see Orphanides and van Norden (2002).

« insert Figure 3 »

The next two figures display graphs of the recursive estimates of some of the hyperparameters of the model and show how the estimated parameters of the economic model (12) and (13) change as additional and more accurate data becomes available. Figure 4 contains estimates of the autoregressive coefficients on the output gap ϕ_1 and ϕ_2 and of the drift of potential output μ_y. The dashed lines are bands of 1.96 standard deviations around the point estimates. All parameter estimates are statistically significant throughout. Both autoregressive parameters are relatively stable over time and are highly correlated. As shown in the bottom right panel their sum is is roughly constant and highly significant.

« insert Figure 4 »

Of special interest is the Phillips-curve parameter γ which describes the effect of the output gap on the change in the inflation rate. Figure 5 shows the recursive estimate of γ together with error bands of 1.96 standard deviations. Except for two short episodes in the 1970s and in the mid-1980s the Phillips-curve coefficient is significantly different from zero. The size of the effect of the output gap on the change inflation is relatively low with estimates between 0.02 and 0.03 from the mid 1980s up to the present.

« insert Figure 5 »

Figure 6 displays the recursive one-sided estimates of the coefficients in the Taylor rule. Often the coefficient on the inflation forecast (upper right panel) is less than one thus violating the Taylor principle (Taylor (1999)). It sometimes even becomes negative, for example in the mid-1970s, the mid-1990s and after the bursting of the new economy bubble in 2001. The coefficient on the output gap (lower left panel) trends upward from the mid 1980s on but exhibits pronounced cyclical swings. The intercept is extremely high in the high-inflation era of the 1970s and in the early 1980s.[8]

« insert Figure 6 »

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[8]The time-varying intercept $\beta_{0,t}$ is affected by ρ_t, by the equilibrium real interest rate $\bar{r}_t$ and by the time-varying inflation target $\bar{\pi}_t$. Retrieving an estimate of the inflation target would require apart from an estimate of rho_t information about the real equilibrium interest rate.

5 Federal Funds Rate forecast uncertainty

5.1 The one-period ahead interest-rate forecast

Forecast uncertainty about the Federal Funds Rate in the next quarter is defined as

$$\mathbf{E}_t \left[(i_{t+1} - \hat{i}_{t+1|t})^2 | \Omega_t \right]. \tag{16}$$

Define $b_t = (\beta_{0,t} \quad \beta_{\pi,t} \quad \beta_{z,t} \quad \rho_t)'$ and $x_t = (1 \quad \pi_{t+2|t-1} \quad z_{t+2|t-1} \quad i_{t-1})'$, then

$$\hat{i}_{t+1|t} = \mathbf{E}_t \left[i_{t+1} | \Omega_t \right] = \mathbf{E}_t \left[x'_{t+1} b_{t+1} | \Omega_t \right]. \tag{17}$$

Ω_t represents the information available to market participants immediately <u>after</u> the interest rate is set at time t. This information set consists of the estimated reaction function in (3) - (7), of the estimated output gap/inflation model in (12) and (13), and of the series of current and past interest rates and output gap and inflation forecasts.

Since b and x are uncorrelated the one-step ahead forecast for the Federal Funds Rate is

$$\hat{i}_{t+1|t} = \mathbf{E}_t \left[x'_{t+1} | \Omega_t \right] \mathbf{E}_t \left[b_{t+1} | \Omega_t \right] = \hat{x}'_{t+1|t} b_{t+1|t}. \tag{18}$$

Note that since $x_t = (1 \quad \pi_{t+2|t-1} \quad z_{t+2|t-1} \quad i_{t-1})'$, the forecast of x_{t+1} based on Ω_t, is $\hat{x}_{t+1|t} = (1 \quad \pi_{t+3|t-1} \quad z_{t+3|t-1} \quad i_t)'$. However the forecast of b_{t+1} based on Ω_t is $b_{t+1|t}$ as i_t is part of the information set in period t.

Combining (16), (17) and (18) leads to

$$
\begin{aligned}
\mathbf{E}_t \left[(i_{t+1} - \hat{i}_{t+1|t})^2 | \Omega_t \right] &= \mathbf{E}_t \left[(x'_{t+1} b_{t+1} - \hat{x}'_{t+1|t} b_{t+1|t})^2 | \Omega_t \right] \\
&= \hat{x}'_{t+1|t} P_{b,t+1|t} \hat{x}'_{t+1|t} + b'_{t+1|t} P_{x,t+1|t} b_{t+1|t} + \sigma^2_{\epsilon,t+1|t}.
\end{aligned} \tag{19}
$$

$P_{b,t+1|t} = \mathbf{E}_t \left[(b_{t+1} - b_{t+1|t})(b_{t+1} - b_{t+1|t})' \right]$ is obtained from the Kalman filter. The first term in (19) is the component of the overall interest rate forecast uncertainty related

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to possible changes in the Fed's reaction function.

$P_{x,t+1|t} = \mathbf{E}_t\left[(x_{t+1} - x_{t+1|t})(x_{t+1} - x_{t+1|t})'|\Omega_t\right]$ represents uncertainty about the forecast of the economic variables the interest rate responds to. A detailed derivation of this expression can be found in Appendix C. The last term in (19) represents uncertainty caused by the Taylor-rule residual ϵ with $\sigma^2_{\epsilon,t+1|t}$ being the forecast of the variance of the approximation error using the estimated GARCH coefficients.

The results for the one-quarter ahead forecast uncertainty in (19) are presented in Figure 7. The solid line represents aggregate interest rate uncertainty while the other two lines represent uncertainty about the reaction coefficients in the Taylor rule (long dashes, first term in (19)) and uncertainty about economic fundamentals in the next quarter (short dashes, second term in (19)).

« insert Figure 7 »

Figure 7 indicates considerable changes in uncertainty about one-quarter ahead forecasts of the Federal Funds Rate. Peaks in forecast uncertainty were in the mid 1970s, in the early 1980s, in 1984Q4 and in 2002Q2. The lower panel shows a truncated version of the graph excluding the very high estimated uncertainty in 1980Q3. Even without looking at the extreme values in the early 1980s, uncertainty about the one-quarter-ahead Federal Funds Rate was much higher in the 1970s and 1980s than in the 1990s and 2000s.

The first strong increase in uncertainty in the early 1970s is dominated by rising uncertainty about the reaction coefficients in the Taylor rule. After a brief decline in the late 1970s uncertainty about future reaction function coefficients increases once more after 1977 and remains high up to the early 1980s. The extreme hike in forecast uncertainty in the early 1980s, however, can only partially explained by uncertainty about the coefficients in the Fed's reaction function. Its primary cause is a strong increase in residual uncertainty, i.e. a massive deterioration of Taylor rule's ability to track the actual Federal Funds Rate.[9] The same applies to the peak in uncertainty in 1984Q4. Uncertainty about future economic fundamentals, i.e. output gap and inflation forecasts, increases only moderately in the mid-1970s. It remains relatively low throughout

---

[9]Residual uncertainty is the difference between aggregate uncertainty and the sum of the other two components.

the remainder of the sample period. While uncertainty about the Taylor rule coefficients and uncertainty about future fundamentals are of similar magnitude from the late 1980s on, the first uncertainty component is quantitatively much more important in the preceding part of the sample period.

The observed decline in macroeconomic volatility in the U.S. since the mid-1980s has been attributed to various sources: a decline in the size and frequency of exogenous shocks, changes in the structure of the U.S. economy, and improvements in monetary policy, e.g. by enhanced transparency (e.g. Gordon (2005), Stock and Watson (2003)). The results in this paper shed some light on the relative importance of these explanations by highlighting changes in the predictability of economic fundamentals and monetary policy in the U.S. Fewer and smaller shocks or a smaller impact of shocks on the economy due to changes in the economic structure would lead to a smaller forecast uncertainty about future output gaps and inflation. A more systematic and thus more predictable monetary policy would result in a decline in uncertainty about the approximation error and uncertainty about the coefficients in the Fed's policy rule. Figure 8 graphs one-step-ahead Federal Funds Rate forecasts and forecast errors from the estimated time-varying Taylor rule. Clearly, forecast errors for the Federal Funds Rate have been much smaller since 1985 than before (RMSE: post-1985: 0.43, pre-1985: 1.46).

« insert Figure 8 »

Figure 9 shows the time series for uncertainty about fundamentals and uncertainty about the monetary policy reaction function in greater detail. The top panel displays the results for uncertainty about future economic fundamentals. Forecast uncertainty about the output gap and inflation in the late 1980s and in the 1990s is not consistently smaller than before, in particular if compared to the late 1970s and the early 1980s. Uncertainty about fundamentals dropped almost to zero in the mid 1980s but rose again at the start of the 1990s. Throughout the 1990s it follows a declining trend interrupted by a brief hike in 1995. In 2001 uncertainty about economic fundamentals increased again. As far as uncertainty about future output gaps and inflation is concerned the post-1985 period compares favorably only to the early 1970s.

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The lower panel shows the time series for uncertainty about the coefficients in the Fed's reaction function and overall uncertainty about the reaction function. The latter is defined as the sum of coefficient uncertainty and uncertainty about the approximation error, i.e. the Taylor rule residual variance $\sigma^2_{\epsilon,t}$. The lower panel uses a scale different from the upper one. For much of the time the two graphs are very close to each other. The residual variance component becomes important in times when the Taylor rule does not render a plausible description of monetary policy, particularly in the early 1980s in 1985 and in 2001. The results shown in this panel are consistent with a quick and persistent decline in uncertainty about the Fed's reaction function taking place in the early to mid 1980s. Uncertainty about monetary policy was extremely low throughout the 1990s and only rose slightly after 2001. The hike in uncertainty about the Fed's reaction function in 2001 is due to the high Taylor-rule residual caused by the Fed drastically lowering the Federal Funds Rate after the bursting of the New Economy bubble. Overall, the results in Figure 9 show the "good policy" argument being more important than "good luck": The predictability of the Fed's monetary policy improved strongly and persistently in the mid-1980s while there is no evidence for such an improvement in forecast uncertainty about economic fundamentals.

5.2   The two-period ahead interest-rate forecast

Uncertainty about the two-quarters ahead forecast of the Federal Funds Rate is

$$\mathbf{E}_t\left[(i_{t+2} - \hat{i}_{t+2|t})^2|\Omega_t\right], \tag{20}$$

where

$$
\begin{aligned}
\hat{i}_{t+2|t} &= \mathbf{E}_t\left[i_{t+2}|\Omega_t\right] = \mathbf{E}_t\left[x'_{t+2}b_{t+2}|\Omega_t\right] \\
&= \mathbf{E}_t\left[x'_{t+2}|\Omega_t\right]\mathbf{E}_t\left[b_{t+2}|\Omega_t\right] = \hat{x}'_{t+2|t}b_{t+2|t}.
\end{aligned}
\tag{21}
$$

Expanding (21) yields

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$$\mathbf{E}_t\left[(i_{t+2} - \hat{i}_{t+2|t})^2|\Omega_t\right] = \mathbf{E}_t\left[(x'_{t+2}b_{t+2} - \hat{x}'_{t+2|t}b_{t+2|t})^2|\Omega_t\right]$$

$$= \hat{x}'_{t+2|t}P_{b,t+2|t}\hat{x}_{t+2|t} + b'_{t+2|t}P_{x,t+2|t}b'_{t+2|t} + \sigma^2_{\epsilon,t+2|t}. \quad (22)$$

$P_{b,t+2|t} = \mathbf{E}_t\left[(b_{t+2} - b_{t+2|t})(b_{t+2} - b_{t+2|t})'|\Omega_t\right]$ can be computed as $P_{b,t+2|t} = P_{b,t+1|t} + \Sigma_{w,t+1|t}$. For the derivation of $P_{x,t+2|t}$ refer to Appendix C.

As expected, the results for (22) shown in Figure 10 demonstrate forecast uncertainty over two quarters to be generally higher than that over one quarter. The relative importance of residual uncertainty declines while uncertainty about future economic fundamentals becomes more important in explaining periods of high forecast uncertainty. Uncertainty about the Taylor rule coefficients is still the main cause for the increase in overall forecast uncertainty in the mid-1970s and around 1980. For the longer forecast horizon uncertainty about output gap and inflation forecasts is quantitatively more important than uncertainty about the future policy reaction function for most of the time after 1980.

« insert Figure 10 »

6   Conclusion

This paper has presented a simple model of monetary policy in the U.S. that separates the forecast uncertainty about future values of the Federal Funds Rate into uncertainty about the state of the economy in the future and uncertainty about how the central bank will react to it.

The results from real-time U.S. data indicate important time variation in the parameters of the policy rule as well as marked changes in the components of Federal Funds rate forecast uncertainty. In particular, uncertainty about the strength of the Fed's future responses to economic fundamentals changed drastically through time and was most pronounced in mid-1970s and from the late 1970s through the early 1980s. For a forecast horizon of one quarter, uncertainty about future output gaps and inflation rates has only a modest impact on Federal Funds Rate forecast uncertainty. For a

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longer forecast horizon its contribution to forecast uncertainty becomes relatively more important.

Results focusing on changes in the predictability of future output gaps and inflation and of the Fed's monetary policy show uncertainty about monetary policy falling to unprecedented low levels in the mid 1980s and remaining very low while uncertainty about economic fundamentals declined only temporarily in the late 1980s. This points to an important role of increased predictability of monetary policy in the U.S. in explaining the observed decline in macroeconomic volatility since the mid-1980s.

Appendix A: The State-space model for output and inflation

The observation equation (12) is

$$Y_t = \mu + H\tilde{x}_t + e_t, \tag{A1}$$

with

$$Y_t = \begin{bmatrix} \Delta y_t \\ \Delta \pi_t \end{bmatrix}, \mu = \begin{bmatrix} \mu_y \\ 0 \end{bmatrix}, e_t = \begin{bmatrix} e_t^n \\ 0 \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & \gamma & 1 & \delta_1 & \delta_2 & \delta_3 \end{bmatrix}$$

$$\mathbf{E}e_t e_t' = \Sigma_Y = \begin{bmatrix} \sigma_{e,n}^2 & 0 \\ 0 & 0 \end{bmatrix},$$

The transition equation for the state variables can be written as

$$\tilde{x}_{t+1} = F\tilde{x}_t + \zeta_{t+1}, \tag{A2}$$

with

$$\tilde{x}_t = \begin{bmatrix} z_t & z_{t-1} & \nu_t & \nu_{t-1} & \nu_{t-2} & \nu_{t-3} \end{bmatrix}',$$

$$\zeta_t = \begin{bmatrix} e_t^z & 0 & e_t^\nu & 0 & 0 & 0 \end{bmatrix}',$$

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$$F = \begin{bmatrix} \phi_1 & \phi_2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{E}\zeta_t\zeta_t' = \Sigma_\zeta = \begin{bmatrix} \sigma_{e,z}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{e,\nu}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The shocks e^ν, e^n and e^z are assumed to be serially and mutually uncorrelated.

Appendix B: The linearized state-space model for the Taylor rule

After replacing the expectations of the output gap and inflation rate with the model-generated forecasts the Taylor rule becomes

$$i_t = \beta_{0,t} + \beta_{\pi,t}\pi_{t+2|t-1} + \beta_{z,t}z_{t+2|t-1} + f(i_{t-1}, \beta_{\rho,t}) + \epsilon_t, \tag{B1}$$

with

$$f(i_{t-1}, \beta_{\rho,t}) = \frac{1}{1 + \exp(-\beta_{\rho,t})} i_{t-1} \equiv \rho_t i_{t-1},$$

and

$$\beta_{t+1} = \beta_t + w_{t+1}, \tag{B2}$$

where

$$\beta_t = (\beta_{0,t} \quad \beta_{\pi,t} \quad \beta_{z,t} \quad \beta_{\rho,t})'.$$

The Kalman filter is applied to a linearized version of (B1) (see Harvey et al. (1992)):
A linear Taylor approximation to (B1) around $\beta_{\rho,t} = \beta_{\rho,t|t-1}$ results in

$$
\begin{aligned}
i_t &= \beta_{0,t} + \beta_{\pi,t}\pi_{t+2|t-1} + \beta_{z,t}z_{t+2|t-1} + \frac{1}{1 + \exp(-\beta_{\rho,t|t-1})}i_{t-1} \\
&\quad + \frac{\exp(-\beta_{\rho,t|t-1})i_{t-1}}{(1 + \exp(-\beta_{\rho,t|t-1}))^2}(\beta_{\rho,t} - \beta_{\rho,t|t-1}) + \epsilon_t.
\end{aligned}
\tag{B3}
$$

This can be written as

$$\tilde{i}_t = \beta_{0,t} + \beta_{\pi,t}\pi_{t+2|t-1} + \beta_{z,t}z_{t+2|t-1} + \frac{\exp(-\beta_{\rho,t|t-1})i_{t-1}}{(1 + \exp(-\beta_{\rho,t|t-1}))^2}\beta_{\rho,t} + \epsilon_t, \tag{B4}$$

with

$$\tilde{i}_t = i_t - \frac{i_{t-1}}{1 + \exp(-\beta_{\rho,t|t-1})} + \frac{\exp(-\beta_{\rho,t|t-1})i_{t-1}}{(1 + \exp(-\beta_{\rho,t|t-1}))^2}\beta_{\rho,t|t-1}.$$

In each iteration of the Kalman filter there is now an additional step to compute $\tilde{i}$ using the estimate from the previous estimation $\tilde{\beta}_{\rho,t|t-1}$.

The modifications that result from the assumption of a GARCH process for the error term are as shown in Kim and Nelson (2006). The error term is included in the unobserved component. Thus

$$
\begin{aligned}
\tilde{i}_t &= \begin{bmatrix} 1 & \pi_{t+2|t} & z_{t+2|t} & \frac{\exp(-\beta_{\rho,t|t-1})i_{t-1}}{(1+\exp(-\beta_{\rho,t|t-1}))^2} & 1 \end{bmatrix} \begin{bmatrix} \beta_t \\ \epsilon_t \end{bmatrix} \tag{B5} \\
&= \tilde{x}_t'\tilde{\beta}_t, \tag{B6}
\end{aligned}
$$

and

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$$\tilde{\beta}_t = G\tilde{\beta}_{t-1} + \tilde{w}_t, \tag{B7}$$

where

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{B8}$$

$$\tilde{w}_t = \begin{bmatrix} w_t & \epsilon_t \end{bmatrix}, \tag{B9}$$

and

$$E\tilde{w}_t\tilde{w}_t' = \Sigma_{\tilde{w},t} = \begin{bmatrix} \Sigma_w & 0 \\ 0 & \sigma_{\epsilon,t}^2 \end{bmatrix}, \tag{B10}$$

$$\sigma_{\epsilon,t}^2 = \kappa_0 + \kappa_1\epsilon_{t-1}^2 + \kappa_2\sigma_{\epsilon,t-2}^2. \tag{B11}$$

The forecasting equations of the Kalman filter become

$$\tilde{\beta}_{t|t-1} = G\tilde{\beta}_{t-1|t-1}, \tag{B12}$$

$$P_{\tilde{\beta},t|t-1} = GP_{\tilde{\beta},t-1|t-1}G' + \Sigma_{\tilde{w},t}. \tag{B13}$$

After i_t is observed the estimates are updated as

$$\tilde{\beta}_{t|t} = \tilde{\beta}_{t|t-1} + P_{\tilde{\beta},t|t-1}\tilde{x}_t[\tilde{x}_t'P_{\tilde{\beta},t|t-1}\tilde{x}_t]^{-1}(\tilde{i}_t - \tilde{x}_t'\tilde{\beta}_{t|t-1}), \tag{B14}$$

$$P_{\tilde{\beta},t|t} = P_{\tilde{\beta},t|t-1} - P_{\tilde{\beta},t|t-1}\tilde{x}_t[\tilde{x}_t'P_{\tilde{\beta},t|t-1}\tilde{x}_t]^{-1}\tilde{x}_t'P_{\tilde{\beta},t|t-1}. \tag{B15}$$

The covariance matrix of the unobserved states $P_{\tilde{\beta}}$ is based on $\tilde{\beta} = (\beta_{0,t} \quad \beta_{\pi,t} \quad \beta_{z,t} \quad \beta_{\rho,t})'$. It can be transformed to the covariance matrix for $(\beta_{0,t} \quad \beta_{\pi,t} \quad \beta_{z,t} \quad \rho_t)'$ by use of the delta method.

Appendix C: Uncertainty measures

Uncertainty about economic conditions in the one-period case

Derivation of (19):

$$
\begin{aligned}
\mathbf{E}_t\left[(i_{t+1}-\hat{i}_{t+1|t})^2|\Omega_t\right] &= \mathbf{E}_t\left[(x'_{t+1}b_{t+1}-\hat{x}'_{t+1|t}b_{t+1|t})^2|\Omega_t\right] \\
&= \mathbf{E}_t\left[b'_{t+1}x_{t+1}x'_{t+1}b_{t+1}|\Omega_t\right]-b'_{t+1|t}\hat{x}_{t+1|t}\hat{x}'_{t+1|t}b_{t+1|t} \\
&\quad +\sigma^2_{\epsilon,t+1|t} \tag{C1}
\end{aligned}
$$

I use a Taylor-Approximation to write

$$
\begin{aligned}
\mathbf{E}\left[b'_{t+1}x_{t+1}x'_{t+1}b_{t+1}|\Omega_t\right] &\approx b'_{t+1|t}\hat{x}_{t+1|t}\hat{x}'_{t+1|t}b_{t+1|t} \tag{C2} \\
&\quad +\hat{x}'_{t+1|t}\mathbf{E}\left[(b_{t+1}-b_{t+1|t})(b_{t+1}-b_{t+1|t})'|\Omega_t\right]\hat{x}_{t+1|t} \\
&\quad +b'_{t+1|t}\mathbf{E}\left[(x_{t+1}-\hat{x}_{t+1|t})(x_{t+1}-\hat{x}_{t+1|t})'|\Omega_t\right]b_{t+1|t}.
\end{aligned}
$$

Substituting this expression into (C1) yields (19)

$$
\begin{aligned}
\mathbf{E}_t\left[(i_{t+1}-\hat{i}_{t+1|t})^2|\Omega_t\right] &= \hat{x}'_{t+1|t}\mathbf{E}_t\left[(b_{t+1}-b_{t+1|t})(b_{t+1}-b_{t+1|t})'|\Omega_t\right]\hat{x}_{t+1|t} \\
&\quad +b'_{t+1|t}\mathbf{E}_t\left[(x_{t+1}-\hat{x}_{t+1|t})(x_{t+1}-\hat{x}_{t+1|t})'|\Omega_t\right]b_{t+1|t} \\
&\quad +\sigma^2_{\epsilon,t+1|t} \\
&= \hat{x}'_{t+1|t}P_{b,t+1|t}\hat{x}'_{t+1|t}+b'_{t+1|t}P_{x,t+1|t}b_{t+1|t}+\sigma^2_{\epsilon,t+1|t}, \tag{C3}
\end{aligned}
$$

with $P_{b,t+1|t}=\mathbf{E}_t\left[(b_{t+1}-b_{t+1|t})(b_{t+1}-b_{t+1|t})'\right]$ and
$P_{x,t+1|t}=\mathbf{E}_t\left[(x_{t+1}-x_{t+1|t})(x_{t+1}-x_{t+1|t})'|\Omega_t\right]$.

Since $x_{t+1}=(1 \quad \pi_{t+3|t} \quad z_{t+3|t} \quad i_t)'$ and $\hat{x}_{t+1}=(1 \quad \pi_{t+3|t-1} \quad z_{t+3|t-1} \quad i_t)'$ I can write

$$P_{x,t+1|t} = \mathbf{E}_t\left[(x_{t+1} - \hat{x}_{t+1|t})(x_{t+1} - \hat{x}_{t+1|t})'|\Omega_t\right]$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & p_{\pi,\pi,t+1} & p_{\pi,z,t+1} & 0 \\ 0 & p_{\pi,z,t+1} & p_{z,z,t+1} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{C4}$$

where $p_{\pi,\pi,t+1} = \mathbf{E}\left[(\pi_{t+3|t} - \pi_{t+3|t-1})^2|\Omega_t\right]$, $p_{z,z,t+1} = \mathbf{E}\left[(z_{t+3|t} - z_{t+3|t-1})^2|\Omega_t\right]$, and $p_{\pi,z,t+1} = \mathbf{E}\left[(\pi_{t+3|t} - \pi_{t+3|t-1})(z_{t+3|t} - z_{t+3|t-1})|\Omega_t\right]$.

The individual elements can be derived as follows: The inflation forecast the central bank will react to in the next period is $\pi_{t+3|t} = \pi_{t-1} + \Delta\pi_t + \sum_{i=1}^{3}\Delta\pi_{t+i|t}$ while the forecast of this variable based on information dated $t-1$ is $\pi_{t+3|t-1} = \pi_{t-1} + \sum_{i=0}^{3}\Delta\pi_{t+i|t-1}$. Hence, using (A1) the forecast error is

$$
\begin{aligned}
\pi_{t+3|t} - \pi_{t+3|t-1} &= = (\Delta\pi_t - \Delta\pi_{t|t-1}) + \sum_{i=1}^{3}(\Delta\pi_{t+i|t} - \Delta\pi_{t+i|t-1}) \\
&= \mathbf{1}_2'\left[(Y_t - Y_{t|t-1}) + \sum_{i=1}^{3}(Y_{t+i|t} - Y_{t+i|t-1})\right] \\
&= \mathbf{1}_2'\left[H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t + \sum_{i=1}^{3}H(\tilde{x}_{t+i|t} - \tilde{x}_{t+i|t-1})\right] \tag{C5} \\
&= \mathbf{1}_2'\left[H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t + H(F + FF + FFF)(\tilde{x}_{t|t} - \tilde{x}_{t|t-1})\right],
\end{aligned}
$$

with $\mathbf{1}_2 = (0 \quad 1)'$.

At the time the policy rate in period t is announced, uncertainty about $\pi_{t+3|t}$, the estimate of inflation the central bank will react to in the next period stems from two sources: First, $(\Delta\pi_t - \Delta\pi_{t|t-1})$ is the error made in estimating the change in the inflation rate from the previous to the current period. Second, $\sum_{i=1}^{3}(\Delta\pi_{t+i|t} - \Delta\pi_{t+i|t-1})$ is the difference between the changes in inflation from period $t+1$ to $t+3$ as forecast by the central bank at the time it has to set i_{t+1} – and thus formed with knowledge of π_t – and the forecast of the changes in inflation made by the public in $t-1$ without knowing π_t.

Use of the Kalman-filter updating equations results in

$$
\begin{aligned}
\pi_{t+3|t} - \pi_{t+3|t-1} &= \mathbf{1}_2'\big[H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t + H(F + FF + FFF)K_{t|t-1}(H(\tilde{x}_t - \tilde{x}_{t|t-1}) \\
&\quad + e_t)\big] \\
&= \mathbf{1}_2'\big[H(I + (F + FF + FFF)K_{t|t-1}H)(\tilde{x}_t - \tilde{x}_{t|t-1}) \\
&\quad + (I + H(F + FF + FFF)K_{t|t-1})e_t\big],
\end{aligned}
\tag{C6}
$$

with $K_{t|t-1} = P_{\tilde{x},t|t-1}H'\big[HP_{\tilde{x},t|t-1}H' + \Sigma_Y\big]^{-1}$. This can be written as

$$
\pi_{t+3|t} - \pi_{t+3|t-1} = \mathbf{1}_2'\Big[D_{1,\tilde{x}}(\tilde{x}_t - \tilde{x}_{t|t-1}) + D_{1,e_t}e_t\Big].
\tag{C7}
$$

Using this expression the result for $p_{\pi,\pi,t+1}$ is

$$
\begin{aligned}
p_{\pi,\pi,t+1} &= \mathbf{E}\big[(\pi_{t+3|t} - \pi_{t+3|t-1})^2|\Omega_t\big] \\
&= \mathbf{1}_2'\Big[D_{1,\tilde{x}}P_{\tilde{x},t|t-1}D_{1,\tilde{x}}' + D_{1,e_t}\Sigma_Y D_{1,e_t}'\Big]\mathbf{1}_2.
\end{aligned}
\tag{C8}
$$

$z_{t+3|t}$ is the (1,1) element of $\tilde{x}_{t+3|t} = FFF\tilde{x}_{t|t}$, while $z_{t+3|t-1}$ is the (1,1) element of $\tilde{x}_{t+3|t-1} = FFF\tilde{x}_{t|t-1}$. Hence,

$$
\begin{aligned}
z_{t+3|t} - z_{t+3|t-1} &= \mathbf{1}_1'FFF(\tilde{x}_{t|t} - \tilde{x}_{t|t-1}) \\
&= \mathbf{1}_1'FFFK_{t|t-1}(H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t),
\end{aligned}
\tag{C9}
$$

where the last step makes use of the Kalman filter updating equation for $\tilde{x}$.

Defining

$$
z_{t+3|t} - z_{t+3|t-1} = \mathbf{1}_1'\Big[B_{1,\tilde{x}}(\tilde{x}_t - \tilde{x}_{t|t-1}) + B_{1,e_t}e_t\Big],
\tag{C10}
$$

with the respective coefficients shown in (C9) leads to

$$p_{z,z,t+1} = \mathbf{E}\left[(z_{t+3|t} - z_{t+3|t-1})^2|\Omega_t\right]$$

$$= \mathbf{1}_1'\mathbf{E}\left[(\tilde{x}_{t+1|t} - \tilde{x}_{t+1|t-1})(\tilde{x}_{t+1|t} - \tilde{x}_{t+1|t-1})'|\Omega_t\right]\mathbf{1}_1$$

$$= \mathbf{1}_1'\left[B_{1,\tilde{x}}P_{\tilde{x},t|t-1}B_{1,\tilde{x}}' + B_{1,e_t}\Sigma_Y B_{1,e_t}'\right]\mathbf{1}_1, \tag{C11}$$

with $\mathbf{1}_1 = (1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0)'$. Uncertainty about the central bank's forecast for the output gap is due to the fact that when policy is set next period additional information in form of observations of π_t and y_t will be available.

Finally, combining (C7) with (C10) yields

$$p_{\pi,z,t+1} = \mathbf{E}\left[(\pi_{t+3|t} - \pi_{t+3|t-1})(z_{t+3|t} - z_{t+3|t-1})|\Omega_t\right]$$

$$= \mathbf{1}_2'\left[D_{1,\tilde{x}}P_{\tilde{x},t|t-1}B_{1,\tilde{x}}' + D_{1,e_t}\Sigma_Y B_{1,e_t}'\right]\mathbf{1}_1. \tag{C12}$$

All these expressions can be evaluated using the parameter estimates from section 3 and the results from the Kalman filter algorithm applied to the model from Appendix A.

Uncertainty about economic conditions in the two-period case

(22) in the paper is derived analogous to (19). To evaluate (22) the following expression is required

$$P_{x,t+2|t} = \mathbf{E}_t\left[(x_{t+2} - \hat{x}_{t+2|t})(x_{t+2} - \hat{x}_{t+2|t})'|\Omega_t\right]$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & p_{\pi,\pi,t+2} & p_{\pi,z,t+2} & p_{\pi,i,t+2} \\ 0 & p_{\pi,z,t+2} & p_{z,z,t+2} & p_{i,z,t+2} \\ 0 & p_{\pi,i,t+2} & p_{i,z,t+2} & p_{i,i,t+2} \end{bmatrix}, \tag{C13}$$

where

$$
\begin{aligned}
p_{\pi,\pi,t+2} &= \mathbf{E}\left[(\pi_{t+4|t+1} - \pi_{t+4|t-1})^2 | \Omega_t\right], \\
p_{z,z,t+2} &= \mathbf{E}\left[(z_{t+4|t+1} - z_{t+4|t})^2 | \Omega_t\right], \\
p_{\pi,z,t+2} &= \mathbf{E}\left[(\pi_{t+4|t+1} - \pi_{t+4|t-1})(z_{t+4|t+1} - z_{t+4|t}) | \Omega_t\right], \\
p_{i,i,t+2} &= \mathbf{E}\left[(i_{t+1} - \hat{i}_{t+1|t})^2 | \Omega_t\right], \\
p_{\pi,i,t+2} &= \mathbf{E}\left[(\pi_{t+4|t+1} - \pi_{t+4|t-1})(i_{t+1} - \hat{i}_{t+1|t}) | \Omega_t\right], \\
p_{i,z,t+2} &= \mathbf{E}\left[(i_{t+1} - \hat{i}_{t+1|t})(z_{t+4|t+1} - z_{t+4|t}) | \Omega_t\right].
\end{aligned}
$$

The inflation forecast the central bank will react to two periods in the future is

$\pi_{t+4|t+1} = \pi_{t-1} + \Delta\pi_t + \Delta\pi_{t+1} + \sum_{i=2}^{4} \Delta\pi_{t+i|t+1}$,

while $\pi_{t+4|t-1} = \pi_{t-1} + \Delta\pi_{t|t-1} + \Delta\pi_{t+1|t-1} + \sum_{i=2}^{4} \Delta\pi_{t+i|t-1}$. Thus,

$$
\begin{aligned}
\pi_{t+4|t+1} - \pi_{t+4|t-1} &= (\Delta\pi_t - \Delta\pi_{t|t-1}) + (\Delta\pi_{t+1} - \Delta\pi_{t+1|t-1}) \\
&\quad + \sum_{i=2}^{4}(\Delta\pi_{t+i|t+1} - \Delta\pi_{t+i|t-1}) \\
&= \mathbf{1}_2'\left[(Y_t - Y_{t|t-1}) + (Y_{t+1} - Y_{t+1|t-1}) + \sum_{i=2}^{4}(Y_{t+2|t+1} - Y_{t+2|t-1})\right] \\
&= \mathbf{1}_2'\left[H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t + H(\tilde{x}_{t+1} - \tilde{x}_{t+1|t-1}) + e_{t+1} \right. \\
&\quad \left. + \sum_{i=2}^{4} H(\tilde{x}_{t+i|t+1} - \tilde{x}_{t+i|t-1})\right] \\
&= \mathbf{1}_2'\left[H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t + H(\tilde{x}_{t+1} - \tilde{x}_{t+1|t-1}) \right. \\
&\quad \left. + e_{t+1} + H(F + FF + FFF)(\tilde{x}_{t+1|t+1} - \tilde{x}_{t+1|t-1})\right] \\
&= \mathbf{1}_2'\left[H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t + H(\tilde{x}_{t+1} - \tilde{x}_{t+1|t-1}) + e_{t+1} \right. \\
&\quad + H(F + FF + FFF)(\tilde{x}_{t+1|t+1} - \tilde{x}_{t+1|t} \\
&\quad \left. + F(\tilde{x}_{t|t} - \tilde{x}_{t|t-1}))\right], \qquad\qquad\qquad\qquad\text{(C14)}
\end{aligned}
$$

with $\mathbf{1}_2 = (0 \quad 1)'$ and using the fact that $(Y_t - Y_{t|t-1}) = H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t$ Using (12) and (13) and the updating equations from the Kalman-filter algorithm leads to

$$\pi_{t+4|t+1} - \pi_{t+4|t-1} = \mathbf{1}_2'\big[H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t + H(F(\tilde{x}_t - \tilde{x}_{t|t-1}) + \zeta_{t+1})$$

$$+e_{t+1}$$

$$+H(F + FF + FFF)[K_{t+1|t}(H(\tilde{x}_{t+1} - \tilde{x}_{t+1|t}) + e_{t+1})$$

$$+FK_{t|t-1}(H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t))]\big]$$

$$\pi_{t+4|t+1} - \pi_{t+4|t-1} = \mathbf{1}_2'\bigg[H(I + F + (F + FF + FFF)[K_{t+1|t}(HF(I - K_{t|t-1}H))$$

$$+FK_{t|t-1}H])(\tilde{x}_t - \tilde{x}_{t|t-1})$$

$$+(I + H(F + FF + FFF)(FK_{t|t-1} - K_{t+1|t}HFK_{t|t-1}))e_t$$

$$+(I + H(F + FF + FFF)K_{t+1|t})e_{t+1} \qquad\qquad (\text{C15})$$

$$+H(I + (F + FF + FFF)K_{t+1|t}H)\zeta_{t+1}\bigg].$$

Define

$$\pi_{t+4|t+1} - \pi_{t+4|t-1} = \mathbf{1}_2'\bigg[D_{2,\tilde{x}}(\tilde{x}_t - \tilde{x}_{t|t-1}) + D_{2,e_t}e_t + D_{2,e_{t+1}}e_{t+1} \qquad (\text{C16})$$

$$+D_{2,\zeta}\zeta_{t+1}\bigg],$$

where the respective coefficients are shown in (C15). This leads to

$$p_{\pi,\pi,t+2} = \mathbf{E}\left[(\pi_{t+4|t+1} - \pi_{t+4|t-1})^2|\Omega_t\right] \qquad\qquad\qquad (\text{C17})$$

$$= \mathbf{1}_2'\bigg[D_{2,\tilde{x}}P_{\tilde{x},t|t-1}D_{2,\tilde{x}}' + D_{2,e_t}\Sigma_Y D_{2,e_t}' + D_{2,e_{t+1}}\Sigma_Y D_{2,e_{t+1}}' + D_{2,\zeta}\Sigma_\zeta D_{2,\zeta}'\bigg]\mathbf{1}_2.$$

$z_{t+4|t+1}$ is the (1,1) element of $\tilde{x}_{t+4|t+1} = FFF\tilde{x}_{t+1|t+1}$, while $z_{t+4|t-1}$ is the (1,1) element of $\tilde{x}_{t+4|t-1} = FFF\tilde{x}_{t+1|t-1}$. Hence,

$$z_{t+4|t+1} - z_{t+4|t-1} = \mathbf{1}_1' FFF(\tilde{x}_{t+1|t+1} - \tilde{x}_{t+1|t-1})$$

$$= \mathbf{1}_1' FFF(\tilde{x}_{t+1|t+1} - \tilde{x}_{t+1|t} + \tilde{x}_{t+1|t} - \tilde{x}_{t+1|t-1})$$

$$= \mathbf{1}_1' \Big[FFF[K_{t+1|t}HF(I - K_{t|t-1}H) + FK_{t|t-1}H](\tilde{x}_t - \tilde{x}_{t|t-1})$$

$$+ FFF[FK_{t|t-1} - K_{t+1|t}HFK_{t|t-1}]e_t$$

$$+ FFFK_{t+1|t}e_{t+1} + FFFK_{t+1|t}H\zeta_{z+1} \Big]. \tag{C18}$$

Define

$$z_{t+4|t+1} - z_{t+4|t-1} = \mathbf{1}_1' \Big[B_{2,\tilde{x}}(\tilde{x}_t - \tilde{x}_{t|t-1}) + B_{2,e_t}e_t + B_{2,e_{t+1}}e_{t+1} \tag{C19}$$

$$+ B_{2,\zeta}\zeta_{t+1} \Big],$$

with the respective coefficients shown in (C18). Hence

$$p_{z,z,t+2} = \mathbf{E}\left[(z_{t+4|t+1} - z_{t+4|t-1})^2 | \Omega_t \right] \tag{C20}$$

$$= \mathbf{1}_1' \mathbf{E}\left[B_{2,\tilde{x}} P_{\tilde{x},t|t-1} B_{2,\tilde{x}}' + B_{2,e_t}\Sigma_Y B_{2,e_t}' + B_{2,e_{t+1}}\Sigma_Y B_{2,e_{t+1}}' + B_{2,\zeta}\Sigma_\zeta B_{2,\zeta}' \right] \mathbf{1}_1.$$

From (C16) and (C19) it follows that

$$p_{\pi,z,t+2} = \mathbf{E}\left[(\pi_{t+4|t+1} - \pi_{t+4|t-1})(z_{t+4|t+1} - z_{t+4|t-1}) | \Omega_t \right] \tag{C21}$$

$$= \mathbf{1}_2' \Big[D_{2,\tilde{x}} P_{\tilde{x},t|t-1} B_{2,\tilde{x}}' + D_{2,e_t}\Sigma_Y B_{2,e_t}' + D_{2,e_{t+1}}\Sigma_Y B_{2,e_{t+1}}' + D_{2,\zeta}\Sigma_\zeta B_{2,\zeta}' \Big] \mathbf{1}_1.$$

Next are the correlations of the forecast errors for the output gap and inflation with the forecast error for the interest rate. The latter one is

$$i_{t+1} - \hat{i}_{t+1|t} = x_{t+1}'b_{t+1} - \hat{x}_{t+1|t}'b_{t+1|t} + \epsilon_{t+1}$$

$$= x_{t+1}'(b_t + v_{t+1}) - \hat{x}_{t+1|t}'b_{t|t} + \epsilon_{t+1}$$

$$= (x_{t+1} - \hat{x}_{t+1|t})'b_{t|t} + x_{t+1}'(b_t + v_{t+1} - b_{t|t}) + \epsilon_{t+1}, \tag{C22}$$

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with v_t being the vector of innovations to the Taylor rule coefficients. The first three elements of v are the first three innovations in w_t in (5) with the fourth being the innovation to ρ_t.

Since $x'_{t+1} = (1 \quad \pi_{t+3|t} \quad z_{t+3|t} \quad i_t)$ and $\hat{x}'_{t+3|t} = (1 \quad \pi_{t+3|t-1} \quad z_{t+3|t-1} \quad i_t)$ the above expression can be expanded to

$$
\begin{aligned}
i_{t+1} - \hat{i}_{t+1|t} = {} & (\pi_{t+3|t} - \pi_{t+3|t-1})\beta_{\pi,t|t} + (z_{t+3|t} - z_{t+3|t-1})\beta_{z,t|t} \\
& + (\beta_{0,t} - \beta_{0,t|t}) + \pi_{t+3|t}(\beta_{\pi,t} - \beta_{\pi,t|t}) \\
& + z_{t+3|t}(\beta_{z,t} - \beta_{z,t|t}) + i_t(\rho_t - \rho_{t|t}) \\
& + x'_{t+1}v_{t+1} + \epsilon_{t+1}.
\end{aligned}
\tag{C23}
$$

The inflation forecast made in period $t + 1$ is

$$
\begin{aligned}
\pi_{t+3|t} = {} & \pi_{t-1} + \Delta\pi_t + \sum_{i=1}^{3} \Delta\pi_{t+i|t} \\
= {} & \pi_{t-1} + 1'_2 \Big[4\mu + H(I - K_{t|t-1}H + (I + F + FF + FFF)K_{t|t-1}H) \\
& (\tilde{x}_t - \tilde{x}_{t|t-1}) + (I - HK_{t|t-1} + H(I + F + FF + FFF)K_{t|t-1})e_t \\
& + H(I + F + FF + FFF)\tilde{x}_{t|t-1} \Big],
\end{aligned}
\tag{C24}
$$

and $(\pi_{t+3|t} - \pi_{t+3|t-1})$ is shown in (C7).

$$
\begin{aligned}
z_{t+3|t} = {} & 1'_1 \tilde{x}_{t+3|t} \\
= {} & 1'_1 FFF\tilde{x}_{t|t} \\
= {} & 1'_1 FFF(\tilde{x}_{t|t-1} + K_{t|t-1}(H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t)) \\
= {} & 1'_1 \Big[FFFK_{t|t-1}H(\tilde{x}_t - \tilde{x}_{t|t-1}) + FFFK_{t|t-1}e_t + FFF\tilde{x}_{t|t-1} \Big],
\end{aligned}
\tag{C25}
$$

and $(z_{t+3|t} - z_{t+3|t-1})$ is shown in (C9).

Hence, using (C7), (C10), and (C23)-(C25)

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$$p_{\pi,i,t+2} = \mathbf{E}\left[(\pi_{t+4|t+1} - \pi_{t+4|t-1})(i_{t+1} - i_{t+1|t})|\Omega_t\right]$$

$$= \mathbf{1}_2'\left[D_{2,\tilde{x}}P_{\tilde{x},t|t-1}D_{1,\tilde{x}}'\beta_{\pi_{t|t}} + D_{2,e_t}\Sigma_Y D_{1,e_t}'\beta_{\pi_{t|t}}\right]\mathbf{1}_2, \qquad (\text{C26})$$

$$+\mathbf{1}_2'\left[D_{2,\tilde{x}}P_{\tilde{x},t|t-1}B_{1,\tilde{x}}'\beta_{z_{t|t}} + D_{2,e_t}\Sigma_Y B_{1,e_t}'\beta_{z_{t|t}}\right]\mathbf{1}_1,$$

and

$$p_{i,z,t+2} = \mathbf{E}\left[(z_{t+4|t+1} - z_{t+4|t-1})(i_{t+1} - i_{t+1|t})|\Omega_t\right]$$

$$= \mathbf{1}_1'\left[B_{2,\tilde{x}}P_{\tilde{x},t|t-1}D_{1,\tilde{x}}'\beta_{\pi_{t|t}} + B_{2,e_t}\Sigma_Y D_{1,e_t}'\beta_{\pi_{t|t}}\right]\mathbf{1}_2, \qquad (\text{C27})$$

$$+\mathbf{1}_1'\left[B_{2,\tilde{x}}P_{\tilde{x},t|t-1}B_{1,\tilde{x}}'\beta_{z_{t|t}} + B_{2,e_t}\Sigma_Y B_{1,e_t}'\beta_{z_{t|t}}\right]\mathbf{1}_1.$$

Finally, $p_{i,i} = \mathbf{E}\left[(i_{t+1|t} - \hat{i}_{t+1|t-1})^2|\Omega_t\right]$ is known from the one-step-ahead forecast uncertainty.

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Figure 1: Output gap estimates from historical and real time data
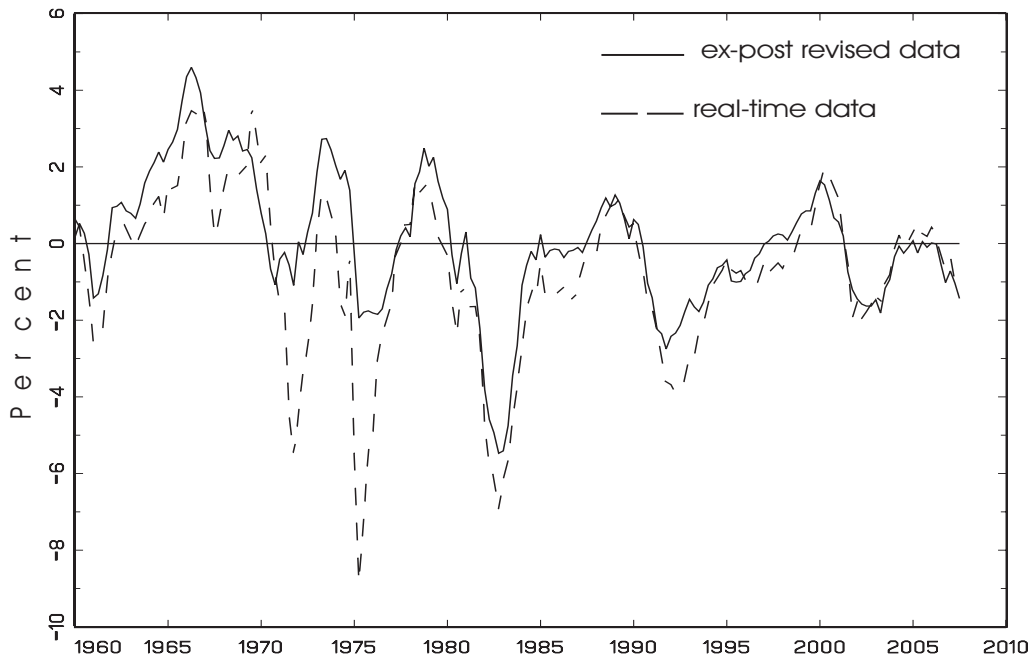
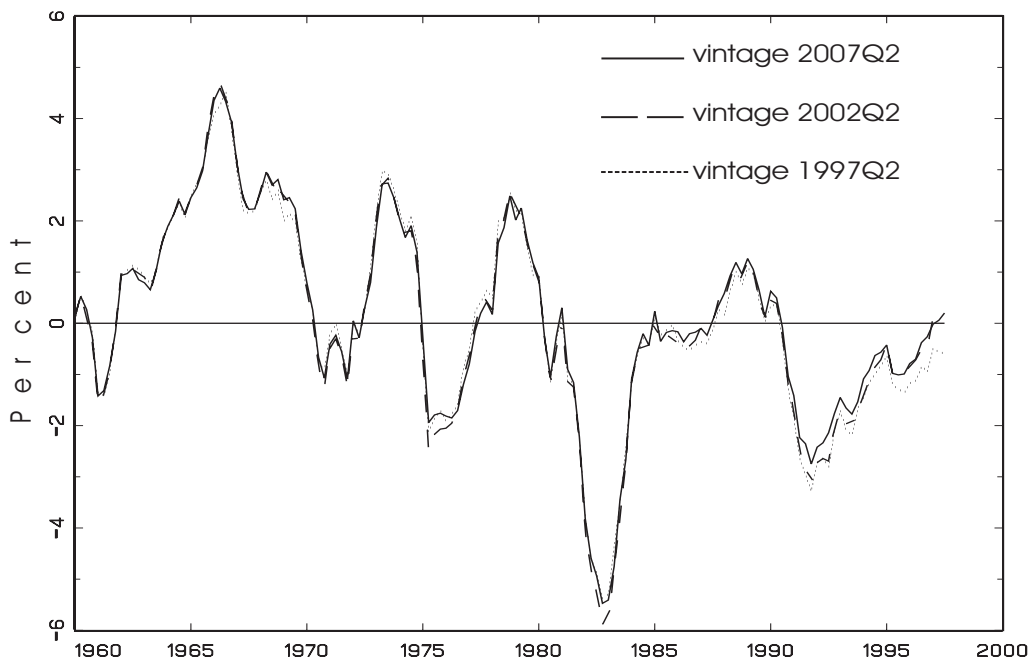Figure 2: Output gap estimates from different vintages of real time data

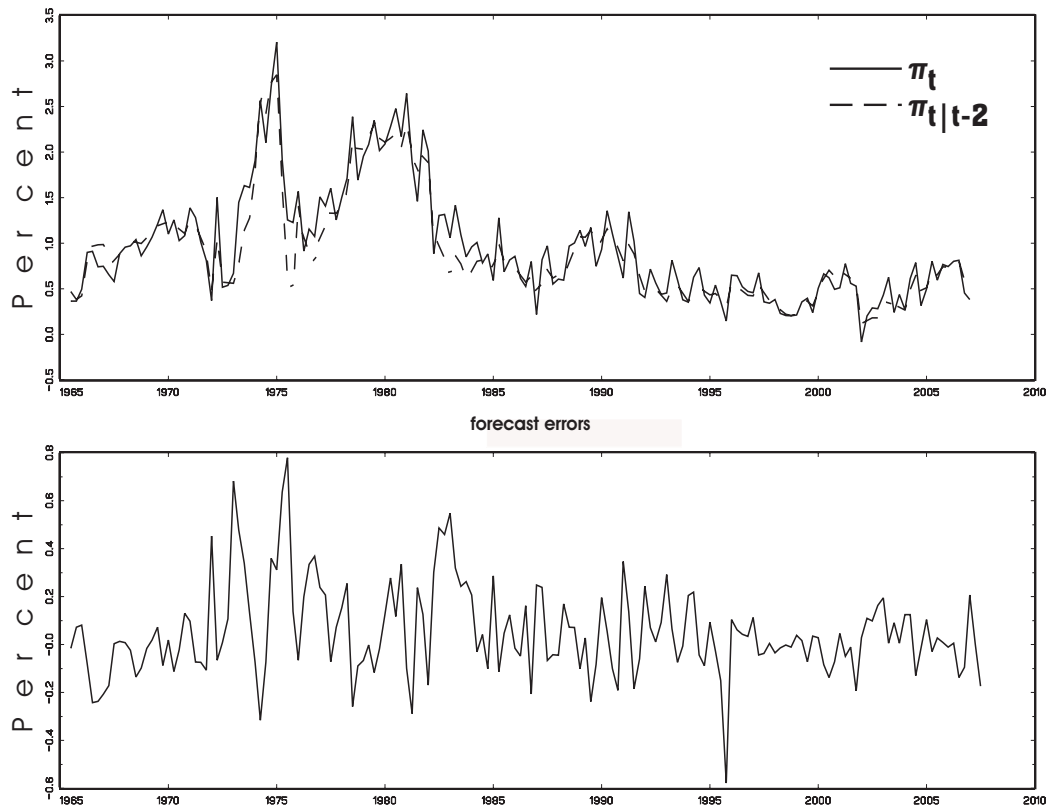Figure 3: Actual inflation and real-time inflation forecasts

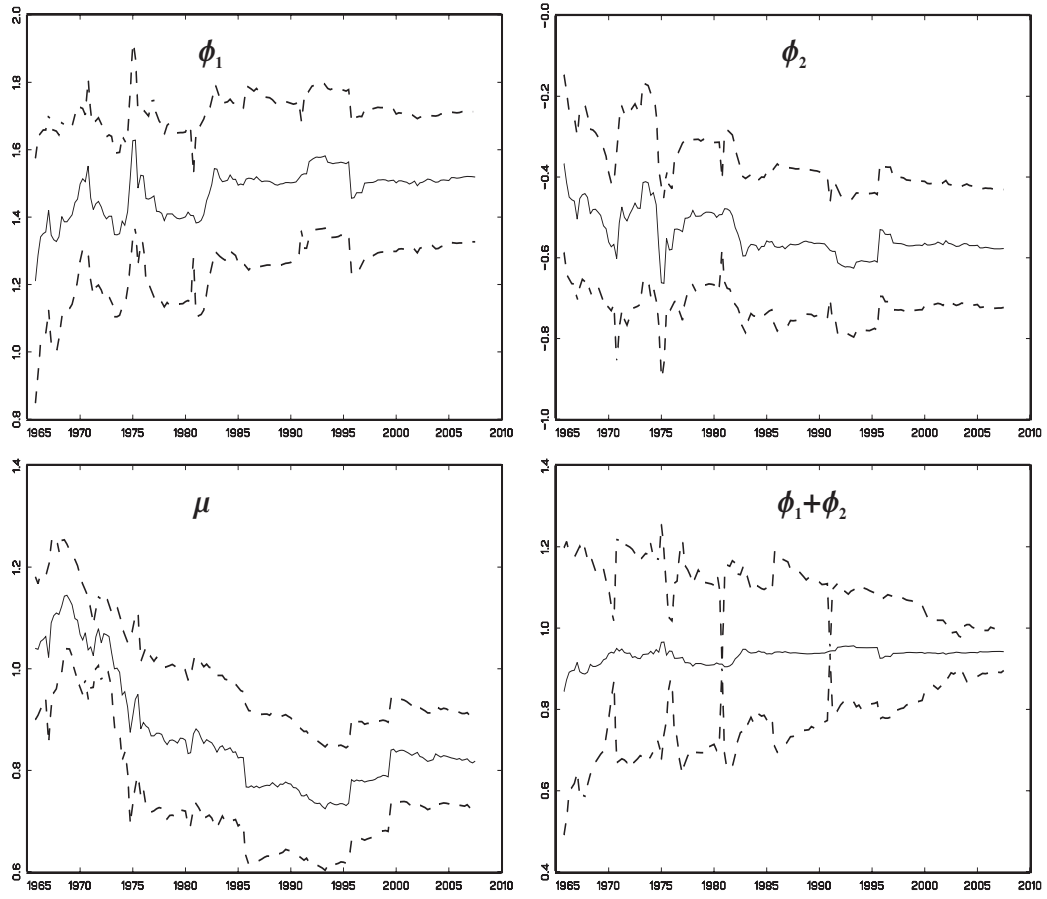Figure 4: Real-time estimates of output-inflation equation coefficients
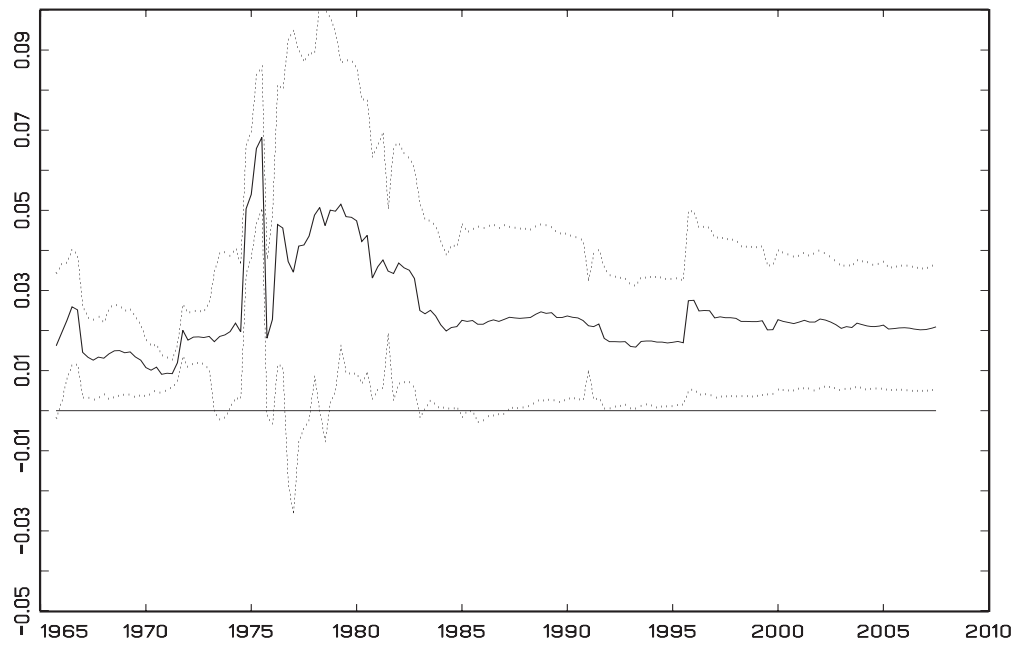

Figure 5: Real-time estimates of γ
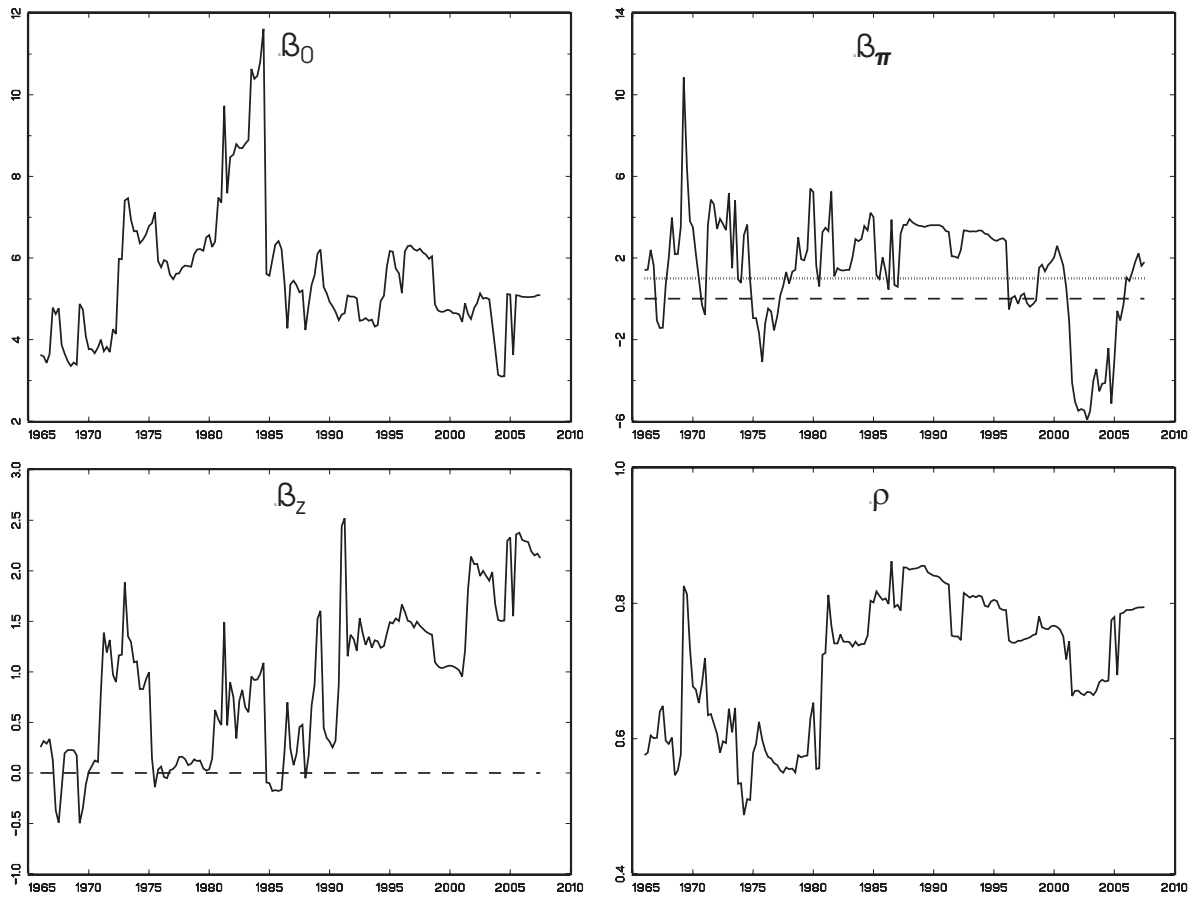
Figure 6: One-sided coefficient estimates

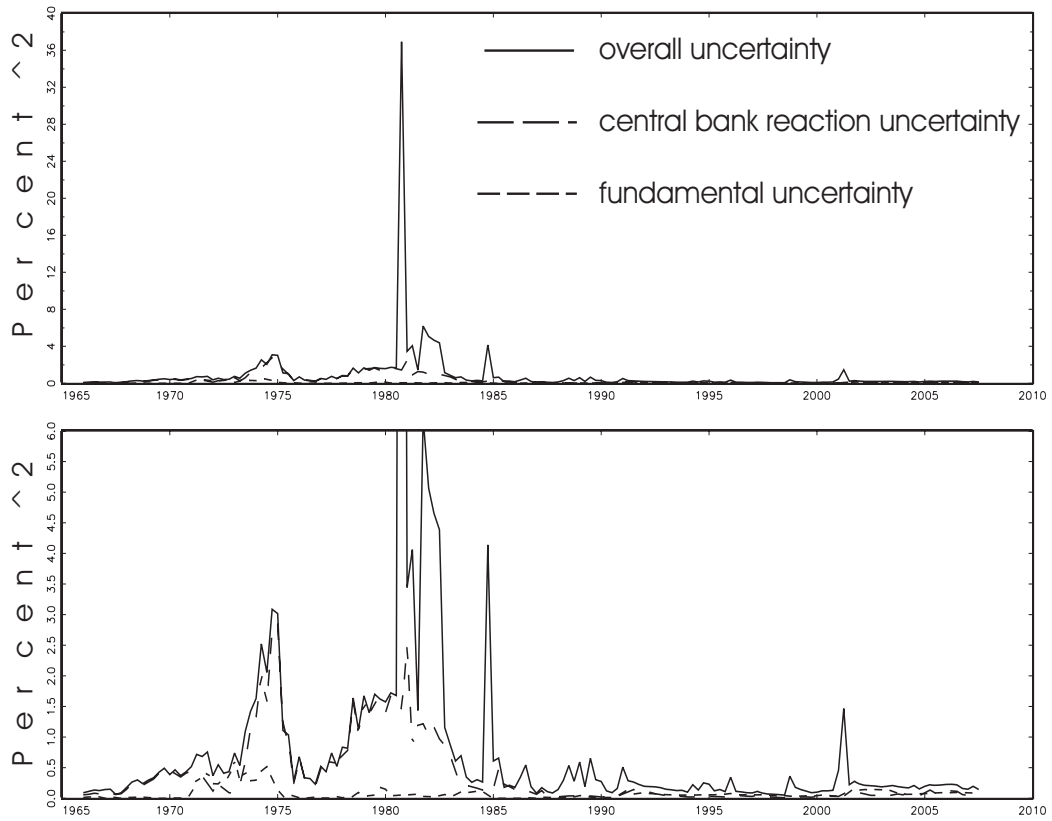Figure 7: One-quarter ahead forecast uncertainty for Federal Funds Rate

Figure 8: Federal Funds Rate, one-quarter ahead Federal Funds Rate forecasts and forecast errors
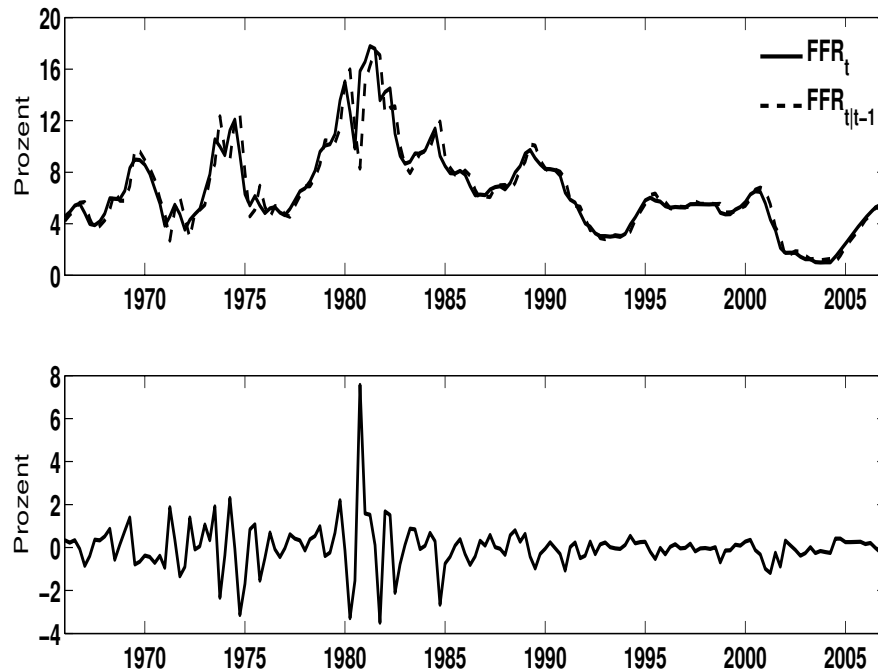

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Figure 9: Fundamental and monetary policy uncertainty

uncertainty about future fundamentals
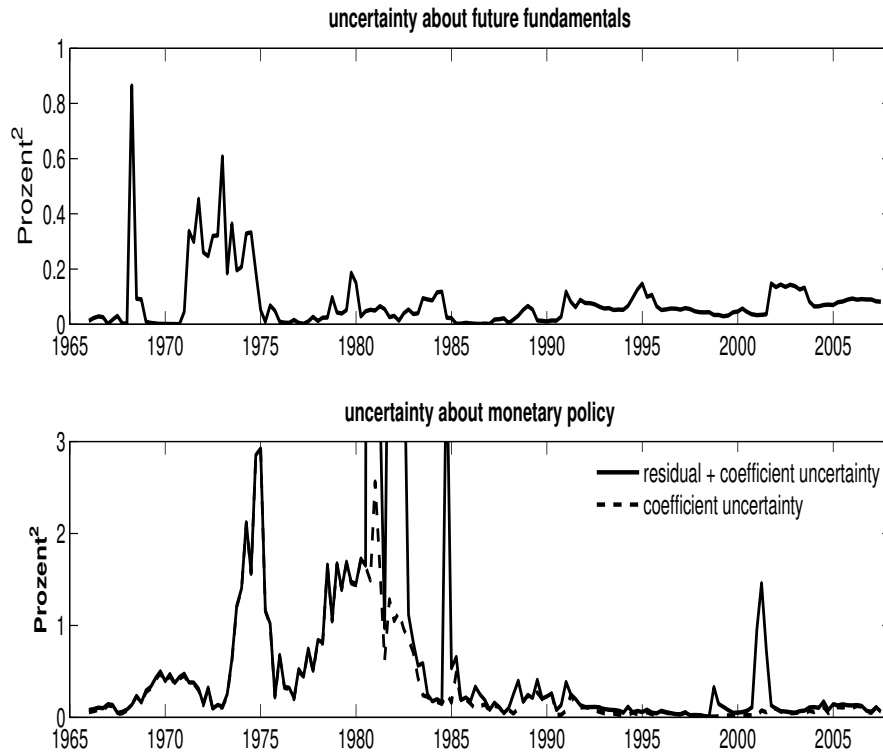
uncertainty about monetary policy

Figure 10: Two-quarter ahead forecast uncertainty for Federal Funds Rate