Lifetime Uncertainty and the Optimal Replacement Rate of urban Public Pension in China

Yang, Zaigui

December 2008

Lifetime Uncertainty and the Optimal Replacement Rate of Urban Public Pension in China

Zaigui Yang
School of Insurance, Central University of Finance and Economics, P.R.China
yangzaigui@hotmail.com

Abstract

By considering lifetime uncertainty, this paper employs an OLG model within general equilibrium framework to analyze China's urban public pension system. Using the condition for the steady-state of market economy to satisfy the social welfare maximization, we solve the optimal social pool benefit replacement rate. This optimal replacement rate depends on the population growth rate, survival probability in retirement period, capital share of income, individual discount rate and social discount rate. The simulations show that the optimal social pool benefit replacement rate rises with the life expectancy, whereas falls with the population growth rate. It should decrease when the life expectancy has risen and the population growth rate fallen because it is much more sensitive to the latter than the former.

1. Introduction

China reformed its urban public pension system in the beginning of 2006. The target replacement rate of pension benefits is adjusted. The social pool benefit replacement rate is raised from 20% to 35%, and the individual account benefit replacement rate is reduced from 38.5% to 24.2%. What should be the optimal level of replacement rate, what are the determinant elements, and how should the optimal replacement rate be calculated?

There are two phenomena tightly related with public pension in China: The population growth rate has fallen, and the life expectancy has risen. Pecchenino and Pollard[1], Pecchenino and Utendorf[2], Pecchenino and Pollard[3], Zhang *et al.*[4], etc. have used overlapping generations (OLG) model with lifetime uncertainty to study pay-as-you-go or fully funded public pension systems.

Based on the literature, this paper investigates the partially funded public pension system in China, and explores the optimal benefit replacement rates.

2. The model

A closed economy is composed of numerous individuals and firms and a government. The generation born at the beginning of period t is called generation t. The population grows at the rate of $n=(N_t/N_{t-1})-1$, where N_t is the population of generation t.

2.1. Individuals

Each individual survives to the end of her working period certainly. Her survival probability in the retirement period is $p\in[0,1]$. In the working period, each individual earns wage by supplying inelastically one unit of labor and makes pension contributions. She consumes part of her incomes and saves the rest. If she survives in her retirement period, then she consumes her savings with accrued interest, individual account benefits and social pool benefits. If she dies at the beginning of her retirement period, then her savings with accrued interest and funded pension benefits are inherited equally by her children as unintentional bequests.

Each individual derives utility from her working-period consumption c_{1t} and possible retirement-period consumption c_{2t+1}. The utility is described by an additively separable logarithmic function. Thus, each individual solves the following maximization problem:

$$\max_{\{c_{1t},c_{2t+1},s_t\}} U_t=\ln c_{1t}+\theta p\ln c_{2t+1} \quad (1)$$

$$s.t.\quad c_{1t}=(1-p)b_t+(1-\tau)w_t-s_t \quad (2)$$

$$c_{2t+1}=(1+r_{t+1})s_t+B_{t+1}+P_{t+1} \quad (3)$$

$$(1+n)b_{t+1}=(1+r_{t+1})s_t+B_{t+1} \quad (4)$$

where $\theta\in(0,1)$ denotes the individual discount rate, w_t the wage, τ the individual contribution rate, s_t the savings, r_{t+1} the interest rate, B_{t+1} the individual account benefits, P_{t+1} the social pool benefits and b_{t+1} the unintentional bequests inherited by each child.

The first-order condition for the utility maximization problem is

$$-c_{2t+1} + \theta p(1 + r_{t+1})c_{1t} = 0 \quad (5)$$

This familiar expression implies that the utility loss from reducing one unit of working-period consumption is equal to the utility gain from increasing ($1+r_{t+1}$) units of possible retirement-period consumption discounted by θ.

2.2. Firms

Firms produce homogenous commodity in competitive markets. The production is described by Cobb-Douglas function $Y_t = AK_t^{\alpha} N_t^{1-\alpha}$ or $y_t = Ak_t^{\alpha}$, where Y_t is the output in period t, K_t the capital stock, $\alpha \in (0,1)$ the capital share of income, A the productivity, $k_t=K_t/N_t$ the capital-labor ratio, and y_t the output-labor ratio.

Firms make pension contributions at the rate of $\eta \in (0,1)$ on their payroll. According to the product distribution, one can get $AK_t^{\alpha} N_t^{1-\alpha} = r_t K_t + (1+\eta) w_t N_t$. The first-order conditions for the profit maximization are

$$r_t = \alpha A k_t^{\alpha-1} \quad (6)$$

$$w_t = (1-\alpha)Ak_t^{\alpha} / (1+\eta) \quad (7)$$

2.3. The government

The social pool fund is paid to the retirees in the current period as pay-as-you-go pension benefits: $pN_{t-1}P_t = \eta w_t N_t$. Using the concept of social pool benefit replacement rate ξ gives

$$P_t = \xi w_t = (1 + n)\eta w_t / p \quad (8)$$

thus

$$\eta = p\xi / (1 + n) \quad (9)$$

The accumulation in the individual account is used to pay the individual when she retires in the next period as funded pension benefits. Using the concept of individual account benefit replacement rate μ gives

$$B_{t+1} = \mu w_{t+1} = (1 + r_{t+1})\tau w_t \quad (10)$$

thus

$$\tau = [\mu / (1 + r_{t+1})] \cdot (w_{t+1} / w_t) \quad (11)$$

2.4. Dynamic Equilibrium System

The savings and the individual pension contributions in period t generate the capital stock in period $t+1$ (See Blanchard and Fischer[5] or Barro and Sala-I-Martin[6] for details):

$$s_t + \tau w_t = (1 + n)k_{t+1} \quad (12)$$

Combining equations (2)-(12) gives a dynamic equilibrium system described by the following difference equation:

$$-\left(k_{t+1} + \alpha A k_{t+1}^{\alpha}\right) - \frac{\xi}{1+n+p\xi}(1-\alpha)Ak_{t+1}^{\alpha} + \theta p\left(1 + \alpha A k_{t+1}^{\alpha-1}\right)\left[\frac{1-p}{1+n}\left(k_t + \alpha A k_t^{\alpha}\right) + \frac{1}{1+n+p\xi}(1-\alpha)Ak_t^{\alpha} - k_{t+1}\right] = 0 \quad (13)$$

The social pool benefit replacement rate has effect on the capital-labor ratio because it appears in the dynamic system. However, the individual account benefit replacement rate has no effect on the capital-labor ratio because the mandatory savings (individual pension contributions) crowds out the voluntary savings by one-for-one.

3. Social optimum

The social welfare is the sum of the lifetime utilities of all current and future generations (Blanchard and Fischer[5] and Groezen *et al.*[7] also use an analogous social welfare function):

$$W = \theta p \ln c_{20} + \sum_{i=0}^{\infty} \rho^i \left(\ln c_{1i} + \theta p \ln c_{2i+1}\right) \quad (14)$$

where $\rho \in (0,1)$ is the social discount rate, which reflects the preference of the social planner. The resource constraint is

$$k_i + Ak_i^{\alpha} = (1+n)k_{i+1} + c_{1i} + p\,c_{2i}/(1+n) \quad (15)$$

The initial condition is that k_0 is given.

The social planner maximizes the social welfare subject to the resource constraint and initial condition. The first-order conditions for this maximization problem are:

$$\theta(1+n)c_1^* = \rho c_2^* \quad (16)$$

$$k^* = [(1+n-\rho)/\rho\alpha A]^{\frac{1}{\alpha-1}} \quad (17)$$

where the superscript * denotes the optimal steady state values of variables. The capital-labor ratio satisfying equation (17) is at the modified golden rule level, which means that the social welfare achieves the maximum.

In order to maximize the social welfare of the market economy in the steady state, we control the policy variable to adjust the capital-labor ratio of the market economy in steady state to the modified golden rule level, namely, $k = k^*$. Substituting equation (17) into equation (13) and arranging gives

$$\xi^* = \frac{\theta p(1-\alpha)(1+n-\rho)-\alpha(1+n)[\rho-\theta p(1-p-\rho)]}{p\alpha[\rho-\theta p(1-p-\rho)]+\rho(1-\alpha)(1+n-\rho)/(1+n)} \quad (18)$$

4. Optimal social pool benefit replacement rate

Differentiating ξ^* with respective to p and n gives that the signs of the derivatives are not determinate. This implies that the effects of the life expectancy and population growth rate on the optimal social pool benefit replacement rate are ambiguous, which can be checked by simulating.

4.1. Estimation of parameter values

The capital share of income, α, is usually to be estimated as 0.3 in developed countries (e.g., Pecchenino and Pollard[3] and Zhang *et al.*[4]). The labor in China is comparatively cheaper, and thus the labor share of income is lower, while the capital share of income is higher than that in developed countries. Hence, we assume that α in China could be 0.35.

Suppose that a period is 26 years length. There are two reasons for this: One is that the length is usually in the interval of 25-30 years in the literature on OLG model. The other is the structure of the data in China. Assume that the individual discount rate per year is 0.98, which is similar to that used by Pecchenino and Utendorf[2]. Hence, the individual discount rate per period is $\theta = 0.98^{26}$.

There are several calibers for population statistics in China. Since the public pension system in urban area is different from that in rural area, and only the former is studied in this paper, so the caliber of "Urban Population" is selected. The population growth rate during the period 1978-2004 is computed at $n\approx2.148$ according to the "Population and Its Composition" in China Statistical Yearbook.

The survival probability in retirement period is estimated by the life expectancy. According to UN Secretariat[8], the life expectancy of Chinese people in 2000-2005 is 72.0 years old. Since one period length is 26 years, the life-span from birth to the end of working-period is 52 years. The life-span from birth to the end of retirement period is 78 years. According to the concept of life expectancy, one can get $(1-p)\times52+p\times78=72.0$, which gives $p\approx76.92\%$. Although the choice of period length is arbitrary, it has to obey the following rule: Three times of the period length should be longer than or equal to the life expectancy to ensure $p \leq 1$.

The social discount rate indicates how much the government weights different generations in its social welfare calculations. It should be estimated according to the government's regulations. The target replacement rate of social pool benefits is 35%, which is the optimal social pool benefit replacement rate thought by the government. Substituting the above related parameter values into equation (18) and calculating repeatedly until the equation holds, we get $\rho\approx0.4739$. These estimated values are baseline values.

4.2. Risen life expectancy

Risen life expectancy means that the survival probability in retirement period increases. According to the prediction of UN Secretariat[8], the life expectancy of Chinese people in 2005-2010 is 73.0 years old, and that in 2010-2015 is 74.0 years old. Hence, the survival probabilities in retirement period are computed to be 80.77% and 84.62%, respectively. When the survival probability is 80.77%, substituting the baseline values of the other parameters into equation (18) gives ξ^*=36.86%. Analogous simulation gives that ξ^* is 38.08% when the survival probability is 84.62%. Shown as Table 1, the optimal social pool benefit replacement rate rises with the life expectancy.

Table 1. ξ^* under different life expectancies

p	76.92%	80.77%	84.62%
ξ^*	35.00%	36.86%	38.08%

4.3. Fallen population growth rate

We predict the urban population in each year from 2007 to 2015 with the sample of the urban population in each year from 1978 to 2006. Using the TREND function in Excel gives the result shown in Appendix A. The R^2 value is 0.9725, which implies a very good fit. Computing the population growth rates yields that the rate during period 1984-2010 is 1.500, and that during period 1989-2015 is 1.272. Simulating with the baseline values and the estimated population growth rates gives the results shown in Table 2. The optimal social pool benefit replacement rate falls with the population growth rate.

Table 2. ξ^* under different population growth rates

n	2.148	1.500	1.272
ξ^*	35.00%	21.53%	16.67%

4.4. Risen life expectancy and fallen population growth rate

ρ, θ and α are at baseline values. When the survival probability in retirement period is 80.77% (corresponding to 2005-2010), simulating with the population growth rates, 1.500 and 1.272, respectively, gives the optimal social pool benefit replacement rates, 22.93% and 17.90%. When the survival probability in retirement period is 84.62% (corresponding to 2010-2015), simulating analogously gives the result shown in Table 3. The optimal social pool benefit replacement rate falls when the life expectancy has risen and the population growth rate has fallen.

Table 3. ξ^* under different life expectancies and population growth rates

p	n	ξ^*
80.77%	1.500	22.93%
80.77%	1.272	17.90%
84.62%	1.500	23.82%
84.62%	1.272	18.68%

Computing the elasticity of ξ^* with respect to p and n gives the result shown in Table 4. The elasticity of ξ^* with respect to n is much higher than that of ξ^* with respect to p. This implies that the optimal social pool benefit replacement rate is much more sensitive to the population growth rate than life expectancy.

Table 4. Elasticities of ξ^* with respect to p and n

ξ^* to p	ξ^* to n
0.815	1.442
0.913	1.419

5. Conclusions

Taking lifetime uncertainty into account, this paper employs the OLG model with general equilibrium to analyze the urban public pension in China. It finds the optimal social pool benefit replacement rate by comparing the market economy in steady state with the social optimum. The optimal social pool benefit replacement rate is shown to depend on the individual discount rate, survival probability in retirement period, capital share of income, population growth rate and social discount rate. Simulations show the following results: The optimal social pool benefit replacement rate rises with the life expectancy, while falls with the population growth rate. It should fall when the life expectancy has risen and the population growth rate fallen because it is much more sensitive to the population growth rate than life expectancy.

Appendix A

Predicted urban population in each year from 2007 to 2015 (10000 persons)

Year	2007	2008	2009	2010	2011
Population	55794	57208	58621	60035	61448
Year	2012	2013	2014	2015	
Population	62862	64276	65689	67103	

Acknowledgement

Financial support from the Social Science Foundation of the Ministry of Education of China (No. 06JA630079) is gratefully acknowledged.

References

[1] Pecchenino, R. and P. Pollard, "The effects of annuities, bequests, and aging in an overlapping generations model of endogenous growth", *The Economic Journal* 107, 1997, pp. 26-46.

[2] Pecchenino, R., and K. Utendorf, "Social security, social welfare and the aging population", *Journal of Population Economics* 12, 1999, pp. 607-623.

[3] Pecchenino, R. and P. Pollard, "Dependent children and aged parents: funding education and social security in an aging economy", *Journal of Macroeconomics* 24, 2002, pp. 145-169.

[4] Zhang, J., J. Zhang and R. Lee, "Mortality decline and long-run economic growth", *Journal of Public Economics* 80, 2001, pp. 485-507.

[5] Blanchard, O.J. and S. Fischer, *Lectures on Macroeconomics*, London: MIT Press, 1989.

[6] Barro, R. J. and X. Sala-I-Martin, *Economic Growth*, Cambridge: MIT Press, 2004.

[7] Groezen, B., T. Leers and L. Meijdam, "Social security and endogenous fertility: pensions and child allowances as siamese twins", *Journal of Public Economics* 87, 2003, pp. 233-251.

[8] UN Secretariat, "World Population Prospects: the 2006 Revision", 2007, PLACE: the Population Division of the Department of Economic and Social Affairs of the UN Secretariat [http://earthtrends.wri.org/text/population-health/variable-379.html] (accessed on April 1, 2008).