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Abstract

In the US, defense R&D share of GDP has decreased significantly since 1960. To analyze the implications on growth and welfare, we develop an R&D-based growth model that features the *crowding-out* and *spillover* effects of defense R&D on civilian R&D. The model also captures the effects of defense technology on (i) national security resembling consumption-type public goods and (ii) aggregate productivity via the *spin-off* effect resembling productive public goods. In this framework, economic growth is driven by market-based civilian R&D as in standard R&D-based growth models and government-financed public goods (i.e., defense R&D) as in Barro (1990). We find that defense R&D has an inverted-U effect on growth, and the growth-maximizing level of defense R&D is increasing in the spillover and spin-off effects. As for the welfare-maximizing level of defense R&D, it is increasing in the *security-enhancing* effect of defense technology, and there exists a critical degree of this security-enhancing effect below (above) which the welfare-maximizing level is below (above) the growth-maximizing level.

Keywords: defense R&D, public goods, economic growth, social welfare

JEL classification: H41, H56, O38, O40

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1. Introduction

In the US, defense R&D as a percentage of GDP decreased from 1.29% in 1961 to 0.57% in 2008 (see Figure 1). This phenomenon has led to a number of studies on the effects of defense R&D on economic growth. For example, a recent empirical study by Goel *et al.* (2008) finds that defense R&D has a positive and significant effect on growth in the US. While it is interesting to analyze the growth effects of defense R&D, it is also important to consider its welfare effects given that growth maximization may not be equivalent to welfare maximization. To shed some light on this issue, we develop an R&D-based growth model to explore the different channels through which defense R&D affects growth and welfare.

Specifically, we develop an R&D-based growth model that captures the commonly discussed *crowding-out* and *spillover* effects of defense R&D on civilian R&D.[1] The crowding-out effect refers to the case in which an increase in defense R&D reduces the factor inputs available for civilian R&D and hence has a negative effect on the growth of civilian technology. For example, Hartley (2006) notes that "[d]efence R&D has obvious opportunity costs through the use of scarce scientific personnel and assets that could be used on civilian research." Also, Gullec and van Pottelsberghe (2003) analyze a group of OECD countries and find that defense R&D indeed has a crowding-out effect on civilian R&D. The spillover effect refers to the case in which defense R&D contributes to the performance of civilian R&D and hence has a positive effect on growth. For example, Chakrabarti and Anyanwu (1993) find that defense R&D has an indirect positive effect on the growth rate of civilian output in the US through technological change for which the number of patents serves as a proxy, and they discuss how this empirical finding supports the presence of a spillover effect from defense R&D to the civilian economy.

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[1] See, for example, Cowan and Foray (1995) and Dunne and Braddon (2008) for a discussion.

In addition to the crowding-out and spillover effects of defense R&D, our model also features two important effects of defense technology (which is accumulated by investment in defense R&D). Firstly, higher defense technology improves national security and increases the utility of households resembling consumption-type public goods. For example, Hartley (2006) argues that "[d]efense R&D increases a nation's military capability so improving its national security through using technology (quality) rather than increasing the quantity of arms." Secondly, defense technology improves aggregate productivity, resembling growth-enhancing public goods, through the development of general-purpose technologies (GPTs) that have civil applications. This effect is referred to as the *spin-off* effect in the defense literature. For example, Ruttan (2006) argues that research in defense has played an important role in the development of some major GPTs, such as (i) interchangeable parts and mass production, (ii) military and commercial aircraft, (iii) nuclear energy and electric power, (iv) computers and semiconductors, and (v) the internet.

In our theoretical framework, economic growth is driven by market-based civilian R&D as in standard R&D-based growth models and government-financed public goods (i.e., defense R&D) as in Barro (1990). We find that a reduction in defense R&D leads to contrasting effects on the growth of output and consumption. In particular, starting at a high (low) level of defense R&D, reducing defense R&D has a positive (negative) effect on growth. Therefore, there exists a growth-maximizing level of defense R&D that is increasing in the spillover and spin-off effects. As for the welfare-maximizing level of defense R&D, it is increasing in the *security-enhancing* effect of defense technology, and there is a critical degree of this security-enhancing effect below (above) which the welfare-maximizing level is below (above) the growth-maximizing level.

This study contributes to the literature on defense and economic growth by providing a tractable growth-theoretic framework that formalizes the commonly discussed crowding-out and spillover effects of defense R&D. To our knowledge, our study is the first attempt to model defense R&D within an R&D-based growth model. Previous studies analyze the dynamic effects of defense spending either in an endowment economy or in a capital-accumulation-driven growth model.[2] For example, Shieh *et al.* (2002) perform a similar growth-welfare analysis on defense spending in an AK growth model and find that the welfare-maximizing level of defense spending is always above the growth-maximizing level. We derive a similar result under a special case of our growth model in which the production sector only employs unskilled labor. However, when the production sector also uses skilled labor, there is an additional crowding-out effect of defense R&D on the production of final goods. Consequently, whether the welfare-maximizing level of defense R&D is above or below the growth-maximizing level depends on the relative magnitude of this crowding-out effect versus the security-enhancing effect of defense technology.

Our study also relates to the literature on productive government spending and economic growth initiated by Barro (1990). While Barro (1990) models public inputs as a flow variable, our study follows the formulation in Futagami *et al.* (1993) to model public inputs as a stock variable.[3] The resulting spin-off effect of defense technology leads to an extra channel of growth via productive public goods in addition to market-based civilian R&D. Furthermore, we consider the security-enhancing effect of defense technology resembling consumption-type public goods as in Turnovsky (1996) and Shieh *et al.* (2002), among others.

The theoretical implications of our study rationalize the results of previous empirical studies that find ambiguous growth effects of defense spending/R&D. In an early study, Benoit

[2] See, for example, Brito (1972), Deger and Sen (1983, 1984), Zou (1995), Chang *et al.* (1996, 2002), Shieh *et al.* (2002, 2007) and Aizenman and Glick (2006).
[3] See Irmen and Kuehnel (2009) for a recent survey of this literature.

(1973) finds that defense spending has a positive effect on growth in developing countries. However, upon surveying the follow-up studies, Ram (1995) concludes that defense spending has opposing effects on growth and the overall effect is ambiguous.[4] Similarly, Lichtenberg (1995) finds that defense R&D has opposing effects on growth and the net effect is ambiguous. In contrast, Goel *et al.* (2008) finds that defense R&D has a positive and significant effect on growth in the US, and surprisingly, this effect is even stronger than private industrial R&D.

The rest of this study is organized as follows. Section 2 presents the model. Section 3 defines the equilibrium and analyzes the balanced-growth path. Section 4 examines the growth and welfare effects of defense R&D. The final section concludes with policy implications.

2. A quality-ladder growth model with defense R&D

We incorporate defense R&D into a modified version of the Grossman-Helpman (1991) quality-ladder model, which is a workhorse model in the R&D-based growth literature. There are four effects of defense R&D in the model. Firstly, it has a positive spillover effect on civilian R&D. Secondly, it has a crowding-out effect on factor inputs for civilian R&D and production. Thirdly, defense technology improves national security and has a positive effect on households' welfare. Finally, defense technology improves aggregate productivity through the spin-off effect. It is worth noting that defense R&D is a flow variable while defense technology is a stock variable. Although the quality-ladder model features only labor inputs, it is appropriate for our analysis because R&D scientists and engineers are the crucial inputs for innovation in civilian and defense technologies. Given that the quality-ladder model has been well-studied, the familiar

[4] For example, Macnair *et al.* (1995), Brumm (1997) and Murdoch *et al.* (1997) find a positive relationship between defense spending and growth as in Benoit (1973) while Deger and Smith (1983), Faini *et al.* (1984) and Deger (1986) find a negative relationship. Also, some studies, such as Biswas and Ram (1986) and Huand and Mintz (1990, 1991), find an insignificant effect of defense spending on growth.

components of the model will be briefly described to conserve space while the new features will be described in more details below.

2.1. Households

There is a unit continuum of identical households, who have a standard log utility function.

$$U=\int_{0}^{\infty} e^{-\rho t}(\ln c_t+\delta \ln d_t)dt\,, \tag{1}$$

where $\rho > 0$ is the discount rate, and c_t is the consumption of final goods. d_t is the level of defense technology, and its law of motion is $\dot{d}_t = d_t f(h_{d,t})$ given an initial d_0 that is normalized to unity. The accumulation of defense technology is driven by defense R&D labor denoted by $h_{d,t}$ (to be discussed in more details later). As mentioned above, we follow previous studies to assume that households derive utility from national security for which the level of defense technology d_t serves as a proxy.[5] In other words, defense technology resembles consumption-type public goods, and the parameter $\delta \geq 0$ determines the degree of this *security-enhancing* effect of defense technology.

Households maximize utility subject to a sequence of budget constraints given by

$$\dot{v}_t = r_t v_t + w_{h,t} + w_{l,t} l - c_t - \tau_t\,. \tag{2}$$

v_t is the value of assets owned by households, and r_t is the real rate of return. Each household is endowed with one unit of high-skill labor (for production, civilian R&D, and defense R&D) and l units of low-skill labor for production only. The market wage rates for high-skill and low-skill

[5] This formulation captures Hartley's (2006) argument (quoted in the introduction) that higher defense technology improves a country's ability in defending its national interests against the threat of foreign rivals and hence increases the utility of households.

labors are $w_{h,t}$ and $w_{l,t}$ respectively. The government levies a lump-sum tax τ_t to finance defense R&D.[6] From utility maximization, the familiar Euler equation is $\dot{c}_t / c_t = r_t - \rho$.

2.2. Final goods

Final goods y_t are produced by a standard Cobb-Douglas aggregator over a unit continuum of differentiated intermediates goods $x_t(i)$ indexed by $i \in [0,1]$.

$$y_t = \exp\left(\int_0^1 \ln x_t(i) di \right). \tag{3}$$

This sector is perfectly competitive, and the producers take the output price and the input prices $p_t(i)$ for $i \in [0,1]$ as given. From profit maximization, the conditional demand function for $x_t(i)$ is $x_t(i) = y_t / p_t(i)$ for $i \in [0,1]$.

2.3. Intermediate goods

There is a unit continuum of industries $i \in [0,1]$ producing the differentiated intermediate goods. In each industry *i*, there is a temporary monopolistic leader, who holds a patent on the latest innovation and dominates the market until the next innovation occurs. The production function for the leader in industry *i* is

$$x_t(i) = z^{n_t(i)} d_t^{\alpha} h_{x,t}^{\theta}(i) l_{x,t}^{1-\theta}(i), \tag{4}$$

where $h_{x,t}(i)$ and $l_{x,t}(i)$ are respectively high-skill and low-skill production labors in industry *i*. The parameter $\theta \in (0,1)$ determines the intensity of high-skill labor in production, and we will

[6] We focus on a lump-sum tax to highlight the crowding-out effect of defense R&D. In the case of distortionary taxes, increasing defense R&D would naturally lead to other distortionary effects on the economy in addition to the crowding-out effect.

show that θ captures the crowding-out effect of defense R&D on consumption. As discussed in Ruttan (2006), defense technology d_t facilitates the development of GPTs and improves aggregate productivity as growth-enhancing public goods. The parameter $\alpha \in (0,1)$ captures the degree of this *spin-off* effect of defense technology. As for technological progress from civilian R&D, $z > 1$ is the exogenous step size of each quality improvement, and $n_t(i)$ is the number of quality improvements that have occurred in industry *i* as of time *t*.

From cost minimization, the marginal cost of production for the leader in industry *i* is

$$mc_t(i) = \frac{1}{z^{n_t(i)} d_t^{\alpha}} \left(\frac{w_{h,t}}{\theta} \right)^{\theta} \left(\frac{w_{l,t}}{1-\theta} \right)^{1-\theta}. \tag{5}$$

As is standard in the literature, the current and former leaders engage in Bertrand competition,[7] and the profit-maximizing price for the current leader is a constant markup (given by the quality step size z) over the marginal cost (i.e., $p_t(i) = z\, mc_t(i)$).[8] As a result, the amount of flow profit in industry $i \in [0,1]$ is

$$\pi_{x,t}(i) = (z-1) mc_t(i) x_t(i) = \left(\frac{z-1}{z} \right) y_t, \tag{6}$$

where the second equality makes use of the conditional demand function $x_t(i) = y_t / p_t(i)$.

2.4. Civilian R&D

Denote the value of an invention in industry *i* as $\tilde{v}_t(i)$. From (6), the amount of profit is the same across industries. As a result, $\tilde{v}_t(i) = \tilde{v}_t$ in a symmetric equilibrium (i.e., an equal arrival rate of

[7] Grossman and Helpman (1991) show that the next innovation must come from another innovator due to the Arrow displacement effect.
[8] Li (2001) considers a CES production function. In this case, the monopolistic markup is determined by either the quality step size or the elasticity of substitution depending on whether innovations are drastic or non-drastic.

innovation across industries).[9] Because inventions are the only assets in the economy, their market value equals the aggregate value of households' assets (i.e., $\tilde{v}_t = v_t$). The no-arbitrage condition for v_t is

(7) $$r_t v_t = \pi_{x,t} + \dot{v}_t - \lambda_t v_t .$$

The left-hand side of (7) is the return on this asset. The right-hand side of (7) is the sum of (i) the profit $\pi_{x,t}$ generated by this asset, (ii) the potential capital gain $\dot{v}_t$, and (iii) the expected capital loss due to creative destruction $\lambda_t v_t$, where λ_t is the aggregate Poisson arrival rate of innovation.

There is a unit continuum of R&D entrepreneurs indexed by $j \in [0,1]$, and they hire high-skill labor to create inventions. The expected profit for R&D entrepreneur *j* is

(8) $$\pi_{r,t}(j) = v_t \lambda_t(j) - w_{h,t} h_{r,t}(j) .$$

$h_{r,t}(j)$ is the number of civilian R&D workers hired by entrepreneur *j*, and the arrival rate of innovation for entrepreneur *j* is $\lambda_t(j) = \overline{\varphi}_t h_{r,t}(j)$, where $\overline{\varphi}_t$ is the productivity of civilian R&D. Free entry leads to zero expected profit in the R&D sector such that

(9) $$v_t \overline{\varphi}_t = w_{h,t} .$$

This condition determines the allocation of high-skill labor between production and civilian R&D. To formalize the *spillover* effect of defense R&D, $\overline{\varphi}_t$ is modeled as an increasing function in defense R&D $h_{d,t}$.[10] For analytical tractability, we consider the following functional form

(10) $$\overline{\varphi}_t = \varphi h_{d,t}^{\phi} .$$

[9] We follow the standard approach in the literature to focus on the symmetric equilibrium. See Cozzi et al. (2007) for a theoretical justification for the symmetric equilibrium in quality-ladder models.
[10] For example, James (2004, p. 37-38) argues that defense R&D spending provides a crucial source of seed funding for civilian technology companies.

This functional form is tractable because the spillover effects of defense R&D is captured by a single parameter $\phi \in (0,1)$. When ϕ equals zero, the R&D sector reduces to the setup in the Grossman-Helpman model in which the productivity of civilian R&D is determined by φ.

2.5. Defense R&D

Government invests in defense R&D to improve defense technology according to

(11) $$\dot{d}_t = d_t f(h_{d,t}).$$

$g_{d,t} \equiv \dot{d}_t / d_t$ denotes the growth rate of defense technology, and the function $f(.)$ satisfies the following regularity conditions $f(0)=0$, $f'>0$ for $h_{d,t} \in [0,1)$, $f'(1)=0$ and $f'' \leq 0$. The government's balanced-budget condition is

(12) $$\tau_t = w_{h,t} h_{d,t}.$$

This setup can be interpreted as the case in which defense R&D is performed by the government and (11) is the government's production function of defense technology. Alternatively, (12) can be viewed as cost-reimbursement contracts with defense firms. In this case, $f(.)$ is also affected by the incentives of defense firms in doing efficient R&D.[11] Under either interpretation, a higher level of defense R&D increases tax burden and reduces available high-skill labor for production and civilian R&D (i.e., the *crowding-out* effect of defense R&D).

[11] See, for example, Rogerson (1995) for a discussion on defense firms' incentives under cost-based contracts.

3. Decentralized equilibrium

This section firstly defines the equilibrium and then characterizes the balanced-growth path. The equilibrium is a sequence of allocations $\{c_t, y_t, x_t(i), l_{x,t}(i), h_{x,t}(i), h_{r,t}(j), h_{d,t}\}_{t=0}^{\infty}$, a sequence of prices $\{w_{l,t}, w_{h,t}, r_t, v_t, p_t(i)\}_{t=0}^{\infty}$ and a sequence of tax policies $\{\tau_t\}_{t=0}^{\infty}$. Also, at each instant of time,

a. households choose $\{c_t\}$ to maximize utility subject to (2) taking $\{w_{l,t}, w_{h,t}, r_t, \tau_t\}$ as given;

b. competitive final-goods firms produce $\{y_t\}$ to maximize profit taking $\{p_t(i)\}$ as given;

c. the leader in industry i produces $\{x_t(i)\}$ and chooses $\{p_t(i), l_{x,t}(i), h_{x,t}(i)\}$ to maximize profit according to the Bertrand competition and taking $\{w_{l,t}, w_{h,t}\}$ as given;

d. R&D entrepreneur j chooses $\{h_{r,t}(j)\}$ to maximize profit taking $\{w_{h,t}, v_t\}$ as given;

e. the market for final goods clears such that $c_t = y_t$;

f. the market for high-skill labor clears such that $h_{x,t} + h_{r,t} + h_{d,t} = 1$;

g. the market for low-skill labor clears such that $l_{x,t} = l$;

h. the government balances its budget constraint such that $\tau_t = w_{h,t} h_{d,t}$.

3.1. Balanced-growth path

As in Grossman and Helpman (1991), the dynamics of the model is characterized by saddle-point stability, so that the economy jumps to a unique and stable balanced-growth path.

Lemma 1: *Given a constant h_d, the economy is on a unique and stable balanced-growth path.*

Proof: See Appendix A.□

Given that the economy is on a balanced-growth path, we next derive the stationary equilibrium allocation of civilian R&D labor for a given h_d. In summary, we use the zero-profit condition from the R&D sector and the resource constraint for high-skill labor to solve for h_r.

Lemma 2: *The equilibrium allocation of civilian R&D labor is stationary and given by*

$$h_r = (1-h_d)\left(\frac{z-1}{z-1+\theta}\right) - \frac{\rho}{\varphi h_d^{\phi}}\left(\frac{\theta}{z-1+\theta}\right). \tag{13}$$

Proof: See Appendix A.□

The properties of h_r are quite intuitive. A larger markup z increases the amount of monopolistic profit and the incentives for civilian R&D; therefore, h_r is increasing in z. A larger discount rate decreases the present value of an invention; therefore, h_r is decreasing in ρ. A larger θ increases the usage of high-skill labor in production; therefore, h_r is decreasing in θ. A larger φ increases R&D productivity and the incentives for civilian R&D; therefore, h_r is increasing in φ for a given h_d. As for defense R&D, it has contrasting effects on civilian R&D. On one hand, a larger h_d reduces the supply of high-skill labor in the market (i.e., the crowding-out effect), and this is a negative effect on civilian R&D. On the other hand, a larger h_d raises R&D productivity $\overline{\varphi} = \varphi h_d^{\phi}$ (i.e., the spillover effect), and this is a positive effect on civilian R&D. We impose a lower bound on φ to ensure that h_r is positive.

Condition R (R&D productivity): $\varphi > \dfrac{\rho\theta}{h_d^{\phi}(1-h_d)(z-1)}$.

Substituting (4) into (3) yields the aggregate production function $y_t = Z_t d_t^{\alpha} h_x^{\theta} l_x^{1-\theta}$, where the aggregate level of civilian technology is defined as

(14) $$Z_t \equiv \exp\left(\int_0^1 n_t(i) di \ln z\right) = \exp\left(\int_0^t \lambda_s ds \ln z\right).$$

The second equality of (14) is obtained by using the law of large numbers. From (14), the growth rate of civilian technology is

(15) $$g_z \equiv \dot{Z}_t / Z_t = \lambda \ln z,$$

where $\lambda = \overline{\varphi} h_r$ is the aggregate arrival rate of innovation. Although the innovation process of each R&D entrepreneur is stochastic, the idiosyncratic uncertainty washes out at the aggregate level, and aggregate technology increases at a constant rate along the balanced-growth path.

4. Growth and welfare effects of defense R&D

Before analyzing the growth and welfare effects of defense R&D, we first establish a connection between defense R&D labor h_d in the model and defense R&D share of GDP that we observe in the data.

Lemma 3: *Defense R&D share of GDP is monotonically increasing in* h_d.

Proof: See Appendix A.□

Next, we consider how h_d affects growth and welfare. Let's denote the balanced-growth rate of consumption and output by $g_c \equiv \dot{c}_t / c_t = \dot{y}_t / y_t$. From the aggregate production function,

(16) $$g_c = g_z + \alpha\, g_d = \left(\frac{\varphi h_d^{\phi}(1-h_d)(z-1) - \rho\theta}{z-1+\theta}\right) \ln z + \alpha\, f(h_d).$$

In this model, economic growth is driven by g_z (i.e., market-based civilian R&D as in standard R&D-based growth models) and g_d (i.e., government-financed public goods as in Barro (1990)). Suppose we suppress the spillover effect (i.e., $\phi = 0$) and the spinoff effect (i.e., $\alpha = 0$). Then, (16) shows that h_d only has a negative effect on growth through crowding out civilian R&D. When we allow for either a positive spillover effect (i.e., $\phi > 0$) or a positive spinoff effect (i.e., $\alpha > 0$), (16) shows that h_d has a countervailing positive effect on growth.

Proposition 1: *There exists a growth-maximizing level of defense R&D h_d^* that is increasing in ϕ (the spillover effect of defense R&D) and α (the spin-off effect of defense technology). A decrease in defense R&D has a positive (negative) effect on growth if h_d is above (below) h_d^*.*

Proof: See Appendix A.□

Proposition 1 suggests that the reduction in defense R&D in the US should have ambiguous effects on growth. These ambiguous effects arise from the opposing forces of the crowding-out effect on civilian R&D versus the spillover and spin-off effects. Therefore, the growth-maximizing level of defense R&D is increasing in ϕ (the spillover effect) and α (the spin-off effect). Figure 2 plots the growth rate of consumption/output as a function of h_d.

[Insert Figure 2 here]

We next evaluate the effects of defense R&D on social welfare. Imposing the balanced-growth condition on (1) simplifies the lifetime utility of households to

$$(17)\qquad U = \frac{1}{\rho}\left(\ln c_0 + \frac{g_c}{\rho}\right) + \frac{\delta}{\rho}\left(\ln d_0 + \frac{g_d}{\rho}\right) = \frac{1}{\rho}\left((1-\theta)\ln l_x + \theta \ln h_x + \frac{g_z + \alpha g_d}{\rho}\right) + \frac{\delta}{\rho}\left(\frac{g_d}{\rho}\right),$$

where the second equality is obtained by dropping the exogenous Z_0 and d_0. Differentiating (17) with respect to h_d yields

$$\rho\frac{\partial U}{\partial h_d}=\frac{\theta}{h_x}\left(\frac{\partial h_x}{\partial h_d}\right)+\frac{1}{\rho}\left(\frac{\partial g_z}{\partial h_d}+\alpha\frac{\partial g_d}{\partial h_d}\right)+\frac{\delta}{\rho}\left(\frac{\partial g_d}{\partial h_d}\right). \tag{18}$$

The last term in (18) captures the (positive) security-enhancing effect of defense technology on welfare. The first term in (18) captures the (negative) crowding-out effect of defense R&D on production that leads to a lower initial consumption $c_0=(1-\theta)\ln l_x+\theta\ln h_x$. Combining (13) and the resource constraint for high-skill labor yields

$$h_x=1-h_d-\left((1-h_d)\left(\frac{z-1}{z-1+\theta}\right)-\frac{\rho}{\varphi h_d^{\phi}}\left(\frac{\theta}{z-1+\theta}\right)\right)=\left(\frac{\theta}{z-1+\theta}\right)\left(1-h_d+\frac{\rho}{\varphi h_d^{\phi}}\right), \tag{19}$$

which is decreasing in h_d.

Suppose we suppress the security-enhancing effect (i.e., $\delta=0$) and the crowding-out effect on production (i.e., $\theta=0$). In this case, (18) shows that the growth-maximizing level of defense R&D is equivalent to the welfare-maximizing level. Suppose we allow $\theta>0$ while keeping $\delta=0$. In this case, defense R&D has a crowding-out effect on initial consumption. As a result, the welfare-maximizing level of defense R&D is below the growth-maximizing level. Suppose we allow $\delta>0$ while keeping $\theta=0$. In this case, defense R&D has a positive effect on nation security. As a result, the welfare-maximizing level of defense R&D is above the growth-maximizing level. This special result resembles the one in Shieh *et al.* (2002). In general, when both the security-enhancing effect and the crowding-out effect on consumption are present, the welfare-maximizing level of defense R&D can be above or below the growth-maximizing level.

To derive the cutoff value $\bar{\delta}$ at which the growth-maximizing and welfare-maximizing levels of defense R&D coincide, we firstly set (18) equal zero to derive the first-order condition for welfare maximization. Then, we substitute the growth-maximizing h_d^* into this condition to set $\partial g_z / \partial h_d + \alpha \partial g_d / \partial h_d$ equal to zero. Finally, we rearrange terms to obtain the threshold

$$\bar{\delta} \equiv -\frac{\partial h_x}{\partial h_d}\left(\frac{\rho\theta}{h_x f'}\right)\Bigg|_{h_d = h_d^*} > 0. \tag{20}$$

Proposition 2: *The welfare-maximizing level of defense R&D is increasing in* δ*. Also, there exists a critical value* $\bar{\delta}$ *below (above) which the welfare-maximizing level is below (above) the growth-maximizing level. As* θ *approaches zero, the welfare-maximizing level of defense R&D is above the growth-maximizing level for any value of* $\delta > 0$*.*

Proof: See Appendix A.□

Proposition 2 shows that there is a welfare-maximizing level of defense R&D denoted by h_d^{**} that is increasing in δ, and there exists a critical value $\bar{\delta}$ below (above) which h_d^{**} is below (above) the growth-maximizing level h_d^* as illustrated in Figure 3. Intuitively, in addition to the growth effects, defense R&D has two additional effects on welfare (a) a negative effect on initial consumption and (b) a positive effect on national security. If national security is not very important to households (i.e., $\delta < \bar{\delta}$), then (a) dominates (b) such that the welfare-maximizing level of defense R&D is below the growth-maximizing level. Otherwise, (b) dominates (a) such that the opposite is true.

[Insert Figure 3 here]

5. Conclusion

This paper develops an R&D-based growth model to analyze the effects of defense R&D on growth and welfare. We find that the growth effect of defense R&D follows an inverted-U shape reflecting the opposing forces of the crowding-out effect on civilian R&D versus the spillover and spin-off effects. Also, whether or not the reduction in defense R&D in the US should have improved economic growth (social welfare) depends on the level of defense R&D in the economy relative to its growth-maximizing (welfare-maximizing) level, which in turn is determined by the spillover and spin-off effects (the security-enhancing effect). We also find that the welfare-maximizing level of defense R&D can be above or below the growth-maximizing level. These theoretical results imply that even if defense R&D contributes to growth as suggested by recent empirical evidence, reducing defense R&D can still be consistent with either a positive or negative effect on social welfare. This finding suggests the importance of investigating beyond the growth effects when policymakers perform a cost-benefit analysis on defense R&D. Finally, the canonical quality-ladder model is a first-generation R&D-based growth model that exhibits scale effects (i.e., a larger economy experiences a higher growth rate). In this study, we set aside the issue of scale effects by normalizing the supply of skilled labor to unity.[12]

[12] See Jones (1999) for an excellent discussion on scale effects in R&D-based growth model.

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Appendix A

Proof of Lemma 1: Given a constant h_d (and hence a constant $\overline{\varphi}$), the resource constraint for high-skill labor is $1-h_d = h_{x,t} + h_{r,t}$. The high-skill-labor share of final goods is $w_{h,t}h_{x,t} = \theta c_t / z$. The arrival rate of innovation is $\lambda_t = \overline{\varphi}\, h_{r,t}$. The zero-profit condition for R&D is $v_t\overline{\varphi} = w_{h,t}$. Substituting these conditions into $1-h_d = h_{x,t} + h_{r,t}$ yields

(A1) $$\lambda_t = \overline{\varphi}(1-h_d) - \theta\xi_t / z,$$

where $\xi_t \equiv c_t / v_t$ is a transformed variable. The law of motion for ξ_t is

(A2) $$\frac{\dot{\xi}_t}{\xi_t} = \frac{\dot{c}_t}{c_t} - \frac{\dot{v}_t}{v_t} = \frac{\pi_{x,t}}{v_t} - \lambda_t - \rho,$$

where the second equality of (A2) uses (7) and the Euler equation. Substituting the profit share of final goods $\pi_{x,t} = c_t(z-1)/z$ and (A1) into (A2) yields

(A3) $$\frac{\dot{\xi}_t}{\xi_t} = \left(\frac{z-1+\theta}{z}\right)\xi_t - \overline{\varphi}(1-h_d) - \rho.$$

The phase diagram for this simple differential equation is plotted in Figure 4. Figure 4 shows that ξ_t must jump to its non-zero steady state given by $\xi^* = (\overline{\varphi}(1-h_d)+\rho)z/(z-1+\theta)$. Otherwise, it would violate households' utility maximization or firms' profit maximization.□

Proof of Lemma 2: Imposing the balanced-growth condition on (7) yields

(A4) $$v_t = \pi_{x,t} / (\rho + \lambda).$$

Substituting (A4), $\pi_{x,t} = c_t(z-1)/z$ and $\lambda = \overline{\varphi}\, h_r$ into ξ^* yields (13).□

Proof of Lemma 3: In the model, defense R&D share of GDP is $w_h h_d /(y + w_h h_d)$. Substituting $y = w_l l_x + w_h h_x + \pi_x$ into this expression yields $w_h h_d /(y + w_h h_d) = w_h h_d /(w_l l_x + w_h h_x + \pi_x + w_h h_d)$, where $\pi_x = w_h h_r (\rho + \lambda)/\lambda$ because $\lambda \pi_x /(\rho + \lambda) = \lambda v = w_h h_r$ from (A4) and (9). Furthermore, using $w_l l_x /(1-\theta) = w_h h_x / \theta$, we have $w_h h_d /(y + w_h h_d) = w_h h_d /(w_h h_x (1-\theta)/\theta + w_h + w_h h_r (\rho / \lambda))$. Finally, $w_h h_d /(y + w_h h_d) = h_d /(h_x (1-\theta)/\theta + 1 + \rho / \overline{\varphi})$ is increasing in h_d.□

Proof of Proposition 1: Recall that $\phi \in (0,1)$. Differentiating (16) with respect to h_d yields

(A5) $$\frac{\partial g_c}{\partial h_d} = h_d^{\phi} \left(\frac{\phi(1-h_d)}{h_d} - 1 \right) \left(\frac{(z-1)\varphi \ln z}{z-1+\theta} \right) + \alpha f'(h_d) = 0,$$

(A6) $$\frac{\partial^2 g_c}{\partial h_d^2} = -\frac{\phi}{h_d^{1-\phi}} \left((1-\phi) \left(\frac{1-h_d}{h_d} \right) + 2 \right) \left(\frac{(z-1)\varphi \ln z}{z-1+\theta} \right) + \alpha f''(h_d) < 0.$$

(A6) shows that g_c is strictly concave in h_d. Therefore, the solution to (A5) denoted by h_d^* is a global maximum. Also, h_d^* is an interior solution because (i) $\partial g_c / \partial h_d > 0$ at $h_d = 0$ and (ii) $\partial g_c / \partial h_d < 0$ at $h_d = 1$. Rearranging (A5) yields

(A7) $$\left(\frac{\phi(1-h_d)}{h_d} - 1 \right) \left(\frac{(z-1)\varphi \ln z}{z-1+\theta} \right) + \frac{\alpha f'(h_d)}{h_d^{\phi}} = 0.$$

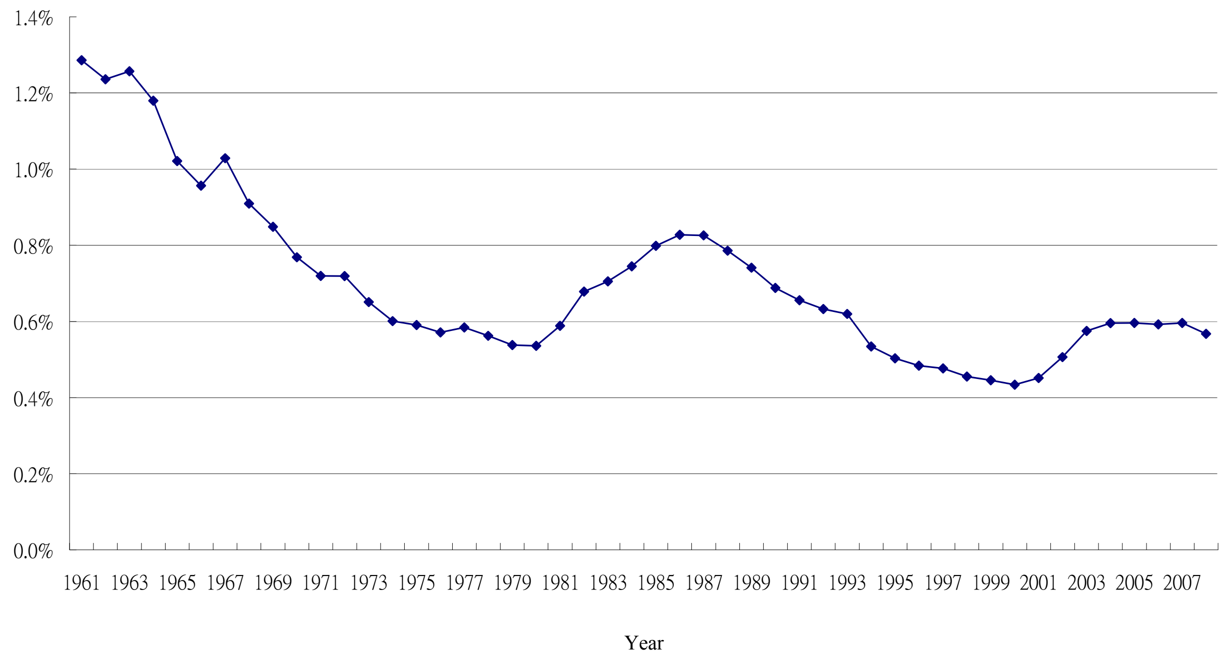
Because the LHS of (A7) is decreasing in h_d and increasing in α and ϕ, h_d^* must be increasing in α and ϕ. Recall that $h_d \in (0,1)$ so that h_d^{ϕ} is decreasing in ϕ.□

Proof of Proposition 2: From (16) and (19), we know that both h_x and g_c are independent of δ. Therefore, in (18), the marginal benefit of h_d is increasing in δ while the marginal cost of h_d is independent of δ. As a result, the welfare-maximizing h_d^{**} must be increasing in δ. Also, h_d^{**} is less than one because when h_d equals one, consumption equals zero given $\theta > 0$. When δ equals zero, $h_d^{**} < h_d^{*}$ because $\partial h_x / \partial h_d < 0$ in (18). Let's recall the definition of $\bar{\delta}$ from (20). When δ equals $\bar{\delta}$, we have $h_d^{**} = h_d^{*}$ because $\partial U / \partial h_d = 0$ at $h_d = h_d^{*}$ in this case. When $\delta > \bar{\delta}$, we have $h_d^{**} > h_d^{*}$ because $\partial h_d^{**} / \partial \delta > 0$. From (20), $\delta > \bar{\delta}\big|_{\theta=0} = 0$ because

(A8) $$\frac{1}{h_x}\left(\frac{\partial h_x}{\partial h_d}\right) = -\left(1 + \phi \frac{\rho}{\varphi h_d^{1+\phi}}\right) \Bigg/ \left(1 - h_d + \frac{\rho}{\varphi h_d^{\phi}}\right)$$

from (19). Therefore, $h_d^{**} > h_d^{*}$ when $\delta > 0$ and $\theta = 0$. □

Figure 1: Federal R&D on national defense as a percentage of US GDP



Data sources: (a) National Science Foundation, and (b) Bureau of Economic Analysis.

Figure 2: Growth effects of defense R&D

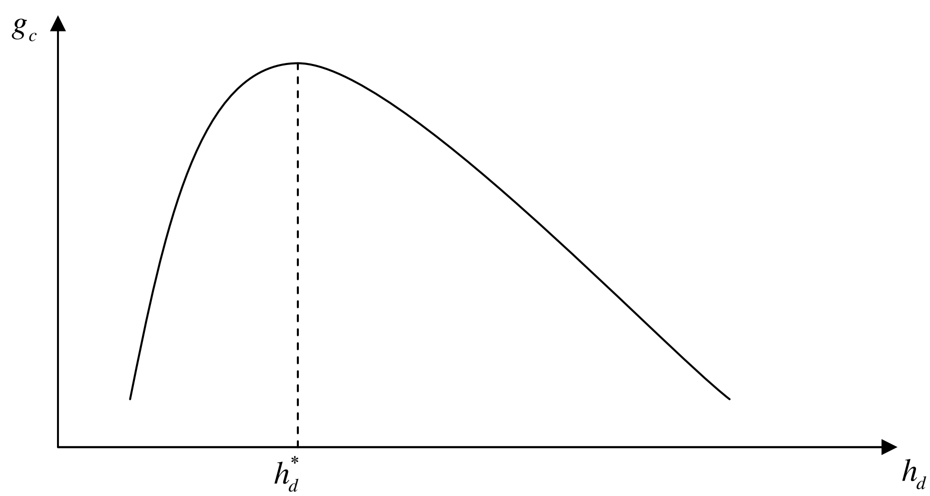


Figure 3: Growth-maximizing versus welfare-maximizing defense R&D

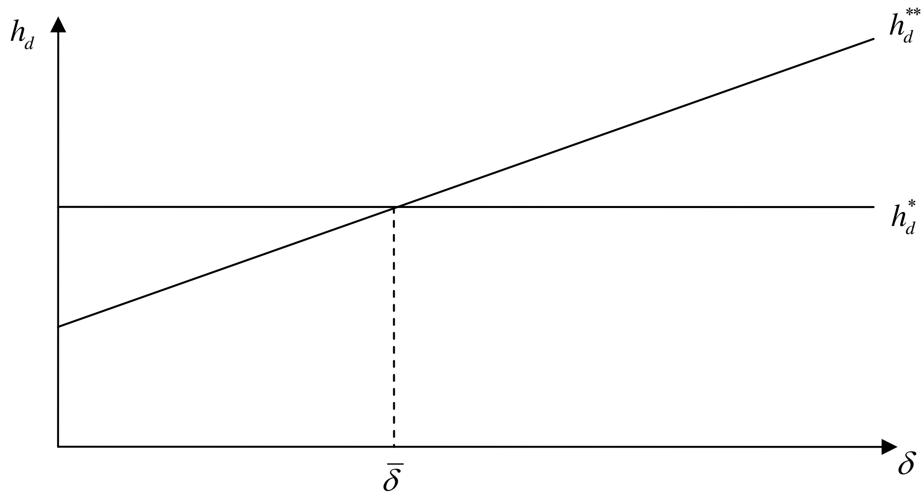


Figure 4: Phase diagram