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24 November 2009

Equilibrium Sovereign Default with Endogenous Exchange Rate Depreciation*

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November 18, 2009

Abstract

Sovereign default is often associated with disturbances in a country's trade relations. Often the defaulter's currency depreciates while trade volume falls drastically. This paper develops a model to incorporate real depreciation along with sovereign bankruptcy. The exchange rate is determined in equilibrium as the relative price of imports. We demonstrate that a default episode can imply up to a 30% real depreciation. This matches the depreciations observed in crisis events for developing countries. We argue that much of the exchange rate movement is explained by market clearing adjustments to trade disruptions in the aftermath of default.

Keywords: endogenous default, endogenous exchange rate, trade balance.

JEL: F34, F11, F17.

*We are grateful to Dan Bernhardt, Anne Villamil, Rui Zhao and Makoto Nakajima for productive discussions and numerous comments; we thank Volodymyr Lugovskyy, Anton Cheremukhin, Daniil Manaenkov, Stephen Morris and Demian Pouzo for their careful revisions to early versions. We also thank participants of Missouri Economics Conference 2008, 4th Annual Graduate Student Conference in Washington University and Presenting Papers seminar series in University of Illinois for useful suggestions. All errors are ours.

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1 Introduction

Sovereign defaults are neither random nor isolated from other macroeconomic events. In particular, fiscal insolvency often coincides with exchange rate depreciations. In this study, we motivate this association between the endogenous default decision and currency depreciation. We investigate the channel through which a government bankruptcy can disrupt trade relations abroad and quantify its contribution to real depreciation. Then, if default implies a terms-of-trade penalty, we examine how it can determine a country's default decision. We find that a default penalty through this "trade channel" can create realistic depreciations. This introduces a borrower's preference for imports as a factor in default. From this mechanism, the defaulter's decision is not necessarily monotone in income fluctuations, which partially answers why defaults are not always during recession.

Establishing a theoretical connection between default and trade disruption is important because empirical literature has often noted that sovereign default and currency crises — rapid exchange rate depreciation — strike together. However, quantitative models typically treat sovereign default in isolation of other crises. In these models, default happens in equilibrium when debt service becomes more onerous than the penalty, which is often reduced to restricted access to credit and a reduction in output. Instead, we focus on the disruption to trade in data that is regularly associated with default. While there is vigorous debate about what adverse effects confront a would-be sovereign defaulter, the effect on the pattern of trade cannot be ignored. In this spirit, we introduce a model in which a country desires imported goods, but these become more expensive if it reneges on its debt. Our country has open markets for financial and goods flows, and to preserve relatively cheap international trade it must meet its financial obligations.

Since exchange rates have floated, defaulters have experienced large nominal exchange depreciations that are mostly coincident to real depreciations. If sovereign governments internalize their citizens' desire for imported goods, averting this depreciation motivates a country to keep its debt service current. In default, countries' terms of trade deteriorate, which means that more domestic production is diverted towards exports in exchange for the same amount of imports. Our model captures the features of the external sector in default: the currency depreciates and exports rise, forcing households to consume less.

In this paper, we introduce this trade channel penalty to mimic the behavior of the external sector in defaulting countries. New insights into how defaulters are punished are particularly important because there is no widespread, codified rule for dealing with defaulters. In focusing on the trade channel, we link a country's preferences for imports and the demand for its exports to its propensity to default. This helps one to understand why similar fluctuations of domestic output may lead to a default episode in some countries but not in others. We attain a reasonable real depreciation prediction without calibrating for one, by fitting the international trade mechanism and representative consumer's preferences to real data. We attain sound comparative statics predictions (particularly, we show how sudden change of tastes for import can lead to default). We measure welfare gains from international trade, and compare them to first-best case, effectively positioning the economy we study between financial autarky and perfect capital markets.

Our paper will proceed first by reviewing the empirical data on sovereign defaults and the external-sector results. In Section 2 we present our model. We calibrate it to Argentina and test its implications in Section 3.

1.1 Models of Default

Our study of default extends earlier work on sovereign default in a small economy. At its root, our model closely resembles the quantitative methods of consumer default models (see Chatterjee et al. (2007)) that introduce a continuum of agents with stochastic income but imperfect risk sharing. In the international setting, we follow the tradition of Eaton and Gersovitz (1981), in which countries' idiosyncratic income can only be buffered by defaultable bonds. Our work's closest antecedent is Arellano (2008), who computes a similar model with realistic business-cycle fluctuations.

In Arellano (2008), default depends on the history of output shocks and is punished by losing a fixed percentage of output and by exclusion from borrowing for some random period. The income penalty is motivated by an empirical finding that output falls after default on average. Tomz and Wright (2007) suggest this mechanism is inadequate because the link between "bad times" and default to be "surprisingly weak"—there is a great deal of variance around this negative mean. They find that countries often default even in relatively good times and remain in default after output has recovered, suggesting that the negative lagged correlation between default and output is neither causal nor very significant. In fact, about 40% of the default episodes that Tomz and Wright (2007) chronicle end with above trend output. Clearly, this evidence about domestic output is not entirely consistent with models in which the cost of default is a penalty to income process, indicating the importance of of alternative punishments.

Tomz (2007) expands on this notion that recession and default are imperfectly correlated, describing some defaults as “inexcusable,” or unexplained by economic fluctuations. Instead, he posits that some regimes have an innate propensity to default. Indeed, since 1975 Standard and Poor’s counts three foreign currency defaults by Argentine, Jamaica, Gabon, Indonesia, Peru, and South Africa and four by Uruguay. We also see defaults precipitated by political changes (see e.g. Todd (1991)). In a half century, Mexico defaulted twice when a new regime took over. In both 1861 under Benito Juarez and 1914 in the chaos after General Diaz, populist leaders put less value on maintaining good standing internationally and defaulted.

Responding to Tomz and Wright (2007), which called for research on “mixed-models” of default, these examples should suggest that less outward-oriented sovereigns might be more willing to default. We consider this scenario as a change in the preference for imported goods, so a government with a populist constituency tries to maximize a consumption basket with a relatively smaller portion of imported goods. In our comparative statics analysis, we begin to consider how a populist revolt with a reduced desire for foreign goods might spur default.

Other quantitative models have also tried to make the punishment mechanism more realistic; Yue (2005) introduced Nash bargaining, and Bi (2008) made this bargaining game repeated. At their root, however, defaulters suffer due to loss of output and exclusion from credit markets for some time — they are still single-motive models. These models have no explicit role for the international flow of goods, even though they describe open economies with free flow of capital. Because there is no demand for foreign goods, only foreign borrowing, financial autarky only hurts the bankrupt country because it may need to smooth over a future recession.

1.2 Motivating the Trade Channel Penalty

As described by Bulow and Rogoff (1989), “reputational” penalties alone are insufficient to support positive debt in equilibrium. They suggest that trade sanctions might sufficiently discourage default as to support realistic levels of debt. They justify such trade embargoes by speculating that a country might rely on its reputation to maintain trade flows, and it would be damaged by a default episode. They were not, however, modeling the trade channel penalty, or making quantitative statements about its implications. Investigating the role of trade penalties further, Wright (2001) discusses the similarities between penalties to trade flows, assets or income. In some environments, there is an equivalences in that any of these penalties can create a welfare loss that dominates the gain from a country defaulting; we show how the trade channel has particularly interesting side effects on trade patterns.

To drive our trade channel penalty, we must impose a deterioration in the country's terms-of-trade. At its extreme, this would be an absolute embargo, which, were not uncommon historically when countries refused to pay their creditors. In the 1861 Mexican default, creditors actually seized the port of Veracruz (see Todd (1991)). Circumnavigating gunboats certainly increased the "iceberg costs" on Mexican exports. However, Tomz (2007) argues that even in the 19th century heyday of gun boat diplomacy, outright military force only played a minor role in punishing defaulters. Though debt repudiation was sometimes correlated with military action, violence usually ended quickly.

Still, there is evidence that *something* impedes trade from bankrupt countries even today when debt arbitration clearly does not carry the threat of cannon fire. Rose (2005) documents that default reduces international trade by 8% for an extended period after default. He speculates that trade suffers after default because international flows require short-term financing. In the aftermath of default, trade partners will not extend this credit and raise the cost of cross-border trade. This conjecture is consistent with Arteta and Hale (2008), who demonstrate that private firms find international credit scarce after a sovereign default. That is, traders, who are private borrowers, are punished for their government's transgression. We do not explicitly model the traders' demand for short-run credit, but it could underly our mechanism.

Aside from disruptions to financing, we can root the trade penalty within the context of already imperfect trade. Exporters are at the whim of other country's trade officers, who can greatly increase the cost of trading goods by overzealous customs inspections or other non-tariff trade barriers. The time cost, deriving from uncooperative trade officials can add 10-30% to the cost of imports, as estimated by Hummels (2001). Many of these costly bureacratic barriers must be fungible and can be made more stringent for less-favored trade partners.

Furthermore, many developing economies depend on favorable trade agreements and sourcing partnerships with the developed world. 195 active bilateral trade agreements have been reported to the World Trade Organization by January 2009, any of which could be terminated in response to a financial episode and would effectively increase tariffs faced by a country's exporters. Without favored market access or the use of a trade partner transport infrastructure, the effect on the cost of exporting goods is similar to additional "iceberg" costs. Default, as it mostly strikes poorer countries, might also reduce the exports because most rich country imports from poor countries tend to be at arms-length, rather than intra-firm, as noted by Antràs (2003). Default may sever these inter-firm trade links more easily than if trade was intra-firm.

1.3 Patterns of Trade in Default

Empirical international studies have shown that dual crises, debt and currency crises, are often tied. De Paoli and Hoggarth (2006) explore defaults since 1975 and find a strong link between currency crises and sovereign default. They propose an informal explanation for this observation related to nominal rigidities and potential central bank insolvency. Here, we expand on this finding: Table 1 shows that even when depreciation might not qualify as a crisis, defaulters usually experience a decline in their terms of trade. Furthermore, we demonstrate how the trade channel penalty can generate a *real* depreciation and substitution towards domestic goods. While other literature often keys on "hot" flows of funds and large changes to the nominal exchange rate, the real depreciation that we model actually accounts for most of the change in relative prices.

Sovereign default also brings adjustments to trade volumes, both in the model and data. Due to the depreciation repricing, domestic households substitute away from imports. In other words, when sovereign default precipitates a change in price incentives with respect to the external sector, households in our model adjust consistently with the data.

Table 1 presents the gross changes over a year for defaulters' nominal effective exchange rate (NEER), real effective exchange rate (REER), export price and the fraction of expenditure on imports. Notice that the median defaulter has nearly the same depreciation in real exchange rate as in nominal exchange rate after a year's time. With higher frequency data, the NEER depreciation tends to be sharper, but the REER quickly catches up. Table 1 suggests that our approach, which models depreciation in bankruptcy in real terms, captures the essence. In some cases, the nominal change is greater than the real, but in many of these instances, narrative evidence suggests mismanagement by the monetary authority or financial market overreaction. Table 1 reveals that this behavior is rather uncommon, and this leads us to model the more "routine" defaults.

Gross change	NEER	REER	Export Prices	Imports/GDP
Mean	0.8492	0.8900	0.9184	0.8968
Median	0.9102	0.9220	0.9075	0.9211
Pr <.2	0.6835	0.7861	0.8291	0.7773
Pr <.4	0.8729	0.8688	0.8984	0.8978
Pr <.6	0.9505	0.9429	0.9269	0.9436
Pr <.8	1.0168	0.9755	1.0068	1.0070

Note: Exchange rate listed as foreign goods per home currency unit. Our data comes from the IMF's IFS database and looks at defaulters since 1975, as identified by Beers and Chambers (2003).

Table 1: One year effect of sovereign default

Countries in default suffer a consistent drop of about 10% in the price of their exports in foreign markets and the distribution of price changes is more tightly clustered than the depreciation distribution. Because the REER weights their trading partners by volume of trade, it suggests that the fall in export prices is more pronounced with a country's more active trading partners. Our model captures this price change observation. Defaulters receive fewer imports in return for the same number of export goods, a fall in export price, which might stimulate domestic households to substitute away from foreign goods, further affecting the exchange rate.

Even at high frequency, the depreciation is abrupt and follows the same trend as above, with REER slightly trailing NEER. In Figure 1, we show the exchange rate dynamics in two recent defaults, Ukraine and Paraguay. Note that in the Ukraine, real and nominal exchange rates move almost indistinguishably. This suggests that post-default currency depreciations are not merely evidence that shaky sovereigns impel hot capital to flee, instead something more basic happens to cross border exchange of real goods.

The data also reveal that households with bankrupt governments also spend relatively less on imports, which have become more expensive following the price changes. This fact is an amalgam of competing forces: the substitution effect would decrease imports, and the income effect would also diminish the demand for imports, with imports as normal goods. On the other hand, average income is lower in default. The falling share of imports suggests that this latter effect is not so strong. In our model too, imports fall as a share of spending. Though Tomz and Wright (2007) suggest that income is not a dependable correlator with default, it does seem to reliably change

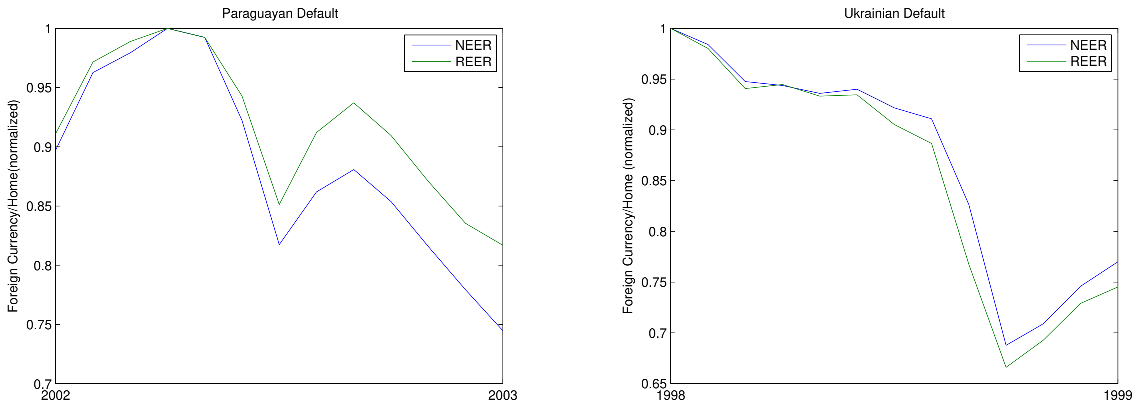


Figure 1: Dynamics of exchange rates in Paraguay and Ukraine, nominal and real. Exchange rates are normalized so that maximum value on sample is 1.

trade flows.

2 The Model

Our model of default extends the models of Eaton and Gersovitz (1981) and Arellano (2008), integrating a commodity space with both domestic and foreign goods. We describe a small open economy in which a government internalizes its citizens preferences over domestic goods, c_t, and imports, m_t. These goods are imperfect substitutes with constant elasticity, $\frac{1}{1-\kappa}$ in our formulation. The relative price of exports is the exchange rate, e_t. Imports are exchanged for exports, x_t by import firms according to $m_t = f(x_t)$. Their profit, Π_t, goes back to households.

Asset markets are incomplete with one period bonds serving as the only insurance against shocks to the income stream, y_t. The sovereign borrows b_t on behalf of its citizens from an international market paying a coupon q_t. Debt contracts are not enforceable, so the country may default. As punishment, the country suffers a deterioration in terms of trade and financial autarky for a random period of time.

The domestic country maximizes

$$U(c_t, m_t; \kappa, \alpha), \tag{1}$$

subject to the income process and budget constraint

$$c_t + e_t m_t + b_t = y_t + q_t b_{t+1} + \Pi_t, \tag{2}$$

$$\log y_t = \rho \log y_{t-1} + \epsilon_t,\ \ \epsilon_t \sim \mathcal{N}(0, s^2). \tag{3}$$

The importer transforms exports to imports according to

$$m_t = f(x_t)\ :\ f'(\cdot) \geq 0, f''(\cdot) \leq 0. \tag{4}$$

In particular, we specify the import export technology to be $f(x_t) = \theta_1(x_t - \theta_0)^{\theta}$. This technology is a reduced-form representation of the rest of the world's demand for domestic goods. For $\theta < 1$, the home country's exports receive diminishing returns in terms of imports. This is equivalent to assuming diminishing marginal utility of the country's goods to the foreign consumers. θ_0 allows for fixed costs to exporting, though in our calibration we find its value to be small.

Certainly, we could motivate the demand coming from the rest of the world (ROW) with a structural formulation for preferences. However, from the point of view of the small open economy, only the ROW demand can be observed, and this demand function is sufficient to pick the optimal quantity of exports. Both estimating and imposing an explicit form for the ROW utility seems treacherous. From an economist's perspective, calibrating the preferences of the ROW is daunting because one must aggregate structural preferences, but the weighting of the relative importance of a trade partner is only observed in equilibrium. Aesthetically, a full optimization by the ROW might be preferable, but we choose to keep our model focused and parsimonious; our contribution is not on why countries trade, but rather on why they default.

Much of our analysis would go through with a simpler linear technology, $f(x_t) = \theta_1 x_t$. With a linear technology, the trade channel penalty would cause depreciation in default, but the depreciation would be one-for-one with the size of the penalty. We would essentially be directly imposing the size of depreciation, instead of allowing it to be determined endogenously by preferences. Our preferred form better matches the stylized facts. In particular, (with a linear technology) the depreciation would be the same size every time, but Table 1 reveals that this is not the case.

2.1 Timing

Time is infinite and discrete. At the beginning of each period, a country's state variables — income, borrowing to repay, and default status — are common knowledge. The first choice is whether to default; then country chooses consumption, trade and borrowing policies. If the country is in good standing, it faces a coupon price schedule from its creditors who will charge a premium over the fixed world interest rate based upon the probability of default in the next period.

If the country defaults, then it experiences a terms-of-trade deterioration before its trade policy is set. The equilibrium exchange rate clears the international trade markets. As in Eaton and Gersovitz (1981), countries in default are in financial autarky, and their bond position reverts to zero. Unlike the aforementioned article, countries can leave this defaulted state each period with an exogenous probability of being "forgiven".

2.2 Representative Household and Government

The sovereign government internalizes the problem of the representative household. In particular, household preferences take the form $u(c_t, m_t; \kappa, \alpha) = [\alpha c_t^\kappa + (1-\alpha) m_t^\kappa]^{\frac{1-\sigma}{\kappa}}/(1-\sigma)$. We will formulate the representative household problem recursively. Given income realization, y, bond position, b, and the indicator of whether it is being punished, taking as exogenous the price of bonds, $q(\cdot,\cdot)$ and imports, $e(\cdot)$, the government solves two subproblems, one if it decides to default and one if it does not default. The solution to the non-default problem is summarized by the value function $V(b, y)$ and the defaulter's problem has value function $W(y)$. The default decision is the argmax of these two functions, and the government's value function, $U(b, y)$ is the envelope over the subproblem value functions, $V(\cdot,\cdot), W(\cdot)$. Formally, the government solves the **Household Problem**:

$$U(b, y) = \max_{h \in \{0,1\}} hW(y) + (1-h)V(b, y),$$

where $h \in \{0, 1\}$ indicates default. Conditional on not defaulting this period, the **Household's Problem In Good Standing** is:

$$\begin{aligned} V(b, y) &= \max_{c,m,b'} u(c, m) + \beta EU(b', y'), \qquad (5) \\ \text{s.t.} & \\ c + em + b &= y + q(y, b')b' + \Pi, \\ \ln y' &= \rho \ln y + \epsilon, \ \epsilon \sim \mathcal{N}(0, s^2). \end{aligned}$$

The debt discount $q(y, b')$ adjusts the price of borrowing to accommodate the probability of default. By choosing $b' = b_0$, the country accepts the contract that gives it $q(y, b_0)b_0$ units of home good this period and takes away b_0 units of home goods next period. Using discounts instead of returns permits banks to decide not lend at all, which may happen when default is nearly certain because of an extremely high level of debt, which imply an infinite interest rate.

If country chooses to default, then its value function is an optimal solution to **The Household Problem In Default**:

$$
\begin{aligned}
W(y) &= \max_{c,m} u(c,m) + \beta E\left(\phi W(y') + (1-\phi)U(0,y')\right), \qquad (6)\\
&\text{s.t.}\\
c + em &= y + \Pi,\\
\ln y' &= \rho \ln y + \epsilon,\ \epsilon \sim \mathcal{N}(0, s^2).
\end{aligned}
$$

2.3 Importers

The importers face a trade demand given by equation (4) from the rest of the world. Note, that the country does *not* face a perfectly elastic demand from the rest of the world. Instead, this is a "small country model" only in that the decisions of the country do not affect the demand of the rest of the world. In case of default, equation (4) is shifted to generate the terms of trade shock that serves as punishment.

Domestic importers take exports x from their countrymen at the price of the consumption good and exchange them for m, taking their price e as given. If the country is being punished for default, it receives fraction $1-\pi$ fewer imports. Their problem is summarized by **Importer's Problem**:

$$
\begin{aligned}
\Pi(x,m,h) &= em - x,\\
&\text{s.t.}\\
m &= (1-\pi)^h f(x).
\end{aligned}
$$

Notice that the trade channel penalty, $1-\pi$ enters as if there were a sudden increase in iceberg costs. This is a conscious choice, intended to expose defaulters to trade barriers that increase the cost of sending their goods abroad. For example, country's exporters may find that its exports sit in customs longer after default, that foreign trade inspectors are more deliberate or, more benignly, that favorable trade agreements are canceled; this can be an efficient way of adding real costs of exporting, and Hummels (2001) shows how more time en route can be a trade barrier. Alternatively, one can think of country getting π less imports for the same amount of exports as a crude defaulted debt renegotiation process.

Figure 2 depicts the effect of the trade penalty, given the concave functional form of $f(x)$. The export quantity is a distance from disposable income to consumption of home produced goods. The endogenous exchange rate, e is the slope of the tangent.

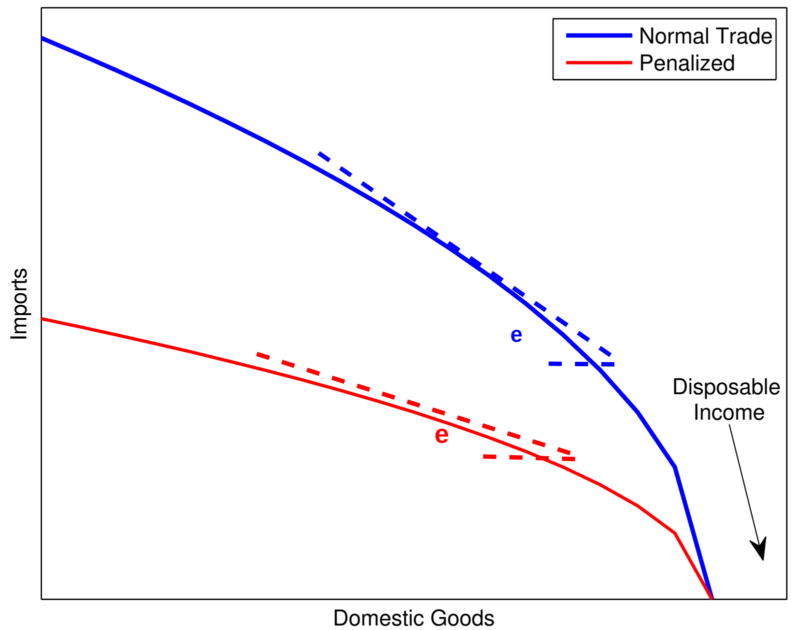


Figure 2: The export-import transformation function and the effect of the trade channel penalty

2.4 International Financial Markets

Denote the default decision control $h(y, b) = \mathbb{I}(W(y) > V(y, b))$. It is equal to 1 when country announces default. World financial markets are risk-neutral but cannot enforce their debt contracts. They have perfect knowledge of the sovereign's problem, so they have zero profits and set $q(y, b')$ so that the expected return equals the international risk-free rate of return R. The credit market **Zero Profit Condition** is:

$$q(y, b') = \frac{1 - E[h(y', b')]}{1 + R}.$$

2.5 Recursive Equilibrium

The Recursive Competitive Equilibrium is a collection of

- consumer choice functions $(c_V(y, b), b'_V(y, b), m_V(y, b))$ when the country is not in default,
- consumer choice functions $(c_W(y, b), m_W(y, b))$ when the country is in default,
- consumer's default choice function $h(y, b)$,
- consumer's value functions $(V(y, b), W(y), U(y, b))$,
- importer choice variables in no default state $(x_{Vm}(y, b), m_{Vm}(y, b), \Pi_V(y, b))$
- importer choice variables in the default state $(x_{Wm}(y), m_{Wm}(y), \Pi_W(y))$,
- and price variables $(e_V(y, b), e_W(y), q(y, b'))$

such that:

- $(c_V(y, b), b'_V(y, b), m_V(y, b))$ solve the **Household's Problem In Default**, given $\Pi_V(y, b)$, $e_V(y, b)$ and $U(y, b)$, and $V(y, b)$ is the value function of this problem.
- $(c_W(y, b), m_W(y, b))$ solve the **Household's Problem In Good Standing**, conditional on $\Pi_W(y)$, $e_W(y)$ and $U(y, b)$, and $W(y)$ is the value function of this problem.
- $(h(y, b))$ solve the Household Problem conditional on $V(y, b)$ and $W(y)$, and $U(y, b)$ is the value function of this problem.
- $(x_{Vm}(y, b), m_{Vm}(y, b))$ solve the Importer's Problem conditional on $e_V(y, b)$, and $\Pi_V(y, b)$ is the value function of this problem.
- $(x_{Wm}(y), m_{Wm}(y))$ solve the Importer's Problem conditional on $e_W(y)$, and $\Pi_W(y)$ is the value function of this problem.
- $q(y, b')$ satisfies the Zero Profit Condition given on $h(y, b)$.

- $e_V(y,b)$ is such that import market clearing condition $m_V(y,b) = m_{Vm}(y,b)$ holds.
- $e_W(y)$ is such that import market clearing condition $m_W(y) = m_{Wm}(y)$ holds.

Equilibrium exists for the same reason as in Arellano (2008); our problem is separable between borrowing and consumption. After the value of borrowing for the next period is chosen, the allocation of available income between consumption and imports is maximizing monotone function upon compact set.

3 Quantitative Evaluations

To evaluate the predictive power of our model, we calibrate our baseline version to Argentine's historical data, following much other related research. Argentine experienced three international default occurrences, in 1982, 1989 and 2002. None of the restructuring periods were particularly long. All these defaults were accompanied by nominal exchange rate depreciation and non-zero trade balances.

3.1 Quarterly Data

Our parameter values come from our own estimates and Arellano (2008). Specifically, we estimated the parameters of income time series, the goods relative preference parameters, and import-export relationship parameters and borrowed the values of R, β, ϕ and σ. INDEC, National Institute of Statistics and Censuses, provides quarterly estimates of GDP composites, deseasonalized and in same-year prices, for years of 1993-2008. To make per capita values, we divide by the annual population of Argentina, obtained from CIA Factbook. To convert the import data from peso expenditure into quantities of foreign goods we used the real exchange rate taken from the European Central Bank website[1].

The parameters that we used were estimated on the quarterly dataset from 1993 to 2008, and are provided in a following table.

Most of our estimates seem close to similar estimations by others. In the international trade literature, most parameter estimates seem to be vigorously debated, though Ruhl (2003) nicely

[1]From Statistical Data Warehouse section, located at http://sdw.ecb.europa.eu/

Parameter	Value	Explanation	Rationale
R	0.017	Unconditional expected rate of return required by international banking system.	Quarterly return on US 5 year bond
ρ	0.9849	Autocorrelation of log-output.	Estimated
s	0.0258	Standard Deviation of log-output.	Estimated
π	Varying	Trade penalty for defaulting.	
ϕ	0.7180	Probability of being not forgiven on the next period.	Sandleris et al. (2004)
β	0.953	Subjective time discount factor.	Arellano (2008)
α	0.5859	Parameter of the instantaneous utility function; weight of home good consumption.	Estimated
κ	0.8447	Parameter of the instantaneous utility function; corresponds to 6.441 elasticity of substitution of export to import.	Estimated
σ	2	Parameter of the instantaneous utility function; corresponds to -1 elasticity of intertemporal substitution.	Arellano (2008)
θ	0.2082	Curvature of export-import transformation function.	Estimated
θ_0	0.0467	Relative position of export-import transformation function.	Estimated
θ_1	0.1959	Scale of export-import transformation function.	Estimated

Table 2: Parameter values, quarterly data.

situates the discussion on the proper elasticity parameter κ. Compared to other international business cycle models, our elasticity estimate is slightly high.

For details about the estimation of our import-export parameters, $\theta, \theta_0, \theta_1$, see Appendix B.3. We are not aware of others who have estimated a comparable functional. However, Das et al. (2001) find fixed costs for export supply close to zero using firm-level data. This is consistent with our value for θ_0. Hummels and Klenow (2005) analyzed the growth of exports from developing countries and found that up to 60% of the increase comes from the "extensive margin." Rather than deepening existing trade relationships, countries increase exports by diversifying their goods or partners. This seems to be evidence that export markets quickly become satiated and justifies small θ.

3.2 Solving for the Recursive Equilibrium

Since competition in the home market is perfect, the solution to the equilibrium is the same as solving with a centralized home economy in which the importer's problem is internal to the household. We call this the **Centralized Equilibrium**, and describe it in Appendix A.

Every period the country faces an import-generating technology, $f(x)$ as in equation 4 and allocates its consumption between imports and domestic production, as illustrated on Figure 2. Disposable income is $y + b - q(b'(y,b))b'(y,b)$ if country can borrow and y if the country cannot.

To numerically solve this problem, we use value function iteration to solve each subproblem of the Centralized Equilibrium, which is equivalent to solving The Household Problem In Default and The Household Problem In Good Standing. Each of these value functions converges trivially. The Household Problem is characterized by the max of these two value functions, and default occurs where W exceeds V. Because both functions are monotone in y and $-b$, for a given value of b, they may cross. As discussed in Arellano (2008), we should have a single crossing, which means that there is a single $b(y)$ that defines the maximum level of debt before the country defaults, given a realization of y.

In the Argentine default example, the estimate of $\ln(1-\pi)$ is -0.6939, which implies $\pi \approx 0.5$. We take this estimate as illustrative as there is no international policy for punishing defaulters, so there is no guarantee that one estimate for π should hold in other contexts. We evaluate the model's predictions at various punishment levels, to obtain bounds on plausible outcomes, and to account for potential short-sample biases. Varying the value of $\pi \in \{0.2, 0.5, 0.8\}$ also serves to demonstrate the effect of the trade channel punishment. We see that the country's default policies and behavior in default is sensitive to the magnitude of the trade channel punishment.

3.3 Welfare Comparisons

To understand our model with incomplete markets, we compare its agents' welfare to two extremes: complete asset markets and absent asset markets. With incomplete markets, bankruptcy provides some insurance against uninsurable, idiosyncratic risk, as discussed in Livshits et al. (2007). π determines the cost of such insurance, and affects both consumption and expected utility. For comparisons, we set $\pi = 0.5$ and other parameters follow Table 2. The expected value function in stationary distribution is -18.4318 and the average volume of the consumption aggregate, $(\alpha c^{\kappa} + (1-\alpha)m^{\kappa})^{1/\kappa}$, is 0.5868. The certainty equivalent of aggregate consumption is 0.5358. The risk premium is 9%, defined as $\frac{E\tilde{c}}{c_{CE}} - 1$ where $\tilde{C}$ is the actual aggregate consumption and $c_C E$ is an aggregate consumption certainty equivalent.

	Expected utility	Aggregare consumption CE	Risk premium	CE difference w/ benchmark
Benchmark	-18.4318	0.5358	9%	
Financial Autarky	-19.7300	0.5189	3%	-4%
First Best	-15.3818	0.5804	0%	+8%

Table 3: Welfare comparison.

The first best scenario provides agents with complete asset markets, which implies perfect consumption smoothing. At the other extreme, in financial autarky, the country cannot save or borrow and cannot insure against income shocks. We elaborate on the results in Appendix C and they are summarized in Table 3.

3.4 Equilibrium Results

We obtain three interesting equilibrium predictions. First, country borrows more if it had a history of negative income shocks, i.e. high current borrowing. Second, the current account is countercyclical, in line with data and Aguiar and Gopinath (2006). Third, positive shocks to income depreciate the home currency. The borrowing policy result is immediate, as it is a consequence of risk-aversity of a representative consumer. Most endogenous default papers have this result.

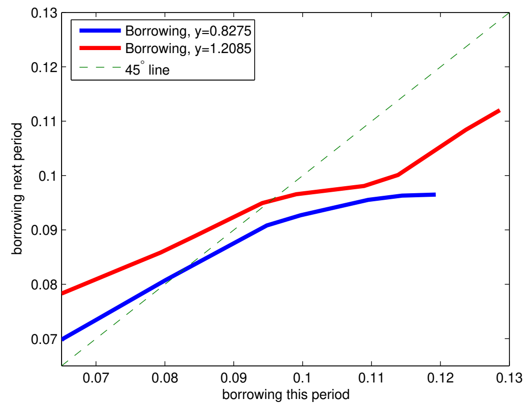


Figure 3: Borrowing policy. Curves stop on right at the point of default.

Our countercyclical current account (Figure 4) is consistent with data and, as Aguiar and Gopinath (2006) point out, nontrivial. In a partial equilibrium, positive shocks would induce savings and a positive current account, but if the interest rate is endogenous, it falls with positive income shocks (because the probability of default is counter cyclical) and spurs borrowing. Arellano (2008) has a pro-cyclical current account, and Aguiar and Gopinath (2006) shows that in their framework, without demand for trade, the current account's correlation depends on the specification of income process. In our model, the current account is governed by both consumer preferences for foreign goods, the importing technology and consumption smoothing. In this richer trade environment, the behavior of the current account does not just mirror the agents' borrowing policy. As our major contribution relies on the trading process that we model, we treat this result as a sign that our trading process properly approximates reality.

The counter cyclical exchange rate is a new result in the endogenous default literature, though Chari et al. (2002) and others establish a similar result in data and in various models. Chari et al. (2002) shows that a fairly standard RBC can generate counter cyclical real exchange rate movements. The intuition within our model is that with an increase in y, the country's possibility frontier in terms of (c, m) moves outwards. Consumption of both goods increase because the goods are normal. Since the marginal utility of consumers abroad goes down with additional exports, the exchange rate must rise with the increase in demand for foreign goods. It is important, however, to

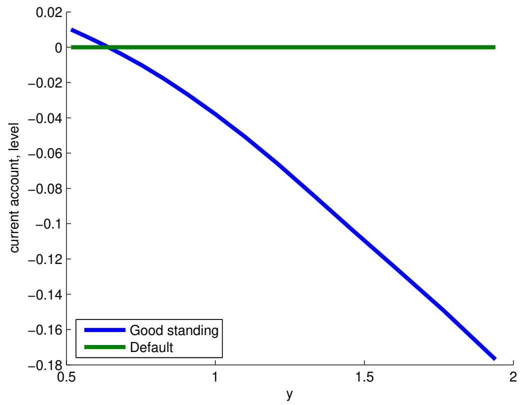


Figure 4: Current account conditional on output in stationary distribution.

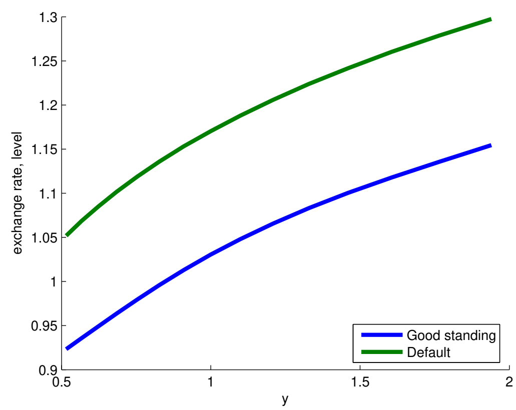


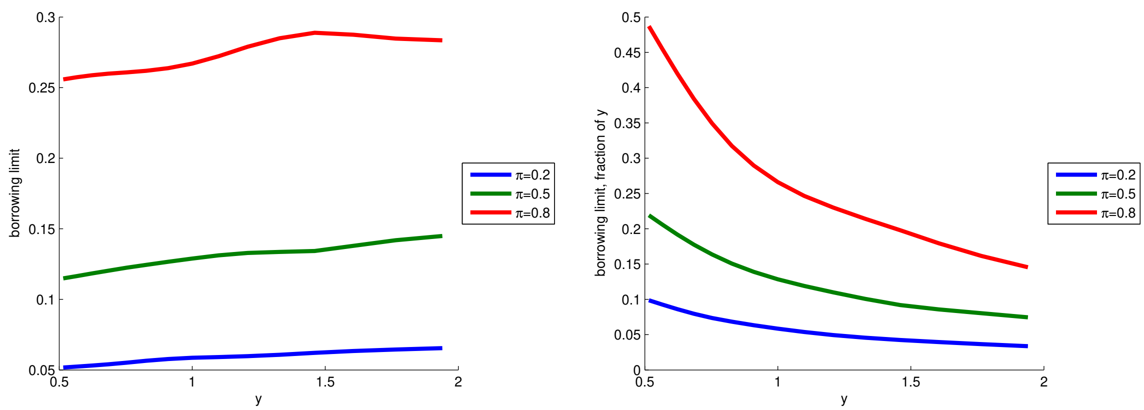
Figure 5: Exchange rate conditional on output in stationary distribution.

understand that foreign consumers in the model have no income shocks. To compare our result to the data, one must consider potential shifts in the world's demand function, which would manifest itself in Equation (4) and may shift the tangency point in Figure 2.

Notice also that the exchange rate is higher in default, though not $\frac{1}{1-\pi} = 2$ times higher — that happens because of the change in trade volumes due to substituting away from imports. The penalty shifts the line down, but the households are also able to adjust their consumption basket, and due to the curvature of Equation 4, this affects the exchange rate. As discussed earlier, we could generate a depreciation with linear trade technology, but the nonlinearity due to $\theta < 1$ allows for this additional freedom. As the data reveal, there is variation in the size of depreciation and trade volumes tend to shift, both consistent with our preferred parameterization.

3.5 The Effect Of Penalty Levels

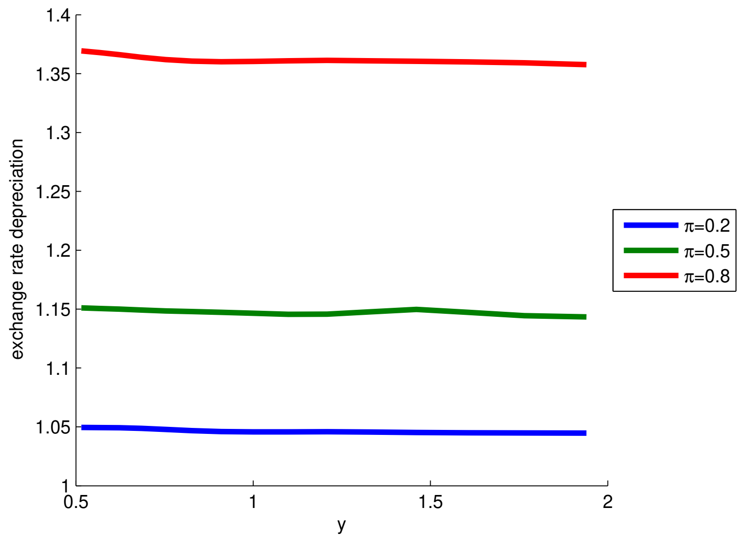
Figure 6 depicts the default decision threshold, the debt such that the country defaults if borrowing goes any higher. The small penalty case, $\pi = 0.2$, in which very little debt can be supported recalls the result of Bulow and Rogoff (1989). As in Aguiar and Gopinath (2006) and Arellano (2008), the sustainable debt increases in y; which makes the interest rate spread counter cyclical. The right panel of Figure 6, demonstrates, that the borrowing limit does not increase as fast as y, so that an expansion does not imply that a borrower can support a higher percentage of debt. This monotone slope, however, reflects the particular parameterization. With different specification (namely, low θ), it can yield an interestingly U-shaped borrowing limit. With this result, recessionary countries with low y, have greater incentive to stay in good standing because they expect the negative shock to persist; and countries with high output realizations are also less likely to default; however, in the middle, countries default on much smaller debt level.



Note: Argentine had debt of around 30% of GDP at the end of 2001, and normalized detrended GDP of 0.95.

Figure 6: Default decision borrowing threshold.

The penalty value changes the exchange rate adjustment post default, as seen in Figure 7. The exchange rate change pictured is calculated as the difference between the exchange rate in a default state and the exchange rate just before the default. That is, if the country has a debt that leaves it indifferent between defaulting and remaining in good standing, what would be the difference of exchange rates in these two cases? This thought experiment about the moment of default is close to thinking about two otherwise equal countries, one slightly below the default threshold, and one just above. Such a comparison isolates the change purely as a response to default decision. The figure reveals that the level of penalty determines the size of the depreciation. However, the country moderates some welfare damage by substituting domestic product for import consumption. The amount of depreciation does not depend much on the country's position in the business cycle.



1.15 on this figure means that foreign goods cost 15% more after default than before default. Penn World Tables report 18% real exchange rate depreciation in 2002 in Argentine.

Figure 7: Exchange rate drop due to default.

Figure 8 (left) illustrates the substitution behavior of domestic households. Country seem to have the same consumption policy at the moment of default independently of the level of π, that's why consumption policy before default is represented by a single line (this qualitatively holds for other parameterizations). As the trade channel penalty rises, the substitution effect becomes stronger. Figure 2 reveals the link between the change in the makeup of the consumption basket and the exchange rate that we highlighted earlier. Home good consumption is a normal good, and in a default state, an increase in income leads to an increase in consumption; while in the good standing state, higher income allows the country to borrow more, somewhat negating the decreasing returns to scale of import-export relation. A positive shock to income in a defaulted country will lead to increased consumption of home goods; a positive shock in non-defaulted country can lead to a more than one-for-one increase in exports.

Figure 8 (right) demonstrates the pattern of import's share in consumption. Decreasing returns to scale of the export-import relation make the dependence on y negative: countries in expansion

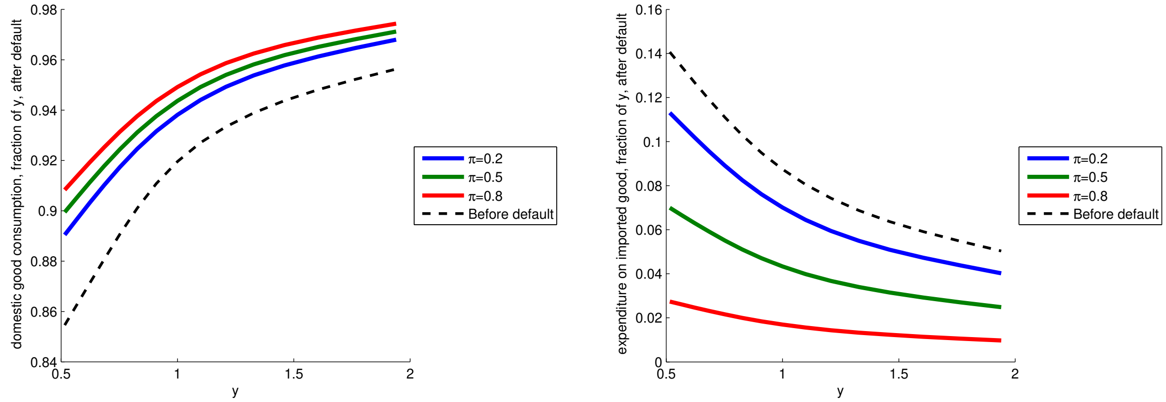


Figure 8: Domestic good consumption (left) and foreign good expenditure (right).

export ever greater amounts to get additional units of imports. Trade channel penalties have both direct effects on the level of import (by construction of the model), and indirect effects through substitution. As in the data, proportional expenditure on imports falls following default; countries in expansion after default benefit from not servicing the debt (which they'd have to if they did not defaulted), which instead they can spend on consumption, including imports. Notice that share of imports for high y countries exceeds $(1-\pi)$ times the corresponding value before default.

Figure 9 shows the change in capital account and trade balance at the moment of default decision. The trade balance improves more drastically in countries in downturn and the penalty level contributes significantly to the change size. The difference between the change in trade balance and change in capital account is the change in assets inflow. This might involve foreign currency reserves of respective central bank. Particularly, it implies that a country that defaults in a cyclical expansion and faces $\pi = 0.8$ would have to come up with additional assets equal up to 5% of GDP. Combined with a policy of fixed exchange rate and capital controls, that extra inflow can create a greater than necessary nominal money mass, resulting in inflation.

To summarize, a higher trade channel penalty leads to: (a) greater household adjustment through the domestic consumption basket, and trade quantities, (b) higher sustainable debt, and (c) greater exchange rate depreciation.

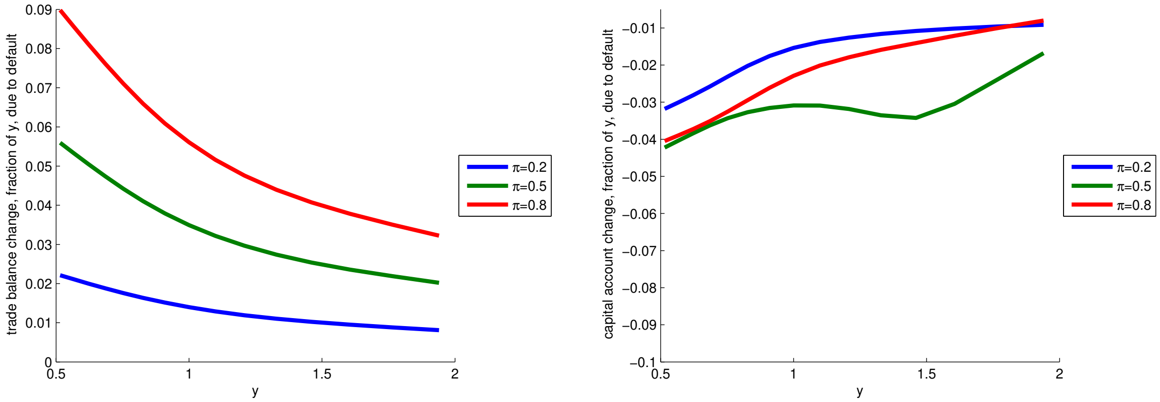


Figure 9: Trade balance (left) and capital account (right) changes at default.

3.6 Comparative Statics

Figure 10 summarizes the expected differences that should arise in cross-country data. The parameters we consider are a change in preferences for imports (α decreased by 0.1), a change in ability to forgive (ϕ increased by 0.1) and a change in variance of income (s^2 doubled the value). The penalty π in benchmark case is set at 0.5.

Changing α affects the willingness to default. A country that is less import-oriented defaults sooner and consumes less import in all states. A rapid change in tastes toward home consumption could precipitate default. Consider a government with debt just below the maximum allowable with a certain value for α; then, if this sovereign suddenly cared less about the import component of consumption, the level of allowable debt will shift down, possibly below the existing stock. In a populist coup, the new government might internalize the preferences of a different social group, one which consumes relatively fewer imports. This abrupt change of tastes of the sovereign's constituency — if combined with severe pre-existing debt — could result in default and explain some of the "inexcusable" defaults of Tomz (2007).

Loading this experiment on α with so much meaning is knowingly problematic. It is not actually a story consistent with our model, as our CES preferences are homothetic, so a poorer social strata should still have the same demand for imports. Further, our country's household is representative. If a coup implies a change to the preferences that the sovereign internalizes we should have to justify this with some explicit strategy for aggregation a diverse population. In the simplest case, our result

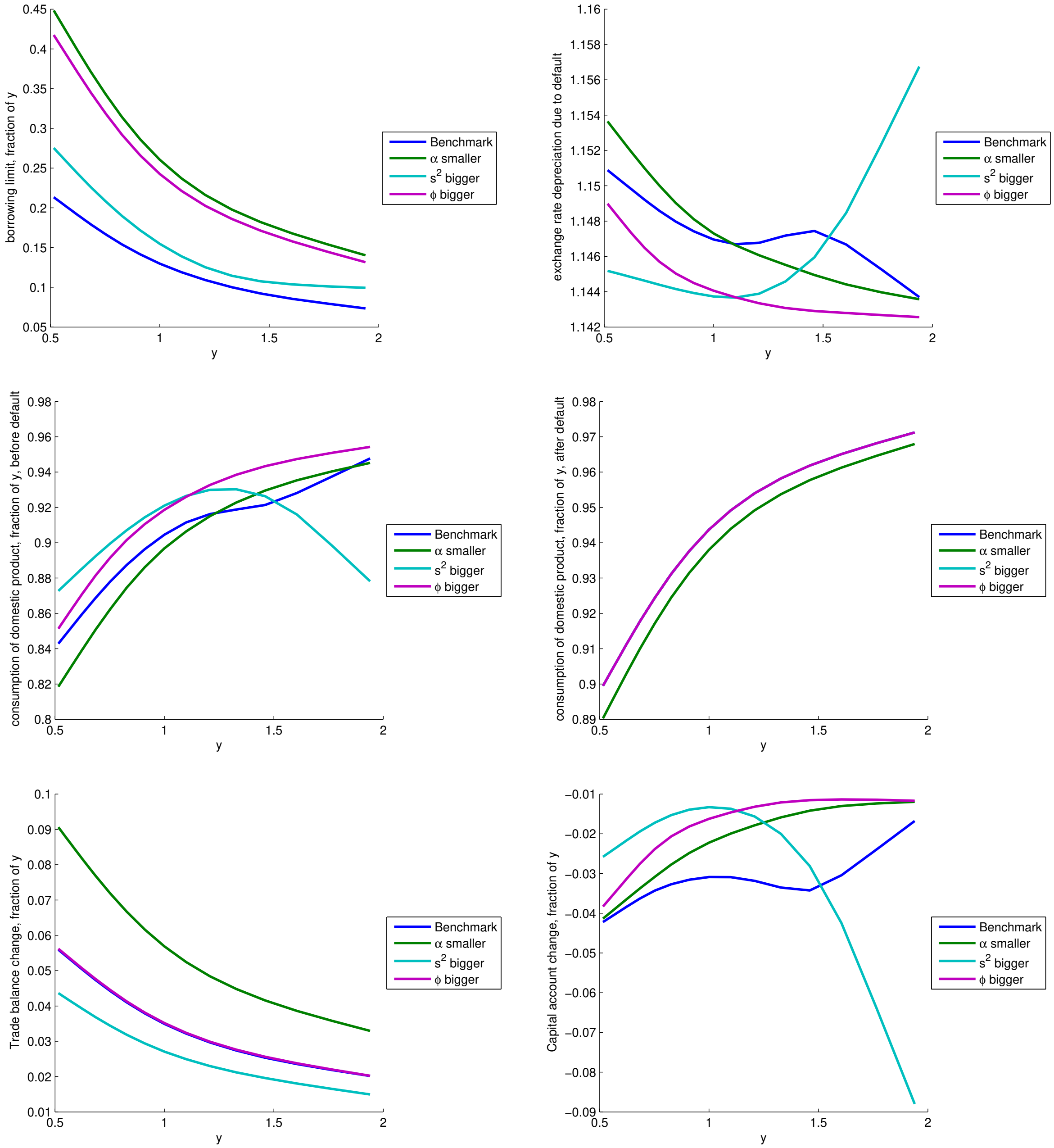


Figure 10: Comparative statics.

is equivalent to a model in which a sovereign that puts full weight on his median constituent. Then, if this groups changes, the preferences upon which he acts would also change.

After default, there is a stronger improvement in trade balance when α is lower, because a lower α country will forego more home consumption in both relative and absolute respects. The new preference regime, by consuming fewer imports, moves down the decreasing returns import-export technology, and so in the aftermath of a default, can sell into a "steeper" market.

The variance of the income process does not notably affect the patterns of trade adjustment relation, either before or after default. However, it affects the default decision through the financial exclusion penalty. This does not necessarily conflict with the Bulow and Rogoff (1989) finding that financial exclusion cannot sustain debt, because we also exclude countries from saving while in bankruptcy. Countries in expansion are more motivated to sustain debt than medium-range countries. The intuition is that high income variance countries obtain higher benefits from maintaining good standing in order to be able to smooth consumption in the future.

Also, observe that the variance of the composition of import and home goods is higher when y is high, and therefore the impact of increased variance is stronger on the right tail of income distribution. That leads to greater aversion to financial autarky, and consequently, to bigger tolerance of debt. Therefore, countries with higher income volatility might actually be safer for investors than steady economies.

The value of ϕ affects the cost of default, but once the choice between defaulting and staying prudent has been made, ϕ does not affect other tradeoffs, and therefore does not change the policies significantly. The bigger is ϕ, the worse is a country's cost of default and this its aversion becomes even stronger if it is currently experiencing an expansion. However, bigger ϕ demands more defiance from international lenders: longer punishment means loss of positive NPV projects and restraining selves from exports of the punished country. These concerns over enforcement are explored more in Wright (2001), but we just take as given that penalties are credible.

The comparative statics demonstrate that our model is qualitatively robust to parameter values, and provide additional insights into parallel problems (such as the consequences of populist uprisings and international risk management). The existing literature noted that business cycle fluctuations do not account for a significant fraction of historical default episodes. Our comparative statics demonstrate how changes in underlying parameters immediately result in significantly different equilibrium strategies — and confuse the identification of the business cycle contribution. Moreover, we can see that default boundary condition on debt shape differently with respect to output fluctuations with respect to these parameters; countries will demonstrate different default behavior. Even “inexcusable” in Tomz (2007) sense behavior is not outside of offered model’s scope; it only implies certain heterogeneity of parameters.

The top right plot of Figure 10 demonstrates that changing parameters does not change equilibrium depreciation much (and parameters that are changed, are changed a lot). In fact, only two parameters seem to matter for depreciation: κ and π. Other parameters, chosen to match Arellano (2008) do not affect equilibrium default depreciation, which seems to be arising from the combination of CES utility function and the proportional penalty. With respect to other parameters, our model is (knowingly) ill-suited to match certain moments of the data. For example, without long-term contracts Chatterjee and Eyigungor (2009) suggests models will not be able to match default frequency without undue manipulation of other paramters. In the interest of conservatism, we kept many parameters from the literature. Equilibrium depreciation, our chief concern, depends mostly upon the few parameters that we have discussed.

4 Conclusion

Our paper introduces a model that accounts for the systematic linkage between sovereign default and real exchange rate deprecation. Prior research has noted this empirical connection, but has not explained it quantitatively. Other quantitative models that capture the contribution of output fluctuations to the default strategy, they do not consider the open economy for what it is: a market for the exchange of differentiated goods rather than merely imperfect insurance against stochastic income shocks. Incorporating this goods trade we developed a model in which penalties to the trade channel create realistic patterns of default and adjustment within bankruptcy. In particular, we capture real depreciation without apealing to monetary transmission. From our results, real depreciation can account for most of the depreciation that is observed post-default. The domestic consumption allocations respond realistically to price incentives after default and imply enough loss of welfare to dissuade sovereigns from defaulting.

As a model of equilibrium sovereign default, our trade channel penalty allows a country to support a realistic level of debt before default. But in keeping with our focus on the trade in goods, the country's theoretical maximum borrowing burden is not necessarily a monotonic function of output. We explore the model's response to changes in variables other than income by computing its comparative statics. Generally, these parameter manipulations are intuitive, and help to identify several factors in the default decision. We and investigate their effects on default and trade policy and generate policy recommendations.

The model is naturally sensitive to specification. We chose a form that is consistent with pre-existing literature in trade and default, but these functions are essentially arbitrary choices from an infinite set of possible forms. However, the model is parsimonious and complete enough to generate qualitative and quantitative predictions in line with reality.

In this exercise, we calibrated to Argentine in order to demonstrate the import-export mechanism, which is the real innovation of our paper. This calibration gave us realistic parameters and we succeeded in generating reasonable predictions for the default decision and subsequent international adjustments. Future research might try to match individual default episodes more closely. We are also interested in integrating the nominal issues and financial flows in the model to capture effects like inflation and stochastic variations in interest and exchange rates. This paper, however, elides these concerns, to emphasize and clarify the real factors behind the default-linked depreciation.

We have begun to discuss how parameter changes could change the probability of default. Future work should more formally consider a mixed-model of default. A particularly promising avenue is to integrate parameter risk into endogenous variables, e.g. how does the probability of a coup effecting α modify the interest rate's default premium? Our discussion also raises the potential of models with heterogeneous agents and competing interests. Our simple model, considering trade and default, might be fruitfully combined with various other branches of the study of crisis.

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A Centralized Equilibrium

At the beginning of the game consumer chooses whether he wants to default or not is **Household's New Problem**:

$$U(b, y) = \max_{h \in \{0,1\}} hW(y) + (1 - h)V(b, y).$$

The borrowers's problem conditional on not defaulting this period is **New Problem With No Default**:

$$\begin{aligned}
V(b, y) &= \max_{c,x,m,b'} u(c, m) + \beta EU(b', y'), \\
\text{s.t.} & \\
c + x + b &= y + q(y, b')b', \\
m &= f(x), \\
\ln y' &= \rho \ln y + \epsilon, \ \epsilon \sim \mathcal{N}(0, s^2).
\end{aligned} \tag{7}$$

If country chooses to default, then its value function is a solution to **New Problem In Default**:

$$\begin{aligned}
W(y) &= \max_{c,x,m} u(c, m) + \beta E \left(\phi W(y') + (1 - \phi)U(0, y')\right), \\
\text{s.t.} & \\
c + x &= y + \Pi, \\
m &= (1 - \pi)f(x), \\
\ln y' &= \rho \ln y + \epsilon, \ \epsilon \sim \mathcal{N}(0, s^2).
\end{aligned} \tag{8}$$

Combined with the Zero Profit Condition, solution to this problem will give the same values as the Equilibrium we want to study. However, it does not give the value of the exchange rate. We recover this from the first-order conditions to the New household choice problems:

$$e_V(y, b) = \frac{\partial u(c_V(y, b), m_V(y, b))}{\partial m} / \frac{\partial u(c_V(y, b), m_V(y, b))}{\partial c}$$

$$e_W(y) = \frac{\partial u(c_W(y), m_W(y))}{\partial m} / \frac{\partial u(c_W(y), m_W(y))}{\partial c}.$$

B Estimation

To test our model's implications, we estimated various parameters using Argentine data. In many cases, we could have better characterized the Argentine data generating process with different parametric functional forms, but this would distract from the model we introduce. Our goal is not to match Argentine perfectly, though there is a certain virtue in doing so, but we fear that additional complexity will impede one's intuition for the model. Rather we decided to stay in the simple world of AR(1) processes and (relatively) linear functions to provide a reasonable test of the quantitative implications of our model .

B.1 Output Time Series

Output y was calibrated to quarterly deseasoned per-capita GDP in constant prices, which was assumed to follow AR(1) process. It was not detrended because trend seems to be too small. Regression equation is

$$\ln y_t - m = \rho(\ln y_{t-1} - m) + \epsilon_t, \epsilon_t \sim N(0, s^2).$$

m is the normalizing coefficient. Estimates are following:

Variable	**Coefficient**	**(Std. Err.)**
m	9.0669*	(0.1464)
ρ	0.9878*	(0.0081)
s	0.0258	

Table 4: Estimation results, output dynamics, One star denotes 1% significance.

A Dickey-Fuller test does not reject a unit root hypothesis for this process, therefore, given standard deviations may be biased downward. Controlling a linear trend does not help to reject the unit root hypothesis, nor does it give much different estimates of other parameters.

B.2 Consumer's Utility Parameters

Utility function parameters were estimated from the first-order condition of consumer. c was taken to be equal to consumption (both private and public) plus investment from INDEC data. Nominal imports quantity was deduced from import value from INDEC data divided by the exchange rate, obtained from European Bank.

$$\ln e = \ln \frac{1-\alpha}{\alpha} + (\kappa - 1)\left(\ln \frac{c/y}{m/y}\right).$$

Estimates are following:

Table 5: Estimation results, output dynamics.

Variable	**Coefficient**	**(Std. Err.)**
$1-\kappa$	0.1553*	(0.0440)
$\ln \frac{1-\alpha}{\alpha}$	-0.3472*	(0.0972)

Within a 95% condfidence interval, we can conclude that $\kappa < 1$, so goods are not perfect substitutes, and $\alpha > 0.5$. With the obvious transformations of the estimation in Table 5, $\kappa = 0.8447$ and $\alpha = 0.5859$. Again, the regression is not perfect. An ARIMA(2,1,1) model specification seems to perform better to correct for the nonstationarity for which exchange rates are notorious.

B.3 Import-Export Equation

The most interesting regression seems to be the trade equation. It was estimated by nonlinear least squares:

$$\ln m_t = \ln(1-\pi)I(\text{punished at } t) + \ln\theta_1 + \theta \ln(x_t - \theta_0) + \epsilon_t.$$

Here we decided to allow the import-export equation to be nonlinear (θ not necessarily equal to 1), have a fixed cost (θ_0 is not necessarily equal to 0). We do not explore why it happens that firms can earn positive profits in equilibrium; we just allow estimates to signal us about that. These degrees of freedom are not necessary, and they don't drive our main result. However, they certainly help to achieve better fit of default responses.

For numbers on imports and exports, we smoothed quarterly fluctuations on the INDEC data using an HP filter Hodrick and Prescott (1997) with smoothing parameter 400.

The estimate of $\ln(1-\pi)$ suggests that π is equal to 0.5 with surprising precision. We will use this number in model outcome calculation; to make sure we don't fall victim of regression's non-robustness, we also solve for the model with $\pi = 0.8$ and $\pi = 0.2$, so whatever is the real value

Table 6: Estimation results, import-export conditions.

Variable	Coefficient	(Std. Err.)
θ	0.2082*	(0.0763)
θ_0	0.0467*	(0.0070)
θ_1	0.1959*	(0.0409)
$\ln(1-\pi)$	-0.6939*	(0.0256)

of π, the real outcome will be lying in between these two models' outcomes. As for values of θs, they are not too far away from the ones a person can get by estimating the 1993-2000 subsample of "no default."

We estimate this equation on a subsample of 1993-2003. The reason why we do not continue on the sample of 2004 and further is that it seems that Argentine does not have the same instant recovery of terms of trade after default as we have assumed in the model. Consider the time series of $u_t = \frac{\Delta m_t}{\Delta x_t}\frac{x_t}{m_t}$, a measure of elasticity of change in import with the change of export, presented on Figure 11. One can see an approximately constant elasticity during most of 1990s, then a decline, a series of definite changes in structure of equation, and an increase in 2004, with a stable more than 2 elasticity after default. The big jump down in 2001 is what we try to capture with $(1-\pi)$ multiplier. In this estimation, we allow for a period of adjustment, a process of re-establishing of connections lost in 2001. Imports do not only increase due to an increase in x, but also from a renewed efficiency of trading. Summarizing, we don't use the data after default because we believe that after forgiving the default import depends not only on export on the same period, but also on export on previous periods.

C Solving for First Best

Solving the "first best" general equilibrium is equivalent to solving the following convex programming problem, instead of the stochastic problem established above. Values for endogenous variables

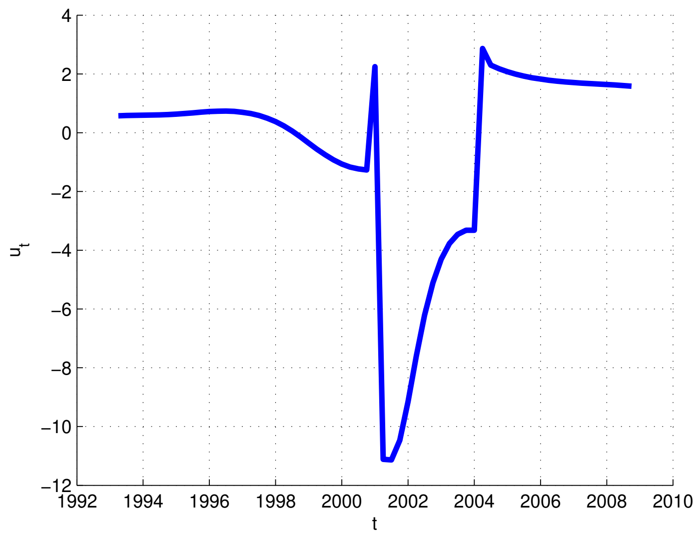

Figure 11: Time series of $u_t = \frac{\Delta m_t}{\Delta x_t}\frac{x_t}{m_t}$, "sample elasticity."

carry the subscript z_{fb}

$$\begin{aligned} V(\bar{y}) &= \max_{c_{fb}, m_{fb}} \{\alpha c_{fb}^{\kappa} + (1-\alpha) m_{fb}^{\kappa}\}^{(1-\sigma)/\kappa} + \beta V(\bar{y}), \\ &\text{subject to:} \\ c_{fb} + e_{fb} m_{fb} + b &= \bar{y} + \Pi, \\ m_{fb} &= \theta_1 (x_{fb} + \theta_0)^{\theta}. \end{aligned}$$

The solution to which gives us

$$\bar{y} + \Pi = c_{fb} + e_{fb}\theta_1 (x_{fb} + \theta_0)^{\theta}, \tag{9}$$

$$\alpha c_{fb}^{\kappa-1} = e_{fb}^{-1}(1-\alpha)(x_{fb} + \theta_0)^{\theta(\kappa-1)}, \tag{10}$$

$$m_{fb} = \theta_1 (x_{fb} + \theta_0)^{\theta}. \tag{11}$$

The importer's problem, gives two more conditions for the optimal quantity of exports and the level of profits.

$$\begin{aligned} \Pi_{fb} &= e_{fb} m_{fb} - x_{fb}, \\ m_{fb} &= \theta_1 (x_{fb} + \theta_0)^{\theta}, \end{aligned}$$

which gives optimal levels of of x_{fb}, Π_{fb}:

$$x = (e_{fb}\theta_1\theta)^{1/(\theta-1)} - \theta_0, \tag{12}$$

$$\Pi = (e\theta_1)^{-1/(\theta-1)}(\theta^{-\theta/(\theta-1)} - \theta^{-1/(\theta-1)}) + \theta_0. \tag{13}$$