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Changes in the productivity of labour and vertically integrated sectors — an empirical study for Italy

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Abstract The object of this paper is to derive measures for the changes in physical labour productivity and to apply them to the case of Italy during the 1995-2000 period. Firstly, section 1, introduces the historical development of selected literature on labour productivity measurement using the notion of vertically integrated sectors and derives measures computable from actual data. In the second place, section 2 describes the series utilised and presents the computation of the previously derived measures. Thirdly, section 3 presents the main results, introducing a typology according to which characterize the determinants behind changes in physical labour productivity, and draws implications for the particular case under study. Some final comments are given in section 4.

Keywords Labour productivity measurement, Vertically integrated sectors, Input-Output analysis.

1 Historical development of selected literature and analytical derivation of productivity measures

The measurement of changes in the physical productivity of labour and its determinants involves the definition of appropriate measures of output and labour-input corresponding to a particular unit of analysis (industry, system or *sub-system*), and the ability to trace its physical movement over time. Moreover, the very notion

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of technical change has been subject of sharp disagreement and the inability to accomplish generally accepted conventions on this issue can be explained, at the most fundamental level, by the existence of differing (and incompatible) theories of production, value and distribution in economic theory.

The literature on the measurement of changes in the productivity of labour explicitly emphasizing the role of reproducible intermediate (fixed and circulating) capital goods can be traced to the work of Pasinetti (1959). Furthermore, the specific view of a production system in terms of subsystems was first introduced by Sraffa (1960), and analytically developed by Pasinetti (1973, 1981) by the formulation of the analytical device of a vertically integrated sector.

From an applied perspective, pioneering works on the use of the notion of subsystem were those by Gossling & Dovring (1966) for the study of the agricultural subsystem in the US in the period 1919-1957 and by Gupta & Steedman (1971) for studying the changes in labour productivity in the UK economy in the period 1954-1966. This last work characterized the general pattern of technical change in terms of direct and indirect use of labour and intensity in the use of intermediate produced inputs.

Ten years later, the work of Rampa (1981) studied the case of Italy for the five-year period 1970-1975 and Rampa & Rampa (1982) analysed the Italian economy for the period 1959-1975. The first of these works and the paper by Siniscalco (1982) introduced an interesting methodology which will be followed quite closely in the present paper. In Rampa (1981) we can find disaggregated measures of labour productivity at the sub-system level complemented by a purely technological (independent of relative prices and the composition of the net product) measure of intensity of use of intermediate commodities, obtained by giving a particular interpretation to the maximum eigenvalue associated with the matrix of interindustry coefficients.

The synthetic description of a production system in terms of observable components of an Input-Output table is the starting point for most of the empirical studies of the type previously mentioned. The present work will consider single-product industries and only circulating capital goods entering interindustry transactions. In this case, consider the accounting identities representing the expenditure side of a symmetric table consisting of m industries for two (accounting) time periods $\{0, t\}$:

$$\begin{aligned} \mathbf{x}_0 &\equiv \mathbf{X}_0 + \mathbf{y}_0 \\ \mathbf{x}_t &\equiv \mathbf{X}_t + \mathbf{y}_t \end{aligned} \tag{1.1}$$

where $\mathbf{x}$ is a column vector representing the value of gross output, $\mathbf{X}$ is a symmetric matrix of dimension m representing the value of interindustry transactions of commodities locally produced (x_{ij} being the value of input produced by industry i

sold to industry j), and $\mathbf{y}$ is a column vector representing the value total domestic final demand (value of the net product).

The magnitudes observed are all expressed in nominal terms. Indicating magnitudes in physical quantities with an overbar, had we had vectors of absolute prices for each period ($\mathbf{p}_t$ and $\mathbf{p}_0$), the following relations would hold:

$$\begin{aligned} \mathbf{x}_k &= \widehat{\mathbf{p}}_k \overline{\mathbf{x}}_k \\ \mathbf{y}_k &= \widehat{\mathbf{p}}_k \overline{\mathbf{y}}_k \\ \mathbf{X}_k &= \widehat{\mathbf{p}}_k \overline{\mathbf{X}}_k \end{aligned} \tag{1.2}$$

for $k = \{0, t\}$.

However, considering period $k = 0$ as base year, we can define a price index $\widehat{\mathbf{i}}_t = \widehat{\mathbf{p}}_t \left(\widehat{\mathbf{p}}_0\right)^{-1}$ to deflate nominal magnitudes of period $k = t$ so as to obtain:

$$\mathbf{x}_{t0} = \widehat{\mathbf{p}}_0 \overline{\mathbf{x}}_t = \widehat{\mathbf{i}}_t^{-1} \mathbf{x}_t \tag{1.3}$$

$$\mathbf{y}_{t0} = \widehat{\mathbf{p}}_0 \overline{\mathbf{y}}_t = \widehat{\mathbf{i}}_t^{-1} \mathbf{y}_t \tag{1.4}$$

$$\mathbf{X}_{t0} = \widehat{\mathbf{p}}_0 \overline{\mathbf{X}}_t = \widehat{\mathbf{i}}_t^{-1} \mathbf{X}_t \tag{1.5}$$

where $\mathbf{x}_{t0}$, $\mathbf{y}_{t0}$ and $\mathbf{X}_{t0}$ represent constant price magnitudes.

Consider a given quantity of gross output $\mathbf{x}$. In this case, a matrix of interindustry requirements per unit of gross output can be defined for each period:

$$\mathbf{A}_0 = \mathbf{X}_0 (\widehat{\mathbf{x}}_0)^{-1} \tag{1.6}$$

$$\mathbf{A}_t = \mathbf{X}_t (\widehat{\mathbf{x}}_t)^{-1} \tag{1.7}$$

So far, defined in purely nominal terms, matrix (1.7) is an expenditure coefficient matrix at current prices. However, by replacing $\mathbf{X}_t$ and $\mathbf{x}_t$ in (1.7) with their definition in constant prices derived from (1.5) and (1.3) we shall obtain:

$$\mathbf{A}_{t0} = \widehat{\mathbf{i}}_t^{-1} \mathbf{A}_t \widehat{\mathbf{i}}_t \tag{1.8}$$

a direct requirements matrix of intermediate inputs per unit of gross output at constant prices. Strictly speaking, when thinking of a technique, we would ideally refer to:

$$\overline{\mathbf{A}}_k = \left(\widehat{\mathbf{p}}_k\right)^{-1} \mathbf{A}_k \widehat{\mathbf{p}}_k \tag{1.9}$$

for $k = \{0, t\}$. However, using (1.9) and the definition of $\widehat{\mathbf{i}}_t$, following Rampa & Rampa (1982, p. 311) it can be seen that:

$$\mathbf{A}_{k0} = \widehat{\mathbf{i}}_k^{-1} \mathbf{A}_k \widehat{\mathbf{i}}_k = \left(\widehat{\mathbf{p}}_k \left(\widehat{\mathbf{p}}_0\right)^{-1}\right)^{-1} \mathbf{A}_k \left(\widehat{\mathbf{p}}_k \left(\widehat{\mathbf{p}}_0\right)^{-1}\right) = \widehat{\mathbf{p}}_0 \overline{\mathbf{A}}_k \left(\widehat{\mathbf{p}}_0\right)^{-1} \tag{1.10}$$

i.e. matrices $\mathbf{A}_k$, $\mathbf{A}_{k0}$ and $\overline{\mathbf{A}}_k$ are similar, for $k = \{0, t\}$. This is an interesting property to be used below.

More fundamentally, in order to study labour productivity, a measure of labour input has to be introduced in the analytical framework. In this case we shall consider the labour per unit of gross output given by:

$$\mathbf{a}_{n0} = \widehat{\mathbf{x}}_0^{-1}\mathbf{l}_0 \tag{1.11}$$

$$\mathbf{a}_{nt} = \widehat{\mathbf{x}}_t^{-1}\mathbf{l}_t \tag{1.12}$$

representing the absolute measure of labour input (to be effective hours of work per year in this study). Alternatively, by using a constant price measure of gross output we obtain:

$$\mathbf{a}_{nt0} = \widehat{\mathbf{x}}_{t0}^{-1}\mathbf{l}_t \tag{1.13}$$

Similarly as we proceeded with matrix $\mathbf{A}$, we would ideally require our labour-input vectors to be defined as:

$$\overline{\mathbf{a}}_{n0} = \widehat{\overline{\mathbf{x}}}_0^{-1}\mathbf{l}_0 \tag{1.14}$$

$$\overline{\mathbf{a}}_{nt} = \widehat{\overline{\mathbf{x}}}_t^{-1}\mathbf{l}_t \tag{1.15}$$

However, noting from (1.3), (1.12) and (1.13) that $\mathbf{a}_{nt}^T = \mathbf{a}_{nt0}^T\widehat{\mathbf{i}}_t^{-1}$, and from (1.2) and (1.15) that $\mathbf{a}_{nt}^T = \overline{\mathbf{a}}_{nt}^T\left(\widehat{\mathbf{p}}_t\right)^{-1}$ for period t, we obtain:

$$\begin{aligned} \mathbf{a}_{n0} &= \left(\widehat{\mathbf{p}}_0\right)^{-1}\overline{\mathbf{a}}_{n0} \\ \mathbf{a}_{nt0} &= \left(\widehat{\mathbf{p}}_0\right)^{-1}\overline{\mathbf{a}}_{nt} \end{aligned} \tag{1.16}$$

as computable labour-input vectors related to physical quantities by means of the price structure of the base year.

In this way, the basic objects presented so far provide us with two techniques $(\overline{\mathbf{A}}_0, \overline{\mathbf{a}}_{n0}^T)$ and $(\overline{\mathbf{A}}_t, \overline{\mathbf{a}}_{nt}^T)$ with which to measure the *changes* in the productivity of labour.

So far the argument has been carried out in terms of domestically produced intermediate inputs. It is our contention that a correct way to consider imported intermediate commodities in the study of labour productivity is to consider import requirements per unit of gross output as a non-produced (at least, locally) input.

Adding a matrix of intermediate import requirements to each interindustry transactions matrix $\mathbf{X}$ would have been like assuming, for the purpose of measuring labour productivity, that the labour required to produce imported commodities can be obtained by looking at the domestic direct labour-input vector. We consider this procedure to be inadequate, as technological heterogeneity, characterised by unequal changes in the productivity of labour of different sectors in different countries, is one of the main determinants of the existence of trade relations among countries[1]. Therefore, assuming technical homogeneity when considering traded commodities seems counter-intuitive.

Consider therefore the vectors of total import requirements in each industry, $\mathbf{f}_0$ and $\mathbf{f}_t$. Then, we shall define two import requirement vectors per unit of gross output:

$$\mathbf{m}_0 = \widehat{\mathbf{x}}_0^{-1}\mathbf{f}_0 \tag{1.17}$$

$$\mathbf{m}_t = \widehat{\mathbf{x}}_t^{-1}\mathbf{f}_t \tag{1.18}$$

The following rate of change accounts for the change in the use of import requirements per value unit of gross output between 0 and t:

$$\left(\Delta\widehat{\mathbf{m}}\right)\left(\widehat{\mathbf{m}}_0\right)^{-1} = \widehat{\mathbf{m}}_t\left(\widehat{\mathbf{m}}_0\right)^{-1} - \mathbf{I} \tag{1.19}$$

It should be clear that throughout the text we will aim at singling out *changes* in productivity, and we shall not interpret absolute values, except otherwise stated. This is so because constant price magnitudes are different from physical quantities (clear to be seen in (1.16)). As the absolute value of a constant price magnitude depends upon the particular price structure of the base year[2], by limiting ourselves to interpreting the rate of change of a constant price variable, we are ruling out the corresponding price structure, allowing for an interpretation in real physical terms.

Hence, in traditional Input-Output analysis, a comparison of both equations in (1.16) with reference to one of them provides us with a measure of the changes in direct labour productivity, at a strictly disaggregated level:

$$\begin{aligned}\left(\Delta\widehat{\boldsymbol{\mu}}\right)\left(\widehat{\boldsymbol{\mu}}_0\right)^{-1} &= \widehat{\mathbf{a}}_{nt0}\widehat{\mathbf{a}}_{n0}^{-1} - \mathbf{I}\\ &= \left(\widehat{\mathbf{p}}_0\right)^{-1}\widehat{\overline{\mathbf{a}}}_{nt}\left(\left(\widehat{\mathbf{p}}_0\right)^{-1}\widehat{\overline{\mathbf{a}}}_{n0}\right)^{-1} - \mathbf{I}\\ &= \widehat{\overline{\mathbf{a}}}_{nt}\widehat{\overline{\mathbf{a}}}_{n0}^{-1} - \mathbf{I}\end{aligned} \tag{1.20}$$

This measure, however, is defined with respect to gross output, and not to sectoral value added. This aspect is crucial to point out. Following Rampa (1981, p. 5) “there is no obvious reason why value added should be the most important technical measure of output by sector”. In this way, many studies based on the notion of a sectoral neoclassical production function using non-reproducible capital and labour obtaining a value added output measure would imply completely

[1] See Pasinetti (1981, Chapter XI) for details.

[2] A constant price expenditure coefficients matrix is different from a technical coefficients matrix, as it still depends on $\mathbf{p}_0^T$, which (in a long-period price equation system) depends on $\overline{\mathbf{A}}_0$, the ruling wage rate and rate of profit of the base year.

different (and to our judgement, incorrect) measures of direct labour productivity. In an Input-Output framework, gross output (at constant prices) is a more appropriate technical measure of output at the industry level for computing changes in direct labour productivity.

However, an interesting thing occurs at the system level. The notion of net product assumes importance as it stands out as the given final (effective) demand of the system. At this level, as regards the input of labour, we aim at measuring the change in the use of total labour per unit of net product (at constant prices). Therefore, while at the industry level a measure of direct labour productivity involved the sectoral gross output, at the system level we have $\mathbf{a}_{nk}^T \mathbf{x}_k = \mathbf{l}_k^T \mathbf{u} = L_k$ for $k = \{0, t\}$, where $\mathbf{u}$ is an appropriate unit vector, and we can define:

$$\mu_0 = (\mathbf{l}_0^T \mathbf{u})(\mathbf{y}_0^T \mathbf{u})^{-1} \tag{1.21}$$

$$\mu_{t0} = (\mathbf{l}_t^T \mathbf{u})(\mathbf{y}_{t0}^T \mathbf{u})^{-1} \tag{1.22}$$

as average measures of total labour productivity, involving the net product of each year (at constant prices).

There is an important connection between this type of reasoning and the notion of a subsystem introduced by Sraffa and its refinement by Pasinetti, synthesized and expanded with the analytical device of vertical integration. When the (physical) net product vector $\overline{\mathbf{y}}$ is taken as a reference point by which to reorganize general interdependence in an Input-Output system, the notion of labour productivity at the sectoral level shall not be confined only to its direct measure, as the total labour embodied in the production of a specific part of the net product acquires essential importance.

As rightly pointed out by Rampa (1981, p. 14), following Sraffa (1960, p. 89): "a system can be subdivided into as many parts as there are commodities in its net product, in such a way that each part forms a smaller self-replacing system the net product of which consists of only one kind of commodity. These parts we shall call 'sub-systems'.". The sub-system, then, turns out to be a unit of analysis by which the disaggregated description of a technique $(\overline{\mathbf{A}}, \overline{\mathbf{a}}_n^T)$ is reorganized into as many parts as there are final commodities in $\overline{\mathbf{y}}$.

Assume a *viable* economic system, as defined in Pasinetti (1973, p. 2). For the particular case in which there are only circulating capital goods entering the interindustry transactions matrix, by considering the $n \leq m$ final commodities for which $\overline{\mathbf{y}}_j > 0$ we can define n vertically integrated sectors as:

$$\begin{aligned} \overline{\mathbf{x}}_k^{(j)} &= \left(\mathbf{I} - \overline{\mathbf{A}}_k\right)^{-1} \widehat{\overline{\mathbf{y}}}_k \mathbf{e}_j \\ \mathbf{l}_k^{(j)} &= \overline{\mathbf{a}}_{nk}^T \left(\mathbf{I} - \overline{\mathbf{A}}_k\right)^{-1} \widehat{\overline{\mathbf{y}}}_k \mathbf{e}_j \\ \overline{\mathbf{K}}_k^{(j)} &= \overline{\mathbf{A}}_k \left(\mathbf{I} - \overline{\mathbf{A}}_k\right)^{-1} \widehat{\overline{\mathbf{y}}}_k \mathbf{e}_j \end{aligned} \tag{1.23}$$

for $k = \{0, t\}$, $j = \{1, \ldots, n\}$, with $\mathbf{e}_j$ being a column null vector except for an entry equal to one in row j. We can further define the following magnitudes present in (1.23) as:

$$\overline{\mathbf{v}}_k^T = \overline{\mathbf{a}}_{nk}^T \left(\mathbf{I} - \overline{\mathbf{A}}_k\right)^{-1} \tag{1.24}$$

$$\overline{\mathbf{H}}_k = \overline{\mathbf{A}}_k \left(\mathbf{I} - \overline{\mathbf{A}}_k\right)^{-1} \tag{1.25}$$

for $k = \{0, t\}$. Each coefficient in vector (1.24) represents the quantity of labour directly and indirectly required to obtain one physical unit of commodity j as a final good, and it will so be called "*vertically integrated labour coefficient*" for commodity j. Furthermore, each column of matrix (1.25), denoted as vector $\overline{\mathbf{h}}_{kj}$, represents a series of heterogeneous physical quantities (a particular composite commodity) which are directly and indirectly required as intermediate circulating capital goods to obtain one unit of commodity j as a final good, and it will be referred to as a "*unit of vertically integrated productive capacity*" for commodity j (Pasinetti 1973, p. 6).

In this way, $(\overline{v}_j, \overline{\mathbf{h}}_j)$ represents a *vertically integrated sector* defined with respect to final commodity j, and we can obtain an alternative description of the technique of the economic system by considering $(\overline{\mathbf{v}}^T, \overline{\mathbf{H}})$. As stated by Pasinetti (1973, p. 6) "A vertically integrated sector is therefore a compact way of representing a subsystem".

To the particular purpose of measuring changes in the physical productivity of labour, we will adopt the vertically integrated sector as our disaggregated unit of analysis from this moment on. A discussion of this point is carried out in Siniscalco (1982, p. 484-485). Essentially, though changes in productivity *originate* at the industry level, it is quite unlikely that the effects of these changes are all kept to itself. General interdependence makes a labour-saving improvement in one industry to induce technical change in all those industries buying the input produced by the technically improving branch. A consistent way of taking into account the cumulative effect of all these interdependencies is to work with subsystems as the disaggregated unit of analysis.

Furthermore, as has been advocated by Rampa (1981, p. 11-12) and De Juan & Febrero (2000, p. 67), it is our contention that a consistent measure of changes in total labour productivity at the disaggregated level can be obtained by studying the changes in the total labour requirements of each vertically integrated sector.

To arrive at our desired measure, we shall first relate direct and indirect labour. Consider (1.14), (1.15), (1.24) and (1.25). By working as a series expansion the definition of (1.24):

$$\begin{aligned}\overline{\mathbf{v}}_k^T &= \overline{\mathbf{a}}_{nk}^T + \overline{\mathbf{a}}_{nk}^T \overline{\mathbf{A}}_k + \overline{\mathbf{a}}_{nk}^T \overline{\mathbf{A}}_k^2 + \overline{\mathbf{a}}_{nk}^T \overline{\mathbf{A}}_k^3 \ldots \\ &= \overline{\mathbf{a}}_{nk}^T + \overline{\mathbf{a}}_{nk}^T \overline{\mathbf{A}}_k \left(\mathbf{I} + \overline{\mathbf{A}}_k + \overline{\mathbf{A}}_k^2 \ldots\right) \\ &= \overline{\mathbf{a}}_{nk}^T + \overline{\mathbf{a}}_{nk}^T \overline{\mathbf{A}}_k \left(\mathbf{I} - \overline{\mathbf{A}}_k\right)^{-1} \\ &= \overline{\mathbf{a}}_{nk}^T + \overline{\mathbf{a}}_{nk}^T \overline{\mathbf{H}}_k \\ &= \overline{\mathbf{a}}_{nk}^T + \overline{\mathbf{a}}_{nik}^T\end{aligned} \tag{1.26}$$

for $k = \{0, t\}$. When comparing the first and the last term of (1.26), total labour can be decomposed into a direct component ($\overline{\mathbf{a}}_{nk}^T$) and an indirect component ($\overline{\mathbf{a}}_{nik}^T = \overline{\mathbf{a}}_{nk}^T \overline{\mathbf{H}}_k$), in which direct labour embodied weights the participation of each element of the composite commodity $\overline{\mathbf{h}}_{jk}$ in the indirect labour required to obtain a unit of final commodity j.

Our discussion so far has been carried out in terms of physical quantities. However, in order to work out measures based on actual data, we have to reintroduce nominal magnitudes. Considering $\mathbf{A}_k$ from (1.9), it is possible to obtain the following similarity result[3]:

$$(\mathbf{I} - \mathbf{A}_k)^{-1} = \widehat{\mathbf{p}}_k \left(\mathbf{I} - \overline{\mathbf{A}}_k\right)^{-1} (\widehat{\mathbf{p}}_k)^{-1} \tag{1.27}$$

for $k = \{0, t\}$. But we also know from (1.2), (1.15) and (1.14) that $\overline{\mathbf{a}}_{nk}^T = \mathbf{a}_{nk}^T \widehat{\mathbf{p}}_k$, for $k = \{0, t\}$. Therefore, starting from (1.24) we get:

$$\begin{aligned}\overline{\mathbf{v}}_k^T &= \overline{\mathbf{a}}_{nk}^T \left(\mathbf{I} - \overline{\mathbf{A}}_k\right)^{-1} \\ &= \mathbf{a}_{nk}^T \widehat{\mathbf{p}}_k (\widehat{\mathbf{p}}_k)^{-1} (\mathbf{I} - \mathbf{A}_k)^{-1} \widehat{\mathbf{p}}_k \\ &= \mathbf{a}_{nk}^T (\mathbf{I} - \mathbf{A}_k)^{-1} \widehat{\mathbf{p}}_k\end{aligned} \tag{1.28}$$

for $k = \{0, t\}$. Furthermore, for the case where $k = t$, considering (1.8), and (1.10) and the fact that $\mathbf{a}_{nt0}^T = \mathbf{a}_{nt}^T \widehat{\mathbf{i}}_t$, we have:

$$\begin{aligned}\overline{\mathbf{v}}_t^T &= \mathbf{a}_{nt}^T (\mathbf{I} - \mathbf{A}_t)^{-1} \widehat{\mathbf{p}}_t \\ &= \mathbf{a}_{nt}^T \widehat{\mathbf{i}}_t (\mathbf{I} - \mathbf{A}_{t0})^{-1} \widehat{\mathbf{i}}_t^{-1} \widehat{\mathbf{p}}_t \\ &= \mathbf{a}_{nt0}^T (\mathbf{I} - \mathbf{A}_{t0})^{-1} \widehat{\mathbf{p}}_0\end{aligned} \tag{1.29}$$

In this way, considering (1.28) for $k = 0$ and the last expression of (1.29) we can compute the appropriate measures for the total requirements of labour of each vertically integrated sector in each period:

$$\overline{\mathbf{v}}_0^T (\widehat{\mathbf{p}}_0)^{-1} = \mathbf{a}_{n0}^T (\mathbf{I} - \mathbf{A}_0)^{-1} = \mathbf{v}_0^T \tag{1.30}$$

[3] See Rampa (1981, p. 30, Theorem 3) for a proof.

$$\overline{\mathbf{v}}_t^T\left(\widehat{\mathbf{p}}_0\right)^{-1} = \mathbf{a}_{nt0}^T\left(\mathbf{I}-\mathbf{A}_{t0}\right)^{-1} = \mathbf{v}_{t0}^T \tag{1.31}$$

both equations relate labour coefficients in terms of physical quantities with nominal ones by means of the same price structure of the base year. Hence, we shall define:

$$\begin{aligned}\left(\Delta\widehat{\mathbf{v}}\right)\left(\widehat{\mathbf{v}}_0\right)^{-1} &= \widehat{\mathbf{v}}_{t0}\widehat{\mathbf{v}}_0^{-1} - \mathbf{I}\\ &= \left(\widehat{\mathbf{p}}_0\right)^{-1}\widehat{\overline{\mathbf{v}}}_t\left(\left(\widehat{\mathbf{p}}_0\right)^{-1}\widehat{\overline{\mathbf{v}}}_0\right)^{-1} - \mathbf{I}\\ &= \widehat{\overline{\mathbf{v}}}_t\widehat{\overline{\mathbf{v}}}_0^{-1} - \mathbf{I}\end{aligned} \tag{1.32}$$

as our measure of changes in total physical labour productivity.

As might be expected, the same argument can be carried out with respect to indirect labour as well. In this case, considering $\overline{\mathbf{a}}_{nik}^T = \overline{\mathbf{a}}_{nk}^T\overline{\mathbf{H}}_k$ for $k=\{0,t\}$, $\overline{\mathbf{a}}_{nit}^T\left(\widehat{\mathbf{p}}_0\right)^{-1} = \overline{\mathbf{a}}_{nt0}^T\overline{\mathbf{H}}_{t0} = \mathbf{a}_{nit0}^T$ and $\overline{\mathbf{H}}_{t0} = \widehat{\mathbf{i}}_t^{-1}\mathbf{H}_t = \widehat{\mathbf{p}}_0\overline{\mathbf{H}}_t$ we obtain a measure for the changes in the physical requirements of indirect labour, defined as:

$$\begin{aligned}\left(\Delta\widehat{\mathbf{a}}_{ni}\right)\left(\widehat{\mathbf{a}}_{ni0}\right)^{-1} &= \widehat{\mathbf{a}}_{nit0}\widehat{\mathbf{a}}_{ni0}^{-1} - \mathbf{I}\\ &= \left(\widehat{\mathbf{p}}_0\right)^{-1}\widehat{\overline{\mathbf{a}}}_t\left(\left(\widehat{\mathbf{p}}_0\right)^{-1}\widehat{\overline{\mathbf{a}}}_{ni0}\right)^{-1} - \mathbf{I}\\ &= \widehat{\overline{\mathbf{a}}}_{nit}\widehat{\overline{\mathbf{a}}}_{ni0}^{-1} - \mathbf{I}\end{aligned} \tag{1.33}$$

An alternative analytical description of a sub-system has been developed by Gossling (1972, Appendix A), and taken by Rampa (1981, p. 14) in order to reorganize general interdependence in terms of vertically integrated sectors. This approach basically consists in the definition of the following linear operator to map industry information into sub-systems:

$$\mathbf{S}_0 = \widehat{\mathbf{x}}_0^{-1}(\mathbf{I}-\mathbf{A}_0)^{-1}\widehat{\mathbf{y}}_0 \tag{1.34}$$

$$\mathbf{S}_t = \widehat{\mathbf{x}}_t^{-1}(\mathbf{I}-\mathbf{A}_t)^{-1}\widehat{\mathbf{y}}_t \tag{1.35}$$

which can be decomposed in $\mathbf{B}_k = \left(\mathbf{I}-\mathbf{A}_k\right)^{-1}\widehat{\mathbf{y}}_k$, for $k=\{0,t\}$ (where b_{ij} describes the value of gross output of industry i directly and indirectly necessary to produce the value of net product of final commodity j) and $\widehat{\mathbf{x}}_k^{-1}$, which can be obtained by diagonalizing vector $\mathbf{B}_k\mathbf{u} = \mathbf{x}_k$. Therefore, the first component of $\mathbf{S}_k$ makes each row of $\mathbf{B}_k$ to be expressed as a proportion of the value of gross output of industry i. In this way, s_{ij} represents the proportion of the value of gross output of industry i for the production of the value of net product of final commodity j. It can be seen that the rows of $\mathbf{S}_k$ sum up to one. Furthermore, it can be proved that the following holds:

$$\mathbf{S}_k = \widehat{\mathbf{x}}_k^{-1}(\mathbf{I}-\mathbf{A}_k)^{-1}\widehat{\mathbf{y}}_k = \widehat{\overline{\mathbf{x}}}_k^{-1}(\mathbf{I}-\overline{\mathbf{A}}_k)^{-1}\widehat{\overline{\mathbf{y}}}_k \tag{1.36}$$

for $k = \{0, t\}$, i.e. the operator $\mathbf{S}_k$ is independent of prices[4]. It must be noted, however, that it does depend on the composition of the net product of each period.

The operator $\mathbf{S}_k$ can be applied to the matrix of interindustry transactions in value terms $\mathbf{X}_k$ so as to obtain:

$$\begin{aligned} \mathbf{X}_k\mathbf{S}_k &= \mathbf{X}_k \left(\widehat{\mathbf{x}}_k\right)^{-1} \left(\mathbf{I} - \mathbf{A}_k\right)^{-1} \widehat{\mathbf{y}}_k \\ &= \mathbf{A}_k \left(\widehat{\mathbf{x}}_k\right)^{-1} \left(\mathbf{I} - \mathbf{A}_k\right)^{-1} \widehat{\mathbf{y}}_k \\ &= \widehat{\mathbf{p}}_k \overline{\mathbf{A}}_k \left(\mathbf{I} - \overline{\mathbf{A}}_k\right)^{-1} \widehat{\overline{\mathbf{y}}}_k \\ &= \widehat{\mathbf{p}}_k \overline{\mathbf{H}}_k \widehat{\overline{\mathbf{y}}}_k \end{aligned} \tag{1.37}$$

for $k = \{0, t\}$. It can be seen that an immediate connection arises between matrix $\overline{\mathbf{H}}$ (in physical terms) developed by Pasinetti (1973) and the operator $\mathbf{S}$ formulated by Gossling (1972). It must be noted, however, that both devices were obtained with different aims, and with different implications[5]. Moreover, it stands out from the right hand-side of the last equality in (1.37) that while operator $\mathbf{S}$ is independent of prices, this is not the case for matrix $\mathbf{XS}$. Therefore, to be consistent with the procedures developed so far, when utilising this operator, we shall always pre-multiply by constant price magnitudes.

For the particular case at hand, we will use operator $\mathbf{S}$ in order to obtain direct requirement matrices for each sub-system. By post-multiplying a direct requirement matrix in constant prices by the diagonal matrix obtained from each of the columns $\mathbf{s}_j$ of matrix $\mathbf{S}$ we shall obtain a series of n matrices $\mathbf{A}^{(j)}$ (one for each vertically integrated sector j). Each element $a_{ik}^{(j)}$ of $\mathbf{A}^{(j)}$ stands for the value of input i that industry k uses for the production of the value of the net product of sector j, expressed as a proportion of the value of the gross output of industry k (with all absolute magnitudes in constant prices). Therefore, we shall compute:

$$\mathbf{A}_0^{(j)} = \mathbf{A}_0 \widehat{\mathbf{s}}_0^{(j)} = \mathbf{X}_0 \left(\widehat{\mathbf{x}}_0\right)^{-1} \widehat{\mathbf{s}}_0^{(j)} = \mathbf{X}_0^{(j)} \left(\widehat{\mathbf{x}}_0\right)^{-1} \tag{1.38}$$

$$\mathbf{A}_{t0}^{(j)} = \mathbf{A}_{t0} \widehat{\mathbf{s}}_t^{(j)} = \mathbf{X}_{t0} \left(\widehat{\mathbf{x}}_{t0}\right)^{-1} \widehat{\mathbf{s}}_t^{(j)} = \mathbf{X}_{t0}^{(j)} \left(\widehat{\mathbf{x}}_{t0}\right)^{-1} \tag{1.39}$$

for $j = \{1, \ldots, n\}$. It can be seen that the sum of all matrices $\mathbf{A}^{(j)}$ add up to matrix $\mathbf{A}$. Each of these matrices distributes the use of produced intermediate commodities of the whole system as it is required by each vertically integrated sector in order to produce its net product.

The objective of having computed (1.38) and (1.39) has been to obtain from them a purely technological indicator measuring the weight that produced intermediate commodities have in the total input requirements of each vertically

[4]See Rampa (1981, p. 30, Theorem 4) for a proof.

[5]It suffices only to mention the implications of matrix $\overline{\mathbf{H}}$ in the theory of value and distribution after the work of Sraffa (1960). See Pasinetti (1973, p. 7-9).

integrated sector. As we had previously assumed our single-product system to be *viable*, and the net product associated to each sub-system to be strictly positive, an appropriate measure for this task is the maximum eigenvalue associated to $\mathbf{A}^{(j)}$: $\lambda^*(\mathbf{A}^{(j)})$[6].

To justify the contention that this eigenvalue summarizes the intensity in the use of intermediate domestically produced inputs, following Rampa (1981, p. 28, Corollary 3), we shall adopt a particular normalization and replace in the definition of the eigenvalue problem. Consider a direct requirements matrix $\overline{\mathbf{A}}$. Take from the subspace generated by the eigenvector associated to $\lambda^* = \lambda^*(\overline{\mathbf{A}})$ that particular $\mathbf{x}^*$ such that $\mathbf{u}^T\mathbf{x}^* = 1$. Let $\overline{\mathbf{A}}\mathbf{x}^* = \lambda^*\mathbf{x}^*$. By pre-multiplying both sides by $\mathbf{u}^T$, we shall obtain: $\lambda^* = \mathbf{u}^T\overline{\mathbf{A}}\mathbf{x}^*$, as $\mathbf{u}^T\mathbf{x}^* = 1$. Therefore, given that $\mathbf{u}^T\overline{\mathbf{A}}$ is a vector whose elements represent the sum of the columns of $\overline{\mathbf{A}}$, we conclude that λ^* can be interpreted as a convex linear combination of the proportion of domestically produced inputs to gross output in each industry, where the weights are given by vector $\mathbf{x}^*$.

This is a particularly interesting result considering the similarity property obtained in (1.10). As similar matrices have the same eigenvalues, the meaning of λ^* remains unaltered whether we work with current, constant price, or even physical quantities direct requirement matrices. We must be aware that, even though the maximum eigenvalue is a purely technological indicator, (1.38) and (1.39) depend on the particular price structure of the base year (because they have been obtained by pre-multiplying matrix $\mathbf{S}$ by a matrix expressed in constant prices), and on a particular composition of the net product (as can be seen from the definition of $\mathbf{S}$). Therefore, we shall compute $\lambda^*(\mathbf{A}_0^{(j)})$ and $\lambda^*(\mathbf{A}_{t0}^{(j)})$ and calculate the rate of change observed between period 0 and t.

Hence, our indicator synthesizing the intensity of intermediate absorptions for each vertically integrated sector in each period will be:

$$\left(\Delta\lambda^*(\mathbf{A}^{(j)}\right)/\lambda^*(\mathbf{A}_0^{(j)}) = \frac{\lambda^*(\mathbf{A}_{t0}^{(j)}) - \lambda^*(\mathbf{A}_0^{(j)})}{\lambda^*(\mathbf{A}_{t0}^{(j)})} \tag{1.40}$$

for $j = \{1, \ldots, n\}$.

With all the elements defined so far, a deeper analysis of the changes in physical labour productivity at the disaggregated level can be carried out. We shall compute in the following section the indicators provided by (1.20) (changes in direct labour requirements per unit of gross output), (1.32) (changes in total labour requirements per unit of net product), (1.33) (changes in indirect labour requirements per unit of

[6]According to Perron-Frobenius theorems, this eigenvalue will also be the maximum modulus one and the only one for which we can find an associated eigenvector with all non-negative components. For details and proofs, see Pasinetti (1977, p. 267-276).

net product) and (1.40) (changes in the intensity of use of produced intermediate commodities) for each vertically integrated sector in two (national accounting) time periods.

As has been argued, the magnitude of the reduction (increase) of total labour requirements will measure the increase (decrease) of physical labour productivity. It is our contention that this movement may be understood by interpreting the co-movement of its proposed determinants (changes in direct and indirect labour requirements and the maximum eigenvalue of the associated sub-system matrix $\mathbf{A}^{(j)}$).

2 Data description and computation of measures

The empirical study has been conducted with Input-Output and National Account disaggregated (using the NACE classification) data for Italy for the 1995-2000 period, which has been obtained from EUROSTAT. As regards Input-Output tables we have considered symmetric tables at current purchasers' prices for 1995 and 2000. All the work (where necessary) has been carried out in constant prices. The year 1995 has been adopted as base year, so a price index vector was obtained in order to deflate the relevant components of the 2000 IO table: interindustry transactions matrix ($\mathbf{X}_t$), net product vector ($\mathbf{y}_t$, domestic final demand), gross domestic output vector ($\mathbf{x}_t$) and imported inputs vector ($\mathbf{f}_t$). The price index vector ($\mathbf{i}_t$) was built from two series of disaggregated net output at current and past year prices, constructing a chain-price index at the 4-digit industry level.

As for the net product vector $\mathbf{y}$, it consists of domestic final demand excluding the 'changes in inventories and valuables' component of capital formation.

As regards the labour component of each technique, the labour input vector ($\mathbf{e}_t$) in thousand of effective hours per year has been obtained from the EUROSTAT National Accounts database. Originally, employment data was obtained for the 58 (4-digit) industries considered in terms of thousand of employees, while the total effective hours were only available at a more aggregate (2-digit) industry level. Therefore, total effective hours in each industry at the 4-digit level were computed by distributing the effective hours of each 2-digit aggregate over the corresponding 4-digit sub-items according to the participation in the total number of employees of each 4-digit entry in the 2-digit aggregate.

In this way, by assuming period $0 = 1995$ and period $t = 2000$ in all the equations above, and applying them to the actual data we obtain a full series of matricial objects with which to analyse the changes in the productivity of labour between 1995 and 2000 in Italy.

Out of the 58 industries present in the Input-Output tables, non-basic commodities producing industries ('Private households with employed persons') and industries producing commodities whose net product (excluding changes in inventories and valuables) in the Italian economy is zero ('Extraction of crude petroleum and natural gas; service activities incidental to oil and gas extraction excluding surveying' and 'Recycling') were not considered as vertically integrated sectors.

The complete results of the computations performed are presented in a synthetic way in Table 6, which is included at the end of the paper, accompanied by a named description of the industry coding classification adopted. The columns of Table 6 can be described as follows:

- $\frac{\Delta a_{nj}}{a_{nj,95}}$: stands for each element of the diagonal matrix (1.20).
- $\frac{\Delta(\mathbf{a_n}^T \mathbf{h}_j)}{\mathbf{a_n}_{95}^T \mathbf{h}_{j,95}}$: stands for each element of the diagonal matrix (1.33).
- $\frac{\Delta v_j}{v_{j,95}}$: stands for each element of the diagonal matrix (1.32).
- $\frac{\Delta \mathbf{u}^T \mathbf{a}_j}{\mathbf{u}^T \mathbf{a}_{j,95}}$: stands for the rate of change of the sum of intermediate domestic-produced purchases per unit of gorss output between 1995 and 2000 for each vertically integrated sector.
- $\frac{\Delta\lambda^*(\mathbf{A}^{(j)})}{\lambda^*(\mathbf{A}_{95}^{(j)})}$: stands for each element of vector (1.40).
- $\frac{y_{j,95}}{\mathbf{u}^T \mathbf{y}_{95}}$: stands for the participation of sector j in the net product of the base year 1995.
- $\frac{\Delta y_j}{y_{j,95}}$: stands for the rate of change of the net product of sector j between 1995 and 2000.

3 Discussion of results

Table 6 can be studied more in depth by looking, for each sector, at the changes in total labour requirements $\mathbf{v}^T$; in indirect labour requirements $\mathbf{a}_n^T \mathbf{H} = \mathbf{a}_{ni}^T$; and in the maximum eigenvalue of the coefficient matrix of the associated subsystem $\lambda^*(\mathbf{A}^{(j)})$.

In each vertically integrated sector, a decrease (increase) in total labour $\mathbf{v}$ means an increase (decrease) in labour productivity. However, it does not provide enough information about the *determinants* of such an increase (decrease).

Studying the relative movements of direct and indirect labour lying behind such total variation, together with the movements of $\lambda^*(\mathbf{A}^{(j)})$, can provide useful insights for understanding the characteristics of the process of technical change and structural dynamics which have taken place between the two periods.

As explained in section 1, the $\lambda^*(\mathbf{A}^{(j)})$'s are indicators of the intensity of the use of intermediate (domestic-)produced commodities in the various production processes, conforming each vertically integrated sector. An increase in the maximum eigenvalue of the associated subsystem coefficient matrix means an increase in the weight of (domestic-)produced intermediate inputs with respect to non-produced ones — labour and imported inputs. Such an increase can be due either to a quantity increase or to a quality improvement of used up commodities.

Anyway, an increase in $\lambda^*(\mathbf{A}^{(j)})$ does not necessarily imply higher indirect labour requirements. It could also be the case that such an increase is accompanied by a shift to less labour-using intermediate inputs, to such an extent that, as a final result, indirect labour decreases instead of rising. Or else, it could be the case that such an increase is accompanied by a growth of total labour productivity in the sectors producing intermediate inputs, so as to more than compensate the increased usage of intermediate inputs themselves. According to whether this is, or is not, the case, we can infer important information about what has happened to the production processes we are analysing.

In order to exploit such information, a classification has been made between sectors in which total labour productivity has increased (43 sectors, tables 1, 2, 3) or decreased (12 sectors, tables 4, 5(a), 5(b), 5(c) and 5(d)). Both categories have been further sub-classified in four groups according to the relative movements of $\mathbf{a}_{nj}^T$ and $\lambda^*(\mathbf{A}^{(j)})$.[7]

3.1 Productivity increases

Those sectors whose total labour coefficient has decreased have experienced an increase in labour productivity. In the present case, they constitute 43 sectors out of 55, accounting for 77.22% of total net product.[8]

For sectors listed in table 1, both a_{nij} and $\lambda^*(\mathbf{A}^{(j)})$ increase. In these sectors the augmented usage of produced intermediate inputs is accompanied by an

[7] In addition to the columns already described in 2 for Table 6, each of the following tables includes a last column measuring the rate of change of direct import requirements per unit of gross output (import coefficients) in each industry producing the final commodity defining the vertically integrated sector.

[8] Belonging to them, we have all the vertically integrated sectors derived from the following single-product industries: 'Agriculture, hunting and forestry'; 'Mining and quarrying' (except 'Mining of coal and lignite, extraction of peat'); 'Manufacturing' (with the exception of 'Manufacture of office machinery and computers' and 'Manufacture of other transport equipment'); 'Electricity, gas, steam and hot water supply'; 'Construction'; 'Wholesale and retail trade; repair of motor vehicles, motorcycles and personal and household goods'; 'Hotels and restaurants'; 'Transport, storage and communication' (except 'Supporting and auxiliary transport activities'); 'Financial intermediation' (except 'Insurance and pension funding'); 'Renting of machinery and equipment without operator and of personal and household goods' and 'Computer and related activities'; 'Public administration and defence; compulsory social security'; and 'Other community, social, personal service activities'.

increase of indirect labour. Notwithstanding this, total labour productivity has risen, showing that direct labour must have decreased to such an extent as to offset the negative indirect effect.

The fact that $\lambda^*(\mathbf{A}^{(j)})$ has increased, as well as both direct and indirect labour, suggests that new intermediate commodities might have been introduced as produced inputs. This may have led to a decrease in direct labour on the one hand — as a consequence of the fact that such inputs require less direct labour — and an increase of indirect labour on the other hand — as a consequence of the fact that their production is more labour-intensive. Probably they are entirely new commodities, and not improved old ones, and this explains why their production requires a greater amount of labour input.

Looking at the table, we can see that all listed sectors (with the only exception of 'Manufacture of furniture') pertain to the production of energy, commerce, transport and communication services, and to real estate and renting activities. These are sectors in which it is likely to see the introduction of new intermediate commodities, produced elsewhere in the economy with an intensive use of labour (think, for example, of research efforts). Moreover, they are all sectors in strong expansion (with the exceptions of 'Manufacture of furniture', 'Electricity, gas, steam and hot water supply' and 'Land transport'), in which one expects to see a high pace of technical change.

To complete the picture, it is worth analysing the import profiles of these sectors. In only three of them ('Sale, maintenance and repair of motor vehicles', 'Wholesale trade and commission trade' and 'Electricity, gas, steam and hot water supply') the import coefficients of the corresponding industries have decreased, while in all the others they have quite strongly increased, especially 'Post and telecommunications' (+58.89%). This means that not only they are increasing the usage of domestic-produced intermediate inputs, but also of imported ones. As a result, the proportion of indirect to total labour is higher than what data suggest.

Table 2 lists those sectors characterised by a decrease of both a_{nij} and $\lambda^*(\mathbf{A}^{(j)}$, i.e. by a decrease in the usage of intermediate produced inputs accompanied by a reduction of indirect labour requirements.

In this case, direct labour requirements and import coefficients could have either decreased or increased, provided that their increase has not been as strong as to offset the positive indirect effects.

The fact that both $\lambda^*(\mathbf{A}^{(j)})$ and a_{nij} have decreased suggests that these sectors have been employing in 2000 a similar bundle of intermediate inputs than in 1995, but in a smaller proportion. The reduction of indirect labour also suggests that, quite likely, an improvement of the labour productivity in the industries producing such intermediate inputs has taken place.

A decrease in direct labour requirements might be due to a more efficient organ-

Table 1: Labour productivity increase with $\Delta a_{nij} > 0$ and $\Delta\lambda^*(\mathbf{A}^{(j)}) > 0$

Sectors	$\frac{y_{j,95}}{\mathbf{u}^T\mathbf{y}_{95}}$	$\frac{\Delta y_j}{y_{j,95}}$	$\frac{a_{nij,95}}{v_{j,95}}$	$\frac{\Delta a_{nij}}{a_{nij,95}}$	$\frac{\Delta a_{nj}}{a_{nj,95}}$	$\frac{\Delta m_j}{m_{j,95}}$
Post and telecommunications	0.86	77.07	72.71	0.74	-44.62	58.89
Computer and related activities	0.54	43.57	68.28	1.07	-8.12	22.46
Sale, maintenance and repair of motor vehicles	2.95	12.57	63.96	11.85	-11.04	-9.18
Recreational, cultural and sporting activities	0.87	29.19	57.18	0.10	-7.44	21.12
Manufacture of furniture; manufacturing n.e.c.	2.55	5.94	54.83	4.73	-16.53	14.09
Electricity, gas, steam and hot water supply	1.12	6.77	48.42	9.91	-36.51	-0.49
Wholesale trade and commission trade, except of motor and motorcycles	4.47	21.34	46.24	2.06	-2.39	-1.10
Retail trade, except of motor vehicles, motorcycles; repair of personal and household goods	6.86	15.74	37.75	1.96	-18.60	14.24
Land transport; transport via pipelines	2.45	5.80	22.37	1.48	-13.63	11.79

isation of the production process and a better use of the existing technology. On the contrary, were we in presence of an increase in direct labour — but this seems quite an unlikely case — we would be induced to think of a sort of 'technological regress'.

Looking at the data we can see that, actually, what we have indicated as a case of 'technological regress' has taken place only in the sector of 'Construction' (+0.92% of direct labour in 2000 with respect to 1995), which is the most important — in terms of participation in the net product (8.27%) — of the 21 sectors listed here.[9] Anyway, it is worth saying that this is a sector where it is quite easy to employ low-cost labour force, also due to the strong migration flows. Moreover, the principal industry conforming the vertically integrated sector has experienced a strong increment of import coefficients (+31.94% between 1995 and 2000), suggesting that part of the reduction in the usage of domestic-produced intermediate commodities is due to their substitution with imported ones.

Apart from this exception, all other sectors have seen a decrease in direct as well as indirect labour requirements, the former effect thus reinforcing the latter.

[9]Such participation has further increased between the two periods under consideration (+7.76%).

The majority of such sectors (12 out of 20) are manufactures.[10] Therefore, they are sectors where a reorganisation of the production process, i.e. a better use of the *existing* technology and, therefore, an increase in direct labour productivity, may play a major role in the increase of total labour productivity. Moreover, they are all sectors in which direct labour has decreased more than average.[11] This, together with the fact that the main diagonal elements of the coefficient matrix corresponding to these sectors is quite strong, i.e. their intermediate inputs are to a great extent made up of their own output (from 9.53% for 'Manufacture of radio, television and communication equipment and apparatus' to 50.85% for 'Manufacture of textiles'), may support the hypothesis that in these sectors there has been technical progress in the production of intermediate inputs.

As to import coefficients, they have decreased in four industries,[12] characterised by a high participation in the net product (accounting for 8.22%) — which has sharply increased during the five years under consideration (27.94% on average) — and have increased in the remaining eight ones,[13] characterised by a smaller participation in the net product (10.18%), which has grown by less (+18.41% on average). Hence, import coefficients are sensibly decreasing in the most important and dynamic industries, indicating that the associated vertically integrated sectors are not only improving their productivity to a great extent, but also exerting backward linkages.

As to the remaining eight sectors, five of them have a very small impact on aggregate net product.[14]

[10]'Tanning, dressing of leather; manufacture of luggage'; 'of electrical machinery and apparatus', 'of chemicals and chemical products', 'of wearing apparel; dressing; dyeing of fur', 'of other non-metallic mineral products', 'of coke, refined petroleum products and nuclear fuel', 'of radio, television and communication equipment and apparatus', 'of basic metals', 'of medical, precision and optical instruments, watches and clocks', 'of food products and beverages', 'of pulp, paper and paper products' and 'of textiles'.

[11]On average, in the whole economic system, the variation of direct labour is around -4.57%. Looking at these 11 sectors, we see that 'Manufacture of medical, precision and optical instruments, watches and clocks' — the one in which direct labour requirements have decreased less — has experienced a decrease of 8.20%, while 'Manufacture of pulp, paper and paper products' — whose decrease has been of -26.27% — is the one in which these have decreased more.

[12]'Manufacture of electrical machinery and apparatus ', -2.37%; 'Manufacture of coke, refined petroleum products and nuclear fuel', -4.22%; 'Manufacture of food products and beverages', -5.24%; and 'Manufacture of textiles', -15.56%.).

[13]from 1.82% for 'Manufacture of basic metals' to 47.97% for 'Manufacture of chemicals and chemical products'.

[14]'Mining of metal ores', 'Renting of machinery and equipment without operator and of personal and household goods', 'Activities of membership organization', 'Other mining

Among the remaining sectors, we have 'Water transport', accounting for 0.35% of the net product and growing at a pace of 12.47% — which has also sensibly decreased its import coefficients (-13.85%) — and 'Air transport', accounting for 0.36% of the net product, growing at a very strong pace (62.04%) and increasing its import coefficients by 10.84%.

Finally, we are left with 'Public administration and defence; compulsory social security'. After 'Construction', it is the most important sector out of the 21 listed here as to its participation to the net output (7.061%). We can see that its participation is growing, even if not in a particularly important way (+2.74%), and that it is reducing its import coefficients (-12.97%).

Table 3 presents those sectors whose decrease in total labour requirements has been accompanied by a decrease in indirect labour and by an increase in $\lambda(\mathbf{A}^{(j)})$. When this combination couples with a decrease in direct labour, we are induced to think of sectors that are using intermediate goods of an improved quality in terms of the quantity of direct labour that they allow to save. On the one hand, in fact, the increase in $\lambda^*(\mathbf{A}^{(j)})$ indicates either an increase in the quantity or an improvement in the quality of used up inputs. On the other hand, the fact that direct labour requirements have decreased induces to think of a quality improvement rather than a quantity increase. Moreover, the fact that indirect labour has decreased suggests that such intermediate commodities in 2000 have a similar composition to that of 1995, but there has been an improvement in labour productivity in the sectors producing them.

The case of increasing direct labour, on the contrary, seems quite unlikely, as it would suggest an increase in the usage of intermediate inputs but coupled with a shift towards less efficient commodities, allowing a reduction of indirect labour — being technologically less advanced — but needing more direct labour to be operated.

And actually, looking at the data, we see that in no case direct labour has increased. On the contrary, it has decreased in all of the seven sectors, usually more than average.

Looking at the table more closely, we can see, similarly to the previous case, that these sectors are mostly manufactures.[15] Moreover, the proportion of own purchases over total intermediate commodity usage is rather high (from 8.34% in

and quarrying' and 'Forestry, logging and related service activities', accounting for 0.35%, though growing quite fast (except the third and the fourth ones).

[15] Five out of seven: 'Manufacture of tobacco products', 'of fabricated metal products, except machinery and equipment', 'Manufacture of rubber and plastic products', 'Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials' and 'Manufacture of motor vehicles, trailers and semi-trailers'.

Table 2: Labour productivity increase with $\Delta \mathbf{a}_{ni}^T < 0$ and $\Delta\lambda^*(\mathbf{A}^{(j)}) < 0$

Sectors	$\frac{y_{j,95}}{\mathbf{u}^T\mathbf{y}_{95}}$	$\frac{\Delta y_j}{y_{j,95}}$	$\frac{a_{nij,95}}{v_{j,95}}$	$\frac{\Delta a_{nij}}{a_{nij,95}}$	$\frac{\Delta a_{nj}}{a_{nj,95}}$	$\frac{\Delta m_j}{m_{j,95}}$
Manufacture of electrical machinery and apparatus n.e.c.	1.169	27.89	71.19	-12.14	-15.11	-2.37
Manufacture of chemicals and chemical products	2.497	35.41	70.39	-12.28	-18.35	47.97
Manufacture of wearing apparel; dressing; dyeing of fur	2.225	8.86	51.59	-12.24	-20.75	11.75
Manufacture of other non-metallic mineral products	0.836	11.48	50.29	-3.05	-18.28	4.66
Manufacture of coke, refined petroleum products and nuclear fuel	0.819	62.41	49.59	-6.73	-12.51	-4.22
Manufacture of radio, television and communication equipment and apparatus	1.053	17.07	49.40	-7.06	-19.19	8.13
Manufacture of basic metals	0.931	24.64	48.23	-17.82	-8.98	1.82
Construction	8.270	7.76	46.69	-3.05	0.92	31.94
Public administration and defence; compulsory social security	7.061	2.74	43.96	-0.11	-6.20	-12.97
Tanning, dressing of leather; manufacture of luggage	1.471	1.64	43.30	-7.17	-8.21	41.42
Manufacture of medical, precision and optical instruments, watches and clocks	0.652	20.78	43.28	-8.04	-8.20	14.63
Manufacture of food products and beverages	4.488	7.61	42.94	-19.07	-12.78	-5.24
Manufacture of pulp, paper and paper products	0.515	27.35	41.12	-15.46	-26.27	5.23
Air transport	0.358	62.04	40.63	-37.01	-50.88	10.84
Water transport	0.352	12.47	32.84	-12.91	-5.48	-13.85
Mining of metal ores	0.001	35.07	31.57	-69.93	-17.45	144.59
Renting of machinery and equipment without operator and of personal and household goods	0.082	32.70	29.22	-5.46	-39.31	-10.77
Manufacture of textiles	1.746	13.83	25.92	-14.48	-12.76	-15.56
Activities of membership organization n.e.c.	0.212	6.01	22.90	-11.61	-7.48	8.52
Other mining and quarrying	0.035	6.50	22.87	-7.21	-26.51	-63.81
Forestry, logging and related service activities	0.020	27.32	3.16	-20.35	-22.73	13.67

'Manufacture of motor vehicles, trailers and semi-trailers' to 64.92% in 'Manufacture of tobacco products') and their direct labour requirements are decreasing more than average,[16] supporting the hypothesis of total labour productivity increase as a result of labour saving at the level of the intermediate inputs-producing industries.

One of these sectors,'Manufacture of tobacco products', had quite a small participation in the 1995 net product (0.09%), which has decreased even further in 2000 (-5.75%).

As to the others, the most important ones as to their participation in the net product are 'Manufacture of motor vehicles, trailers and semi-trailers' (2.23%) and 'Manufacture of fabricated metal products, except machinery and equipment' (1.25%). The four sectors, taken altogether, account for 4.52% of aggregate net product, and are growing, on average, at a pace of 22.19%. Anyway, they are all increasing their import coefficients (+10.86% on average), indicating that they also are increasing the usage of imported inputs. It is also worth noticing that the proportion of indirect to total labour, for all these five sectors, was quite high even in 1995 (from 39.08% in 'Manufacture of motor vehicles, trailers and semi-trailers' to 67.94% in 'Manufacture of fabricated metal products, except machinery and equipment').

The only two sectors in this group that are not manufactures are 'Hotels and restaurants' and 'Agriculture, hunting and related service activities'. Having the latter quite a small participation in the total product (0.77%, even if increasing from 1995 to 2000 at a pace of 27.38%), consider 'Hotels and restaurants', which, on the contrary, has quite a high participation in total net output (4.47%), which has further increased within the period we are considering (+16.68%). The decrease in direct labour is above average (-10.77% against -4.57%) and the proportion of indirect to total labour was quite high in 1995 as well (48.05%). Within the period under consideration, import coefficients have increased (+24.15%), showing that the sector's indirect labour requirements are growing more than what data allow to see.

Finally, table 4 shows the last case of total labour productivity increase, i.e. the one characterised by an increase in indirect labour requirements and a decrease in $\lambda^*(\mathbf{A}_j)$. Clearly, in this case, direct labour requirements *must* decrease in order to offset the increase in indirect ones. This kind of co-movements suggests that the intermediate (produced) commodities used up in these sectors in 2000 are similar to those of 1995, but have been produced using more labour, both direct and

[16] From -8.72% in 'Manufacture of rubber and plastic products' to -36.78% in 'Manufacture of tobacco products' (with the exception of 'Manufacture of fabricated metal products, except machinery and equipment', only -1.13%).

Table 3: Labour productivity increase with $\Delta\mathbf{a}_{ni}^T < 0$ and $\Delta\lambda^*(\mathbf{A}^{(j)}) > 0$

Sectors	$\frac{y_{j,95}}{\mathbf{u}^T\mathbf{y}_{95}}$	$\frac{\Delta y_j}{y_{j,95}}$	$\frac{a_{nij,95}}{v_{j,95}}$	$\frac{\Delta a_{nij}}{a_{nij,95}}$	$\frac{\Delta a_{nj}}{a_{nj,95}}$	$\frac{\Delta m_j}{m_{j,95}}$
Manufacture of tobacco products	0.09	-5.75	78.14	-3.35	-36.78	-4.55
Manufacture of fabricated metal products, except machinery and equipment	1.25	12.84	67.94	-0.02	-1.13	13.81
Manufacture of rubber and plastic products	0.83	26.04	61.92	-7.15	-8.72	6.70
Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials	0.21	23.00	57.31	-13.46	-24.06	21.02
Hotels and restaurants	4.47	16.68	48.05	-4.60	-10.77	24.15
Manufacture of motor vehicles, trailers and semi-trailers	2.23	26.88	39.08	-8.43	-24.97	11.90
Agriculture, hunting and related service activities	0.77	27.38	20.89	-20.59	-23.49	1.43

indirect.[17]

Let us now analyse the data.

First, these six sectors together represent 9.88% of the net product. Four of them have quite a small participation: 'Publishing, printing, reproduction of recorded media', 0.74%, 'Other service activities', 1.05%, 'Activities auxiliary to financial intermediation', 0.1%, and 'Sewage and refuse disposal, sanitation and similar activities', 0.39%. In particular, 'Sewage and refuse disposal, sanitation and similar activities' and 'Publishing, printing, reproduction of recorded media' have seen a further reduction in such percentage from 1995 to 2000 (-3.84% and -4.33%), inducing us to consider them as near irrelevant sectors.

As to the 'Activities auxiliary to financial intermediation', its participation in the net product is really small, but it could be considered together with 'Financial intermediation, except insurance and pension funding'. Altogether, they account for 1.83% of the net product, a percentage that has decreased by 8.07%. Their

[17] The decrease in $\lambda^*(\mathbf{A}^{(j)})$ suggests a decrease in the *quantity* of used up intermediate inputs. But indirect labour is increasing. This means that, *in the sectors producing such intermediate commodities*, there has been an increase in indirect labour requirements, due to the introduction of some new, and better, intermediate produced inputs. If this were not the case, we should conclude that the same has happened in the 'final' sectors. But this would mean using intermediate inputs improved to such extent as to be considered entirely new commodities. In such a case, $\lambda^*(\mathbf{A}^{(j)})$ would increase, and not decrease as it is happening here.

Table 4: Labour productivity increase with $\Delta \mathbf{a}_{ni}^T > 0$ and $\Delta\lambda^*(\mathbf{A}^{(j)}) < 0$

Sectors	$\frac{y_{j,95}}{\mathbf{u}^T\mathbf{y}_{95}}$	$\frac{\Delta y_j}{y_{j,95}}$	$\frac{a_{nij,95}}{v_{j,95}}$	$\frac{\Delta a_{nij}}{a_{nij,95}}$	$\frac{\Delta a_{nj}}{a_{nj,95}}$	$\frac{\Delta m_j}{m_{j,95}}$
Publishing, printing, reproduction of recorded media	0.74	-4.33	60.36	1.13	-9.56	-1.61
Financial intermediation, except insurance and pension funding	1.73	-9.74	53.50	23.84	-28.89	-5.37
Manufacture of machinery and equipment n.e.c.	5.87	10.52	42.19	1.71	-2.99	-2.29
Activities auxiliary to financial intermediation	0.10	23.90	29.00	0 1.94	-26.52	-20.06
Other service activities	1.05	15.49	17.93	7.20	-15.61	26.43
Sewage and refuse disposal, sanitation and similar activities	0.39	-3.84	7.48	3.25	-5.18	8.57

direct labour requirements are decreasing more than average (-10.41%) and their import coefficients are decreasing as well (-6.1%).

We are therefore left with 'Manufacture of machinery and equipment', alone accounting for 5.87% of the net product in 1995, a percentage that has increased by 10.52% in 2000. It is a very important sector not only for how much it produces, but mostly for *what* it does produce: equipment and machinery, i.e investment goods. An increase in (labour) productivity in the production of such commodities surely have backwards linkages, inducing a reduction in indirect labour in all those sectors making use of them as intermediate commodities. Moreover, we can see from the data that, in the period considered, such sector has also decreased its import coefficients (-2.29%) adding further backwards linkages to the ones mentioned above.

3.2 Productivity decreases

Those vertically integrated sectors whose total labour coefficient has increased have experienced a decrease in labour productivity. In the present case, they are 12 sectors out of 55, together accounting for 22.78% of total net product.[18] They are listed in tables 5(a), 5(b), 5(c) and 5(d). The sub-cases distinguishing

[18] They are 'Fishing, operation of fish hatcheries and fish farms; service activities incidental to fishing'; 'Mining of coal and lignite; extraction of peat'; 'Manufacture of office machinery and computers'; 'Manufacture of other transport equipment'; 'Collection, purification and distribution of water'; 'Supporting and auxiliary transport activities; activities of travel agencies'; 'Insurance and pension funding, except compulsory social security'; 'Real estate activities'; 'Research and development'; 'Other business activities'; 'Education'; and 'Health and social work'.

different combinations in the co-movements of $\mathbf{a}_{ni}^T$ and $\lambda(\mathbf{A}^{(j)})$ are the same as those described above for total labour productivity increase.

Table 5(a) shows sectors characterised by an increase of both $\mathbf{a}_{ni}^T$ and $\lambda^*(\mathbf{A}^{(j)})$, indicating that the usage of intermediate produced inputs is more intensive, and that has likely been implemented through the introduction of entirely new, and technically more advanced, commodities (as indirect labour is increasing as well). The fact that this produces a decrease in labour productivity can have three kinds of explanations.

The first one (hopefully the one holding in the majority of cases) is that such commodities are technologically more advanced than the ones previously in use to such an extent that their introduction requires a period of learning, and training, in order for them to be efficiently operated by the labour force. This should mean that these sectors should finally end up increasing their total labour productivity, after one or more periods of decrease.

The second one is that the introduction of these intermediate commodities *is not* accompanied by a period of skills training, with the result that the gains that could be obtained by using such commodities are lost because those who have to operate them are not capable of doing it in an appropriate way.

The third and last explanation is that in these sectors the introduction of more advanced intermediate inputs does not change substantially the physical process of output production.

Let us now look at the data.

The first thing that it is worth noticing is that 'Education' and 'Research and development' figure among the sectors whose labour productivity is decreasing. While the former had quite a great weight in the net product of the base year (4.55%) but has decreased from 1995 to 2000 (-1.50%), the latter has opposite characteristics (0.39% of total net output in 1995, +25% during the considered period). Both of them have increased their import coefficients (+2.81% for 'Education', +15.95% for 'Research and development') and have experienced an increment in direct labour requirements, especially the latter (+5.07% and +41.76%, respectively).

We are therefore induced to conclude that the most likely explanations for what has happened are the third one (or the second one) for 'Education', and (hopefully) the first one for 'Research and development'.

As to the other three sectors, two of them, namely 'Collection, purification and distribution of water' and 'Insurance and pension funding, except compulsory social security' have quite a small participation in the net product (0.39% and 0.65%, respectively). The nature of such sectors, however, induces us to attribute the decrease in total labour productivity to the third one of the three explanations provided above.

The case of 'Insurance and pension funding', anyway, is particularly interesting. Displaying a decrease of direct labour requirements, it would be classified by *traditional* theory as a sector in which labour productivity has increased. But, according to the present analysis, this is not the case. Actually, this is a sector in which there was, in 1995, a great prevalence of direct over indirect labour. Reducing the latter by intensifying the usage of intermediate goods would be considered as a productivity gain with respect to the initial situation. Import coefficients have risen as well, further increasing the weight of circulating capital goods in the production process. The present approach allows us to see this underlying decreasing — rather than increasing — movement in total labour productivity.

Finally, we have 'Real estate activities', which accounts for 6.99% of the 1995 net product, a weight which has further increased by 4.55% from the base year to 2000. This sector has decreased its import coefficients (-4.61%) and sensibly increased direct labour requirements (+6.86%). Also in this case, the most likely explanation for this tendency of labour productivity change is the third one, coupled with the fact that in sectors like the present one a precise measurement of output is quite difficult to obtain, thus favouring an inefficient utilisation of the labour force.

Table 5(b) includes sectors in which both $\lambda^*(\mathbf{A}^{(j)})$ and a_{nij} have decreased, notwithstanding this, total labour productivity has reduced. This is of course due to a great increase of direct labour requirements, offsetting the positive effect of the reduction in the labour embodied in circulating capital goods. The comovement of $\lambda^*(\breve{A}^{(j)})$ and a_{nij} suggests that a similar bundle of intermediate produced commodities is in use both in 1995 and in 2000, but their *quantities* have been reduced. This, coupled with the fact that direct labour requirements have increased, suggests two kinds of explanations.

The first one is that these sectors have reduced their 'degree of mechanisation', shifting the composition of total labour from indirect to direct labour.

The second one is that domestic-produced intermediate commodities have been replaced by analogous imported ones. As a result, the usage of intermediate inputs itself has increased, rather than decreased, therefore increasing direct labour requirements (necessary to operate such intermediate commodities).

By looking at the data, we have to notice, first of all, that the two sectors ('Manufacture of office machinery and computers' and 'Fishing, operation of fish hatcheries and fish farms') belonging to this group account for only 0.48% of total net product. Both of them are increasing their import coefficients (+10.04% and +33.42%, respectively), the first starting from a great proportion of indirect to total labour (57.62%). This induces us to think that, in both cases, the most likely explanation is the second one (even if, for 'Fishing, operation of fish hatcheries and fish farms', the first one is plausible as well).

Table 5(c) lists sectors in which indirect labour requirements have decreased, while $\lambda^*(\mathbf{A}^{(j)})$ has increased. Also here, direct labour must have increased to such an extent as to offset the positive effect induced by the reduction of indirect labour. The co-movement of $\lambda^*(\mathbf{A}^{(j)})$ and a_{nij} suggests that these sectors are using more technologically advanced intermediate commodities, but produced using less labour. Hence, we are induced to infer that an increase in the productivity of labour in the sectors providing inputs for the production of such intermediate commodities has taken place. Nonetheless, total labour productivity has decreased. Again, we have two kind of explanations for this fact.

The first one is linked to the same learning costs already mentioned for the first case of productivity reduction.

The second one is linked to the possible increase in imported goods per unit of output, that has consequently increased the number of workers necessary to operate them.

Looking at the table, we find only one sector, 'Health and social work', though a qualitative and quantitatively very important one, representing 5.57% of aggregate net product in 1995, participation which has further increased by 8.68% between the two considered periods. We immediately notice that imports have sensibly increased (+31.98%), a fact that suggests us to adopt, at least partially, the second explanation proposed. Anyway, this is likely a sector where intermediate commodities have a very high technological content. Therefore, it is probable that some (strong) learning costs are associated with the introduction of new, more 'complicated', intermediate commodities.

Finally, we have table 5(d), presenting those sectors for which $\lambda^*(\mathbf{A}^{(j)})$ has decreased but indirect labour requirements have increased. The reduction of $\lambda^*(\mathbf{A}^{(j)})$ suggests that similar intermediate commodities used in the base year have been used in 2000 as well, but in smaller quantities. The increase in indirect labour requirements suggests that the sectors producing such intermediate commodities have experienced a decrease in total labour productivity, either coupling with a further worsening of productivity in the 'final' sector (if direct labour increases as well) or offsetting the improvement in productivity in the 'final' sector (if direct labour decreases).

By looking at the data, we see that in all four sectors listed in this table[19] direct labour has increase.[20]

[19]'Manufacture of other transport equipment'; 'Supporting and auxiliary transport activities; activities of travel agencies'; 'Other business activities'; and 'Mining of coal and lignite; extraction of peat'.

[20]Incredibly so in 'Mining of coal and lignite': +351.42%, in addition to an increase of the 117.40% in import coefficients, suggesting that the sectors producing intermediate commodities used by it are experiencing a heavy decline in productivity. Anyway, this

As to import coefficients, they have increased in all sectors, in quite a sensible way. Moreover, in 'Manufacture of other transport equipment' and in 'Supporting and auxiliary transport activities; activities of travel agencies', the proportion of indirect to total labour was very high even in 1995 (63.34% and 63.18%, respectively), suggesting that a decrease in the labour productivity of intermediate commodities-producing sectors could well have had a strong negative impact on their total labour productivity.

4 Final comments

A final point is to be made about the rationale for using vertically integrated sectors in the measurement of changes in labour productivity.

Firstly, it is the variation of total labour embodied in one unit of net product and not that of direct labour requirements per unit of gross output that determines the changes in physical labour productivity. More specifically, total labour requirements $\overline{\mathbf{v}}^T$ can be decomposed into a direct $\overline{\mathbf{a}}_n^T$ and an indirect $\overline{\mathbf{a}}_n^T\overline{\mathbf{H}}$ labour component. This decomposition provides us with much more information *without* loosing the Input-Output character of the economic system: direct labour is a *uni-dimensional* magnitude. Total labour is a *multi-dimensional* one. Changes in the productivity of labour in an industry producing a basic commodity will affect every other industry in the system. The decomposition between direct and indirect labour allows us to infer in what stage of the production process a productivity increase has taken place.

A direct implication of this remark is that it is difficult to accept the usefulness of the search for a unique synthetic index of labour productivity changes that can also describe the structural processes of technical change lying behind them. On the contrary, we think that is very useful to dispose of a set of related measures allowing us to uncover such structural processes. As an example, this paper has combined four measures: $\overline{\mathbf{v}}^T$, $\overline{\mathbf{a}}_n^T\overline{\mathbf{H}}$, $\overline{\mathbf{a}}_n^T$ and $\lambda^*\left(\mathbf{A}^{(j)}\right)$.

By no means we state that the chosen set of measures completely describes the process under study. However, its use has allowed us to draw more complete implications than those which could have been drawn by using them separately.

sector had a very small participation in the 1995 net product (0.002%), which has further decreased in 2000 by -92.44%.

Table 5: Labour productivity decrease

(a) with $\Delta a_{ni} > 0, \Delta\lambda^*(\mathbf{A}^{(j)}) > 0$

Sectors	$\frac{y_{j,95}}{\mathbf{u}^T\mathbf{y}_{95}}$	$\frac{\Delta y_j}{y_{j,95}}$	$\frac{a_{nij,95}}{v_{j,95}}$	$\frac{\Delta a_{nij}}{a_{nij,95}}$	$\frac{\Delta a_{nj}}{a_{nj,95}}$	$\frac{\Delta m_j}{m_{j,95}}$
Research and development	0.39	25.00	80.35	16.48	41.76	15.95
Collection, purification and distribution of water	0.17	-4.92	78.41	50.13	21.91	-8.89
Real estate activities	6.99	4.55	68.32	27.96	6.86	-4.61
Education	4.55	-1.50	33.63	17.00	5.07	2.81
Insurance and pension funding, except compulsory social security	0.65	-7.11	28.71	- 34.66	-4.39	28.12

(b) with $\Delta a_{ni} < 0, \Delta\lambda^*(\mathbf{A}^{(j)}) < 0$

Sectors	$\frac{y_{j,95}}{\mathbf{u}^T\mathbf{y}_{95}}$	$\frac{\Delta y_j}{y_{j,95}}$	$\frac{a_{nij,95}}{v_{j,95}}$	$\frac{\Delta a_{nij}}{a_{nij,95}}$	$\frac{\Delta a_{nj}}{a_{nj,95}}$	$\frac{\Delta m_j}{m_{j,95}}$
Manufacture of office machinery and computers	0.36	-1.16	57.62	-2.02	12.09	10.04
Fishing, operation of fish hatcheries and fish farms; service activities incidental to fishing	0.12	16.91	16.93	-22.62	23.22	33.42

(c) with $\Delta a_{ni} < 0, \Delta\lambda^*(\mathbf{A}^{(j)}) > 0$

Sectors	$\frac{y_{j,95}}{\mathbf{u}^T\mathbf{y}_{95}}$	$\frac{\Delta y_j}{y_{j,95}}$	$\frac{a_{nij,95}}{v_{j,95}}$	$\frac{\Delta a_{nij}}{a_{nij,95}}$	$\frac{\Delta a_{nj}}{a_{nj,95}}$	$\frac{\Delta m_j}{m_{j,95}}$
Health and social work	5.57	8.68	20.18	-0.12	0.55	31.98

(d) with $\Delta a_{ni} > 0, \Delta\lambda^*(\mathbf{A}^{(j)}) < 0$

Sectors	$\frac{y_{j,95}}{\mathbf{u}^T\mathbf{y}_{95}}$	$\frac{\Delta y_j}{y_{j,95}}$	$\frac{a_{nij,95}}{v_{j,95}}$	$\frac{\Delta a_{nij}}{a_{nij,95}}$	$\frac{\Delta a_{nj}}{a_{nj,95}}$	$\frac{\Delta m_j}{m_{j,95}}$
Manufacture of other transport equipment	0.654	-2.01	63.34	23.82	6.10	11.73
Supporting and auxiliary transport activities; activities of travel agencies	1.320	-1.22	63.18	5.02	10.88	38.59
Other business activities	1.262	16.10	37.14	4.56	10.96	9.25
Mining of coal and lignite; extraction of peat	0.002	-92.44	8.67	97.63	351.42	117.40

Table 6: Summary of Measures at the Vertically Integrated Sector Level (in %)

VI sectors	$\frac{\Delta a_{nj}}{a_{nj,95}}$	$\frac{\Delta(\mathbf{a_n}^T\mathbf{h}_j)}{\mathbf{a_{n}}_{95}^T\mathbf{h}_{j,95}}$	$\frac{\Delta v_j}{v_{j,95}}$	$\frac{\Delta \mathbf{u}^T\mathbf{a}_j}{\mathbf{u}^T\mathbf{a}_{j,95}}$	$\frac{\Delta\lambda^*(\mathbf{A}^{(j)})}{\lambda^*(\mathbf{A}_{95}^{(j)})}$	$\frac{y_{j,95}}{\mathbf{u}^T\mathbf{y}_{95}}$	$\frac{\Delta y_j}{y_{j,95}}$
f45	0.921	-3.047	-0.914	4.167	-7.026	8.270	7.761
l75	-6.201	-0.109	-4.971	4.976	-8.174	7.061	2.738
k70	6.856	27.960	21.266	39.355	40.550	6.988	4.550
g52	-18.601	1.959	-14.003	7.970	10.048	6.859	15.741
dk29	-2.990	1.705	-0.285	6.558	-1.509	5.868	10.522
n85	0.555	-0.124	0.400	6.801	17.102	5.565	8.678
m80	5.071	17.000	5.964	10.400	32.068	4.550	-1.497
da15	-12.785	-19.072	-17.698	1.481	-12.709	4.488	7.605
g51	-2.390	2.062	-0.251	7.596	7.856	4.471	21.340
h55	-10.772	-4.599	-8.745	11.204	7.134	4.470	16.680
g50	-11.040	11.852	-2.399	15.487	15.547	2.952	12.572
dn36	-16.535	4.733	-6.237	15.162	9.783	2.552	5.943
dg24	-18.353	-12.275	-14.589	-5.530	-5.910	2.498	35.407
i60	-13.633	1.483	-7.492	4.638	8.128	2.448	5.801
dm34	-24.968	-8.427	-14.491	-3.067	23.802	2.229	26.880
db18	-20.746	-12.237	-17.061	-1.748	-25.218	2.225	8.858
db17	-16.317	-12.757	-14.480	-1.384	-2.522	1.746	13.830
j65	-28.886	23.843	-13.593	39.611	-11.454	1.733	-9.744
dc19	-8.205	-7.174	-7.614	1.643	-9.528	1.471	1.644
i63	10.878	5.021	7.745	10.994	-4.234	1.320	-1.215
k74	10.957	4.561	8.806	7.501	-5.067	1.262	16.103
dj28	-1.129	-0.024	-0.663	5.057	1.603	1.254	12.841
dl31	-15.113	-12.139	-13.644	-4.339	-6.267	1.169	27.890
e40	-36.508	9.906	-14.836	13.583	29.272	1.121	6.774
dl32	-19.193	-7.056	-13.940	-3.290	-87.227	1.053	17.068
o93	-15.615	7.196	-13.118	14.293	-6.838	1.052	15.494
dj27	-8.978	-17.824	-14.988	-18.074	-39.740	0.932	24.642
o92	-7.445	0.097	-4.446	3.213	3.652	0.870	29.196
i64	-44.618	0.742	-31.595	6.180	38.287	0.860	77.069
di26	-18.279	-3.048	-10.933	4.705	-4.267	0.836	11.485
dh25	-8.719	-7.151	-7.931	-3.108	0.083	0.826	26.045
df23	-12.506	-6.732	-8.441	-19.389	-55.282	0.819	62.408
a01	-23.493	-20.587	-22.886	-7.551	18.754	0.774	27.380
de22	-9.562	1.131	-4.259	10.316	-14.280	0.739	-4.327
dm35	6.103	23.823	15.818	26.114	-0.142	0.654	-2.014
dl33	-8.201	-8.044	-8.140	-4.381	-30.050	0.653	20.785
j66	-4.388	34.656	22.286	60.297	15.889	0.650	-7.114
k72	-8.121	1.073	-4.706	4.848	43.811	0.543	43.575
de21	-26.270	-15.459	-19.745	-7.227	-2.471	0.515	27.347
o90	-5.175	3.254	-0.356	6.544	-22.938	0.394	-3.835
k73	41.763	16.482	30.649	19.874	46.827	0.386	25.003
i62	-50.883	-37.013	-40.798	-37.789	-10.317	0.358	62.039
dl30	12.091	-2.021	2.045	0.643	-81.787	0.358	-1.163
i61	-5.483	-12.906	-10.173	-12.285	-22.188	0.352	12.476
o91	-7.483	-11.609	-8.223	-11.359	-20.831	0.212	6.008
dd20	-24.062	-13.464	-19.705	0.317	2.240	0.211	23.005
e41	21.915	50.130	39.960	60.381	20.294	0.174	-4.920
b05	23.222	-22.625	15.458	-15.720	-8.891	0.122	16.910
j67	-26.523	1.940	-18.205	13.547	-3.039	0.101	23.896
da16	-36.785	-3.347	-28.117	27.465	36.490	0.094	-5.748
k71	-39.309	-5.458	-12.111	-2.734	-1.551	0.083	32.700
cb14	-26.515	-7.214	-18.228	1.040	-16.290	0.035	6.503
a02	-22.727	-20.355	-22.652	-6.212	-19.017	0.020	27.325
ca10	351.416	97.632	329.422	222.670	-51.118	0.002	-92.441
cb13	-17.451	-69.929	-29.453	-72.817	-63.211	0.002	35.068

Source: Own calculations based on EUROSTAT: SUIOT95 and National Accounts Database.

Table 7: NACE industry classification, EUROSTAT

tables/sectors	sector
a01	Agriculture, hunting and related service activities
a02	Forestry, logging and related service activities
b05	Fishing, operation of fish hatcheries and fish farms; service activities incidental to fishing
ca10	Mining of coal and lignite; extraction of peat
cb13	Mining of metal ores
cb14	Other mining and quarrying
da15	Manufacture of food products and beverages
da16	Manufacture of tobacco products
db17	Manufacture of textiles
db18	Manufacture of wearing apparel; dressing; dyeing of fur
dc19	Tanning, dressing of leather; manufacture of luggage
dd20	Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials
de21	Manufacture of pulp, paper and paper products
de22	Publishing, printing, reproduction of recorded media
df23	Manufacture of coke, refined petroleum products and nuclear fuel
dg24	Manufacture of chemicals and chemical products
dh25	Manufacture of rubber and plastic products
di26	Manufacture of other non-metallic mineral products
dj27	Manufacture of basic metals
dj28	Manufacture of fabricated metal products, except machinery and equipment
dk29	Manufacture of machinery and equipment n.e.c.
dl30	Manufacture of office machinery and computers
dl31	Manufacture of electrical machinery and apparatus n.e.c.
dl32	Manufacture of radio, television and communication equipment and apparatus
dl33	Manufacture of medical, precision and optical instruments, watches and clocks
dm34	Manufacture of motor vehicles, trailers and semi-trailers
dm35	Manufacture of other transport equipment
dn36	Manufacture of furniture; manufacturing n.e.c.
e40	Electricity, gas, steam and hot water supply
e41	Collection, purification and distribution of water
f45	Construction
g50	Sale, maintenance and repair of motor vehicles
g51	Wholesale trade and commission trade, except of motor and motorcycles
g52	Retail trade, except of motor vehicles, motorcycles; repair of personal and household goods
h55	Hotels and restaurants

Table 7: NACE industry classification, EUROSTAT

tables/sectors	sector
i60	Land transport; transport via pipelines
i61	Water transport
i62	Air transport
i63	Supporting and auxiliary transport activities; activities of travel agencies
i64	Post and telecommunications
j65	Financial intermediation, except insurance and pension funding
j66	Insurance and pension funding, except compulsory social security
j67	Activities auxiliary to financial intermediation
k70	Real estate activities
k71	Renting of machinery and equipment without operator and of personal and household goods
k72	Computer and related activities
k73	Research and development
k74	Other business activities
l75	Public administration and defence; compulsory social security
m80	Education
n85	Health and social work
o90	Sewage and refuse disposal, sanitation and similar activities
o91	Activities of membership organization n.e.c.
o92	Recreational, cultural and sporting activities
o93	Other service activities

Source: EUROSTAT, NACE Industry classification.

References

De Juan, O. & Febrero, E. (2000). Measuring Productivity from Vertically Integrated Sectors. *Economic Systems Research*, 12(1):65–82.

Gossling, W. (1972). *Productivity Trends in a Sectoral Macro-Economic Model*. Input-Output Publishing, London.

Gossling, W. & Dovring, F. (1966). Labor Productivity Measurement: The Use of Subsystems in the Interindustry Approach and Some Approximating Alternatives. *Journal of Farm Economics*, 48(2):369–377.

Gupta, S. & Steedman, I. (1971). An Input-Output Study of Labour Productivity in the British Economy. *Oxford Bulletin of Economics and Statistics*, 33:21–34.

Pasinetti, L.L. (1973). The Notion of Vertical Integration in Economic Analysis. *Metroeconomica*, 25:1–29.

——— (1977). *Lectures on the Theory of Production*. Columbia University Press, New York.

——— (1981). *Structural Change and Economic Growth*. Cambridge University Press, Cambridge.

——— (1959). On Concepts and Measures of Changes in Productivity. *The Review of Economics and Statistics*, 41(3):270–286.

Rampa, G. (1981). The Concept and Measurement of Productivity in an Input-Output Framework. *University of Cambridge, Working Papers*.

Rampa, G. & Rampa, L. (1982). Sul mutamento tecnologico nell'economia Italiana, 1959-1975: un'analisi input-output. *Ricerche Economiche*, XXXVI(4):303–341.

Siniscalco, D. (1982). Il sistema produttivo: analisi per industrie e subsistemi. *Ricerche Economiche*, XXXVI(4):475–488.

Sraffa, P. (1960). *Production of Commodities by Means of Commodities*. Cambridge University Press, Cambridge.