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MONEY, TOBIN EFFECT, AND INCREASING RETURNS*

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Money, Tobin Effect, and Increasing Returns

Abstract: This paper shows that unregulated decentralized equilibrium is viable under increasing returns technologies in an overlapping generations model of production with cash-in-advance constraints. We also demonstrate that the model exhibits both the Tobin effect and the reverse Tobin effect.

Keywords: Increasing returns, cash-in-advance constraints, overlapping generations, Phillips curve.

JEL Codes: D51, D52, D91, E52.

1 Introduction

Competitive equilibrium has mostly coexisted with constant or decreasing returns to scale technologies in the economic literature. Welfare theorems and existence results of Walrasian general equilibrium theory (Arrow and Debreu (1954), Debreu (1959)) build upon the concavity assumption on production functions. The reason is the presence of a major obstacle against the operation of the price system under increasing returns (strictly convex production functions), namely the problem of unbounded input demands and output supplies for any given non-zero output prices.

Despite difficulties, there have been some attempts to introduce increasing returns into Arrow-Debreu general equilibrium. The common practice has been to regulate economic activity through marginal cost pricing or average cost pricing (See Beato (1982), Brown and Heal (1983), Guesnerie (1975), Khan and Vohra (1987) and Vohra (1988, 1992).) But, this reputable remedy also yields some side effects. In marginal cost pricing equilibrium, producers face operating losses that must be subsidized by the regulatory body, whereas in average cost pricing equilibrium they have an incentive to expand outputs at the quoted prices. One must also take into account the welfare losses caused by the informational rents offered to the regulated producers under an incentive-compatible mechanism when either demand or cost information is private.

This paper shows that unregulated decentralized equilibrium is viable in an increasing returns economy under cash-in-advance constraints.[1] We assume that our economy operates with outside money and we get rid of unbounded input demands by constraining firms to use money as working capital in financing their transactions in the factor market. However, there still remains a problem as the first-order necessary condition (Euler equation) is not sufficient for a maximum when the production function is strictly convex. We overcome this technical obstacle by imposing assumptions on the basic settings of the model. Thereby, we ensure that (i) corner solutions to Euler equation cannot be optimal, (ii) interior solution to Euler equation exists and is unique.

Cash-in-advance constraints have extensively been used in the literature to model the transactions demand for money as these constraints require that individuals hold cash to finance some or all of their transactions. Clower (1967) is the first to present a basic cash-in-advance model in which individuals are liquidity constrained to purchase consumption goods. In such a setting the classical result of superneutrality continues to hold, i.e. anticipated inflation yields no welfare effects. In Lucas (1980, 1984, 1990) and Lucas and Stokey (1983, 1987), liquidity constraints are similarly imposed only on consumers' purchases of a subset of commodities or assets. In this class of studies money inflation leads to a fall in real money holdings and hence a reduction in the quantity of cash goods consumed. Such a distortion in output is shown to be eliminated by a deflationary money supply rule in line with the representative agent's rate of time preference.

Another strand of the same literature looks at the influences of cash-in-advance constraints on inifinite-horizon economies with production. Some examples are Cooley and Hansen (1989), Fuerst (1992), Carlstrom and Fuerst (1995), Christiano, Eichenbaum, Evans (1997, 1998) and Basci and Saglam (1999, 2003a, 2003b). These studies all establish an operational Phillips curve between anticipated inflation and employment that is downward-sloping along with the presence of a working capital premium as the gap between real wage and productivity. This gap can be completely eliminated, as proposed by numerous studies, by a deflationary policy (Friedman rule) that equates the

[1] The existence problem of competitive equilibrium under increasing returns to scale technologies was recently studied by Ata and Basci (2004) in a finite-horizon representative-agent framework. In that paper money is not fiat and does not grow; the utility function is logarithmic, the production function is quadratic and no welfare analysis for monetary policy is conducted.

real rate of return on money to the time preference of the representative agent. A similar rule maximizing employment and output is obtained as the discount rate of the most patient agent in the model of Basci and Saglam (2005), which allows for heterogeneous producers.

The current paper also demonstrates that the slope of the long-run Phillips curve may depend on the way monetary policy is conducted. We show that anticipated money inflation may have expansionary as well as contractionary effects on equilibrium employment and output depending on the allocation of money transfers among producers in the economy. We obtain this result in a model with overlapping heterogenous producers who are distinguished as 'young' and 'old' and assign unequal valuations to monetary savings in each period.

It is worth emphasizing that the incorporation of cash-in-advance constraints in an overlapping generations framework attempts to complete the missing part of 'monetary' models. In traditional overlapping generations models with money, agents hold money as an asset that helps to smooth out consumption over time and it is possible that the money is not valued when the rate of return on other assets is greater than that of money. One can argue that it is inappropriate to classify these conventional models as 'monetary', and there exists room for reintroducing money into OLG models by means of liquidity constraints. Indeed, the studies by Crettez, Michel and Wigniolle (1999) and Erdogan and Saglam (2006) are two recent examples of this endeavor. Crettez, Michel and Wigniolle (1999) consider a Diamond OLG model with cash-in-advance constraints in the good market transactions and study conditions under which money is neutral and is not neutral. They also characterize the monetary policy that implements the optimal allocation of resources. Erdogan and Saglam (2006) impose liquidity constraints on both factor market and good market transactions in order to analyze the full effects of money inflation in a simple production economy with decreasing returns, and obtain a striking dependence of the shape of long-run Phillips curves on the form of the monetary policy.

Our model considers overlapping generations of consumer-producers and consumer-workers living for two periods. We deviate from the setup of Erdogan and Saglam (2006) in that producers own increasing returns to scale (IRTS) technologies during their lifetime to convert the single factor of production, labor, into a single good of consumption. Just before the end of their lives, producers transfer their technologies as bequest to the generation after next so that in every period each newborn producer is

endowed with a production plant. We also assume that the labor market opens before the good market in each period and transactions in both markets are payable on a cash basis, only.[2]

Private money endowments through bequests are absent in the model, thus money transfers from the government to at least one of the two groups of agents, producers and workers, are indispensable for the operation of the economy. By the assumed sequencing of the two markets, workers can, in every period, earn the cash they need for the planned purchases in the good market by selling labor in the factor market. Therefore, we assume for simplicity that workers do not receive money transfers from the government either when young or when old.

Unlike workers, producers begin their lives by entering first a market (labor market) in which they choose to buy rather than to sell. After production takes place with the employed labor, the good market opens and transactions determine the end-of-period cash balance of each producer. This balance is chosen as positive by liquidity-constrained producers, when they are young, to finance the planned factor payments in the next period. It is now clear that money as a working capital needs to be transfered to newborn producers, only. Nonetheless, we also let old producers receive (pay) money transfers (taxes) not only to allow the monetary authority to budget transfers but also to show that the distribution of money matters for the real effects of money inflation.

For a definition of monetary competitive equilibrium where nominal prices are stationary when normalized with respect to the money growth rate, we are able to give the full characterization under increasing returns if and only if the money growth rate is not too low. We argue that sufficiently high levels of money inflation make cash-in-advance constraints binding and eliminate both workers' and producers' incentives for hoarding money. Money is used as a working capital in all types of transactions.

We analyze the competitive equilibrium and immediately note that the quantity of money in circulation only determines nominal price levels, i.e. money is neutral. But, money is not superneutral because the rate of money growth influences real variables as well. However, we deviate from the traditional non-superneutrality result which states

[2]Basci and Saglam (2003a) paper shows that if the good market opens before the factor market, the competitive outcomes are the same as those obtained in the absence of such cash constraints. Liquidity constraints have some real effects only if the factor market opens before the good market. Barth and Ramey (2001) provides empirical support for the relevance of such liquidity constrained models at the industry and aggregate levels.

that money inflation in the presence of liquidity constraints curbs the commodity supply and demand. Indeed, we show that the impact of money inflation on the equilibrium outcome can be in both directions depending upon the dynamic allocation of money transfers between young and old producers.

An increase in the growth rate of money supply, through a rise in old producers' share in money transfers, decreases the equilibrium real wage rate and employment, which can be called as a modified 'Tobin (1965) effect' with capital - the intensive factor of production in the literature - being replaced in our model by labor that is indeed the single factor. Thus, we obtain between anticipated inflation and employment the conventional downward-sloping Phillips curve commonly derived in the cash-in-advance literature.

At the opposite extreme, an increase in the growth rate of money supply through a rise in young producers' share in money transfers leads to an increase in the equilibrium real wage rate and employment (a 'reverse Tobin effect'). We thus recover an upward-sloping Phillips curve relation between anticipated inflation and employment analytically. While inflation increases, we show that employment converges to its upper-bound determining the frontier of the economy.

In order to analyze the impact of money inflation on aggregate output, we simulate the equilibrium with an artificial set of model parameters. We find that the Phillips curve relations between inflation and employment extend to similar relations between inflation and output for each transfer allocation rule.

The sensitivity of our results to the distribution of money transfers completely stems from both the heterogeneity of producers and the fact that generations overlap in each period. The fact that generations overlap renders useless making an assumption of money being tax backed since there is always production on the part of young producers, and thus leaves room for the monetary policy to affect the slope of the long-run Phillips curve. As for the importance of heterogenity of producers, notice that in the absence of money bequests, producers never supply the good market in the last period of their lives, as it is optimal for them to consume the whole produced output before dying. On the contrary, young producers, awaiting to live one more period, sell in the good market to smooth out consumption as well as to collect cash for the expected wage payments in the next period. So, in every period only young producers sell consumption good to workers. Since the tightness of cash-in-advance requirements young producers face in

the factor market determine the quantity of output they supply in the good market, an increase in money inflation relieves the liquidity constrained economy in terms of a rising employment and output level only if young producers' relative share of money transfers is increased.

The paper proceeds as follows. Section 2 introduces the basic settings of the model. Section 3 defines and characterizes the monetary competitive equilibrium. Section 4 provides monetary analysis of the equilibrium. Section 5 contains some concluding remarks. Finally, proofs are relegated to the Appendix.

2 The Model

The basing settings of the model are described as follows.

Agents: There are overlapping generations of two types of agents, 'workers' and 'producers', distinguished by the index $i \in \{w, p\}$. Each generation lives for two periods. At the beginning of each period t appears a new generation, called generation t. The subscripts {1,t} and {2,t} respectively stand for a 'young' agent of generation t and an 'old' agent of generation $t-1$ who meet in period t. No starting point is assumed for the time horizon, therefore at each period there are young and old agents of consecutive generations. There is no population growth either, so at each period there are equal numbers of young and old agents of each type that we denote by n^i for $i \in \{w, p\}$.

Commodities: There are two commodities in each period: the single factor of production, labor, and a nonstorable consumption good that can be produced with labor.

Factor Endowments: Workers have equal amounts of labor endowments in the two periods of their lives, denoted by $\bar{l}_1^w = \bar{l}_2^w = \bar{l}^w > 0$. Producers do not have labor endowments.

Valuation of Leisure: Workers value leisure in units of the consumption good through the function $v^w(.)$. Producers do not value leisure.

Production Technology: Each old producer of generation t, just before dying, leaves his technology $f^p(.)$ as bequest to a distinct member of generation $t+2$ so that each producer is born with a technology available for use during his lifetime. We assume that the technology $f^p(.)$ has increasing returns to scale (IRTS), as $f^p(.)$, $f^{p'}(.)$ and $f^{p''}(.) > 0$.

We also assume that workers do not own production technology, i.e. $f^w(.) = 0$ in every period.

Utilities: The representative worker and producer of generation t have the lifetime utilities $U^w(c^w_{1,t} + v^w(e^w_{1,t})) + \beta^w U^w(c^w_{2,t+1} + v^w(e^w_{2,t+1}))$ and $U^p(c^p_{1,t}) + \beta^p U^p(c^p_{2,t+1})$, respectively. Here, $c^i_{1,t}$ and $e^i_{1,t}$ respectively denote consumption and leisure of the representative agent of type-i when young, and $c^i_{2,t+1}$ and $e^i_{2,t+1}$ consumption and leisure of the same agent when old. We assume that $U^i(.)$ and $v^w(.)$ are twice continuously differentiable, increasing and strictly concave. We also assume

A0. $v^{w'}(0) = \infty$ and $v^{w'}(\bar{l}^w) = 0$.

A1. $\lim_{x \to 0} \dfrac{v^{w'}(\bar{l}^w - \psi x)}{f^{p'}(x)} = 0$ for all $\psi > 0$.

A2. $U^{p'}(0) = \infty$.

A3. $U^{p'}$ satisfies multiplicative separability, i.e. for any $c_1, c_2 \in I\!R_{++}$, $U^{p'}(c_1 c_2) = U^{p'}(c_1)U^{p'}(c_2)$.

A4. $f^p(l)/g(f^{p'}(l))$ is increasing in $l \in I\!R_{++}$, where g is the inverse function of $1/U^{p'}$.

Assumptions $A0 - A3$ help to make sure that an interior solution to the optimization problem of producers exists and corner solutions are not optimal. Assumption $A4$ is very crucial for the existence result of this paper as it ensures that the interior solution is unique and Euler equations are sufficient for optimality.

Note that IRTS production functions $f^p(L) = \theta l^\gamma$ together with the leisure function $v^w(e) = \sqrt{e} - e/(2\bar{l}^w)$ and the CRRA utility functions $U^p(c) = c^\eta/\eta$, where $\theta > 0$, $\gamma \in (1, 2)$ and $\eta \in (0, 1/\gamma)$, satisfy assumptions $A0 - A4$. The function $U^p(c) = \ln(c)$ is also admissible. In the light of these special classes of technologies and utilities, assumption $A4$ can be interpreted as that for a given convex production technology, utility function of producers must be sufficiently concave.

Money and the Government: The economy operates with fiat money. By m_t, we denote the aggregate money stock at the end of period t that evolves over time according to

$$m_{t+1} = (1 + \alpha)m_t,$$

where $\alpha > -1$. Government changes the money stock in the economy through lump-sum transfers/taxes at the beginning of each period. While no worker receives money transfer during his lifetime, each of the young and old producers living in period t receive $x^p_{1,t} = \alpha_1 m_{t-1}/n^p$ and $x^p_{2,t} = \alpha_2 m_{t-1}/n^p$ units of money transfer, respectively. We assume $\alpha_1 + \alpha_2 = \alpha$ so that money transfers received lead to the desired money inflation. We allow the taxation of old producers as long as $\alpha_2 > -1$. But, we must have $\alpha_1 > 0$ inevitably, since newborn producers need cash for their wage expenses in the factor market.

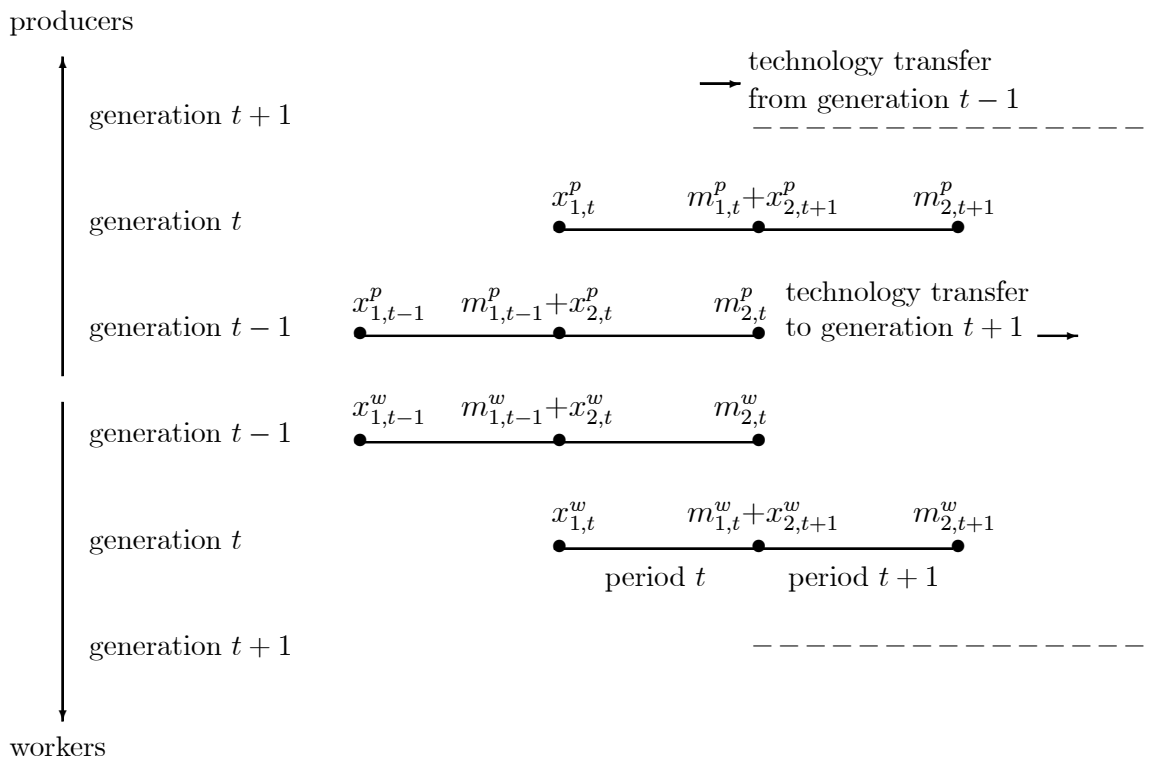

FIGURE 1.— Time flow of cash and technology

Now we describe the flow of money in the economy (that we depict in Figure 1 along with the flow of technology across generations). A type-i agent of generation $t-1$ starts his first period with the initial money transfer $x^i_{1,t-1}$ and ends it with the balance $m^i_{1,t-1}$. The same agent receives the transfer $x^i_{2,t}$ at the start of his second period, and ends his life with the balance $m^i_{2,t}$. (Note that $x^w_{1,t-1} = x^w_{2,t} = 0$ for all t, by asssumption.)

The following table summarizes the basic structures of the model:

TABLE I

SUMMARY OF BASIC STRUCTURES

	Workers		Producers	
	Young	Old	Young	Old
Labor Endowment	$\bar{l}_1^w = \bar{l}^w > 0$	$\bar{l}_2^w = \bar{l}^w > 0$	$\bar{l}_1^p = 0$	$\bar{l}_2^p = 0$
Valuation of Leisure	v^w (Concave)	v^w (Concave)	$v^p = 0$	$v^p = 0$
Production Technology	$f^w = 0$	$f^w = 0$	f^p (IRTS)	f^p (IRTS)
Period$-t$ Money Transfer	$x_{1,t}^w = 0$	$x_{2,t}^w = 0$	$x_{1,t}^p = \alpha_1 m_{t-1}/n^p$	$x_{2,t}^p = \alpha_2 m_{t-1}/n^p$

Trade Institution: For a given *economy* $\mathcal{E} = \{\, n^i,\, \beta^i,\, U^i,\, v^i,\, f^i,\, \bar{l}_1^i,\, \bar{l}_2^i,\, x_{1,t}^i,\, x_{2,t}^i \mid$ for all t and $i \in \{w, p\}\,\}$, a trade institution is the description of choice variables for each type of agents, price variables, constraints on the given choice variables determined by the given prices, and a feasibility requirement for the collective choices of agents.

Choice variables of type i agents when they are young and old respectively:

$c_{1,t}^i$, $c_{2,t+1}^i$: consumption
$q_{1,t}^i$, $q_{2,t+1}^i$: (+) commodity demand, (-) commodity supply
$l_{1,t}^i$, $l_{2,t+1}^i$: (+) labor demand, (-) labor supply
$m_{1,t}^i$, $m_{2,t+1}^i$: end-of-period money holding

Money prices:

ω_t, ω_{t+1} : nominal wage rates per unit of labor
p_t, p_{t+1} : money prices per unit of good

All prices and wages are expressed in terms of money, which serves as numeraire.

Timing of Transactions: Under the assumption that the labor market opens before the good market, the sequence of transactions that take place in the economy in period t can be listed under the following four subperiods:

T1. Type i agent starts period t with a money balance that is equal to the sum of money transfers/taxes and the balance carried from the end of period $t-1$. (This sum is $x^i_{1,t}$ for each young agent whereas $m^i_{1,t-1} + x^i_{2,t}$ for each old agent in period t, where $m^i_{1,t-1}$ represents the end-of-period $t-1$ money holding.)

T2. Labor market opens. Factor trade takes place at the nominal wage rate ω_t and all wage bills are paid in advance of production.

T3. Good production occurs with the labor $l^p_{1,t}$ and $l^p_{2,t}$ employed by young and old producers, respectively.

T4. Good market opens. Transactions are made by cash at the nominal good price p_t. Hence, the end-of-period t money balances realize as

$$m^i_{1,t} = x^i_{1,t} - \omega_t l^i_{1,t} - p_t q^i_{1,t}$$

$$m^i_{2,t} = x^i_{2,t} + m^i_{1,t-1} - \omega_t l^i_{2,t} - p_t q^i_{2,t}$$

for each of the young and old agents, respectively.

Agents' Problems: The representative agent of type i faces the following utility maximization problem given his lifetime endowment structure $(\bar{l}^i_1, \bar{l}^i_2)$ and strictly positive prices $\{\omega_t, \omega_{t+1}, p_t, p_{t+1}\}$:

$$\max \quad U^i(c^i_{1,t} + v^i(\bar{l}^i_1 + l^i_{1,t})) + \beta^i U^i(c^i_{2,t+1} + v^i(\bar{l}^i_2 + l^i_{2,t+1}))$$

subject to:

$$m^i_{1,t} = x^i_{1,t} - \omega_t l^i_{1,t} - p_t q^i_{1,t} \tag{1}$$

$$m^i_{2,t+1} = m^i_{1,t} + x^i_{2,t+1} - \omega_{t+1} l^i_{2,t+1} - p_{t+1} q^i_{2,t+1} \tag{2}$$

$$-\bar{l}^i_1 \leqslant l^i_{1,t} \leqslant \frac{x^i_{1,t}}{\omega_t} \tag{3}$$

$$-\bar{l}^i_2 \leqslant l^i_{2,t+1} \leqslant \frac{m^i_{1,t} + x^i_{2,t+1}}{\omega_{t+1}} \tag{4}$$

$$-f^i(\bar{l}^i_1 + l^i_{1,t}) \leqslant q^i_{1,t} \leqslant \frac{x^i_{1,t} - \omega_t l^i_{1,t}}{p_t} \tag{5}$$

$$-f^i(\bar{l}_2^i + l_{2,t+1}^i) \leqslant q_{2,t+1}^i \leqslant \frac{m_{1,t}^i + x_{2,t+1}^i - \omega_{t+1} l_{2,t+1}^i}{p_{t+1}} \tag{6}$$

$$c_{1,t}^i = f^i(\bar{l}_1^i + l_{1,t}^i) + q_{1,t}^i \tag{7}$$

$$c_{2,t+1}^i = f^i(\bar{l}_2^i + l_{2,t+1}^i) + q_{2,t+1}^i \tag{8}$$

Equations (1) and (2) describe the end-of-period cash holdings. Constraints (3) and (4) must be read as that in each period supply of labor is bounded from above by the labor endowments of workers whereas demand for labor is constrained by the amount of cash with which producers enter the factor market. Similarly, constraints (5) and (6) respectively impose on good supply and good demand both technology and liquidity constraints. Finally, equations (7) and (8) state that per-period consumption of each agent equals the sum of whatever he has produced and purchased.

3 Monetary Competitive Equilibrium

We are now ready to define our equilibrium concept.

Definition 3.1. The list $\{\, \omega_t,\, \omega_{t+1},\, p_t,\, p_{t+1},\, c_{1,t}^i,\, c_{2,t+1}^i,\, l_{1,t}^i,\, l_{2,t+1}^i,\, q_{1,t}^i,\, q_{2,t+1}^i,\, m_{1,t}^i,\, m_{2,t+1}^i \,|\, \text{for all } t \text{ and } i \in \{w,p\}\,\}$ is a *Monetary Competitive Equilibrium* (MCE) for the economy $\mathcal{E}$, if $\omega_t, \omega_{t+1}, p_t, p_{t+1} > 0$ and

(i) $\{c_{1,t}^i, c_{2,t+1}^i, l_{1,t}^i, l_{2,t+1}^i, q_{1,t}^i, q_{2,t+1}^i, m_{1,t}^i, m_{2,t+1}^i\}$ solves the maximization problem of each type-i agent for all $i \in \{w,p\}$ under the sequence $\{\omega_t, \omega_{t+1}, p_t, p_{t+1}\}$

(ii) $n^w(l_{1,t}^w + l_{2,t}^w) + n^p(l_{1,t}^p + l_{2,t}^p) = 0$

(iii) $n^w(q_{1,t}^w + q_{2,t}^w) + n^p(q_{1,t}^p + q_{2,t}^p) = 0$

(iv) $n^w(m_{1,t}^w + m_{2,t}^w) + n^p(m_{1,t}^p + m_{2,t}^p) = m_t$

(v) $\omega_{t+1}/\omega_t = p_{t+1}/p_t = m_{t+1}/m_t = 1 + \alpha$

(vi) $c_{j,t+1}^i/c_{j,t}^i = l_{j,t+1}^i/l_{j,t}^i = q_{j,t+1}^i/q_{j,t}^i = 1$ for all $i \in \{w,p\}$ and $j \in \{1,2\}$.

(vii) $m_{1,t}^w = m_{2,t+1}^w = 0$.

Condition (i) states that representative agents make optimal choices under perfect foresight of future prices and price taking behaviour. Conditions (ii)-(iv) ensure the clearing of the three markets. Condition (v) is the stationarity of the nominal variables as normalized by the money growth rate. Condition (vi) denotes the symmetry of real variables across generations. Finally, condition (vii) calls for workers not to hold any end-of-period money balances after the good market transactions, which ensures that every unit of currency in the economy can get its proper use as a working capital in the factor market in every period.

To characterize equilibrium, we shall first obtain the reduced-form problem of a type-i agent. For this purpose, we eliminate $c^i_{1,t}$, $c^i_{2,t+1}$ and $q^i_{1,t}$, $q^i_{2,t+1}$ from the respective maximization problems, using the equality constraints (1)-(2) and (7)-(8). Then, we restrict ourselves to the clearing of the labor and money markets alone, since the good market will automatically clear as well, thanks to a version of Walras' law applicable to our case.

Using $x^w_{1,t} = x^w_{2,t+1} = f^w(.) = 0$, we obtain the reduced-form problem faced by each worker of generation t as:

$$\max_{\{m^w_{1,t}, m^w_{2,t+1}, l^w_{1,t}, l^w_{2,t+1}\}} U^w\left(\frac{-m^w_{1,t} - \omega_t l^w_{1,t}}{p_t} + v^w(\bar{l}^w_1 + l^w_{1,t})\right)$$

$$+ \beta^w U^w\left(\frac{m^w_{1,t} - m^w_{2,t+1} - \omega_{t+1} l^w_{2,t+1}}{p_{t+1}} + v^w(\bar{l}^w_2 + l^w_{2,t+1})\right)$$

subject to:

$$0 \leqslant m^w_{1,t} \leqslant -\omega_t l^w_{1,t} \tag{9}$$

$$0 \leqslant m^w_{2,t+1} \leqslant m^w_{1,t} - \omega_{t+1} l^w_{2,t+1} \tag{10}$$

$$-\bar{l}^w_1 \leqslant l^w_{1,t} \leqslant 0 \tag{11}$$

$$-\bar{l}^w_2 \leqslant l^w_{2,t+1} \leqslant \frac{m^w_{1,t}}{\omega_{t+1}} \tag{12}$$

Similarly, the reduced-form problem faced by each producer of generation t becomes:

$$\max_{\{m^p_{1,t}, m^p_{2,t+1}, l^p_{1,t}, l^p_{2,t+1}\}} U^p\left(\frac{x^p_{1,t} - m^p_{1,t} - \omega_t l^p_{1,t}}{p_t} + f^p(\bar{l}^p_1 + l^p_{1,t})\right)$$

$$+ \beta^p U^p\left(\frac{m^p_{1,t} + x^p_{2,t+1} - m^p_{2,t+1} - \omega_{t+1} l^p_{2,t+1}}{p_{t+1}} + f^p(\bar{l}^p_2 + l^p_{2,t+1})\right)$$

subject to:

$$0 \leqslant m^p_{1,t} \leqslant x^p_{1,t} - \omega_t l^p_{1,t} + p_t f^p(\bar{l}^p_1 + l^p_{1,t}) \tag{13}$$

$$0 \leqslant m^p_{2,t+1} \leqslant m^p_{1,t} + x^p_{2,t+1} - \omega_{t+1} l^p_{2,t+1} + p_{t+1} f^p(\bar{l}^p_2 + l^p_{2,t+1}) \tag{14}$$

$$0 \leqslant l^p_{1,t} \leqslant \frac{x^p_{1,t}}{\omega_t} \tag{15}$$

$$0 \leqslant l^p_{2,t+1} \leqslant \frac{m^p_{1,t} + x^p_{2,t+1}}{\omega_{t+1}} \tag{16}$$

The next proposition addresses two issues: the set of money growth rates sustaining an equilibrium and the complete characterization of the unique eqilibrium.

Proposition 3.1. *Monetary Competitive Equilibrium* $\{p_t, \omega_t, p_{t+1}, \omega_{t+1}, c^i_{1,t}, c^i_{2,t+1}, l^i_{1,t}, l^i_{2,t+1}, q^i_{1,t}, q^i_{2,t+1}, m^i_{1,t}, m^i_{2,t+1}, \mid i = w, p\}$ *of the economy* $\mathcal{E}$

(i) exists if for all t

$$1 + \alpha \geqslant \max\{\beta^w,\ \beta^p U^{p'}(c^p_{2,t+1})/U^{p'}(c^p_{1,t})\} \quad \textit{and} \tag{17}$$

$$1 + \alpha_2 \in \left(\beta^p \frac{f^{p'}(l^p_{2,t+1}) l^p_{2,t+1}}{f^p(l^p_{1,t})} \frac{U^{p'}(c^p_{2,t+1})}{U^{p'}(c^p_{1,t})}, 1\right] \tag{18}$$

(ii) is uniquely characterized by (19)-(34) for all t:

$$\frac{\omega^*_t}{p^*_t} = \frac{\beta^p}{1+\alpha} f^{p'}(l^{*p}_{2,t+1}) \frac{U^{p'}(c^{*p}_{2,t+1})}{U^{p'}(c^{*p}_{1,t})} \tag{19}$$

$$\frac{\omega^*_t}{p^*_t} = v^{w'}(\bar{l}^w_1 + l^{*w}_{1,t}) \tag{20}$$

$$\frac{\omega^*_{t+1}}{p^*_{t+1}} = v^{w'}(\bar{l}^w_2 + l^{*w}_{2,t+1}) \tag{21}$$

$$n^w(l_{1,t}^{*w} + l_{2,t}^{*w}) = -n^p \left(\frac{x_{1,t}^p}{\omega_t^*} + \frac{m_{1,t-1}^{*p} + x_{2,t}^p}{\omega_t^*} \right) \tag{22}$$

$$q_{1,t}^{*w} = -\frac{\omega_t^*}{p_t^*} l_{1,t}^{*w} \tag{23}$$

$$q_{2,t+1}^{*w} = -\frac{\omega_{t+1}^*}{p_{t+1}^*} l_{2,t+1}^{*w} \tag{24}$$

$$q_{1,t}^{*p} = -\frac{m_{1,t}^{*p}}{p_t^*} \tag{25}$$

$$q_{2,t+1}^{*p} = 0 \tag{26}$$

$$l_{1,t}^{*p} = \frac{x_{1,t}^p}{\omega_t^*} \tag{27}$$

$$l_{2,t+1}^{*p} = \frac{m_{1,t}^{*p} + x_{2,t+1}^p}{\omega_{t+1}^*} \tag{28}$$

$$c_{1,t}^{*i} = q_{1,t}^{*i} + f^i(\bar{l}_1^i + l_{1,t}^{*i}), \qquad i = w, p \tag{29}$$

$$c_{2,t+1}^{*i} = q_{2,t+1}^{*i} + f^i(\bar{l}_2^i + l_{2,t+1}^{*i}), \qquad i = w, p \tag{30}$$

$$m_{1,t}^{*w} = 0 \tag{31}$$

$$m_{2,t+1}^{*w} = 0 \tag{32}$$

$$m_{1,t}^{*p} = \frac{m_t}{n^p} \tag{33}$$

$$m_{2,t+1}^{*p} = 0 \tag{34}$$

Proof. See Appendix.

The two conditions in Proposition 3.1-(i) are sufficient for the existence of MCE. In addition, inequality (17) is also necessary for that the money holding plans stated in equations (31)-(34) are optimal for workers and producers. The part of (18) that is not necessary for the existence of MCE is that $1 + \alpha_2 \leq 1$, which restricts the monetary authority to inflationary or deflationary policies that never transfer money to the old producers. The other part of (18) is both necessary and sufficient for that the optimal

consumption of each young producer is positive.

A traditional result we obtain in line with the previous literature on cash-in-advance models (Basci and Saglam, 1999, 2003a, 2003b, 2005; Crettez, Michel and Wigniolle, 1999) is that the real wage rate never exceeds the marginal product of labor, apparent from Euler equation (19) along with condition (17) in Proposition 3.1-(i). The intuition underlying Euler equation is that increasing consumption in period t by reducing savings for the next period by Δm units yields to a young producer a marginal utility of $U^{p\prime}(c^p_{1,t})(\Delta m/p_t)$, where $\Delta m/p_t$ is the amount of additional consumption. On the other hand, with Δm units of reduction in period $t+1$ money holdings, the labor demand of the producer, when old, falls by $\Delta m/w_{t+1}$ units, which implies a reduction of $f^{p\prime}(l^p_{2,t+1})\Delta m/w_{t+1}$ units in output. The decrease in the utility of the producer due to the fall in output in period $t+1$ then becomes $\beta^p U^{p\prime}(c^p_{2,t+1})f^{p\prime}(l^p_{2,t+1})\Delta m/w_{t+1}$. In equilibrium, where money prices grow at the money inflation rate, the net marginal benefit of transferring money from period t to period $t+1$ is zero only if equation (19) is satisfied.

The observed wage-productivity gap in (19) can be interpreted as a premium accruing to producers from the use of money as a working capital. Indeed, it is the presence of this gap that makes cash-in-advance constraints binding for producers and money be held for transaction motive.

By Proposition 3.1-(i), money growth rate $1+\alpha$ cannot fall below the marginal rate of substitution, β^w, of workers at a stationary consumption plan; therefore no young worker has any incentive to hoard money by carrying cash balances to the next period. Instead, each young worker spends his entire wage earning in the good market. Additionally, by the optimality of consumption plans both workers and producers leave the economy at the end of their second periods without any cash balances. So, as clear from equations (31)-(34), end-of-period money balances are nonzero only for young producers, who have chosen to hold cash to pay wage receipts in the coming period.

The difference in the attitudes of young and old producers toward holding money gives rise to the nonstationarity of the monetary competitive equilibrium within generations. While the quantity of output supplied by producers in the good market is nil in the second period, it is not so in the first period as dictated by the aforementioned optimal money holding plan.

As we shall see in the next section, the crucial part of the above proposition is

labor demand by each old and young producer as given by equations (19) and (27), respectively. On the opposite side of the factor market, labor supply curves of each young and old worker are given by equations (20) and (21), respectively. It is easy to see that we have a conventional upward-sloping aggregate labor supply curve due to the strict concavity of the leisure function. In addition, supply of labor does not change over time, since workers own the same amount of labor endowment in each period of their lives. The equilibrium real wage rate is therefore constant. In result, equations (23), (24), (29) and (30) altogether imply the same amount of consumption for workers in every period.

4 Monetary Analysis of Equilibirum

This section seeks to investigate the interaction between money and real equilibrium variables in the economy.

For ease of notation, we will henceforth suppress the superscript (*) that marks equilibrium variables. We recall from the previous section that aggregate labor supply is increasing in the real wage rate. We also observe from equations (20) and (21) that supply of labor depends on neither the level nor the growth rate of money stock in the economy.

To analyze the equilibrium in the labor market, let us now consider the demand side. We rewrite equation (19), which represents labor demand of an old producer, as

$$\frac{\omega_t}{p_t} = \frac{\beta^p}{1+\alpha} f^{p\prime}(l^p_{2,t+1}) \frac{U^{p\prime}(f^p(l^p_{2,t+1}))}{U^{p\prime}(-l^p_{2,t+1}(\omega_t/p_t)(1+\alpha)/(1+\alpha_2) + f^p(l^p_{1,t}))} \tag{35}$$

using (25), (26), (29) and (30). Hence we observe that the rate of growth, but not the level, of money stock affects the labor demand by an old producer. Similarly, the labor demand of each young producer depends on the money growth rate if old producers' ratio of money transfers to the money stock remains constant. This is evident from $l^p_{1,t} = \alpha_1 m_{t-1}/(n^p\omega_t)$ obtained by using (27) and the definition of $x^p_{1,t}$. Now using $l^p_{2,t+1} = (m^p_{1,t} + x^p_{2,t+1})/\omega_{t+1} = (1+\alpha_2)m_{t-1}/(n^p\omega_t)$, we get

$$l^p_{1,t} = \frac{\alpha_1}{1+\alpha_2} l^p_{2,t+1}. \tag{36}$$

However, even with this simple intertemporal relation that producers' demand for labor must satisfy in equilibrium, it is still quite difficult to analyze the demand by an old

producer as it stands in equation (35). So, to simplify matters further, we henceforth assume $f^p(L) = l^\gamma$, where $\gamma > 1$, and $U^p(c) = \ln(c)$. Then, equation (35) becomes

$$\frac{\omega_t}{p_t} = \frac{\beta^p \gamma}{1 + \alpha_2 + \beta^p \gamma} \frac{1 + \alpha_2}{1 + \alpha} \frac{(l^p_{1,t})^\gamma}{l^p_{2,t+1}}. \tag{37}$$

Using (36), labor demand curve of an *old* producer further reduces to

$$\frac{\omega_t}{p_t} = \frac{\beta^p \gamma}{1 + \alpha_2 + \beta^p \gamma} \frac{\alpha_1^\gamma}{(1 + \alpha)(1 + \alpha_2)^{\gamma - 1}} (l^p_{2,t+1})^{\gamma - 1}. \tag{38}$$

Similarly, the reduced-form labor demand of a *young* producer satisfies

$$\frac{\omega_t}{p_t} = \frac{\beta^p \gamma}{1 + \alpha_2 + \beta^p \gamma} \frac{\alpha_1}{1 + \alpha} (l^p_{1,t})^{\gamma - 1}. \tag{39}$$

Finally, we obtain *aggregate* labor demand $l^d_t = n^p[l^p_{1,t} + l^p_{2,t+1}] = n^p[(1+\alpha)/(1+\alpha_2)]l^p_{2,t+1}$ in reduced form as

$$\frac{\omega_t}{p_t} = \frac{\beta^p \gamma}{1 + \alpha_2 + \beta^p \gamma} \frac{\alpha_1^\gamma}{(1 + \alpha)^\gamma} \frac{(l^d_t)^{\gamma - 1}}{(n^p)^{\gamma - 1}}. \tag{40}$$

Clearly, aggregate labor demand is increasing in the real wage rate. This result is against the common intuition in economics that demand by a profit maximizing firm for a factor of production falls if its unit cost has increased in real terms. However, we should be aware that such an intuition rests upon the assumption of diminishing marginal products, which has mostly been taken for granted in the litareture. Here, the presence of increasing returns to scale (or rising marginal products) leads producers to respond to wage rises by expanding their aggregate demand for labor while the boundedness of producers' choices is ensured by cash-in-advance constraints.

Figure 2 below depicts workers' total supply of labor, $l^s_t = -n^w(l^w_{1,t} + l^w_{2,t+1})$, and producers' total demand for labor, l^d_t with respect to the real wage rate w_t/p_t. (Henceforth, we assume $v_1^{'''} > 0$ and $f^{p'''} < 0$ for a convex labor supply function and concave labor demand function, respectively. Note that assumption $A3$ checks that labor demand is steeper than labor supply at the origin, ensuring a unique equilibrium in the labor market.)

Our first remark is that money is neutral, i.e. the quantity of money has no effect on the equilibrium real wage rate, employment and output, for neither supply nor demand curve in the labor market depends upon the level of money.

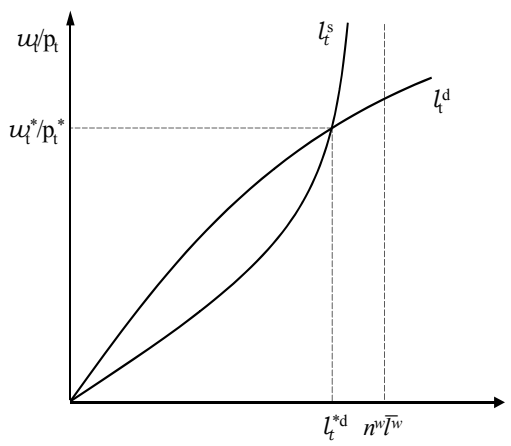

FIGURE 2. — Labor demand and supply curves

The next corollary aims to address a related issue: the impact of money inflation on the equilibrium outcome under two extreme distributions of money transfers.

Corollary 4.1. *An increase in the growth rate α of money supply (i) decreases the equilibrium real wage rate, aggregate employment and young producer's output if the ratio α_1 of young producers' transfers to money stock remains constant (ii) increases the equilibrium real wage rate, agregate employment and young producer's output if the ratio α_2 of old producers' transfers to money stock remains constant.*

Proof. See Appendix.

The first part of the above result is familiar to us from the previous literature on cash-in-advance models with infinitely-lived representative agents owning decreasing returns to scale (DRTS) technologies.[3] Even the (nonconventional) second part of the corollary is not due the availability of increasing returns in production but due to the presence of heterogenous producers in the economy, as Erdogan and Saglam (2006) obtains a similar result in an OLG framework with DRTS technologies. It is evident in our model that in any time period only young producers supply the good market (for the sole purpose of collecting cash required for the next period's factor payments), since old producers find

[3]See, for example, Basci and Saglam (2005).

it optimal to consume the last period's output entirely before dying. Then, it is intuitive that money inflation is productive in terms of rising employment only if it relaxes young producers' liquidity constraints.

If the ratio of transfers received by young producers to the existing money stock is held constant (Corollary 4.1-(i)), an increase (a decrease) in the money inflation (deflation) relieves old producers in the factor market and consequently all workers in the labor market. However, the resulting rise in nominal wages makes young producers more liquidity constrained in the labor market and this affects planned aggregate supply adversely. In effect, good prices rise faster than factor prices, leading to a decrease in the real wage rate and in the aggregate employment. Hence we obtain a Phillips curve with a stable negative tradeoff between anticipated inflation and employment.

Oppositely, if the ratio of transfers received by old producers to the money stock is held constant (Corollary 4.1-(ii)), an increase in the money inflation relaxes the liquidity constraints faced by young producers as well as all workers. The expansion in supply plans increases real wage rate, hence aggregate employment, inspite of the upward pressure in the good prices resulting from the increase in planned money spendings by workers. Now, we obtain an upward-sloping Phillips curve between anticipated inflation and employment.

We should note from the equilibrium relation $l^p_{2,t+1} = l^p_{1,t}(1+\alpha_2)/\alpha_1$ that the terms $l^p_{1,t}$ and $(1+\alpha_2)/\alpha_1$ always move in opposite directions under the two described transfer rules in Corollary 4.1. So, the influences of money growth on old producers' labor demand and output are ambiguous. We cannot draw any conclusions about the impact of inflation on the aggregate output, either.

In order to uncover the otherwise unobvious interaction between inflation and output, we simulate the model for the set of parameters $\langle \beta^p = \beta^w = 0.95,\ n^p = n^w = 100,\ \gamma = 1.5,\ \bar{l}^w = 1\rangle$ using the leasure function $v^w(e) = \sqrt{e} - e/(2\sqrt{\bar{l}^w})$, where $e \in [0, \bar{l}^w]$.

We first fix α_1 to 0.1, and vary the money growth rate α by changing the 'transfer to stock ratio' α_2 of old producers in increments of 0.20 between -0.90 and -0.10. The results reported in Table II below show that decreased money deflation curbs the aggregate labor demand l^d_t and individual labor demands $l^p_{1,t}$ and $l^p_{2,t+1}$ of each young and old producer. In effect, the individual output levels in columns five and six and aggregate output $Q_t = n^p[f^p(l^p_{1,t}) + f^p(l^p_{2,t+1})]$ in column seven fall. Thus, we get a downward-sloping Philips Curve relation between anticipated inflation and output. Consumptions

of young and old producers also decrease with falling money deflation. This is apparent from columns six and eight, recalling that $c^p_{2,t+1} = f^p(l^p_{2,t+1})$.

TABLE II

Equilibrium Outcome Associated with Various Levels of Transfer/Stock Ratio for Old Producers

α_2	l^d_t	$l^p_{1,t}$	$l^p_{2,t+1}$	$f^p(l^p_{1,t})$	$f^p(l^p_{2,t+1})$	Q_t	$c^p_{1,t}$	ω_t/p_t	$(rrwg)_t$
-0.90	136.25	0.68	0.68	0.56	0.56	112.46	0.04	0.39	0.69
-0.70	46.21	0.12	0.35	0.04	0.20	24.33	0.01	0.07	0.92
-0.50	14.51	0.02	0.12	0.00	0.04	4.58	0.00	0.02	0.96
-0.30	5.40	0.01	0.05	0.00	0.01	1.08	0.00	0.01	0.98
-0.10	2.36	0.00	0.02	0.00	0.00	0.32	0.00	0.00	0.99

Table II also verifies a direct implication of Corollary 4.1 for the influence of inflation on the consumption of each young and old worker. Falling money deflation leads to a dramatic fall in the real wage rate. Since per-period labor supplied by each worker $l^d_t/(2n^w)$ falls with reduced aggregate demand l^d_t, workers consume less in equilibrum. The final column of Table II reports the widening relative real wage rate gap, $(rrwg)_t$, defined as the wage-productivity gap $(\omega_t/p_t) - f^{p'}(l^p_{2,t+1})$ as a fraction of the productivity $f^{p'}(l^p_{2,t+1})$.

We now simulate the artificial economy described above for a money transfer rule lying at the other extreme. We set α_2 to -0.1, and vary young producers' money transfer to money stock ratio α_1 in increments of 0.20 between 0.10 and 0.90. This change in the distribution of money transfers affects results in the direction one would partially infer from Corollary 4.1-(ii). Table III shows that inflating money now stimulates aggregate output. Hence, we can derive an upward-sloping Phillips curve between anticipated inflation and output.

Decomposing the positive impact of inflation with respect to the two types of producers at an individual level, we observe in the above table that demand for labor as well as supply and consumption of the final good by each young and old producer increase with money inflation. Our simulations also demonstrate a rise in the real wage rate. In effect, labor supplied and output consumed by young and old workers increase. Finally,

we observe that the wage-productivity gap in relative terms narrows down as the money inflation becomes higher.

TABLE III

Equilibrium Outcome Associated with Various Levels of Transfer/Stock Ratio for Young Producers

α_1	l_t^d	$l_{1,t}^p$	$l_{2,t+1}^p$	$f^p(l_{1,t}^p)$	$f^p(l_{2,t+1}^p)$	Q_t	$c_{1,t}^p$	ω_t/p_t	$(rrwg)_t$
0.10	2.36	0.00	0.02	0.00	0.00	0.32	0.00	0.00	0.99
0.30	29.59	0.07	0.22	0.02	0.10	12.47	0.01	0.04	0.94
0.50	62.79	0.22	0.40	0.11	0.26	36.26	0.04	0.10	0.89
0.70	87.12	0.38	0.49	0.24	0.34	57.84	0.09	0.17	0.84
0.90	103.74	0.52	0.52	0.37	0.37	74.71	0.14	0.22	0.80

What we learn from the above results is that optimal money supply rule depends on the way transfers are allocated. A monetary authority must set the inflation rate α at the level that is produced by the the highest α_1 and lowest α_2 in their admissible domains, so as to maximize the aggregate employment (and possibly the aggregate output as suggested by the simulations in Tables II and III.) The lower bound for α_2 is explicit in (18) whereas there exists no upper bound for α_1. So, under the suggested money allocation which sets the transfer to stock ratio of old producers at the lowest possible rate, the optimal growth rate of money (the upper bound for money transfers received by young producers) is limited by only the printing capacity of the monetary authority. But, we should notice that the marginal impact of inflation on the equilibrium outcome is diminishing. As the inflation rate α increases, the varying term $\alpha_1/(1+\alpha)$ in equation (40) approaches to 'one', yielding the frontier of the aggregate demand curve. Clearing the supply in the labor market with this limit curve of demand, we can find the finite supremum of the output level that an ever-growing money inflation would lead to in equilibrium.

The unboundedness of the optimal inflation rule must not be very annoying though, for we should notice that our model is missing costs of inflation other than the direct welfare costs (benefits in our case). Enriching our model with ingredients such as menu costs, competitiveness in the world market and reputation effects, we may also get

a finite positive rate of money growth that balances the trade-off between the direct welfare benefits and indirect costs of inflation.

5 Conclusions

This paper shows that unregulated decentralized equilibrium is viable in an IRTS economy involving overlapping generations of producers and workers who face liquidity constraints in both factor and good markets. Monetary competitive equilibrium is found to exist if and only if the rate of money growth is sufficiently high. The well-known wage-productivity gap as a working capital premium is also established.

It is crucial for our results that firms cannot 'perfectly' commit to pay unlimited amounts of wages to workers in goods, as soon as production has taken place. Obviously, the introduction of commodity contracts (IOU's by firms) or even a credit market (IOU's by banks) into our model to intermediate the timing mismatch between payments to workers and production would not change the nature of our existence result as long as firms were also facing constraints in issuing contracts or borrowing credits, which may for example arise due to the need to collateralize loans. Yet, in order to simplify the characterization of equilibirum we restrict our attention in this study to 'outside money' as the single working capital.

We find that money is neutral but not superneutral. As in Erdogan and Saglam (2006), we show that the effects of money inflation on the aggregate employment can be in both directions depending upon the rule according to which the monetary authority allocates money transfers between young and old producers. Conducting simulations, we also get the missing analytical interaction between inflation and output. By the observed monotonic relation between employment and output in equilibrium, the Phillips curve relation between inflation and output is similar to the relation between inflation and employment for each money transfer rule.

We should note that the sensitivity of the distribution of social welfare to the division of monetary transfers between generations is not really new. As a matter of fact, Drazen (1981) obtains a similar result in a finite life-cycle model with money in the utility function. He shows that if seignorage goes to the the young then a Tobin effect arises; i.e. money inflation unambigously increases the demand for capital (per labor). He also establishes that if seignorage goes to the old, then a reverse Tobin effect arises. The

possibility of a reverse Tobin effect is also obtained by Ghossoub and Reed (2005) in an infinite horizon model involving cash-in-advance constraints on consumption purchases. Our integrated model which imposes cash-in-advance constraints in a finite life-cycle (OLG) model clearly demonstrates that unregulated decentralized equilibrium is viable and that the results of Drazen (1981) and Ghossoub and Reed (2005) may remain to hold under increasing returns technologies, too.

Appendix

Lemma A.1. Let $h : I\!R_{++} \to I\!R_{++}$ be a function that is strictly increasing (decreasing) and multiplicatively separable. Define $g := 1/h$. Then the inverse function g^{-1} is strictly decreasing (increasing) and separable.

Proof. Both g and g^{-1} are strictly decreasing (increasing), since h is strictly increasing (decreasing). It is also obvious that g is separable, since h is separable. Now pick any $c_1, c_2 \in I\!R_{++}$. Suppose

$$g^{-1}(c_1c_2) \neq g^{-1}(c_1)g^{-1}(c_2).$$

Then the monotonicity and separability of g implies

$$g(g^{-1}(c_1c_2)) \neq g(g^{-1}(c_1)g^{-1}(c_2))$$

or

$$c_1c_2 \neq g(g^{-1}(c_1))g(g^{-1}(c_2)) = c_1c_2$$

which is a contradiction. Therefore, g^{-1} is separable. *Q.E.D.*

Proof of Proposition 3.1. We will consider the two parts of the proposition separately.

Part-i: We consider the reduced form maximization problem of workers:

$$\max_{\{m^w_{1,t}, m^w_{2,t+1}, l^w_{1,t}, l^w_{2,t+1}\}} U^w\left(\frac{-m^w_{1,t} - \omega_t l^w_{1,t}}{p_t} + v^w(\bar{l}^w_1 + l^w_{1,t})\right) +$$

$$\beta^w U^w\left(\frac{m^w_{1,t} - m^w_{2,t+1} - \omega_{t+1} l^w_{2,t+1}}{p_{t+1}} + v^w(\bar{l}^w_2 + l^w_{2,t+1})\right)$$

The first-order conditions for $l^w_{1,t}$ and $l^w_{2,t+1}$ yield the respective labor supply curves $\omega_t/p_t = v^{w'}(\bar{l}^w_1 + l^\omega_{1,t})$ and $\omega_{t+1}/p_{t+1} = v^{w'}(\bar{l}^w_2 + l^\omega_{2,t+1})$. So, cash-in-advance constraint is not binding for workers in the factor market.

Note that if $1+\alpha \geqslant \beta^w$, the Euler condition associated with the control $m^w_{1,t}$, becomes

$$-\frac{1}{p_t}U^{w'}\big(c^w_{1,t} + v^w(\bar{l}^w_1 + l^w_{1,t})\big) + \frac{\beta^w}{p_{t+1}}U^{w'}\big(c^w_{2,t+1} + v^w(\bar{l}^w_2 + l^w_{2,t+1})\big) \leqslant 0,$$

using $l^w_{1,t} = l^w_{2,t+1}$ by (20) and (21); $c^w_{1,t} = c^w_{2,t+1}$ by (23), (24), (29) and (30), and additionally $p_{t+1} = p_t(1+\alpha)$ by the stationarity of nominal prices. So, the money holding plan $m^w_{1,t} = 0$ is optimal if $1+\alpha \geqslant \beta^w$.

If, on the contrary, $1+\alpha < \beta^w$, the Euler condition becomes

$$-\frac{1}{p_t}U^{w'}\big(c^w_{1,t} + v^w(\bar{l}^w_1 + l^w_{1,t})\big) + \frac{\beta^w}{p_{t+1}}U^{w'}\big(c^w_{2,t+1} + v^w(\bar{l}^w_2 + l^w_{2,t+1})\big) > 0.$$

Then, by slightly increasing $m^w_{1,t}$ over initial money endowment of $x^w_{1,t} = 0$ (hence slightly increasing $c^w_{2,t+1}$ over $c^w_{1,t}$) workers can be better off. In that case, the plan $m^w_{1,t} = 0$ could not be optimal.

The proof of the 'if' statement for inequality (17) is implicit in the proof of Part-ii-(b).

Finally, notice that the inequality $1+\alpha_2 > \beta^p \frac{f^{p'}(l^p_{2,t+1})l^p_{2,t+1}}{f^p(l^p_{1,t})} \frac{U^{p'}(c^p_{2,t+1})}{U^{p'}(c^p_{1,t})}$ in condition (18) is sufficient for the equilibrium consumption

$$\begin{aligned} c^p_{1,t} &= f^p(l^p_{1,t}) - l^p_{2,t+1}\frac{\omega_t}{p_t}(1+\alpha)/(1+\alpha_2) \\ &= f^p(l^p_{1,t}) - \frac{\beta^p}{1+\alpha_2} f^{p'}(l^p_{2,t+1})l^p_{2,t+1}\frac{U^{p'}(c^p_{2,t+1})}{U^{p'}(c^p_{1,t})} \end{aligned}$$

of each young producer to be positive. The remaining part of condition (18), namely $\alpha_2 \leq 0$, suffices for the existence of a solution to the Euler equation associated with the producers' problem, which is explicit in the proof of Part-ii-(b).

Part-ii: The proof consists of two parts: (a) Every MCE satisfies (19)-(34); (b) the plan (19)-(34) is a MCE.

(a) Let $\{p_t, \omega_t, p_{t+1}, \omega_{t+1}, c^i_{1,t}, c^i_{2,t+1}, l^i_{1,t}, l^i_{2,t+1}, q^i_{1,t}, q^i_{2,t+1}, m^i_{1,t}, m^i_{2,t+1}, \mid i = w, p\}$ be a MCE. In Part-i of the proof, we showed that labor supply of each worker must satisfy

(20) and (21) when young and old, respectively. Labor market clearing implies (22). Equations (23),(24),(25),(26) follow from (1) and (2) given the optimal choices of money holding, whereas (29) and (30) are restatements of (7) and (8) in equilibrium.

To derive the rest of the MCE plan, consider the reduced form maximization problem of producers:

$$\max_{\{m^p_{1,t}, m^p_{2,t+1}, l^p_{1,t}, l^p_{2,t+1}\}} U^p\left(\frac{x^p_{1,t} - m^p_{1,t} - \omega_t l^p_{1,t}}{p_t} + f^p(l^p_{1,t})\right) +$$

$$\beta^p U^p\left(\frac{m^p_{1,t} + x^p_{2,t+1} - m^p_{2,t+1} - \omega_{t+1} l^p_{2,t+1}}{p_{t+1}} + f^p(l^p_{2,t+1})\right)$$

The first-order conditions for $l^p_{1,t}$ and $l^p_{2,t+1}$, under the assumption that $\omega_t/p_t \leqslant f^{p\prime}(l^p_{2,t+1})$, become $l^p_{1,t} = x^p_{1,t}/\omega_t$ and $l^p_{2,t+1} = (m^p_{1,t} + x^p_{2,t+1})/\omega_{t+1}$. That is, cash-in-advance constraints are binding for producers in the factor market.

The objective of producers, then, reduces to

$$\max_{\{m^p_{1,t}, m^p_{2,t+1}\}} U^p\left(\frac{-m^p_{1,t}}{p_t} + f^p\left(\frac{x^p_{1,t}}{\omega_t}\right)\right) + \beta^p U^p\left(\frac{-m^p_{2,t+1}}{p_{t+1}} + f^p\left(\frac{m^p_{1,t} + x^p_{2,t+1}}{\omega_{t+1}}\right)\right).$$

The Euler condition associated with the control $m^p_{1,t}$ becomes

$$-\frac{1}{p_t} U^{p\prime}(c^p_{1,t}) + \frac{\beta^p f^{p\prime}(l^p_{2,t+1})}{\omega_{t+1}} U^{p\prime}(c^p_{2,t+1}) = 0.$$

Using the stationarity condition $\omega_{t+1} = (1+\alpha)\omega_t$, we obtain (19).

(b) We have to prove that the plan (19)-(34) is optimal, individually feasible, stationary, symmetric across generations and satisfies aggregate feasibility (market clearing) conditions.

(b-i) Optimality: We will check that both types of agents optimize under the proposed prices and plans of action. For ease of notation, suppress the superscript $(*)$ in equilibrium variables, hereafter. The optimality of $l^w_{1,t}$ and $l^w_{2,t+1}$ were shown in Part-i of the proof. Denote the objective function of type i agents as $V^i(m^i_{1,t}, m^i_{2,t+1})$ for $i = w, p$. Define $V^i_1(m^i_{1,t}, m^i_{2,t+1}) = \partial V^i(m^i_{1,t}, m^i_{2,t+1})/\partial m^i_{1,t}$ and $V^i_2(m^i_{1,t}, m^i_{2,t+1}) = \partial V^i(m^i_{1,t}, m^i_{2,t+1})/\partial m^i_{2,t+1}$.

Now, we will verify that $V^w(m^w_{1,t}, m^w_{2,t+1})$ is jointly concave in $m^w_{1,t}$ and $m^w_{2,t+1}$. First recall that

$$V^w(m^w_{1,t}, m^w_{2,t+1}) = U^w\left(-\frac{m^w_{1,t}}{p_t} - \frac{\omega_t}{p_t} l^w_{1,t} + v^w(\bar{l}^w_1 + l^w_{1,t})\right)$$

$$+ \beta^w U^w\left(\frac{m^w_{1,t} - m^w_{2,t+1}}{p_{t+1}} - \frac{\omega_{t+1}}{p_{t+1}} l^w_{2,t+1} + v^w(\bar{l}^w_2 + l^w_{2,t+1})\right).$$

Then

$$V^w_1 = -\frac{1}{p_t} U^{w\prime}\left(c^w_{1,t} + v^w(\bar{l}^w_1 + l^w_{1,t})\right) + \frac{\beta^w}{p_{t+1}} U^{w\prime}\left(c^w_{2,t+1} + v^w(\bar{l}^w_2 + l^w_{2,t+1})\right) \leqslant 0,$$

since $1 + \alpha \geqslant \beta^w$ and $c^w_{1,t} + v^w(\bar{l}^w_1 + l^w_{1,t}) = c^w_{2,t+1} + v^w(\bar{l}^w_2 + l^w_{2,t+1})$. Moreover,

$$V^w_2 = -\frac{\beta^w}{p_{t+1}} U^{w\prime}\left(c^w_{2,t+1} + v^w(\bar{l}^w_2 + l^w_{2,t+1})\right) < 0$$

and

$$V^w_{11} = \frac{1}{p_t^2} U^{w\prime\prime}\left(c^w_{1,t} + v^w(\bar{l}^w_1 + l^w_{1,t})\right) + \frac{\beta^w}{p_{t+1}^2} U^{w\prime\prime}\left(c^w_{2,t+1} + v^w(\bar{l}^w_2 + l^w_{2,t+1})\right) < 0$$

$$V^w_{22} = \frac{\beta^w}{p_{t+1}^2} U^{w\prime\prime}\left(c^w_{2,t+1} + v^w(\bar{l}^w_2 + l^w_{2,t+1})\right) < 0$$

$$V^w_{12} = V^w_{21} = -\frac{\beta^w}{p_{t+1}^2} U^{w\prime\prime}\left(c^w_{2,t+1} + v^w(\bar{l}^w_2 + l^w_{2,t+1})\right) > 0.$$

Hence, the Hessian matrix is negative semi-definite, and $V^w(m^w_{1,t}, m^w_{2,t+1})$ is concave.

Similarly, the objective function of a representative producer is

$$V^p(m^p_{1,t}, m^p_{2,t+1}) = U^p\left(-\frac{m^p_{1,t}}{p_t} + f^p\left(\frac{x^p_{1,t}}{w_t}\right)\right)$$

$$+\beta^p U^p\left(-\frac{m^p_{2,t+1}}{p_{t+1}} + f^p\left(\frac{m^p_{1,t} + x^p_{2,t+1}}{\omega_{t+1}}\right)\right).$$

It follows that

$$V^p_2 = -\frac{\beta^p}{p_{t+1}} U^{p\prime}\left(q^p_{2,t+1} + f^p\left(\frac{m^p_{1,t} + x^p_{2,t+1}}{\omega_{t+1}}\right)\right) < 0$$

$$V^p_{22} = \frac{\beta^p}{p_{t+1}^2} U^{p\prime\prime}\left(q^p_{2,t+1} + f^p\left(\frac{m^p_{1,t} + x^p_{2,t+1}}{\omega_{t+1}}\right)\right) < 0.$$

So, we must have $m^p_{2,t+1} = 0$.

Define $\tilde{m}^p_{1,t} := m^p_{1,t}/w_{t+1}$, $\tilde{x}^p_{1,t} := x^p_{1,t}/w_t$ and $\tilde{x}^p_{2,t} := x^p_{2,t}/w_t$ for all t. It then follows that $V^p(m^p_{1,t}, m^p_{2,t+1}) = V^p(\tilde{m}^p_{1,t} w_{t+1}, 0)$ that we simply denote by

$$V^p(\tilde{m}^p_{1,t}) = U^p\left(f^p(\tilde{x}^p_{1,t}) - \frac{w_{t+1}}{p_t}\tilde{m}^p_{1,t}\right) + \beta^p U^p\left(f^p(\tilde{m}^p_{1,t} + \tilde{x}^p_{2,t+1})\right)$$

with an abuse of notation. Then we have

$$\begin{aligned} V^{p\prime}(\tilde{m}^p_{1,t}) &= -\frac{w_{t+1}}{p_t}U^{p\prime}\left(f^p(\tilde{x}^p_{1,t}) - \frac{w_{t+1}}{p_t}\tilde{m}^p_{1,t}\right) \\ &\quad +\beta^p f^{p\prime}(\tilde{m}^p_{1,t} + \tilde{x}^p_{2,t+1})U^{p\prime}\left(f^p(\tilde{m}^p_{1,t} + \tilde{x}^p_{2,t+1})\right). \end{aligned}$$

The rest of the proof aims to show the optimality of MCE plan for producers, and involves three steps: First, we will show that there exists an interior solution to the first-order condition for the problem of producers for all money growth rates satisfying conditions (17) and (18). Second, we will prove that corner solutions cannot be optimal and hence the optimal solution must be an interior one. The last step verifies that the interior solution is unique.

Step 1. Note that $-\tilde{x}^p_{2,t+1} \geq 0$, since $\alpha_2 \leq 0$ by (18). Then

$$\tilde{m}^p_{1,t} \to -\tilde{x}^p_{2,t+1} \text{ implies } V^{p\prime} \to \infty, \text{ and}$$

$$\tilde{m}^p_{1,t} \to \frac{p_t}{w_{t+1}} f^p(\tilde{x}^p_{1,t}) \text{ implies } V^{p\prime} \to -\infty.$$

Now denote the set of feasible money holding plans by $\mathcal{A} = [-\tilde{x}^p_{2,t+1}, p_t f^p(\tilde{x}^p_{1,t})/w_{t+1}]$ and its interior by $int(\mathcal{A})$. One can easily show that $int(\mathcal{A}) \neq \emptyset$ if and only if

$$-\frac{\alpha_2}{1+\alpha_2} l^p_{2,t+1} < \frac{f^p(l^p_{1,t})}{\beta^p f^{p\prime}(l^p_{2,t+1})} \frac{U^{p\prime}(c^p_{1,t})}{U^{p\prime}(c_{2,t+1})}$$

which holds true thanks to condition (18). Then, we conclude by the continuity of $V^{p\prime}$ that there exists $\tilde{m}^p_{1,t} \in int(\mathcal{A})$ such that $V^{p\prime}(\tilde{m}^p_{1,t}) = 0$.

Step 2. First consider the consumption plan $c^p_{1,t} = f^p(\tilde{x}^p_{1,t}) + \tilde{x}^p_{2,t+1} w_{t+1}/p_t$ and $c^p_{2,t+1} = 0$ associated with the corner solution $\tilde{m}^p_{1,t} = -\tilde{x}^p_{2,t+1}$. Assumption $A2$ ensures that producers have an incentive to slightly increase $\tilde{m}^p_{1,t}$ and hence $c^p_{2,t+1} = 0$ if $int(\mathcal{A}) \neq \emptyset$.

Next consider the consumption plan $c^p_{1,t} = 0$ and $c^p_{2,t+1} = f^p(\tilde{x}^p_{2,t+1} + f^p(\tilde{x}^p_{1,t})p_t/w_{t+1})$ associated with the corner solution $\tilde{m}^p_{1,t} = p_t f^p(\tilde{x}^p_{1,t})/w_{t+1}$. Assumption $A2$ now ensures that by slightly decreasing $\tilde{m}^p_{1,t}$, hence by slightly increasing $c^p_{1,t}$, producers become better-off if $int(\mathcal{A}) \neq \emptyset$. Therefore, no corner solution can be optimal.

Step 3. At an interior solution, the Euler condition becomes

$$U^{p'}\left(f^p(\tilde{x}^p_{1,t}) - \frac{w_{t+1}}{p_t}\tilde{m}^p_{1,t}\right) = \frac{\beta^p}{w_{t+1}/p_t} f^{p'}(\tilde{m}^p_{1,t} + \tilde{x}^p_{2,t+1}) U^{p'}\left(f^p(\tilde{m}^p_{1,t} + \tilde{x}^p_{2,t+1})\right).$$

Define $g := (1/U^{p'})^{-1}$. The function $U^{p'}$ is strictly decreasing and separable, by assumption. Thus, by Lemma A.1., g is strictly increasing and separable. Then

$$\begin{aligned} f^p(\tilde{m}^p_{1,t} + \tilde{x}^p_{2,t+1}) &= g\left(\frac{\beta^p}{w_{t+1}/p_t} f^{p'}(\tilde{m}^p_{1,t} + \tilde{x}^p_{2,t+1}) \frac{1}{U^{p'}\left(f^p(\tilde{x}^p_{1,t}) - \tilde{m}^p_{1,t} w_{t+1}/p_t\right)}\right) \\ &= g(\beta^p p_t/w_{t+1})\, g(f^{p'}(\tilde{m}^p_{1,t} + \tilde{x}^p_{2,t+1}))\, (f^p(\tilde{x}^p_{1,t}) - \tilde{m}^p_{1,t} w_{t+1}/p_t). \end{aligned}$$

Then

$$0 = f^p(\tilde{m}^p_{1,t} + \tilde{x}^p_{2,t+1}) - g(\beta^p p_t/w_{t+1})\, g(f^{p'}(\tilde{m}^p_{1,t} + \tilde{x}^p_{2,t+1}))\, (f^p(\tilde{x}^p_{1,t}) - \tilde{m}^p_{1,t} w_{t+1}/p_t).$$

Denoting RHS of the above equation by $h(\tilde{m}^p_{1,t})$, we have $h(\tilde{m}^p_{1,t}) = 0$, as a restatement of Euler equation. By step 1, there exists at least one choice of $\tilde{m}^p_{1,t}$ that solves Euler equation. We have $h(\tilde{m}^p_{1,t}) > 0$ if and only if

$$\frac{f^p(\tilde{m}^p_{1,t} + \tilde{x}^p_{2,t+1})}{g(\beta_2 p_t/w_{t+1}) g(f^{p'}(\tilde{m}^p_{1,t} + \tilde{x}^p_{2,t+1}))} + \tilde{m}^p_{1,t}\, w_{t+1}/p_t > f^p(\tilde{x}^p_{1,t}).$$

Note also that

$$\begin{aligned} h'(\tilde{m}^p_{1,t}) &= f^{p'}(\tilde{m}^p_{1,t} + \tilde{x}^p_{2,t+1}) \\ &\quad - g(\beta^p p_t/w_{t+1}) g'(f^{p'}(\tilde{m}^p_{1,t} + \tilde{x}^p_{2,t+1})) f^{p''}(\tilde{m}^p_{1,t} + \tilde{x}^p_{2,t+1})(f^p(\tilde{x}^p_{1,t}) \\ &\quad - \tilde{m}^p_{1,t}\, w_{t+1}/p_t) + g(\beta_2 p_t/w_{t+1}) g(f^{p'}(\tilde{m}^p_{1,t} + \tilde{x}^p_{2,t+1})) w_{t+1}/p_t, \end{aligned}$$

and we have $h'(\tilde{m}^p_{1,t}) > 0$ if and only if

$$\frac{f^{p'}(\tilde{m}^p_{1,t} + \tilde{x}^p_{2,t+1})}{g(\beta^p p_t/w_{t+1}) g'(f^{p'}(\tilde{m}^p_{1,t} + \tilde{x}^p_{2,t+1})) f^{p''}(\tilde{m}^p_{1,t} + \tilde{x}^p_{2,t+1})}$$

$$+ \frac{g(f^{p'}(\tilde{m}^p_{1,t} + \tilde{x}^p_{2,t+1}))\, w_{t+1}/p_t}{g'(f^{p'}(\tilde{m}^p_{1,t} + \tilde{x}^p_{2,t+1})) f^{p''}(\tilde{m}^p_{1,t} + \tilde{x}^p_{2,t+1})} + \tilde{m}^p_{1,t}\, w_{t+1}/p_t > f^p(\tilde{x}^p_{1,t}).$$

By assumption $A4$, $f^p(\tilde{m}^p_{1,t} + \tilde{x}^p_{2,t+1})/g(f^{p'}(\tilde{m}^p_{1,t} + \tilde{x}^p_{2,t+1}))$ is increasing in $\tilde{m}^p_{1,t} + \tilde{x}^p_{2,t+1}$, hence in $\tilde{m}^p_{1,t}$. Thus we have

$$\frac{f^p(\tilde{m}^p_{1,t} + \tilde{x}^p_{2,t+1})}{g(f^{p'}(\tilde{m}^p_{1,t} + \tilde{x}^p_{2,t+1}))} < \frac{f^{p'}(\tilde{m}^p_{1,t} + \tilde{x}^p_{2,t+1})}{g'(f^{p'}(\tilde{m}^p_{1,t} + \tilde{x}^p_{2,t+1}))f^{p''}(\tilde{m}^p_{1,t} + \tilde{x}^p_{2,t+1})}.$$

From the above inequalities we conclude that $h'(\tilde{m}^p_{1,t}) > 0$ for all $\tilde{m}^p_{1,t}$ such that $h(\tilde{m}^p_{1,t}) > 0$, and therefore the solution of Euler equation is unique.

(b-ii) Individual feasibility: On the workers' side, money demands in (31) and (32) respectively satisfy constraints (9) and (10) at the lower bounds. Conditions (11) and (12) are satisfied in the interior.

On the producers' side, constraint (13) reduces to $m^p_{1,t} \leqslant p_t f^p(\bar{l}_1 + l^p_{1,t})$ at (27), which holds true since $c^p_{1,t} > 0$. Whereas constraint (14) holds at the lower boundary, (15) holds at the upper bound. Finally, (28) satisfies constraint (16) at the boundary.

(b-iii) Aggregate feasibility: By equation (22), labor market clears. The plans (23), (24), (25) and (26) clear the good market, whereas the plans (31), (32), (33) and (34) are consistent with the money market clearing.

(b-iv) Symmetry and stationarity: One can easily verify that in equilibrium (19)-(34) the nominal variables are stationary whereas the real variables are symmetric across generations. *Q.E.D.*

Proof of Corollary 4.1. We prove the two parts of the Corollary separately.

Part-i: Using $\alpha_1 + \alpha_2 = \alpha$, we can check that an increase in α causes aggregate labor demand l^d_t in equation (40) to increase for all values of the real wage rate when α_1 is constant. Then the aggregate labor demand curve (40) shifts rightward while the aggregate supply curve keeps its position in a plane where the real wage lies on the vertical axis. Given the positive slope of the demand curve, the excess demand for labor is eliminated by a lower real wage rate in equilibrium, the result being a fall in aggregate employment. In effect, labor demand $l^p_{1,t}$ and the implied output $f^p(l^p_{1,t})$ of each young producer will both decrease, which follows from the fact that $l^p_{1,t} = [\alpha_1/(1+\alpha)]l^d_t/n^p$ along with $\alpha_1/(1+\alpha) > 0$.

Part-ii: When α_2 is constant, an increase in α causes aggregate labor demand l_t^d to decrease for all values of the real wage rate. Following the leftward shift in the aggregate labor demand curve (40), the excess supply of labor is eliminated by a higher real wage rate in equilibrium. Consequently, aggregate employment is higher in equilibrium. By a reasoning similar to that in the proof of Part-i, labor demand $l_{1,t}^p$ and the implied output $f^p(l_{1,t}^p)$ of each young producer are higher, too. *Q.E.D.*

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