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Precise finite-sample quantiles of the Jarque-Bera adjusted Lagrange multiplier test

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Abstract:

It is well known that the finite-sample null distribution of the Jarque-Bera Lagrange Multiplier (LM) test for normality and its adjusted version (ALM) introduced by Urzua differ considerably from their asymptotic $\chi^2(2)$ limit. Here, we present results from Monte Carlo simulations using 10^7 replications which yield very precise numbers for the LM and ALM statistic over a wide range of critical values and sample sizes. Depending on the sample size and values of the statistic we get p values which significantly deviate from numbers previously published and used in hypothesis tests in many statistical software packages. The p values listed in this short Letter enable for the first time a precise implementation of the Jarque-Bera LM and ALM tests for finite samples.

1 Introduction

The Jarque-Bera (1980, 1987) Lagrange multiplier test is likely the most widely used procedure for testing normality of economic time series returns. The algorithm provides a joint test of the null hypothesis of normality in that the sample skewness b_1 equals zero and the sample kurtosis b_2 equals three. The null is rejected when the Lagrange multiplier statistic

$$LM = N\left(\frac{(b_1^{1/2})^2}{6} + \frac{(b_2 - 3)^2}{24}\right)$$
(1)

exceeds some critical value, which is taken in the asymptotic limit from the $\chi^2(2)$ distribution. N is the sample size, $b_1^{1/2} = m_3/m_2^{3/2}$, $b_2 = m_4/m_2^2$ where m_i is the *i*-th central moment of the observations $m_i = \Sigma(x_j - \overline{x})^i/N$, and \overline{x} the sample mean.

Urzua (1996) modified the Jarque-Bera test replacing the asymptotic means and variances by their exact finite-sample values yielding

$$ALM = N\left(\frac{(b_1^{1/2})^2}{c_1} + \frac{(b_2 - c_2)^2}{c_3}\right) .$$
(2)

Here the parameters $c_{1,2,3}$ are given by the expectation value and variances of the skewness and kurtosis

$$c_1 = var(b_1^{1/2}) = \frac{6(N-2)}{(N+1)(N+3)} ,$$

$$c_2 = E(b_2) = \frac{3(N-1)}{(N+1)} ,$$

$$c_3 = var(b_2) = \frac{24N(N-2)(N-3)}{(N+1)^2(N+3)(N+5)} .$$

Note, that the ALM has the same asymptotic distribution as the LM statistic.

The work of Urzua (1996) as well as the work by Deb and Sefton (1996) already warn about the incorrect use of the Jarque-Bera test in the case of small- and medium-sized samples. The authors performed Monte Carlo simulations and tabulated significance points for 5% and 10%, on a series of sample sizes ranging between 10 and 800. Deb and Sefton used 600'000 replications in their Monte Carlo simulations and Urzua used 10'000 replications and added results for the 1%, 15% and 20% significance points. Very recently Lawford (2004) developed an accurate response surface approximation for the 5% and 10% critical values of the Jarque-Bera test based on Monte Carlo simulations using 1 Million replications. The tables for the LM and ALM statistic values presented in these papers are restricted usually to a small set of parameters and the precision is in most cases limited to two digits. Furthermore, for small N we observe significant differences in comparison to previously published values. For some parameter settings the differences are so large, that this may result in inaccurate hypothesis tests or evenmore this may lead to situations with wrong decisions.

In this Letter we present tables with very precise values for both, the LM and ALM statistic. Since the slow convergence of the Monte Carlo simulation is well known we extend the simulations to 10 Million replications and enhance the mesh of p-values and sample sizes considerably.

The results have been used to implement R functions for the finite sample Jarque-Bera test and the distribution itself, using either the LM or ALM statistic. R (2004) is a powerful and widely used GPL-licensed statistical software environment based on the S language. In this sense our functions can also be called from the commercial S-Plus software package. The R functions are part of the Rmetrics software project, www.rmetrics.org. The software is GPL licensed and can be downloaded from the CRAN Server www.r-project.org.

2 Monte Carlo Simulation

We performed Monte Carlo simulations of the LM and ALM statistic using 10^7 replications. The results are summarized in Table 1 for both the LM and ALM statistic.

LM: p∖N	10	20	35	50	75	100	150	200	300	500	800	1000	1600	2400	10000
0.01%	15.345	46.996	66.612	71.734	69.910	68.032	60.632	54.736	47.572	38.847	33.247	31.213	26.956	24.249	19.940
0.05%	12.444	31.159	40.759	43.256	41.909	40.430	37.229	34.330	30.561	26.270	23.045	21.979	19.760	18.366	16.052
0.10%	10.995	24.970	31.969	33.753	32.738	31.840	29.547	27.551	24.830	21.812	19.521	18.736	17.150	16.083	14.397
0.50%	7.3004	13.471	16.414	17.281	17.305	16.959	16.257	15.638	14.669	13.583	12.726	12.366	11.762	11.384	10.792
1.00%	5.7029	9.7182	11.736	12.392	12.586	12.491	12.185	11.882	11.3580	10.778	10.299	10.117	9.8095	9.6084	9.3128
5.00%	2.5247	3.7954	4.5929	4.9757	5.2777	5.4300	5.5984	5.6758	5.7732	5.8551	5.9103	5.9242	5.9569	5.9671	5.9857
10.00%	1.6232	2.3470	2.8814	3.1834	3.4862	3.6734	3.9041	4.0327	4.1891	4.3317	4.4274	4.4568	4.5132	4.5424	4.5888
15.00%	1.2826	1.8230	2.2533	2.5094	2.7713	2.9390	3.1416	3.2580	3.4003	3.5312	3.6198	3.6507	3.7016	3.7309	3.7778
20.00%	1.1236	1.5623	1.9162	2.1278	2.3463	2.4865	2.6558	2.7559	2.8764	2.9882	3.0645	3.0909	3.1360	3.1611	3.2036
30.00%	0.9389	1.2516	1.4997	1.6466	1.7975	1.8944	2.0112	2.0807	2.1639	2.2427	2.2962	2.3153	2.3460	2.3650	2.3968
40.00%	0.8077	1.0360	1.2115	1.3128	1.4165	1.4828	1.5619	1.6087	1.6649	1.7175	1.7547	1.7679	1.7889	1.8024	1.8248
50.00%	0.6950	0.8574	0.9771	1.0447	1.1126	1.1563	1.2076	1.2385	1.2752	1.3101	1.3338	1.3420	1.3568	1.3655	1.3808
60.00%	0.5885	0.6948	0.7699	0.8114	0.8529	0.8800	0.9105	0.9292	0.9518	0.9732	0.9882	0.9931	1.0024	1.0085	1.0181
70.00%	0.4801	0.5378	0.5769	0.5985	0.6202	0.6348	0.6508	0.6610	0.6730	0.6851	0.6940	0.6965	0.7018	0.7056	0.7108
80.00%	0.3618	0.3777	0.3896	0.3969	0.4046	0.4105	0.4168	0.4213	0.4267	0.4325	0.4368	0.4376	0.4402	0.4421	0.4451
85.00%	0.2950	0.2938	0.2958	0.2982	0.3010	0.3044	0.3071	0.3096	0.3130	0.3163	0.3189	0.3194	0.3209	0.3221	0.3245
90.00%	0.2192	0.2047	0.2002	0.1997	0.1997	0.2006	0.2016	0.2024	0.2040	0.2060	0.2071	0.2074	0.2081	0.2089	0.2106
95.00%	0.1272	0.1084	0.1022	0.1005	0.0995	0.0996	0.0992	0.0995	0.1000	0.1005	0.1010	0.1012	0.1013	0.1019	0.1024
99.00%	0.0304	0.0230	0.0208	0.0203	0.0198	0.0197	0.0196	0.0196	0.0197	0.0198	0.0197	0.0199	0.0020	0.0020	0.0020
99.50%	0.0156	0.0116	0.0104	0.0101	0.0099	0.0098	0.0098	0.0098	0.0099	0.0099	0.0098	0.0099	0.0099	0.0099	0.0100
99.90%	0.0032	0.0023	0.0021	0.0020	0.0020	0.0019	0.0019	0.0019	0.0020	0.0019	0.0020	0.0020	0.0020	0.0020	0.0020
99.95%	0.0016	0.0012	0.0010	0.0010	0.0010	0.0009	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010
99.99%	0.0003	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
ALM: p\N	10	20	35	50	75	100	150	200	300	500	800	1000	1600	2400	10000
ALM: p\N 0.01%	10 51.600	20 91.217	35 99.883	50 96.158	75 85.696	100 79.523	150 67.685	200 59.600	300 50.530	500 40.472	800 34.231	1000 31.993	1600 27.454	2400 24.575	10000 20.040
ALM: p\N 0.01% 0.05%	10 51.600 41.502	20 91.217 60.508	35 99.883 60.927	50 96.158 58.013	75 85.696 51.413	100 79.523 47.444	150 67.685 41.713	200 59.600 37.524	300 50.530 32.579	500 40.472 27.431	800 34.231 23.751	1000 31.993 22.545	1600 27.454 20.128	2400 24.575 18.621	10000 20.040 16.114
ALM: p\N 0.01% 0.05% 0.10%	10 51.600 41.502 36.538	20 91.217 60.508 48.399	35 99.883 60.927 47.780	50 96.158 58.013 45.318	75 85.696 51.413 40.266	100 79.523 47.444 37.414	150 67.685 41.713 33.106	200 59.600 37.524 30.158	300 50.530 32.579 26.498	500 40.472 27.431 22.785	800 34.231 23.751 20.147	1000 31.993 22.545 19.244	1600 27.454 20.128 17.474	2400 24.575 18.621 16.314	10000 20.040 16.114 14.457
ALM: p\N 0.01% 0.05% 0.10% 0.50%	10 51.600 41.502 36.538 23.831	20 91.217 60.508 48.399 25.963	35 99.883 60.927 47.780 24.569	50 96.158 58.013 45.318 23.229	75 85.696 51.413 40.266 21.334	100 79.523 47.444 37.414 19.986	150 67.685 41.713 33.106 18.285	200 59.600 37.524 30.158 17.156	300 50.530 32.579 26.498 15.689	500 40.472 27.431 22.785 14.211	800 34.231 23.751 20.147 13.129	1000 31.993 22.545 19.244 12.694	1600 27.454 20.128 17.474 11.971	2400 24.575 18.621 16.314 11.525	10000 20.040 16.114 14.457 10.827
ALM: p\N 0.01% 0.05% 0.10% 0.50% 1.00%	10 51.600 41.502 36.538 23.831 18.374	20 91.217 60.508 48.399 25.963 18.643	35 99.883 60.927 47.780 24.569 17.540	50 96.158 58.013 45.318 23.229 16.659	75 85.696 51.413 40.266 21.334 15.506	100 79.523 47.444 37.414 19.986 14.719	150 67.685 41.713 33.106 18.285 13.707	200 59.600 37.524 30.158 17.156 13.042	300 50.530 32.579 26.498 15.689 12.149	500 40.472 27.431 22.785 14.211 11.271	800 34.231 23.751 20.147 13.129 10.616	1000 31.993 22.545 19.244 12.694 10.372	1600 27.454 20.128 17.474 11.971 9.9667	2400 24.575 18.621 16.314 11.525 9.7158	10000 20.040 16.114 14.457 10.827 9.3386
ALM: p\N 0.01% 0.05% 0.10% 0.50% 1.00% 5.00%	10 51.600 41.502 36.538 23.831 18.374 7.4161	20 91.217 60.508 48.399 25.963 18.643 6.9317	35 99.883 60.927 47.780 24.569 17.540 6.6788	50 96.158 58.013 45.318 23.229 16.659 6.5533	75 85.696 51.413 40.266 21.334 15.506 6.4144	100 79.523 47.444 37.414 19.986 14.719 6.3192	150 67.685 41.713 33.106 18.285 13.707 6.2182	200 59.600 37.524 30.158 17.156 13.042 6.1493	300 50.530 32.579 26.498 15.689 12.149 6.0925	500 40.472 27.431 22.785 14.211 11.271 6.0497	800 34.231 23.751 20.147 13.129 10.616 6.0309	1000 31.993 22.545 19.244 12.694 10.372 6.0218	1600 27.454 20.128 17.474 11.971 9.9667 6.0182	2400 24.575 18.621 16.314 11.525 9.7158 6.0077	10000 20.040 16.114 14.457 10.827 9.3386 5.9961
ALM: p\N 0.01% 0.05% 0.10% 0.50% 1.00% 5.00% 10.00%	10 51.600 41.502 36.538 23.831 18.374 7.4161 4.1769	20 91.217 60.508 48.399 25.963 18.643 6.9317 3.9657	35 99.883 60.927 47.780 24.569 17.540 6.6788 3.9612	50 96.158 58.013 45.318 23.229 16.659 6.5533 3.9977	75 85.696 51.413 40.266 21.334 15.506 6.4144 4.0664	100 79.523 47.444 37.414 19.986 14.719 6.3192 4.1256	150 67.685 41.713 33.106 18.285 13.707 6.2182 4.2180	200 59.600 37.524 30.158 17.156 13.042 6.1493 4.2718	300 50.530 32.579 26.498 15.689 12.149 6.0925 4.3547	500 40.472 27.431 22.785 14.211 11.271 6.0497 4.4336	800 34.231 23.751 20.147 13.129 10.616 6.0309 4.4923	1000 31.993 22.545 19.244 12.694 10.372 6.0218 4.5095	1600 27.454 20.128 17.474 11.971 9.9667 6.0182 4.5462	24.575 18.621 16.314 11.525 9.7158 6.0077 4.5650	10000 20.040 16.114 14.457 10.827 9.3386 5.9961 4.5941
ALM: p\N 0.01% 0.05% 0.10% 0.50% 1.00% 5.00% 10.00%	10 51.600 41.502 36.538 23.831 18.374 7.4161 4.1769 2.8110	20 91.217 60.508 48.399 25.963 18.643 6.9317 3.9657 2.7736	35 99.883 60.927 47.780 24.569 17.540 6.6788 3.9612 2.8895	50 96.158 58.013 45.318 23.229 16.659 6.5533 3.9977 2.9935	75 85.696 51.413 40.266 21.334 15.506 6.4144 4.0664 3.1215	100 79.523 47.444 37.414 19.986 14.719 6.3192 4.1256 3.2150	150 67.685 41.713 33.106 18.285 13.707 6.2182 4.2180 3.3356	200 59.600 37.524 30.158 17.156 13.042 6.1493 4.2718 3.4086	300 50.530 32.579 26.498 15.689 12.149 6.0925 4.3547 3.5045	500 40.472 27.431 22.785 14.211 11.271 6.0497 4.4336 3.5952	800 34.231 23.751 20.147 13.129 10.616 6.0309 4.4923 3.6611	1000 31.993 22.545 19.244 12.694 10.372 6.0218 4.5095 3.6833	1600 27.454 20.128 17.474 11.971 9.9667 6.0182 4.5462 3.7228	24.575 18.621 16.314 11.525 9.7158 6.0077 4.5650 3.7450	10000 20.040 16.114 14.457 10.827 9.3386 5.9961 4.5941 3.7812
ALM: p\N 0.01% 0.05% 0.10% 0.50% 1.00% 5.00% 15.00% 20.00%	10 51.600 41.502 36.538 23.831 18.374 7.4161 4.1769 2.8110 2.1830	20 91.217 60.508 48.399 25.963 18.643 6.9317 3.9657 2.7736 2.2164	35 99.883 60.927 47.780 24.569 17.540 6.6788 3.9612 2.8895 2.3547	50 96.158 58.013 45.318 23.229 16.659 6.5533 3.9977 2.9935 2.4616	75 85.696 51.413 40.266 21.334 15.506 6.4144 4.0664 3.1215 2.5881	100 79.523 47.444 19.986 14.719 6.3192 4.1256 3.2150 2.6767	150 67.685 41.713 33.106 18.285 13.707 6.2182 4.2180 3.3356 2.7895	200 59.600 37.524 30.158 17.156 13.042 6.1493 4.2718 3.4086 2.8582	300 50.530 32.579 26.498 15.689 12.149 6.0925 4.3547 3.5045 2.9462	500 40.472 27.431 22.785 14.211 11.271 6.0497 4.4336 3.5952 3.0321	800 34.231 23.751 20.147 13.129 10.616 6.0309 4.4923 3.6611 3.0923	1000 31.993 22.545 19.244 12.694 10.372 6.0218 4.5095 3.6833 3.1132	1600 27.454 20.128 17.474 11.971 9.9667 6.0182 4.5462 3.7228 3.1502	24.575 18.621 16.314 11.525 9.7158 6.0077 4.5650 3.7450 3.1706	10000 20.040 16.114 14.457 10.827 9.3386 5.9961 4.5941 3.7812 3.2058
ALM: p\N 0.01% 0.05% 0.10% 0.50% 1.00% 5.00% 15.00% 20.00% 30.00%	10 51.600 41.502 36.538 23.831 18.374 7.4161 4.1769 2.8110 2.1830 1.6376	20 91.217 60.508 48.399 25.963 18.643 6.9317 3.9657 2.7736 2.2736 2.2164 1.6569	35 99.883 60.927 47.780 24.569 17.540 6.6788 3.9612 2.8895 2.3547 1.7585	50 96.158 58.013 45.318 23.229 16.659 6.5533 3.9977 2.9935 2.4616 1.8388	75 85.696 51.413 40.266 21.334 15.506 6.4144 4.0664 3.1215 2.5881 1.9325	100 79.523 47.444 19.986 14.719 6.3192 4.1256 3.2150 2.6767 1.9986	150 67.685 41.713 33.106 18.285 13.707 6.2182 4.2180 3.3356 2.7895 2.0826	200 59.600 37.524 30.158 17.156 13.042 6.1493 4.2718 3.4086 2.8582 2.1350	300 50.530 32.579 26.498 15.689 12.149 6.0925 4.3547 3.5045 2.9462 2.2003	500 40.472 27.431 22.785 14.211 11.271 6.0497 4.4336 3.5952 3.0321 2.2646	800 34.231 23.751 20.147 13.129 10.616 6.0309 4.4923 3.6611 3.0923 2.3098	1000 31.993 22.545 19.244 12.694 10.372 6.0218 4.5095 3.6833 3.1132 2.3262	1600 27.454 20.128 17.474 11.971 9.9667 6.0182 4.5462 3.7228 3.7228 3.1502 2.3528	24.575 18.621 16.314 11.525 9.7158 6.0077 4.5650 3.7450 3.1706 2.3694	10000 20.040 16.114 14.457 10.827 9.3386 5.9961 4.5941 3.7812 3.2058 2.3980
ALM: p\N 0.01% 0.05% 0.10% 0.50% 1.00% 5.00% 10.00% 15.00% 20.00% 30.00% 40.00%	10 51.600 41.502 36.538 23.831 18.374 7.4161 4.1769 2.8110 2.8310 1.6376 1.3166	20 91.217 60.508 48.399 25.963 18.643 6.9317 3.9657 2.7736 2.2164 1.6569 1.3130	35 99.883 60.927 47.780 24.569 17.540 6.6788 3.9612 2.8895 2.3547 1.7585 1.3785	50 96.158 58.013 45.318 23.229 16.659 6.5533 3.9977 2.9935 2.4616 1.8388 1.432	75 85.696 51.413 40.266 21.334 15.506 6.4144 4.0664 3.1215 2.5881 1.9325 1.4975	100 79.523 47.444 37.414 19.986 14.719 6.3192 4.1256 3.2150 2.6767 1.9986 1.5435	150 67.685 41.713 33.106 18.285 13.707 6.2182 4.2180 3.3356 2.7895 2.0826 1.6021	200 59.600 37.524 30.158 17.156 13.042 6.1493 4.2718 3.4086 2.8582 2.1350 1.6382	300 50.530 32.579 26.498 15.689 12.149 6.0925 4.3547 3.5045 2.9462 2.2003 1.6840	500 40.472 27.431 22.785 14.211 11.271 6.0497 4.4336 3.5952 3.0321 2.2646 1.7287	800 34.231 23.751 20.147 13.129 10.616 6.0309 4.4923 3.6611 3.0923 2.3098 1.7613	1000 31.993 22.545 19.244 12.694 10.372 6.0218 4.5095 3.6833 3.1132 2.3262 1.7731	1600 27.454 20.128 17.474 11.971 9.9667 6.0182 4.5462 3.7228 3.1502 2.3528 1.7917	24.575 18.621 16.314 11.525 9.7158 6.0077 4.5650 3.7450 3.1706 2.3694 1.8042	10000 20.040 16.114 14.457 10.827 9.3386 5.9961 4.5941 3.7812 3.2058 2.3980 1.8251
ALM: pN 0.01% 0.05% 0.10% 0.50% 1.00% 15.00% 20.00% 30.00% 40.00% 50.00%	10 51.600 41.502 36.538 23.831 18.374 7.4161 4.1769 2.8110 2.8310 1.6376 1.3166 1.0658	20 91.217 60.508 48.399 25.963 18.643 6.9317 3.9657 2.7736 2.2164 1.6569 1.3130 1.0460	35 99.883 60.927 47.780 24.569 17.540 6.6788 3.9612 2.8895 2.3547 1.7585 1.3785 1.0844	50 96.158 58.013 45.318 23.229 16.659 6.5533 3.9977 2.9935 2.4616 1.8388 1.432 1.1183	75 85.696 51.413 40.266 21.334 15.506 6.4144 4.0664 3.1215 2.5881 1.9325 1.4975 1.1604	100 79.523 47.444 37.414 19.986 14.719 6.3192 4.1256 3.2150 2.6767 1.9986 1.5435 1.1904	150 67.685 41.713 33.106 18.285 13.707 6.2182 4.2180 3.3356 2.7895 2.0826 1.6021 1.2290	200 59.600 37.524 30.158 17.156 13.042 6.1493 4.2718 3.4086 2.8582 2.1350 1.6382 1.2536	300 50.530 32.579 26.498 15.689 12.149 6.0925 4.3547 3.5045 2.9462 2.2003 1.6840 1.2846	500 40.472 27.431 22.785 14.211 11.271 6.0497 4.4336 3.5952 3.0321 2.2646 1.7287 1.3150	800 34.231 20.147 13.129 10.616 6.0309 4.4923 3.6611 3.0923 2.3098 1.7613 1.3367	1000 31.993 22.545 19.244 12.694 10.372 6.0218 4.5095 3.6833 3.1132 2.3262 1.7731 1.3441	1600 27.454 20.128 17.474 11.971 9.9667 6.0182 4.5462 3.7228 3.1502 2.3528 1.7917 1.3579	24.575 18.621 16.314 11.525 9.7158 6.0077 4.5650 3.7450 3.7450 3.1706 2.3694 1.8042 1.3663	10000 20.040 16.114 14.457 10.827 9.3386 5.9961 4.5941 3.7812 3.2058 2.3980 1.8251 1.3808
ALM: p\N 0.01% 0.05% 0.10% 0.50% 1.00% 10.00% 10.00% 10.00% 0.00%	10 51.600 41.502 36.538 23.831 18.374 7.4161 4.1769 2.8110 2.1830 1.6376 1.3166 1.0658 0.8464	20 91.217 60.508 48.399 25.963 18.643 6.9317 3.9657 2.7736 2.2164 1.6569 1.3130 1.0460 0.8165	 35 99.883 60.927 47.780 24.569 17.540 6.6788 3.9612 2.8895 2.3547 1.7585 1.3785 1.0844 0.8344 	50 96.158 58.013 45.318 23.229 6.5533 3.9977 2.9935 2.4616 1.8388 1.432 1.1183 0.8532	75 85.696 51.413 40.266 21.334 15.506 6.4144 4.0664 3.1215 2.5881 1.9325 1.4975 1.1604 0.8781	100 79.523 47.444 37.414 19.986 14.719 6.3192 4.1256 3.2150 2.6767 1.9986 1.5435 1.1904 0.8971	150 67.685 41.713 33.106 18.285 13.707 6.2182 4.2180 3.3356 2.7895 2.0826 1.6021 1.2290 0.9200	200 59.600 37.524 30.158 17.156 13.042 6.1493 4.2718 3.4086 2.8582 2.1350 1.6382 1.2536 0.9359	300 50.530 32.579 26.498 15.689 12.149 6.0925 4.3547 3.5045 2.9462 2.2003 1.6840 1.2846 0.9554	 500 40.472 27.431 22.785 14.211 11.271 6.0497 4.4336 3.5952 3.0321 2.2646 1.7287 1.3150 0.9746 	800 34.231 23.751 20.147 13.129 10.616 6.0309 4.4923 3.6611 3.0923 2.3098 1.7613 1.3367 0.9888	1000 31.993 22.545 19.244 12.694 10.372 6.0218 4.5095 3.6833 3.1132 2.3262 1.7731 1.3441 0.9936	1600 27.454 20.128 17.474 11.971 9.9667 6.0182 4.5462 3.7228 3.1502 2.3528 1.7917 1.3579 1.0027	24.575 18.621 16.314 11.525 9.7158 6.0077 4.5650 3.7450 3.1706 2.3694 1.8042 1.3663 1.0087	10000 20.040 16.114 14.457 10.827 9.3386 5.9961 4.5941 3.7812 3.2058 2.3980 1.8251 1.3808 1.0179
ALM: p\N 0.01% 0.05% 0.10% 5.00% 10.00% 15.00% 10.00% 30.00% 40.00% 50.00% 60.00% 70.00%	10 51.600 41.502 36.538 23.831 18.374 7.4161 4.1769 2.8110 2.1830 1.6376 1.3166 1.0658 0.8464 0.6406	20 91.217 60.508 48.399 25.963 18.643 6.9317 3.9657 2.7736 2.2164 1.6569 1.3130 1.0460 0.8165 0.6065	35 99.883 60.927 47.780 24.569 17.540 6.6788 3.9612 2.8895 2.3547 1.7585 1.3785 1.0844 0.8344 0.6100	50 96.158 58.013 45.318 23.229 6.5533 3.9977 2.9935 2.4616 1.8388 1.432 1.1183 0.8532 0.6183	75 85.696 51.413 40.266 21.334 15.506 6.4144 4.0664 3.1215 2.5881 1.9325 1.4975 1.1604 0.8781 0.6309	100 79.523 47.444 37.414 19.986 14.719 6.3192 4.1256 3.2150 2.6767 1.9986 1.5435 1.1904 0.8971 0.6416	150 67.685 41.713 33.106 18.285 13.707 6.2182 4.2180 3.3356 2.7895 2.0826 1.6021 1.2290 0.9200 0.6538	200 59.600 37.524 30.158 17.156 13.042 6.1493 4.2718 3.4086 2.8582 2.1350 1.6382 1.2536 0.9359 0.6625	300 50.530 32.579 26.498 15.689 12.149 6.0925 4.3547 3.5045 2.9462 2.2003 1.6840 1.2846 0.9554 0.6735	 500 40.472 27.431 22.785 14.211 11.271 6.0497 4.4336 3.5952 3.0321 2.2646 1.7287 1.3150 0.9746 0.6848 	800 34.231 23.751 20.147 13.129 10.616 6.0309 4.4923 3.6611 3.0923 2.3098 1.7613 1.3367 0.9888 0.6937	1000 31.993 22.545 19.244 12.694 10.372 6.0218 4.5095 3.6833 3.1132 2.3262 1.7731 1.3441 0.9936 0.6963	1600 27.454 20.128 17.474 11.971 9.9667 6.0182 4.5462 3.7228 3.1502 2.3528 1.7917 1.3579 1.0027 0.7014	24.575 18.621 16.314 11.525 9.7158 6.0077 4.5650 3.7450 3.1706 2.3694 1.8042 1.3663 1.0087 0.7054	10000 20.040 16.114 14.457 10.827 9.3386 5.9961 4.5941 3.7812 3.2058 2.3980 1.8251 1.3808 1.0179 0.7108
ALM: p\N 0.01% 0.05% 0.10% 0.50% 10.00% 10.00% 15.00% 20.00% 30.00% 50.00% 60.00% 80.00%	10 51.600 41.502 36.538 23.831 18.374 7.4161 2.8110 2.8110 2.1830 1.6376 1.3166 1.3166 1.3166 0.8464 0.6406 0.4376	20 91.217 60.508 48.399 25.963 18.643 6.9317 2.7736 2.2736 2.2164 1.6569 1.3130 1.0460 0.8165 0.6065 0.4056	35 99.883 60.927 47.780 24.569 17.540 6.6788 3.9612 2.8895 2.3547 1.7585 1.3785 1.3785 1.3785 1.3785 1.3844 0.8344 0.8344 0.6100 0.4005	50 96.158 58.013 45.318 23.229 16.659 6.553 3.9977 2.9935 2.4616 1.8388 1.432 1.1183 0.8532 0.6183 0.4022	75 85.696 51.413 40.266 21.334 15.506 6.4144 3.1215 2.5881 1.9325 1.4975 1.4975 1.4975 1.4975 1.4975 0.6309 0.4064	100 79.523 47.444 19.986 14.719 6.3192 4.1256 3.2150 2.6767 1.9986 1.5435 1.1904 0.8971 0.6416 0.4111	150 67.685 41.713 33.106 18.285 13.707 6.2182 4.2180 3.3356 2.7895 2.0826 1.6021 1.2290 0.9200 0.9200 0.6538 0.4163	200 59.600 37.524 30.158 17.156 13.042 6.1493 4.2718 3.4086 2.8582 2.1350 1.6382 1.2536 0.9359 0.6625 0.4203	300 50.530 32.579 26.498 15.689 12.149 6.0925 4.3547 3.5045 2.9462 2.2003 1.6840 1.2846 0.9554 0.6735 0.4257	500 40.472 27.431 22.785 14.211 11.271 6.0497 4.4336 3.5952 2.2646 1.7287 1.3150 0.9746 0.6848 0.4316	800 34.231 23.751 20.147 13.129 10.616 6.0309 4.4923 3.6611 3.0923 2.3098 1.7613 1.3367 0.9888 0.6937 0.4359	1000 31.993 22.545 19.244 12.694 10.372 6.0218 4.5095 3.6833 3.1132 2.3262 1.7731 1.3441 0.9936 0.6963 0.4369	1600 27.454 20.128 17.474 11.971 9.9667 6.0182 4.5462 3.7228 2.3528 1.7917 1.3579 1.0027 0.7014 0.4397	24.575 18.621 16.314 11.525 9.7158 6.0077 4.5650 3.7450 2.3694 1.8042 1.3663 1.0087 0.7054 0.4420	10000 20.040 16.114 14.457 10.827 9.3386 5.9961 4.5941 3.7812 3.2058 2.3980 1.8251 1.3808 1.0179 0.7108 0.4451
ALM: pN 0.01% 0.05% 0.10% 0.50% 1.00% 5.00% 15.00% 20.00% 30.00% 60.00% 80.00% 85.00%	10 51.600 41.502 36.538 23.831 18.374 7.4161 4.1769 2.8110 2.1830 1.6376 1.3658 0.8464 0.6406 0.4376 0.3344	20 91.217 60.508 48.399 25.963 18.643 6.9317 3.9657 2.7736 2.2164 1.6569 1.3130 1.0460 0.8165 0.6065 0.6056 0.3061	35 99.883 60.927 47.780 24.569 17.540 6.6788 3.9612 2.8895 2.3547 1.7585 1.3785 1.0844 0.8344 0.6100 0.4005 0.2994	50 96.158 58.013 45.318 23.229 16.659 6.5533 3.9977 2.9935 2.4616 1.8388 1.432 1.1183 0.8532 0.6183 0.6183 0.4022 0.2991	75 85.696 51.413 40.266 21.334 15.506 6.4144 4.0664 3.1215 2.5881 1.9325 1.4975 1.1604 0.8781 0.6309 0.4064 0.3005	100 79.523 47.444 37.414 19.986 6.3192 4.1256 3.2150 2.6767 1.9986 1.5435 1.1904 0.8971 0.6416 0.4111 0.3029	150 67.685 41.713 33.106 18.285 13.707 6.2182 4.2180 3.3356 2.7895 2.0826 1.6021 1.2290 0.9200 0.6538 0.4163 0.3058	200 59.600 37.524 30.158 17.156 13.042 6.1493 4.2718 3.4086 2.8582 2.1350 1.6382 1.2536 0.9359 0.6625 0.4203 0.3082	300 50.530 32.579 26.498 15.689 12.149 6.0925 4.3547 3.5045 2.9462 2.2003 1.6840 1.2846 0.9554 0.6735 0.4257 0.3117	500 40.472 27.431 22.785 14.211 11.271 6.0497 4.4336 3.5952 3.0321 2.2646 1.7287 1.3150 0.9746 0.6848 0.4316 0.3154	800 34.231 23.751 20.147 13.129 10.616 6.0309 4.4923 3.6611 3.0923 2.3098 1.7613 1.3367 0.9888 0.6937 0.4359 0.3181	1000 31.993 22.545 19.244 12.694 10.372 6.0218 4.5095 3.6833 3.1132 2.3262 1.7731 0.3936 0.6963 0.6963 0.4369 0.3187	1600 27.454 20.128 17.474 11.971 9.9667 6.0182 4.5462 3.7228 3.1502 2.3528 1.7917 1.3579 1.0027 0.7014 0.4397 0.3204	24.575 18.621 16.314 11.525 9.7158 6.0077 4.5650 3.7450 2.3694 1.8042 1.3663 1.0087 0.7054 0.4420 0.3219	10000 20.040 16.114 14.457 10.827 9.3386 5.9961 4.5941 3.7812 3.2058 2.3980 1.8251 1.3808 1.0179 0.7108 0.4451 0.3243
ALM: pN 0.01% 0.05% 0.10% 0.50% 1.00% 5.00% 10.00% 20.00% 30.00% 40.00% 70.00% 80.00% 80.00%	10 51.600 41.502 36.538 23.831 18.374 7.4161 4.1769 2.8110 2.8100 1.6376 1.3166 1.3166 0.8464 0.6406 0.6406 0.4376 0.3344 0.2284	20 91.217 60.508 48.399 25.963 18.643 6.9317 3.9657 2.7736 2.2164 1.6569 1.3130 1.0460 0.8165 0.6065 0.4056 0.3061 0.2060	35 99.883 60.927 47.780 24.569 17.540 6.6788 3.9612 2.8895 2.3547 1.7585 1.3785 1.0844 0.8344 0.6100 0.4005 0.2994 0.1992	50 96.158 58.013 45.318 23.229 16.655 2.4616 1.8388 1.432 1.1183 0.8532 0.6183 0.4022 0.2991 0.1982	75 85.696 51.413 40.266 21.334 15.506 6.4144 4.0664 3.1215 2.5881 1.9325 1.4975 1.1604 0.8781 0.6309 0.4064 0.3005 0.1979	100 79.523 47.444 37.414 19.986 14.719 6.3192 4.1256 3.2150 2.6767 1.9986 1.5435 1.1904 0.8971 0.6416 0.6416 0.64111 0.3029 0.1991	150 67.685 41.713 33.106 18.285 13.707 6.2182 4.2180 3.3356 2.7895 2.0826 1.6021 1.2290 0.9200 0.6538 0.4163 0.3058 0.2000	200 59.600 37.524 30.158 17.156 6.1493 4.2718 3.4086 2.8582 2.1350 1.6382 1.2536 0.9359 0.6625 0.4203 0.3082 0.2012	300 50.530 32.579 26.498 15.689 12.149 6.0925 4.3547 3.5045 2.9462 2.2003 1.6840 1.2846 0.9554 0.6735 0.4257 0.3117 0.2029	500 40.472 27.431 22.785 14.211 11.271 6.0497 4.4336 3.5952 3.0321 2.2646 1.7287 1.3150 0.9746 0.6848 0.4316 0.63154 0.2051	800 34.231 23.751 20.147 13.129 10.616 6.0309 4.4923 3.6611 3.0923 2.3098 1.7613 1.3367 0.9888 0.6937 0.4359 0.3181 0.2065	1000 31.993 22.545 19.244 10.372 6.0218 4.5095 3.6833 3.1132 2.3262 1.771 1.3441 0.9936 0.69633 0.4369 0.3187 0.2071	1600 27.454 20.128 17.474 11.971 9.9667 6.0182 4.5462 3.7228 3.1502 2.3528 1.7917 1.3579 1.0027 0.7014 0.4397 0.3204 0.2078	2400 24.575 18.621 16.314 11.525 9.7158 6.0077 4.5650 3.7450 3.7450 3.1706 2.3694 1.8042 1.3663 1.0087 0.7054 0.7054 0.3219 0.2087	10000 20.040 16.114 14.457 10.827 9.3386 5.9961 4.5941 3.7812 3.2058 2.3980 1.8251 1.3808 1.0179 0.7108 0.4451 0.3243 0.2105
ALM: pN 0.01% 0.05% 0.10% 0.50% 5.00% 10.00% 15.00% 20.00% 30.00% 40.00% 50.00% 80.00% 85.00% 90.00% 95.00%	10 51.600 41.502 36.538 23.831 18.374 7.4161 4.1769 2.8110 2.1830 1.6376 1.3166 1.0658 0.8464 0.6406 0.4376 0.3344 0.2284 0.1177	20 91.217 60.508 48.399 25.963 18.643 6.9317 3.9657 2.7736 2.2164 1.6569 1.3130 1.0460 0.8165 0.4056 0.4056 0.4056 0.4056	35 99.883 60.927 47.780 24.569 17.540 6.6788 3.9612 2.8895 2.3547 1.7585 1.3785 1.0844 0.8344 0.6100 0.4005 0.2994 0.1992 0.0996	50 96.158 58.013 45.318 23.229 16.6553 2.4616 1.8388 1.432 1.1183 0.8532 0.6183 0.4022 0.2991 0.1982 0.0986	75 85.696 51.413 40.266 21.334 15.506 6.4144 4.0664 3.1215 2.5881 1.9325 1.4975 1.1604 0.8781 0.6309 0.4064 0.3005 0.4095 0.40980	100 79.523 47.444 37.414 19.986 14.719 6.3192 4.1256 3.2150 2.6767 1.9986 1.5435 1.1904 0.8971 0.6416 0.4111 0.3029 0.1991 0.0981	150 67.685 41.713 33.106 18.285 13.707 6.2182 4.2180 3.3356 2.7895 2.0826 1.6021 1.2290 0.9200 0.6538 0.4163 0.3058 0.2000 0.0982	200 59.600 37.524 30.158 17.156 13.042 6.1493 4.2718 3.4086 2.8582 2.1350 1.6382 1.2536 0.9359 0.6625 0.4203 0.3082 0.2012 0.0985	300 50.530 32.579 26.498 15.689 12.149 6.0925 4.3547 3.5045 2.9462 2.2003 1.6840 1.2846 0.9554 0.4257 0.4257 0.4257 0.2029 0.0993	500 40.472 27.431 22.785 14.211 11.271 6.0497 4.4336 3.5952 3.0321 2.2646 1.7287 1.3150 0.9746 0.4316 0.4316 0.4316 0.43154 0.2051 0.1001	800 34.231 23.751 20.147 13.129 10.616 6.0309 4.4923 3.6611 3.0923 2.3098 1.7613 1.3367 0.9888 0.6937 0.4359 0.4359 0.4359 0.4359 0.4359	1000 31.993 22.545 19.244 10.372 6.0218 4.5095 3.6833 3.1132 2.3262 1.7731 1.3441 0.9936 0.4369 0.4369 0.3187 0.2071 0.1009	1600 27.454 20.128 17.474 11.971 9.9667 6.0182 4.5462 3.7228 3.1502 2.3528 1.7917 1.3579 1.0027 0.7014 0.4397 0.3204 0.4397 0.3204 0.4397	2400 24.575 18.621 16.314 11.525 9.7158 6.0077 4.5650 3.7450 3.7450 3.1706 2.3694 1.8042 1.3663 1.0087 0.7054 0.4220 0.3219 0.2087 0.1017	10000 20.040 16.114 14.457 10.827 9.3386 5.9961 4.5941 3.7812 3.2058 2.3980 1.8251 1.3808 1.0179 0.7108 0.4451 0.3243 0.2105 0.1023
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Table 1: Top: Significance points for the finite sample Jarque-Bera test. Bottom: Same values for the adjusted Jarque-Bera Test. The numbers are based on Monte Carlo simulations using 10^7 replications. Note, that the p values are listed in reverse order as 1 - p. The three major levels, 1%, 5% and 10%, are written in bold face.

Figure 1 illustrates the results in a graph. The simulated p values and the deviations from the asymptotic $\chi^2(2)$ limit are shown. The curves belong to the same values of sample sizes N as listed in table 1.



Figure 1: LM (left) and ALM (right) finite sample p values and their differences with respect to the asymptotic limit. The upper bundle of curves shows the p values. The lower bundle of curves measures the difference $p_N - p_{\infty}$ to the asymptotic limit. The graph clearly demonstrates that the adjusted Jarqua-Bera test outperforms the original version of the test. The three dotted vertical lines mark the 1% (99%), 5% (95%) and 10% (95%) levels in the asymptotic limit, respectively.

3 Response Surface and Hypothesis Test

To compute the LM and ALM statistic for a wide range of quantiles and sample sizes one usually approximates the response surface for a fixed value of p as a series in powers of 1 over N

$$q(p,N) = q(p,\infty) + \sum_{k=1}^{K} \beta_k N^{-k} .$$
(3)

Lawford (2004) has done this for the 5% and 10% quantile lines. He fitted his Monte Carlo data based on 1 Million replications for K = 9. The regression coefficients β are listed in the aforementioned paper. We have done fits over a wide range of p-values. The results are shown in figure 2 in comparison with those obtained by Lawford. Note that Lawford's fit becomes less reliable for small lengths where the convergence of the Monte Carlo simulation slows down.

Another approach would be an Edgeworth (1917) expansion of the distribution in 1/N. Unfortunately, we found out that the expansion converges extremely slow. So we applied "Curve Fitting", as suggested by Rothenberg (1984), to approximate the response surface. Simple linear interpolation, 2-dimensional splines or connectionist function approximators are only three possibilities from many others. We followed the first approach fitting on logarithmic scales. The results are shown in Figure 3 for both the traditional Jarque-Bera test as well as its adjusted version.

We have implemented the Jarque-Bera test for finite samples into S functions using the statistical software packages R and SPlus, but it can be done very easily in any other software environment like Matlab, Eviews, or SAS among others. The underlying simulations with 10^7 replications were done with a separate C program using a multiplicative lagged Fibonacci random number generator with a lag of size 1279. The software allows to compute the distribution function and the quantile function for finite samples and the asymptotic limit either for the *LM* or *ALM* test version. These functions are used to derive the *p* values by the hypothesis test function.



Figure 2: The figures show the LM (left) and ALM (right) statistic for a wide range of p values as a function of sample sizes. The dots show the results from the Monte Carlo simulations using 10^7 replications together with the asymptotic limit (marked by the open circles). The dotted lines are fitted series expansions of order K = 6 in 1/N. The two thick lines in the left LM graph display the results of Lawford for the 5% and 10% levels.



Figure 3: The figures show the LM (left) and ALM (right) surface of p values for a wide range of statistics (0.4 ... 100) and sample sizes (10 ... 10'000). Note, that the x- and y-axis are on logarithmic scales. The inputs consist of almost 2000 p-values ranging between 0.0001 and 0.9999.

4 Summary

This Letter tabulates precise p-values for the Jarque-Bera finite sample normality test. In addition to the original version of the Lagrange Multiplier test we have also computed finite sample p-values for its adjusted version formulated by Urzua (1996). In contrast to previous investigations the results were derived from a MC simulation with 10^7 replications. To our knowledge this is one of the largest simulations ever done in statistics. The outcome of the simulation are very precise values for finite samples which we have tabulated and can now be used for an improved hypothesis testing.

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