Endogenous Growth Models in Open Economies: A Possibility of Permanent Current Account Deficits

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Abstract

The paper examines the impacts of heterogeneity in the degree of relative risk aversion on the balance on current account in the framework of endogenous growth, and concludes that, like heterogeneity in demographic changes, heterogeneity in the degree of relative risk aversion generates persisting current account imbalances. The imbalance continues permanently, but its ratio to outputs stabilizes. With evidence in many empirical studies that the degree of relative risk aversion in Japan is relatively higher than that in the U.S., the paper argues that the persisting bilateral trade deficit of the U.S. with Japan is partially generated by this mechanism.

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1. INTRODUCTION

The large current account deficit in the U.S. and the large current account surplus in Japan have continued during the past three decades. This phenomenon is usually explained by the intertemporal approach to the current account based on overlapping-generations variants of the intertemporal models. The intertemporal approach explains persistent current account imbalances by heterogeneous demographic changes. In a more rapidly aging country, e.g., Japan, current account surpluses persist. On the other hand, in a less rapidly aging country, e.g., the U.S. current account deficits persist. The basic idea behind the explanation is simple. National savings moves heterogeneously under the heterogeneous demographic changes while national investments are affected less by the heterogeneous demographic changes because they are determined basically by the world real interest rate, and thus the heterogeneous demographic changes generate heterogeneous movements of the balance on current account, i.e., heterogeneous movements of national savings minus national investments. There have been voluminous works that conduct simulations and project the impacts of heterogeneous demographic changes in the U.S., Japan and other countries based on the intertemporal approach (e.g. Kotlikoff et al., 2001; Brooks, 2003; Faruqee, 2003). Many of the simulations project that the current account in Japan shows surpluses for several decades and then will turn to persisting deficits in the near future owing to the rapid demographical change in Japan.

However, although theoretical projections based on demographic changes have been numerously shown, few systematic empirical examinations into the relation between the balance on current account and demographic changes have studied. Poterba (2001) is one of the few such studies and concludes that although theoretical models generally suggest that equilibrium returns on financial assets will vary in response to changes in population age structure, it is difficult to find robust evidence of such relationships in the time series data. In addition, Obstfeld and Rogoff (1995) argue that the conventional intertemporal approach to the current account can not explain the persisting large current account imbalances. These arguments suggest that the explanation based on heterogeneous demographic changes is still merely a theoretical possibility and there may be another heterogeneity that generates the persisting current account imbalance.

This paper examines heterogeneity in the degree of risk aversion as an alternative source of persistent current account imbalances. The reason why attention is directed to the degree of risk aversion is firstly that in endogenous growth models the degree of relative risk aversion plays a crucial role for growth rates and thus its heterogeneity significantly complicates movements of international transactions. The familiar Euler condition with the Harrod neutral production function such that! $y_t = \frac{Y_t}{L_t} = A_t^{\alpha} k_t^{1-\alpha} = A_t^{\alpha}\left(\frac{K_t}{L_t}\right)^{1-\alpha}$ is $\frac{\dot{c}_t}{c_t} = \frac{(1-\alpha)\left(\frac{A_t}{k_t}\right)^{\alpha} - n_t - \theta}{\varepsilon}$ where Y_t is outputs, K_t is capital inputs, L_t is labor inputs, A_t is technology, y_t is output per capita, c_t is consumption per capita, k_t is capital per capita, $n_t = \frac{\dot{L}_t}{L_t}$ is the growth rate of population in period t. In addition, θ is the rate of time preference, ε is the degree of relative risk aversion, and α is a constant. In most endogenous growth models, $\frac{A_t}{k_t}$ is modeled to be constant, and thus the growth rate of consumption becomes constant (e.g. Romer, 1990; Aghion and Howitt, 1998; Jones, 2003). Hence, in endogenous growth models, the constant growth rate of consumption $\frac{\dot{c}_t}{c_t} = \frac{(1-\alpha)\left(\frac{A_t}{k_t}\right)^{\alpha} - n_t - \theta}{\varepsilon}$ crucially depends on the value of the degree of relative risk aversion ε, and thus its heterogeneity significantly complicates balanced growth paths in the world of free trade.

The second reason why this paper directs its attention to the degree of relative risk aversion is because it has been reported that the degree of relative risk aversion in Japan is relatively higher than that in the U.S. It is another important heterogeneity than demographic changes between the U.S. and Japan. Szpiro's (1986) well-known empirical studies on international comparison of the degree of risk aversion conclude that, of the nine industrialized countries studied, the Japanese have the highest degree of relative risk aversion, e.g. the degree of relative risk aversion in Japan is 2.76 while that in the U.S. is 1.19. Furthermore, it is a well-known fact that compared with the households in the U.S., the households in Japan invest their financial assets much less in risky investments, which clearly indicates that the degree of risk aversion in Japan is much higher than that in the U.S. (e.g., Nakagawa and Shimizu, 2000). In addition, heterogeneity in risk averse behavior has recently been reported from the medical or genetical point of view. Ono et al. (1997) and Nakamura et al. (1997) show that the genetic composition of the receptor for brain chemicals such as serotonin or dopamine differs widely among human races, and that most Japanese have inherited a certain type of receptor composition that produces more cautious and therefore more risk averse characteristics, while many Americans have inherited the other type that produces less risk averse characteristics. Harashima (1998) argues that the so-called "Japanese economic system" or "Japanese capitalism" originates in the higher degree of relative risk aversion in the Japanese.

The model in this paper indicates that heterogeneity in the degree of relative risk aversion can generate persistent current account imbalances. The balance on current account in a less risk averse country shows deficits permanently, and in reverse that in a more risk averse country shows surpluses permanently. Nevertheless, current account deficits and surpluses do not explode but the ratio of deficits or surpluses to outputs asymptotically approach unique finite value and stabilize in both countries. The model predicts that if the degree of relative risk aversion in Japan is truly relatively higher than that in the U.S. as many empirical studies conclude, there is a possibility that the current account surplus persists in Japan permanently and the current account deficit persists in the U.S. permanently.

The paper is organized as follows. In section 2, a two-country endogenous growth model, in which international transactions are incorporated, is constructed. In section 3, the basic nature of the model is examined. There is a balanced growth path on which the limits of growth rates of consumption, capital, technology, and output are all equal and they are equal in both countries. In section 4, the balance of payments is examined based on the model. It is shown that the balance on current account in the less risk averse country shows deficits permanently and *vice versa*. Finally, some concluding remarks are offered in section 6.

2. THE MODEL

2.1 The base model

As shown in Introduction, the degree of relative risk aversion plays a crucial role for growth rates in most endogenous growth models. In this sense, most of the endogenous growth models may be used for the analysis in this paper if international transactions are incorporated in them. However, at the same time, they commonly have the problems of scale effects and/or the influence of population growth (e.g., Jones, 1995a, b). Hence, this paper specifically uses the model shown in Harashima (2004) that is free from both problems (see also e.g. Jones, 1995a; Aghion and Howitt, 1998; Peretto and Smulders, 2002). Being free from the problems is very advantageous when a factor other than demographic changes is examined since we can extract the effect of the factor that is independent of effects of population.

The production function is $Y_t = F(A_t, K_t, L_t)$. The accumulation of capital is

$$\dot{K}_t = Y_t - C_t - v\dot{A}_t - \delta K_t \tag{1}$$

where C_t is consumption, δ is the rate of depreciation, $v(>0)$ is a constant, and a unit of K_t and $\frac{1}{v}$ of a unit of A_t are equivalent, i.e., they are produced using the same quantities of inputs. Every firm is identical and has the same size, and for any period,

$$m = \frac{M_t^{\rho}}{L_t} \tag{2}$$

where M_t is the number of firms and m and $\rho(>1)$ are constants. In addition, the relation

$$\frac{\partial Y_t}{\partial K_t} = M_t^{-\rho} \frac{\partial Y_t}{\partial (vA_t)} \tag{3}$$

and thus

$$\frac{\partial y_t}{\partial k_t} = (mv)^{-1} \frac{\partial y_t}{\partial A_t} \tag{4}$$

is always kept. Equation (2) indicates that the number of population and the number of firms in an economy are positively related. Equations (3) and (4) indicate that returns on investing in K_t and investing in A_t for a firm are kept equal, and also that a firm that invents a new technology cannot obtain all the returns on investing in A_t. This means that investing in A_t increases Y_t but returns of an individual firm that invests in A_t is only a fraction of the increase of Y_t such that $M_t^{-\rho}\frac{\partial Y_t}{\partial(vA_t)} = (mL_t)^{-1}\frac{\partial Y_t}{\partial(vA_t)}$. The reason is uncompensated knowledge spillovers to other firms.

Broadly speaking, there are two types of uncompensated knowledge spillovers: the first is the intra-sectoral knowledge spillover, i.e. MAR externalities, and the second is the inter-sectoral knowledge spillover, i.e. Jacobs externalities. The theory of MAR assumes that knowledge spillovers between homogenous firms work out most effectively and thus spillovers primarily emerge within one sector. As a result, uncompensated knowledge spillovers will be more active if the number of firms within one sector is larger. On the other hand, Jacobs (1969) argues that knowledge spillovers are most effective among firms that practice different activities, and thus diversification, i.e. variety of sectors, is important for spillovers. As a result, uncompensated knowledge spillovers will be more active if the number of sectors is larger in an economy.

If it is assumed that all the sectors have the same number of firms, an increase of the number of firms in an economy results in more active knowledge spillovers owing to either MAR externalities or Jacobs externalities. That is, if an increase of the number of firms in an economy is a result of an increase of the number of firms in each sector, uncompensated knowledge spillovers will become more active by MAR externalities, and if an increase of the number of firms in an economy is a result of an increase of the number of sectors, uncompensated knowledge spillovers will become more active by Jacobs externalities. In either case, an increase of the number of firms in an economy leads to more active uncompensated knowledge spillovers.

Furthermore more active uncompensated knowledge spillovers will reduce the returns of a firm that invests in A_t. $\frac{\partial Y_t}{\partial A_t}$ indicates the over all increase in Y_t in an economy by an additional A_t, that consists of both increase in production in the firm that invented the new technology and increase in production in other firms that use the newly invented technology that the firms obtained either compensating for it to the firm or by uncompensated knowledge spillovers. If the number of firms becomes larger and thus uncompensated knowledge spillovers becomes more active, the compensated fraction in $\frac{\partial Y_t}{\partial A_t}$ that the firm can obtain will become smaller and thus the returns of the firm will become also smaller. Equations (3) and (4)) simply describes this mechanism.

The production function is specified as $Y_t = A_t^{\alpha} f(K_t, L_t)$, where $\alpha(0 < \alpha < 1)$ is a constant. Let $y_t = \frac{Y_t}{L_t}$, $k_t = \frac{K_t}{L_t}$, $c_t = \frac{C_t}{L_t}$ and $n_t = \frac{\dot{L}_t}{L_t}$, and assume that $f(K_t, L_t)$ is homogenous of degree one. Thereby $y_t = A_t^{\alpha} f(k_t)$, and

$$\dot{k}_t = y_t - c_t - \frac{v\dot{A}_t}{L_t} - n_t k_t - \delta k_t \,.$$

By equations (2) and (3), $A_t = \frac{\alpha f(k_t)}{mvf'(k_t)}$ because $\frac{\partial y_t}{mv\partial A_t} = \frac{\partial y_t}{\partial k_t} \Leftrightarrow \frac{\alpha}{mv} A_t^{\alpha-1} f(k_t) =$

$A_t^{\alpha} f'(k_t)$. Since $A_t = \dfrac{\alpha f}{mvf'}$, then $y_t = A_t^{\alpha} f = \left(\dfrac{\alpha}{mv}\right)^{\alpha} \dfrac{f^{1+\alpha}}{f'^{\alpha}}$ and $\dot{A}_t = \dfrac{\alpha}{mv} \dot{k}_t \left(1 - \dfrac{f f''}{f'^2}\right)$.

2.2 Endogenous growth model in open economies

Suppose that there are only two countries in the world: country 1 and country 2. In both countries, the values of parameters as well as population are identical except the degree of relative risk aversion, and the growth rate of population is zero, i.e., $n_t = 0$. The degree of relative risk aversion in country 1 is ε_1 and that in country 2 is ε_2, and $\varepsilon_1 < \varepsilon_2$. Goods and services and capitals are freely traded but labor is immobilized in each country. The balance on current account in country 1 is τ_t and the balance on current account in country 2 is $-\tau_t$. The production function in country 1 is $y_{1,t} = A_t^{\alpha} f(k_{1,t})$, and that in country 2 is $y_{2,t} = A_t^{\alpha} f(k_{2,t})$ where $y_{i,t}$ and $k_{i,t}$ are output and capital per capita in country i in period t for i = 1, 2. The number of population is equally $\dfrac{L_t}{2}$ in both countries and thus the total number of population in the world is L_t. The number of firms in both countries is M_t and firms operate in both countries. Because a balanced growth path requires Harrod neutral technological progress, the production functions are specified as $y_{i,t} = A_t^{\alpha} k_{i,t}^{1-\alpha}$ and thus $Y_{i,t} = K_{i,t}^{1-\alpha} (A_t L_t)^{\alpha}$ $(i = 1,2)$.[1]

Because both countries are free open economies, returns on investments in both countries are kept equal through international arbitration such that

$$\frac{\partial y_{1,t}}{\partial k_{1,t}} = (2mv)^{-1} \frac{\partial (y_{1,t} + y_{2,t})}{\partial A_t} = \frac{\partial y_{2,t}}{\partial k_{2,t}}. \tag{5}$$

That is, an increase in A_t enhances outputs in both countries such that $\dfrac{\partial Y_{i,t}}{\partial K_{i,t}} = M_t^{-\rho} \dfrac{\partial (Y_{1,t} + Y_{2,t})}{\partial (vA_t)}$, and because the number of population is equally $\dfrac{L_t}{2}$ in both countries, then $\dfrac{\partial Y_{i,t}}{\partial K_{i,t}} = \dfrac{\partial y_{i,t}}{\partial k_{i,t}} = M_t^{-\rho} \dfrac{\partial (Y_{1,t} + Y_{2,t})}{\partial (vA_t)} = (mL_t)^{-1} \dfrac{\partial (y_{1,t} + y_{2,t})}{\partial (vA_t)} \dfrac{L_t}{2} =$ $(2mv)^{-1} \dfrac{\partial (y_{1,t} + y_{2,t})}{\partial A_t}$. Thus, $A_t = \dfrac{\alpha [f(k_{1,t}) + f(k_{2,t})]}{2mvf'(k_{1,t})} = \dfrac{\alpha [f(k_{1,t}) + f(k_{2,t})]}{2mvf'(k_{2,t})}$. Because equation (5) is always held through international arbitration, equations $k_{1,t} = k_{2,t}$, $\dot{k}_{1,t} = \dot{k}_{2,t}$, $y_{1,t} = y_{2,t}$ and $\dot{y}_{1,t} = \dot{y}_{2,t}$ are also held. In addition, because $\dfrac{\partial (y_{1,t} + y_{2,t})}{\partial A_{1,t}}$ $= \dfrac{\partial (y_{1,t} + y_{2,t})}{\partial A_{2,t}}$ through international arbitration, then $\dot{A}_{1,t} = \dot{A}_{2,t}$ is held by equation (5). Hence, $A_t = \dfrac{\alpha f(k_{1,t})}{mvf'(k_{1,t})} = \dfrac{\alpha f(k_{2,t})}{mvf'(k_{2,t})}$.

[1] As is well known, only Harrod neutral technological progress matches the stylized facts presented by Kaldor (1961).

The accumulated current account balance $\int_0^t \tau_s ds$ mirrors international capital flows owing to current account imbalances. That is, the country with current account surpluses invests them in the other country. Since $\frac{\partial y_{1,t}}{\partial k_{1,t}} \left(= \frac{\partial y_{2,t}}{\partial k_{2,t}} \right)$ are returns on investments, $\left(\frac{\partial y_{1,t}}{\partial k_{1,t}} - \delta \right) \int_0^t \tau_s ds$ and $\left(\frac{\partial y_{2,t}}{\partial k_{2,t}} - \delta \right) \int_0^t \tau_s ds$ represent international income receipts on assets or income payments on assets. Hence, $\tau_t - \left(\frac{\partial y_{2,t}}{\partial k_{2,t}} - \delta \right) \int_0^t \tau_s ds$ is the balance on goods and services of country 1, and $\left(\frac{\partial y_{1,t}}{\partial k_{1,t}} - \delta \right) \int_0^t \tau_s ds - \tau_t$ is that of country 2. Because the balance on current account mirrors international capital flows, then it is a function of capitals in both countries such that $\tau_t = g(k_{1,t}, k_{2,t})$.

The representative household in country 1 maximizes the expected utility

$$E \int_0^\infty u_1(c_{1,t}) \exp(-\theta t) dt ,$$

subject to

$$\dot{k}_{1,t} = y_{1,t} + \left(\frac{\partial y_{2,t}}{\partial k_{2,t}} - \delta \right) \int_0^t \tau_s ds - \tau_t - c_{1,t} - v \dot{A}_{1,t} \left(\frac{L_t}{2} \right)^{-1} - \delta k_{1,t} , \tag{6}$$

and the representative household in country 2 maximizes the expected utility

$$E \int_0^\infty u_2(c_{2,t}) \exp(-\theta t) dt ,$$

subject to

$$\dot{k}_{2,t} = y_{2,t} - \left(\frac{\partial y_{1,t}}{\partial k_{1,t}} - \delta \right) \int_0^t \tau_s ds + \tau_t - c_{2,t} - v \dot{A}_{2,t} \left(\frac{L_t}{2} \right)^{-1} - \delta k_{2,t} , \tag{7}$$

where $u_{i,t}$, $c_{i,t}$ $\dot{A}_{i,t}$ are the utility function, consumption and the increase of A_t by R&D activities in country i in period t for i = 1, 2, $\dot{A}_t = \dot{A}_{1,t} + \dot{A}_{2,t}$, and E is the expectation operator. Equations (6) and (7) implicitly assume that at t = 0 each country does not have any foreign asset.

3. THE BASIC NATURE OF THE MODEL

3.1 The growth rate of consumption

Because the production function is Harrod neutral such that $y_{i,t} = A_t^\alpha k_{i,t}^{1-\alpha}$

and thus $Y_{i,t} = K_{i,t}^{1-\alpha}\left(A_t L_t\right)^{\alpha}$, and because $A_t = \frac{\alpha f\left(k_{1,t}\right)}{mvf'\left(k_{1,t}\right)} = \frac{\alpha f\left(k_{2,t}\right)}{mvf'\left(k_{2,t}\right)}$ and $f = k_{i,t}^{1-\alpha}$, then $A_t = \frac{\alpha}{mv(1-\alpha)}k_{i,t}$ and $\frac{\partial y_{i,t}}{\partial k_{i,t}} = \left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}$. Because $\dot{A}_{1,t} = \dot{A}_{2,t}$ and $\frac{\partial y_{1,t}}{\partial k_{1,t}} = \frac{\partial y_{2,t}}{\partial k_{2,t}}$, then $\dot{k}_{1,t} = y_{1,t} + \left(\frac{\partial y_{1,t}}{\partial k_{1,t}} - \delta\right)\int_0^t \tau_s ds - \tau_t - c_{1,t} - \frac{v\dot{A}_t}{2}\left(\frac{L_t}{2}\right)^{-1} - \delta k_{1,t}$ $= \left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}k_{1,t} + \left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\int_0^t \tau_s ds - \tau_t - c_{1,t} - \frac{\alpha}{mL_t(1-\alpha)}\dot{k}_{1,t} - \delta k_{1,t}$. Hence,

$$\dot{k}_{1,t} = \frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\alpha}\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta\right]k_{1,t} + \left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\int_0^t \tau_s ds - \tau_t - c_{1,t}\right\}.$$

Since the problem of scale effects in endogenous growth models is not a focal point in this paper, L_t is assumed to be sufficiently large for simplicity and thus $\frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\alpha} = 1$ is satisfied hereafter in this paper.

Therefore, the optimization problem of country 1 can be rewritten as

$$Max\ E_0 \int_0^{\infty} u_1\left(c_{1,t}\right)\exp(-\theta t)dt,$$

subject to

$$\dot{k}_{1,t} = \left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta\right]k_{1,t} + \left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\int_0^t \tau_s ds - \tau_t - c_{1,t}.$$

Let Hamiltonian H_1 be

$$H_1 = u_1\left(c_{1,t}\right)\exp(-\theta t) + \lambda_{1,t}\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta\right]k_{1,t} + \left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\int_0^t \tau_s ds - \tau_t - c_{1,t}\right\}$$

where λ_{1t} is a costate variable, thus the optimality conditions for country 1 are

$$\frac{\partial u_1\left(c_{1,t}\right)}{\partial c_{1,t}}\exp(-\theta t) = \lambda_{1,t}, \tag{8}$$

$$\dot{\lambda}_{1,t} = -\frac{\partial H_1}{\partial k_{1,t}}, \tag{9}$$

$$\dot{k}_{1,t} = \left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta\right]k_{1,t} + \left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\int_0^t \tau_s ds - \tau_t - c_{1,t}, \tag{10}$$

$$\lim_{t\to\infty}\lambda_{1,t}\,k_{1,t}=0\,. \tag{11}$$

Similarly, let Hamiltonian H_2 be

$$H_2=u_2(c_{2,t})\exp(-\theta t)+\lambda_{2,t}\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]k_{2,t}-\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\int_0^t\tau_s ds+\tau_t-c_{2,t}\right\}$$

where λ_{2t} is a costate variable, thus the optimality conditions for country 2 are

$$\frac{\partial u_2(c_{2,t})}{\partial c_{2,t}}\exp(-\theta t)=\lambda_{2,t}\,, \tag{12}$$

$$\dot{\lambda}_{2,t}=-\frac{\partial H_2}{\partial k_{2,t}}\,, \tag{13}$$

$$\dot{k}_{2,t}=\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]k_{2,t}-\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\int_0^t\tau_s ds+\tau_t-c_{2,t}\,, \tag{14}$$

$$\lim_{t\to\infty}\lambda_{2,t}\,k_{2,t}=0\,. \tag{15}$$

Hence, by equations (8), (9) and (10), the growth rate of consumption in country 1 is

$$\frac{\dot{c}_{1,t}}{c_{1,t}}=\varepsilon_1^{-1}\left\{\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta+\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial k_{1,t}}-\frac{\partial\tau_t}{\partial k_{1,t}}-\theta\right\} \tag{16}$$

and, by equations (12), (13) and (14), the growth rate of consumption in country 2 is

$$\frac{\dot{c}_{2,t}}{c_{2,t}}=\varepsilon_2^{-1}\left\{\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial k_{2,t}}+\frac{\partial\tau_t}{\partial k_{2,t}}-\theta\right\}. \tag{17}$$

where $\varepsilon_1=-\dfrac{c_{1,t}\,u''}{u'}$ and $\varepsilon_2=-\dfrac{c_{2,t}\,u''}{u'}$. A constant growth rate such that $\dfrac{\dot{c}_{1,t}}{c_{1,t}}=\dfrac{\dot{c}_{2,t}}{c_{2,t}}$ is possible if

$$\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left[\varepsilon_2\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial k_{1,t}}+\varepsilon_1\frac{\partial\left(\int_0^t\tau_s ds\right)}{\partial k_{2,t}}\right]+(\varepsilon_2-\varepsilon_1)\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta\right]$$
$$=\varepsilon_2\frac{\partial\tau_t}{\partial k_{1,t}}+\varepsilon_1\frac{\partial\tau_t}{\partial k_{2,t}} \tag{18}$$

is satisfied. This possibility is examined in the following sections.

3.2 Transversality condition

Transversality conditions are satisfied if the following conditions are satisfied.

Lemma 1: Unless $\lim_{t\to\infty}\frac{\dot{\lambda}_{1,t}}{\lambda_{1,t}} < -1$, $\lim_{t\to\infty}\frac{\dot{\lambda}_{2,t}}{\lambda_{2,t}} < -1$, $\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}} < -1$, or $\lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}} < -1$, the transversality conditions (equations (11) and (15)) are satisfied if

$$\lim_{t\to\infty}\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]\left[\frac{\int_0^t \tau_s ds}{k_{1,t}}-\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1,t}}\right]-\left(\frac{\tau_t}{k_{1,t}}-\frac{\partial \tau_t}{\partial k_{1,t}}\right)-\frac{c_{1,t}}{k_{1,t}}\right\}<0 \text{ and}$$

$$-\lim_{t\to\infty}\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]\left[\frac{\int_0^t \tau_s ds}{k_{2,t}}-\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{2,t}}\right]-\left(\frac{\tau_t}{k_{2,t}}-\frac{\partial \tau_t}{\partial k_{2,t}}\right)+\frac{c_{2,t}}{k_{2,t}}\right\}<0 \cdot$$

Proof: $\frac{\dot{k}_{1,t}}{k_{1,t}}=\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]+\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\int_0^t \tau_s ds}{k_{1,t}}-\frac{\tau_t+c_{1,t}}{k_{1,t}}$ by equation (10). .

On the other hand, $\frac{\dot{\lambda}_{1,t}}{\lambda_{1,t}}=-\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]+\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1,t}}-\frac{\partial \tau_t}{\partial k_{1,t}}\right\}$ by equation (9). Here,

$$\lim_{t\to\infty}\left(\frac{\dot{\lambda}_{1,t}}{\lambda_{1,t}}+\frac{\dot{k}_{1,t}}{k_{1,t}}\right)=\lim_{t\to\infty}\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]\left[\frac{\int_0^t \tau_s ds}{k_{1,t}}-\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1,t}}\right]-\left(\frac{\tau_t}{k_{1,t}}-\frac{\partial \tau_t}{\partial k_{1,t}}\right)-\frac{c_{1,t}}{k_{1,t}}\right\}.$$

Hence, unless $\lim_{t\to\infty}\frac{\dot{\lambda}_{1,t}}{\lambda_{1,t}} < -1$, $\lim_{t\to\infty}\frac{\dot{\lambda}_{2,t}}{\lambda_{2,t}} < -1$, $\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}} < -1$, or $\lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}} < -1$, then $\lim_{t\to\infty}\left(\frac{\dot{\lambda}_{1,t}}{\lambda_{1,t}}+\frac{\dot{k}_{1,t}}{k_{1,t}}\right)<0$ if

$$\lim_{t\to\infty}\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]\left[\frac{\int_0^t \tau_s ds}{k_{1,t}}-\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1,t}}\right]-\left(\frac{\tau_t}{k_{1,t}}-\frac{\partial \tau_t}{\partial k_{1,t}}\right)-\frac{c_{1,t}}{k_{1,t}}\right\}<0 \cdot$$

Similarly, if

$$-\lim_{t\to\infty}\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]\left[\frac{\int_0^t \tau_s ds}{k_{2,t}}-\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{2,t}}\right]-\left(\frac{\tau_t}{k_{2,t}}-\frac{\partial \tau_t}{\partial k_{2,t}}\right)+\frac{c_{2,t}}{k_{2,t}}\right\}<0,\ \lim_{t\to\infty}\left(\frac{\dot{\lambda}_{2,t}}{\lambda_{2,t}}+\frac{\dot{k}_{2,t}}{k_{2,t}}\right)<0$$

■

Lemma 1 indicates that if τ_t is not significantly large compared with $c_{1,t}$ and $c_{2,t}$, the transversality conditions are satisfied. Note that the case of $\lim_{t\to\infty}\frac{\dot{\lambda}_{1,t}}{\lambda_{1,t}} < -1$, $\lim_{t\to\infty}\frac{\dot{\lambda}_{2,t}}{\lambda_{2,t}} < -1$, $\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}} < -1$, or $\lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}} < -1$ is extremely unusual and thus these cases are excluded hereafter in this paper.

3.3 Growth path

Balanced growth is the focal point for the analysis of growth path. Therefore, the following analyses focus on the steady state such that $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}$, $\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}$, $\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}$, $\lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}}$ and $\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}$ are constants. Using lemma 1, the following important nature of the model is proved.

Lemma 2: If $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=$ constant, then $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}=\lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}}=$ $\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\left(\int_0^t\tau_s ds\right)}{dt}}{\int_0^t\tau_s ds}$.

Proof: $\lim_{t\to\infty}\frac{\dot{k}_{1t}}{k_{1t}}=\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta+\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\lim_{t\to\infty}\frac{\int_0^t\tau_s ds}{k_{1t}}-\lim_{t\to\infty}\frac{\tau_t+c_{1t}}{k_{1t}}$ and $\lim_{t\to\infty}\frac{\dot{k}_{2t}}{k_{2t}}=\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\lim_{t\to\infty}\frac{\int_0^t\tau_s ds}{k_{2t}}+\lim_{t\to\infty}\frac{\tau_t-c_{2t}}{k_{2t}}$ by equations (10) and (14). By equations (6) and (7), $c_{1,t}-c_{2,t}=2\left(\frac{\partial y_{1,t}}{\partial k_{1,t}}\int_0^t\tau_s ds-\tau_t\right)=$ $2\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\int_0^t\tau_s ds-\tau_t\right]$ because $\frac{\partial y_{1,t}}{\partial k_{1,t}}=\frac{\partial y_{2,t}}{\partial k_{2,t}}$, $k_{1,t}=k_{2,t}$, $y_{1,t}=y_{2,t}$, $\dot{A}_{1t}=\dot{A}_{2t}$ and $\frac{\partial y_{i,t}}{\partial k_{i,t}}=\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}$ for $i=1,2$. Hence, if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=$ constant, then $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}=\lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}}=\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\left(\int_0^t\tau_s ds\right)}{dt}}{\int_0^t\tau_s ds}$. ■

Lemma 3: If and only if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=$ constant, all the optimality conditions are satisfied at the steady state.

Proof: By Lemma 2, if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=$ constant, then $\lim_{t\to\infty}\frac{\tau_t}{k_{1,t}}=\lim_{t\to\infty}\frac{\tau_t}{k_{2,t}}=\varXi$ where $\varXi$

is a constant. In addition, because $\lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds} = \lim_{t\to\infty}\frac{\tau_t}{\int_0^t \tau_s ds} = \lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}$, then $\lim_{t\to\infty}\frac{\int_0^t \tau_s ds}{k_{1,t}}$ $= \lim_{t\to\infty}\frac{\int_0^t \tau_s ds}{k_{2,t}} = \Xi\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}$. Therefore, $\lim_{t\to\infty}\frac{\tau_t}{k_{1,t}} = \lim_{t\to\infty}\frac{\partial\tau_t}{\partial k_{1,t}} = \lim_{t\to\infty}\frac{\tau_t}{k_{2,t}} = \lim_{t\to\infty}\frac{\partial\tau_t}{\partial k_{2,t}}$ and $\lim_{t\to\infty}\frac{\int_0^t \tau_s ds}{k_{1,t}} = \lim_{t\to\infty}\frac{\partial\int_0^t \tau_s ds}{\partial k_{1,t}} = \lim_{t\to\infty}\frac{\int_0^t \tau_s ds}{k_{2,t}} = \lim_{t\to\infty}\frac{\partial\int_0^t \tau_s ds}{\partial k_{2,t}}$, and thus,

$$\lim_{t\to\infty}\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]\left[\frac{\int_0^t \tau_s ds}{k_{1,t}}-\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1,t}}\right]-\left(\frac{\tau_t}{k_{1,t}}-\frac{\partial\tau_t}{\partial k_{1,t}}\right)-\frac{c_{1,t}}{k_{1,t}}\right\} = -\lim_{t\to\infty}\frac{c_{1,t}}{k_{1,t}} < 0$$ and

$$-\lim_{t\to\infty}\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]\left[\frac{\int_0^t \tau_s ds}{k_{2,t}}-\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{2,t}}\right]-\left(\frac{\tau_t}{k_{2,t}}-\frac{\partial\tau_t}{\partial k_{2,t}}\right)+\frac{c_{2,t}}{k_{2,t}}\right\} = -\lim_{t\to\infty}\frac{c_{2,t}}{k_{2,t}} < 0.$$ Hence,

by Lemma 1, the transversality conditions (equations (11) and (15)) are satisfied while all the other optimality conditions are satisfied.

On the other hand, if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} \neq \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}$, then $\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t} \neq \lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds}$. Thus by Lemma 1, for both countries to satisfy the transverality conditions, it is necessary that $\lim_{t\to\infty}\frac{c_{1,t}}{k_{1,t}} = \infty$ or $\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{k_{2,t}} = \infty$, which violates equations (10) or (14). As a result, if and only if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} =$ constant, all the optimality conditions are satisfied at the steady state. ■

By Lemmas, it is proved that, if all the optimality conditions are satisfied, both countries grow on the following balanced growth path while satisfying all the optimality conditions.

Proposition 1: If and only if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} =$ constant, then $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} = \lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}$ $= \lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}} = \lim_{t\to\infty}\frac{\dot{y}_{1,t}}{y_{1,t}}\lim_{t\to\infty}\frac{\dot{y}_{2,t}}{y_{2,t}} = \lim_{t\to\infty}\frac{\dot{A}_t}{A_t} =$ constant.

Proof: As for $y_{1,t}$, because $y_{1,t} = A_t^{\alpha} k_{1,t}^{1-\alpha}$, $\dot{y}_{1,t} = \left(\frac{A_t}{k_{1,t}}\right)^{\alpha}\left[(1-\alpha)\dot{k}_{1,t} + \alpha\frac{k_{1,t}}{A_t}\dot{A}_t\right]$. Because $A_t = \frac{\alpha\left[f(k_{1,t})+f(k_{2,t})\right]}{mvf'(k_{1,t})} = \frac{\alpha}{mv(1-\alpha)}k_{1,t}$ and thus because $\dot{A}_t = \frac{\alpha}{mv(1-\alpha)}\dot{k}_{1,t}$, then $\dot{y}_{1,t} =$

$\dot{k}_{1,t}\left(\frac{A_t}{k_{1,t}}\right)^{\alpha}\left[(1-\alpha)+\frac{\alpha^2}{mv(1-\alpha)}\frac{k_{1,t}}{A_t}\right]$, and thus $\frac{\dot{y}_{1,t}}{y_{1,t}}=\frac{\dot{k}_{1,t}}{k_{1,t}}\left[(1-\alpha)+\frac{\alpha^2}{mv(1-\alpha)}\frac{k_{1,t}}{A_t}\right]$. Because $A_t=\frac{\alpha}{mv(1-\alpha)}k_{1,t}$, $\frac{\dot{y}_{1,t}}{y_{1,t}}=\frac{\dot{k}_{1,t}}{k_{1,t}}[(1-\alpha)+\alpha]=\frac{\dot{k}_{1,t}}{k_{1,t}}$. Hence, by Lemma 2, $\lim_{t\to\infty}\frac{\dot{y}_{1,t}}{y_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}$. Because $y_{1,t}=y_{2,t}$, then $\lim_{t\to\infty}\frac{\dot{y}_{1,t}}{y_{1,t}}=\lim_{t\to\infty}\frac{\dot{y}_{2,t}}{y_{2,t}}=\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}=\lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}}$.

As for A_t, by $\dot{y}_{1,t}=\left(\frac{A_t}{k_{1,t}}\right)^{\alpha}\left[(1-\alpha)\dot{k}_{1,t}+\alpha\frac{k_{1,t}}{A_t}\dot{A}_t\right]$ and $\dot{A}_t=\frac{\alpha}{mv(1-\alpha)}\dot{k}_{1,t}$, $\dot{y}_{1,t}=$! $\dot{A}_t\left(\frac{A_t}{k_{1,t}}\right)^{\alpha}\left[\frac{mv(1-\alpha)^2}{\alpha}+\alpha\frac{k_{1,t}}{A_t}\right]$ and thus $\frac{\dot{y}_{1,t}}{y_{1,t}}=\frac{\dot{A}_t}{k_{1,t}}\frac{mv(1-\alpha)^2}{\alpha}+\alpha\frac{\dot{A}_t}{A_t}$. Because $\dot{A}_t=\frac{\alpha}{mv(1-\alpha)}\dot{k}_{1,t}$, then $\frac{\dot{y}_{1,t}}{y_{1,t}}=(1-\alpha)\frac{\dot{k}_{1,t}}{k_{1,t}}+\alpha\frac{\dot{A}_t}{A_t}$. Hence, $\frac{\dot{y}_{1,t}}{y_{1,t}}=\frac{\dot{k}_{1,t}}{k_{1,t}}=(1-\alpha)\frac{\dot{k}_{1,t}}{k_{1,t}}+\alpha\frac{\dot{A}_t}{A_t}$ and thus $\frac{\dot{k}_{1,t}}{k_{1,t}}=\frac{\dot{A}_t}{A_t}$. Because $k_{1,t}=k_{2,t}$, then $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}=\lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}}=\lim_{t\to\infty}\frac{\dot{y}_{1,t}}{y_{1,t}}=\lim_{t\to\infty}\frac{\dot{y}_{2,t}}{y_{2,t}}=\lim_{t\to\infty}\frac{\dot{A}_t}{A_t}$. ■

Corollary 1: If and only if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=$ constant, then $\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\int_0^t\tau_s ds}{dt}}{\int_0^t\tau_s ds}=$

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}=\lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}}=\lim_{t\to\infty}\frac{\dot{y}_{1,t}}{y_{1,t}}=\lim_{t\to\infty}\frac{\dot{y}_{2,t}}{y_{2,t}}=\lim_{t\to\infty}\frac{\dot{A}_t}{A_t}=\text{a positive constant.}$$

Proof: By Lemma 2, $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\int_0^t\tau_s ds}{dt}}{\int_0^t\tau_s ds}$. Hence, by Proposition 1, $\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\int_0^t\tau_s ds}{dt}}{\int_0^t\tau_s ds}=\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}=\lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}}=\lim_{t\to\infty}\frac{\dot{y}_{1,t}}{y_{1,t}}=\lim_{t\to\infty}\frac{\dot{y}_{2,t}}{y_{2,t}}=\lim_{t\to\infty}\frac{\dot{A}_t}{A_t}=$ a positive constant. ■

Because eventually current account imbalances grow at the same rate with output, consumption and capital, then the ratio of the balance on current account to output do not explode but stabilizes as shown in the proof of Lemma 3, i.e., $\lim_{t\to\infty}\frac{\tau_t}{k_{1,t}}=\lim_{t\to\infty}\frac{\tau_t}{k_{2,t}}=\Xi$.

Because technology will not decrease persistently, i.e., $\lim_{t\to\infty}\frac{\dot{A}_t}{A_t}>0$, then only the case such that $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}=\lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}}=\lim_{t\to\infty}\frac{\dot{y}_{1,t}}{y_{1,t}}=\lim_{t\to\infty}\frac{\dot{y}_{2,t}}{y_{2,t}}=\lim_{t\to\infty}\frac{\dot{A}_t}{A_t}>0$ is examined hereafter in this paper.

3.4 Unilateral balanced growth path

Although the balanced growth path shown in Proposition 1 satisfies all the optimality conditions, the representative households in both countries may not necessarily behave consistently with the balanced growth path because they are heterogeneous. Becker (1980) shows that if households have heterogeneous rates of time preference, the most patient household owns all wealth if households are purely price takers. Ghiglino (2002) predicts that it is likely that under appropriate assumptions the results in Becker (1980) still hold in endogenous growth models. Farmer and Lahiri (2004) show that in general, balanced growth equilibria do not exist in a multi-agent economy except for the special case that all agents have the same constant rate of time preference. The similar argument may hold for the heterogeneous degrees of relative risk aversion.

Lemma 4: If the representative household in each country sets τ_t without regarding the other country's optimality conditions, then it is not possible that all the optimality conditions of both countries are satisfied.

Proof: In this case, τ_t can be seen as a control variable for each country. Hence, the same optimality condition $\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial \tau_t}=1$ is added to the optimality conditions of each of the two countries. Here, by Lemmas 3, if all the optimality conditions are satisfied, then $\lim_{t\to\infty}\frac{\tau_t}{k_{1,t}}=\lim_{t\to\infty}\frac{\tau_t}{k_{2,t}}=\Xi$ and $\lim_{t\to\infty}\frac{\int_0^t \tau_s ds}{k_{1,t}}=\lim_{t\to\infty}\frac{\int_0^t \tau_s ds}{k_{2,t}}=\Xi\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}$ where Ξ is a constant. By condition $\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial \tau_t}=1$, $\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta=\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}$. Hence, $\lim_{t\to\infty}\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1,t}}-\frac{\partial \tau_t}{\partial k_{1,t}}\right\}=\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)\Xi\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}-\Xi=0$. Therefore, $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\varepsilon^{-1}\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta_1\right]$ and $\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=\varepsilon^{-1}\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta_2\right]$. Thereby $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}>\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}$, which contradicts the conditions $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=$ constant shown in Lemma 3. ■

The proof of Lemma 4 indicates that country 1 can satisfy all its optimality conditions only if either $\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds}=\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}$ or $\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds}$ $=\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta$ because $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}$ can be constant only in both cases. The former case corresponds to the case Proposition 1 shows (hereafter called a "multilateral balanced growth path"), and both countries can satisfy all the optimality conditions. On the other hand, in the latter case, although country 1 can achieve all its optimality conditions, country 2 cannot (hereafter called a "unilateral balanced growth path"). In this case, $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\neq\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds}$ and $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}>\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}$. Here, by equations (6) and (7), $c_{1,t}-c_{2,t}=2\left(\frac{\partial y_{1,t}}{\partial k_{1,t}}\int_0^t \tau_s ds-\tau_t\right)=2\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\int_0^t \tau_s ds-\tau_t\right]$, and thus a unilateral balanced growth path requires $\lim_{t\to\infty}(c_{1,t}-c_{2,t})=0$ because $\lim_{t\to\infty}\frac{\tau_t}{\int_0^t \tau_s ds}=$ $\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta$. However, because $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}>\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}$, then country 2 must initially sets consumption such that $c_{2,0}=\infty$ that violates the optimality condition of country 2. Therefore, unlike multilateral balanced growth path, country 2 cannot satisfy all its optimal conditions even though country 1 can.

How should country 2 respond to the unilateral balanced growth path of country 1? Possibly, both countries negotiate for the trade between them, and some agreements may be reached. Nevertheless, if no agreement is reached and country 1 never regards the country 2's optimality conditions, country 2, in general, will fall into the following uncomfortable situation.

Remark 1: If the representative household in country 1 does not regard the country 2's optimality conditions, all capitals in country 2 will be eventually owned by country 1.

The reason for Remark 1 is as follows. Suppose first that country 1 chooses the unilateral balanced growth path and sets $c_{1,0}$ so as to achieve this path. There are two options for country 2. The first option is that country 2 also pursues its own optimality without regarding country 1, i.e., chooses its own unilateral balanced growth path. The second option is to adapt to the behavior of country 1 as a follower. If country 2 takes the first option, it sets $c_{2,0}$ without regarding $c_{1,0}$ like country 1. As Lemma 4 indicates, unilaterally optimal growth rates are different between the two countries and $\frac{\dot{c}_{1,t}}{c_{1,t}}>\frac{\dot{c}_{2,t}}{c_{2,t}}$, and thus initial consumptions are set as $c_{1,0}<c_{2,0}$. Because $\frac{\partial y_{1,t}}{\partial k_{1,t}}=(2mv)^{-1}\frac{\partial(y_{1,t}+y_{2,t})}{\partial A_t}$

$= \frac{\partial y_{2,t}}{\partial k_{2,t}}$ and $k_{1,t} = k_{2,t}$ must be kept, capitals and technology are equal and grow at the same rate in both countries. Hence, because $c_{1,0} < c_{2,0}$, more capitals are initially produced in country 1 than country 2 and thus some of them need be exported to country 2. As a result, $\frac{\dot{c}_{1,t}}{c_{1,t}} > \frac{\dot{k}_{1,t}}{k_{1,t}} = \frac{\dot{k}_{2,t}}{k_{2,t}} > \frac{\dot{c}_{2,t}}{c_{2,t}}$, which means that each of both countries equally cannot satisfy all its own optimality conditions. Because $\lim_{t\to\infty} \frac{\dot{c}_{1,t}}{c_{1,t}} > \lim_{t\to\infty} \frac{\dot{k}_{1,t}}{k_{1,t}}$ $= \lim_{t\to\infty} \frac{\dot{k}_{2,t}}{k_{2,t}} > \lim_{t\to\infty} \frac{\dot{c}_{2,t}}{c_{2,t}}$, capital soon becomes abundant in country 2, and thus unutilized goods and services are produced in country 2. These unutilized products are exported to and utilized in country 1. This process escalates as time passes because $\lim_{t\to\infty} \frac{\dot{c}_{1,t}}{c_{1,t}} >$ $\lim_{t\to\infty} \frac{\dot{k}_{1,t}}{k_{1,t}} = \lim_{t\to\infty} \frac{\dot{k}_{2,t}}{k_{2,t}} > \lim_{t\to\infty} \frac{\dot{c}_{2,t}}{c_{2,t}}$ and eventually almost all of consumer goods and services produced in country 2 are consumed by the household in country 1. This consequence will be uncomfortable for country 2.

Next, if country 2 takes the second option, country 2 should set $c_{2,0} = \infty$ to satisfy all its optimality conditions as Lemma 4 shows. Setting $c_{2,0} = \infty$ is impossible, but country 2 as a follower will initially set as large $c_{2,t}$ as possible. This action gives country 2 the higher expected utility than that when taking the first option because consumption of country 2 in this case is always higher than that when taking the first option. As a result, country 2 imports as large goods and services as possible from country 1, and the trade deficit of country 2 pile up until $\left(\frac{\alpha}{mv}\right)^{\alpha} (1-\alpha)^{1-\alpha} \int_0^t \tau_s ds = \tau_t$ is achieved, i.e., until $\frac{\dot{\tau}_t}{\tau_t} = \frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds}$ is achieved. In other words, the trade balance of country 2 never becomes surpluses. The current account deficits and the accumulated debts of country 2 to country 1 continue to increase indefinitely. Furthermore, it increases more rapidly than the growth rate of outputs ($\lim_{t\to\infty} \frac{\dot{y}_{2,t}}{y_{2,t}}$) because in general $\lim_{t\to\infty} \frac{\dot{c}_{1,t}}{c_{1,t}} < \lim_{t\to\infty} \frac{\dot{\tau}_t}{\tau_t}$, i.e., $(1-\varepsilon)\left[\left(\frac{\alpha}{mv}\right)^{\alpha} (1-\alpha)^{1-\alpha} - \delta\right] < \theta_1 (< \theta_2)$. Then, soon, all capitals in country 2 are owned by country 1.[2] This consequence will be also uncomfortable for country 2.

[2] Note that even though the households in country 2 possess no capital, the capital stock in country 2 is still kept to be $k_{2,t} = k_{1,t}$ and thus $y_{2,t} = y_{1,t}$. Point is that all the capital in country 2 is owned by foreigners.

As a result, country 2 cannot satisfy all its optimality conditions in any case if country 1 takes a unilateral balanced growth path, and both options to counter the unilateral action of country 1 are uncomfortable for country 2. However, the expected utility of country 2 is higher if it takes the second option than the first option. Hence, under the circumstance that country 2 cannot satisfy all its optimality conditions in any case, country 2 will choose the second option that gives the higher expected utility. Thus, if country 1 does not regard country 2's optimality conditions, all capitals in country 2 will be eventually owned by country 1. This result corresponds to the consequence in an economy with households that have heterogeneous rates of time preference shown in Becker (1980).

3.5 *Multilateral balanced growth path*

Nevertheless, country 2 may refuse to trade and isolate itself if country 1 takes the unilateral balanced growth path Furthermore, if country 2 shows intention to isolate itself, country 1 may change its behavior because the isolation of country 2 is also uncomfortable for country 1. The isolation of country 2 indicates that country 1 must allocate more resources for the generation of technology, and as a result, consumption and the expected utility of the representative household in country 1 will decline by the isolation of country 2. Hence, country 1 may compromise to cooperate with country 2. Sorger (2002) shows that if a government levies a progressive income tax, or if there are few households of each type and thus they are not simple price takers but play a Nash equilibrium, the results shown in Becker (1980) do not hold anymore. Ghiglino (2002) argues that the latter case in Sorger (2002) can be interpreted as a model of international trade with a common market simply by associating each household's type to a country with a national central planner or a representative household.

The above arguments suggest that it is not unnatural that the representative households in both countries play a Nash equilibrium with regard to the sequence of τ_t. Lemma 3 shows that, if and only if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} =$ constant, all the optimality conditions in both countries are satisfied. Therefore, if the representative households in both countries behave so as to satisfy $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} =$ constant at the Nash equilibrium, the growth path shown in Proposition 1 i.e., the multilateral balanced growth path, is achieved. Both countries can satisfy all the optimality conditions simultaneously.

4. THE BALANCE OF PAYMENTS

In this section, the balance of payment when the multilateral balanced growth path is achieved is examined. The balance on current account shows deficits in one country and surpluses in the other country. The natural question is which country experiences deficits. As shown in the proof of Lemma 3, $\lim_{t\to\infty}\frac{\tau_t}{k_{1,t}} = \lim_{t\to\infty}\frac{\tau_t}{k_{2,t}} = \Xi$ and

$\lim_{t\to\infty}\frac{\int_0^t \tau_s ds}{k_{1,t}}=\lim_{t\to\infty}\frac{\int_0^t \tau_s ds}{k_{2,t}}=\Xi\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}$ on the multilateral balanced growth path, and because $k_{i,t}$ is positive, if the sign of Ξ is negative, the current account of economy 1 shows deficits eventually and permanently and *vice versa*. On the multilateral balanced growth path, the value of Ξ is uniquely determined as follows.

Lemma 5: $\Xi=\dfrac{(\varepsilon_1-\varepsilon_2)\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta\right]}{(\varepsilon_1+\varepsilon_2)\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left(\frac{\varepsilon_1+\varepsilon_2}{2}\right)\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta\right]^{-1}-1\right\}}$.

Proof: Because $\lim_{t\to\infty}\frac{\partial\tau_t}{\partial k_{1,t}}=\lim_{t\to\infty}\frac{\partial\tau_t}{\partial k_{2,t}}=\Xi$ and $\lim_{t\to\infty}\frac{\partial\int_0^t \tau_s ds}{\partial k_{1,t}}=\lim_{t\to\infty}\frac{\partial\int_0^t \tau_s ds}{\partial k_{2,t}}=\Xi\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}$, $\left(1+\frac{\varepsilon_1}{\varepsilon_2}\right)\Xi\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}-1\right\}=-\left(1-\frac{\varepsilon_1}{\varepsilon_2}\right)\left\{\left(\frac{2\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta\right\}$ by equation (18), and therefore, $\Xi=\dfrac{(\varepsilon_1-\varepsilon_2)\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta\right]}{(\varepsilon_1+\varepsilon_2)\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}-1\right\}}$. By this result and equations (16) and (17), the limit of the growth rate is $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=\left(\frac{\varepsilon_1+\varepsilon_2}{2}\right)^{-1}\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta\right]$. Thereby,

$$\Xi=\frac{(\varepsilon_1-\varepsilon_2)\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta\right]}{(\varepsilon_1+\varepsilon_2)\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left(\frac{\varepsilon_1+\varepsilon_2}{2}\right)\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta\right]^{-1}-1\right\}}. \quad ! \quad ■$$

Hence, the value of Ξ is uniquely determined. In addition, the sign of Ξ is uniquely determined by the relative difference of the degree of risk aversion between country 1 and 2 as follows.

Proposition 2: If $\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta\right]\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]^{-1}<\frac{\varepsilon_1+\varepsilon_2}{2}$, then $\Xi<0$. That is, the current account deficits of country 1 continue indefinitely and *vice versa*.

Proof: $\lim_{t\to\infty}\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1,t}}-\frac{\partial \tau_t}{\partial k_{1,t}}\right\}<0$ for $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}$ by equations (16) and (17), and $\lim_{t\to\infty}\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1,t}}-\frac{\partial \tau_t}{\partial k_{1,t}}\right\}=$ $\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\Xi\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}-\Xi=\Xi\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}-1\right\}<0$. Because $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\left(\frac{\varepsilon_1+\varepsilon_2}{2}\right)^{-1}\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta\right]$ as shown in the proof of Lemma 5, then $\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}-1=\left(\frac{\varepsilon_1+\varepsilon_2}{2}\right)\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta\right]^{-1}-1$. Therefore, if $\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta\right]\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]^{-1}<\frac{\varepsilon_1+\varepsilon_2}{2}$, then $0<\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}-1$ and thereby $\Xi<0$. ■

Proposition 2 indicates the permanent current account deficits in less risk averse country 1 and the permanent current account surpluses in more risk averse country 2. The condition $\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta\right]\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]^{-1}<\frac{\varepsilon_1+\varepsilon_2}{2}$ is generally satisfied for reasonable parameter values. Therefore, the model predicts that if the degree of relative risk aversion in Japan is truly relatively higher than that in the U.S. as many empirical studies conclude, current account surpluses continue in Japan permanently and current account deficits continue in the U.S. permanently.

On the other hand, the opposite is true for the trade balance.

Corollary 2: If $\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta\right]\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]^{-1}<\frac{\varepsilon_1+\varepsilon_2}{2}$, then $\lim_{t\to\infty}\left[\tau_t-\left(\frac{\partial y_{2,t}}{\partial k_{2,t}}-\delta\right)\int_0^t \tau_s ds\right]>0$. That is, the trade surpluses of country 1 continue indefinitely and *vice versa*.

Proof: The balance on goods and services in country 1 is $\lim_{t\to\infty}\left[\tau_t-\left(\frac{\partial y_{2,t}}{\partial k_{2,t}}-\delta\right)\int_0^t \tau_s ds\right]$.

Here, $\lim_{t\to\infty}\frac{\tau_t-\left(\frac{\partial y_{2,t}}{\partial k_{2,t}}-\delta\right)\int_0^t\tau_s ds}{k_{1,t}}=-\left[\left(\frac{\partial y_{2,t}}{\partial k_{2,t}}-\delta\right)\lim_{t\to\infty}\frac{\int_0^t\tau_s ds}{k_{1,t}}-\lim_{t\to\infty}\frac{\tau_t}{k_{1,t}}\right]=-\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\varXi\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}-\varXi\right\}$

$=-\varXi\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}-1\right\}$. As shown in the proof of Proposition 2, if

$\frac{\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta}{\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta}<\frac{\varepsilon_1+\varepsilon_2}{2}$, then $\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}-1>0$ and

$\varXi<0$, and thus $\lim_{t\to\infty}\left[\tau_t-\left(\frac{\partial y_{2,t}}{\partial k_{2,t}}-\delta\right)\int_0^t\tau_s ds\right]>0$ because $\lim_{t\to\infty}k_{1,t}>0$. ■

Corollary 2 indicates the permanent trade surpluses in less risk averse country 1. That is, goods and services are transferred from country 1 to country 2 in each period indefinitely in exchange for the return to the accumulated current account deficits in country 1. Nevertheless, the trade balance of country 1 is not surplus from the beginning. Before Corollary 1 is satisfied, negative $\int_0^t\tau_s ds$ should be piled up. In the early periods with the small amount of $\int_0^t\tau_s ds$, the balance on goods and services in country 1 $\tau_t-\left(\frac{\partial y_{2,t}}{\partial k_{2,t}}-\delta\right)\int_0^t\tau_s ds$ continues to be negative. That is, country 1 experiences continuous trade deficits for the time being, and after negative $\int_0^t\tau_s ds$ piles up sufficiently, the trade balance of country 1 changes to surpluses. Therefore, the model predicts that if the degree of relative risk aversion in Japan is truly relatively higher than that in the U.S. as many empirical studies conclude, trade surpluses continue in Japan and trade deficits continue in the U.S. for a long while, but after a sufficiently long period, the trade surpluses in Japan turn to deficits and the trade deficits in the U.S. turn to surpluses and the trade surpluses and deficits continue permanently.

5. CONCLUDING REMARKS

The large current account deficit of the U.S. and the large current account surplus of Japan have continued during the past several decades, and the large bilateral trade deficit of the U.S. with Japan has also persisted. The conventional intertemporal approach to the current account can not explain these persisting large current account imbalances as Obstfeld and Rogoff (1995) argue. This paper examines heterogeneity in the degree of risk aversion as an alternative source of persistent current account imbalances. The reason why the paper directs its attention to the degree of risk aversion is because in endogenous growth models the degree of relative risk aversion plays a crucial role for growth rates, and thus its heterogeneity significantly complicates movements of international transactions. Another reason is because it has been reported that the degree of relative risk aversion in Japan is truly relatively higher than that in the U.S. as many empirical studies conclude, which implies that the large current account deficits in the U.S. and the large current account surpluses in Japan can be explained by the difference of the degree of risk aversion between Japan and the U.S.

The model in the paper shows that if the less risk averse country behaves unilaterally, all capitals in the more risk averse country are eventually owned by the less risk averse country. This result corresponds to the consequence of heterogeneous rates of time preference Becker (1980) shows. However, if both countries behave in multilaterally optimal ways as Sorger (2002) and Ghiglino (2002) suggest, the multilateral balanced growth path is achieved. On this path, heterogeneity in the degree of risk aversion generates persistent current account imbalances. The balance on current account in the less risk averse country shows deficits permanently, and in reverse surpluses permanently in the more risk averse country. On the other hand, the trade balance in the less risk averse country shows surpluses permanently and *vice verse*. The trade imbalances do not explode but the ratios of deficits or surpluses to outputs asymptotically approach unique finite value and stabilize eventually. Note however that the less risk averse country does not experience trade surpluses from the beginning. Initially, the trade balance of the less risk averse country shows deficits, but after its current account deficits pile up sufficiently, its trade balance changes to surpluses. The model therefore predicts that if the degree of relative risk aversion in Japan is truly relatively higher than that in the U.S. as many empirical studies conclude, current account surpluses persist in Japan and current account deficits persist in the U.S. permanently.

The mechanism of trade imbalances presented in the paper does not deny the possibility of trade imbalances caused by heterogeneous demographic changes. Both mechanisms have probably worked simultaneously. Furthermore, other heterogeneous parameters may play important roles for international transactions, e.g. heterogeneous technologies.

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