Trade Liberalization and Heterogeneous Rates of Time Preference across Countries: A Possibility of Trade Deficits with China

Harashima, Taiji

16 December 2009

Trade Liberalization and Heterogeneous Rates of Time Preference across Countries: A Possibility of Trade Deficits with China

Taiji Harashima[†]

December 2009

Version 2.0

Abstract

Strategies for trade liberalization when the rates of time preference are heterogeneous across countries are examined in the framework of endogenous growth. The paper argues that the best strategy for a country with the relatively higher rate of time preference is the strategy of free trade with wielding market power if the country is large enough to wield market power because all the optimality conditions are satisfied in this case. By this strategy, the current account of the country shows persisting surpluses, which implies a possibility that China has taken this strategy.

JEL Classification code: F10, F21, F43, O24
Keywords: Trade Liberalization; Time preference; Heterogeneity; Trade deficits; China

[†]Correspondence: Science Council of Japan, 7-22-34, Roppongi, Minato-ku, Tokyo, 106-8555, Japan
Email: taiji.harashima@cao.go.jp
t-harashima@mve.biglobe.ne.jp

*The views expressed herein are those of the author and not necessarily those of Science Council of Japan.

1. INTRODUCTION

The trade liberalization in developing countries has been actively studied in the last several decades. It has been argued that trade liberalization promotes growth because openness raises the steady state level of income. Many empirical studies support this argument although there are many econometric difficulties to establish an empirical link between trade liberalization and economic performance. Winters (2004) concludes, after surveying the recent literature on this issue (e.g. Easterly and Levine,2001; Dollar and Kraay, 2004), that the weight of evidence is quite clearly in the direction that openness enhances growth. However, the actual processes of growth through trade liberalization do not seem so clear-cut. For example, if preferences of households are heterogeneous across countries, the link between trade liberalization and economic performance is not so simple as the case of the identical preferences across countries. Owing to some disturbing factors, the actual processes of growth initiated by trade liberalization may not proceed on a straight course but be amalgamation of complex processes. This paper studies these complex processes of growth initiated by trade liberalization, and directs its attention to heterogeneity in the rate of time preference rate in the framework of endogenous growth.

It has been argued that people in poor countries have the higher rate of time preference. Importance of this factor is stressed particularly in the literature of environmental economics. Lawrance (1991) concludes that time preference rates have a strong negative correlation with labor income. Cuesta et al. (1997) concludes that there is some evidence of declining discount rates with increasing income based on empirical research in Costa Rica and a review of 14 other empirical studies.[1] Mink (1993) suggests that an inherently short time-horizon of the poor produces environmental degradation. The notion that the poor has the higher rate of time preference is implicitly argued in the broader literature of sustainable development (e.g. World Bank, 1992). This paper commences its analysis starting from the fact that people in poor countries have the higher rate of time preference, but does not examine why the poor has the higher rate of time preference. The paper merely examines theoretical consequences of trade liberalization when the rates of time preference are heterogeneous across countries based on an endogenous growth model.

Becker (1980) argues that the heterogeneous rate of time preference results in an unfavorable consequence for relatively more impatient households because the whole capital is eventually owned only by the most patient household. Similar consequences may be observed between heterogeneous countries. However, the model in this paper predicts different consequences. Firstly, if a relatively more impatient country is large enough and can wield market power, the best strategy for it is the strategy of free trade with wielding market power, because only this strategy can satisfy all the optimality conditions. With this strategy, the balance on current account of the relatively more impatient country shows persisting surpluses while it owning all its capitals. This strategy may provide insights into the recent trade behavior of China whose economy clearly appears to be large enough to wield market power. The large bilateral current

[1] The arguments over the reason why the poor has the higher rate of time preference are inconclusive. Pender (1996) concludes that credit constraints are the main reason. Some economists argue that they have the higher rates of time preference because they are poor.

account deficit of the U.S. with China has been persisting. The model in the paper predicts that the current account deficit of the U.S. with China will be observed if the rate of time preference in China is relatively higher than that in the U.S. and if China is wielding market power. Secondly, when a relatively more impatient country is not large enough and cannot wield market power, no strategy can achieve optimality. Nevertheless, if many small countries with similar preferences can cooperate with each other and integrate their economies, and if they can wield a combined market power that is strong enough like a large country, they can also choose the strategy of free trade with wielding market power, and thus all their optimality conditions are satisfied.

The paper is organized as follows. In Section 2, a two-country endogenous growth model in which international transactions and heterogeneous time preference rates are incorporated is constructed. In Section 3, the basic nature of the model is examined. In Section 4, three strategies for a relatively more impatient country (the strategy of free trade without wielding market power, the strategy of trade protection, and the strategy of free trade with wielding market power) are examined. In Section 5, the three strategies are compared with regard to optimality, the level of output, long-run growth rates, and the balance on current account, and the best strategy for the country is examined. Finally some concluding remarks are offered in Section 6.

2. THE MODEL

2.1 The base model

In most endogenous growth models, $\frac{A_t}{k_t}$ is kept constant by some mechanisms that are different according to the type of models, and the growth rate of consumption is commonly expressed as $\frac{\dot{c}_t}{c_t} = \frac{(1-\alpha)\left(\frac{A_t}{k_t}\right)^{\alpha} - n_t - \theta}{\varepsilon}$ where c_t, k_t, A_t and n_t are consumption per capita, capital per capita, technology and the growth rate of population in period t respectively, and θ is the rate of time preference, ε is the degree of relative risk aversion, and α is a constant (e.g. Romer, 1990; Aghion and Howitt, 1998; Jones, 2003). Thus, in most of the models, the rate of time preference plays a crucial role for growth rates. In this sense, most of the endogenous growth models may be used for the analysis in this paper if international transactions are incorporated in them. However, at the same time, they commonly have the problems of scale effects and/or the influence of population growth (e.g., Jones, 1995a, b). Hence, this paper specifically uses the model shown in Harashima (2004) that is free from both problems (see also e.g. Jones, 1995a; Aghion and Howitt, 1998; Peretto and Smulders, 2002; Harashima, 2005)..

Let Y_t, C_t, K_t, L_t and A_t be outputs, consumption, capital inputs, labor inputs and technology in period t respectively. The production function is $Y_t = F(A_t, K_t, L_t)$. The accumulation of capital is

$$\dot{K}_t = Y_t - C_t - v\dot{A}_t - \delta K_t \tag{1}$$

where δ is the rate of depreciation, $v(>0)$ is a constant, and a unit of K_t and $\frac{1}{v}$ of a unit of A_t are equivalent, i.e., they are produced using the same quantities of inputs. Every firm is identical and has the same size, and for any period,

$$m = \frac{M_t^{\rho}}{L_t} \tag{2}$$

where M_t is the number of firms and m and $\rho(>1)$ are constants. In addition, the relation

$$\frac{\partial Y_t}{\partial K_t} = M_t^{-\rho} \frac{\partial Y_t}{\partial (vA_t)} \tag{3}$$

and thus

$$\frac{\partial y_t}{\partial k_t} = (mv)^{-1} \frac{\partial y_t}{\partial A_t} \tag{4}$$

is always kept where y_t, is output per capita in period t. Equation (2) indicates that the number of population and the number of firms in an economy are positively related. Equations (3) and (4) indicate that returns on investing in K_t and investing in A_t for a firm are kept equal, and also that a firm that invents a new technology cannot obtain all the returns on investing in A_t. This means that investing in A_t increases Y_t but returns of an individual firm that invests in A_t is only a fraction of the increase of Y_t such that $M_t^{-\rho} \frac{\partial Y_t}{\partial (vA_t)} = (mL_t)^{-1} \frac{\partial Y_t}{\partial (vA_t)}$. The reason is uncompensated knowledge spillovers to other firms.

Broadly speaking, there are two types of uncompensated knowledge spillovers: the first is the intra-sectoral knowledge spillover, i.e. MAR externalities, and the second is the inter-sectoral knowledge spillover, i.e. Jacobs externalities. The theory of MAR assumes that knowledge spillovers between homogenous firms work out most effectively and thus spillovers primarily emerge within one sector. As a result, uncompensated knowledge spillovers will be more active if the number of firms within one sector is larger. On the other hand, Jacobs (1969) argues that knowledge spillovers are most effective among firms that practice different activities, and thus diversification, i.e. variety of sectors, is important for spillovers. As a result, uncompensated knowledge spillovers will be more active if the number of sectors is larger in an economy.

If it is assumed that all the sectors have the same number of firms, an increase of the number of firms in an economy results in more active knowledge spillovers owing to either MAR externalities or Jacobs externalities. That is, if an increase of the number of firms in an economy is a result of an increase of the number of firms in each sector, uncompensated knowledge spillovers will become more active by MAR externalities, and if an increase of the number of firms in an economy is a result of an increase of the number of sectors, uncompensated knowledge spillovers will become more active by Jacobs externalities. In either case, an increase of the number of firms in an economy leads to more active uncompensated knowledge spillovers.

Furthermore more active uncompensated knowledge spillovers will reduce the returns of a firm that invests in A_t. $\frac{\partial Y_t}{\partial A_t}$ indicates the over all increase in Y_t in an economy by an additional A_t, that consists of both increase in production in the firm that invented the new technology and increase in production in other firms that use the newly invented technology that the firms obtained either compensating for it to the firm or by uncompensated knowledge spillovers. If the number of firms becomes larger and thus uncompensated knowledge spillovers becomes more active, the compensated fraction in $\frac{\partial Y_t}{\partial A_t}$ that the firm can obtain will become smaller and thus the returns of the firm will become also smaller. Equations (3) and (4)) simply describes this mechanism.

The production function is specified as $Y_t = A_t^{\alpha} f(K_t, L_t)$, where $\alpha(0<\alpha<1)$ is a constant. Let $y_t = \frac{Y_t}{L_t}$, $k_t = \frac{K_t}{L_t}$, $c_t = \frac{C_t}{L_t}$ and $n_t = \frac{\dot{L}_t}{L_t}$, and assume that $f(K_t, L_t)$ is homogenous of degree one. Thereby $y_t = A_t^{\alpha} f(k_t)$, and $\dot{k}_t = y_t - c_t - \frac{v\dot{A}_t}{L_t} - n_t k_t - \delta k_t$. By equations (2) and (3), $A_t = \frac{\alpha f(k_t)}{mvf'(k_t)}$ because $\frac{\partial y_t}{mv\partial A_t} = \frac{\partial y_t}{\partial k_t} \Leftrightarrow \frac{\alpha}{mv} A_t^{\alpha-1} f(k_t)$ $= A_t^{\alpha} f'(k_t)$. Because $A_t = \frac{\alpha f}{mvf'}$, then $y_t = A_t^{\alpha} f = \left(\frac{\alpha}{mv}\right)^{\alpha} \frac{f^{1+\alpha}}{f'^{\alpha}}$ and $\dot{A}_t = \frac{\alpha}{mv}\dot{k}_t\left(1 - \frac{ff''}{f'^2}\right)$.

2.2 Endogenous growth model in open economies

Suppose that there are only two countries in the world: country 1 and country 2. In both countries, the values of parameters as well as population are identical except the rate of time preference, and the growth rate of population is zero, i.e., $n_t = 0$. The time preference rate of the representative household in country 1 is θ_1 and that in country 2 is θ_2, and $\theta_1 < \theta_2$. Goods and services and capitals are freely traded but labor is immobilized in each country. The balance on current account in country 1 is τ_t and the balance on current account in country 2 is $-\tau_t$. The production function in country 1 is $y_{1,t} = A_t^{\alpha} f(k_{1,t})$, and that in country 2 is $y_{2,t} = A_t^{\alpha} f(k_{2,t})$ where $y_{i,t}$ and $k_{i,t}$ are output and capital per capita in country i in period t for i = 1, 2. The number of population is equally $\frac{L_t}{2}$ in both countries and thus the total number of population in the world is L_t. The number of firms in both countries is M_t and firms operate in both countries. Because a balanced growth path requires Harrod neutral technological progress, the production functions are specified as $y_{i,t} = A_t^{\alpha} k_{i,t}^{1-\alpha}$ and thus $Y_{i,t} = K_{i,t}^{1-\alpha}(A_t L_t)^{\alpha}$ $(i=1,2)$.[2]

Because both countries are free open economies, returns on investments in both countries are kept equal through international arbitration such that

[2] As is well known, only Harrod neutral technological progress matches the stylized facts presented by Kaldor (1961).

$$\frac{\partial y_{1,t}}{\partial k_{1,t}} = (2mv)^{-1} \frac{\partial (y_{1,t} + y_{2,t})}{\partial A_t} = \frac{\partial y_{2,t}}{\partial k_{2,t}} . \tag{5}$$

That is, an increase in A_t enhances outputs in both countries such that $\frac{\partial Y_{i,t}}{\partial K_{i,t}} = M_t^{-\rho} \frac{\partial (Y_{1,t} + Y_{2,t})}{\partial (vA_t)}$, and because the number of population is equally $\frac{L_t}{2}$ in both countries, then $\frac{\partial Y_{i,t}}{\partial K_{i,t}} = \frac{\partial y_{i,t}}{\partial k_{i,t}} = M_t^{-\rho} \frac{\partial (Y_{1,t} + Y_{2,t})}{\partial (vA_t)} = (mL_t)^{-1} \frac{\partial (y_{1,t} + y_{2,t})}{\partial (vA_t)} \frac{L_t}{2} =$ $(2mv)^{-1} \frac{\partial (y_{1,t} + y_{2,t})}{\partial A_t}$. Thus, $A_t = \frac{\alpha \lfloor f(k_{1,t}) + f(k_{2,t}) \rfloor}{2mvf'(k_{1,t})} = \frac{\alpha \lfloor f(k_{1,t}) + f(k_{2,t}) \rfloor}{2mvf'(k_{2,t})}$. Because equation (5) is always held through international arbitration, equations $k_{1,t} = k_{2,t}$, $\dot{k}_{1,t} = \dot{k}_{2,t}$, $y_{1,t} = y_{2,t}$ and $\dot{y}_{1,t} = \dot{y}_{2,t}$ are also held. In addition, because $\frac{\partial (y_{1,t} + y_{2,t})}{\partial A_{1,t}}$ $= \frac{\partial (y_{1,t} + y_{2,t})}{\partial A_{2,t}}$ through international arbitration, then $\dot{A}_{1,t} = \dot{A}_{2,t}$ is held by equation (5). Hence, $A_t = \frac{\alpha f(k_{1,t})}{mvf'(k_{1,t})} = \frac{\alpha f(k_{2,t})}{mvf'(k_{2,t})}$.

The accumulated current account balance $\int_0^t \tau_s ds$ mirrors international capital flows owing to current account imbalances. That is, the country with current account surpluses invests them in the other country. Since $\frac{\partial y_{1,t}}{\partial k_{1,t}} \left(= \frac{\partial y_{2,t}}{\partial k_{2,t}} \right)$ are returns on investments, $\left(\frac{\partial y_{1,t}}{\partial k_{1,t}} - \delta \right) \int_0^t \tau_s ds$ and $\left(\frac{\partial y_{2,t}}{\partial k_{2,t}} - \delta \right) \int_0^t \tau_s ds$ represent international income receipts on assets or income payments on assets. Hence, $\tau_t - \left(\frac{\partial y_{2,t}}{\partial k_{2,t}} - \delta \right) \int_0^t \tau_s ds$ is the balance on goods and services of country 1, and $\left(\frac{\partial y_{1,t}}{\partial k_{1,t}} - \delta \right) \int_0^t \tau_s ds - \tau_t$ is the balance on goods and services of country 2. Because the balance on current account mirrors international capital flows, then it is a function of capitals in both countries such that $\tau_t = g(k_{1,t}, k_{2,t})$.

The representative household in country 1 maximizes the expected utility

$$E \int_0^\infty u_1(c_{1,t}) \exp(-\theta_1 t) dt ,$$

subject to

$$\dot{k}_{1,t} = y_{1,t} + \left(\frac{\partial y_{2,t}}{\partial k_{2,t}} - \delta \right) \int_0^t \tau_s ds - \tau_t - c_{1,t} - v\dot{A}_{1,t} \left(\frac{L_t}{2} \right)^{-1} - \delta k_{1,t} , \tag{6}$$

and the representative household in country 2 maximizes the expected utility

$$E\int_0^\infty u_2(c_{2,t})\exp(-\theta_2 t)dt ,$$

subject to

$$\dot{k}_{2,t} = y_{2,t} - \left(\frac{\partial y_{1,t}}{\partial k_{1,t}} - \delta\right)\int_0^t \tau_s ds + \tau_t - c_{2,t} - v\dot{A}_{2,t}\left(\frac{L_t}{2}\right)^{-1} - \delta k_{2,t} , \tag{7}$$

where $u_{i,t}$, $c_{i,t}$ $\dot{A}_{i,t}$ are the utility function, consumption and the increase of A_t by R&D activities in country i in period t for i = 1, 2, $\dot{A}_t = \dot{A}_{1,t} + \dot{A}_{2,t}$, and E is the expectation operator. Equations (6) and (7) implicitly assume that at t = 0 each country does not have any foreign asset.

3. THE BASIC NATURE OF THE MODEL

3.1 *The growth rate of consumption*

Because the production function is Harrod neutral such that $y_{i,t} = A_t^\alpha k_{i,t}^{1-\alpha}$ and thus $Y_{i,t} = K_{i,t}^{1-\alpha}(A_t L_t)^\alpha$, and because $A_t = \frac{\alpha f(k_{1,t})}{mvf'(k_{1,t})} = \frac{\alpha f(k_{2,t})}{mvf'(k_{2,t})}$ and $f = k_{i,t}^{1-\alpha}$, then $A_t = \frac{\alpha}{mv(1-\alpha)}k_{i,t}$ and $\frac{\partial y_{i,t}}{\partial k_{i,t}} = \left(\frac{\alpha}{mv}\right)^\alpha (1-\alpha)^{1-\alpha}$. Because $\dot{A}_{1,t} = \dot{A}_{2,t}$ and $\frac{\partial y_{1,t}}{\partial k_{1,t}} = \frac{\partial y_{2,t}}{\partial k_{2,t}}$, then $\dot{k}_{1,t} = y_{1,t} + \left(\frac{\partial y_{1,t}}{\partial k_{1,t}} - \delta\right)\int_0^t \tau_s ds - \tau_t - c_{1,t} - \frac{v\dot{A}_t}{2}\left(\frac{L_t}{2}\right)^{-1} - \delta k_{1,t}$ $= \left(\frac{\alpha}{mv}\right)^\alpha (1-\alpha)^{-\alpha} k_{1,t} + \left[\left(\frac{\alpha}{mv}\right)^\alpha (1-\alpha)^{1-\alpha} - \delta\right]\int_0^t \tau_s ds - \tau_t - c_{1,t} - \frac{\alpha}{mL_t(1-\alpha)}\dot{k}_{1,t} - \delta k_{1,t}$. Hence,

$$\dot{k}_{1,t} = \frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\alpha}\left\{\left[\left(\frac{\alpha}{mv}\right)^\alpha (1-\alpha)^{-\alpha} - \delta\right]k_{1,t} + \left[\left(\frac{\alpha}{mv}\right)^\alpha (1-\alpha)^{1-\alpha} - \delta\right]\int_0^t \tau_s ds - \tau_t - c_{1,t}\right\} .$$

Since the problem of scale effects in endogenous growth models is not a focal point in this paper, L_t is assumed to be sufficiently large for simplicity and thus $\frac{mL_t(1-\alpha)}{mL_t(1-\alpha)+\alpha} = 1$ is satisfied hereafter in this paper.

Therefore, the optimization problem of country 1 can be rewritten as

$$Max\ E_0\int_0^\infty u_1(c_{1,t})\exp(-\theta_1 t)dt ,$$

subject to

$$\dot{k}_{1,t} = \left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta\right]k_{1,t} + \left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\int_0^t \tau_s ds - \tau_t - c_{1,t}\,.$$

Let Hamiltonian H_1 be

$$H_1 = u_1(c_{1,t})\exp(-\theta_1 t) + \lambda_{1,t}\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta\right]k_{1,t} + \left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\int_0^t \tau_s ds - \tau_t - c_{1,t}\right\}$$

where λ_{1t} is a costate variable, thus the optimality conditions for country 1 are

$$\frac{\partial u_1(c_{1,t})}{\partial c_{1,t}}\exp(-\theta_1 t) = \lambda_{1,t}\,, \tag{8}$$

$$\dot{\lambda}_{1,t} = -\frac{\partial H_1}{\partial k_{1,t}}\,, \tag{9}$$

$$\dot{k}_{1,t} = \left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta\right]k_{1,t} + \left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\int_0^t \tau_s ds - \tau_t - c_{1,t}\,, \tag{10}$$

$$\lim_{t\to\infty}\lambda_{1,t}\,k_{1,t} = 0\,. \tag{11}$$

Similarly, let Hamiltonian H_2 be

$$H_2 = u_2(c_{2,t})\exp(-\theta_1 t) + \lambda_{2,t}\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta\right]k_{2,t} - \left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\int_0^t \tau_s ds + \tau_t - c_{2,t}\right\}$$

where λ_{2t} is a costate variable, thus the optimality conditions for country 2 are

$$\frac{\partial u_2(c_{2,t})}{\partial c_{2,t}}\exp(-\theta_1 t) = \lambda_{2,t}\,, \tag{12}$$

$$\dot{\lambda}_{2,t} = -\frac{\partial H_2}{\partial k_{2,t}}\,, \tag{13}$$

$$\dot{k}_{2,t} = \left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta\right]k_{2,t} - \left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\int_0^t \tau_s ds + \tau_t - c_{2,t}\,, \tag{14}$$

$$\lim_{t\to\infty}\lambda_{2,t}\,k_{2,t} = 0\,. \tag{15}$$

Hence, by equations (8), (9) and (10), the growth rate of consumption in country 1 is

$$\frac{\dot{c}_{1,t}}{c_{1,t}} = \varepsilon^{-1}\left\{\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta + \left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1,t}} - \frac{\partial \tau_t}{\partial k_{1,t}} - \theta_1\right\} \tag{16}$$

and, by equations (12), (13) and (14), the growth rate of consumption in country 2 is

$$\frac{\dot{c}_{2,t}}{c_{2,t}} = \varepsilon^{-1}\left\{\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta - \left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{2,t}} + \frac{\partial \tau_t}{\partial k_{2,t}} - \theta_2\right\}. \tag{17}$$

where $\varepsilon = -\frac{c_{1,t}\, u''}{u'} = -\frac{c_{2,t}\, u''}{u'}$ is the household's degree of relative risk aversion that is constant. A constant growth rate such that $\frac{\dot{c}_{1,t}}{c_{1,t}} = \frac{\dot{c}_{2,t}}{c_{2,t}}$ is possible if

$$\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\left[\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1,t}} + \frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{2,t}}\right] - \left(\frac{\partial \tau_t}{\partial k_{1,t}} + \frac{\partial \tau_t}{\partial k_{2,t}}\right) = \theta_1 - \theta_2 \qquad ! \tag{18}$$

is satisfied. This possibility is examined in the following sections.

3.2 Transversality condition

Transversality conditions are satisfied if the following conditions are satisfied.

Lemma 1: Unless $\lim_{t\to\infty}\frac{\dot{\lambda}_{1,t}}{\lambda_{1,t}} < -1$, $\lim_{t\to\infty}\frac{\dot{\lambda}_{2,t}}{\lambda_{2,t}} < -1$, $\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}} < -1$, or $\lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}} < -1$, the transversality conditions (equations (11) and (15)) are satisfied if

$$\lim_{t\to\infty}\left\{\left(\frac{\partial \tau_t}{\partial k_{1,t}} - \frac{\tau_t}{k_{1,t}}\right) - \left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\left[\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1,t}} - \frac{\int_0^t \tau_s ds}{k_{1,t}}\right] - \frac{c_{1,t}}{k_{1,t}}\right\} < 0 \text{ and}$$

$$\lim_{t\to\infty}\left\{\left(\frac{\tau_t}{k_{2,t}} - \frac{\partial \tau_t}{\partial k_{2,t}}\right) - \left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\left[\frac{\int_0^t \tau_s ds}{k_{2,t}} - \frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{2,t}}\right] - \frac{c_{2,t}}{k_{2,t}}\right\} < 0 \cdot$$

Proof: $\frac{\dot{k}_{1,t}}{k_{1,t}} = \left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta\right] + \left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\frac{\int_0^t \tau_s ds}{k_{1,t}} - \frac{\tau_t + c_{1,t}}{k_{1,t}}$ by equation (10). On the other hand, $\frac{\dot{\lambda}_{1,t}}{\lambda_{1,t}} = -\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta\right] + \left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\frac{\partial\int_0^t \tau_s ds}{\partial k_{1,t}} - \frac{\partial \tau_t}{\partial k_{1,t}}\right\}$ by equation (9). Here,

$$\lim_{t\to\infty}\left(\frac{\dot{\lambda}_{1t}}{\lambda_{1t}} + \frac{\dot{k}_{1t}}{k_{1t}}\right) = -\lim_{t\to\infty}\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta\right] + \left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right]\frac{\partial\int_0^t \tau_s ds}{\partial k_{1,t}} - \frac{\partial \tau_t}{\partial k_{1,t}}\right\}$$

$$+\lim_{t\to\infty}\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]+\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\int_0^t \tau_s ds}{k_{1,t}}-\frac{\tau_t+c_{1,t}}{k_{1,t}}\right\}$$

$$=\lim_{t\to\infty}\left\{\left(\frac{\partial\tau_t}{\partial k_{1,t}}-\frac{\tau_t}{k_{1,t}}\right)-\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left[\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1,t}}-\frac{\int_0^t \tau_s ds}{k_{1,t}}\right]-\frac{c_{1,t}}{k_{1,t}}\right\}.$$ Therefore, unless $\lim_{t\to\infty}\frac{\dot{\lambda}_{1,t}}{\lambda_{1,t}}<-1$, $\lim_{t\to\infty}\frac{\dot{\lambda}_{2,t}}{\lambda_{2,t}}\geq -1$, $\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}<-1$, or $\lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}}<-1$, then $\frac{\dot{\lambda}_{1,t}}{\lambda_{1,t}}+\frac{\dot{k}_{1,t}}{k_{1,t}}<0$ if

$$\lim_{t\to\infty}\left\{\left(\frac{\partial\tau_t}{\partial k_{1,t}}-\frac{\tau_t}{k_{1,t}}\right)-\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left[\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1,t}}-\frac{\int_0^t \tau_s ds}{k_{1,t}}\right]-\frac{c_{1,t}}{k_{1,t}}\right\}<0.$$ Similarly,

$$\lim_{t\to\infty}\left(\frac{\dot{\lambda}_{2,t}}{\lambda_{2,t}}+\frac{\dot{k}_{2,t}}{k_{2,t}}\right)<0 \text{ if } \lim_{t\to\infty}\left\{\left(\frac{\tau_t}{k_{2,t}}-\frac{\partial\tau_t}{\partial k_{2,t}}\right)-\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left[\frac{\int_0^t \tau_s ds}{k_{2,t}}-\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{2,t}}\right]-\frac{c_{2,t}}{k_{2,t}}\right\}<0.$$

■

Lemma 1 indicates that if τ_t is not significantly large compared with $c_{1,t}$ and $c_{2,t}$, the transversality conditions are satisfied. Note that the case of $\lim_{t\to\infty}\frac{\dot{\lambda}_{1,t}}{\lambda_{1,t}}<-1$, $\lim_{t\to\infty}\frac{\dot{\lambda}_{2,t}}{\lambda_{2,t}}<-1$, $\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}<-1$, or $\lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}}<-1$ is extremely unusual and thus these cases are excluded hereafter in this paper.

3.3 Growth path

Balanced growth is the focal point for the analysis of growth path. Therefore, the following analyses focus on the steady state such that $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}$, $\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}$, $\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}$, $\lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}}$ and $\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}$ are constants. Using Lemma 1, the following important nature of the model is proved.

Lemma 2: If $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=$ constant, then $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}=\lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}}=$

$$\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds}.$$

Proof: $\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}=\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta+\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\lim_{t\to\infty}\frac{\int_0^t \tau_s ds}{k_{1,t}}-\lim_{t\to\infty}\frac{\tau_t+c_{1,t}}{k_{1,t}}$ and $\lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}}=\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\lim_{t\to\infty}\frac{\int_0^t \tau_s ds}{k_{2,t}}+\lim_{t\to\infty}\frac{\tau_t-c_{2,t}}{k_{2,t}}$ by equations (10) and (14). By equations (6) and (7), $c_{1,t}-c_{2,t}=2\left(\frac{\partial y_{1,t}}{\partial k_{1,t}}\int_0^t \tau_s ds-\tau_t\right)=2\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\int_0^t \tau_s ds-\tau_t\right]$ because $\frac{\partial y_{1,t}}{\partial k_{1,t}}=\frac{\partial y_{2,t}}{\partial k_{2,t}}$, $k_{1,t}=k_{2,t}$, $y_{1,t}=y_{2,t}$, $\dot{A}_{1,t}=\dot{A}_{2,t}$ and $\frac{\partial y_{i,t}}{\partial k_{i,t}}=\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}$ for $i=1,2$. Hence, if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=$ constant, then $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}$

$$=\lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}}=\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds}. \qquad \blacksquare$$

Lemma 3: If and only if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=$ constant, all the optimality conditions are satisfied at the steady state.

Proof: By Lemma 2, if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=$ constant, then $\lim_{t\to\infty}\frac{\tau_t}{k_{1,t}}=\lim_{t\to\infty}\frac{\tau_t}{k_{2,t}}=\Xi$ where Ξ is a constant. In addition, because $\lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds}=\lim_{t\to\infty}\frac{\tau_t}{\int_0^t \tau_s ds}=\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}$, then $\lim_{t\to\infty}\frac{\int_0^t \tau_s ds}{k_{1,t}}$

$=\lim_{t\to\infty}\frac{\int_0^t \tau_s ds}{k_{2,t}}=\Xi\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}$. Therefore, $\lim_{t\to\infty}\frac{\tau_t}{k_{1,t}}=\lim_{t\to\infty}\frac{\partial\tau_t}{\partial k_{1,t}}=\lim_{t\to\infty}\frac{\tau_t}{k_{2,t}}=\lim_{t\to\infty}\frac{\partial\tau_t}{\partial k_{2,t}}$ and $\lim_{t\to\infty}\frac{\int_0^t \tau_s ds}{k_{1,t}}=\lim_{t\to\infty}\frac{\partial\int_0^t \tau_s ds}{\partial k_{1,t}}=\lim_{t\to\infty}\frac{\int_0^t \tau_s ds}{k_{2,t}}=\lim_{t\to\infty}\frac{\partial\int_0^t \tau_s ds}{\partial k_{2,t}}$, and thus,

$$\lim_{t\to\infty}\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]\left[\frac{\int_0^t \tau_s ds}{k_{1,t}}-\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1,t}}\right]-\left(\frac{\tau_t}{k_{1,t}}-\frac{\partial\tau_t}{\partial k_{1,t}}\right)-\frac{c_{1,t}}{k_{1,t}}\right\}=-\lim_{t\to\infty}\frac{c_{1,t}}{k_{1,t}}<0, \text{ and}$$

$$-\lim_{t\to\infty}\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta\right]\left[\frac{\int_0^t \tau_s ds}{k_{2,t}}-\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{2,t}}\right]-\left(\frac{\tau_t}{k_{2,t}}-\frac{\partial\tau_t}{\partial k_{2,t}}\right)+\frac{c_{2,t}}{k_{2,t}}\right\}=-\lim_{t\to\infty}\frac{c_{2,t}}{k_{2,t}}<0.$$

Hence, by Lemma 1, the transversality conditions (equations (11) and (15)) are satisfied while all the other optimality conditions are satisfied.

On the other hand, if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} \neq \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}$, then $\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t} \neq \lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds}$. Thus by Lemma 1, for both countries to satisfy the transverality conditions, it is necessary that $\lim_{t\to\infty}\frac{c_{1,t}}{k_{1,t}} = \infty$ or $\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{k_{2,t}} = \infty$, which violates equations (10) or (14). As a result, if and only if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} =$ constant, all the optimality conditions are satisfied at the steady state. ■

By Lemmas, it is proved that, if all the optimality conditions are satisfied, both countries grow on the following balanced growth path while satisfying all the optimality conditions.

Proposition 1: If and only if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} =$ constant, then $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} = \lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}$ $= \lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}} = \lim_{t\to\infty}\frac{\dot{y}_{1,t}}{y_{1,t}} \lim_{t\to\infty}\frac{\dot{y}_{2,t}}{y_{2,t}} = \lim_{t\to\infty}\frac{\dot{A}_t}{A_t} =$ constant.

Proof: As for $y_{1,t}$, because $y_{1,t} = A_t^{\alpha} k_{1,t}^{1-\alpha}$, $\dot{y}_{1,t} = \left(\frac{A_t}{k_{1,t}}\right)^{\alpha}\left[(1-\alpha)\dot{k}_{1,t} + \alpha\frac{k_{1,t}}{A_t}\dot{A}_t\right]$. Because $A_t = \frac{\alpha\left[f(k_{1,t}) + f(k_{2,t})\right]}{mvf'(k_{1,t})} = \frac{\alpha}{mv(1-\alpha)}k_{1,t}$ and thus because $\dot{A}_t = \frac{\alpha}{mv(1-\alpha)}\dot{k}_{1,t}$, then $\dot{y}_{1,t} =$ $\dot{k}_{1,t}\left(\frac{A_t}{k_{1,t}}\right)^{\alpha}\left[(1-\alpha) + \frac{\alpha^2}{mv(1-\alpha)}\frac{k_{1,t}}{A_t}\right]$, and thus $\frac{\dot{y}_{1,t}}{y_{1,t}} = \frac{\dot{k}_{1,t}}{k_{1,t}}\left[(1-\alpha) + \frac{\alpha^2}{mv(1-\alpha)}\frac{k_{1,t}}{A_t}\right]$. Because $A_t = \frac{\alpha}{mv(1-\alpha)}k_{1,t}$, $\frac{\dot{y}_{1,t}}{y_{1,t}} = \frac{\dot{k}_{1,t}}{k_{1,t}}\left[(1-\alpha) + \alpha\right] = \frac{\dot{k}_{1,t}}{k_{1,t}}$. Hence, by Lemma 2, $\lim_{t\to\infty}\frac{\dot{y}_{1,t}}{y_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} =$ $\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}$. Because $y_{1,t} = y_{2,t}$, then $\lim_{t\to\infty}\frac{\dot{y}_{1,t}}{y_{1,t}} = \lim_{t\to\infty}\frac{\dot{y}_{2,t}}{y_{2,t}} = \lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} = \lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}} = \lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}}$.

As for A_t, by $\dot{y}_{1,t} = \left(\frac{A_t}{k_{1,t}}\right)^{\alpha}\left[(1-\alpha)\dot{k}_{1,t} + \alpha\frac{k_{1,t}}{A_t}\dot{A}_t\right]$ and $A_t = \frac{\alpha}{mv(1-\alpha)}k_{1,t}$, $\dot{y}_{1,t} =$! $\dot{A}_t\left(\frac{A_t}{k_{1,t}}\right)^{\alpha}\left[\frac{mv(1-\alpha)^2}{\alpha} + \alpha\frac{k_{1,t}}{A_t}\right]$ and thus $\frac{\dot{y}_{1,t}}{y_{1,t}} = \frac{\dot{A}_t}{k_{1,t}}\frac{mv(1-\alpha)^2}{\alpha} + \alpha\frac{\dot{A}_t}{A_t}$. Because $\dot{A}_t = \frac{\alpha}{mv(1-\alpha)}\dot{k}_{1,t}$, then $\frac{\dot{y}_{1,t}}{y_{1,t}} = (1-\alpha)\frac{\dot{k}_{1,t}}{k_{1,t}} + \alpha\frac{\dot{A}_t}{A_t}$. Hence, $\frac{\dot{y}_{1,t}}{y_{1,t}} = \frac{\dot{k}_{1,t}}{k_{1,t}} = (1-\alpha)\frac{\dot{k}_{1,t}}{k_{1,t}} + \alpha\frac{\dot{A}_t}{A_t}$ and thus $\frac{\dot{k}_{1,t}}{k_{1,t}} = \frac{\dot{A}_t}{A_t}$. Because $k_{1,t} = k_{2,t}$, then $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} = \lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}} = \lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}} = \lim_{t\to\infty}\frac{\dot{y}_{1,t}}{y_{1,t}} = \lim_{t\to\infty}\frac{\dot{y}_{2,t}}{y_{2,t}} = \lim_{t\to\infty}\frac{\dot{A}_t}{A_t}$. ■

Corollary 1: If and only if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} =$ constant, then $\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t} = \lim_{t\to\infty}\frac{\frac{d\int_0^t \tau_s ds}{dt}}{\int_0^t \tau_s ds} =$ $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} = \lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}} = \lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}} = \lim_{t\to\infty}\frac{\dot{y}_{1,t}}{y_{1,t}} = \lim_{t\to\infty}\frac{\dot{y}_{2,t}}{y_{2,t}} = \lim_{t\to\infty}\frac{\dot{A}_t}{A_t} =$ a positive constant.

Proof: By Lemma 2, $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} = \lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t} = \lim_{t\to\infty}\frac{\frac{d\int_0^t \tau_s ds}{dt}}{\int_0^t \tau_s ds}$. Hence, by Proposition 1, $\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t} = \lim_{t\to\infty}\frac{\frac{d\int_0^t \tau_s ds}{dt}}{\int_0^t \tau_s ds} = \lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} = \lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}} = \lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}} = \lim_{t\to\infty}\frac{\dot{y}_{1,t}}{y_{1,t}} = \lim_{t\to\infty}\frac{\dot{y}_{2,t}}{y_{2,t}} = \lim_{t\to\infty}\frac{\dot{A}_t}{A_t} =$ a positive constant. ■

Because eventually current account imbalances grow at the same rate with output, consumption and capital, then the ratio of the balance on current account to output do not explode but stabilizes as shown in the proof of Lemma 3, i.e., $\lim_{t\to\infty}\frac{\tau_t}{k_{1,t}} = \lim_{t\to\infty}\frac{\tau_t}{k_{2,t}} = \varXi$.

Because technology will not decrease persistently, i.e., $\lim_{t\to\infty}\frac{\dot{A}_t}{A_t} > 0$, then only the case such that $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} = \lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}} = \lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}} = \lim_{t\to\infty}\frac{\dot{y}_{1,t}}{y_{1,t}} = \lim_{t\to\infty}\frac{\dot{y}_{2,t}}{y_{2,t}} = \lim_{t\to\infty}\frac{\dot{A}_t}{A_t} > 0$ is examined hereafter in this paper.

4. THREE STRATEGIES

The strategy of trade liberalization for a relatively more impatient country is examined in the following sections based on the model shown in Section 3.

4.1 The strategy of free trade

Although the balanced growth path shown in Proposition 1 satisfies all the optimality conditions, the representative households in both countries may not necessarily behave consistently with the balanced growth path because they are heterogeneous. Becker (1980) shows that if households have heterogeneous rates of time preference, the most patient household owns all wealth if households are purely price takers. Ghiglino (2002) predicts that it is likely that under appropriate assumptions the results in Becker (1980) still hold in endogenous growth models. Farmer and Lahiri (2004) show that in general, balanced growth equilibria do not exist in a multi-agent economy except for the special case that all agents have the same constant rate of time preference. This argument may still hold in the model in this paper.

Lemma 4: If the representative household in each country sets τ_t without regarding the other country's optimality conditions, then it is not possible that all the optimality conditions of both countries are satisfied.
Proof: In this case, τ_t can be seen as a control variable for each country. Hence, the same optimality condition $\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial \tau_t}=1$ is added to the optimality conditions of each of the two countries. Here, by Lemma 3, if all the optimality conditions are satisfied, then $\lim_{t\to\infty}\frac{\tau_t}{k_{1,t}}=\lim_{t\to\infty}\frac{\tau_t}{k_{2,t}}=\Xi$ and $\lim_{t\to\infty}\frac{\int_0^t \tau_s ds}{k_{1,t}}=\lim_{t\to\infty}\frac{\int_0^t \tau_s ds}{k_{2,t}}=$ $\Xi\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}$ where Ξ is a constant. By condition $\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial \tau_t}=1$, $\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta=\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}$. Hence, $\lim_{t\to\infty}\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1,t}}-\frac{\partial \tau_t}{\partial k_{1,t}}\right\}=$ $\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)\Xi\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}-\Xi=0$. Therefore, $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\varepsilon^{-1}\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta_1\right]$ and $\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=\varepsilon^{-1}\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\theta_2\right]$. Thereby $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}>\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}$, which contradicts the conditions $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=$ constant shown in Lemma 3. ■

The proof of Lemma 4 indicates that country 1 can satisfy all its optimality conditions only if either $\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds}=\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}$ or $\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds}$ $=\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta$ because $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}$ can be constant only in both cases. The former case corresponds to the case Proposition 1 shows (hereafter called a "multilateral balanced growth path"), and both countries can satisfy all the optimality conditions. On the other hand, in the latter case, although all the optimality conditions are satisfied in country 1, they cannot in country 2 (hereafter called a "unilateral balanced growth path"). In this case, $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\neq\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds}$ and $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}>\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}$. Here, by equations (6) and (7), $c_{1,t}-c_{2,t}=2\left(\frac{\partial y_{1,t}}{\partial k_{1,t}}\int_0^t \tau_s ds-\tau_t\right)=2\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\int_0^t \tau_s ds-\tau_t\right]$, and thus a unilateral balanced growth path requires $\lim_{t\to\infty}(c_{1,t}-c_{2,t})=0$ because $\lim_{t\to\infty}\frac{\tau_t}{\int_0^t \tau_s ds}=\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta$. However, because $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}>\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}$, then country 2 must initially sets consumption such that $c_{2,0}=\infty$ that violates the optimality condition of country 2. Therefore, unlike multilateral balanced growth path, country 2 cannot satisfy all its optimal conditions even though country 1 can.

How should country 2 respond to the unilateral balanced growth path of country 1? Possibly, both countries negotiate for the trade between them, and some agreements may be reached. Nevertheless, if no agreement is reached and country 1 never regards the country 2's optimality conditions, country 2, in general, will fall into the following uncomfortable situation.

Remark 1: If the representative household in country 1 does not regard the country 2's optimality conditions, all capitals in country 2 will be eventually owned by country 1.

The reason for Remark 1 is as follows. Suppose first that country 1 chooses the unilateral balanced growth path and sets $c_{1,0}$ so as to achieve this path. There are two options for country 2. The first option is that country 2 also pursues its own optimality without regarding country 1, i.e., chooses its own unilateral balanced growth path. The second option is to adapt to the behavior of country 1 as a follower. If country 2 takes the first option, it sets $c_{2,0}$ without regarding $c_{1,0}$ like country 1. As Lemma 4 indicates, unilaterally optimal growth rates are different between the two countries and $\frac{\dot{c}_{1,t}}{c_{1,t}}>\frac{\dot{c}_{2,t}}{c_{2,t}}$, and thus initial consumptions are set as $c_{1,0}<c_{2,0}$. Because $\frac{\partial y_{1,t}}{\partial k_{1,t}}=(2mv)^{-1}\frac{\partial(y_{1,t}+y_{2,t})}{\partial A_t}=\frac{\partial y_{2,t}}{\partial k_{2,t}}$ and $k_{1,t}=k_{2,t}$ must be kept, capitals and technology are equal and grow at the same rate in both countries. Hence, because $c_{1,0}<c_{2,0}$, more capitals are initially produced in country 1 than country 2 and thus some of them need be exported to country 2. As a result, $\frac{\dot{c}_{1,t}}{c_{1,t}}>\frac{\dot{k}_{1,t}}{k_{1,t}}=\frac{\dot{k}_{2,t}}{k_{2,t}}>\frac{\dot{c}_{2,t}}{c_{2,t}}$, which means that each of both countries equally cannot satisfy all its own optimality conditions. Because $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}>\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}}=\lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}}>\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}$, capital soon becomes abundant in country 2, and thus unutilized goods and services are produced in country 2. These unutilized products are exported to and utilized in country 1. This process escalates as time passes because $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}>$

$\lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}} = \lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}} > \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}$ and eventually almost all of consumer goods and services produced in country 2 are consumed by the household in country 1. This consequence will be uncomfortable for country 2.

Next, if country 2 takes the second option, country 2 should set $c_{2,0} = \infty$ to satisfy all its optimality conditions as Lemma 4 shows. Setting $c_{2,0} = \infty$ is impossible, but country 2 as a follower will initially set as large $c_{2,t}$ as possible. This action gives country 2 the higher expected utility than that when taking the first option because consumption of country 2 in this case is always higher than that when taking the first option. As a result, country 2 imports as large goods and services as possible from country 1, and the trade deficit of country 2 pile up until $\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\int_0^t \tau_s ds = \tau_t$ is achieved, i.e., until $\frac{\dot{\tau}_t}{\tau_t} = \frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds}$ is achieved. In other words, the trade balance of country 2 never becomes surpluses. The current account deficits and the accumulated debts of country 2 to country 1 continue to increase indefinitely. Furthermore, it increases more rapidly than the growth rate of outputs ($\lim_{t\to\infty}\frac{\dot{y}_{2,t}}{y_{2,t}}$) because in general $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} < \lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}$, i.e., $(1-\varepsilon)\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta\right] < \theta_1(<\theta_2)$. Then, soon, all capitals in country 2 are owned by country 1.[3] This consequence will be also uncomfortable for country 2.

As a result, country 2 cannot satisfy all its optimality conditions in any case if country 1 takes a unilateral balanced growth path, and both options to counter the unilateral action of country 1 are uncomfortable for country 2. However, the expected utility of country 2 is higher if it takes the second option than the first option. Hence, under the circumstance that country 2 cannot satisfy all its optimality conditions in any case, country 2 will choose the second option because of the higher expected utility. Thus, if country 1 does not regard country 2's optimality conditions, all capitals in country 2 will be eventually owned by country 1. This result corresponds to the consequence in an economy with households that have heterogeneous rates of time preference shown in Becker (1980). As a result, the consequences of the strategy of adopting free trade suggest that this strategy is not necessarily favorable for more impatient country 2, and thus country 2 may search for an alternative more comfortable strategy.

4.2 The strategy of trade protection

[3] Note that even though the households in country 2 possess no capital, the capital stock in country 2 is still kept to be $k_{2,t} = k_{1,t}$ and thus $y_{2,t} = y_{1,t}$. Point is that all the capital in country 2 is owned by foreigners.

To avoid the uncomfortable strategy of free trade, country 2 may take the strategy of trade protection. If this strategy is taken, the model has to be modified to exclude the variable τ_t. In addition, the returns on investing in $A_{i,t}$ need to be modified to $(m\,v)^{-1}\dfrac{\partial y_{i,t}}{\partial A_{i,t}}$. If both countries are open, the relation $m = \dfrac{M_t^{\rho}}{L_t}$ is held for the combined numbers of population and firms, but if a country is isolated, this relation is completed within its economy, and thus $m = M_{i,t}^{\ \rho}\left(\dfrac{L_t}{2}\right)^{-1}$ is held in each of both countries. Hence, $\dfrac{\partial Y_{i,t}}{\partial K_{i,t}} = M_{i,t}^{\ -\rho}\dfrac{\partial Y_{i,t}}{\partial (vA_{i,t})}$ and $\dfrac{\partial y_{i,t}}{\partial k_{i,t}} = \left(m\dfrac{L_t}{2}\right)^{-1}\dfrac{\partial y_{i,t}}{\partial vA_{i,t}}\dfrac{L_t}{2} = (m\,v)^{-1}\dfrac{\partial y_{i,t}}{\partial A_{i,t}}$. The optimization problem of the representative household in country i (i = 1, 2) is;

$$Max\ E_0\int_0^{\infty} u_i(c_{i,t})\exp(-\theta_i t)dt\ ,$$

subject to

$$\dot{k}_{i,t} = y_{i,t} - c_{i,t} - v\dot{A}_{i,t}\left(\frac{L_t}{2}\right)^{-1} - \delta k_{i,t}\ ,$$

Because $\dot{k}_{i,t} = \left[\left(\dfrac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta\right]k_{i,t} - c_{i,t}$, then $\dfrac{\dot{c}_{i,t}}{c_{i,t}} = \varepsilon^{-1}\left[\left(\dfrac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta - \theta_i\right]$, and the balanced growth path in each country is $\dfrac{\dot{c}_{i,t}}{c_{i,t}} = \dfrac{\dot{k}_{i,t}}{k_{i,t}} = \dfrac{\dot{y}_{i,t}}{y_{i,t}} = \dfrac{\dot{A}_{i,t}}{A_{i,t}}$.

Because $\theta_1 < \theta_2$, then $\dfrac{\dot{c}_{1,t}}{c_{1,t}} > \dfrac{\dot{c}_{2,t}}{c_{2,t}}$. Hence, although country 2 can satisfy all its optimality conditions, its growth rate is always lower than country 1 and thus its outputs become far smaller than country 1. This consequence of the trade protection strategy will be also not favorable for country 2.

4.3 The strategy of free trade with wielding market power

Both strategies (free trade and trade protection) are uncomfortable for country 2. Nevertheless, this does not mean that country 2 has no escape because the isolation of country 2 is also not comfortable for country 1 and thus country 1 may change its behavior if country 2 shows its intention to isolate itself. The isolation of country 2 indicates that country 1 must allocate more resources for the generation of technology, and as a result, consumption and the expected utility of the representative household in country 1 will decline by the isolation of country 2. Hence, country 1 may compromise to cooperate with country 2. Sorger (2002) shows that if a government levies a progressive income tax, or if there are few households of each type and thus they are not simple price takers but play a Nash equilibrium, the results shown in Becker (1980) do not hold anymore. Ghiglino (2002) argues that the latter case in Sorger (2002) can be interpreted as a model of international trade with a common market simply by associating each household's type to a country with a national central planner or a representative household.

The above arguments suggest that it is not unnatural that the representative households in both countries play a Nash equilibrium with regard to the sequence of τ_t. As Sorger (2002) argues, if a household in a country behaves as a member of a large group of households and know demand functions in markets, the households in the country can wield market power. Therefore, if a relatively more impatient country is large enough, it may be possible to wield market power against the less impatient country. Lemma 3 shows that if and only if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} =$ constant, all the optimality conditions in both countries are satisfied. Therefore, if the representative households in both countries behave so as to satisfy $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} =$ constant at the Nash equilibrium, the growth path shown in Proposition 1, i.e., the multilateral balanced growth path, is achieved. Both countries can satisfy all the optimality conditions simultaneously.

As shown in the proof of Lemma 3, $\lim_{t\to\infty}\frac{\tau_t}{k_{1,t}} = \lim_{t\to\infty}\frac{\tau_t}{k_{2,t}} = \Xi$ and $\lim_{t\to\infty}\frac{\int_0^t \tau_s ds}{k_{1,t}} = \lim_{t\to\infty}\frac{\int_0^t \tau_s ds}{k_{2,t}} = \Xi\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}$ on the multilateral balanced growth path, and because $k_{i,t}$ is positive, if the sign of Ξ is negative, the current account of economy 1 shows deficits eventually and permanently and *vice versa*. On the multilateral balanced growth path, the value of Ξ is uniquely determined as follows.

Lemma 5: $\Xi = \frac{\theta_1-\theta_2}{2}\left\{\varepsilon\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\frac{\theta_1+\theta_2}{2}\right]^{-1}-1\right\}^{-1}$.

Proof: Because $\lim_{t\to\infty}\frac{\partial\tau_t}{\partial k_{1,t}} = \lim_{t\to\infty}\frac{\partial\tau_t}{\partial k_{2,t}} = \Xi$ and $\lim_{t\to\infty}\frac{\partial\int_0^t \tau_s ds}{\partial k_{1,t}} = \lim_{t\to\infty}\frac{\partial\int_0^t \tau_s ds}{\partial k_{2,t}} = \Xi\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}$, $2\Xi\left\{\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]-1\right\} = \theta_1-\theta_2$ by equation (18), and thus, $\Xi = \frac{\theta_1-\theta_2}{2}\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left[\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}\right]-1\right\}^{-1}$. Here, the limit of the growth rate is $\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} = \lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \frac{\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}+\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}}{2} = \varepsilon^{-1}\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\frac{\theta_1+\theta_2}{2}\right]$ by equations (16) and (17). Hence, $\Xi = \frac{\theta_1-\theta_2}{2}\left\{\varepsilon\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\frac{\theta_1+\theta_2}{2}\right]^{-1}-1\right\}^{-1}$. ■

Therefore, the value of Ξ is uniquely determined. In addition, the sign of Ξ is uniquely determined by the relative difference of the rate of time preference between country 1 and 2 as follows.

Proposition 2: If $\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\varepsilon\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]<\frac{\theta_1+\theta_2}{2}$, then $\Xi<0$. That is, the current account deficits of country 1 continue indefinitely and *vice versa*.

Proof: $\lim_{t\to\infty}\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1,t}}-\frac{\partial \tau_t}{\partial k_{1,t}}\right\}<0$ for $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}$ by equations (16) and (17), and $\lim_{t\to\infty}\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1,t}}-\frac{\partial \tau_t}{\partial k_{1,t}}\right\}=\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\Xi\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}-\Xi$

$=\Xi\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}-1\right\}<0$. Because $\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=\varepsilon^{-1}\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\frac{\theta_1+\theta_2}{2}\right]$ as shown in the proof of Lemma 5, then $\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}-1=$

$\frac{\varepsilon\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]}{\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\frac{\theta_1+\theta_2}{2}}-1$. Thus, if $\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\varepsilon\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]<\frac{\theta_1+\theta_2}{2}$, then $0<\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}-1$ and thus $\Xi<0$. ■

Proposition 2 indicates the permanent current account deficits in less impatient country 1. The condition $\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\varepsilon\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]<\frac{\theta_1+\theta_2}{2}$ is generally satisfied for reasonable parameter values.

On the other hand, the opposite is true for the trade balance.

Corollary: If $\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\varepsilon\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]<\frac{\theta_1+\theta_2}{2}$, $\lim_{t\to\infty}\left[\tau_t-\left(\frac{\partial y_{2,t}}{\partial k_{2,t}}-\delta\right)\int_0^t \tau_s ds\right]>0$. That is, the trade surpluses of country 1 continue indefinitely and *vice versa*.

Proof: The balance on goods and services in country 1 is $\lim_{t\to\infty}\left[\tau_t-\left(\frac{\partial y_{2,t}}{\partial k_{2,t}}-\delta\right)\int_0^t \tau_s ds\right]$.

Here, $\lim_{t\to\infty}\frac{\tau_t-\left(\frac{\partial y_{2,t}}{\partial k_{2,t}}-\delta\right)\int_0^t\tau_s ds}{k_{1,t}}=-\left[\left(\frac{\partial y_{2,t}}{\partial k_{2,t}}-\delta\right)\lim_{t\to\infty}\frac{\int_0^t\tau_s ds}{k_{1,t}}-\lim_{t\to\infty}\frac{\tau_t}{k_{1,t}}\right]=-\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\Xi\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}-\Xi\right\}$

$=-\Xi\left\{\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}-1\right\}$. As shown in the proof of Proposition 2, if

$\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\delta-\varepsilon\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]<\frac{\theta_1+\theta_2}{2}$, then $\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}-\delta\right]\left(\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}\right)^{-1}-1>0$

and $\Xi<0$, and thus $\lim_{t\to\infty}\left[\tau_t-\left(\frac{\partial y_{2,t}}{\partial k_{2,t}}-\delta\right)\int_0^t\tau_s ds\right]>0$ because $\lim_{t\to\infty} k_{1,t}>0$. ■

Corollary 2 indicates the permanent trade surpluses in less impatient country 1. That is, goods and services are transferred from country 1 to country 2 in each period indefinitely in exchange for the return to the accumulated current account deficits in country 1.

Nevertheless, the trade balance of country 1 is not surplus from the beginning. Before Corollary 1 is satisfied, negative $\int_0^t\tau_s ds$ should be piled up. In the early periods with the small amount of $\int_0^t\tau_s ds$, the balance on goods and services in country 1 $\tau_t-\left(\frac{\partial y_{2,t}}{\partial k_{2,t}}-\delta\right)\int_0^t\tau_s ds$ continues to be negative. That is, country 1 experiences continuous trade deficits for the time being, and after negative $\int_0^t\tau_s ds$ piles up sufficiently, the trade balance of country 1 changes to be surpluses.

5. DISCUSSION

5.1 The comparison of strategies

The summary of the three strategies for more impatient country 2 is as follows.

1) The optimality conditions
- The strategy of free trade without wielding market power
  Not satisfied
- The strategy of trade protection
  Satisfied
- The strategy of free trade with wielding market power
  Satisfied

2) Outputs
- The strategy of free trade without wielding market power
  $y_{2,t}=y_{1,t}$
- The strategy of trade protection

$y_{2,t} < y_{1,t}$

- The strategy of free trade with wielding market power

$y_{2t} = y_{1t}$

3) The long-run growth rate of output

- The strategy of free trade without wielding market power

$$\lim_{t\to\infty}\frac{\dot{y}_{2,t}}{y_{2,t}} = \lim_{t\to\infty}\frac{\dot{y}_{1,t}}{y_{1,t}} = \varepsilon^{-1}\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta - \theta_1\right]$$

- The strategy of trade protection

$$\frac{\dot{y}_{2,t}}{y_{2,t}} = \varepsilon^{-1}\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta - \theta_2\right] < \frac{\dot{y}_{1,t}}{y_{1,t}} = \varepsilon^{-1}\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \delta - \theta_1\right]$$

- The strategy of free trade with wielding market power

$$\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} = \lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \varepsilon^{-1}\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \delta - \frac{\theta_1+\theta_2}{2}\right]$$

4) The balance on current account

- The strategy of free trade without wielding market power

  Deficits

- The strategy of trade protection

  No trade

- The strategy of free trade with wielding market power

  Surpluses

5) The ownership of capitals

- The strategy of free trade without wielding market power

  No

- The strategy of trade protection

  All

- The strategy of free trade with wielding market power

  All

5.2 The best strategy

5.2.1 The best strategy for country 2 that is large enough and can wield market power.

If country 2 is large enough and can wield market power, country 2 can choose the strategy that satisfies all its optimality conditions, i.e., the strategy of free trade with wielding market power. In this sense, the strategy of free trade with wielding market power is preferable for country 2. Nevertheless, although the optimality conditions are not satisfied, the strategy of free trade without wielding market power shows the highest long-run growth rate and thus the highest long-run level of output and consumption. From this point of view, country 2 may choose the strategy of free trade without wielding market power. However, this choice indicates that households in country 2 do not care about optimality, i.e., they are irrational. If households are rational, they will give priority to its satisfying optimality conditions even though the growth rate is low to some extent. Even if choosing the strategy of free trade with wielding market power, the growth rates of country 1 and 2 are equal, which implies that country 2 will not be so uncomfortable for choosing this strategy. As a whole, if country 2 is large enough and if the households in country 2 behave rationally, the best strategy will be the strategy of free trade with wielding market power.

This conclusion provides insights into the recent trade behavior of China whose economy is clearly large enough to wield market power. The large bilateral current account deficit of the U.S. with China has been persisting and is a big political issue between the U.S. and China. The reason why the large bilateral current account deficit of the U.S. with China has been persisting has been debated actively and many economists argue that the problem is China's currency manipulation. Probably China’s currency manipulation has truly distorted markets significantly and may explain a large part of the deficit of the .U.S. with China, but some other ingredients may also have influence to some extent. The model in the paper indicates that if the rate of time preference in China is higher compared with the U.S. and if China is wielding market power, the balance on current account in China shows surpluses permanently as a result of rational behavior in both countries.

5.2.2 The best strategy for country 2 that is not large enough and can not wield market power.

If country 2 is not large enough and cannot wield market power, country 2 has only two options: the strategy of trade protection and the strategy of free trade without wielding market power. The former strategy satisfies all its optimality conditions, but the latter strategy provides the much higher growth rate. If households are rational, they will give priority to the former strategy. Nevertheless, protecting trade results in the permanently lower growth rate and thus far lower consumption level compared with country 1. The gap of outputs between both countries widens exponentially forever.

One way to evaluate which is the best strategy is to simply compare the expected utility when choosing each of the strategies. Nevertheless, unless country 2 has the very high rate of time preference, it is not easy to say which strategy provides the higher expected utility. As a whole, this problem — which strategy is the best — may not be solved purely from the economic point of view. It may be solved from the political point of view, e.g., the national economic security or the pride of the nation that may be hurt by “economical occupation” by foreigners, although it is a very hard choice.

Nevertheless, there is a chance to evade the hard choice. If many small countries with similar preferences cooperate with each other and integrate their economies, they can wield a combined market power. If their market power is strong enough like a large country, they can choose the strategy of free trade with wielding market power. As a result, they can satisfy all their optimality conditions. Therefore, integrating economies by regional Free Trade Agreements among small countries may be a way to obtain their optimal situation.

6. CONCLUDING REMARKS

This paper studies the impact of trade liberalization in a country with the relatively higher rate of time preference in the framework of endogenous growth. Based on a two-country endogenous growth model, the strategy for a relatively more impatient country to deal with trade liberalization is examined. The results are summed up as follows: (1) when a relatively more impatient country is large enough and can wield market power, its best strategy is the strategy of free trade with wielding market power because only by this strategy, all the optimality conditions can be satisfied, (2) when a relatively more impatient country is not large enough and can not wield market power, it is very difficult to say which strategy is the best. Nevertheless, if many small countries with similar preferences cooperate with each other and integrate their economies, they can choose the strategy of free trade with wielding market power like a large country.

The strategy of free trade with wielding market power provides insights into the recent trade behavior of China whose economy is large enough to wield market power. The large bilateral current account deficit of the U.S. with China has been persisting. The model in this paper predicts that the current account deficit of the U.S. with China will be observed if the rate of time preference in China is relatively higher than that in the U.S. and if China is wielding market power. The trade imbalance may be mainly caused by China’s currency manipulation as many economists argue, but, considering the importance of this issue, the mechanism of trade imbalances shown in this paper should also be studied.

References

Aghion, Philippe and Peter Howitt. (1998). *Endogenous Growth Theory,* Cambridge, MA, MIT Press.

Becker, Robert A. (1980). "On the long-run steady state in a simple dynamic model of equilibrium with heterogeneous households," *The Quarterly Journal of Economics*, Vol. 95, 375-382.

Cuesta, Mauricio, Gerald Carlson, and Ernst Lutz. (1997). *An Empirical Assessment of Farmers' Discount Rates in Costa Rica*, Environment Department, World Bank, Washington D.C.

Dollar, David and Aart Kraay. (2004). "Trade, Growth, and Poverty," *The Economic Journal*, Vol. 114, pp. F22-F49.

Easterly, W. and R. Levine. (2001). "It's not factor accumulation: stylized facts and growth models," *The World Bank Economic Review*, Vol. 15(2), pp. 177-220.

Farmer, Roger E. A. and Amartya Lahiri. (2004). "Recursive Preferences and Balanced Growth," *mimeo*.

Ghiglino, Christian. (2002). "Introduction to a General Equilibrium Approach to Economic Growth," *Journal of Economic Theory*, Vol. 105, 1-17.

Harashima, Taiji. (2004). "A New Asymptotically Non-Scale Endogenous Growth Model," *EconWPA Working Papers*, ewp-dev/0412009.

Harashima, Taiji. (2005). "Endogenous Growth Models in Open Economies: A Possibility of Permanent Current Account Deficits," *EconWPA Working Papers*, ewp-it/ 0502001.

Jacobs, Jane. (1969). *The Economy of Cities,* Random House, New York.

Jones, Charles I. (1995a). "*Time series test of endogenous growth models,*" Quarterly Journal of Economics, Vol. 110, pp. 495-525.

Jones, Charles I. (1995b). "R&D-Based Models of Economic Growth," *Journal of Political Economy*, Vol. 103, pp. 759-784.

Jones, Charles I. (2003). "Population and Ideas: A Theory of Endogenous Growth," in P. Aghion, R. Frydman, J. Stiglitz and M. Woodford, eds., *Knowledge, Information, and Expectations in Modern Macroeconomics: In honor of Edmund S. Phelps*, Princeton University Press.

Kaldor, N. (1961). "Capital Accumulation and Economic Growth," Chap. 10 of A. Lutz and D. C. Hague (eds.), *The Theory of Capital*, St. Martin's Press, New York.

Lawrance, Emily C. (1991) "Poverty and the rate of time preference: evidence from panel data," *Journal of Political Economy*, vol. 99, no. 1, 54-77.

Mink, S. D. (1993). "Poverty, Population and the Environment," *World Bank Discussion Paper* 189, World Bank.

Pender, J. L. (1996). "Discount rates and credit markets: Theory and evidence from rural India," *Journal of Development Economics*, Vol. 50, pp. 257-96.

Peretto, Pietro and Sjak Smulders. (2002). "Technological Distance, Growth and Scale Effects," *The Economic Journal*, Vol. 112, pp. 603-624.

Romer, P. M. (1990). "Endogenous Technological Change," *Journal of Political Economy*, Vol. 98, pp. S71-S102.

Sorger, Gerhard. (2002). "On the Long-run Distribution of Capital in the Ramsey Model," *Journal of Economic Theory*, Vol. 105, 226-243.

Winters, L Alan. (2004). "Trade Liberalisation and Economic Performance: An

Overview," *The Economic Journal*, Vol. 114, pp. F4-F21.
World Bank. (1992). *World Development Report 1992. Development and the Environment*, Oxford University Press, New York.