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The Unequal Treatment of Voters Under a Single Tranferable Vote: Implications for Electoral Welfare with an Application to the 2003 Northern Ireland Assembly Elections

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Abstract

The method of Single Transferable Voting (STV) underpins elections in several countries. The advantages claimed for STV are that, firstly, it allows each voter to express his/her preferences over all the candidates and, secondly, it takes account of each voter's range of preferences in determing the electoral outcome. A disquieting feature of STV - and one that has hardly been commented upon - is that the second point is not true: some voters have more than just their first preference taken account of; for other voters, it is only their first preference votes which are counted, their remaining preferences being ignored. This creates two classes of voters - termed in this paper as 'further-preference' and 'first-preference only' voters. Applying these concepts to the (STV based) Northern Ireland Assembly elections of 2003, this paper shows that over half of all voters were 'first-preference only' voters. Moreover, the different parties had different endowments of voters from these groups: in particular, the Unionist parties had a disproprtionately larger share of 'further-preference' voters compared to the Nationalist parties. This might go some way to explaining why, even though the vote share of the Democratic Unionist Party was only slightly higher - and the vote share of the Ulster Unionist Party was actually lower - than that of Sinn Féin, both parties had disproportionately more seats in the Assembly. The paper proceeds to argue that, if society is averse to inter-voter inequality, it might prefer a voting method which treated all voters equally - even though it allowed them a more limited expression of preferences over candidates - to the STV method.

1   Introduction

The Single Transferable Vote (STV) is a method of voting that allows voters to rank candidates (as opposed to parties) in order of preference. Under STV, each voter is allowed to write a number against the name of each candidate listed on the ballot paper, where this number expresses the voter's preference for the candidate: the most preferred candidate has a '1' against his/her name, the next most preferred a '2', and so on. The first stage of the count is to ascertain the total number of first preference votes for each candidate. Any candidate who has more first preference votes than the 'quota'[1] is immediately elected. If no candidate achieves the quota, the candidates with the lowest number of votes is eliminated and the second preferences on his/her ballot papers are assigned to the remaining candidates. If a candidate is elected at a particular count, the surplus votes (i.e. votes in excess of the quota) are redistributed, according to the subsequent (or 'next available') preferences on the ballot papers, to the remaining candidates[2]. National parliamentary elections in Ireland - and elections to the Assembly and to local District Councils in Northern Ireland - are underpinned by STV. It is also used in Malta and for elections to the Tasmanian - and the Australian Capital Territory - Legislative Assembly.

The rationale for the STV method is two-fold. First, each ballot paper is capable of expressing the preference ordering of voters over *all* the candidates though, needless to say, voters - by truncating the ordering - may express preferences over only a subset of the available candidates. Second, the preference rankings of the voters are taken into account in determining the successful candidates. This occurs because when a candidate is elected at a count - or when a candidate with the lowest number of votes is eliminated, in the event of no election at a count - votes are transferred to the candidates next in the order of preference on the successful (or eliminated) candidate's ballot papers[3].

However, an anomaly of the method is that when a candidate (or candidates) are elected at the first count, the transfer of votes to the remaining candidates is effected by examining the second preferences of *all* the ballot papers of the elected candidate(s). However, for candidates elected at subsequent counts, only the transfers made at the count of election are examined for subsequent preferences. This rule for effecting transfers creates two classes of voters. Some voters - 'further-preference' voters - after declaring their preferences across the candidates, have a range of their preferences taken account of in determining the successful candidates. For other voters ('first-preference only' voters), it is only their first preference votes which influence the election outcomes, their remaining preferences being ignored. This is what is meant by the 'unequal treatment of voters' under STV. How this discriminatory treatment comes about - and the

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[1] This quota is also known as the "Droop" quota and is the analogue of a simple majority in a single member constituency. A quota is not the same as a "threshold" since, on the last count, a candidate may be elected without reaching the quota: it is, therefore, a suficient, but not a necessary, condition for election.

[2] For a fuller discussion of STV, including its merits and demerits, see Hallett (1984), Katz (1984), Amy (1993) and Bowles and Grofman (2000).

[3] For elected canidates it is the votes in excess of the quota that are transferred.

voters who comprise the 'further-preference' and 'first-preference only' groups - is discussed in the next section. Suffice it to say here, the value of the STV method is diminished by the fact that all voters are not treated equally. As a corollary of this, there may be a case for supporting alternative election methods which, while allowing voters only a limited expression of their preferences across candidates, ensures that all votes count equally towards the electoral outcome. In other words, it can be argued that, if electoral welfare depends on both the full expression of preferences by voters *and* on the equal treatment of all voters, there may well be a trade-off between 'full expression' and 'equal treatment'. The purpose of this paper is to analyse the nature of such a trade-off and to apply it to results from the (STV-based) elections of November 2003 to the Northern Ireland Assembly.

2 The Analytical Framework

There are N voters in a constituency from which, without loss of generality, six members are to be elected[4] using the method of STV. The ("Droop") quota is, therefore, $Q = (N/7) + 1$: any candidate receiving Q or more votes, at any count, is deemed to be elected. If C represents the set of candidates, let V_X^r represent the number of votes counted for $X \in C$ at the rth count, where V_X^1 - the number of votes counted for X at the *first* count - represents the number of *first-preference* votes received by the candidate.

2.1 Case 1: a candidate(s) is elected at the first count

Suppose that X, at the first count, receives votes equal to, or in excess of, the quota ($V_X^1 \geq Q$) and is, therefore, elected. The 'surplus' of X, denoted S_X, is the excess of votes received over the quota: $S_X = V_X^1 - Q \geq 0$. This surplus is then to be distributed over the remaining candidates.

Let V_{Xj}^1 (> 0) represent the number of voters - of the V_X^1 voters who had X as their most preferred candidate - who ranked j ($j \in C, j \neq X$) as their *second-preference* candidate and let $V_{X0}^1 \geq 0$ be the number of X's *non-transferable* votes (that is, votes received by X in which no further preference was expressed). By definition:

$$V_X^1 = \sum_{j \neq X} V_{Xj}^1 + V_{X0}^1 \tag{1}$$

Now define the proportions[5]:

$$\pi_{Xj}^1 = V_{Xj}^1 / V_X^1 \tag{2}$$

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[4] Six members are elected from each of the 18 constituencies to the Northern Ireland Assembly.

[5] By equation (1), $\sum_{j \neq X} \pi_{Xj}^1 \leq 1$, $\sum_{j \neq X} \pi_{Xj}^1 = 1 \Leftrightarrow V_{X0}^1 = 0$ This method of defining proportions where non-transfers are taken into account is used in Northern Ireland. In the Republic of Ireland, non-transfers are set aside and the proportions are defined as: $\pi_{Xj}^1 = V_{Xj}^1 / (V_X^1 - V_{X0}^1)$.

Then, at the second count, candidate j ($j \notin X$) receives the proportion π^1_{Xj} of X's surplus of S_X votes. Consequently, the transfers to candidate j (denoted T^2_{Xj}), *made at the second count*, from the surplus votes of X are computed as[6]:

$$T^2_{Xj} = \pi^1_{Xj} S_X \qquad (3)$$

If, at the first count, another candidate, Y, was also elected, each of the remaining candidates j would receive transfers from Y - computed analogously to transfers from X - as $T^2_{Yj} = \pi^1_{Yj} S_Y$. Consequently, at the end of the second count, the votes counted for candidate j are:

$$V^2_j = V^1_j + T^2_j \qquad (4)$$

where T^2_j represents the total of transfers received by j from *all* the candidates who were elected at the first count.

In the Republic of Ireland, the T^2_{Xj} and T^2_{Yj} ballot papers, representing the transferred votes to j from, respectively, X and Y, are drawn randomly from the total of V^1_{Xj} and V^1_{Yj} ballot papers in which voters, after giving their first preference to X and Y, respectively, gave j their second preference. In Northern Ireland, *all* the V^1_{Xj} and V^1_{Yj} second-preference ballot papers are transferred to candidate j at the appropriate fractions of their value[7]. This difference in procedure between the two parts of Ireland has no implications for the votes received by any of the candidates at this count, but it does have implications for subsequent counts when later preferences may have to be counted. This is because while all the V^1_{Xj} and V^1_{Yj} ballot papers have the same second-preference (namely, candidate j), they will quite likely differ in terms of subsequent preferences.

2.2 Case 2: a candidate(s) is elected at the second or later count

Suppose that a candidate, X, *after receiving transfers from earlier counts*, is elected at the $R^{th}(> 1)$ count. These transfers may come either from candidates elected or eliminated at previous counts.

If no candidate was elected at the previous count $(R-1) \geq 1$ - because none achieved the quota Q - then the candidate with the lowest number of votes is eliminated and *all* of his/her votes are distributed to the other candidates. Since the candidate was eliminated at count $R - 1$, it is the Rth preference on that candidate's ballot that is the relevant ('next available') preference.

Suppose, without loss of generality, that Y received the lowest number of votes at the $(R-1)^{th}$ count, in which no candidate was elected. Then Y is

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[6] Note that, because of the presence of non-transferable votes, the entire surplus of X may not be distributed among the remaining candidates. However, it will be in the Republic of Ireland where non-transfers are set aside.

[7] These fractions are $T^2_{Xj}/V^1_{Xj} = \pi^1_{Xj}S_X/V^1_{Xj} = S_X/V^1_X$ and $T^2_{Yj}/V^1_{Yj} = \pi^1_{Yj}S_Y/V^1_{Yj} = S_Y/V^1_Y$

eliminated at that count and the votes counted for the remaining candidates - of which X is one - at the subsequent - R^{th} - count are:

$$V_X^R = V_X^{R-1} + T_{YX}^R \qquad (5)$$

where T_{YX}^R are the transfers received by X at the Rth count from Y, the candidate eliminated in the previous count[8].

On the other hand, the transfers to X could have come from the surplus votes of a candidate (or candidates) elected at the previous $(R-1)^{th}$ count. Irrespective of how these transfers arrive, since $V_X^R > Q$, X is elected at the $R^{th}(> 1)$ count and X's surplus votes, $(S_X = V_X^R - Q)$, are then tranferred to the remaining candidates.

Now the votes counted for X at the Rth count are the sum of the X's votes at the first count (V_X^1) and the transfer votes received by X at subsequent counts $(T_X^r, r = 2...R)$:

$$V_X^R = V_X^1 + \sum_{r=2}^{R} T_X^r \qquad (6)$$

Then X's surplus is distributed among the remaining candidates by calculating, for each of the remaining candidates j, the proportion of the transfer votes that X received *at the count at which he/she was elected* - which, in this case is the Rth count - which had candidate j as their 'next available preference' (i.e. $(R+1)^{th}$) preference. Let σ_{Xj}^R denote this proportion.

At the end of the $(R+1)^{th}$ count, each candidate j receives a total of T_{Xj}^{R+1} transfer votes from the surplus votes of X where:

$$T_{Xj}^{R+1} = \sigma_{Xj}^R S_X \qquad (7)$$

If, at the Rth count, another candidate, Y, was also elected, each of the remaining candidates j would receive transfers from Y - computed analogously to transfers from X - as $T_{Yj}^{R+1} = \sigma_{Yj}^R S_Y$. Consequently, at the end of the $(R+1)^{th}$ count, the votes counted for candidate j are:

$$V_j^{R+1} = V_j^R + T_j^{R+1} \qquad (8)$$

where T_j^{R+1} represents the total of all transfers received by j at the $(R+1)^{th}$ count.

3   The relative importance of different voters

Lying at the source of these *surplus transfers, described above,* is/are the candidate(s) elected at the first count (i.e. those whose first-preference votes exceeded

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[8] It is possible, in order to speed the counting, to eliminate more than one candidate at the same count. For example, suppose for three candidates, A, B, and C : $V_A^R < V_B^R < V_C^R$, $V_C^R - V_B^R > V_A^R > and V_C^R + V_A^R < Q$. Then even if all of A's votes went to B, B would still have the lowest number of votes, after candidate A was eliminated; and even if all of A's votes went to candidate C, C would still not be elected at the next count. Consequently, B is bound to be eliminated at the next count and could, as well, be dropped at the current count.

the quota) for it is he/she/they who triggers the chain of surplus vote transfers. If X was elected at the first count, then *all* the votes received by X are inspected in effecting the transfer of his/her surplus at the second count to the remaining candidates.

However, the distribution of the surplus votes of a candidate, Y - elected at a later count - to the remaining candidates at the next count, are drawn *entirely* from the *transfer votes* received by the elected candidates *at the count at which they were elected.* Consequently, once Y has been elected, the subsequent preferences of those voters for whom Y was the most prefered candidate play no role in influencing subsequent results. In contrast, as long as candidates are elected at successive counts, the subsequent preferences of those voters for whom X - who was elected at the first count - was the most prefered candidate play a role in influencing subsequent results.

In that sense, while all voters are treated equally in terms of first-preference votes, it is the subsequent preferences of voters who plumped for a candidate(s) who was elected at the first count that thread their way into later counts. The subsequent preferences of voters whose most-preferred candidates were eliminated also matter since these preferences show up as votes for the remaining candidates[9]. But the subsequent preferences of those voters who gave their first-preference vote to a candidate who was elected, but *not* at the first count - as well as the subsequent preferences voters who gave their first-preference vote to a candidate who was neither elected nor eliminated[10] - *are entirely disregarded.* This anomaly springs from the fact that vote transfers under STV are effected solely by reference to the votes received by elected candidates at the count at which they were elected.

<Table 1>

Table 1 shows that in the November 2003 elections to the Northern Ireland Assembly - conducted using STV - only 28 per cent of voters gave their first preference votes to candidates who were elected at the first count, and only 18 per cent of voters gave their first preference votes to candidates who were eliminated during the counts: such voters are termed 'further-preference' voters because their preferences - beyond their first preference - influence the electoral outcome. Conversely, 46 percent of voters gave their first preference votes to candidates who were elected at later counts, and 8 per cent of voters gave their first preference votes to candidates who were *not* eliminated during the counts. Consequently, over half (54 per cent) of voters at the 2003 Northern Ireland Assembly elections influenced the electoral outcome *solely* through their first preference vote; their further preferences were ignored. Such voters are termed 'first-preference only' voters.

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[9] The only way way the influence of the X voters is broken is if there is no election at a count so that a candidate is eliminated. In that case, subsequent transfers are drawn from the votes of the eliminated candidates.

[10] i.e. a candidate who was still in the contest at the count at which the final candidate was elected.

Voters comprising the 'further-preference' group constituted two extreme subgroups. At one extreme, there were those voters who supported (i.e. gave their first preference votes to) strong candidates who were elected at the first count: a little over 28 per cent of all voters at the 2003 Northern Ireland Assembly elections fell into this category. At the other extreme of the 'further-preference' group were those who supported weak candidates who were eliminated before the election was concluded: nearly 18 per cent of voters were in this category. 'First-preference only' voters also comprised two subgroups. There were those who supported candidates who were strong enough to be elected, but who lacked the necesary support to be elected at the first count: nearly: 46 per cent of all voters were in this category. There were also those who supported candidates who did not have enough support to be elected, but who were not weak enough to be eliminated before the election was concluded: such persons comprised 8 per cent of all voters.

<Table 2>

Northern Ireland has four main parties. Of these, two - the Democratic Unionist Party (DUP) and the Ulster Unionist Party (UUP) - represent unionist aspirations for Northern Ireland to remain within the United Kingdom and two - Sinn Féin (SF) and the Social Democratic Labour Party (SDLP) - represent nationalist aspirations for a united Ireland. In addition to these four big parties, there are a range of smaller parties clubbed together in this study under the rubric 'Other parties'. With this background, one may examine the division into 'further-preference' and 'first-preference only' voters in terms of party support.

Table 2 shows that of the 177,944 voters who gave their first preference vote to the DUP, 44 per cent (77,478 voters) cast their votes for DUP candidates who were elected at the first count. Similarly, of the 156,931 voters who gave their first preference vote to the UUP, 43 percent (66,741 voters) cast their votes for UUP candidates who were elected at the first count. Consequently, nearly a half of unionist voters were 'further-preference' voters whose further preferences percolated through the subsequent counts, influencing their outcomes. By contrast, only 24 per cent of SF voters - and only 6 per cent of SDLP voters - were in a position to influence electoral outcomes beyond the first count.

In the 2003 Northern Ireland Assembly elections, the DUP received 15,186 more first preference votes (representing 2 per cent of the total of 692,028 votes) - while the UUP received 5,827 fewer votes - than SF. Yet, the DUP won six more seats - and the UUP won three more seats - than SF. At least part of this can be ascribed to the fact that the preferences of DUP and UUP supporters were more effective in making their way through the electoral system compared to the preferences of SF supporters.

More generally, the fact that voters under STV are treated differently, in terms of how their preferences influence electoral outcomes, may also explain why a perceived weakness of the STV system is its lack of proportionality[11].

[11]See The Electoral Reform Society, http://www.electoral-reform.org.uk/voting systems/systems3.htm

Indeed, using Gallaher's (1991) "least squares" index for disproportionality[12], the 2003 Northern Ireland Assembly elections returned a value of 3.1 for this index[13]. This represented the same level of disporoportionality as Ireland and a higher level of disproportionality than Germany, Netherlands, Austria, Denmark, Sweden, Italy and Iceland (Gallagher, 1991).

4 Welfare Implications of Inequality in Electoral Influence

We suppose that every voter, $i = 1...N$, has a preference ranking over all the candidates. Suppose the election extends over R counts. Let $k_i = 1..R$ represent the number of counts at which a preference from voter i's ballot paper is recorded: $1 \leq k_i \leq R$. Hereafter, k_i is referred to as voter i's *electoral influence*: the higher the value of k_i, the greater the influence. At one extreme, if $k_i = 1$, it is only voter i's first preference that matters; at the other extreme, if $k_i = R$, voter i's preferences are taken into account in all the counts. For the 'first-preference only' group of voters (defined in the previous section), $k_i = 1$ since, by construct of STV, every (first preference) vote is counted at the first count. For 'further preference' voters, $k_i > 1$.

We assume that the utility (U) a voter derives from participating in an STV-based election depends postively on the number of counts at which a preference from his ballot is recorded:

$$U = U(k_i) \tag{9}$$

where, by assumption: marginal utility is positive ($U'(k_i) > 0$) but diminishing in k_i ($U''(k_i) < 0$). If *electoral welfare, W,* is represented as the sum of the utilities of the individual voters:

$$W = \sum_{i=1}^{N} U(k_i) \tag{10}$$

then electoral welfare is maximised under an electoral system in which $k_i = R, \forall i$.

Suppose that, because of the nature of the voting method used, electoral welfare cannot be maximised. Assume that, in the most general case, the vector $\mathbf{k} = \{k_i\}$ represents the distribution of electoral influence across the $i = 1...N$ voters, where $k_i < R$ for some i. The average level of electoral influence under the voting method is:

$$\overline{k} = \sum_{i=1}^{N} k_i \tag{11}$$

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[12]Defined as: $L = \left[\frac{1}{2} \left(\sum_{k=1}^{K} (v_k - s_k)^2 \right) \right]^{\frac{1}{2}}$, where v_k and s_k are the vote and seat shares of party k.

[13]This was lower than the value of 4.2 for the 1998 Assembly elections.

In the context of the distinction, under STV, between 'further-preference' and 'first-preference only' voters, $k_i > 1$ for 'further-preference' voters and $k_i = 1$ for 'first-preference only' voters. If α represents the proportion of 'further-preference' voters, and $\mu > 1$ represents their average electoral influence (i.e. the average number of counts at which their votes were counted) then the average level of electoral influence under STV is $\overline{k} = \alpha\mu + (1 - \alpha)$ and electoral welfare under STV is $W = N[\alpha U(\mu) + (1 - \alpha)U(1)]$.

Following Atkinson (1970), let $k^* \leq \overline{k}$ represent the average level of electoral influence which, *if equally distributed across the voters*, would yield the same level of electoral welfare as the existing distribution of electoral influence, represented by the vector $\mathbf{k} = \{k_i\}$. In other words:

$$W = N \times U(k^*) = \sum_{i=1}^{N} U(k_i) \qquad (12)$$

Then k^* may be termed the *equally distributed equivalent electoral influence*. Atkinson's (1970) inequality index may be applied to the distribution, $\mathbf{k} = \{k_i\}$, of electoral influence across the voters by defining the inequality index:

$$I(\mathbf{k}; \mathbf{N}) = 1 - (k^*/\overline{k}) = 1 - \left[N^{-1} \sum_{i=1}^{N} \left(k_i/\overline{k} \right)^{1-\varepsilon} \right]^{1/(1-\varepsilon)} , \ \varepsilon > 0 \ \varepsilon \neq 1 \quad (13)$$

The parameter ε - which ranges from 0 to ∞, so that the values of $I(\mathbf{k}; N)$ range from 0 to 1 - is a measure of society's *aversion to inequality* in electoral influence. When $\varepsilon = 0$, society is indifferent as to how a given average of electoral influence (represented by $\overline{k}$) is distributed across the voters: $k^* = \overline{k}$ and $I(\mathbf{k}; N) = 0$. For $\varepsilon > 0$, $k^* < \overline{k}$ and $I(\mathbf{k}; N) > 0$. This means that society would be prepared to adopt a voting method that embodies a lower average electoral influence than the current method, provided that the reduction in the average value is accompanied by an equal distribution of electoral influence across the voters. The higher the value of the inequality aversion parameter, ε, the smaller will be the value of k^* and higher will be the value of $I(\mathbf{k}; N)$.

In order to make comparisons of loss (or gain) of welfare across different values of $\overline{k}$, a specific transformation, linking the inequality measure I to the different values of $\overline{k}$, is needed. One obvious transformation is the reverse of the Atkinson transformation which yields the welfare function (Sen, 1973):

$$W = \overline{k}(1 - I) \qquad (14)$$

which is homogenous of degree one in the k_i.

The electoral welfare function in equation (14) has a natural interpretation: electoral welfare from a given average electoral influence, $\overline{k}$, is reduced by the extent of inequality in the distribution of electoral influence between voters . Given a value of $\overline{k}$, equation (14) says that electoral welfare depends upon the degree of inequality in the distribution of electoral influence across voters - as measured by the Atkinson index (I) - and this inequality, in turn, is determined

by the inter-voter distribution of average electoral influence *and* upon the degree to which society is averse to inequality in electoral influence.

One may compare electoral welfare under STV to that under a "naive" method in which only first preference votes matter: for example, the six candidates in a constituency receiving the largest number of first preference votes are elected as its representatives. Under STV, $\overline{k} = 1 + \alpha(\mu - 1)$ and electoral welfare under STV and plurality - denoted, W^S and W^P, respectively - are, from equation (14):

$$W^S = [1 + \alpha(\mu - 1)][1 - I(\mathbf{k};N)] \text{ and } W^P = 1 \qquad (15)$$

where, from equation (13):

$$
\begin{aligned}
I(\mathbf{k};N) &= 1 - \left[N \left\{ \alpha \left(\frac{\mu}{k} \right)^\varepsilon + (1 - \alpha) \left(\frac{1}{k} \right)^\varepsilon \right\} \right]^{1/(1+\varepsilon)} \\
&= 1 - \left[N \left\{ \left(\frac{1}{\overline{k}} \right)^\varepsilon + \alpha \left(\frac{\mu - 1}{\overline{k}} \right)^\varepsilon \right\} \right]^{1/(1+\varepsilon)} \qquad (16)
\end{aligned}
$$

Then setting $W^S = W^P$ (from equation (15)) yields:

$$[1 + \alpha(\mu - 1)][1 - I(\mathbf{k};N)] = 1 \qquad (17)$$

and using the expression for $I(\mathbf{k};N)$ from equation (16), allows one to solve for the degree of inequality aversion (ε^*) which would make STV and plurality to be welfare equivalent. In determining the outcome of an election, when $\varepsilon = \varepsilon^*$, society is indifferent between: an electoral method which allows voters a fuller representation of their preferences than simply a single preference, but takes unequal account of these representations; and a method which allows voters a single representation of their preferences, but takes equal account of this representation.

<Table 3>

Table 3 shows what the outcome of the 2003 Northern Ireland Assembly elections would have been if the "naive" method had been used to determine the results. The number of seats to the DUP and the SDLP would have remained unchanged at, respectively, 30 and 18. However, the UUP would have won four fewer seats (down from 27 to 23), 'Other' Parties would have won two fewer seats (down from 9 to 7) and Sinn Féin would have won six more seats (up from 24 to 30) to make it, along with the DUP, the largest party in the Assembly[14].

4.1   Diagrammatic Representation

It may be useful to present an intuitive understanding of the approach towards measuring inter-voter inequality in electoral influence, set out in the previous

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[14] The degree of disproportionality, as measured by the least squares index would rise to 4.7. However, needless to say, this is not a fair reflection of the outcome under plurality since party strategies in the selection of candidates were made on the basis of STV.

sections, by means of a diagram. Figure 1 portrays a world of two voters, 1 and 2, who "share" a given average electoral influence, $\overline{k}$ to obtain, respectively, k_1 and k_2. The horizontal axis measures k_1 and the vertical axis measures k_2. The sharing equation is $\overline{k} = (k_1 + k_2)/2$ and this is represented by the "electoral-possibility" line PQ in the diagram. Each point on PQ represents a (k_1, k_2) combination that yields the value $\overline{k}$. At any point on the 45^0 line OM, passing through the origin, $k_1 = k_2$.

Superimposed upon the electoral-possibility line in the diagram are the electoral welfare indifference curves: each curve shows the different k_1, k_2 combinations that yield the same level of electoral welfare, defined in equation (10). If the voting system delivers the point X, then the average electoral influence OC is distributed bewteen the two voters so that $k_1 = OC$ and $k_2 = OD$. In welfare terms this is equivalent to the outcome at point Y (since X and Y lie on the same indifference curve) at which $k_1 = k_2 = OB$ where OB is the "equally distributed equivalent" electoral influence (k^*). The degree of inequality in the voting method is, from equation (13), given by $(1 - OB/OA)$.

<Figure 1>

Under STV, $k_1 = \mu$ and $k_2 = 1$ at the point X in Figure 1. Under plurality, $k_1 = k_2 = 1$ and this represented at the point Y in Figure 1. Since Y and X lie on the same indifference curve, the degree of inequallity aversion - as represented by how "bowed" the indifference curve is - is such as to make plurality and STV welfare equivalent.

5   A Proposal for Reform of STV

As section 3 made clear, the unequal treatment of voters under STV arises because, in effecting the transfer of surplus votes of candidates elected at the first count to the remaining candidates, *all* the votes received by the successful candidates are inspected. However, in effecting the transfer of surplus votes of candidates elected at later counts, *only* the transfers received by such candidates, at the count at which they were elected, are inspected. This inter-voter inequality of treatment can be overcome by ensuring that, *even for candidates elected at later counts*, the transfer of surplus votes is based on an inspection of *all* the votes received by the successful candidates. Call this method *Extended STV* (abbreviated to ESTV). Under ESTV, there would be no change in the way that the surplus of candidates elected at the first count was effected. Nor would there be any change in the analysis relating to the elimination of candidates. However, under ESTV, candidates elected at subsequent counts would have their surplus votes transferred to the remaining candidates in the same way that such transfers are effected for candidates elected at the first count.

Under ESTV, the proportion of the total votes received by X at the Rth count that would accrue to each of the remaining candidates is calculated for each of the remaining candidates, denoted by j, as:

$$\rho_{Xj}^R = V_{Xj}^R / V_X^R \tag{18}$$

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where V_{Xj}^R are the number of votes that j would receive from V_X^R, the total votes received by X at the Rth count. The transfers received by j (from X, who was elected at the Rth count[15]) would then be:

$$T_j^R = \rho_{Xj}^R S_X \tag{19}$$

and the total votes counted for j at the $(R+1)^{th}$ would be: $V_j^{R+1} = V_j^R + T_j^R$

In order to effect ESTV, *all* the V_X^R ballots from which X received votes, upto the Rth count, would have to be inspected: the V_X^1 ballots listing X as the first-preference candidate would have to be inspected for the second-preference candidate, and the T_X^r ballots that listed X as the rth preference candidate would have to be inspected for their $(r+1)^{th}$ preference $(r = 2..R)$.

In contrast to ESTV, STV, as discussed earlier, effects the transfer of the surplus of candidates, elected at counts later than the first, only in terms of the transfers received at the count at which they were elected.

6   Conclusions

Issues relating to equality loom larger in the public concsiousness in Northern Ireland - and play a more important role in the making of public policy - than they do elsewhere in the UK. In large measure, this is due to the turbulent history of Northern Ireland where, as is well known, tensions and animosities between the Catholic and Protestant communities have, since the inception of the state in 1920, run deep. Many events have coalesced to produce this state of affairs in Northern Ireland and some have acquired more prominence in the popular consciousness than others. One event was the abolition in 1922 - by the Government of Northern Ireland, under the Prime Ministership of Sir James Craig - of proportional representation for local council elections (Bogdanor, 2001). This replacement, both at the level of district council and of parliamentary elections, of proportional representation by plurality election established a Protestant hegemony over affairs in Northern Ireland[16] and the exercise of this hegemony in discriminating against Catholics in public sector employment and housing culminated in the civil rights protests of the late 1960s and thus began the spiral of violence that, notwithstanding the current cease-fire, continues to splutter on.

However, it is the contention of this paper that by underpinning elections in Northern Ireland - both to district councils and to the Northern Ireland Assembly - by STV, a different kind of inequality has been created. This inequality - which remains largely unoted and, therefore, is not a cause for public comment - is inequality *between* voters in the way their votes influence the electoral outcome. Under STV, voters with different preferences are treated differently in terms of their impact on the electoral outcome. In essence, STV creates a two

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[15] It is assumed, without loss of generality, that no other candidate was elected at that count.

[16] "A Protestant Parliament for a Protestant People" as one Unionist Prime Minister memorably expressed it.

classes of voters. In determining the list of successful candidates, some voters - 'further-preference' voters - have more than their first preference vote taken into account, while for other voters - 'first preference' voters - it is only their first preference that is counted. So, while all voters may express their preferences over all the candidates, such expression is meanginful for only some voters and meaningless for the remainder. If society was averse to inter-voter inequality in electoral influence, it might prefer a voting system which, while allowing a voters a more limited expression of preferences, allowed each voter's preferences to count equally towards the outcome of the election.

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Table 1
Percentage of Voters Whose Full Range of Preferences Influenced Electoral Outcomes in Northern Ireland's 2004 Assembly Elections

	Percentage of voters giving 1st preference to candidates elected at first count	Percentage of voters giving 1st preference to candidates elected at later counts *	Percentage of voters giving 1st preference to candidates who were eliminated	Percentage of voters giving 1st preference to candidates who were neither eliminated nor elected
Belfast East	50.8 (UUP: 20.9; DUP: 29.9)	30.9	13.4	4.9
Belfast North	46.9 (DUP: 29.4; SF: 17.5)	33.8	15.0	4.3
Belfast South	17.2 (UUP: 17.2)	55.6	20.3	6.9
Belfast West	18.9 (SF: 18.9)	56.0	16	9.1
East Antrim	31.4 (UUP: 16.7; DUP: 14.7)	33.1	27.7	7.8
East Londonderry	No election at first count	63.7	29.3	7.0
Fermanagh South Tyrone	No election at first count	71.4	19.1	9.5
Foyle	46.5 (SDLP: 16.7; DUP: 15.0; SF: 14.8)	23.7	20.8	9.0
Lagan Valley	34.2 (UUP: 34.2)	41.2	16.6	8.0
Mid Ulster	36.8 (DUP: 18.5; SF 18.3)	44.3	10.2	8.7
Newry & Armagh	48.6 (DUP: 17.1; SF: 16.0; UUP: 15.5)	32.5	10.2	8.7
North Antrim	52.2 (DUP: 19.8; DUP: 17.9; UUP: 14.5)	30.5	10.2	7.1
North Down	No election at first count	64.4	27.3	8.3
South Antrim	18.9 (UUP: 18.9)	51.4	18.2	11.5
South Down	15.0 (DUP: 15.0)	53.3	23.0	8.7
Strangford	38.1 (DUP: 22.9; UUP: 15.2)	40.5	13.6	7.8
Upper Baan	21.1 (UUP: 21.1)	54.6	15.2	9.1
West Tyrone	29.2 (Ind: 14.8; SF: 14.4)	44.5	15.6	10.7
Northern Ireland	28.4	45.8	17.5	8.3

DUP=Democratic Unionist Party UUP=Ulster Unioinist Party; SF=Sinn Féin; SDLP=Social Democratic Labour Party; Ind=Independent

Table 2
'Further-Preference' Voters in Northern Ireland, by Party Support
Northern Ireland Assembly Elections, November 2003

Constituency	Number of voters giving first preference to party elected at first count				
	DUP	*UUP*	*SF*	*SDLP*	*Other Parties*
Belfast East	9254	6459	-	-	-
Belfast North	9276	-	5524	-	-
Belfast South	-	5389	-	-	-
Belfast West	-	-	6199	-	-
East Antrim	4544	5175	-	-	-
East Londonderry	-	-	-	-	-
Fermanagh South Tyrone	-	-	-	-	-
Foyle	6101	-	6036	6806	-
Lagan Valley	-	14,104	-	-	-
Mid Ulster	8211	-	8128	-	-
Newry & Armagh	8125	7347	7595	-	-
North Antrim	16630	6385	-	-	-
North Down	-	-	-	-	-
South Antrim	-	7066	-	-	-
South Down	6789	-	-	-	-
Strangford	8548	5658	-	-	-
Upper Baan	-	9158	-	-	-
West Tyrone	-	-	6019	-	6158
Total 'first count' votes	*77478*	*66741*	*39502*	*6806*	*6158*
Total first preference votes	*177944*	*156931*	*162758*	*117547*	*76848*
Share of further-preference votes in 1st preference votes	*44*	*43*	*24*	*6*	*8*
Share of 1st preference votes in total votes cast (692,028)	*25.71*	*22.67*	*23.52*	*16.98*	*11.12*
Number (and share) of Assembly seats	*30/108 (28)*	*27/108 (25)*	*24/108 (22)*	*18/108 (17)*	*9/108 (8)*

Table 3
Party Positions Under STV and Plurality

Constituency	Number of voters giving first preference to party elected at earliest count									
	DUP		UUP		SF		SDLP		Others	
	STV	PL	STV	PL	STV	PL	STV	PL	STV	PL
Belfast East	2	1	2	3	0	0	0	0	2	2
Belfast North	2	1	1	1	2	2	1	2	0	0
Belfast South	1	2	2	1	1	1	2	2	0	0
Belfast West	1	0	0	0	4	5	1	1	0	0
East Antrim	3	3	2	1	0	0	0	1	1	1
East Londonderry	2	3	2	1	1	1	1	1	0	0
Fermanagh and South Tyrone	1	1	2	2	2	2	1	1	0	0
Foyle	1	1	0	0	2	3	3	2	0	0
Lagan Valley	1	2	3	1	0	1	1	1	1	1
Mid Ulster	1	1	1	1	3	3	1	1	0	0
Newry & Armagh	1	1	1	1	3	3	1	1	0	0
North Antrim	3	3	1	1	1	1	1	1	0	0
North Down	2	2	2	3	0	0	0	0	2	1
South Antrim	2	2	2	2	0	1	1	0	1	1
South Down	1	1	1	1	2	2	2	2	0	0
Strangford	3	3	2	2	0	0	0	1	1	0
Upper Baan	2	2	2	1	1	2	1	1	0	0
West Tyrone	1	1	1	1	2	3	1	0	0	1
Total Seats	30	30	27	23	24	30	18	18	9	7

Figure 1
Inequality in the Distribution of Electoral Influence