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Abstract This article analyses the dynamics of an overlapping generations economy (Diamond, 1965) with pay-as-you-go financed public pensions and myopic expectations. It is shown that large PAYG pensions may trigger economic fluctuations depending on the mutual relationship between technology and preference parameters. Our findings constitute a policy warning about the size of social security and provide another explanation of the occurrence of persistent cycles.

Keywords Myopic foresight; PAYG pensions; Stability; OLG model

JEL Classification C62; H55; J26

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1. Introduction

Social security in many developed countries is based on pay-as-you-go (PAYG) public pensions. Motivated by the thrift of ageing on the viability of the widespread PAYG schemes in the long-run, social security reforms are currently high in the political agenda. While a growing body of economic literature dealing with the relationship between pensions, fertility, longevity and economic growth has been developed in the last decades (see, amongst many others, Zhang et al., 2001, 2003; van Groezen et al. 2003; Pecchenino and Pollard, 2005), less attention has been paid to the dynamical effects of the PAYG systems in the overlapping generations (OLG) context.

Moreover, while the effects of higher longevity as a threat for the sustainability of public pensions has been extensively debated, its role in determining whether and how PAYG pensions can affect economic stability has not been so far investigated. In this paper we show that if, as commonly retained, a rise in longevity can threaten the balancing of the PAYG budget,[1] it can favour, however, the stability of the economy.

The aim of this paper, therefore, is to provide a stability analysis of the conventional general equilibrium OLG economy with PAYG pensions, showing the conditions under which economic cycles can occur and also how the latter are affected by the size of the pension system as well as by other economic variables. In particular, we focus on the PAYG defined-contribution scheme,[2] which is the most largely investigated in literature and also seems to be increasingly implemented in

[1] By passing, we note that Fanti and Gori (2008) showed that in many cases a rise in longevity may favour the viability of the PAYG plan in the long run.

[2] In order to preserve solvency, a government that wishes to redistribute across generations with a PAYG pensions needs to adjust either contribution rates (defined-benefit scheme) or pension promises (defined-contribution scheme).

the recent years.[3] We contribute some findings to the existing literature on OLG economies and social security. First, we find that the size of the PAYG defined-contribution scheme may be responsible of the existence of endogenous cycles, adding another possible explanation to the cyclical behaviour in OLG models.[4] Second, we show that countries with a high capital share (e.g., Italy, Japan and Spain) may support larger PAYG systems than countries with low capital shares (e.g., Germany and the US) without destabilising the economy.[5] Third, pensioners in countries with a relatively high individual degree of thriftiness as well as with long-lived agents can receive a more generous pension arrangement than pensioners in countries where individuals are relatively impatient and adult mortality is much larger.

As is known, cyclical behaviour can occur in many-good OLG models (Grandmont, 1985) as well as in the one-good Diamond-type OLG context (Farmer, 1986; Reichlin, 1986). On the one hand, however, a low value of the elasticity of substitution in production is required (in particular, a value well below unity) in order to make cycles possible.[6] On the other hand, with myopic foresight, the steady state equilibrium may be oscillatory and exhibit deterministic complex cycles (Michel and de la Croix, 2000, de la Croix and Michel, 2002; Fanti and Spataro, 2008), but only

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[3] As noted by Wagener (2003), several countries recently switched to a defined-contribution scheme (e.g., Sweden), or chose the fixed-contribution option when newly designing their pension system in the late 1990s, such as Latvia and Poland, or at least moved into that direction (e.g., Italy and Germany).

[4] Wagener (2003) investigates a PAYG defined-replacement-ratio scheme (i.e., pensions as a given fraction of wages during working age) with perfect foresight, and shows hat changes in the replacement rate may expose the economy to periodic or unstable dynamics, because – different from a defined-contribution system – the defined-benefit system is characterized by a second order difference equation which is “intrinsically” more prone to generate unstable dynamics. Different from Wagener (2003), however, in this paper we focused on a defined-contribution PAYG system with myopic foresighted individuals also to investigate the effects of longevity on the stability of the economy.

[5] For recent estimates of the capital share in several countries, see, e.g., Jones (2003) and Rodriguez and Ortega (2006).

[6] Reichlin (1986) discussed the Leontief case (with no substitutability), while Farmer (1986) showed that cycles occur only when technologies exhibit lower factor substitutability than the Cobb-Douglas function and discussed the CES example.

when the inter-temporal elasticity of substitution in the utility function is higher than unity (i.e., higher than the inter-temporal elasticity of substitution in the Cobb-Douglas utility function).

Therefore, although it is well known that OLG economies with myopic expectations and elasticity of substitution in the production and in the utility functions relatively low and high, respectively, may show cyclical (and even chaotic) dynamics, in this paper we show that the size of PAYG pensions may play a crucial role in determining the stability of the economy even in a double Cobb-Douglas context and, in particular, large PAYG systems may cause persistent cycles. This is, to the best of our knowledge, a novel example of the possibility of cyclical behaviour in OLG models.

The remainder of the paper is organised. In Section 2 we present the model. In Section 3 we analyse the dynamics of the economy showing that the size the PAYG system may trigger economic fluctuations showing also an example of chaotic dynamics. Section 4 concludes.

2. The model

2.1. Individuals

Consider a two-period overlapping generations economy (Diamond, 1965) with stationary population and identical individuals.[7] In the first period of life (working period) young individuals belonging to generation t (N_t) are endowed with one unit of time supplied inelastically on the labour market, while receiving wage income at the competitive rate w_t. This income is used to consume, to save and to support material consumption of the elderly (through a public PAYG-based pension system). In the second period of life (retirement period) old-age individuals are retired and

[7] Assuming a constant rate of population growth does not alter any of the substantive conclusions of the model and, hence, it is not included here.

live on the proceeds of their savings (s_t) plus the expected interest accrued at the rate $r^e{}_{t+1}$ as well as on the expected pension benefit, $p^e{}_{t+1}$. Moreover, we suppose young individuals survive to the first period with (constant) probability $0<\pi<1$ (i.e., $1-\pi$ is the probability of dying after one period only). The existence of a perfect annuity market implies old survivors will benefit not only from their own past saving plus interest, but also from the saving plus interest of those who have deceased.

Each young born at time t must choose how much to save out of wage income so as to maximise a homothetic and separable (lifetime) utility function (U_t) defined over young-aged and old-aged consumption, $c_{1,t}$ and $c_{2,t+1}$, respectively, subject to the first and second period of life budget constraints. Assuming logarithmic preferences, the representative individual born at time t solves the following constrained maximisation programme:

$$\max_{\{s_t\}} U_t = \ln(c_{1,t}) + \pi\beta \ln(c_{2,t+1}), \tag{P}$$

subject to

$$\begin{aligned} c_{1,t} + s_t &= w_t(1-\theta) \\ c_{2,t+1} &= \frac{1+r^e{}_{t+1}}{\pi} s_t + p^e{}_{t+1} \end{aligned},$$

where $0<\theta<1$ is the payroll tax paid by the young contributors to finance pension arrangements to the current old-aged and $0<\beta<1$ is the subjective discount factor, that is, β represents the degree of individual (im)patience to consume over the life cycle. The higher β is the more individuals are patient and prefer to smooth consumption over the retirement period.

Maximisation of (P) thus gives the following saving function:

$$s_t = \frac{\pi\beta w_t(1-\theta)}{1+\pi\beta} - \frac{\pi p^e{}_{t+1}}{(1+\pi\beta)(1+r^e{}_{t+1})}. \tag{1}$$

2.2. Firms

As regards the production sector, we assume that firms are identical and act competitively on the market. The (aggregate) constant returns to scale technology is $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, where Y_t, K_t and $L_t = N_t$ are output, capital and the time-t labour input respectively, $A > 0$ represents a scale parameter and $0 < \alpha < 1$ is the output elasticity of capital. Defining $k_t := K_t / N_t$ and $y_t := Y_t / N_t$ as capital and output per worker, respectively, the intensive form production function may be written as $y_t = Ak_t^{\alpha}$. Assuming that capital fully depreciates at the end of each period and normalising the price of final output to unity, profit maximisation implies that factor inputs are paid their marginal products, that is:

$$r_t = \alpha A k_t^{\alpha-1} - 1, \tag{2}$$

$$w_t = (1-\alpha) A k_t^{\alpha}. \tag{3}$$

2.3. Government

The government redistributes between generations through an unfunded PAYG social security scheme. Therefore, in every period the benefit received by current pensioners is entirely financed by current workers whose working income is taxed away at the constant rate $0 < \theta < 1$. Therefore, the (per worker) government pension budget at t reads as

$$\pi p_t = \theta w_t, \tag{4}$$

the left-hand side being the social security expenditure and the right-hand side the tax receipt.

Now, inserting the one-period-forward pension accounting rule Eq. (4) into Eq. (1), the saving rate is

$$s_t = \frac{\pi \beta w_t (1-\theta)}{1+\pi \beta} - \frac{\theta}{1+\pi \beta} \cdot \frac{w^e{}_{t+1}}{1+r^e{}_{t+1}}. \tag{5}$$

From Eq. (5) it can readily be seen that savings are divided in two components: (*i*) the private saving component (the first term on the right-hand side of Eq. 5), which depends exclusively on the marginal willingness to save out of wage income (under the hypothesis of Cobb-Douglas utility), and (*ii*) the public pension component (the second term on the right-hand side of Eq. 5), which depends on both the expected pension benefit and expected interest rate. Notice that the lower adult mortality and the higher the individual subjective discount factor, the lower (higher) the relative weight of the public pension component (private saving component) in savings.

2.4. Equilibrium

Given the government budget Eq. (4) and knowing that $N_{t+1} = N_t$ (i.e. stationary population hypothesis), market-clearing condition in goods and capital markets is

$$k_{t+1} = s_t \, , \tag{6}$$

that is, the stock of capital installed at time $t+1$ is determined by the amount of resources saved in at time t. Combining Eqs. (5) and (6), equilibrium implies:

$$k_{t+1} = \frac{\pi \beta w_t (1-\theta)}{1+\pi \beta} - \frac{\theta}{1+\pi \beta} \cdot \frac{w^e{}_{t+1}}{1+r^e{}_{t+1}} \, . \tag{7}$$

Depending on whether individuals are perfect or myopic foresighted the dynamics of capital dramatically changes. Below we show that while the same unique positive steady-state is preserved in both cases (see Michel and de La Croix, 2000), (*i*) the dynamics of capital with rational expectations is always monotonic and convergent to the steady-state, and (*ii*) the dynamics of capital with myopic expectations may be non-monotonic and divergent to the steady-state even with Cobb-Douglas utility and production functions. It is worth noting that the non-monotonic behaviour of the economy relies on (*a*) the size of the (fixed) payroll tax paid by the young to finance pensions to the old, and (*b*) the mutual relationship between technology and preference parameters.

2.4.1. Perfect foresight

With perfect foresight, both the expected interest and wage rates depend on the future value of the stock of capital per young, that is

$$\begin{cases} 1+r^{e}{}_{t+1}=\alpha A k_{t+1}{}^{\alpha-1} \\ w^{e}{}_{t+1}=(1-\alpha)A k_{t+1}{}^{\alpha} \end{cases}, \tag{8}$$

Therefore, exploiting Eqs. (2), (3), (7) and (8), the dynamic equilibrium sequence of capital can be written as

$$k_{t+1}=\frac{\pi\beta(1-\theta)\alpha(1-\alpha)A}{\alpha(1+\pi\beta)+\theta(1-\alpha)}\cdot k_t{}^{\alpha}. \tag{9}$$

Steady-state implies $k_{t+1}=k_t=k^*$, so that:

$$k^*=\left[\frac{\pi\beta(1-\theta)\alpha(1-\alpha)A}{\alpha(1+\pi\beta)+\theta(1-\alpha)}\right]^{\frac{1}{1-\alpha}}. \tag{10}$$

2.4.2. Myopic foresight

With myopic foresight, both the expected interest and wage rates depend on the current value of the stock of capital per person, that is

$$\begin{cases} 1+r^{e}{}_{t+1}=\alpha A k_t{}^{\alpha-1} \\ w^{e}{}_{t+1}=(1-\alpha)A k_t{}^{\alpha} \end{cases}. \tag{11}$$

Using (2), (3), (7) and (11), the dynamic path of capital accumulation is now given by:

$$k_{t+1}=\frac{\pi\beta(1-\theta)(1-\alpha)A}{1+\pi\beta}k_t{}^{\alpha}-\frac{\theta}{1+\pi\beta}\cdot\frac{1-\alpha}{\alpha}k_t, \tag{12}$$

and the steady state is still determined by Eq. (10).

3. Local stability with myopic foresight

In this section we study the dynamics of the economy with PAYG pensions and myopic expectations. First, we study the case with generic utility and production functions. Second, we analyse the local stability properties of the double Cobb-Douglas economy.[8]

Assuming generic utility and production functions (as well as the existence and uniqueness of the steady state equilibrium), the market-clearing condition Eq. (7) can be written as

$$k_{t+1} = S\left\{ \underbrace{w_t(k_t), r^e{}_{t+1}(k_t)}_{private \quad saving}, \underbrace{P\left[w^e{}_{t+1}(k_t), r^e{}_{t+1}(k_t)\right]}_{public \quad pension} \right\}, \tag{13}$$

where the saving function S depends on both the private saving and public pension components. The private saving component is determined by the individual's willingness to save (which, in turn, depends on both the current wage and the expected interest rate), while the (PAYG) public pension function P depends on the expected values of both the wage and interest rates.[9]

Therefore, totally differentiating Eq. (13) with respect to k_t yields

$$\frac{dk_{t+1}}{dk_t} = \underbrace{\overbrace{\frac{\partial S}{\partial w_t}}^{+} \cdot \overbrace{\frac{\partial w_t}{\partial k_t}}^{+}}_{+} + \underbrace{\overbrace{\frac{\partial S}{\partial r^e{}_{t+1}}}^{+/-} \cdot \overbrace{\frac{\partial r^e{}_{t+1}}{\partial k_t}}^{-}}_{+/-} + \underbrace{\overbrace{\frac{\partial S}{\partial P}}^{-} \cdot \left[\overbrace{\frac{\partial P}{\partial w^e{}_{t+1}}}^{+} \cdot \overbrace{\frac{\partial w^e{}_{t+1}}{\partial k_t}}^{+} + \overbrace{\frac{\partial P}{\partial r^e{}_{t+1}}}^{-} \cdot \overbrace{\frac{\partial r^e{}_{t+1}}{\partial k_t}}^{-} \right]}_{-}, \tag{14}$$

where $\partial S / \partial w_t > 0$ represents the marginal propensity to save out of wage income, $\partial S / \partial r^e{}_{t+1}$ captures the effects of the expected rate of interest within the private component of total savings (both describe how aggregate saving varies along with the private marginal willingness to save),

[8] The (local) stability properties of a double Cobb-Douglas economy with PAYG pensions and perfect foresight is briefly presented in Appendix A. Different from the case with myopia, an economy with perfect foresighted individuals does not exhibit any interesting dynamical feature.

[9] With myopic expectations individuals will expect the future values of the wage and the interest rate to be exactly the same than those prevailing in the current period. Therefore, both private savings and the public pension function P depend exclusively on the level of the stock of capital per person installed in the current period.

and $\partial S / \partial P < 0$ reflects the (negative) public pension effect on S, i.e., the disincentive to save caused by public social security. In particular, during the working period the young know that the government will provide a benefit to support material consumption when old, and this crowds out private savings.

As regards the effects of the private saving component on the accumulation of capital, we note that with a generic utility function the individual propensity to save depends of course on the interest rate. As a consequence, a reduction in the expected rate of interest – due to a marginal increase in the level of the future stock of capital – positively (negatively) affects aggregate savings depending on whether the substitution effect is dominated by (dominates) the income effect, i.e., $\partial S / \partial r^{e}_{t+1} < 0$ ($\partial S / \partial r^{e}_{t+1} > 0$). The substitution effect describes the advantages or disadvantages to substitute consumption between youth and oldness, while the income effect captures the increased or reduced revenue from savings, other things being equal (a lower rate of interest, therefore, makes less profitable to substitute consumption bundles over the life cycle and decreases the direct revenue from savings).

From Eq. (14) we see that a marginal increase in the stock of capital in period t ambiguously affects the value of the capital stock installed in the subsequent period. Therefore, both monotonic and oscillatory dynamics are possible. In particular, the final effect is threefold.

As regards the private saving component there exist (*i*) a positive effect that increases the current wage, and thus the marginal propensity to save out of wage income and the capital stock will be installed in the future, and (*ii*) an ambiguous effect of the reduced expected rate of interest. Therefore, (*ii*.1) if the substitution effect dominates the income effect (i.e., $\partial S / \partial r^{e}_{t+1} > 0$), then a reduced expected interest rate – by decreasing the relative weight of the private saving component – tends to reduce the value of the stock of capital installed in the subsequent period; (*ii*.2) if the substitution effect is dominated by the income effect (i.e., $\partial S / \partial r^{e}_{t+1} < 0$), then a reduced expected interest rate – by increasing the relative weight of the private saving component – raises future capital. Moreover, as regards the public pension component, a marginal increase in the current stock of capital causes (*iii*) a negative effect due to the increased relative weight of the public pension function P in total savings, that reduces, in turn, the stock of capital installed in the future. In fact, the increased expected future marginal productivity of labour (the wage rate effect) as well as the reduced expected marginal productivity of capital (the interest rate effect) tend to enhance the expected pension benefit will be received by current pensioners, while also reducing the private marginal willingness to save.

If the substitution effect dominates the income effect, then the positive effect of the increased private marginal willingness to save on the accumulation of capital is counterweighted by the reduced interest rate will prevail on the future (the private saving component) as well as by the negative public pension effect (i.e., the relative weight of the public pension function is higher).

If the substitution effect is dominated by the income effect, then the higher marginal willingness to save out of wage income due to a rise k_t is reinforced by the effect played by the lower expected rate of interest so that this force is counterweighted only by the negative public pension effect. Hence, when the substitution effect dominates the income effect, oscillatory dynamics are more likely to occur than when the substitution effect is dominated by the income effect.

If both substitution and income effects exactly cancel out (i.e., $\partial S / \partial r^{e}_{t+1} = 0$ – Cobb-Douglas utility), then the final effect of a rise in the stock of capital at t on the level of the capital stock installed at $t+1$ exclusively depends on two counterbalancing forces: the positive private saving effect (that contributes to increase total savings due to the higher willingness to save when young) and the negative public pension effect (which, instead, crowds out savings due to the higher benefit received during the retirement period). Definitively, in the case of Cobb-Douglas utility the dynamics of capital with myopic expectations is monotonic (oscillatory) if the private saving component dominates (is dominated by) the public pension component.

Below we typify the dynamics of a double Cobb-Douglas OLG economy and showing that the introduction of a PAYG scheme when individuals are myopic foresighted may cause either (convergent) monotonic dynamics or (convergent or divergent) oscillatory dynamics. It is worth noting that the occurrence of temporary or permanent oscillations depends on the generosity of the unfunded social security system as well as on the mutual relationship between both technology and preference parameters. In particular, when the production technology is relatively labour-oriented, then a too large PAYG pensions (that is, a relatively high contribution rate) may destabilise the economy. Moreover, the higher the rate of longevity is and the more individuals prefer to smooth consumption over the retirement period (a higher subjective discount factor), the lower – given the increased relative importance of the private component in total saving – the risk of cyclical instability associated with the PAYG scheme is.

Analysis of Eqs. (10) and (12) gives the following proposition:

Proposition 1. *In a double Cobb-Douglas economy with public PAYG pensions and myopic expectations the dynamics of capital is the following.*

(1) Let $0 < \alpha < \alpha_4$ *hold. Then* $\theta_1 < \theta_2 < 1$, *and*

(1.1) if $0 < \theta < \theta_1$, *the dynamics of capital is monotonic and convergent to* k^*;

(1.2) if $\theta_1 < \theta < \theta_2$, *the dynamics of capital is oscillatory and convergent to* k^*;

(1.3) if $\theta = \theta_2$, *a flip bifurcation emerges;*

(1.4) if $\theta_2 < \theta < 1$, *the dynamics of capital is oscillatory and divergent to* k^*.

(2) Let $\alpha_4 < \alpha < \alpha_2$ *hold. Then* $\theta_1 < 1$, $\theta_2 > 1$, *and*

(2.1) if $0 < \theta < \theta_1$, *the dynamics of capital is monotonic and convergent to* k^*;

(2.2) if $\theta_1 < \theta < 1$, *the dynamics of capital is oscillatory and convergent to* k^*.

(3) Let $\alpha_2 < \alpha < 1$ *hold. Then* $\theta_2 > \theta_1 > 1$*, and the dynamics of capital is monotonic and convergent to* k^* *for any* $0 < \theta < 1$*,*

where

$$\theta_1(\alpha,\beta,\pi) = \theta_1 := \frac{\alpha^2}{(1-\alpha)^2}(1+\pi\beta), \tag{15}$$

$$\theta_2(\alpha,\beta,\pi) = \theta_2 := \frac{\alpha(1+\alpha)}{(1-\alpha)^2}(1+\pi\beta) = \theta_1\frac{1+\alpha}{\alpha}, \tag{16}$$

$$\alpha_2 = \alpha_2(\beta,\pi) := \frac{-1+\sqrt{1+\pi\beta}}{\pi\beta} > 0, \tag{17}$$

$$\alpha_4 = \alpha_4(\beta,\pi) := \frac{-(3+\pi\beta)+\sqrt{\pi^2\beta^2+10\pi\beta+9}}{2\pi\beta} > 0. \tag{18}$$

Proof. See Appendix B.

From Proposition 1 the following remark can be derived.

Remark 1. *The monotonic dynamics of capital in a double Cobb-Douglas PAYG-based economy with myopic expectations is always convergent to the stationary state (i.e., the so-called saddle node bifurcation can never occur, or, in other words, the economy may loose stability only through oscillations).*

Proposition 1 shows that when labour is relatively important in production, large PAYG systems may imply oscillatory movements and also trigger the occurrence of persistent cycles. In particular, two different threshold values of the contribution rate exist that discriminates between (*i*) monotonic and non-monotonic regions, and (*ii*) stable and unstable movements exclusively within the non-monotonic region. If the payroll tax paid by the young contributors is lower (higher) than the threshold θ_1, then the public pension component in total savings is dominated by (dominates) the private component and thus the dynamics path of capital accumulation exhibits monotonic (oscillatory) movements. Rising further the contribution rate, however, may destabilise the economy (this is due to the fact that the relative weight of the public pension component in total savings is higher. This happens when the flip bifurcation value of the payroll tax θ_2 is exceeded).

The following Figure 1 illustrates in the space (α,θ) and for a given value of the rate of longevity and the individual subjective discount factor, the locus which discriminates between monotonic and oscillatory regions (θ_1) as well as the period-doubling flip bifurcation locus (θ_2), which, instead, discriminates between stable and unstable regions. The figure clearly shows that the higher is the distributive capital share the lower is the risk of cyclical instability. Moreover, a rise in either the individual subjective discount factor or life expectancy, or both, shifts upward both loci and thus tends to stabilise the economy shrinking the size of the cyclically unstable region.[10]

[10] We recall that $\alpha_4 < 1/3$ and $\alpha_2 < 1/2$ hold for any $\pi, \beta \in (0,1)$. For the sake of brevity, we do not display here the loci's shifts upward following a rise either in the individual subjective discount factor or in life expectancy.

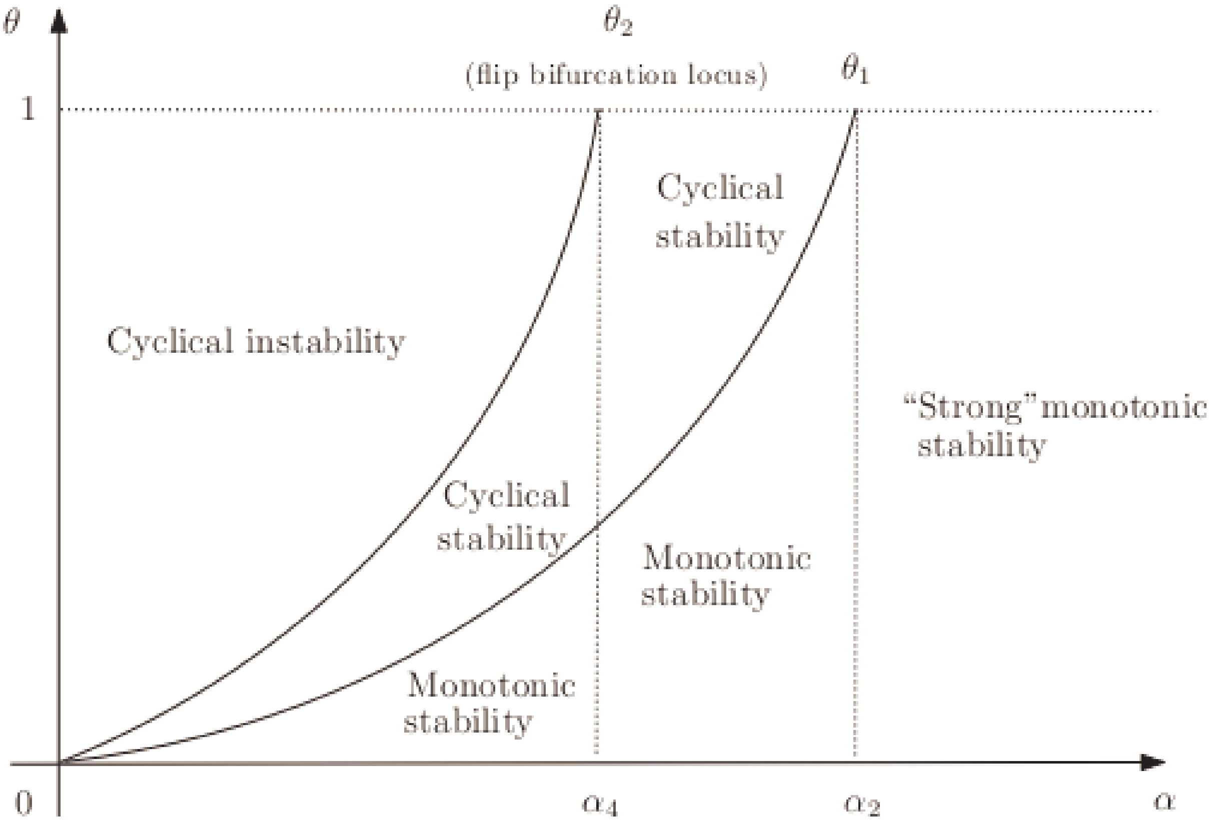

Figure 1. Stability and instability regions in the space (α, θ).

Therefore, countries with a relatively high capital share in production as well as with patient individuals (that ascribe a large enough importance to the private saving component rather then to the public pension component and thus prefer to smooth consumption over the retirement period), may increase the size of the contribution rate paid by the young to fund the benefits to retired people without generating temporary or permanent oscillations. In these countries large PAYG systems do not ever constitute a peril for the economic stability. In contrast, countries with a relatively low distributive capital share and with a low degree of thriftiness (i.e., individuals are impatient and prefer to consume more today rather than tomorrow) are much more prone to economic instability when the government rises the contribution rate to finance PAYG pensions.

In the following proposition we clarify the role played by both the degree of individual thriftiness and the rate of longevity on the stability of the economy.

Proposition 2. *Let individuals be myopic foresighted. Then a rise in either life expectancy or the degree of individual thriftiness, or both, tends to stabilise the economy.*

Proof. The proof can easily be derived by differentiating Eqs. (15) and (16) with respect to π and β, that is $\frac{\partial \theta_1}{\partial \pi} = \frac{\alpha^2}{(1-\alpha)^2}\beta > 0$, $\frac{\partial \theta_1}{\partial \beta} = \frac{\alpha^2}{(1-\alpha)^2}\pi > 0$, $\frac{\partial \theta_2}{\partial \pi} = \frac{\alpha(1+\alpha)}{(1-\alpha)^2}\beta > 0$ and $\frac{\partial \theta_2}{\partial \beta} = \frac{\alpha(1+\alpha)}{(1-\alpha)^2}\pi > 0$. Since the economy is unstable only through oscillations, then a rise in either life expectancy or in the degree of individual thriftiness increases the value the contribution rate beyond which the dynamics of capital is displays oscillatory movements as well as the flip bifurcation value of the payroll tax. This causes a reduction in both the width of the oscillatory region and the width of the cyclical unstable region. **Q.E.D.**

In Figures 2 and 3 we illustrate Proposition 2 showing an example of the different non-monotonic behaviour when adult mortality is relatively high (Figure 2) and low (Figure 3). As an example, we take the following parameter values for both figures: $A = 10$, $\alpha = 0.20$, $\beta = 0.50$. Moreover, we choose $\pi = 0.30$ (Figure 2) and $\pi = 0.90$ (Figure 3). This parameter sets generate the following flip bifurcation values of the contribution rate: $\theta_2 = 0.4312$ (when $\pi = 0.30$) and $\theta_4 = 0.5437$ (when $\pi = 0.90$). Then we take $\theta = 0.53$. The cobweb depicted in Figure 2 shows the non-monotonic divergent dynamics of capital when the rate of longevity is relatively low, while the cobweb in Figure 3 displays the convergent non-monotonic dynamics of capital when the rate of longevity is relatively high. The figures clearly show that a low adult mortality acts as an economic stabiliser.

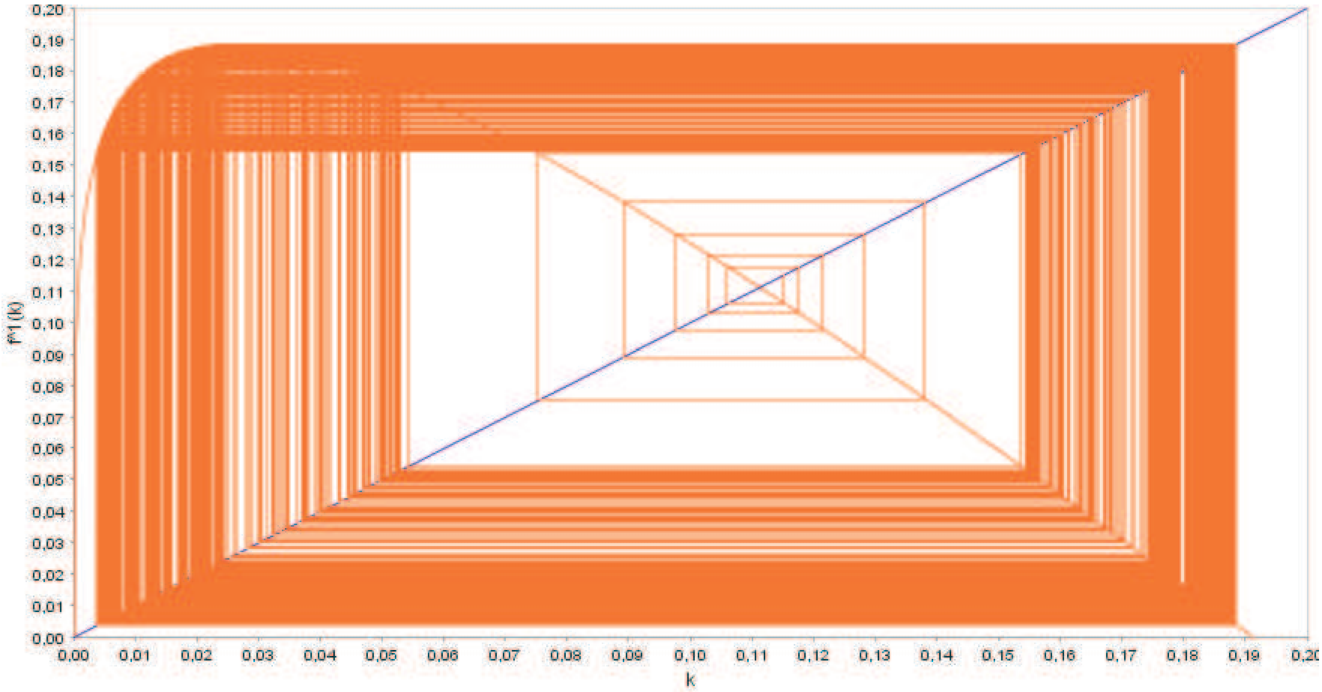

Figure 2. Case $\pi = 0.30$. A pictorial view of cyclical instability ($k^* = 0.1111$, $k_0 = 0.108$).

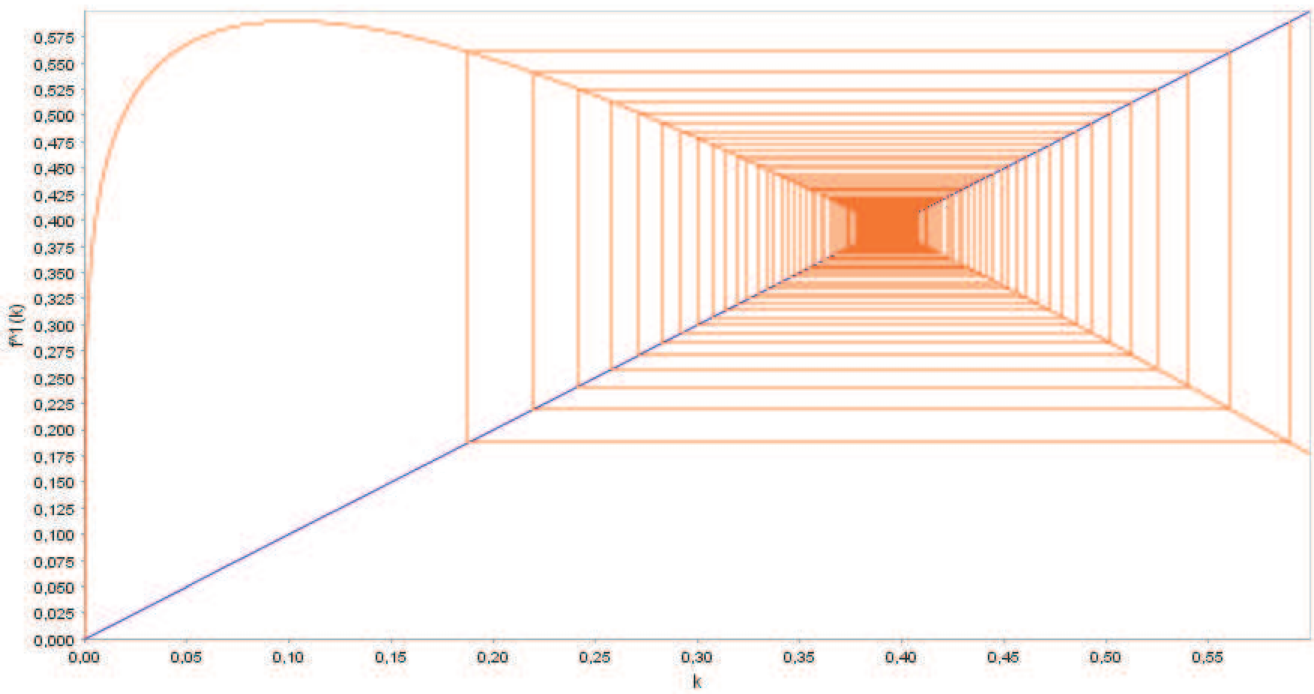

Figure 3. Case $\pi = 0.90$. A pictorial view of cyclical stability ($k^* = 0.3932$, $k_0 = 0.108$).

This model also shows deterministic chaos. Figure 4 depicts the bifurcation diagram for the parameter θ (which lies on the horizontal axis). The parameter values are the same as in Figure 2. On the vertical axis we show the limit points of the equilibrium sequence of capital. When the contribution rate is relatively low (below 0.4312) a unique limit point exists. When $\theta = 0.4312$ a period doubling bifurcation emerges. Then period doubling bifurcations appear more rapidly after $\theta = 0.5125$. Finally, a further increase in the contribution rate brings the economy into the chaotic region. This is an example of the complex dynamics generated by the large PAYG systems in an OLG Cobb-Douglas economy.

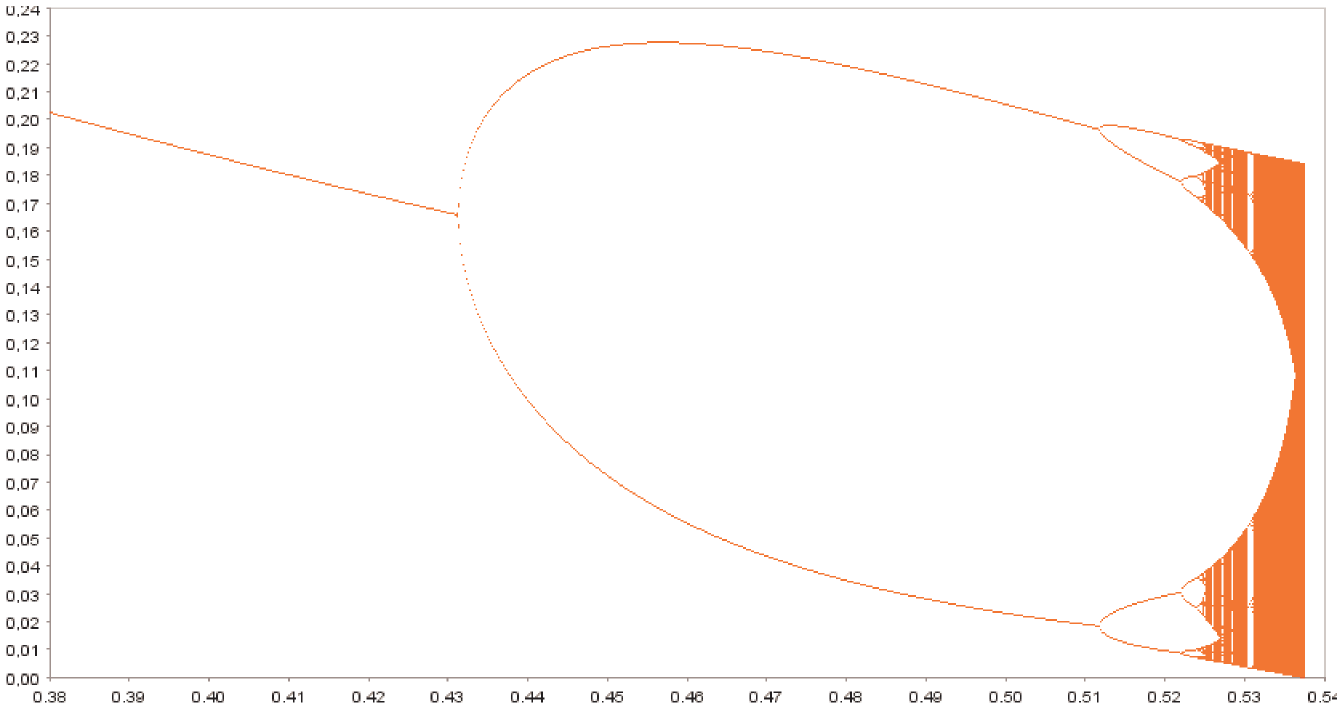

Figure 4. Bifurcation diagram for θ.

4. Conclusions

We analysed the dynamics of an OLG economy with PAYG (defined-contribution-typed) pensions and myopic foresighted individuals.

We showed that countries with a relatively capital-oriented technology and a relatively high degree of parsimony (such as Italy) may support larger public pension systems without perils for the economic stability than countries where technology is relatively labour-oriented and individuals have a low degree of parsimony (such as the US).

Our results have a twofold interpretation: (*i*) constitute a policy warning about the risks of (cyclical) instability caused by the widespread and generous PAYG pension schemes in presence of realistic myopia of individuals, and (*ii*) they represent a further explanation of economic cycles in OLG economies.

Appendix A

We briefly show here that with perfect foresight the dynamics of a PAYG-based Cobb-Douglas OLG economy cannot be cyclical.

Proposition A.1. *In a Cobb-Douglas PAYG-based economy with perfect foresighted individuals the dynamics of capital is always monotonic and convergent to* k^* *irrespectively of the size of the PAYG system.*

Proof. Differentiating (9) with respect to k_t and using (10) we find:

$$\left.\frac{\partial k_{t+1}}{\partial k_t}\right|_{k_t=k^*} = \alpha \frac{\pi\beta(1-\theta)\alpha(1-\alpha)A}{\alpha(1+\pi\beta)+\theta(1-\alpha)}\left(k^*\right)^{\alpha-1} = \alpha .$$

Therefore, $0 < \left.\frac{\partial k_{t+1}}{\partial k_t}\right|_{k_t=k^*} < 1$ for any $0 < \theta < 1$. **Q.E.D.**

Appendix B

Proof of Proposition 1.

Differentiating Eq. (12) in Section 2.4.2. with respect to k_t and evaluating it at the steady-state gives:

$$\left.\frac{\partial k_{t+1}}{\partial k_t}\right|_{k_t=k^*} = \alpha \frac{\pi\beta(1-\theta)(1-\alpha)A}{1+\pi\beta}\left(k^*\right)^{\alpha-1} - \frac{\theta}{1+\pi\beta}\cdot\frac{1-\alpha}{\alpha}. \tag{B1}$$

Using Eq. (10) in Section 2.4.1., Eq. (B1) becomes

$$\left.\frac{\partial k_{t+1}}{\partial k_t}\right|_{k_t=k^*} = \alpha - \frac{\theta}{1+\pi\beta}\cdot\frac{(1-\alpha)^2}{\alpha}. \tag{B2}$$

Monotonic and non-monotonic dynamics

The condition $\left.\frac{\partial k_{t+1}}{\partial k_t}\right|_{k_t=k^*} \underset{<}{\overset{>}{-}} 0$ implies

$$\alpha - \frac{\theta}{1+\pi\beta}\cdot\frac{(1-\alpha)^2}{\alpha} \underset{<}{\overset{>}{-}} 0 \Leftrightarrow \theta \underset{>}{\overset{<}{-}} \theta_1, \tag{B3}$$

where $\theta = \theta_1$ (defined by Eq. 15 in Section 3.) represents the value of the contribution rate below (beyond) which the dynamics of capital is monotonic (oscillatory). In particular, $\theta_1 < 1$ ($\theta_1 > 1$) for any $0 < \alpha < \alpha_2$ ($\alpha_2 < \alpha < 1$).

$\theta_1 < 1$ implies $\alpha_1 < \alpha < \alpha_2$, where

$$\alpha_1 = \alpha_1(\beta,\pi) := \frac{-1-\sqrt{1+\pi\beta}}{\pi\beta} < 0, \tag{B4}$$

$$\alpha_2 = \alpha_2(\beta,\pi) := \frac{-1+\sqrt{1+\pi\beta}}{\pi\beta} > 0. \tag{B5}$$

Since $\alpha_1 < 0$ it can be automatically ruled out. Notice also that $0 < \alpha_2 < 1/2$ for any β and π.

Now, $\left.\frac{\partial k_{t+1}}{\partial k_t}\right|_{k_t=k^*} < 1$ gives

$$\alpha - \frac{\theta}{1+\pi\beta} \cdot \frac{(1-\alpha)^2}{\alpha} < 1 \Rightarrow \theta > -\frac{\alpha}{1-\alpha}(1+\pi\beta). \tag{B6}$$

Therefore, in the case of monotonic behaviour, the dynamics of capital is always convergent to the stationary state, that is, $0 < \frac{\partial k_{t+1}}{\partial k_t}\Big|_{k_t = k^*} < 1$.

Stability and instability analysis in the case of non-monotonic dynamics

The condition $\frac{\partial k_{t+1}}{\partial k_t}\Big|_{k_t = k^*} \gtrless -1$ implies

$$\alpha - \frac{\theta}{1+\pi\beta} \cdot \frac{(1-\alpha)^2}{\alpha} \gtrless -1 \Rightarrow \theta \lessgtr \theta_2, \tag{B7}$$

where $\theta = \theta_2 > \theta_1$ (defined by Eq. 16 in Section 3.) represents the flip bifurcation value of the contribution rate, that is, the value of θ below (beyond) which the equilibrium with oscillatory dynamics is stable (unstable). In particular, $\theta_2 < 1$ ($\theta_2 > 1$) for any $0 < \alpha < \alpha_4$ ($\alpha_4 < \alpha < 1$), with $\alpha_4 < \alpha_2$.

$\theta_2 < 1$ implies $\alpha_3 < \alpha < \alpha_4$, where

$$\alpha_3 = \alpha_3(\beta, \pi) := \frac{-(3+\pi\beta) - \sqrt{\pi^2\beta^2 + 10\pi\beta + 9}}{2\pi\beta} < 0, \tag{B8}$$

$$\alpha_4 = \alpha_4(\beta, \pi) := \frac{-(3+\pi\beta) + \sqrt{\pi^2\beta^2 + 10\pi\beta + 9}}{2\pi\beta} > 0. \tag{B9}$$

Since $\alpha_3 < 0$ it can be automatically ruled out. Notice also that $0 < \alpha_4 < 1/3$ for any β and π.

Therefore, if $0 < \alpha < \alpha_4$ then $\theta_1 < \theta_2 < 1$, and $0 < \frac{\partial k_{t+1}}{\partial k_t}\Big|_{k_t = k^*} < 1$ for any $0 < \theta < \theta_1$, $-1 < \frac{\partial k_{t+1}}{\partial k_t}\Big|_{k_t = k^*} < 0$ for any $\theta_1 < \theta < \theta_2$, $\frac{\partial k_{t+1}}{\partial k_t}\Big|_{k_t = k^*} = -1$ if and only if $\theta = \theta_2$, and $\frac{\partial k_{t+1}}{\partial k_t}\Big|_{k_t = k^*} < -1$ for any $\theta_2 < \theta < 1$.

If $\alpha_4 < \alpha < \alpha_2$ then $\theta_1 < 1$, $\theta_2 > 1$, and $0 < \left.\frac{\partial k_{t+1}}{\partial k_t}\right|_{k_t = k^*} < 1$ for any $0 < \theta < \theta_1$ and $-1 < \left.\frac{\partial k_{t+1}}{\partial k_t}\right|_{k_t = k^*} < 0$ for any $\theta_1 < \theta < 1$.

If $\alpha_2 < \alpha < 1$ then $\theta_2 > \theta_1 > 1$, and $0 < \left.\frac{\partial k_{t+1}}{\partial k_t}\right|_{k_t = k^*} < 1$ for any $0 < \theta < 1$. **Q.E.D.**

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