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Abstract

This paper analyzes a T-stage model of oligopoly where firms build up capacity and conclude forward sales in stages $t < T$, and they choose production quantities in $t = T$. We consider the case of n firms with asymmetric marginal costs. In the two-stage game, the set of outcomes is a quasi-hyperrectangle including Cournot, Allaz-Vila, and all two-stage Stackelberg outcomes. In general, it consists of $T - 1$ such hyperrectangles where the lower bound approaches the Bertrand outcome as T tends to infinity. In the limit, a range of outcomes stretching from Cournot via Stackelberg to Bertrand can result in equilibrium, i.e. the mode of competition is entirely endogenous.

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Keywords: forward sales, capacity precommitment, Cournot, Stackelberg, Bertrand

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1 Introduction

This paper analyzes a T-stage model of oligopoly with the following move structure. In the first $T-1$ stages, firms pre-build capacity and conclude forward contracts. In the final stage, the firms set quantities. Capacities can still be extended in the final stage, but only at incremental costs. Eventually, the market clears in Cournot fashion, with linear inverse demand. The set of equilibrium outcomes is characterized completely for n-firm oligopolies with asymmetric marginal costs. In the two-stage case, it can be characterized as a hyperrectangle in the space of first-order conditions. It corresponds with a quasi-hyperrectangle in the space of payoff profiles containing the Cournot outcome, all two-stage Stackelberg outcomes, and the outcome of Allaz and Vila's (1993) model of forward trades. In general, it is the union of $T-1$ hyperrectangles with a constant upper bound and a lower bound converging to the Bertrand outcome as T tends to infinity.

Our results show that the mode of competition—Cournot, Bertrand, Stackelberg, or Allaz-Vila—may be entirely endogenous in oligopoly in the sense that all these modes are self-sustaining in ex-ante equivalent industries. If firms anticipate Cournot, then they are best off playing according to Cournot, if firms anticipate Stackelberg (with a given leader-follower assignment), then Stackelberg results, and so on. The firms' anticipations may be given by historical standards.

The analysis generalizes several existing models and results of a literature reaching back to Saloner (1987). Saloner analyzed a Cournot duopoly with two production periods and showed that a one-dimensional manifold including Cournot and Stackelberg outcomes may result in equilibrium. Romano and Yildirim (2005) generalize the result and label this phenomenon the "endogeneity of Cournot and Stackelberg equilibria." This "endogeneity" shows that (i) industries need not converge to Cournot equilibrium and non-cost-related size differences may be persistent in equilibrium, and (ii) Stackelberg leadership can be sustained without asynchronous timing and without retaliations against deviations of followers (i.e. in stationary equilibria of repeated games).[1] Pal (1991), however, showed that Saloner's result does not continue to hold if production costs are

[1]Another branch of literature, including e.g. Hamilton and Slutsky (1990), Robson (1990), and van Damme and Hurkens (1999), studies endogenous timing in duopoly. As Matsumura (1999) shows, endogenous Stackelberg does typically not result if there are more than two firms, and in general, models of endogenous timing are restrictive in the sense that firms can produce only in one of two or more initially feasible periods. Romano and Yildirim (2005) discuss this in more detail.

different between the two periods, i.e. the endogeneity seems degenerate. Aside from this, there are open questions as to how the endogeneity generalizes from duopoly to oligopoly, whether it applies if firms choose capacities rather than production quantities in early periods, and in which circumstances it may comprise other modes of competition besides Cournot and Stackelberg (e.g. Bertrand and Allaz-Vila), as well.

To address these open questions, our model combines the notion of production timing analyzed by Saloner (1987) with the notion of sales timing (forward trades) analyzed by Allaz and Vila (1993). The basic idea is that if input can be bought forward (*capacity pre-building* or *production timing*), then output may also be sold forward (*sales timing*), and vice versa. Allaz and Vila consider T-stage games where the firms may sell forward (some of) their eventual output in stages $t < T$, they decide how much to produce in stage $t = T$, and eventually the market clears in Cournot fashion. In the unique equilibrium, competition is more intense than in the Cournot model and it converges to Bertrand as T tends to infinity.[2] We extend the Allaz-Vila model by assuming that in all stages where q' units of output can be sold forward, it is also possible to buy forward the input factors to produce some q'' units of output.

Following Kreps and Scheinkman (1983), Saloner (1987), and many subsequent studies, we assume that the costs of pre-building capacity are sunk in the short term. This implies that capacity is either constant or accumulates along the path of play and relates the present study to the aforementioned studies of two-period Cournot models and "games of accumulation" (Romano and Yildirim, 2005). The difference to these studies is that they consider games of quantity accumulation while we consider games of capacity accumulation. In our model, that is, quantity is entirely flexible until the production period is reached. We are going to show that the results of Saloner and Pal continue to hold as "quantity precommitment" gets exchanged with "capacity precommitment," and also as oligopolistic interactions of more than two firms are considered.

Our main results are as follows. The one-dimensional manifold observed by Saloner (1987) is an n-dimensional manifold in the n-firm oligopoly with both production

[2]Independently, Bolle (1993) and Powell (1993) reached similar conclusions for $T = 2$, and to name a few subsequent studies, Ferreira (2003) derives a Folk theorem for the case that there is no final trading period, Mahenc and Salanié (2004) show that forward trades weaken competition if firms compete in prices (which relates to the fact that price competition exhibits strategic substitutes), and Liski and Montero (2006) show that forward trades simplify penal strategies and tacit collusion in repeated oligopoly.

and sales timing. This manifold contains the Cournot outcome, the Allaz-Vila outcome, and many Stackelberg outcomes, and as T tends to infinity, it additionally contains the Bertrand outcome. Note, however, that we are not going to derive a Folk theorem. The diversity of equilibrium outcomes is restricted to different modes of competition, i.e. tacit collusion cannot be rationalized in this manner and penal strategies are not required to sustain a particular mode.

Section 2 introduces the model and derives a few preliminary results. Section 3 reviews and extends benchmark models known from the literature. Section 4 derives the main result in the two-stage model, Section 5 generalizes it to $T > 2$, and Section 6 concludes. The proofs are relegated to the appendix.

2 Definition of the two-stage model and basic results

Notation Firms are denoted as $i \in N = \{1, \ldots, n\}$. In stage 1, they set capacities z_i and forward sales y_i. In stage 2, they set quantities x_i. The players act simultaneously in each stage, and the choices made in stage 1 are common knowledge in stage 2. The unit costs of pre-building capacity are $\gamma_i > 0$. In case a quantity $x_i > z_i$ is chosen in stage 2, the pre-built capacity is extended at unit costs $c_i \geq \gamma_i$. There are no costs of production besides the costs of capacity. The inverse demand function is $p(\mathbf{x}) = a - b \sum_{i \in N} x_i$. The market for forward trades is competitive and clears between periods 1 and 2 (i.e. before the actual quantities (x_i) are chosen). Throughout this paper scalar values and functions are set in italics, e.g. capacities z_i, vectors are set in boldface type, e.g. $\mathbf{z} = (z_i)_{i \in N}$, sets of scalars are denoted by capital letters, e.g. $Z_i \ni z_i$, and sets of vectors are denoted by capital letters set in boldface type, e.g. $\mathbf{Z} = \times_{i \in N} Z_i$.

Definition 2.1 (Game)**.** Strategy profiles are denoted as triples $(\mathbf{z}, \mathbf{y}, \mathbf{x}) = (z_i, y_i, x_i)_{i \in N}$ where for all $i \in N$: $z_i \in Z_i \subseteq \mathbb{R}_+$, $y_i \in Y_i \subseteq \mathbb{R}_+$, and $x_i : \mathbf{Z} \times \mathbf{Y} \rightarrow X_i \subseteq \mathbb{R}_+$. In stage 2 (the short term), the forward trades had been concluded and the forward trade price p^f is fixed. The short-term profit of i is, as a function of the $\mathbf{x}$ chosen by all players and $(\mathbf{z}, \mathbf{y})$,

$$\Pi_i^S(\mathbf{x}|\mathbf{z}, \mathbf{y}) = (x_i - y_i) \cdot \left(a - b \sum_j x_j\right) + p^f \cdot y_i - c_i \cdot \max\{x_i(\mathbf{z}, \mathbf{y}) - z_i, 0\} - \gamma_i z_i. \qquad (1)$$

Assuming perfect foresight and that profitable arbitrage is impossible in equilibrium, following Allaz and Vila (1993), the market price for forward trades equates with the anticipated market price conditional on the choices of $(\mathbf{z},\mathbf{y})$, i.e.

$$p^f(\mathbf{x}|\mathbf{z},\mathbf{y}) = a - b\cdot\sum_{j\in N} x_j(\mathbf{z},\mathbf{y}). \tag{2}$$

Substituting p^f in Eq. (1), the stage-1 (long term) profit function of $i\in N$ becomes

$$\Pi_i^L(\mathbf{z},\mathbf{y},\mathbf{x}) = x_i(\mathbf{z},\mathbf{y}) * p\big(\mathbf{x}|\mathbf{z},\mathbf{y}\big) - c_i\cdot\max\{x_i(\mathbf{z},\mathbf{y}) - z_i, 0\} - \gamma_i z_i. \tag{3}$$

We determine subgame-perfect equilibria (SPEs) in pure strategies.

Stage 2 analysis In stage 2, the choices of (z_i) and (y_i) are fixed, and the quantities (y_i) had been sold at the forward trade price p^f. The latter is therefore fixed in stage 2. For all $i\in N$, the stage 2 profits Π_i^S are continuous and concave in x_i, which implies existence and uniqueness of best-reply functions. Using the indicator $I_{x_i>z_i}$, which evaluates to 1 iff $x_i>z_i$, the partial derivatives with respect to x_i (for $x_i\neq z_i$) can be expressed as

$$\frac{\partial\Pi_i^S}{\partial x_i} = a - b\sum_j x_j - b(x_i - y_i) - c_i\cdot I_{x_i>z_i}, \qquad\qquad \frac{\partial^2\Pi_i^S}{\partial x_i^2} = -2b. \tag{4}$$

From this, best reply functions can be derived easily. Mutual best replies, i.e. stage 2 equilibria, solve a corresponding equation system, and existence of stage 2 equilibria follows from Brouwer's fixed point theorem. The best replies are only piecewise linear, however, which implies that uniqueness does not follow from standard arguments. Our first result establishes uniqueness of stage 2 equilibria for all $(\mathbf{z},\mathbf{y})$, and it provides an implicit characterization of the stage 2 choices (x_i^*) that is used further below. A simple closed-form characterization does not seem to be available.

Lemma 2.2. *Fix any profile $(\mathbf{z},\mathbf{y})$. The equilibrium quantities $x_i^*(\mathbf{z},\mathbf{y})$ are unique for all $i\in N$ and satisfy, using $r_i = a - b\left(2z_i - y_i + x_{-i}^*\right)$ and $x_{-i}^* = \sum_{j\neq i} x_j^*$,*

$$x_i^*(\mathbf{z},\mathbf{y}) = \begin{cases} z_i + \frac{r_i}{2b}, & \text{if } r_i < 0 \\ z_i, & \text{if } 0\le r_i\le c_i \\ z_i + \frac{r_i - c_i}{2b}, & \text{if } r_i > c_i. \end{cases} \tag{5}$$

The characterization of the stage 2 equilibrium rests on a profile of values denoted by $(r_i)\in\mathbb{R}^N$ that can be interpreted as follows. Firm i's revenue in stage 2 is

$$R_i^S = (x_i - y_i)\cdot\big(a - b\textstyle\sum_j x_j\big) + p^f\cdot y_i, \tag{6}$$

and r_i is i's marginal revenue in stage 2 at $x_i = z_i$ (in response to the opponents' equilibrium quantities). We may distinguish three cases. Firm i does not exploit the pre-built capacity ($x_i < z_i$) if $r_i < 0$, it just does so ($x_i = z_i$) if $r_i \in [0, c_i]$, and it extends capacity ($x_i > z_i$) if $r_i > c_i$. In turn, firm i equates marginal revenue with marginal costs only if $r_i \notin (0, c_i)$. At $x_i = z_i$, i.e. at the soft capacity constraint, the cost function is not differentiable, and the marginal revenue may assume any value in $r_i \in [0, c_i]$ in equilibrium.

The comparative statics of the stage 2 equilibrium are as follows. Varying the capacity z_i of firm i, we observe a *capacity effect*—a tendency to exactly use the pre-built capacity. In essence, this is a commitment effect due to having pre-built capacity. We have seen above that the choice $x_i = z_i$ maximizes stage 2 payoffs whenever the marginal revenue r_i is in the interval $[0, c_i]$. Now assume that r_i is in the interior of $[0, c_i]$. Small increments of z_i toward z_i' in stage 1 reduce r_i in stage 2, but as r_i remains positive, the quantity x_i will be chosen in stage 2 such that $x_i = z_i'$. Similarly, small decrements of z_i induce decrements of x_i. Hence the tendency to match the pre-built capacity. Varying the amount of forward trades y_i by firm i, we observe a *forward trade effect* (as described by Allaz and Vila, 1993). This is a commitment effect due to the possibility of forward trading. As y_i increases, the quantity that is left to be sold in stage 2 shrinks and hence the marginal revenue r_i increases. This induces a tendency to increase production in stage 2. In relation to the Cournot solution, the forward trade effect tends to induce supra-Cournot quantities.

Exploitation of pre-built capacities Let $MR_i^S = \partial R_i^S / \partial x_i$ denote i's short-term marginal revenue, and let $MR_i^L = \partial R_i^L / \partial z_i$ denote i's long-term marginal revenue. They can be defined as

$$MR_i^S = a - b\sum_j x_j - b(x_i - y_i) \qquad\qquad MR_i^L = a - b\sum_j x_j - bx_i. \tag{7}$$

The difference between MR_i^S and MR_i^L is the forward trade effect. It implies $MR_i^S \geq MR_i^L$ in general and $MR_i^S > MR_i^L$ if $y_i > 0$. If y_i is large enough, then $MR^L \leq \gamma_i$ and $MR^S > c_i$ may hold true simultaneously. In this case, firm i would be best off delaying capacity investments until stage 2. The next result shows that capacity investments are never delayed in equilibrium (if $c_i > \gamma_i$), and it also shows that firms do not build up excess capacity in equilibrium. That is, firms exactly exploit their pre-built capacity along the path of play in any SPE.

Lemma 2.3. *Fix any SPE* $(\mathbf{z},\mathbf{y},\mathbf{x})$. *For all* $i \in N$*, the quantity chosen along the equilibrium path satisfies* $x_i(\mathbf{z},\mathbf{y}) \geq z_i$*, and in case* $c_i > \gamma_i$ *it satisfies* $x_i(\mathbf{z},\mathbf{y}) = z_i$*.*

The case $c_i = \gamma_i$ is a little more complex. As the previous result suggests, capacity may be extended in stage 2 if $c_i = \gamma_i$. We will see below (Lemma 4.2), however, that the set of equilibrium outcomes is unaffected by this effect.

3 Interrelations between simplified two-stage models

The present section describes the range of equilibrium outcomes that result when the model is restricted in either of its dimensions. We thereby generalize a few results known from the literature to the case of n firms and asymmetric costs, but the main purpose of this section is to establish a representation of these outcomes in the space of first-order conditions. This alternative representation offers a simple (and novel) way to describe the interrelations between the various models and will be helpful in analyzing the general model further below.

We begin with the case that pre-built capacity cannot be extended in stage 2 (i.e. $c_i = \infty$ for all $i \in N$). The Cournot outcome results. This result is not novel, but it is stated for completeness. It shows that a necessary condition for the competition-enhancing effect of forward trades is that capacity can be extended after output had been sold forward. The possibility of forward trades does not induce the competition-enhancing effect when "only" quantity, as opposed to capacity, is variable in the production period.

Proposition 3.1 (Capacity cannot be extended in stage 2)**.** *Assume "sufficiently similar" marginal costs* (γ_i) *and* $c_i = \infty$ *for all* $i \in N$*. In the unique SPE, the Cournot outcome results, with the equilibrium price*

$$p^C = \frac{a + \sum_{j \in N} \gamma_j}{1+n}. \tag{8}$$

Second consider interactions where capacity cannot be pre-built, i.e. the framework of Allaz and Vila (1993). The following result generalizes Prop. 2.3 of Allaz and Vila to asymmetric costs and n players. We find that forward trades induce competitive behavior in the sense that the Allaz-Vila price is smaller than the Cournot price in general, i.e. $p^{AV} < p^C$. For our purpose, the closed-form characterization of the price in the n-firm framework is more important, however. For notational clarity, $c_i = \gamma_i$ is used here, since γ_i as used before is irrelevant when capacity cannot be pre-built.

Proposition 3.2 (Capacity cannot be pre-built)**.** *Assume "sufficiently similar" marginal costs* (γ_i)*,* $c_i = \gamma_i$ *for all* $i \in N$*, and* $Z_i = \{0\}$ *for all* $i \in N$*. In the unique SPE, the Allaz-Vila outcome results, with equilibrium price*

$$p^{AV} = \frac{a + n\sum_{i \in N} \gamma_i}{1+n^2}. \tag{9}$$

In light of Props. 3.1 and 3.2, let us now look at the common theme underlying these results. The first-order conditions in the Cournot models are

$$p - bz_i - \gamma_i = 0 \quad \forall i \in N, \tag{10}$$

using $p = a - b\sum_j z_j$ as the respective market price, and in reduced form, the first-order conditions in the Allaz-Vila model are

$$p - \frac{1}{n} \cdot bz_i - \gamma_i = 0 \quad \forall i \in N. \tag{11}$$

In general terms, these alternative sets of first-order conditions can be represented as

$$p - \lambda_i bz_i - \gamma_i = 0 \quad \forall i \in N \tag{12}$$

for appropriately chosen $(\lambda_i) \in \mathbb{R}^N_+$. The first-order conditions in the Cournot-model are obtained for $\lambda_1 = \cdots = \lambda_n = 1$, and the Allaz-Vila conditions correspond with $\lambda_1 = \cdots = \lambda_n = 1/n$. That is, we can represent models, and their respective first-order conditions, by profiles $(\lambda_i) \in \mathbb{R}^N_+$. To grasp the underlying idea, let us show how (λ_i) relates to conjectural derivatives. If q_i denotes the quantity of i and q_{-i} the aggregate quantity of i's opponents, and if players compete by choosing quantities in a market with inverse demand $P(q_i + q_{-i})$, then i first-order condition is (assuming constant marginal costs γ_i)

$$P - q_i \cdot P'(q_i + q_{-i}) \cdot \left(1 + \tfrac{dq_{-i}}{dq_i}\right) - \gamma_i = 0. \tag{13}$$

In our model, $P'(q) = -b$ applies, and hence we obtain $\lambda_i = 1 + \frac{dq_{-i}}{dq_i} = \frac{d(q_{-i}+q_i)}{dq_i}$. In words, λ_i measures how much the aggregate market quantity increases if i increases q_i by a unit. In the Cournot model, the aggregate quantity increases by a unit, too, but in the Allaz-Vila model, an increase of the amount of forward trades induces an increase of the own quantity which in turn crowds out the opponents' quantities. The ratio of the increase of the own quantity to the resulting increase of the overall quantity is $\lambda_i = 1/n$ in the Allaz-Vila model.

It is straightforward to verify that in the case of linear demand and constant marginal costs as assumed above, there is a unique equilibrium price and profit profile associated with each (λ_i), i.e. with each set of first-order conditions.

$$p = \frac{a + \sum_i \lambda_i^{-1} \gamma_i}{1 + \sum_i \lambda_i^{-1}}, \qquad \Pi_i = \frac{1}{\lambda_i b} \cdot \left(\frac{a - \gamma_i + \sum_j \lambda_j^{-1}(\gamma_j - \gamma_i)}{1 + \sum_j \lambda_j^{-1}} \right)^2. \tag{14}$$

Hence, it is sufficient to characterize models and their outcomes by the induced profile (λ_i) of conjectural derivatives. To further illustrate the resulting interrelations between models, let us next consider the following class of two-stage Stackelberg games.

Definition 3.3 (Two-stage Stackelberg games)**.** For any partition (N_1, N_2) of N, define the (N_1, N_2)-Stackelberg game as the two-round extensive form game of perfect information where all players $i \in N_1$ simultaneously move (choosing quantities) in round 1, and all players $j \in N_2$ do so in round 2.

Take a partition (N_1, N_2) of N, and consider the respective (N_1, N_2)-Stackelberg game. It is easy to see (see e.g. Lemma 4.4 below) that the first-order conditions of this (N_1, N_2)-Stackelberg game can be characterized as $\lambda_i = 1/(|N_2| + 1)$ for all first-movers $i \in N_1$ and as $\lambda_j = 1$ for all followers $j \in N_2$. That is, Cournot oligopolists have conjectural derivatives ($\lambda_i = 1$) that correspond with those of Stackelberg followers. Allaz-Vila oligopolists have conjectural derivatives that are equivalent to those of first movers in $(\{i\}, N \setminus \{i\})$-Stackelberg games where all opponents are followers.

In the last of our benchmark models, forward trades are impossible. The result that we report relates to the results that Saloner (1987) and Pal (1991) obtain for Cournot duopolies with two production periods. In essence, the relation of (c_i) and (γ_i) becomes relevant. Saloner showed that a continuum of outcomes containing the Stackelberg outcomes and the Cournot outcome may result if $c = \gamma$, and Pal showed that this kind of Cournot-Stackelberg endogeneity disappears as $c \neq \gamma$. We distinguish the cases (i) $c_i > \gamma_i$ for all i and (ii) $c_i = \gamma_i$ for all i and show that the same pair of outcomes can be established if we consider "capacity accumulation" rather than "quantity accumulation." That is, we show that Saloner's result continues to hold when quantity produced in stage 1 can be withheld from being sold on the market in stage 2. To my knowledge, this remained an open question ever since it had been raised by Saloner (1987, p. 186 f.).

Proposition 3.4 (Zero forward trades)**.** *Assume "sufficiently similar" marginal costs* (γ_i) *and* $Y_i = \{0\}$ *for all* $i \in N$.

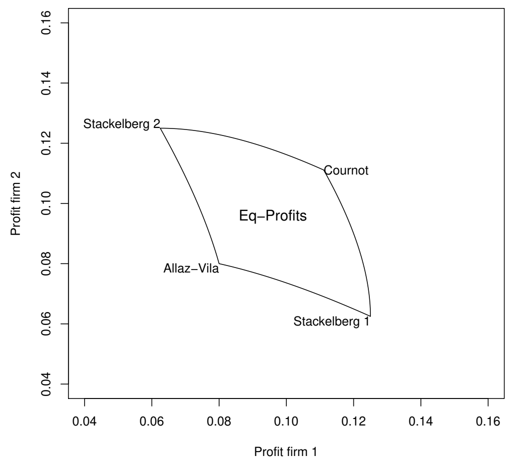
1. *If* $c_i > \gamma_i$ *for all* $i \in N$*, then the Cournot outcome results in the unique SPE.*

2. *If* $c_i = \gamma_i$ *for all* $i \in N$*, then an outcome* $\langle p, (\Pi_i)\rangle$ *can result if and only if there exists a partition* (N_1, N_2) *of* N *such that* $\langle p, (\Pi_i)\rangle$ *satisfies Eq.* (14) *for some* (λ_i) *where* $\lambda_i \in \left[\frac{1}{|N_2|+1}, 1\right]$ *for all* $i \in N_1$ *and* $\lambda_j = 1$ *for all* $j = N_2$.

Prop. 3.4 also shows how the Cournot-Stackelberg endogeneity generalizes from duopoly to oligopoly. In oligopoly, the set of equilibrium outcomes is a continuum containing the outcomes of all *two-stage* Stackelberg games (see Def. 3.3) and the Cournot outcome. The set is not convex in the payoff space, but it is the union of finitely many hyperrectangles in (λ_i)-space. Namely, it is the union, over all (N_1, N_2)-Stackelberg games, of the hyperrectangles containing the respective (N_1, N_2)-Stackelberg outcome and the Cournot outcome. For example, in three-player games, the set of outcomes is the union of six hyperrectangles (as there are six two-stage Stackelberg games between three players), where each hyperrectangle has a Stackelberg solution and the Cournot solution at its corner points.

Briefly, let us illustrate why a continuum of outcomes exists in case $c = \gamma$. Focus on the case of two players and consider the outcome associated with some (λ_1, λ_2) where $\lambda_1 \in [1/2, 1)$ and $\lambda_2 = 1$. In this case, $1/2 \leq \lambda_1 < 1$ implies that firm 1 chooses a capacity somewhere between that of a Stackelberg leader and that of a Cournot duopolist, and $\lambda_2 = 1$ implies that 2 plays the best response. In a pure Cournot framework, firm 1 would benefit by decreasing his quantity, but in the two-stage game considered here, firm 2 would respond by increasing the capacity in stage 2 in case $c_2 = \gamma_2$. Hence, decreasing capacity pays off for firm 1 only if it does so for a Stackelberg leader. The latter, however, cannot be the case if Eq. (12) holds true for $\lambda_1 \geq 1/2$. In turn, player 1 does not benefit from increasing capacity when Eq. (12) is satisfied for some $\lambda_1 \leq 1$. For, firm 2 does not respond to a capacity increase of 1 by decreasing his quantity in stage 2 as long as $\lambda_2 > 0$ (i.e. as long as i's marginal revenue is positive). To summarize: 1's incentives correspond with those of a Cournot duopolist with respect to capacity increases and with those of Stackelberg followers with respect to capacity decreases. Hence, λ_1 may attain any value in $[1/2, 1]$ in equilibrium.

Figure 1: Range of profit profiles that may result in equilibrium (two players, zero costs)



4 Analysis of two-stage games

To summarize the above benchmark results, the Cournot-Stackelberg endogeneity holds equally in both, games of quantity accumulation and games of capacity accumulation, but it seems to be degenerate in that $c_i = \gamma_i$ for all $i \in N$ is a necessary condition, and it connects all two-stage Stackelberg games with Cournot competition. The present section considers the general two-stage interaction with both production timing and sales timing, i.e. with both capacity accumulation and forward trades. Initially we focus on the "generic" case $c_i > \gamma_i$ for all i, the extension toward $c_i = \gamma_i$ for all i is covered below. The result is provided first.

Proposition 4.1. *Assume $c_i > \gamma_i$ for all $i \in N$, the (c_i) are sufficiently close to (γ_i), and the (γ_i) are sufficiently similar. A price $p \in \mathbb{R}$ and a payoff profile $\Pi \in \mathbb{R}^N$ can result in SPE if and only if there exists $(\lambda_i) \in \left[\frac{1}{n}, 1\right]^N$ such that p and Π satisfy Eq.* (14).

The set of equilibrium outcomes is the hyperrectangle, in (λ_i)-space, with the Cournot solution ($\lambda_i = 1$ for all i) and the Allaz-Vila solution ($\lambda_i = 1/n$ for all i) at its opposite corner points. It contains all two-stage Stackelberg outcomes and many intermediate outcomes. Note, however, that the set of equilibrium outcomes is only "quasi-hyperrectangular" in in the payoff space, i.e. neither convex nor hyperrectangular (see also Figure 1).

To gain intuition, let us outline the structure of the equilibria constructed in the proof (further equilibria exist, but they do not induce alternative outcomes). Fix $(\lambda_i) \in \left[\frac{1}{n}, 1\right]^N$ and find the unique capacities $(z_i)_{i\in N}$ such that

$$p - \lambda_i b z_i - \gamma_i = 0 \quad \forall i \in N, \tag{15}$$

at the respective market price $p = a - b\sum_i z_i$. See e.g. Eq. (39). Set $(y_i)_{i\in N}$ such that

$$p - b(z_i - y_i) - c_i = 0 \quad \forall i \in N. \tag{16}$$

Since $\lambda_i > 0$, this is possible even if $c_i > \gamma_i$.[3] All such strategy profiles $(\mathbf{z}, \mathbf{y})$ can be extended to SPEs by choosing appropriate (x_i). By Lemma 2.2, the respective (x_i) are unique in all subgames, and by Lemma 2.3, $x_i = z_i$ results along the path of play for all $i \in N$. Eq. (16) implies that the stage 2 marginal revenue satisfies $r_i = c_i$ for all $i \in N$. When any $i \in N$ deviates unilaterally by increasing z_i in stage 1, then he will be best off exploiting the extended capacity in stage 2 (his marginal revenue falls below c_i but remains positive). Anticipating this quantity increase after observing the capacity "increase" of i, the opponents' marginal revenues fall below marginal costs c_j, but they remain positive, too. Hence, the opponents' quantities are constant in response to i's capacity increase, and hence, i's capacity increase pays off if and only if it would pay off for a Cournot oligopolist. The latter applies iff

$$p - b z_i - \gamma_i \geq 0 \quad \forall i \in N. \tag{17}$$

Alternatively, i may cut capacity. The most profitable capacity cut implies that i simultaneously adjusts y_i so that he will not be best off extending capacity in stage 2 again. Regardless of y_i, however, the opponents' marginal revenues in stage 2 rise above c_j due to the capacity cut (i.e. due to correctly anticipating the quantity cut that follows), and hence they all respond by extending their capacities in stage 2. In turn, capacity cuts pay off if and only if a quantity cut pays for a Stackelberg leader to which all $n-1$ opponents respond by acting simultaneously. This applies iff

$$p - \frac{1}{n} b z_i - \gamma_i \leq 0 \quad \forall i \in N. \tag{18}$$

[3] To be precise, it is possible whenever $c_i \leq p^{AV}$ for all $i \in N$. Eq. (16) cannot be satisfied if $c_i > p$. Assuming $c_i = c$ for all i, $p^{AV} \leq c \leq p^C$ implies that the equilibrium price range is the interval $\left[c, p^C\right]$. If $c > p^C$, short-term capacity extensions become prohibitive in the sense of Prop. 3.1, and the Cournot outcome results in the unique SPE.

Since Eq. (15) is satisfied for some $\lambda_i \in \left[1/n, 1\right]$, neither Eq. (17) and Eq. (18) can be satisfied, i.e. neither capacity cuts nor capacity extensions pay off if $\lambda_i \in \left[1/n, 1\right]$.

The next result establishes that Prop. 4.1 extends to the degenerate case $c_i = \gamma_i$ for all $i \in N$.

Lemma 4.2. *Assume (γ_i) are sufficiently similar. The set outcomes that can be sustained in equilibrium is upper hemicontinuous in (c_i) if $c_i \geq \gamma_i$ for all $i \in N$.*

The next pair of results explores the relation of the equilibrium set to that of general Stackelberg games. The first of these results derives the representation of the equilibrium outcomes of general Stackelberg games in the (λ_i)-space. For notational simplicity, we assume identical marginal costs (γ_i) in the following.

Definition 4.3 (Stackelberg games)**.** For any partition $(N_t)_{t \leq T} = (N_1, N_2, \ldots, N_T)$ of N, define the $(N_t)_{t \leq T}$-Stackelberg game as the T-round extensive form game of perfect information where the players $i \in N_t$ simultaneously move (choosing quantities) in round t, for all $t = 1, \ldots, T$.

Lemma 4.4. *Assume $\gamma_i = \gamma_j$ for all $i, j \in N$. The equilibrium profits of the players in any $(N_t)_{t \leq T}$-Stackelberg game are given by Eq.* (14) *using $\lambda_i = \prod_{t'=t+1}^{T} \frac{1}{|N_{t'}|+1}$ for all $i \in N_t$ and all $t \leq T$. The respective equilibrium price, quantities, and profits correspond with an equilibrium according to Prop. 4.1 if and only if $\prod_{t=2}^{T} \frac{1}{|N_t|+1} \geq \frac{1}{n}$.*

In the eyes of the first mover, the most “desirable” move sequence is that all opponents follow by moving simultaneously. This is a two-stage Stackelberg game, and as indicated before, it is associated with (λ_i) where $\lambda_1 = 1/n$ and $\lambda_i = 1$ for all $i > 1$. Since $\lambda_i \geq 1/n$ for all i applies in this game, this Stackelberg game corresponds with an equilibrium of the general game in Prop. 4.1. As Lemma 4.4 shows, the equilibria of many other Stackelberg games coincide with equilibria of the present model. This does not apply for the equilibria of all Stackelberg games, however. For example, consider the Stackelberg game where $n = 3$ players move strictly sequential (player 1 moves first, 2 moves second, 3 moves third). By Lemma 4.4, its equilibrium is characterized as $\lambda_1 = \frac{1}{4}$, $\lambda_2 = \frac{1}{2}$, and $\lambda_3 = 1$, which violates $\lambda_i \geq \frac{1}{n}$ for all i. Then again, its outcome is weakly Pareto dominated by the Cournot outcome, and it is strictly Pareto dominated by other equilibrium outcomes compatible with Prop. 4.1. The following result establishes that equilibria of Stackelberg outcomes are either compatible with the general game covered by Prop. 4.1 or they are Pareto dominated by an outcome compatible with Prop. 4.1.

Lemma 4.5. *Assume $\gamma_i = \gamma_j$ for all $i, j \in N$. Not all equilibrium outcomes of general $(N_t)_{t \leq T}$-Stackelberg games correspond with equilibrium outcomes according to Prop. 4.1. All those that do not are Pareto dominated by some equilibrium outcome compatible with Prop. 4.1, however.*

5 Analysis of the general T-stage game

Allaz and Vila (1993) and Romano and Yildirim (2005) motivate and analyze the T-round games of forward trading and accumulation, respectively. Their analyses provide an interesting pair of baseline results. Allaz and Vila (1993) show that the forward trade effect becomes more intense and the (unique) equilibrium price converges to the Bertrand price as T approaches infinity in the forward trading game. Romano and Yildirim (2005), in turn, show that solutions of accumulation games are invariant with respect to T, i.e. the set of equilibrium outcomes is independent of the number of accumulation periods T.

In order to analyze the aggregate effect, the notation has to be extended slightly. To keep it as simple as possible, we focus on Markov perfect equilibria (MPEs): the players' strategies depend on the cumulative amounts of pre-built capacity and forward trades, and on the current round index $t \leq T$, but they do not depend on the actual move sequence detailing how the cumulative amounts have been reached. By definition, all MPEs are also SPEs, and thus the set of outcomes that can result in SPEs includes at least the outcomes derived below. The set of "states" in this context is denoted by $\overline{T} \times \mathbf{H}$ with $\overline{T} = \{1, \ldots, T\}$ and $\mathbf{H} = \mathbf{Z} \times \mathbf{Y}$. Given any state (t, h), the accumulated capacity is denoted as $\bar{z}_i(h)$, and following Romano and Yildirim (2005) we assume prior capacity investments are sunk. The capacity choices are therefore strategies satisfying

$$z_i : \overline{T} \times H \to Z_i \qquad \text{s.t.} \qquad z_i(t,h) \geq \bar{z}_i(h) \quad \forall (t,h). \tag{19}$$

Similarly, the accumulated amount of forward trades is denoted as $\bar{y}_i(h)$ for $i \in N$, and following Allaz and Vila (1993), forward trades are cumulative, too.

$$y_i : \overline{T} \times H \to Y_i \qquad \text{s.t.} \qquad y_i(t,h) \geq \bar{y}_i(h) \quad \forall (t,h) \tag{20}$$

Finally, the quantity choice has to match the forward trades.

$$x_i : H \to \mathbb{R} \qquad \text{s.t.} \qquad x_i(h) \geq \bar{y}_i(h) \quad \forall h \tag{21}$$

Strategies are tuples (z_i, y_i, x_i) for all $i \in N$. The T-period game is denoted as $\Gamma(T)$. Two additional assumptions are made. First, identical marginal costs $\gamma_1 = \cdots = \gamma_n$, as we will observe (similarly to Allaz and Vila) a convergence toward competitive pricing, and competitive pricing is not well-defined in the case of heterogenous marginal costs. Second, $c_i = \gamma_i$ for all $i \in N$ is assumed for simplicity (otherwise, slight restrictions are imposed on the set of equilibria).

Our first result relates to the T-invariance derived by Romano and Yildirim (2005). Namely, the set of outcomes that may result in equilibria of $\Gamma(T)$ is a subset of the outcomes that may result in equilibria of $\Gamma(T+1)$, and hence of $\Gamma(T+l)$ for all $l \geq 1$.

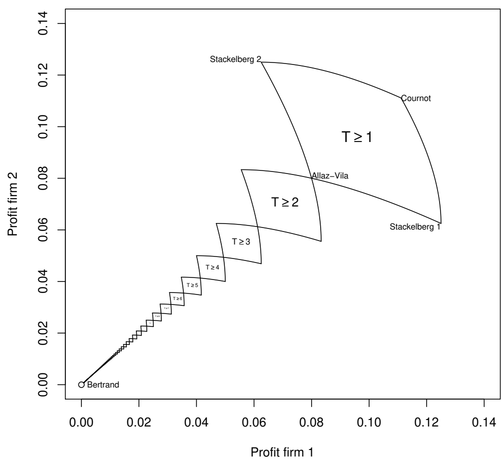
Lemma 5.1. *For all $T \geq 1$ and any payoff profile $\Pi \in \mathbb{R}^N$ that results in an MPE of $\Gamma(T)$, there exists an MPE of $\Gamma(T+1)$ that results in the same payoff profile.*

Hence, the reasoning underlying the time invariance in games of accumulation remains intact in games of capacity accumulation. To gain intuition, consider an MPE of $\Gamma(T)$. At the end of round T, all firms have concluded their planning phase, i.e. their plans (forward trades and pre-built capacities) are mutual best responses. We can now construct a strategy profile of $\Gamma(T+1)$ that replicates the moves in all rounds $t \leq T$ of the considered MPE, and loosely speaking everything is held constant in round $T+1$. It can be shown that by the mutual optimality of the plans in $\Gamma(T)$, the players may not gain by deviating *unilaterally* in round $T+1$ of $\Gamma(T+1)$, and based on this, we can construct a strategy profile that is an MPE of $\Gamma(T+1)$.

However, while the players are best off not to deviate unilaterally from an equilibrium of $\Gamma(T)$ in round $T+1$, they may well be best off deviating from the $\Gamma(T)$-plans in $\Gamma(T+1)$ if all opponents are doing so. That is, either all firms effectively conclude their planning phase after round T or they do so after round $T+1$. Our last result Prop. 5.2 shows that this implies that all equilibria of $\Gamma(T)$ can be characterized by an integer $T^* \leq T$ which denotes the *effective* duration of the planning phase and a vector (λ_i) of implicit conjectural derivatives. By varying T^* and (λ_i), the continuum of equilibrium outcomes is obtained.

Proposition 5.2. *Fix $T \geq 1$ and $\gamma = \gamma_i = c_i$ for all $i \in N$. The Pareto frontier of the equilibrium profits in $\Gamma(T)$ equates with the one of Prop. 4.1, and as T tends to infinity the minimal equilibrium price converges to marginal costs γ. Price p and profit profile $\Pi \in \mathbb{R}^N$ can result in an MPE of $\Gamma(T)$ if and only if there exist $T^* \leq T$ and $\lambda \in \left[\frac{1}{n}, 1\right]^N$*

Figure 2: Set of equilibrium profit profiles for $T \geq 1$ (assuming $a = b = 1$ and $\gamma = 0$)



such that

$$p = \frac{a + \beta^1 \gamma}{1 + \beta^1} \qquad \textit{and} \qquad \Pi_i = \frac{\alpha_i^1 + \lambda_i^{-1}}{b} (p - \gamma)^2 \tag{22}$$

where $\beta^{T^*} = \sum_i \lambda_i^{-1}$ *and for all* $t \leq T^*$,

$$\beta^t = \beta^{T^*} + \left[n + (n-2)\beta^{T^*}\right] \sum_{\tau=1}^{T^*-t} (n-1)^{\tau-1}, \tag{23}$$

$$\alpha_i^t = \sum_{\tau=t+1}^{T^*} (1 + \beta^\tau - 2\lambda_i^{-1}) * (-1)^{T^*-\tau+1}. \tag{24}$$

The respective capacities/quantities and amounts of forward trades can be computed straightforwardly, as a function of $\langle T^*, (\lambda_i) \rangle$, as detailed in the proof of Prop. 5.2. A graphical representation of the set of equilibrium outcomes in a two-player case is provided in Figure 2. It is rather easy to distinguish the component of the set of equilibrium outcomes that relates to equilibria where the effective duration of the planning phase is $T^* = 1$ from the components with duration $T^* = 2$, $T^* = 3$ and so on. The set of equilibrium outcomes corresponding with any $T^* \leq T$ form a hyperrectangle in (λ_i)-space, and the intersection of succeeding hyperrectangles consists of exactly one point (i.e. the components are not disconnected nor do they overlap).[4]

[4] The equilibrium outcome corresponding with T^* and $\lambda_i = 1/n \forall i$ equates with the outcome corresponding with $T^* + 1$ and $\lambda_i = 1 \forall i$.

An simple implication is as follows. In industries where firms participate in forward markets (for inputs and outputs) for many rounds prior to the production period, the market price approaches the Bertrand price. If the effective duration of the planning phase is brief, i.e. if it ends early or starts late, then the market price will be closer to the Cournot price.

6 Concluding remarks

This paper analyzed a model of oligopoly allowing for sales timing (forward trades) and production timing (capacity precommitment). We showed that existing results in games of quantity accumulation apply equivalently in games of capacity accumulation, generalized both the forward-trade analysis of Allaz and Vila (1993) and the Cournot-Stackelberg endogeneity (see e.g. Saloner, 1987, Pal, 1991, and Romano and Yildirim, 2005) to n-firm oligopolies, and then showed that the Cournot-Stackelberg endogeneity is generic in the generalized model considered here. Furthermore, it also covers the Allaz-Vila outcome and even the Bertrand outcome as T tends to infinity.

We have thus shown that the mode of competition may be entirely independent of "objective differences" between markets—to the degree that different focal points or historical standards do not constitute objective differences—and that the different modes are fully *self-sustaining* in equilibrium in the sense that repeated interaction and complex penal codes are not required. Conjectural variations with negative conjectural derivatives rationalized, too, while rationalizable conjectural derivatives of different firms are not entirely independent of one another and collusive actions are not rationalized.

The assumptions made in this study are fairly standard. For example, linear demand and constant marginal costs are standard and can be generalized somewhat. The well-known limitations that Cournot equilibria do not generally exist apply (see e.g. Novshek, 1980). Similarly, the assumption that capacity accumulates, i.e. that capacity investments represent sunk costs at later stages, is a standard assumption. An issue that deserves some discussion relates to the point raised by Pal (1996) who argues, in the context of two-period Cournot games, that asymmetric equilibria seem implausible in symmetric games. Note that we do not argue that say Stackelberg equilibria necessarily result in one-shot oligopoly games, but that asymmetric outcomes may be sustained in stationary equilibrium points of industries with ex-ante symmetric firms. Such industries may well have historically established leadership and follower assignments, even if firms do not move asynchronously, and since the equilibria are self-sustaining, our results show that such role assignments need not disappear over time even if firm owners or managers tend to think myopically (low δ, hence no Folk theorem) or tend to act stationarily or forward-looking (which rules out the possibility of retaliations against firms that deviated from acting as say followers).

To conclude, let us emphasize that the present study discussed a general framework of competition, and as such, it does not rationalize everything. That is, it also generates several falsifiable predictions. As Figure 2 illustrates, the relative profits of different firms are somewhat correlated in equilibrium (but note that the correlation weakens as the number of firms grows). Similarly, there is a falsifiable mapping from implicit conjectural derivatives ($= 1 - \lambda_i$ as discussed above) into the set of outcomes. An investigation of these predictions may be a subject of further research.

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A Relegated Proofs

Proof of Lemma 2.2 Fix $(\mathbf{z},\mathbf{y})$. For all $i \in N$, define the intervals $X_i' = [y_i, \bar{x}_i]$ such that $a - b\bar{x}_i = 0$. The best responses are generally unique, continuous in $\mathbf{x}_i \in \mathbf{X}_{-i}$, and for all $\mathbf{x}_i \in \mathbf{X}_{-i}$ the best response of i necessarily satisfies $x_i \in X_i'$. Since X_i' is compact, closed, and convex for all $i \in N$, existence of a pure strategy equilibrium $(x_i^*)_{i=1,\dots,n}$ thus follows from Brouwer's fixed point theorem. Eq. (5) represents the necessary conditions for a profile of mutual best responses, which therefore have to be satisfied in any equilibrium $\mathbf{x}^*$. We have to show that the equilibrium is unique. For any equilibrium $\mathbf{x}$ there exist sets $N^-, N^+ \subseteq N$, with $N^- \cap N^+ = \emptyset$, such that

$$x_i < 0 \text{ for } i \in N^-, \qquad x_i = 0 \text{ for } i \notin N^- \cup N^+, \qquad x_i > 0 \text{ for } i \in N^+. \tag{25}$$

Define m and k such that, relabeling the players appropriately, the sets are $N^- = \{1,\ldots,m\}$ and $N^+ = \{k+1,\ldots,n\}$. Thus, and due to Eq. (5), $\mathbf{x}$ solves a linear $(m+n-k)$–dimensional equation system with the following unique solution $(x_1,\ldots,x_m,x_{k+1},\ldots,x_n)$.

$$\begin{pmatrix} x_1 \\ \cdot \\ x_m \\ x_{k+1} \\ \cdot \\ x_n \end{pmatrix} = \begin{pmatrix} \frac{2s}{s+1} & \cdot & -\frac{2}{s+1} & -\frac{2}{s+1} & \cdot & -\frac{2}{s+1} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ -\frac{2}{s+1} & \cdot & \frac{2s}{s+1} & -\frac{2}{s+1} & \cdot & -\frac{2}{s+1} \\ -\frac{2}{s+1} & \cdot & -\frac{2}{s+1} & \frac{2s}{s+1} & \cdot & -\frac{2}{s+1} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ -\frac{2}{s+1} & \cdot & -\frac{2}{s+1} & -\frac{2}{s+1} & \cdot & \frac{2s}{s+1} \end{pmatrix} \times \begin{pmatrix} \frac{a+by_1}{2b} \\ \cdot \\ \frac{a+by_m}{2b} \\ \frac{a-c_{k+1}+by_{k+1}}{2b} \\ \cdot \\ \frac{a-c_n+by_n}{2b} \end{pmatrix} \tag{26}$$

using $s := m+n-k$. In turn, for any pair N^-,N^+, there is (at most) one profile x such that Eqs. (5) and (25) are satisfied. Hence, the equilibrium is unique if N^- and N^+ are unique. Fix any pair N^-,N^+ such that an equilibrium is induced. First, note that N^- contains the players i with the lowest values of $\frac{a+by_i}{2b}$, i.e. the following is true.

$$i \in N^- \text{ and } j \notin N^- \qquad \Rightarrow \qquad \frac{a+by_i}{2b} < \frac{a+by_j}{2b}. \tag{27}$$

Similarly, it can be shown that N^+ contains the players with the largest $\frac{a-c_i+by_i}{2b}$, i.e.

$$i \in N^+ \text{ and } j \notin N^+ \qquad \Rightarrow \qquad \frac{a-c_i+by_i}{2b} > \frac{a-c_j+by_j}{2b}. \tag{28}$$

Thus, for all pairs (k,m) such that $m := |N^-|$ and $k := n-|N^+|$, there is a unique solution $\mathbf{x}_i$ satisfying Eqs. (5) and (25). Now pick any equilibrium $\mathbf{x}$, and assume that it is not unique. An equilibrium $\mathbf{x}' \neq \mathbf{x}$ exists which is characterized by M^-,M^+ where

$$x'_i < 0 \text{ for } i \in M^-, \qquad x'_i = 0 \text{ for } i \notin M^- \cup M^+, \qquad x'_i > 0 \text{ for } i \in M^+. \tag{29}$$

Further, define $m' := |M^-|$ and $k' := n-|M^+|$. If $x' \neq x$, then $(m',n') \neq (m,n)$ follows by the previous argument. Without loss of generality, assume $m' > m$ which implies $M^- \supset N^-$. Define $\Delta x_i := x'_i - x_i$ for all i. Note that $M^- \supset N^-$ implies $\Delta x_i < 0$ for all

$i \in M^-$. In turn, $M^- \supset N^-$ requires that $j \notin M^-$ exists such that $\Delta x_j > 0$. Hence, $j \in M^+$, which implies $\Delta x_{j'} > 0$ for all $j' \in M^+$ and thus $M^+ \supseteq N^+$. This can be satisfied only if

$$x_i' = \frac{a+by_i}{2b} - \frac{1}{2}\sum_{j\neq i} x_j', \qquad x_i \leq \frac{a+by_i}{2b} - \frac{1}{2}\sum_{j\neq i} x_j$$

for all $i \in M^-$ (note that the inequality on the right-hand side is an equality iff $i \in N^-$, but not for all $i \in M^-$), and

$$x_i' = \frac{a-c_i+by_i}{2b} - \frac{1}{2}\sum_{j\neq i} x_j', \qquad x_i \geq \frac{a-c_i+by_i}{2b} - \frac{1}{2}\sum_{j\neq i} x_j$$

for all $i \in M^+$. Hence, for $i \in M^-$ and $j \in M^+$, we obtain

$$x_i + \frac{1}{2}\sum_{k\neq i} x_k \leq x_i' + \frac{1}{2}\sum_{k\neq i} x_k', \qquad x_j + \frac{1}{2}\sum_{k\neq j} x_k \geq x_j' + \frac{1}{2}\sum_{k\neq j} x_k' \tag{30}$$

and

$$x_j' - x_j + \frac{1}{2}(x_i' - x_i) \leq \frac{1}{2}\sum_{k\neq i,j} (x_k - x_k') \leq x_i' - x_i + \frac{1}{2}(x_j' - x_j). \tag{31}$$

This is satisfied iff $x_j' - x_j \leq x_i' - x_i$, which contradicts $\Delta x_i < 0$ and $\Delta x_j > 0$. Hence, the equilibrium $\mathbf{x}$ is unique. □

Proof of Lemma 2.3 Assume an SPE $(\mathbf{z}, \mathbf{y}, \mathbf{x})$ exists where $x_i(\mathbf{z}, \mathbf{y}) \neq z_i$ for some $i \in N$. Then i benefits by deviating unilaterally to $z_i' = x_i(\mathbf{z}, \mathbf{y})$ in stage 1. By Lemma 2.2, the choices of $\mathbf{x}(z_i', \mathbf{z}_{-i}, \mathbf{y})$ following this unilateral deviation are unique, and hence the following quantities, which are mutual best responses, must be played: $x_j(z_i', \mathbf{z}_{-i}, \mathbf{y}) = x_j(\mathbf{z}, \mathbf{y})$ for all $j \neq i$, and $x_i'(z_i', z_{-i}, y) = z_i'$. The gain of player i would be $-(x_i - z_i)\gamma_i$ if $x_i < 0$, or $(x_i - z_i) * (c_i - \gamma_i)$ if $x_i > 0$. In turn, the initially assumed strategy profile is not an SPE under the assumptions of the Lemma. □

Proof of Proposition 3.1 Fix any SPE $(\mathbf{z}, \mathbf{y}, \mathbf{x})$, let p denote the corresponding market price, and define $r_i := a - b(z_{-i} + 2z_i - y_i)$ for all i. Eq. (34) must be satisfied again, i.e. $p = b(z_i - y_i) + r_i$ for all $i \in N$. Since $c_i = \infty$ for all $i \in N$, $x_i(z_i', \mathbf{z}_{-i}, \mathbf{y}) \leq z_i'$ follows for all $i \in N$ and all z_i'. By Lemma 2.3, $x_i(\mathbf{z}, \mathbf{y}) = z_i$ holds true in any SPE. Combined, this shows that a decrease of a capacity z_i cannot lead to an increase of the quantity x_j for any $j \neq i$. As a result, $\frac{\partial \Pi_i}{\partial z_i} = p - bz_i - \gamma_i \geq 0$ must be satisfied for $\partial z_i < 0$, i.e. $p \geq bz_i + \gamma_i$, which implies $r_i \geq \gamma_i > 0$, for all $i \in N$. By Lemma 2.2, this implies $\frac{\partial x_j}{\partial z_i} = 0$ for all

$j \neq i$. As in the proof of Prop. 3.4, $\frac{\partial \Pi_i}{\partial z_i} \leq p - bz_i - \gamma_i \leq 0$ has to be satisfied for $\partial z_i > 0$, and thus, Eq. (35) is a necessary and sufficient condition again. The unique solution is given by the Cournot capacities, and the corresponding price. Hence, any strategy profile $(\mathbf{z}, \mathbf{y}, \mathbf{x})$ where $\mathbf{z}$ corresponds with the Cournot capacities and $\mathbf{x}$ corresponds with the solution according Lemma 2.2 can be extended to an SPE by any $\mathbf{y}$ that satisfies the trivial condition $y_i \in [0, z_i]$ for all $i \in N$. □

Proof of Proposition 3.2 Assume that the (x_i) chosen along the path of play represent an interior solution (this will be confirmed by the actual solution below). Given $\mathbf{y}$, the first-order conditions for an equilibrium in $\mathbf{x}$ are $p - b(x_i - y_i) = c_i$ for all i, which implies

$$bx_i = \frac{1}{n+1}\left(a + \sum_j c_j - b\sum_j y_j\right) - c_i + by_i \tag{32}$$

for all $i \in N$ and $p = \frac{1}{n+1}\left(a + \sum_i (c_i - by_i)\right)$. Hence, the optimal forward trade quantities satisfy the following first-order conditions in equilibrium.

$$\frac{\partial \Pi_i}{\partial y_i} = (p - c_i)\left(1 - \tfrac{1}{n+1}\right) - bx_i \cdot \frac{1}{n+1} = 0 \qquad \forall i \in N \tag{33}$$

Given $p = a - b\sum_i x_i$, this immediately implies that the equilibrium price is unique and equates with p^{AV}. □

Proof of Proposition 3.4 *Point 1.* By Lemma 2.3, $x_i(\mathbf{z}, \mathbf{y}) = 0$ for all $i \in N$ holds true along the path of play in any SPE. Thus, $p = a - b\sum_i x_i = a - b\sum_i z_i$ and $r_i = a - b(z_{-i} + 2z_i - y_i)$ imply that equilibrium price can be characterized as follows in any SPE.

$$p = b(z_i - y_i) + r_i \qquad \forall i \in N \tag{34}$$

In any SPE, $\frac{\partial \Pi_i}{\partial z_i} = (p - bz_i) * \frac{\partial x_i}{\partial z_i} - \gamma_i \leq p - bz_i - \gamma_i \leq 0$ holds true for $\partial z_i > 0$. By $y_i = 0$ and Eq. (34), it follows that $r_i \leq \gamma_i < c_i$ for all $i \in N$, and thus $\frac{\partial x_j(\mathbf{z},\mathbf{y})}{\partial z_i} = 0$ for all $i, j \in N$ by Lemma 2.2. Hence, any SPE has to satisfy

$$\frac{\partial \Pi_i}{\partial z_i} = p - bz_i - \gamma_i = 0 \qquad \forall i \in N. \tag{35}$$

Since this condition is also sufficient, a profile $\mathbf{z}$ is part of an SPE iff it is a solution to this equation system. The unique solution is $z_i = \frac{1}{(n+1)b} * \left(a + \sum_{j\in N} \gamma_j\right) - \frac{\gamma_i}{b}$ for all $i \in N$.

Point 2. Fix any strategy profile $(\mathbf{z}, \mathbf{y}, \mathbf{x})$. Initially assume that the capacities are fully pre-built in stage 1 (along the path of play). Due to the assumption of sufficiently similar (γ_i), we can focus on the case that all capacities are positive, which implies that the induced market price is above marginal costs γ_i for all $i \in N$. Let p denote the induced market price. Hence, there exists a profile $(\lambda_i) \in \mathbb{R}_+^N$ such that $p - \lambda_i b z_i - \gamma_i = 0$ for all $i \in N$. Define $k := \#\{j \in N | \lambda_j = 1\}$. First consider the case that $(\mathbf{z}, \mathbf{y}, \mathbf{x})$ is an SPE. This holds true only if $\lambda_i \leq 1$ (for all $i \in N$), since i would otherwise benefit by increasing z_i unilaterally in stage 1 (note that no opponent responds to a small increase of z_i by decreasing quantity since $p > \gamma_j \Leftrightarrow \lambda_j > 0$ for all $j \neq i$). Likewise, $\lambda_i \geq \frac{1}{k+1}$ is necessary (for all $i \in N$), since i would otherwise be best off cutting capacity z_i unilaterally in stage 1 (note that k players respond to i's capacity cut by increasing quantity in stage 2). Second we show that these necessary conditions are also sufficient. Assume (λ_i) satisfies $\lambda_i \in \left[\frac{1}{k+1}, 1\right]$ for all $i \in N$ for $k = \#\{j \in N | \lambda_j = 1\}$. Note that this implies $\exists i \in N : \lambda_i = 1$, i.e. $k \geq 1$. No player may benefit from extending capacity unilaterally in stage 1 because $\lambda_i \leq 1$ for all i. Also, no player may benefit from cutting capacity since $\lambda_i \geq \frac{1}{k+1}$ for all i with $\lambda_i < 1$ and $\lambda_i \geq \frac{1}{k}$ for all i with $\lambda_i = 1$ (note that k and $k-1$ players, respectively, respond to the capacity cut by extending quantity in stage 2).

It remains to show that the initial assumption—capacity be fully pre-built—can be made without loss of generality. On the one hand, this holds true for proving sufficiency of the conditions on (λ_i). For, $p - \lambda_i b z_i - \gamma_i = 0$ for all i implies (by Lemma 2.2) that quantities equate with capacities in the unique stage 2 equilibrium. This confirms that strategy profiles characterized by $p - \lambda_i b z_i - \gamma_i = 0 \forall i \in N$ satisfy the initial assumption in equilibrium, and hence that they are SPEs. Concerning the necessity of the restrictions on (λ_i), on the other hand, first note that fully pre-building capacity is itself not a necessary condition for SPE. It can be shown, however, that $x_i > z_i$ for some $i \in N$ can result in equilibrium only if k as defined above satisfies $k = 1$, $\lambda_i = 1$, and for all $j \neq i$: $\lambda_j = \frac{1}{2}$. Otherwise, some $j \neq i$ would benefit by deviating unilaterally toward a higher z_j in stage 1. This case is compatible with the necessary conditions identified above, and in all other cases, the initial assumption itself is necessary for SPE, and so must be the derived conditions. Finally, it is easy to verify that $p - \lambda_i b z_i - \gamma_i = 0 \forall i \in N$ induces the equilibrium outcome described in the proposition. □

Proof of Proposition 4.1 Fix a strategy profile $(\mathbf{z}, \mathbf{y}, \mathbf{x})$ and let p denote the induced market price. Assume that $x(\mathbf{z}, \mathbf{y})$ constitutes Nash equilibria for all $(\mathbf{z}, \mathbf{y})$. We focus on SPEs where $r_i = c_i$ for all $i \in N$ results along the path of play (it will be shown that such

SPEs exist, and it is easy to see that the set of outcomes of SPEs where $r_i \neq c_i$ for at least one $i \in N$ is a subset of the outcomes derived in the following). To abbreviate notation of directional derivatives, let $\nabla_{(\Delta z_i, \Delta y_i)} f(\mathbf{z}, \mathbf{y})$ denote the change of f (which could be x_i, x_j, or Π_i) if i changes (z_i, y_i) along $(\Delta z_i, \Delta y_i)$. By Lemma 2.3, directions $(\Delta z_i, \Delta y_i)$ that induce $\nabla_{(\Delta z_i, \Delta y_i)} x_i(\mathbf{z}, \mathbf{y}) \neq 0$ are generally dominated. Given the stage 2 solutions $x_i(\mathbf{z}, \mathbf{y})$ from Eq. (26), it follows that we can focus on directions $(\Delta z_i, \Delta y_i)$ such that either (i) $\Delta z_i > 0$ and $\Delta y_i \leq 2\Delta z_i$, or (ii) $\Delta z_i < 0$ and $\Delta y_i \leq \frac{n+1}{n} \Delta z_i$. It further holds that if a deviation in any direction is profitable, then either of the extreme deviations where Δy_i is bound by an equality must be profitable. Consider first $\Delta z_i > 0$ and $\Delta y_i = 2\Delta z_i$. By Eq. (26), this implies $\nabla_{(\Delta z_i, \Delta y_i)} x_i(\mathbf{z}, \mathbf{y}) = 0$ and $\nabla_{(\Delta z_i, \Delta y_i)} x_j(\mathbf{z}, \mathbf{y}) = 0$ for all $j \neq i$, and therefore

$$\nabla_{(\Delta z_i, \Delta y_i)} \Pi_i(\mathbf{z}, \mathbf{y}) = p - b z_i - \gamma_i \leq 0 \tag{36}$$

has to be satisfied in equilibrium. Second consider $\Delta z_i < 0$ and $\Delta y_i = \frac{n+1}{n} \Delta z_i$. By Eq. (26), $\nabla_{(\Delta z_i, \Delta y_i)} x_i(\mathbf{z}, \mathbf{y}) = 0$ and $\nabla_{(\Delta z_i, \Delta y_i)} x_j(\mathbf{z}, \mathbf{y}) = \frac{1}{n}$ for all $j \neq i$ result, which implies that

$$\nabla_{(\Delta z_i, \Delta y_i)} \Pi_i(\mathbf{z}, \mathbf{y}) = -\left(p - \tfrac{1}{n} b z_i - \gamma_i\right) \leq 0 \tag{37}$$

has to be satisfied. In turn, all $(\mathbf{z}, \mathbf{y})$ that satisfy both conditions can be extended by appropriate $\mathbf{x}$ to an SPE. Hence, the necessary and sufficient condition for SPE (conditional on the initial assumption $r_i = c_i$ for all $i \in N$) can be expressed as follows.

$$\forall i \in N \, \exists \lambda_i \in \left[\tfrac{1}{n}, 1\right]: \qquad p - \lambda_i b z_i - \gamma_i = 0 \tag{38}$$

Hence, $b z_i = \lambda_i^{-1} p - \lambda_i^{-1} \gamma_i$ for all i, and since $p = a - b \sum_i z_i$ in equilibrium, this implies $p = \left(a + \sum_i \lambda_i^{-1} \gamma_i\right) / \left(1 + \sum_i \lambda_i^{-1}\right)$. Since $\lambda_i b z_i = p - \gamma_i$, see Eq. (38), it follows that

$$\lambda_i b z_i = \frac{1}{1 + \sum_j \lambda_j^{-1}} \left(a - \gamma_i + \sum_j \lambda_j^{-1} (\gamma_j - \gamma_i)\right) \tag{39}$$

and that $z_i > 0$ are positive for all $i \in N$ and all $(\lambda_i) \in \left[\frac{1}{n}, 1\right]^N$ if the (γ_i) are sufficiently similar. Finally, $r_i = p - b(z_i - y_i)$, see Eq. (34), and $p = \lambda_i b z_i + \gamma_i$, see Eq. (38), imply that the initial condition $r_i = c_i$ is satisfied if $b y_i = c_i - \gamma_i + (1 - \lambda_i) b z_i$. Since $\lambda_i \in \left[\frac{1}{n}, 1\right]$, appropriate $y_i \leq z_i$ exist whenever c_i is sufficiently close to γ_i. It is easy to see that these (y_i) do not contradict payoff maximization, since increasing y_i implies $r_i > c_i$, decreasing y_i is payoff irrelevant, and directional variations of (z_i, y_i) are not profitable due to the arguments made above. □

Proof of Lemma 4.2 By a standard argument of upper hemicontinuity it follows that the set of SPEs constructed for the case $\mathbf{c} \approx \boldsymbol{\gamma}$ in Prop. 4.1 remain SPEs when $\mathbf{c} = \boldsymbol{\gamma}$. Hence, the set of equilibrium outcomes (prices and profits) in case $\mathbf{c} = \boldsymbol{\gamma}$ contains all equilibrium outcomes that may result if $\mathbf{c} \approx \boldsymbol{\gamma}$ (where $\mathbf{c} > \boldsymbol{\gamma}$). It also follows that all outcomes that may result in SPEs in case $\mathbf{c} = \boldsymbol{\gamma}$ but not in case $\mathbf{c} \approx \boldsymbol{\gamma}$ necessitate $x_i > z_i$ for at least one $i \in N$ along the path of play. It has to be shown that the outcomes associated with such equilibria are in the set of equilibrium outcomes even if $\mathbf{c} \approx \boldsymbol{\gamma}$. This follows from an argument closely related to the proof of Prop. 3.2, i.e. it can be shown that all SPEs where $x_i > z_i$ for at least one $i \in N$ along the path of play induce the Allaz-Vila outcome (price and profits). The details are skipped. □

Proof of Lemma 4.4 The proof is made by logical induction starting in round T. Assuming $x_{-T} \in \mathbb{R}$ denotes the aggregate quantity of the players acting in previous rounds, the first-order condition for all $i \in N_T$ is $\Pi'_i = p - bx_i - \gamma_i = 0$, and hence the aggregate quantity of all $i \in N_T$ is $x^a_T = (a - bx_{-T} - \gamma_i)/b * |N_T|/(1 + |N_T|)$. Fix $t \leq T$. Now assume that the aggregate quantity of all players acting in rounds $t' \geq t$ can be expressed as a function of the aggregate quantity x_{-t} of the players acting in earlier rounds as follows.

$$x^a_t = \frac{\beta_t}{1+\beta_t} \cdot \frac{1}{b}\left(a - bx_{-t} - \gamma_i\right). \tag{40}$$

Using $\beta_T = |N_T|$, this applies for $t = T$. The first-order condition for all $i \in N_{t-1}$ is

$$\Pi'_i = p - \frac{1}{\beta_t} \cdot bx_i - \gamma_i = 0, \tag{41}$$

which allows us, in combination with Eq. (40) and $p = a - b\sum_i x_i$, to express the price as a function of $x_{-(t-1)}$ (the aggregate quantity prior to round $t-1$) as follows.

$$p - \gamma_i = \frac{1}{1 + \beta_t * (|N_{t-1}| + 1)} \cdot \left(a - \gamma_i - bx_{-(t-1)}\right) \tag{42}$$

Substituting this for $p - \gamma_i$ in Eq. (41), again using Eq. (40), yields

$$x^a_{t-1} = \frac{\beta_t * (|N_{t-1}| + 1)}{1 + \beta_t * (|N_{t-1}| + 1)} \cdot \frac{1}{b} \cdot \left(a - \gamma_i - bx_{-(t-1)}\right). \tag{43}$$

Hence, $\beta_{t-1} = \beta_t * (|N_{t-1}| + 1) = \prod_{t'=t-1}^{T}(|N_{t'}| + 1)$, which thus applies for all $t \leq T$. For all $t \leq T$ and all $i \in N_t$, λ_i in Eq. (14) corresponds with β^{-1}_{t+1} in Eq. (41), and thus it confirms the first part of the lemma. The second part follows, since an equilibrium corresponding with (λ_i) exists under the conditions of Prop. 4.1 if $\lambda_i \geq \frac{1}{n}$ for all $i \in N$. □

Proof of Lemma 4.5 We show first that all payoff profiles Eq. (14) associated with some $(\lambda_i) \in [0,1]^N$ where $\min_{i\in N} \lambda_i < \frac{1}{n}$ are Pareto dominated by some $(\lambda'_i) \in [0,1]^N$ satisfying $\lambda'_i \geq \lambda_i$ for all $i \in N$ and $\lambda'_i > \lambda_i$ for at least one $i \in N$. Using $r = 0$, the payoff of $i \in N$ at (λ_i) can be expressed as, using $h_i(r) = (r + \lambda_i^{-1})/\lambda_i^{-1}$,

$$\Pi_i(r) = \frac{1}{\lambda_i^{h_i(r)} b} \cdot \left(\frac{a - \gamma_i}{1 + \sum_j \lambda_j^{-h_j(r)}} \right)^2. \tag{44}$$

The first derivative of $\Pi_i(r)$ with respect to r is proportional to

$$\frac{d\Pi_i(r)}{dr} \propto -\lambda_i \cdot \ln \lambda_i + 2 \cdot \frac{\sum_j \ln \lambda_j}{1 + \sum_j \lambda_j^{-1}} \tag{45}$$

and hence negative if $\lambda_i = 1$ (in this case, some $j \neq i$ exists such that $\lambda_j < 1/n < 1$). Considering the case $\lambda_i < 1$, the aforementioned derivative of Π_i is negative if

$$\lambda_i \cdot \left(1 + \sum_j \lambda_j^{-1}\right) < 2 \cdot \sum_j \log_{\lambda_i} \lambda_j, \tag{46}$$

which is generally satisfied if $\min_i \lambda_i < \frac{1}{n}$. As a result of $d\Pi_i/dr < 0$ for all $i \in N$ if $\min_i \lambda_i < \frac{1}{n}$, for any $(\lambda_i) \in [0,1]^N$ where $\min \lambda_i < \frac{1}{n}$ there exists $(\lambda'_i) \in \left[\frac{1}{n}, 1\right]^N$ such that the payoff profile associated with (λ_i) is Pareto dominated by the one associated with (λ'_i). By Lemma 4.4 it thus follows that all outcomes of Stackelberg games are either in the set of outcomes compatible with Prop. 4.1 or Pareto dominated by one of those. □

Proof of Lemma 5.1 Fix $T \geq 1$ and any MPE $(\mathbf{z}, \mathbf{y}, \mathbf{x})$ of $\Gamma(T)$. Construct a strategy profile $(\mathbf{z}', \mathbf{y}', \mathbf{x}')$ of $\Gamma(T+1)$ as follows. (i) For all states (t,h) associated with some $t \leq T$ maintain the strategies from $\Gamma(T)$, i.e. $z'_i(t,h) = z_i(t,h)$ and $y'_i(t,h) = y_i(t,h)$ for all $t \leq T$ and $h \in H$. (ii) In the production period, set x_i according to the unique equilibrium $\mathbf{x}^*(h)$, for all h, derived in Lemma 2.2. (iii) For all states (t,h) associated with $t = T+1$, set z_i equal to the greater of $\bar{z}_i(h)$ and $x_i(h)$, i.e. $z'_i(T+1,h) = \max\{\bar{z}_i(h), x_i(h)\}$, and set $y'_i(T+1,h)$ such that (for all i and h) $p^*(h) - b(x_i(h) - y'_i(T+1,h)) \leq \gamma_i$ where $p^*(h) := a - b\sum_j x_j(h)$. Appropriate $y'_i(T+1,h) \geq \bar{y}_i(h)$ exist for all h since, by Lemma 2.2, the $x_i(h)$ chosen in any SPE imply $p^*(h) - b(x_i(h) - \bar{y}_i(h)) \leq c_i = \gamma_i$ for all i.

Note that $(\mathbf{z}', \mathbf{y}', \mathbf{x}')$ is outcome equivalent to $(\mathbf{z}, \mathbf{y}, \mathbf{x})$. It remains to be shown that it is an MPE of $\Gamma(T+1)$. By construction the latter is satisfied for the production period and also with respect to the y_i' chosen in states (t,h) associated with round $t = T+1$ (they are payoff irrelevant). By Lemma 2.2, the fact that $(\mathbf{z},\mathbf{y},\mathbf{x})$ is an MPE of $\Gamma(T)$ implies $p^* - b(x_i(h) - \bar{y}_i(h)) \in [0,\gamma_i]$ for all i and h, and this in turn implies that $z_i'(T+1,h) = \max\{\bar{z}_i(h), x_i(h)\}$ are mutual best responses in the states associated with period $T+1$. Finally, note that the construction of $(\mathbf{z}',\mathbf{y}',\mathbf{x}')$ implies that for all states (t,h) with $t = T$ and all action profiles viable in this state, the profiles of continuation payoffs are identical under $(\mathbf{z}',\mathbf{y}',\mathbf{x}')$ and $(\mathbf{z},\mathbf{y},\mathbf{x})$. Hence, action profiles that constitute mutual best responses in state (T,h) of $\Gamma(T)$ must also be best responses in state (T,h) of $\Gamma(T+1)$, and by backward induction, this applies in all states (t,h) with $t \leq T$. □

Proof of Proposition 5.2 Fix any $T^* \leq T$. The following derives the conditions under which a given outcome can result in an MPE of $\Gamma(T^*)$ subject to the constraint that the quantity sold forward is increased in every planning period of $\Gamma(T^*)$. By Lemma 5.1, an outcome equivalent MPE of $\Gamma(T)$ exists. Hence, the set of outcomes that can be sustained in MPEs of $\Gamma(T)$ is the union of all outcomes as derived next over all $T^* \leq T$. Considering $\Gamma(T^*)$, fix any state (t,h) where $t = T^*$. Similarly to the argument leading to Eq. (38), it can be shown that the necessary and sufficient condition for MPE is (along the equilibrium path, where $z_i(T^*,h) > \bar{z}_i(h)$ can be assumed w.l.o.g.)

$$\forall i \in N\, \exists \lambda_i \in \left[\tfrac{1}{n},1\right]: \qquad p^*(T^*,h) - \lambda_i b(z_i(T^*,h) - \bar{y}_i(h)) - \gamma_i = 0, \tag{47}$$

where $p^*(T^*,h)$ denotes the market price resulting along the equilibrium path conditional on state (T^*,h). Using $p = a - b\sum_i z_i$ it follows that

$$p^*(T^*,h) = \frac{1}{1+\sum_i \lambda_i^{-1}} \left(a + \sum_i \lambda_i^{-1}\gamma_i - b\sum_i \bar{y}_i(h)\right) \tag{48}$$

Define $\beta^{T^*} := \sum_i \lambda_i^{-1}$. Thus, using $\gamma = \gamma_1 = \cdots = \gamma_n$ and $\bar{y}(h) = \sum_i \bar{y}_i(h)$,

$$p^*(T^*,h) = \left(a + \beta^{T^*}\gamma - b\bar{y}(h)\right)/(1+\beta^{T^*}). \tag{49}$$

We now turn to states (t,h) in arbitrary rounds $t \leq T^*$. Define $\bar{y}_i^{T^*}(t,h)$ as the quantity that is going to be sold forward, prior to round T^* and conditional on the current state (t,h), along the equilibrium path. The induction assumptions are (i) $p^*(t,h) = (a + \beta^t\gamma - b\bar{y}(h))/(1+\beta^t)$, which is satisfied for $t = T^*$ using β^{T^*} as defined above, and (ii) $\bar{y}_i^{T^*}(t,h) = \bar{y}_i(h) + \frac{p^*-\gamma}{b}\cdot\alpha_i^t$, which is satisfied for $t = T^*$ if $\alpha_i^{T^*} = 0$ for all $i \in N$. By definition, the profit of i in state (t,h) is $\Pi_i(t,h) = (z_i^* - \bar{y}_i(h)) * (p - \gamma_i) + p^f * \bar{y}_i(h)$, for some constant p^f and using z_i^* as the capacity that is going to be built eventually conditional on (t,h). Eq. (47) allows us to express z_i^* as a function of $\bar{y}_i^{T^*}(\cdot)$, and the latter can be expressed as $\bar{y}_i^{T^*}(t+1,\cdot) = y_i(t,h) + \frac{p^*-\gamma}{b}\cdot\alpha_i^{t+1}$ by the induction assumption. The following expression of Π_i follows, neglecting the constant term $p^f * \bar{y}_i(h)$.

$$\Pi_i(t,h) = \frac{1}{\lambda_i b}(p^*-\gamma)^2 + \left(y_i(t,h) + \frac{p^*-\gamma}{b}\cdot\alpha_i^{t+1} - \bar{y}_i(h)\right)\cdot(p^*-\gamma) \tag{50}$$

The first-order conditions of maximizing $\Pi_i(t,h)$ over $y_i(t,h)$ yield, for all $i \in N$,

$$y_i(t,h) = \bar{y}_i(h) + \frac{p^*-\gamma}{b}\cdot\left[1+\beta^{t+1} - 2(\alpha_i^{t+1}+\lambda_i^{-1})\right]. \tag{51}$$

Hence, $\alpha_i^t = \alpha_i^{t+1} + \left[1+\beta^{t+1} - 2(\alpha_i^{t+1}+\lambda_i^{-1})\right] = 1+\beta^{t+1} - \alpha_i^{t+1} - 2\lambda_i^{-1}$, and

$$\sum_{i\in N} y_i(t,h) = \sum_{i\in N}\bar{y}_i(h) + \frac{p^*-\gamma}{b}\cdot\left[n*(1+\beta^{t+1}) - 2\sum_i(\alpha_i^{t+1}+\lambda_i^{-1})\right]. \tag{52}$$

Using the induction assumption (i) $p^*(t+1,h) = (a+\beta^{t+1}\gamma - b\sum_i y_i(t,h))/(1+\beta^{t+1})$,

$$p^*(t,h) = \frac{a - b\bar{y}(h) + \gamma\cdot\left[n+(n+1)\beta^{t+1} - 2\sum_i(\alpha_i^{t+1}+\lambda_i^{-1})\right]}{(n+1)*(1+\beta^{t+1}) - 2\sum_i(\alpha_i^{t+1}+\lambda_i^{-1})}$$

It follows that $\beta^t = n+(n+1)\beta^{t+1} - 2\sum_i(\alpha_i^{t+1}+\lambda_i^{-1})$, and recursively both (α_i^t) and β^t are thus well-defined for all $t \le T^*$. Since $\bar{y}_i(h) = 0$ for all $i \in N$ in $t = 1$ (no output is sold forward prior to round 1), Eq. (48) thus yields the equilibrium price, Eq. (47) yields the equilibrium capacity/quantity for all $i \in N$, and $\bar{y}_i^{T^*}(1,h) = 0 + \frac{p^*-\gamma}{b}\cdot\alpha_i^1$ for all $i \in N$. The equilibrium profit Eq. (22) follows from Eq. (50), using $t = 1$ and $\bar{y}_i^1 = 0$ for all i. To see that β^t is increasing in T^*, resolve the recursive definition of β^t. If T^*-t is even,

$$\beta^t = \beta^{t+1} + n(\beta^{t+1} - 2\beta^{t+2} + 2\beta^{t+3} - \cdots + \cdots - 2\beta^{T^*} - 1) + 2\beta^{T^*} \tag{53}$$
$$\beta^{t+1} = \beta^{t+2} + n(\beta^{t+2} - 2\beta^{t+3} + 2\beta^{t+4} - \cdots + \cdots + 2\beta^{T^*} + 1) - 2\beta^{T^*} \tag{54}$$

and (partially) substituting for β^{t+1}, we obtain $\beta^t = \beta^{t+1} + (n-1)(\beta^{t+1} - \beta^{t+2})$ and the expression provided in the proposition. The same applies if T^*-t is odd. Note $\beta^{T^*-1} - \beta^{T^*} = n+(n-2)\beta^{T^*}$ and $\beta^{T^*} = \sum_i\lambda_i^{-1}$. Hence $\beta^t \to \infty$ as well as $p \to \gamma$ when $T^* \to \infty$. Resolving the recursive definition of α_i^t yields, for all $i \in N$, $\alpha_i^t = \sum_{\tau=t+1}^{T^*}(1+\beta^\tau - 2\lambda_i^{-1}) * (-1)^{T^*-\tau+1}$. □