Rent Seeking, Tax Policy, and Economic Growth

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2001

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Working paper #2001/025.

Prepared for the VIII CONFERENCE “TRANSFORMING GOVERNMENT IN ECONOMIES IN TRANSITION”, November 2-4, 2000. The research was fulfilled within the NES Research Program and sponsored by Ford Foundation.

МОСКВА

2001

Polterovich V.M. Rent seeking, tax policy, and economic growth./ Working paper#2001/025.- Moscow, New Economic School, 2001.- 43 p.(Engl.)

It is suggested a Romer-Barro - type model of endogenous economic growth where producers contest for distribution of a fixed share of the government's tax revenue. The proportional contest mechanism is assumed. We studied conditions under which consumers gain or lose due to existence of the rent seeking (RS) opportunities. It is found that RS always decreases rate of growth but nevertheless may raise consumer's overall utility. RS is advantageous if tax rate is too high or rate of production return is too low. The area of parameters, where RS has positive effect, is larger for more impatient consumers.

We study also a static RS production model with heterogeneous producers and show that excessive tax burden creates incentives for RS (which is interpreted as corruption). It is argued that the producers' support of corruption-free regimes depends on the marginal cost elasticity of the production technologies and may be reached due to technical progress.

The results demonstrate that the connection, observed in a number of empirical papers, between economic development and RS may be two - way since it may be caused by factors that influence both RS and economic growth.

Полтерович В.М. Присвоение ренты, налоговая политика и экономический рост./ Препринт # 2001/025 - М.: Российская экономическая школа, 2001 г.- 43 с.(Англ.)

Предложена модификация модели эндогенного экономического роста Ромера – Барро, в которой фирмы состязаются за перераспределение фиксированной доли налогового дохода правительства. Предполагается пропорциональный состязательный механизм. Изучаются условия, при которых подобная деятельность по присвоению ренты (ПР) приводит к выигрышу или проигрышу потребителей.

Анализ показывает, что возможность ПР в производстве уменьшает темп роста экономики, но, тем не менее, может увеличить интегральную полезность потребителей. ПР приводит к выгоде потребителя, если налоговые ставки слишком высоки, или если технологическая продуктивность чересчур низка. Область параметров, в которой эффект ПР положителен, шире для более нетерпеливых потребителей.

В работе изучается также статическая модель производства с разнородными производителями и бюрократом, распределяющим субсидии за взятки. И в этом случае избыточная налоговая нагрузка стимулирует присвоение ренты. Поддержка производителями безкоррупционного режима зависит также от технического прогресса, определяющего характеристики производственных функций.

Полученные результаты показывают, что связь между экономическим развитием и ПР, установленная в ряде эмпирических работ, может быть двусторонней , так как она определяется факторами, которые влияют одновременно и на присвоение ренты, и на экономический рост.

ISBN 5-8211-0137-9

PART I. Rent seeking in a model of exogenous economic growth

I.I. Introduction

Wide-spread expectations prevailed at the beginning of the transition processes in Russia and other East European countries that market institutions would arise spontaneously just after elimination of the centralized control. These expectations were not realized. It is clear now that spontaneous forces push the economic systems towards a different direction: government power is substituted at least partially by mafia control and corruption and rent seeking hampers the creation of western type market relations (Alexeev, Gaddy, Leitzel (1995), Leitzel (1996), Levin, Satarov (1997)). The new situation is somewhat similar to a "bad" equilibrium, when agents do not want any changes or do not able to enforce them.

A number of papers addressed this issue. Most of the authors analyze the costs of rent seeking (RS) activity and stress that higher governance quality and stronger law enforcement are needed to diminish the costs (Shleifer and Vishny (1993), Murphy, Shleifer, and Vishny (1993), Bicchieri and Rovelli (1995), Bac (1996), Gelb, Hillman, and Urspring
(1996)) . The theoretical conclusion, that economic growth is dependent on institutional quality, is supported by empirical researches (McCallum, Blais (1987), Shleifer (1997), Olson Jr., Sarna and Swamy (2000), Popov (2000)). A comprehensive survey of the results is given in Aron (2000).

There are also some recent evidences that economic performance influences institutional quality (Heybey and Murrell (1999), Chong and Calderon (2000), Paldam (2000))[1]. The quality is measured by indices that reflect corruption level, property right enforcement, rule of law, but do not contain information about the government skill to choose macroeconomic policy. However some researchers underline that imperfect economic policy creates a base for RS and corruption. In fact a few papers examine this issue on a theoretical level . Ericson developed a general equilibrium model with bribes and demonstrated that the bribe equilibrium could be a Pareto improvement if price distortions prevailed in an economy (Ericson (1983)). In Loayza (1996), it is shown that distorted tax policy may give incentives for tax evasion.

Another important result was got by Polischuk and Savvateev (1997) (see also Savvateev (1997)). They have shown that "social stability" of a RS regime depends on the elasticity of marginal cost function.

An economy can get out of a rent seeking regime only if a significant part of population recognizes that RS is harmful. Therefore it is very important to know the conditions under which RS is or is not advantageous. This is the main issue of my article. I do not assume that government is perfect. Its non-optimal tax policy and presence of externalities creates possibilities for RS to be advantageous. But every rent seeker has also production opportunities. Therefore the gain from RS depends also on technological efficiency and, in a dynamic framework, on the rate of consumers' time preferences.

I use two models to study connections between tax policy, technological efficiency, and consumers' preferences from one side and social stability of RS

[1] Paldam has found that the corruption scale depends negatively on rate of growth and GDP per capita if one consider these indicators separately. However, the dependence is robust with respect to GDP in multiple regression whereas the coefficient to growth even changes sign. In the framework of our model, this may be partially explained by the presence of common productivity factor which influence both rate of growth and GDP per capita (see below).

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regimes, from the other one. In both models RS is associated with reallocation of the state revenue, the case which is particularly important for transition economies. The benchmark of the first model is the exogenous growth theory developed by Romer (1986) and Barro (1990) (see also Barro and Sala- I- Martin (1992)). I build a proportional content mechanism in a Barro model of economic growth.

This RS scheme is traditional for RS considerations (see Lu (1994), Polischuk and Savvateev (1997)). Endogenous growth models were used in Loayza (1996) to study the shadow economy and in Mohtadi and Roe (1998) where lobbing was considered. The last paper is closer to my approach but it uses quite different contest mechanism and does not admit distorted government policy.

The second model, that I use, is static. It gives possibilities to investigate RS activities of heterogeneous producers and to exogenize the proportion of the state revenue assigned to RS.

The first model is considered in the next two sections of the Part I. Part II is devoted to the second model. Each Part has its own numeration of the sections, statements and formulas.

I.2. A growth model with RS

It is considered a Romer-Barro - type model (Romer (1986), Barro (1990), Barro-Sala-i Martin (1992)) of exogenous economic growth where a representative consumer maximizes overall utility function

$$\max \int_0^\infty u(c)e^{-\rho t}\,dt \quad \text{with respect to } c(t), a(t) \tag{1}$$

subject to the budget constraint

$$c + da/dt = ra + y\ , \tag{2}$$

antd to the No-Ponzi-Game condition

$$a(t)\exp(-\int_0^t r(\xi)d\xi) \to 0 \;\; \text{if } t \to \infty, \tag{2a}$$

where c is consumption, $\rho > 0$ is the constant rate of time preference, a is the quantity of real assets (a(0) is given), r is the real rate of return, and y is the maximal production profit. The consumer chooses c and a taking r and y as given quantity. For simplicity and following a tradition, we take labor force as a constant, and assume that the instantaneous utility function is given by

$$u(\,c\,) = c^{1-\theta}/(1-\theta),$$

where θ is a positive constant, $\theta \neq 1$.

A representative producer distributes rented capital, K, between production and rent seeking (RS) opportunities to find maximal value y of his/her profit function

$$y = \max_{K,\,s}\,[(1-\sigma)F(\,K\text{-}s,\,g) + ps - rK]\,. \tag{3}$$

Here $F(k,g) = A\,k^{1-\alpha}g^{\alpha}$ is a Cobb – Douglas production function that depends on production capital k = K – s, and on the quantity of public services, g. The constants A, α are positive, $\alpha < 1$. Government uses a fixed tax rate, σ, to collect tax revenue that is supposed to be a source of public services g. However, a fixed share, γ, of the revenue collected, turns out to be a subject of the RS activity. Producers choose the quantity s of capital to seek for direct subsidies ps, where p is defined by the proportional contest mechanism. If s_i is RS capital of the producer i then she/he gets

$$s_i\,\gamma\sigma\,\Sigma_j F(\,k_j,g)\,/\Sigma s_j\,,$$

so that $p = \gamma\sigma \, \Sigma_j F(k_j,g) / \Sigma s_j$. The number of producers is supposed to be fixed and all of them are similar. Therefore they make the same decisions in an equilibrium, $s = s_i$, $k = k_i$.

Thus the equilibrium conditions are as follows:

$$a = K = k + s, \tag{4}$$

$$g = (1 - \gamma)\sigma F(k,g), \tag{5}$$

$$ps = \gamma\sigma F(k,g). \tag{6}$$

The model ignores depreciation of capital. This seems to be a usual simplification. However, in our case it includes an implicit assumption that depreciation rates are equal for productive and RS capitals. The RS capital is spent to build lobbing organizations and long-run connections, to pay salaries and bribes. Probably, this kind of capital depreciates faster than productive capital. The difference in depreciation rates may be taken into account in our model but calculation would be more complicated in this case.

I.3. Comparative statics

In this section we study under which conditions consumers gain or lose due to RS activities of producers and how variations of the parameters influence the role of RS.

In view of (3), equations (2) and (4) entail the following balance equation

$$c = (1 - \sigma)F(k,g) + ps - dk/dt - ds/dt. \tag{7}$$

The first order optimality conditions for the problems (3) and (1),(2) involve:

$$p = r = (1 - \sigma) F_k , \tag{8}$$

$$\theta\lambda = (1 - \sigma)F_k - \rho = r - \rho, \tag{9}$$

where subscript denotes a partial derivative with respect to a corresponding parameter, k, and λ is a consumption growth rate.

Using (6), (5), (8), it is simple to check that the following equality is valid for the Cobb-Duglas function F

$$s/k = \gamma\sigma \ /(1- \alpha)(1- \sigma). \tag{10}$$

Taking into account (7), (6), (10) and the equality

$$(1- \sigma)F(k, g) = rk/(1 - \alpha)$$

one has

$$c/k = r/(1-\alpha) - \lambda - (\lambda - r) \ s/k \ . \tag{11}$$

Similar to the benchmark Romer – Barro case, our model has no transitional dynamics. The economy develops with a constant growth rate λ (see Appendix 1). This is a consequence of the facts that the equilibrium quantity, g, of the public good and the total equilibrium quantity, F(k,g), of the good produced are linear functions of capital k (see (5)),

$$g = [A\ (1 - \gamma)\sigma]^{1/(1 - \alpha)}\ k, \tag{12}$$

$$F(k,g(t)) = A[A\ (1 - \gamma)\sigma]^{\alpha/(1 - \alpha)}\ k\ . \tag{13}$$

In view of (8) and (12) the rate of return, r, is constant on the equilibrium trajectories,

$$r = \ (1-\sigma)(1 - \alpha)\ A[A\ (1 - \gamma)\sigma]^{\alpha/(1 - \alpha)}\ . \tag{14}$$

One gets a straightforward conclusion from (9) and (14) that RS hampers economic growth.

Proposition 1. Growth rate, λ, is a decreasing function of the scale, γ, of the RS activity.

However, this does not mean that consumers lose from RS since their overall utility depends not only on λ, but also on initial consumption c_0. RS is a way to get back a part of the tax revenue extracted by the government. One can expect that if the tax burden is too excessive RS may be advantageous.

Let c = $c_0 e^{\lambda t}$. Below we investigate how the maximal value, Φ, of the utility function (1) depends on γ.

One has from (9)

$$\Phi = c_0^{1-\theta} / (1-\theta)\, q, \qquad (15)$$

where $q = r - \lambda$. The integral exists iff $q > 0$ which equivalent to the relation[2]

$$r\,(1-\theta) < \rho. \qquad (16)$$

In view of (14) we get:

$$r_\gamma = -\alpha\; r/(1\text{-}\alpha)(1 - \gamma).$$

Besides, $\theta q = \rho - (1 - \theta)r$.

Therefore $$\theta q_\gamma = - (1 - \theta)r_\gamma = (1 - \theta)\,\alpha r/(1\text{-}\alpha)(1 - \gamma).$$

It follows from (9) that $\theta\lambda_\gamma = r_\gamma$. Therefore $\lambda_\gamma = - r_\gamma/(1-\theta)$.

Let $k_0 = 1$. Using (11) and formulas above, it is simple to check that

$$c_0 = q(s_0 + 1/(1\text{-}\alpha)) + \lambda\alpha/(1-\alpha),$$

$$c_{0\gamma} = q_\gamma[s_0 + 1/(1 - \alpha) - \alpha/(1 - \alpha)(1 - \theta)] + q\sigma/(1 - \alpha)(1 - \sigma),$$

where s_0 is defined by (10) for k = 1 .

Since $q > 0$ and $(1\text{-}\theta)^{-1}q_\gamma > 0$, the sign of the derivative Φ_γ coincides with the sign of the following function

$$\Gamma = [(1\text{-}\theta)c_{0\gamma}/q_\gamma - c_0/q](1 - \alpha)(1\text{-}\sigma)/\theta.$$

After substitution $c_{0\gamma}$, q_γ, c_0, and q, and after some manipulations, we get

$$\Gamma = -1 + \sigma(1-\gamma) - \alpha(1-\sigma)/\theta\;\omega + \omega\sigma/(1 - \alpha)(1 - \gamma)/\alpha, \qquad (17)$$

where

$$\omega = (\rho/r + \theta - 1)/\theta, \qquad (18)$$

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[2] This requirement follows also from the No-Ponzi-Game condition (2a).

and r is defined by (14) (see Proposition A2 in Appendix 2).

Obviously, ω >0 since, by assumption, q > 0.

Our model and the function Γ depend on six parameters γ, σ, A, ρ, α, and θ. For different sets of parameters, the RS activity may influence positively or negatively on the wealth of population. It is natural and convenient to understand advantageousness and harmfulness of RS in a local sense in accordance to the following definition.

Definition 1.We say that RS is (locally) harmful at x =(γ, σ, A, ρ, α, θ) if $\Phi_\gamma < 0$, and RS is (locally) advantageous at x if $\Phi_\gamma > 0$.

One can expect that population would oppose RS if it were harmful and support RS if it were advantageous. If RS is advantageous at x where $\gamma = 0$ then consumers prefer to have a non-zero RS scale.

Definition 2. A variation of a parameter is RS- promoting (RS- opposing) if it may transform an RS-harmful state into advantageous one (an RS- promoting state into RS-harmful one), but not vise versa.

It was mentioned above that the function Γ(x) has the same sign as Φ_γ. Using formula (17), (18), and (14), one can check that Γ(x) grows to infinity if σ approaches 1; Γ(x) decreases with respect to A and increases with respect to ρ. Therefore Propositions 2 and 3 are valid[3].

Proposition 2. For every set of parameters (γ, A, ρ, α, θ) one can find σ_* such that RS is advantageous at all x = (γ, σ, A, ρ, α, θ), $\sigma > \sigma_*$.

Thus consumers are not interested in decreasing of the RS scale if the tax rate is too high.

[3] Propositions 2-5 are proved in Appendix 2.

Proposition 3. An increase of the time preference rate ρ and a decrease of the productivity constant A are RS- promoting. Moreover, let $x_0 = (\gamma_0, A_0, \rho_0, \sigma_0, \theta_0)$ be fixed, and $x'(A) = (\gamma_0, A, \rho_0, \sigma_0, \theta_0)$ and $x''(\rho) = (\gamma_0, A_0, \rho, \sigma_0, \theta_0)$. Then there exist A_*, ρ_* such that RS is advantageous at $x'(A)$ if $A>A_*$, and RS is harmful if $A < A_*$; RS is advantageous at $x''(\rho)$ if $\rho < \rho_*$, and RS is harmful if $\rho > \rho_*$.

The following two propositions can be proved by straightforward calculations if one takes into account that the inequalities $\lambda > 0$ and $\omega < 1$ are equivalent.

Proposition 4. RS is harmful at every state x such that $\lambda > 0$ and $\sigma < \sigma* = (\alpha\theta + \alpha^2) / (\theta + \alpha^2)$.

Note that the last inequality is definitely valid if the tax rate σ is not succeeded its optimal value α since $\alpha < \sigma*$. [4]

Proposition 5. RS is advantageous at every state x such that $\gamma = 0$, $\lambda < 0$, and $\sigma > \sigma*$.

The rate of growth λ is higher if A is larger or ρ is lower. The same mistakes in the tax policy may give rise to RS in one country and do not have this consequence in another one if consumers of the first country are more impatient and its technology is less productive.

Let us assume now that the scale of RS activity γ is endogenous and is changing in accordance to the following differential equation

$$d\gamma/dt = f(\Phi_\gamma), \tag{19}$$

where f is an increasing function, $f(0) = 0$. This relation entails that γ increases if RS is locally advantageous and decreases if RS is locally harmful. In our model,

[4] One can check that α is the optimal tax rate in the corresponding welfare optimization problem (Barro,1990).

the root of RS advantage is excessive tax burden and, as a result, inefficiency of the government services.

As a function of γ, the maximal present value, Φ, of the utility function (1) may have several extrema. An important case is shown on Fig.1. Five parameters α, ρ, θ, σ, A are fixed (α = 0.6, ρ = 0.3, θ = 0.94, σ = 0.8, A = 2.15), and γ is changing from zero to 0.95. We do not consider γ = 1 since our formulas lose their sense. (If γ = 1 then there is no growth). If γ is small then RS is advantageous. One may expect that the scale of RS activity, γ, will increase and will stabilize on the level where Φ reaches its maximum (γ ≅ 0.15). Assume, however, that the scale of RS activity increases due to some causes and became larger than interior minimum (about 0.7 on Fig.1).

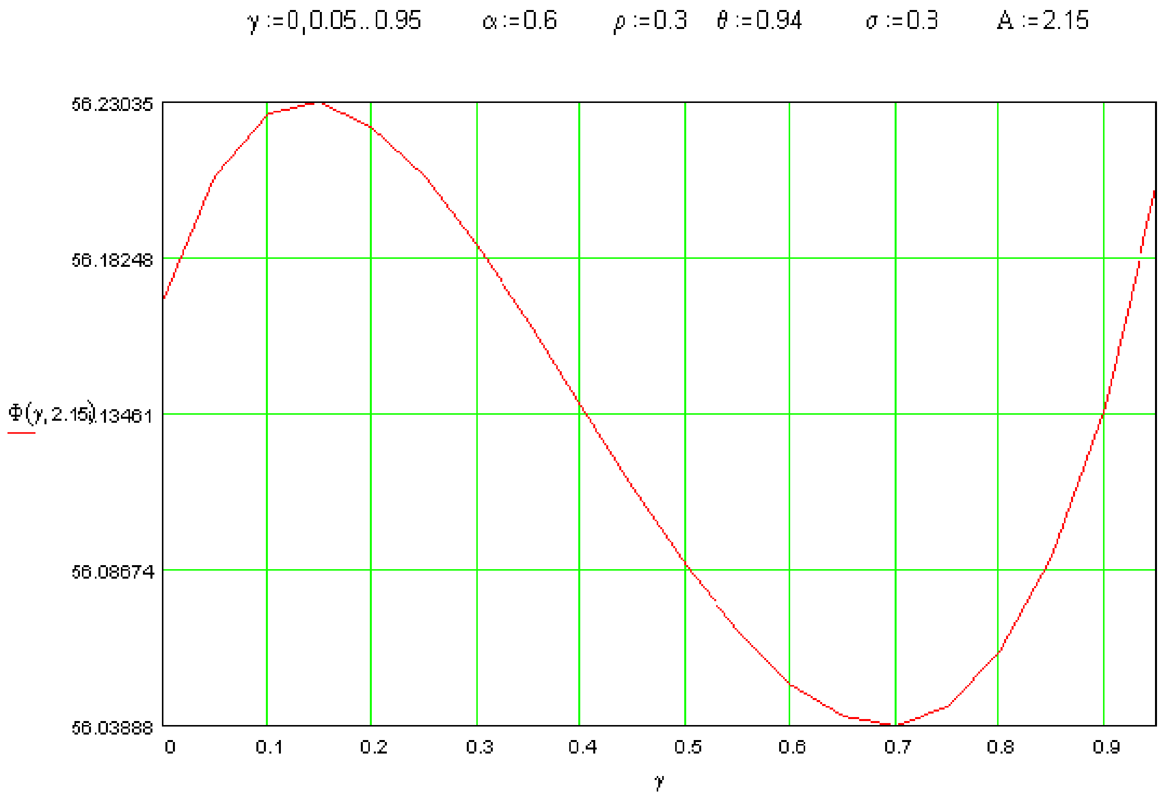

Fig.1

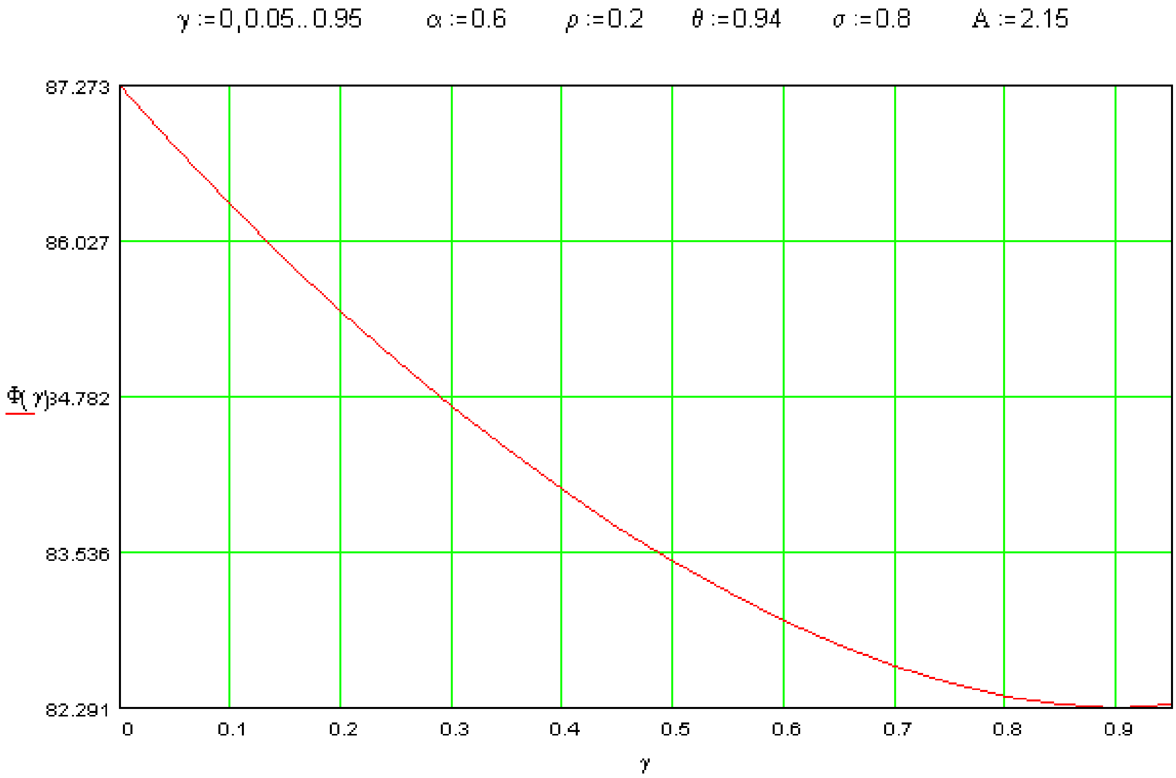

Fig.2

Now RS turns out to be locally advantageous again, and the intrinsic forces have to shift γ to an arbitrary small neighborhood of its maximum value 1. This means that a smaller proportion of the tax collected is used as a source of public service. Therefore investment into production turns out to be inefficient, and the agents should prefer to intensify RS activity, i.e. to increase γ. The more intensive is RS, the less efficient is production, and the stronger incentives exist to intensify RS. This kind of positive feedback is a standard cause of so-called institutional traps (Polterovich, 1999).

Strengthening control and punishments for deviating behavior and development of mechanisms of competition are standard measures usually suggested to exit from institutional traps. However, some other possibilities are much less understood on the theoretical level. An institutional trap can disappear due to technology improvements or decrease of the time preference rate. Fig. 2 and Fig.3 demonstrate these possibilities. The curve of Fig.2 corresponds to the same parameters as Fig.1 for exception of productivity coefficient A, which is larger for Fig.2. The increase of productivity changes drastically the shape of the curve. The interior minimum disappears, and the RS advantageous interval shrinks up to small neighborhood of the point $\gamma = 1$. A similar transformation takes place if the rate of time preference, ρ, decreases, as it is shown on Fig.3.

The propositions above lead to a hypothesis that negative correlation between RS and growth, observed in a number of empirical papers, is not necessarily a result of negative influence of the RS activity on the economic development. It may be, at least partially, caused by factors that influence both RS and the economic growth.

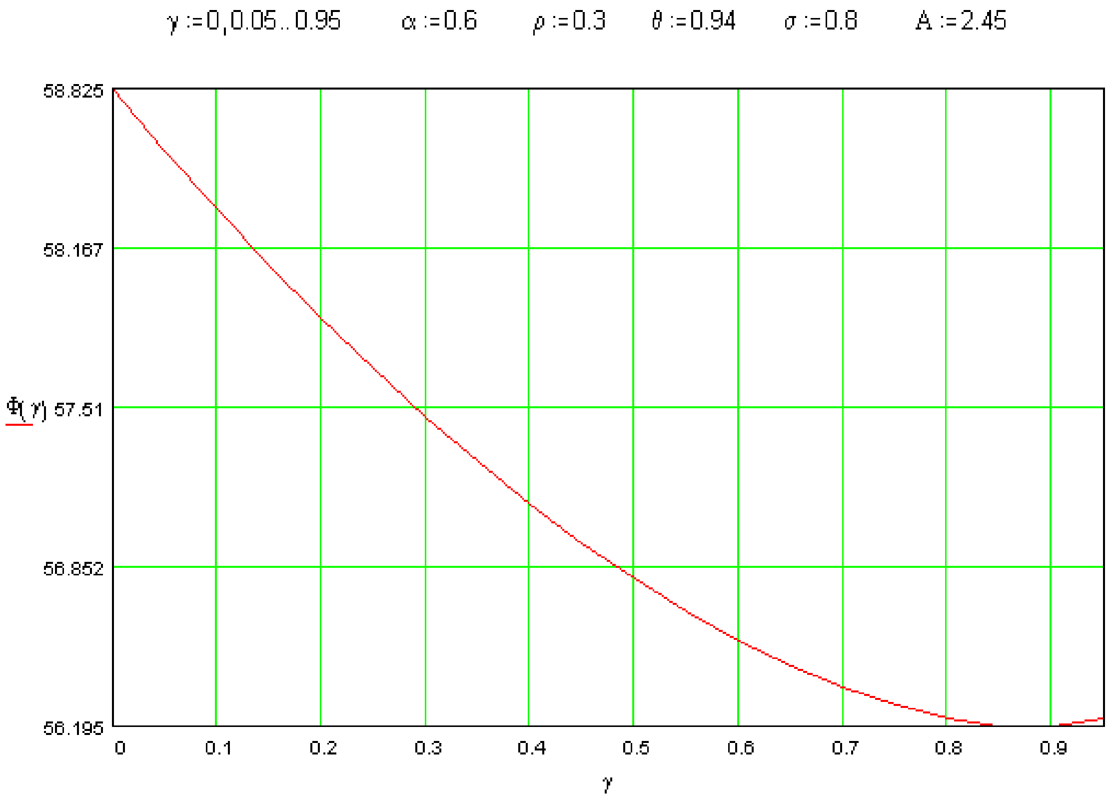

Fig. 3

Appendix 1.

Proposition A1. Equilibrium functions c(t), k(t), s(t) are exponential with equal exponents.

Proof. Euler's equation for the problem for the problem (1), (2), (2a) is as follows $(-u''(c) / u'(c))\ (dc/dt) = r - \rho$.

Since $-u''(c) / u'(c) = \theta / c$, one has the relation (9)

$$\theta\lambda = r - \rho.$$

The equilibrium real rate of return r is constant (see (14)), hence the equilibrium consumption growth rate λ is constant as well.

The equality (10) follows from the equilibrium condition (16) and the first order optimality condition (8). This equation entails that s/a is a constant on the equilibrium trajectories. Then y/a is also a constant in view of (3), (13), and (8). Denote $\mu = r + y/a$. Now the equation (2) can be rewritten as follows

$$da/dt - \mu a = - c_0\ \mathbf{e}^{\lambda t}.$$

Its general solution is as follows : $a = H\, e^{\mu t} + Q\, e^{\lambda t}$.

Since $\mu > r$, the No – Ponzi – Game condition (2a) can be valid only if $H = 0$, $\lambda < r$. Thus $Q = a_0$, and $a(t) = a_0\, e^{\lambda t}$. In view of (10) indicators s and k have the same constant rate of grows λ. The Proposition A1 is proved.

Appendix 2

Proposition A2. The expression Γ, which is is given by the formula (17), has the same sign as Φ_γ .

Let $c = c_0 e^{\lambda t}$, λ = const. Then the overall utility function

$$\Phi = (1 - \theta)^{-1}\int_0^\infty c_0^{1-\theta} \exp[\lambda(1 - \theta)-\rho]tdt = (1 - \theta)^{-1} c_0^{1-\theta}\, 1/[\rho - \lambda(1 - \theta)], \quad \text{(A1)}$$

where $\rho > \lambda(1 - \theta)$. In view of (9),

$$q = r - \lambda = \rho - \lambda(1 - \theta), \quad \text{(A2)}$$

therefore

$$(1 - \theta)\Phi = c_0^{1-\theta} /q. \quad \text{(A3)}$$

Differentiating (A3) and (A2) with respect to γ, one has

$$q^2 (1- \theta)\Phi_\gamma = c_{0\gamma}(1- \theta)\, c_0^{1-\theta}\, q - c_0^{1-\theta} /q_\gamma, \quad \text{(A4)}$$

$$q_\gamma = - \lambda_\gamma(1- \theta). \quad \text{(A5)}$$

Formula (11) and (A2) entail

$$c_0 = q(s_0 + 1/(1 - \alpha)) + \lambda\alpha/(1 - \alpha). \quad \text{(A6)}$$

Therefore

$$c_{0\gamma} = q_\gamma(s_0 + 1/(1 - \alpha)) + q\, s_{0\gamma} + \lambda_\gamma \alpha/(1 - \alpha). \quad \text{(A7)}$$

In view of (10) $s_0 = \gamma\sigma/(1- \alpha)(1 - \sigma)$; $s_{0\gamma} = \sigma/(1- \alpha)(1 - \sigma)$.

Therefore and due to (A6), (A7), and (A5) one has

$$(1- \alpha)(1 - \sigma)c_0 = q(\gamma\sigma + 1 - \sigma) + \lambda\alpha(1 - \sigma), \quad \text{(A8)}$$

$$(1- \alpha)(1 - \sigma)c_{0\gamma} = q_\gamma(\gamma\sigma + 1 - \sigma) + q\sigma - \alpha q_\gamma(1 - \sigma)/(1 - \theta). \quad \text{(A9)}$$

Let us denote

$$\theta\Gamma = q(1 - \theta)(1 - \alpha)(1 - \sigma)\, c_0^{1-\theta}\, \Phi_\gamma/q_\gamma. \quad \text{(A10)}$$

Then, multiplying (A4) by $c_0^{1-\theta}$ $(1 - \alpha)(1 - \sigma)/qq_\gamma\, \theta$ one gets

$$\theta\Gamma = (1 - \theta)(1 - \alpha)(1 - \sigma)\, c_{0\gamma}/q_\gamma - (1- \alpha)(1 - \sigma)c_0/q. \qquad \text{(A11)}$$

Let us substitute (A8) and (A9) into (A11).

$$\theta\Gamma = (1 - \theta)[\gamma\sigma + 1 - \sigma] + (1 - \theta)\sigma\, q/q_{\gamma} - \alpha(1 - \sigma) -$$
$$[\gamma\sigma + (1 - \sigma)] - \alpha(1 - \sigma)\lambda/q. \qquad \text{(A12)}$$

Note that

$$q = r - \lambda = [\rho - (1 - \theta)r]/\theta; \qquad \text{(A13)}$$

$$q_{\gamma} = - (1 - \theta)r_\gamma /\theta; \qquad \text{(A14)}$$

$$r_\gamma = - \alpha r / (1 - \alpha)(1 - \gamma) \qquad \text{(A15)}$$

in view of (14). Therefore

$$q/q_{\gamma} = [\rho - (1 - \theta)r](1 - \alpha)(1 - \gamma) / (1 - \theta)\alpha r;$$
$$\lambda / q = (r - \rho) / [\rho - ((1 - \theta)r].$$

Denote $\omega = (\rho/r - 1 + \theta)/\theta$. Obviously, $\omega > 0$. Then

$$q/q_{\gamma} = \theta\omega (1 - \alpha)(1 - \gamma) /\alpha(1 - \theta); \qquad \text{(A16)}$$

$$\lambda / q = (1 - \rho / r) / \theta\omega. \qquad \text{(A17)}$$

Now, we have from (A12), (A16), (A17)

$$\theta\Gamma = -\theta[1 - (1 - \gamma)\sigma] + \theta\sigma(1 - \alpha)(1 - \gamma)\omega/\alpha - \alpha(1 -\sigma) - \alpha(1 -\sigma)(1- \rho/r)/\theta\omega.$$

Since $\rho/r - 1 = \omega\theta - \theta$, we have the following relation

$$\Gamma = -1 + (1 - \gamma)\sigma + \sigma(1 - \alpha)(1 - \gamma)\omega/\alpha - \alpha(1 -\sigma)/\theta\omega, \qquad \text{(A18)}$$

which coincides with (17).

It follows from (A14) and (A15) that $q_\gamma /(1 - \theta) = - r_\gamma\, \theta > 0$. Therefore Proposition A2 is a consequence of the equality (A10).

Proof of Propositions 2 and 3.

The function $\Gamma(\gamma, \sigma, \alpha, \theta, \omega)$, defined by (17), is increasing with respect to σ, θ, and ω and decreasing with respect to γ and α. The function

$$\omega(\rho, \theta, r(\sigma, \gamma, \alpha, A)) = \omega(\theta, \sigma, \rho, \gamma, \alpha, A),$$

defined by (18), increasing with respect to ρ, and decreasing with respect to γ and A. Therefore $\Gamma(x)$ decreases with respect to A and increases with respect to ρ. This proves the first statement of Proposition 3.

Evidently, $r(\sigma = 1) = 0$, $\omega(\sigma =1) = \infty$, and $\Gamma(\sigma = 1) = \infty$. This entails Proposition 2.

To prove the second statement of Proposition 3, let γ_0, A_0, ρ_0, σ_0, α_0, θ_0 be fixed. Then Γ is an increasing function of ω, and $\Gamma(0) < 0$, $\Gamma(\infty) > 0$. Therefore there exists ω_* such that $\Gamma(\omega_*) = 0$. The function $\omega(A, \rho)$, defined by (18), is increasing with respect to ρ and decreasing with respect to A. It maps each of the intervals of feasible values of A and ρ into $(0,\infty)$. Therefore one can find A_* and ρ_* such that $\omega(A_*, \rho_0) = \omega_*$, $\omega(A_0, \rho_*) = \omega_*$. Proposition 3 is proved.

Proof of Propositions 4 and 5.

Let α and θ be fixed, and $V(\sigma) = -1 + \sigma - \alpha(1 - \sigma)/\theta + \sigma(1 - \alpha)/\alpha$.

The inequality $\lambda = r - \rho > 0$ is equivalent to the inequality

$$\omega = (\rho/r + \theta - 1) /\theta < 1.$$

If $\omega < 1$ then

$$\Gamma < -1 + \sigma(1 - \gamma) - \alpha(1 - \sigma)/\theta + \sigma(1 - \alpha)(1 - \gamma)/\alpha < V(\sigma).$$

If $\lambda < 0$ then $\omega > 1$, and, under $\gamma = 0$, we have: $\Gamma > V(\sigma)$. The function $V(\sigma)$ is increasing with respect to σ and has $\sigma^* = (\alpha\theta + \alpha^2)/(\theta + \alpha^2)$ as its zero. Therefore both Propositions 4 and 5 are valid.

PART II. Corruption and the tax policy[5]

II.1. Preliminary remarks

In the model studied above we assumed that producers are homogenous, the production functions have very simple and special form, and the proportion of government revenue assigned for RS does not depend on rent seekers' efforts. In the second part of the paper we consider a static production model where, however, all these restrictions are removed.

Below an equilibrium model is developed where corruption behavior is described in revealed form and the technology of counter- productive reallocation is derived from behavioral assumptions. The model includes an arbitrary number of producers and a representative bureaucrat- bribe-taker. Production functions depend on two inputs that are a homogeneous resource (money) and a public good. The bureaucrat collects a part of GDP as tax payments and has to transform it into the public good. But he/ she prefers to receive bribes giving some part of the government revenue as subsidies to producers. The bribe-taker decisions are generated by maximization of a goal function that brings into confrontation money utility and disutility of punishment. The corruption equilibrium is compared with an optimal corruption-free regime. I demonstrate that non-optimal tax policy can result in support of a corruption regime by some or even by all producers. It is proved that if the tax rate is not bigger than optimal one and marginal cost elasticity of production functions is not bigger than 1[6] then the corruption regime is Pareto inferior independently on the scale of corruption and

[5] A version of this Part circulated as a manuscript from 1998, and some results were described in Polterovich (1998).

[6] This condition was received by Polishchuk and Savvateev (1997) for a model with homogeneous producers and without taxes and public goods.

initial resource allocation. More general sufficient condition connects the tax rate level and the law enforcement degree. I argue as well that the marginal cost elasticity condition can be supported by fast technical progress.

In the next section I describe the model and prove that the corruption equilibrium resource allocation is a solution of an optimal programming problem. Existence and uniqueness results follows from this observation. I consider also a variant of the model that admits reallocation of initial resources between producers (Section 3). Section 4 contains the main theorem and discussion of the conditions that guarantee social stability of the corruption-free regime independently on the punishment level for corruption. The proof is based on a consideration of a "limit case" when the model is reduced to Polishchuk and Savvateev (1997) construction but with different production functions for different agents. Their main results is generalized in Section 5. Section 6 contains the proof of the main theorem and some comparative statics exercises. Section 7 concludes.

II. 2. Corruption equilibrium

I consider a set of producers indexed by i each of them has initial amount of money M_i . Prices of the production input and outputs are supposed to be fixed and do not figure in the model. The production i is described by a production function F_i (m_i, g) that depends on the amount m_i of money invested and the quantity g of a public good which is free of charge. A representative bureaucrat collects taxes by tax rate σ and has to transform the collected money into public good. But he/she prefers to allocate a part of this money as direct subsidies to producers for bribes[7]. The bribe system is supposed to work as a competitive market so that a bribe price q of the ruble of subsidies is set up to equilibrate the supply and demand for subsidies. Every producer allocates his/her money between production and bribes solving the following maximization problem:

$$\max \ (1-\sigma)F_i (m_i, g) + z_i \quad \text{w. r. to } (m_i , z_i) \tag{2.1}$$

$$qz_i + m_i = M_i \tag{2.2}$$

$$m_i \geq 0 \ , \qquad z_i \geq 0 \tag{2.3}$$

where z_i is the subsidy received.

The bureaucrat compares his/her utility of money m_b received as bribes and the disutility from possible punishment, that depends on the proportion γ of the total government income Z. The amount γZ is assigned to subsidies to producers and the rest for public service. The proportion γ is chosen as a solution of the following problem[8]:

$$\max \ U(m_b , \gamma) \tag{2.4}$$

where

$$m_b = q\gamma Z \ , \ 0 \leq \gamma \leq 1. \tag{2.5}$$

Definition 1. A set of numbers ($m_i , z_i , i \in I , \gamma, g, Z, q$) is said to be a corruption equilibrium if (m_i , z_i) is a solution of (2.1)-(2.3), γ is a solution of (2.4)-(2.5) , and the following equalities hold

$$Z = \sigma \Sigma F_i(m_i, g) \tag{2.6}$$

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[7] This is a very stylized description. Usually total subsidy level and distribution of subsidies are results of interactions among many bureaucrats and lobbing groups.

[8] Two simplest form of the utility function are $u(m_b) - \pi(\gamma)$ or $u(m_b) / \pi(\gamma)$ where u is utility of money and π is a penalty function. Our assumption that disutility of punishment depends not on absolute value of bribes but on the relative scale of bribe activity seems to be reasonable though it needs to be tested.

$$\Sigma z_i = \gamma Z \qquad (2.7)$$

$$g = (1-\gamma)Z\,. \qquad (2.8)$$

In the sequel, the set of producers is supposed either to be finite or to be equal to a segment. In the later case, the symbol Σ will denote integration over the segment.

Let us study the model. Below we use the following assumptions.

A1. The F_i are increasing for $m_i > 0$, $g > 0$ and smooth; $F_i(0,0)=0$; they are strictly concave with respect to m_i; their partial derivatives $F'_{i1}(0, g)=\infty$ under $g>0$.

A2. U is defined for all $m_b \geq 0$ and $\gamma \in [0,1)$, smooth, strictly concave, its partial derivatives satisfy the conditions $U_1' > 0$, $U_2' < 0$, $U_1'(0, \gamma) = \infty$.

A2a. $U_2'(m_b, \gamma) / U_1'(m_b, \gamma) \to -\infty$ as $\gamma \to 1$.

A2b. $U_1'(m_b, \gamma)$ decreases as γ grows.

The condition A2a means that the disutility of punishment increases much faster than utility of money, when the bureaucrat devoted almost all money at his/her disposal to the corruption activity. In accordance to A2b a stronger punishment diminishes marginal utility of money.

From (2.2) and (2.7) the equilibrium price is as follows,

$$\hat{q} = (M - \hat{m}) / \hat{\gamma}\hat{Z} \qquad (2.9)$$

where the hat "^" symbol denotes equilibrium values of the parameters,

$M = \Sigma M_i$, $\hat{m} = \Sigma \hat{m}_i$.

Relations (2.2), (2.5) and (2.7) yield

$$\hat{m}_b = M - \hat{m}\,, \qquad (2.10)$$

and (2.4), (2.5) implies by A2a

$$U_1'(\hat{m}_b, \hat{\gamma})\, \hat{m}_b + U_2'(\hat{m}_b, \hat{\gamma})\hat{\gamma} = 0. \qquad (2.11)$$

A2 implies that the left hand side decreases as a function of γ, and there exists a function Γ defined on $(0, \infty)$ such that

$$\hat{\gamma} = \Gamma(\hat{m}_b),\ 0 < \Gamma(\hat{m}_b) < 1. \qquad (2.12)$$

Solving (2.2) with respect to z_i and using (2.6), (2.9), (2.10), (2.12) we get the following equivalent form of the problem (2.1) - (2.3) at equilibrium

$$\max F_i(m_i, \hat{g}) + \Lambda(\hat{\mu}(.), \hat{g})(M_i - m_i) \qquad (2.13)$$

$$0 \leq m_i \leq M_i. \qquad (2.14)$$

where $\hat{\mu}(.)$ is the vector whose components are $\hat{m}_i$ (a function of i) and

$$\Lambda(\mu(.), g) = \sigma\Sigma F_i(m_i, g)\, \Gamma(M - m) /(M - m)(1 - \sigma). \qquad (2.15)$$

The first order condition for (2.13), (2.14) is written as

$$F'_{i1}(\hat{m}_i, \hat{g}) \geq \Lambda(\hat{\mu}(.), \hat{g}) \qquad (2.16)$$

where (2.16) is satisfied as soon as (as an equality) if $\hat{m}_i < M_i$.

(F'_{i1} represents the derivative of the function F_i with respect to first argument).

Let us introduce a function

$$B(m) = \int_1^{M-m} (\Gamma(x)/x)dx$$

and consider the following problem :

$$\max (1 - \sigma) \ln \Sigma F_i(m_i, \hat{g}) + \sigma B(m), \text{ w. r. to } (m_i) \qquad (2.17)$$

$$m = \Sigma m_i \qquad (2.18)$$

$$0 \leq m_i \leq M_i,\ i \in I. \qquad (2.19)$$

Proposition 1. The function B is concave.

Proof. We need to prove that the function $\psi(x) = \Gamma(x)/x$ is decreasing. From (2.11) we have for $x = \hat{m}_b > 0$, $\hat{\gamma} = \Gamma(\hat{m}_b)$

$$U_1'(x, x\psi(x)) + U_2'(x, x\psi(x))\psi(x) = 0.$$

This can be considered as an identity which defines ψ. Differentiation entails the following equality

$$U_{11}' + U_{12}' (\psi + x\psi') + U_2'\psi' + U_{21}'\psi + U_{22}' (\psi + x\psi') \psi = 0.$$

All second derivatives are negative, by concavity and by A2b. Since $U_2' < 0$ one has to conclude that $\psi' < 0$. Hence B is concave.

The following statement substantially simplifies the exploration of our equilibrium model .

Theorem 1 (Equivalence Theorem). If A1, A2 are valid then an array $(\hat{m}_i, \hat{z}_i, i \in I, \hat{\gamma}, \hat{g}, \hat{Z}, \hat{q})$ is an equilibrium if and only if $(\hat{m}_i, i \in I)$ is a solution of (2.17)- (2.19) and the following equality holds

$$\hat{g} = (1 - \Gamma(M - \hat{m}))\, \sigma\, \Sigma F_i(\hat{m}_i, \hat{g}) . \tag{2.20}$$

We omit the proof for it follows straightforwardly from a comparison of the first order optimality conditions for the optimization problem and the equilibrium conditions including (2.20).

Note that an equilibrium value $\hat{g}$ is a parameter of the maximized function (2.17). A remarkable case arises, if the following assumption is valid,

A3. $F_i(m_i, g) = f_i(m_i)\, \varphi(g)$.

Instead of $\varphi(g)$ one can take $\alpha_i\, \varphi(g)$ and then redefine f_i. Assumption A3, means that the elasticities of the production functions with respect to public good are equal and independent of money expenditures.

The following statement is a straightforward consequence of both A3 and the Equivalence Theorem.

Theorem 2. If A1- A3 hold, then an equilibrium allocation $(\hat{m}_i, i \in I)$ is a solution of the problem

$$\max\ (1 - \sigma) \ln \Sigma f_i(m_i) + \sigma B(m) \tag{2.21}$$

under the constraints (2.18), (2.19). Moreover if $\varphi(g)$ is strictly concave then the corruption equilibrium is unique.

The last statement of Theorem 2 follows from equation (2.20) by monotonicity of the function $g / \varphi(g)$.

Theorem 2 shows that the equilibrium money distribution can be found independently on the public good. After that all other equilibrium parameters can be calculated straightforwardly[9].

Existence of a corruption equilibrium is also a consequence of Equivalence Theorem.

Theorem 3. Assume A1**,** A2 hold, $F_i(m, g) /g \to 0$ as $g \to \infty$, and $F_i(m, g) /g \to \infty$ as $g \to 0$ for every $m > 0$ and i. Then there exists a corruption equilibrium.

Proof. Let $S(\hat{g}) = (S_i(\hat{g}))$ be the solution of the problem (2.17)-(2.19) Denote $G(\hat{g}) = \hat{g} - (1 - \Gamma(M - \Sigma S_i(\hat{g})))\sigma \Sigma F_i(S_i(\hat{g}), \hat{g})$.
Evidently $G(\hat{g})>0$ if $\hat{g}$ is large enough, and $G(\hat{g}) <0$ if $\hat{g}$ is small. By Theorem 1, the proof is complete.

II.3. Corruption equilibrium with markets

In this Section we introduce both credit and borrowing into the corruption system. The agent problem (2.1)-(2.3) has to be modified accordingly to account for this new possibility.

$$\max \ (1-\sigma)F_i(m_i, g) + z_i + ph_i \quad \text{w. r. to } (m_i, z_i, h_i), \qquad (3.1)$$

[9] Theorem 1 stays valid if strict concavity in A1 is substituted for concavity. In this case equilibrium money distributions form a convex set.

$$q z_i + m_i + h_i = M_i \,, \tag{3.2}$$

$$m_i \geq 0 \;, \quad z_i \geq 0 \tag{3.3}$$

where h_i is credit (or borrowing quantity if h_i is negative), p-1 is interest rate. To define the concept of equilibrium one has to add $\Sigma h_i = 0$ to the equilibrium conditions (2.6)-(2.8).

The maximized function (3.1) is equal to $(1-\sigma)F_i(m_i, g) + (M_i - m_i)/q + (p-1/q)h_i$. At equilibrium $p = 1/q$, and the agent problem is reduced to (2.13) but (2.14) does not constrain the choice. Therefore the statement of Equivalence Theorem holds for corruption equilibrium with markets (CEM) if one eliminates the inequalities (2.19).

Suppose A1-A3 obtain. First order conditions for (3.1)-(3.3) at equilibrium as well as for (2.20), (2.18) have the form of equalities (compare (2.16))

$$f'_i(m^{\sim}_i) = \lambda(\mu^{\sim}(.)) \tag{3.4}$$

where $\mu^{\sim}(.) = (m^{\sim}_i, i \in I)$ is CEM- allocation and

$$\lambda(\mu(.)) = \sigma \Sigma f_i(m_i)\Gamma\Gamma(M - m)/(M - m)(1 - s). \tag{3.5}$$

CEM- money allocations $(m^{\sim}_i, i \in I)$ have a remarkable property. Define the feasible initial endowments $(M_i, i \in I)$ such that $\Sigma M_i = M$, and consider the total corruption equilibrium (CE) aggregate output $Y_F = \Sigma F_i(m^{\wedge}_i, g^{\wedge})$ and the aggregate CE bribe spending $H = M - \Sigma m^{\wedge}_i$ as functions of initial endowments.

Proposition 2. Assume A1-A3 obtain and assume j is strictly concave, then the quantity $g^{\wedge}$ of public good, the value $\Sigma f_i(m^{\wedge}_i)$, and CE output $Y_F = \Sigma F_i(m^{\wedge}_i, g^{\wedge})$ reach their maxima and CE bribe spending $H = M - \Sigma m^{\wedge}_i$ reaches its minimum when $M_i \geq m^{\sim}_i$, $i \in I$. In this case, CE allocations coincides with CEM allocations.

Proof. If some F.O.C. are satisfied as strict inequalities, then initial endowments can be redistributed such that $\Sigma f_i(m^{\wedge}_i)$ increases and H decreases. In the new CE allocation, $g^{\wedge}$ is larger by both (2.20) and strict concavity of j. Therefore Y_F is also larger. Hence if Y_F reaches its maximum, then all F.O.C. are satisfied at equality, and CE allocation coincides with CEM allocation.

Proposition 3. Assume A1-A3 obtain and assume j be strictly concave, then every producer prefers a CEM to a corruption equilibrium with the same inital endowments M_i, i.e.

$$F_i(m^{\sim}_i, g^{\sim}) + \Lambda(\mu^{\sim}(.), g^{\sim})(M_i - m^{\sim}_i) \geq F_i(m^{\wedge}_i, g^{\wedge}) + \Lambda(\mu^{\wedge}(.), g^{\wedge})(M_i - m^{\wedge}_i) \qquad (3.6)$$

for all i (see (2.13), (2.15)).

To prove the proposition, let us consider (2.15) and let us write (3.6) in an equivalent form

$$(f_i(m^{\sim}_i) + \lambda(\mu^{\sim}(.))(M_i - m^{\sim}_i))\varphi(g^{\sim}) \geq (f_i(m^{\wedge}_i) + \lambda(m^{\wedge}(.))(M_i - m^{\wedge}_i))\varphi(g^{\wedge}) \qquad (3.7)$$

where λ is defined by (3.5). A3 and Proposition 2 imply that $\varphi(g^{\sim}) \geq \varphi(g^{\wedge})$ and that $\lambda^{\sim} = \lambda(m^{\sim}(.)) \geq \lambda^{\wedge} = \lambda(m^{\wedge}(.))$. The inequality (3.6) is valid since

$f_i(m^{\sim}_i) + \lambda^{\sim}(M_i - m^{\sim}_i) \geq f_i(m^{\wedge}_i) + \lambda^{\sim}(M_i - m^{\wedge}_i) \geq f_i(m^{\wedge}_i) + \lambda^{\wedge}(M_i - m^{\wedge}_i)$.

The main goal of this paper is to analyze determinants of corruption in framework of the model described. First of all we define corruption-free equilibrium and ask if it is Pareto-superior to corruption regimes.

II. 4. Corruption- free equilibrium: the problem of social stability

Let us define a concept of equilibrium without corruption but with markets or **corruption- free equilibrium (CFE)**. If corruption is suppressed and credit market exists then the producer problem is transformed into the following model (compare (3.1)-(3.3))

$$\max\ (1-\sigma)F_i(m_i, g) + p(M_i - m_i) \quad \text{w. R. To } m_i\,, \tag{4.1}$$

where the price p of credit is chosen in order the solutions m^*_i of the problem, (4.1) be balanced: $\Sigma m^*_i = M$, and $g^* = \sigma\Sigma F_i(m^*_i, g^*)$.

If A3 is fulfilled, then the CFE allocation is the solution of the following problem,

$$\max\ \Sigma(m_i),\ \Sigma m_i = M\,.$$

To be short we will say sometimes that a producer " votes against corruption" if the CFE-value of her/his welfare function is not inferior to the CEM-value, respectively that she/he "supports corruption" in the opposite case.

Our purpose is to describe the conditions under which a corruption -free regime is Pareto- superior to a corruption equilibria. In this case the CFE can be considered socially stable. If this condition prevails then there is a hope that a corrupted economy might transform into a corruption-free one.

Let us consider cost function c(y) which is the inverse of the production function y = f(m). Denote by e_i the elasticity of the marginal cost function corresponding to f_i . One can check that

$$e_i = c_i'' y_i / c_i' = -f_i'' f_i / (f_i')^2.$$

In what follows, we asume that

$$\varphi = g^{\alpha},\ \alpha > 0. \tag{4.2}$$

The following theorem is the main result of the Part 2.

Theorem 4. Assume A1-A3 be valid, elasticities e_i of marginal cost functions $e_i \le 1$, and

$$\sigma \le K(\gamma, \alpha) = [1 - (1-\gamma)^{\alpha/(1-\alpha)}] / [1 - (1-\gamma)^{1/(1-\alpha)}] \tag{4.3}$$

where γ is a CEM value of the bribe proportion. Then all agents prefer the CFE to the CEM and hence, to the corruption equilibrium with the same initial amounts of money M_i.

We prove this statement in Section 6 after considering of a special "limit case". The function $K(\gamma,\alpha)$ increases with respect to γ and reaches its infimum α at $\gamma = 0$. Thus criterion (4.3) can be broken even under maximal punishment, if

$$\sigma > \alpha.$$

However one should note that the Government problem can be formulated as the maximization of the following social welfare function

$$(1 - \sigma)\Sigma f_i(m_i)\ \varphi(\sigma\Sigma f_i(m_i)g) = (1 - \sigma)\sigma^{\alpha/(1+\alpha)}\ (\Sigma f_i(m_i))^{(2-\alpha)/(1+\alpha)}$$

under $\Sigma m_i = M$ with respect to m_i and σ. The problem is separated into two distinct problems: a) maximization of the output $\Sigma f_i(m_i)$ and b) maximization of the function $(1 - \sigma)\sigma^{\alpha/(1+\alpha)}$ with respect to σ. The solution of the latter is equal to $\sigma = \alpha$. Therefore Theorem 4 entails the following important statement.

Theorem 5. Let A1-A3 and (4.2) be valid, $e_i \leq 1$, and tax rate σ is not larger than its optimal value $\sigma = \alpha$. Then the condition (4.3) holds, and all agents vote against corruption.

Thus under assumption $e_i \leq 1$ the agent may support corruption only if the tax rate is above its optimal level i.e. if the state pretends to be more influential than it is entailed by existing technology of public service production . In the last case moderate corruption (not very large γ) can be preferable for some or even for all producers. To understand the situation let us take a close to zero. Then increase of public service g above 1 gives a small production effect, so that tax extraction turns out to be a loss for the economy. Therefore the producers prefer to get back a part of the lost money through corruption. Only if the corruption is large enough to fulfill (4.3) then its negative effect outweighs and the producers vote against corruption.

The following example illuminates the situation.

Example 1: initial allocation is optimal, $e_i \le 1$ but all producers gain from corruption since (4.3) is not fulfilled.

Let production function be equal for two producers: $f_i(m) = m^{1/2}$, $i = 1,2$. It means $e_i = 1$. We put $\gamma = \sigma = 1/2$; $M = 32$, $M_i = m_i^* = 16$. For the CF equilibrium we have

$$W_i^{CF} = (1 - \sigma)(g^*)^\alpha[f(m_i^*) + \lambda^*(M_i - m_i^*)] = 2(g^*)^\alpha$$

and $$g^* = \sigma 2\,(m^*)^{1/2}(g^*)^\alpha \text{ so that } g^{*\alpha} = (4)^{\alpha/(1-\alpha)}.$$

Obviously $m_i = m$, $i = 1,2$ for the CEM equilibrium. Hence

$$\lambda = (\sigma Y\gamma)/H(1 - \sigma) = m^{1/2}/(M - 2m) = f_i'(m) = 1/2m^{1/2},$$

therefore $m = M/4 = 8$, and $W_i^{CEM} = (1 - \sigma)g^\alpha\,[m^{1/2} + \lambda(M_i - m)] = (3\sqrt{2}/2)g^\alpha$, where $g = (1 - \gamma)\,\sigma\,2\,g^\alpha m^{1/2} = g^\alpha\sqrt{2}$ so that $g^\alpha = (\sqrt{2})^{\alpha/(1-\alpha)}$. If α is small enough then

$$(g^*)^\alpha \cong g^\alpha \cong 1, \text{ and } W_i^{CF} \cong 2 < W_i^{CEM} \cong 3\sqrt{2}/2.$$

One can note that $e_i = 1$ in this example which is clearly not crucial. Condition (4.3) holds also if the tax rate is larger than its optimal level, but corruption level is large enough so that the corruption activity turns out to be inferior.

There is a common belief that law enforcement is needed to enhance efficiency and to diminish transaction costs. Example 1 and Theorem 4 show that it is not always true (if one says just about a production criterion and does not take into account a moral damage from corruption.)

One can show that the marginal cost elasticity (MCE) condition of Theorem 4 is also substantial (see Savvateev (1997) , Polishchuk and Savvateev (1997) and Example 2 of the next section). If MCE condition is not fulfilled then strong law enforcement can be necessary to avoid corruption.

The MCE condition needs to be discussed in greater details. First of all, note that the "corruption technology " is linear in our model : the allocation of subsidies among producers is proportional to the amount of bribe money paid. The MCE is equal to zero if a production function is linear. One may assume that a relation between MCE's of the production and corruption technologies does matter. This observation leads to the following idea: to fight corruption one needs not just to increase punishment strength (decreasing g) but to change the competitive (linear) corruption mechanism. We do not develop this idea here.

For positively homogeneous functions, the value of MCE depends inversely on the degree of homogeneity and hence reflects "efficiency" under large input values. Probably fast technical progress supports low MCE levels. The following simple argument based on Arrow's learning by doing idea explains why this could be the case. The MCE is equal to $1/\alpha - 1$ for $F(K, L) = K^{\alpha}(A\ L)^{\beta}$ if labor quality A is a constant. But the MCE decreases if, due to technical progress, the labor quality positively depends on capital accumulated: $A = K^{\zeta}$.

To prove Theorem 4, we consider first the following "limit case", which has an interest of its own.

II. 5. A "limit case": no law enforcement, no public service

Let us consider the case of no law enforcement and no public service: $\gamma = 1$, $\alpha = 0$. Then the producer i utility function $V_i(\sigma)$ has the following CEM value (see (3.1),(3.2), (2.9), (2.6))

$$V_i(\sigma) = (1 - \sigma)f_i(m_i) + \sigma r(M_i - m_i), \tag{5.1}$$

where $r = 1/q^{\sim}$,

$$\sigma r = \sigma Y/H = (1 - \sigma)f_i'(m_i), \tag{5.2}$$

$$Y = \Sigma f_i(m_i), \ H = M - \Sigma m_i , \tag{5.3}$$

and m_i, $i \in I$, are CEM-values.

In our original notation $m_i .= m_i^{\sim}$. We omit the symbol $^{\sim}$ to simplify notation. In the "limit case" punishment for corruption is not effective at all and tax collection is distributed totally through bribes. The model considered in Polishchuk and Savvateev (1997) can be interpreted as this "limit case" under additional assumption that all agents have identical production functions.

Let us prove the statement of Theorem 4 for this case[10]. Let (m_i^*, p^*) be CF-equilibrium . Note that if $\sigma \to 0$ then $m_i \to m_i^*$ due to Equivalence Theorem. In view of (5.2) $\sigma r \to f_i'(m_i^*) = \lambda^*$. Therefore

$$V_i^* = f_i(m_i^*) + \lambda^* (M_i - m_i^*) = \lim_{\sigma \to 0} V_i(\sigma).$$

[10]The left hand side of (4.3) is indefinite under $\gamma=1$, $\alpha=0$. The considerations below will show that it has to be taken as 1 for this case. (One can imagine that $1-\gamma$ approaches zero much faster than α, for example $\gamma = 1 - \exp(- 1/\alpha^2)$).

To prove Theorem 4 we calculate $V_{i\sigma}' = dV_i/d\sigma$. It turns out that the derivative $V_{i\sigma}'$ is negative if $e_i \leq 1$ for all i. Therefore V_i reaches its maximum at $\sigma = 0$, that proves the statement.[11]

In fact we will be able to get a little bit stronger result: it is enough that $e_i \leq 1/(1-\sigma)$ for $V_{i\sigma}'$ to be negative. Under this condition a small decrease of the appropriated proportion σ is Pareto-improving.

Let $Q = \sigma r$. Rather straightforward calculations entails successively the following formulas[12]

$$r/(1 - \sigma) + \sigma r'_\sigma = (1 - \sigma)f_i''(m_i)m_{i\sigma}' ; \tag{5.4}$$

$$r_\sigma' = r\, \Sigma m_{i\sigma}' / H(1 - \sigma); \tag{5.5}$$

$$r_\sigma' = r^2\, \eta / (1 - \sigma)[(1 - \sigma)^2 H - \sigma r\eta], \tag{5.6}$$

$$\text{where} \quad \eta = \Sigma 1/f_i''(m_i); \tag{5.7}$$

$$Q_\sigma' = r + \sigma r_\sigma' = [(1-\sigma)Y + \Sigma(f_i')^2/f_i''] / [(1-\sigma)H - \Sigma f_i' / f_i'']. \tag{5.8}$$

Formulas (5.4) – (5.8) are valid independently on conditions concerning elasticity of marginal costs e_i.

If $e_i = - f_i'' f / (f_i')^2 \leq 1/(1-\sigma)$ we have $\Sigma(f_i')^2/f_i'' = - \Sigma f_i/e_i \leq -(1-\sigma)Y$, therefore $Q_\sigma' \leq 0$.

Using (5.2) one has

$$V_{i\sigma}' = -f_i(m_i) + Q_\sigma'(M_i - m_i). \tag{5.9}$$

If $M_i - m_i \geq 0$ then $V_{i\sigma}' < 0$, and the statement is proved.

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[11]There is a way to prove the statement through simpler calculations. But our method permits us to make other useful conclusions.

[12]One can get (5.4) and (5.5) making use from (5.2) after differentiation of identities $\sigma r = (1 - \sigma)f_i'(m_i)$ (see (5.2)) and $\ln r = \ln Y - \ln H$ with respect to σ. The equality (5.6) follows from (5.4) and (5.5). Equality (5.8) can be received if one substitutes (5.6) in the identity $Q_\sigma' = r + \sigma r_\sigma'$ and uses (5.2) again.

The case $M_i - m_i < 0$ does not require the condition $e_i \le 1/(1-\sigma)$. We have

$$Q_\sigma' = (\sigma r)' = ((1 - \sigma)f_i'(m_i))' = -f_i' + (1 - \sigma)f''m_\sigma' \ge -f_i' , \qquad (5.10)$$

since

$$m_\sigma' < 0 \qquad (5.11)$$

in view of (5.4) - (5.7).[13] Therefore $Q_\sigma'(M_i - m_i) \le - f_i'(M_i - m_i)$ if $M_i - m_i < 0$. Let us use (5.9) and concavity of the production function f.

We have

$$V_{i\sigma}' \le - f_i(m_i) - f_i'(m_i)(M_i - m_i) \le -f_i(M_i) < 0. \qquad (5.12)$$

It completes the proof of Theorem 4 for the limit case.

Now it is simple to check that the following statements are valid.

Proposition 4. Let $\gamma = \Gamma(m_b) \equiv 1$ and $\sigma = \sigma^0$ is fixed. If a producer purchases additional amount of money m_i at a CEM equilibrium to use it in production (i.e. $M_i - m_i \le 0$) then her/his welfare function increases as σ diminishes from σ^0 up to zero. The producer votes against corruption.

The Proposition 4 follows from (5.1) and (5.11).

Proposition 5. Let $\gamma = \Gamma(m_b) \equiv 1$, and $e_i \le 1/(1-\sigma)$ for all i. Then producers' welfare functions decrease in a small neighborhood of σ, and one can diminish the proportion of appropriated quantity to reach a Pareto- improvement . If, moreover, $e_i \le 1$ then all producers vote against corruption.

Corruption can root in both types of distortions: wrong initial resource allocation or wrong tax policy. Proposition 5 shows that the special conditions for production functions can compensate both types of distortions in the "limit case".

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[13]Indeed $r_\sigma' < 0$ due to (5.6),(5.7). Therefore $\Sigma m_{i\sigma}' < 0$. But (5.4) entails that all $m_{i\sigma}$ have the same sign.

The “limit case” gives a good opportunity to check the supposition that inefficiency of the tax policy can support corruption. Let us consider the case of optimal initial resource allocation: $M_i = m_i^*$, m_i^* are CFE value. The following proposition is an evident consequence of (5.8), (5.9).

Proposition 6. If $M_i = m_i^*$ and σ is small ($\sigma \approx 0$) or large enough ($\sigma \approx 1$) then all producers vote against corruption.

Indeed, if $\sigma \approx 0$ then $m_i \approx M_i = m_i^*$. Due to (5.11) $m_i \le m_i^*$ and Q_σ' is bounded from above: $Q_\sigma' \le Y/H$. Therefore $V_{i\sigma}' < 0$ due to (5.9).

If $\sigma \approx 1$ then $m_i \approx 0$, $Y \approx 0$, $r \approx 0$, $H = M - \Sigma m_i \approx M$. Hence $Q_\sigma' \le Y/H$, Y/H is a small quantity, and V_i are small for all i, $V_i \le V_i^*$. This proves Proposition 6.

If all producers have the same production functions, $f_i(m_i) = f(m_i)$, then $m_i = m$, $m_i^* = m^*$, $r = Y/H = f(m)/(m^* - m)$, and (5.1) entails that all producers vote against corruption (see also Polishchuk, Savvateev (1997)). One can suppose that the same is true for arbitrary σ. But this guess is not valid as the following example demonstrates.

Example 2. Some agents may prefer corruption even if initial resource allocation is optimal.

Let $f_i(m_i) = h_i - 1/m_i$, $i = 1,2$; $M = h_1 + h_2$; $\sigma = 1/2$. Then $m_1^* = m_2^* = M/2$, and

$$V_1^* = f_1(m_1^*) = h_1 - 2/M, \qquad (5.13)$$

$f_i'(m_i) = r$, therefore $m_i = 1/\sqrt{r} = 1$, $r = [h_1 - 1 + h_2 - 1]/[M - 1 - 1] = 1$. We have

$$V_1 = (1 - \sigma)f_1(m_1) + \sigma r(m_1^* - m) = 3h_1/4 + h_2/4 - 1. \qquad (5.14)$$

If one takes $h_2 \geq 4 + h_1$ then $V_1 > V_1^*$ and the first agent prefers the corruption regime.[14]

One has to take into account that the optimal tax level is equal to zero when public service is absent. The elasticity of marginal costs is bigger than 1 in this example. Therefore the economy is sensible to the policy mistakes. The corruption advantage is rooted in the tax policy imperfection.

II. 6. Proof of Theorem 4 and some comparative statics

<u>Proof</u>. Due to Proposition 3 CEM is Pareto superior to every CE, therefore one has to compare CFE and CEM. For notation convenience in this proof we will omit symbol ~ in notation of CEM parameters. We will prove that

$$W_i^{CF} = (1 - \sigma)\, \varphi(g^*)[f_i(m_i^*) + \lambda^*(M_i - m_i^*)] \geq W_i^{CEM} =$$
$$(1 - \sigma)\, \varphi(g)[f_i(m_i) + \lambda(M_i - m_i)] \qquad (6.1)$$

where

$$\lambda^* = f_i'(m_i^*), \qquad (6.2)$$

$$\lambda = f_i'(m_i) = (\sigma Y \gamma) / H(1 - \sigma), \quad H = M - \Sigma m_i\, ; \; Y = \Sigma f_i(m_i), \qquad (6.3)$$

and
$$g^* = \sigma \Sigma f_i(m_i^*)\, \varphi(g^*), \qquad (6.4)$$

$$g = \sigma(1 - \gamma)\Sigma f_i(m_i)\, \varphi(g)\,. \qquad (6.5)$$

It follows from (6.3) that

[14] The functions f_i do not fulfill condition $f_i(0) = 0$ which is usually demanded. But the same considerations are valid for the following smooth and concave functions:
$f_i(m) = mh_i^2/3 - m^2h_i^3/27$ if $0 \leq m \leq 3/h_i$, and $f_i(m) = h_i - 1/m_i$ if $m \geq 3/h_i$.

$$1 - \sigma = (1 - \sigma(1 - \gamma))\, Y/(\lambda H + Y). \qquad (6.6)$$

From (6.4) and (6.5) we get in view of Proposition 2

$$(g^{*} / g\,) \geq 1/(1 - \gamma)^{1/(1-\alpha)}.$$

Therefore and due to homogeneity of φ one has using (4.3)

$$\eta = (1 - \sigma)\, \varphi(g^*) / [(1 - \sigma(1 - \gamma))\, \varphi(g)] \;\geq$$

$$(1 - \sigma) / [(1 - \gamma)^{\alpha/(1-\alpha)}(1 - \sigma(1 - \gamma))] \;\geq\; 1. \quad (6.7)$$

Let us denote $\qquad \nu = \sigma\gamma[1 - \sigma(1-\gamma)]^{-1}. \qquad (6.8)$

From (6.7) we have $\qquad \varphi(g^{*})/\varphi(g) \geq [1 - \sigma(1 - \gamma)]/(1 - \sigma) = 1/(1 - \nu).$

Let λ be given by (6.3) and let $\qquad r = Y/H. \qquad (6.9)$

Then $\qquad (1 - \nu)\lambda = \sigma\gamma(1 - \nu)r/(1 - \sigma) = \nu r\, . \qquad (6.10)$

Therefore (6.1) can be written as

$$V_i^{*} = f_i(m_i^{*}) + \lambda^{*}(M_i - m_i^{*}) \geq V_i = (1 - \nu)f_i(m_i) + \nu r(M_i - m_i) \qquad (6.11)$$

where $\nu r = (1- \nu)f_i'(m_i) = \nu Y/H$ due to (6.9), (6.10) and (6.3), and

$$\lambda^{*} = f_i'(m_i^{*}). \qquad (6.12)$$

The problem to prove inequality (6.11) under the condition (6.12) coincides with the problem considered in Section 5 if one substitutes ν for σ. The difference is that now ν depends on m_i since $\gamma = \Gamma(M - \Sigma m_i)$. Nevertheless an analogue of

Proposition 5 is applicable to compare CEM and CFE- values of the welfare functions. It entails Theorem 4.

Due to Proposition 4 we conclude that every producer i with $M_i - m_i < 0$ votes against corruption if (4.3) holds. Proposition 6 and (6.8) lead to the conclusion that elasticity conditions can be avoided if the corruption level is small and tax rate and initial allocation are optimal.

Proposition 6'. If $M_i = m_i^*$, γ is small enough and σ is optimal, $\sigma = \alpha$ then all producers prefer corruption - free equilibrium.

To study the influence of the tax rate σ let us assume that $\Gamma(x)$ is an increasing function. Since $\Gamma(x)/x$ is decreasing and A3 is valid one can derive from (2.20) that its solution m_i is a decreasing function of σ. Hence $\gamma = \Gamma$ is an increasing function of σ. Therefore ν increases from zero to one as σ grows. Let the conditions of Theorem 4 hold. Then the welfare functions of producers are decreasing functions of σ due to Proposition 5. This conclusion is not surprising since under our assumption increasing of the tax rate entails higher proportion of the bribes.

Let us consider the results of strengthening of the law enforcement. Let $\Gamma_1(x) > \Gamma_2(x)$ for all x. Using F.O.C. for the problem (2.20), (2.18) one can derive the expected result for CEM equilibrium: all m_i are higher for Γ_2 , equilibrium value of γ is lower, ν is lower and all producers gain under the conditions of Theorem 4. It is clearly not necessary true if the conditions are not valid.

II.7. Concluding remarks

One can ask whether some countries are underdeveloped because their government are not able to suppress RS activities, or RS activities prevail in these countries because they are underdeveloped and use inefficient technology and administration. An answer is very important since it defines a rational strategy of governance. If RS is responsible for low wealth then strengthening law enforcement may be the main issue. But if RS is a consequence of a wrong macroeconomic policy and low productivity then quite different measure may be effective to diminish the RS intensity and rise the wealth: one should improve technologies of production and decision making. The considerations above show that both directions of the causality are important. Therefore campaigns of fighting RS and corruption may be counter-productive if they do not include improvements of the macroeconomic policy. A balanced strategies are needed to enhance attractiveness of investment into production.

The notion of corruption equilibrium was defined above for a special kind of corruption activity connected with government spending for industrial public service and with bribe competition for subsidies. For this setting, we demonstrate that support of corruption is connected not only with imperfections of initial resource allocation, but also with non-optimal tax policies, and that an efficient policy itself does not guarantee social stability of corruption-free equilibria. The conditions were described that entail Pareto-superiority of corruption-free regimes.

These results seem to be important for the understanding of a two-side connection between rent seeking and economic growth. On the one hand rent seeking hampers economic growth. On the other hand quantitative growth with slow change in technologies can entail increase of marginal cost elasticity under large inputs in view of exhaustion of extensive growth factors. Technical progress diminishes this elasticity and creates incentives to dismantle counter-productive regimes. It leads to a testable hypothesis that corruption and RS have to be intensive not only in young low efficient economies, but also in old stagnating systems.

The comparative statics of corruption equilibria was investigated here under very restrictive assumptions. More efforts should be done to understand the behavior of the economic system when conditions of Theorem 4 are not fulfilled and corruption can be supported by some producers. An important task is also the incorporation of a more general RS mechanism, studied in Part II, in a growth model.

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