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1. INTRODUCTION

The theory of manipulation studies how agents choose their announcements about personal characteristics in order to influence the outcome of a given game. Research in this area has focused attention on manipulation of utility functions. Hurwicz (1985) and Thomson (1985) provide good surveys of this literature. In contrast, the theory of how firms manipulate remains largely unexplored (but see Weitzman (1976), Alkan-Sertel (1981), Vickers (1985) and Koray-Sertel (1988)). The question of manipulation in an oligopolistic framework has been studied by Sklivas (1987) and Fershtman and Judd (1987) (F-J in the sequel). In these papers owners design managerial incentives in order to manipulate the outcome of a market game. Since Sklivas' results are a special case of F-J we will concentrate on the latter. F-J show that under quantity (resp. price) setters managers a Manipulative Equilibrium in which owners maximize expected profits involves that a positive (resp. negative) weight is given to sales. Also the equilibrium output is greater than the output it would occur if firms maximized expected profits acting as quantity-setters in a one shot game. F-J's acknowledge that "the analysis made several simplifying assumptions. ... Further research should generalized our analysis" (p. 940). Their assumptions include linear demand, constant returns to scale, duopoly, risk-neutral and profit maximizer owners and that uncertainty affects in a special way either demand or cost functions. In the later case they do not obtain any clear result (p. 935).

The purpose of this note is to generalize the model presented by F-J, allowing for non linear functions, an arbitrary number of players and more general preferences and risk attitudes. We also generalize F-J set up since we allow each firm to choose a one dimensional parameter which can be interpreted as a measure of the aggressivity of the firm in the second stage of the game.

Our main result is that strategic substitution and strategic complements yield opposite results. In fact, the role of manipulation is to reverse the result obtained in the second stage: If the second stage presents a certain degree of implicit collusion (as it happens under strategic substitution) manipulation makes the outcome more competitive. But if the second stage is very competitive (i.e. strategic complementarity) manipulation provides opportunities for implicit collusion. Thus the effect of opening opportunities for manipulation may, or may not, improve resource allocation.

2. THE MODEL AND THE RESULTS

There are n firms. Let x_i ($\in \mathbb{R}_+$) be the action of firm i = 1,...,n. Let x_{-i} be a vector of actions with the i^{th} component deleted. Uncertainty is represented by the set of possible states of the world A with finite cardinality s. Let a be a typical element of A. Firm i is assumed to belong to some type, say t_i which belongs to an abstract set of types T_i. A type specifies the payoff function $U_i(\ )$ of firm i. U_i: $R^n_+ \times A \times T_i \longrightarrow R_+$. This function may be derived from basic data like demand and cost functions. Attitudes towards risk are represented by a s-dimensional vector of subjective probabilities $(q^1_i, ..., q^s_i)$. Agents are assumed to be expected utility maximizers. Let us denote by $U_i(\ ,t^o_i)$ the true payoff function of agent i. Let $T = \times_{i=1}^{n} T_i$ be the type space and let $t = (t_1, ..., t_n)$ be an element of T. t_{-i} denotes a n - 1 dimensional vector in which t_i is deleted.

The game is played in two stages. In the first stage agents announce simultaneously their types not knowing which state of the world will occur. In the second stage uncertainty is removed and agents choose their actions.

Definition 1. *A Nash Equilibrium in the second stage of the game (NE) is a* $(x'_1, ..., x'_n)$ *such that* x'_i *maximizes* $U_i(x_i, x'_{-i}, a, t_i)$, $\forall i$, *given t and a.*

A NE defines a mapping e: $A \times T \longrightarrow R^n_+$, where e(t, a) is the vector of NE actions for given t and a. We will see that under our assumptions e() is single-valued. Let $W_i(t) \equiv \sum_{a \in A} q^a_i \ U_i(e(a, t), a, t^o_i)$ be the (indirect) utility function of agent i. Now let us introduce our main equilibrium concept.

Definition 2. t^* *is said to be a Manipulative Nash Equilibrium (MNE) if* $\forall$ *i = 1,...,n,* t^*_i *maximizes* $W_i(t_i, t^*_{-i})$ $t_i \in T_i$.

In other words, a MNE is a Subgame Perfect Nash equilibrium in which types are the strategies of the game.

We now study the effects of manipulation on resource allocation. This is done by means of two models. In the first one actions are aggregated such that payoffs of each firm can be written as a function of its own action and the sum of the actions of all players. This generalizes the usual model of homogeneous oligopoly in which firms set quantities in the second stage of the game and quantities are **strategic substitutes** (see Bulow, Geanakoplos, Klemperer (1985)) in the sense that an increase in the quantities of all firms except, say, i causes a decrease in the optimal quantity produced by i. The second model generalizes an heterogeneous oligopoly model in which firms set prices in the second stage of the game and prices are **strategic complements** (see Bulow, Geanakoplos, Klemperer (1985)) in the sense that an increase in a price set by a player other than i causes an increase in the price set by i.

We now list assumptions corresponding to the homogeneous oligopoly model.

1.- All relevant functions are twice continuously differentiable.

2.- All allocations which will be considered are interior.

3.- The payoff function for i is $U_i(x_i, x, a, t_i)$, $t_i \in R$, where $x = \sum_{i=1}^{n} x_i$.

4.- $\delta U_i / \delta x < 0$.

In order to simplify notation let:

$$\frac{\delta U_i(x_i, x, t_i, a)}{\delta x_i} + \frac{\delta U_i(x_i, x, t_i, a)}{\delta x} \equiv R_i(x_i, x, t_i, a) \quad (1)$$

5.- $R_i(\ )$ is strictly increasing on t_i, for given a, x_i and x.

6.- $R_i(\ )$ is strictly decreasing on x_i given x and on x given x_i, for given a and t_i.

7.- $\forall$ a $\in$ A, $t_i \in T_i$ and $x_{-i} \in R_+$, $\exists\ \bar{x}_i$ such that $\forall x_i \geq \bar{x}_i$,

$$U_i(x_i, \bar{x}_i + x_{-i}, a, t_i) \leq U_i(0, x_{-i}, a, t_i).$$

A.3 is twofold. On the one hand it assumes that t_i is a real number. On the other hand it says that actions affect payoffs in an additive way. A.4 corresponds to the assumption of a decreasing demand curve in the standard oligopoly model. A.5 means that the x_i that maximizes $U_i(\ )$ is increasing on t_i. Thus t_i measures the aggressivity of firm i. A consequence of A.6 is that $U_i(\ )$ is concave on x_i (see Friedman (1982) p. 496). When p( ) and $c_i(\ )$ are linear, as assumed by F-J, A.6 is automatically satisfied. Finally, A.7 is a boundness assumption. In the Cournot model it corresponds to the assumption that for x_i large enough $p(x) < c_i(x_i)$. A model satisfying A.1-7 will be called a **homogeneous oligopoly model.** Then, we are ready for our first result.

Theorem 1: *In the homogeneous oligopoly model* $\forall i = 1,\ldots,n,\ t^*_i > t^O_i$.
Proof: *We divide the proof of the Theorem in four steps.*

<u>First Step.</u> *e( ) is nonempty-valued, i.e. there is a NE in the second stage of the game.* <u>Proof:</u> *The objective function of each firm is continuous and concave in the variable controlled by this firm. Its strategy set is convex and can be taken to be compact. Therefore the usual argument (e.g. Friedman (1982) p. 496 implies the existence of a Nash Equilibrium.*

<u>Second Step.</u> *e( ) is single valued, i.e. there is a unique NE in the second stage of the game.* <u>Proof:</u> *Suppose it is not. Therefore there are, at least, two different NE. Let us denote them by the superindex 1 and 2. First notice that* $x^1 = x^2$ *is impossible since* $R_i(\ )$ *strictly decreasing on* x_i *will imply that* $x^1_i = x^2_i\ \forall\ i = 1,\ldots,n$, *contradicting that these equilibria are distinct. Then, without loss of generality let us assume that* $x^1 > x^2$. *But assumption 7 again implies that* $x^1_i < x^2_i$ *which is impossible by the definition of x.*

<u>Third Step.</u> $e_i(\ )$ *is increasing on* t_i *and decreasing on* $t_j\ \forall j \neq i$. <u>Proof:</u> *Let J be the matrix with typical elements* $a_{ii} = \partial R_i/\partial x_i + \partial R_i/\partial x$ *and* $a_{ij} = \partial R_i/\partial x\ \forall\ j \neq i$. *Then differentiating first order conditions of a NE we get:*

$$J.u = s$$

where u is a column vector with typical element $\partial x_j/\partial t_i$ *and s is a column vector with first element* $-\partial R_i/\partial t_i$ *and zeros in the rest. Tedious calculations show that the determinant of J is non-vanishing and the inverse matrix of J is a Z-matrix (see Berman & Plemmons (1979)). Application of Cramer's rule yields the result.*

<u>Fourth Step.</u> *Let us assume that* $t^O_i \geq t^*_i$. *Then, equation (1) and assumption 6 imply that* $R_i(x^*_i, x^*, a, t^O_i) \geq 0$. *But first order condition for a MNE are*

$$\Sigma\ q^a_i\ (R_i(x^*_i, x^*, a, t^O_i).\partial e_i/\partial t_j + \Sigma\ q^a_i.\partial U_i\ (x^*_i, x^*, a, t^O_i)/\partial x\ .\ \partial e_j/\partial t_i = 0$$

and from the result obtained in step 3 we reach a contradiction.

Now we present our second model. Assumptions 1, 2, 5 and 7 will continue to hold. The new assumptions are the following:

3'.- The payoff function can be written as $U_i(x_1,\ldots,x_n, a, t_i)$, $t_i \in R$.

4'.- $U_i(\ )$ is strictly increasing on $x_j\ \forall i \neq j$.

5'.- $R_i(\ )$ is strictly decreasing on t_i.

6'.- $R_i(\ )$ is strictly decreasing on x_i strictly increasing on $x_j\ \forall j \neq$ and $\partial R_i/\partial x_i + \Sigma\ |\partial R_i/\partial x_j|_{j\neq i} < 0$.

The last part of A.6' is identical to Assumption 6 in Friedman ((1982) p. 504). A.3' is weaker than A.3 since it does not require that the actions of the other players can be aggregated. A model satisfying assumptions 1, 2, 3', 4', 5', 6' and 7 will be called a **heterogeneous oligopoly model.**

Theorem 2: *In the heterogeneous oligopoly model* $\forall i = 1,...,n,\ t_i^* < t_i^O$.
Proof: *The proof follows the lines of the proof of Theorem 1. The existence and uniqueness of a NE can be proved in the same way. In this case the matrix* $-J$ *is a P-matrix and therefore* J^{-1} *has only negative elements (see Berman & Plemmons) and therefore* $\partial e_i/\partial t_j < 0\ \forall\ i,j$. *The last part of the Theorem follows by a similar argument than in Theorem 1.*

Finally, we compare the sum of actions (i.e. x) in a MNE and in a NE in which firms are truthful. First let us define this last concept.

Definition 3. *A truthful NE (TNE)* $(x_1,..., x_n)$ *is a NE for* $t = t^O$.

Theorem 3: *If* $\partial e_i/\partial t_i$ *is independent of the state a in the the homogeneous oligopoly model then* $x < x^*$.
Proof: *First order conditions of a TNE read* $\sum q_i^a\ R_i(x_i, x, t_i^O, a) = 0$
Therefore if $x^* \leq x$ *there is a firm, say i, such that* $x_i^* \leq x_i$. *But then,* $\sum q_i^a\ R_i(x_i^*, x^*, t_i^O, a) > 0$. *The first order condition of a MNE as written at the end of the proof of Theorem 2.*

$$\sum q_i^a\ R_i(x_i^*, x^*, a, t_i^O).\partial e_i/\ \partial t_i + \sum \partial U_i\ (x_i^*, x^*, a, t_i^O)/\ \partial x\ .\ \partial e_j/\ \partial t_i = 0$$

$$\sum q_i^a\ R_i(x_i^*, x^*, a, t_i^O).\partial e_i/\ \partial t_i = -\sum q_i^a\ \partial U_i\ (x_i^*, x^*, a, t_i^O)/\ \partial x\ .\ \partial e_j/\ \partial t_i$$

By the assumption on the outcome function

$$\sum q_i^a\ \partial U_i(x_i^*, x^*, a, t_i^O)/\partial x\ (\partial e_j/\ \partial t_i) = (\sum q_i^a\ R_i(x_i^*, x^*, a, t_i^O))\partial e_i(t^*, a)/\partial t_i$$

Since $\sum q_i^a\ R_i(x_i^*, x^*, a, t_i^O) \geq 0$ *we have obtained a contradiction since the RHS of the above equality is negative and the LHS is positive.*

Theorem 4: *In the heterogeneous oligopoly model if* $\partial\ e_i/\ \partial x_i$ *are independent of a, the effect of* x_i *dominates on the effect on x and assumption 3 holds then* $x^* > x$*, i.e. the sum of strategies increases under manipulation.*
Proof: *First order condition of a TNE are*

$$\sum q_i^a\ R_i(x_i, x, a, t_i^O) = 0.$$

Therefore if $x^* \leq x^o$, *there is a firm, say i, such that* $x^*_i \leq x^o_i$. *Then, the above expression, evaluated at* (x^*_i, x^*) *will be greater than zero since the effect of* x_i *dominates on the effect on x. By the first order condition of a MNE we know that:*

$$\Sigma\, q^a_i\, \partial U_i(x^*_i, x^*, a, t^o_i)/\partial x(\partial e_j/\ \partial t_i) = (\Sigma\, q^a_i\, R_i(x^*_i, x^*, a, t^o_i))\partial e_i(t^*, a)/\partial t_i$$

Since $\Sigma\, q^a_i\, R_i(x^*_i, x^*, a, t^o_i)) \geq 0$ *we have obtained a contradiction since the RHS of the above equality is positive and the LHS is negative.*

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