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22 January 2010

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January 22, 2010

Abstract

The paper examines the timing of endogenous wage setting under Bertrand competition in a unionized mixed duopoly. The results are that when the public firm chooses the timing of wage setting: (1) sequential wage setting is the outcome and (2) simultaneous wage setting is the outcome. The first result coincides with the choices of the private firm, its union, and the union of the public firm if imperfect substitutability is sufficiently large. This result is in contrast to the findings of prior literature. However, but the second result does not coincide between firms and their unions if imperfect substitutability is sufficiently small. However, simultaneous wage setting is more likely to improve the welfare if imperfect substitutability is sufficiently small. Furthermore, we find that the impact of sequential wage setting on the equilibrium path is lower in terms of improving welfare than the other outcome of sequential wage setting.

JEL: D21, J51, H44, L13.

Keywords: Endogenous Wage Setting, Bertrand Competition, Mixed Duopoly, Social Welfare.

1 Introduction

There are several studies of pure oligopolies where private firms maximize their own profit by adopting the timing of endogenous wage-setting between firms and unions. Theoretical studies that introduce the timing of endogenous wage-setting (i.e., the setting of input costs) into oligopolistic markets include De Fraja (1993a), Corneo (1995), and Barcena-Ruiz and Casado-Izaga (2008) under either Cournot or Bertrand competition. They showed that if private firms decide the timing of wage setting, in equilibrium they will decide to play simultaneously. But if unions choose the timing, in equilibrium, the unions will decide to play sequentially. Specifically, Barcena-Ruiz and Casado-Izaga (2008) extend the findings of Brekke and Straume (2004)[1] by introducing the timing of endogenous wage-setting. Thus, they showed that both unions prefer sequential wage-setting and that the timing of wage setting does not alter the location equilibrium. In addition, Barcena-Ruiz and Casado-Izaga (2008) demonstrate that bargaining over wages is simultaneous if and only if two private firms decide the timing of wage negotiation; otherwise, negotiation takes place sequentially, which is similar to the results derived by De Fraja (1993a) and Corneo (1995). However, none of these papers have considered the case in which both private and public firms choose to bargain over wages in endogenous timing under Bertrand Competition in a unionized mixed duopoly. Hence, we investigate Bertrand competition in a unionized mixed duopoly where wages can be negotiated either simultaneously or sequentially. That is, we extend Barcena-Ruiz and Casado-Izaga (2008) for evaluating Bertrand competition

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[1]In a spatial context, Brekke and Straume (2004) have analyzed how equilibrium locations in location-price games under Hotelling's model are affected when wage negotiations occur simultaneously.

by considering the timing of endogenous wage-setting where outputs are chosen simultaneously in a mixed-duopoly context.

There have been some attempts, namely, De Fraja (1993b), Haskel and Sanchis (1995), Haskel and Szymanski (1993), and Ishida and Mastushima (2009), to introduce the union's utility into a model of mixed-duopoly markets[2]. In particular, De Fraja (1993b) and Haskel and Sanchis (1995) investigate the wage-bargaining process in mixed duopolies for examining the effect of privatization. Furthermore, focusing on wage regulation, Ishida and Mastushima (2009) analyze the optimal framework that is imposed on the public firm and the union. However, few studies have been undertaken on how the effect of the timing of endogenous wage negotiations between the public and private firms influences product-market interactions. Moreover, while many of the previous analyses that have been conducted on union's utility focused on the power of collective bargaining, any discussion of the issue under Bertrand competition in mixed oligopolies ignores the timing of endogenous wage setting.

In fact, we bring together two independent strands of the literature: a mixed-duopoly market under Bertrand competition and wage setting. We consider that differentiated outputs in a mixed duopoly are chosen simultaneously but extend previous works by assuming that the timing of wage setting is endogenously determined under Bertrand competition. Consequently, the present study differs from the existing literature in at least two important ways. First, the existing studies on mixed oligopolies consider simultaneous wage-setting rather than the effects of different timings of wage setting. Second, prior studies on unionized mixed oligopolies mainly focus on Cournot competition, while our study investigates Bertrand competition in a mixed duopoly and social welfare depending on the nature of goods (i.e., imperfect substitutability) and the timing of wage setting. This is why our analysis needs to be extended to the case of Bertrand competition in a unionized mixed duopoly.

Firstly, our findings show that bargaining over wages under Bertrand competition in a unionized mixed duopoly is always sequential if the degree of imperfect substitutability is sufficiently large. Otherwise, the standard results follow if the degree of imperfect substitutability is sufficiently small, as De Fraja (1993a), Corneo (1995), and Barcena-Ruiz and Casado-Izaga (2008) point out[3]. Given that there is a strictly dominant strategy for the private firm, which is to bargain at the second opportunity, and that all unions prefer to play sequentially rather than simultaneously, the social welfare is determined on the basis of the degree of imperfect substitutability. That is, if the degree of imperfect substitutability is sufficiently large, the public and private firms in the sequential case get either greater profits or greater social welfare than those in the simultaneous case, whereas it is desirable in terms of improving the welfare to force all the firms to choose the simultaneous case if the degree of imperfect substitutability is sufficiently small. Therefore, on the one hand, the public firm in the case of sequential wage setting understands that increasing both the output and wage is desirable to maximize the social welfare. On the other hand, the public firm in the case of simultaneous wage setting also considers price decreases for the consumer surplus. Hence, if the degree of imperfect substitutability is

[2]In the literature on unionized oligopolies, the bargaining process between the firm and the union has been developed almost independently. For instance, there is a well-known framework for international unionized oligopolies regarding the issue of foreign direct investment, as identified by Naylor (1998, 1999), Lommerud *et al.* (2003), and Mukherjee and Suetrong (2007). In other related studies, among domestic private firms, the relationship between the production quantity and the union has been developed (Naylor, 2002; Naylor and Santoni, 2003). There are many studies considering unionized (international) oligopoly, See for instance, Straume (2003), Ishida and Matsushima (2008) and references therein.

[3]While the present work needs to look for potential application areas and study the implication of such union preferences (i.e. order of wage setting) in these contexts, it is left to future research to develop the analysis more generally. To the best of the author's knowledge, only one theoretical study is as follows: Barcena-Ruiz and Campo (2009) showed that unions may choose to set wages simultaneous spending most on R&D if the size of the market is small enough and the efficiency of the R&D technology is great enough.

sufficiently small, the public firm appropriately controls its wages and prices as the latter effect dominates the former, and vice versa.

Second, in contrast to the findings of the prior literature, we find that simultaneous wage setting is more likely to improve welfare if imperfect substitutability is sufficiently small. Furthermore, we show that the outcome of sequential wage setting on the equilibrium path is lower in terms of improving welfare than the other outcome of sequential wage setting. This is because each level of social welfare is determined by a representative consumer's utility, which directly depends upon the degree of substitutability under Bertrand competition in a unionized-mixed duopolistic market.

Consequently, when the choice of timing of endogenous wage setting is set in a unionized duopoly, our results differ from the standard findings of De Fraja (1993a), Corneo (1995), and Barcena-Ruiz and Casado-Izaga (2008). They show that regardless of the impact of imperfect substitutability in a pure duopoly, when wage bargaining is decentralized at the level of the private firm, unions prefer to play sequentially and vice versa.

The organization of the paper is as follows. In Section 2, we describe the model. Section 3 presents fixed-timing games regarding the wage setting. Section 4 determines firms' endogenous choices of wage setting and social welfare. Concluding remarks appear in Section 5.

2 The Model

Consider a unionized mixed duopoly situation for a differentiated good that is supplied by a public firm and a private firm $i = 0, 1$. Firm 1 is the profit-maximizing private firm and firm 0 is the public firm that maximizes the social welfare. On the demand side of the market, we assume that the representative consumer's utility is a quadratic function given by

$$U = x_i + x_j - \frac{1}{2}\left(x_i^2 + 2cx_ix_j + x_i^2\right), \quad i \neq j;\ i, j = 0, 1,$$

where x_i denotes the output of firm i ($i = 0, 1$). The parameter $c \in (0, 1)$ is a measure of the degree of substitutability among goods, while a negative $c \in (-1, 0)$ implies that goods are complements. In the main body of analysis we focus on the imperfect substitutability case of $c \in (0, 1)$[4]. Thus, the inverse demand is characterized by

$$p_i = 1 - cx_j - x_i, \quad i \neq j;\ i, j = 0, 1, \tag{1}$$

where p_i is firm i's market price and x_i denotes the output of firm i ($i = 0, 1$). Hence, we can obtain the direct demands as

$$x_i = \frac{1 - c + cp_j - p_i}{1 - c^2}, \quad i \neq j;\ i, j = 0, 1, \tag{2}$$

provided the quantities are positive.

To analyze the union's wage bargaining, we also assume that the public and private firms are unionized and that the wages, w_i are determined as a consequence of bargaining between firms and unions. Let $\overline{w}$ denote the reservation wage. Taking $\overline{w}$ as given, the union's optimal wage-setting strategy, w_i, regarding firm i is defined as:

$$u_i = (w_i - \overline{w})^{\theta} x_i, \quad i = 0, 1, \tag{3}$$

[4]However, we do not analyze imperfect complementarity case of $c \in (-1, 0)$ since we could have similar results when goods are complements. The detailed computations of complements are available from author upon request.

where θ is the weight that the union attaches to the wage level. As suggested by Haucap and Wey (2004), Leahy and Montagna (2000), and Lommerud *et al.* (2003)), for simplicity of exposition, we assume that the union possesses full bargaining power (i.e., $\theta = 1$) for the wage level and $\overline{w} = 0$ to show our results in the simple way[5]. Hence, to determine the wage set at each firm, we consider the monopoly union model, which assume that the union sets the wage while public and private firms unilaterally decide the level of employment (see Booth (1995) for good introduction). On the other hand, the firms are homogeneous with respect to productivity. Each firm adopts a constant returns-to-scale technology where one unit of labor is turned into one unit of the final good. Thus, the utility function of the union at firm is its wage bill: $u_i(w_i, L_i) = w_i L_i = w_i x_i$.

To specify the public firm 0's objective function SW, and each firm's profit π_i, as

$$SW = U - \sum_{i=0}^{1} p_i x_i + \sum_{i=0}^{1} (\pi_i + u_i),$$
$$\pi_i = (p_i - w_i)x_i, \quad i = 0, 1,$$

where $U - \sum_{i=0}^{1} p_i x_i$ is the consumer surplus, and each firm π_i is the profit of both the public and private firms, and u_i is the union's utility for both the private and public firm. The objective function of the public firm is the sum of consumer surplus, profit of all firms and the union's utility for all the firms.

Timing of the second-stage game is as follows. In the first stage, it is simultaneously decided whether to negotiate over wages in either period 1 or period 2. Note that decision of timing of wage setting could be taken in each case by the firm, by the union or as a result of negotiations between a firm and its union. If the periods of negotiation happen to be identical, the wage-setting process is simultaneous; otherwise, the wage-setting process is sequential. In the second stage, each firm simultaneously chooses its price to maximize its respective objective knowing each union's choice of the wage level.

3 Equilibrium Outcomes

Before analyzing the social welfare and the privatization, we first consider all firms' maximization problems. In this paper, since we focus on symmetric Nash equilibrium, we assume that all firms choose the same type of bargaining. Thus, the game is solved by backward induction, i.e., the solution concept used is the subgame perfect Nash equilibrium.

3.1 Price Competition in a Unionized Mixed Duopoly

In the third stage, by maximization social welfare (respectively, profit) each firm sets its price as a best response to any price chosen by its private firm (respectively, the public firm). The public firm's objective is given by

$$\begin{aligned}\max_{p_0}\ SW =& \frac{(1-c) + cp_1 - p_0}{1-c^2} + \frac{(1-c) + cp_0 - p_1}{1-c^2} \\ &- \frac{1}{2}\left\{\frac{[(1-c) + cp_1 - p_0]^2 + [(1-c) + cp_0 - p_1]^2 + 2c[(1-c) + cp_0 - p_1][(1-c) + cp_1 - p_0]}{(1-c^2)^2}\right\} \\ \text{s.t.}\quad & \frac{(p_0 - w_0)[(1-c) + cp_1 - p_0]}{1-c^2} \geq 0.\end{aligned}$$

[5]As Naylor (1998, 1999), Haucap and Wey (2004), Leahy and Montagna (2000), and Lommerud *et al.* (2003) suggested, this is because wage claims are decided by the elasticity of labor demand rather than the firm's profit. See also Oswald and Turnbull (1985).

The constraint implies that there is some lower-bound restriction on the public firm's profit, i.e., the public firm faces a budget constraint[6]. Therefore, since we assume that each firm's output is a positive value as in (2), we can rewrite the budget constraint as follows: $(p_0 - w_0)x_0 \geq 0 \Leftrightarrow p_0 - w_0 \geq 0$.

Denoting the multiplier of the budget constraint λ^B, the first-order conditions are given by

$$\frac{\partial L}{\partial p_0} = 0 \Leftrightarrow \lambda^B = \frac{-(cp_1 - p_0)}{1 - c^2}, \tag{4}$$

$$\frac{\partial L}{\partial \lambda^B} = p_0 - w_0 = 0. \tag{5}$$

On the other hand, the first-order condition for the private firm is given by

$$\frac{\partial \pi_1}{\partial p_1} = 0 \Leftrightarrow p_1 = \frac{1 - c + cp_0 + w_1}{2}. \tag{6}$$

By using x_i and solving the these two equations (5) and (6) problems yields

$$x_0 = \frac{(1-c)(2+c) + cw_1 - (2-c^2)w_0}{2(1-c^2)}, \tag{7}$$

$$x_1 = \frac{1 - c - w_1 + cw_0}{2(1-c^2)}. \tag{8}$$

3.2 Wage Setting in a Unionized Mixed Duopoly

[**Simultaneous Wage Setting**]: In the second stage of this case, each wage is set to maximize the its own firm's union: $u_i = x_i w_i$. In the analysis that follows, we focus on the union's maximization problem. Using (7) and (8), the problems for union i are defined as

$$\max_{w_0} \; u_0 = w_0 x_0 = \frac{w_0[(1-c)(2+c) + cw_1 - (2-c^2)w_0]}{2(1-c^2)}, \tag{9}$$

$$\max_{w_i} \; u_1 = w_1 x_1 = \frac{w_1(1 - c - w_1 + cw_0)}{2(1-c^2)}. \tag{10}$$

The best reply functions for the public firm 0 and the private firm 1 are $w_0 = \frac{(1-c)(2+c)+cw_1}{2(2-c^2)}$ and $w_1 = \frac{1-c+cw_0}{2}$, respectively. Thus, straightforward computation yields each an equilibrium wage, denoted as w_i^b is obtained by maximizing (9) and (10), and substituting w_i^b into (7) and (8) yields the equilibrium output x_i^b and price p_i^b. Thus, we have the following result.

Lemma 1: *Suppose that under Bertrand competition, each firm's union is allowed to engage in decentralized bargaining. Then, the equilibrium wage, output, union's utility, and price levels*

[6] As described in Ishida and Mastushima (2009), if both unions aim at maximizing wage level simultaneously and the public firm's union does not face the budget constraint with a simple Stone-Geary utility function $u_i = (w_i - \overline{w})^\theta x_i$, the public firm's union can unlimitedly raise its wage because the optimal price level of the public firm is independent of the wage.

are given by

$$
\begin{aligned}
w_0^b &= \frac{4-c-3c^2}{8-5c^2}, \quad w_1^b = \frac{4-2c-3c^2+c^3}{8-5c^2}; \\
x_0^b &= \frac{8-2c-10c^2+c^3+3c^4}{2(1-c^2)(8-5c^2)}, \quad x_1^b = \frac{4-2c-3c^2+c^3}{2(1-c^2)(8-5c^2)}; \\
u_0^b &= \frac{32-16c-62c^2+20c^3+41c^4-6c^5-9c^6}{2(1-c^2)(8-5c^2)^2}, \quad u_1^b = \frac{(4-2c-3c^2+c^3)^2}{2(1-c^2)(8-5c^2)^2}; \\
p_0^b &= \frac{4-c-3c^2}{8-5c^2}, \quad p_1^b = \frac{12-6c-9c^2+3c^3}{2(8-5c^2)}.
\end{aligned}
$$

Substituting Lemma 1 into (4) then we have

$$
\lambda^B = \frac{8-14c+9c^3-3c^4}{2(8-5c^2)(1-c^2)} > 0 \quad \text{if} \quad c < 0.90,
$$

which shows that the budget constraint is binding. Since the Lagrange multiplier is marginal social welfare, when $c \geq 0.90$, the Lagrange multiplier is negative, which means that relaxing the constraint makes social welfare even lower.

Finally, noting that $SW^b = U^b$ and π_1^b, we can compute the social welfare and private firm's profit as SW^b and π_1^b respectively;

$$
SW^b = \frac{304-144c-816c^2+316c^3+787c^4-222c^5-320c^6+50c^7+45c^8}{8(1-c^2)^2(8-5c^2)^2}, \tag{11}
$$

$$
\pi_1^b = \frac{(4-2c-3c^2+c^3)^2}{4(1-c^2)(8-5c^2)^2}. \tag{12}
$$

[**Sequential Wage Setting: Public Firm's Leader**]: In this case, we discuss that the public firm or its union acts as the leader regarding wage setting. Public firm's union 0 will choose to maximize its utility taking as given the private firm's wage w_1 set by private firm's union 1. By solving the first-order condition for private firm's union 1, we have already obtained the best response function to be represent as: $w_1 = \frac{1-c+cw_0}{2}$. By using x_i, we obtain that

$$
x_0 = \frac{4-c-3c^2-(4-3c^2)w_0}{4(1-c^2)}. \tag{13}
$$

Thus, the problem for public firm's union 0 is defined as

$$
\max_{w_0} \ u_0 = w_0 x_0 = \frac{w_0[4-c-3c^2-(4-3c^2)w_0]}{4(1-c^2)}.
$$

By solving the first-order condition for the public firm's union 0, we have the following result when the rival firms takes wage as given, superscript l stands for the leader and f for the follower[7]:

[7] The superscripts in which wages are bargained first in the private firm are symmetric.

Lemma 2: *Suppose thatunder Bertrand competition, the public firm or its union acts as a leader when each firm's union is allowed to engage in decentralized bargaining. Then, the equilibrium wage, output, union's utility, and price levels are given by*

$$w_0^l = \frac{4-c-3c^2}{2(4-3c^2)}, \quad w_1^f = \frac{8-4c-7c^2+3c^3}{4(4-3c^2)},$$
$$x_0^l = \frac{16-4c-24c^2+3c^3+9c^4}{8(1-c^2)(4-3c^2)}, \quad x_1^f = \frac{8-4c-7c^2+3c^3}{8(1-c^2)(4-3c^2)},$$
$$u_0^l = \frac{(4-c-3c^2)(16-4c-24c^2+3c^3+9c^4)}{16(1-c^2)(4-3c^2)^2}, \quad u_1^f = \frac{(8-4c-7c^2+3c^3)^2}{32(1-c^2)(4-3c^2)^2}$$
$$p_0^l = \frac{4-c-3c^2}{2(4-3c^2)}, \quad p_1^f = \frac{24-12c-21c^2+9c^3}{8(4-3c^2)}.$$

This Lemma 2 suggests that the budget constraint is binding. That is, substituting Lemma 2 into (4) then we have

$$\lambda^B = \frac{16-28c+21c^3-9c^4}{8(1-c^2)(4-3c^2)} > 0,$$

which shows that the public firm sets the price that yields zero profit in equilibrium[8].

Using equilibrium values when the public firm acts as a leader regarding wage setting, we can compute the social welfare, SW^l and the private firm's profit, π_1^f under the unionized mixed duopoly as follows:

$$SW^l = \frac{1216-576c-3632c^2+1464c^3+4471c^4-1230c^5-1824c^6+342c^7+297c^8}{128(1-c^2)^2(4-3c^2)^2}, \tag{14}$$

$$\pi_1^f = \frac{(8-4c-7c^2+3c^3)^2}{32(1-c^2)(4-3c^2)^2}. \tag{15}$$

[**Sequential Wage Setting: Private Firm's Leader**]: Similar to the previous sequential wage setting of public firm's leader, we can directly compute each equilibrium value w_i^m, x_i^m, p_i^m, and u_i^m where $m = l, f; i = 0, 1$ when the private firm or its union acts as a leader;

Lemma 3: *Suppose that under Bertrand competition, the private firm or its union acts as a leader when each firm's union is allowed to engage in decentralized bargaining. Then, the equilibrium wage, output, union's utility, and price levels are given by*

$$w_0^f = \frac{16-4c-22c^2+3c^3+7c^4}{4(2-c^2)(4-3c^2)}, \quad w_1^l = \frac{4-2c-3c^2+c^3}{2(4-3c^2)},$$
$$x_0^f = \frac{32-8c-60c^2+10c^3+36c^4-3c^5-7c^6}{8(1-c^2)(2-c^2)(4-3c^2)}, \quad x_1^l = \frac{16-8c-24c^2+10c^3+9c^4-3c^5}{8(1-c^2)(2-c^2)(4-3c^2)},$$
$$u_0^f = \frac{512-256c-1632c^2+672c^3+2056c^4-648c^5-1282c^6+272c^7+397c^8-42c^9-49c^{10}}{32(1-c^2)(2-c^2)^2(4-3c^2)^2},$$
$$u_1^l = \frac{64-64c-128c^2+128c^3+80c^4-84c^5-11c^6+18c^7-3c^8}{16(1-c^2)(2-c^2)(4-3c^2)^2},$$
$$p_0^f = \frac{16-4c-22c^2+3c^3+7c^4}{4(2-c^2)(4-3c^2)}, \quad p_1^l = \frac{48-24c-64c^2+26c^3+21c^4-7c^5}{8(2-c^2)(4-3c^2)}.$$

[8]It can be easily checked by which the public firm's profit is zero in equilibrium; $\pi_0^l = [p_0^l - w_0^l]x_0^l = 0 \times x_0^l$.

Similar to Lemma 2, Lemma 3 suggests that the budget constraint is binding. That is, substituting Lemma 3 into (4) then we have

$$\lambda^B = \frac{64 - 64c - 64c^2 + 76c^3 + 2c^4 - 21c^5 + 7c^6}{8(1-c^2)(2-c^2)(4-3c^2)} > 0,$$

which shows that the public firm sets the price that yields zero profit in equilibrium.

Using equilibrium values when the private firm or its union acts as a leader regarding wage setting, we can compute the social welfare, SW^f and the private firm's profit, π_1^l under the unionized mixed duopoly as follows:

$$\begin{aligned} SW^f =& \frac{4864 - 2304c - 19328c^2 + 8192c^3 + 34424c^4 - 11456c^5 - 26040c^6}{128(1-c^2)^2(2-c^2)^2(4-3c^2)^2} \\ &+ \frac{7876c^7 + 11795c^8 - 2662c^9 - 2720c^{10} + 354c^{11} + 245c^{12}}{128(1-c^2)^2(2-c^2)^2(4-3c^2)^2}, \\ \pi_1^l =& \frac{(16 - 8c - 24c^2 + 10c^3 + 9c^4 - 3c^5)^2}{64(1-c^2)(2-c^2)^2(4-3c^2)^2}. \end{aligned} \tag{16}$$

4 Choice of the Timing of Wage Setting and Social Welfare

4.1 Timing of Endogenous Wage-Setting

Having derived the equilibrium for three fixed-timing games in the previous section and using the same notation for the timings as before, we will find the Nash equilibrium in the first stage for any given set of utilities of the unions and profits of the firms in a unionized mixed duopoly.

Let "F" and "S" represent first period and second period with regard to timing choice of wage setting respectively. When agents (the firms or the unions) have chosen "F" or "S", they will play a Cournot-type game of the wage setting in the first period; when the public firm's agent has chosen "F" while the private firm's agent has chosen "S", a public-leader Stackelberg-type game of the wage setting arises in the second period; when the private firm's agent has chosen "F" while the public firm's agent has chosen "S", a private-leader Stackelberg-type game of the wage setting arises in the second period.

From Lemma 1 to Lemma 3, the reduced endogenous-timing game between unions can be represented by the following payoff Table 1.

Table 1: Timing of Wage Setting between Unions

		Union 1	
		F	S
Union 0	F	u_0^b, u_1^b	u_0^l, u_1^f
	S	u_0^f, u_1^l	u_0^b, u_1^b

To find the subgame perfect Nash equilibrium, we need to compare the utilities of unions. Straightforward computations show in Table 1 that[9]

$$\begin{aligned} &u_0^f > u_0^l > u_0^b \quad \text{if} \quad c \in (0, 0.56]; \quad \text{otherwise,} \quad u_0^l > u_0^f > u_0^b \quad \text{if} \quad c \in [0.57, 1), \\ &u_1^f > u_1^l > u_1^b \quad \text{if} \quad c \in (0, 0.64]; \quad \text{otherwise,} \quad u_1^l > u_1^f > u_1^b \quad \text{if} \quad c \in [0.65, 1). \end{aligned}$$

[9]The detailed computations are available from author upon request; The Appendix B will not be included in the main paper. However, to provide correct calculations we present on Appendix B, which is only available for the reviewers and editor.

These inequalities tell us that regardless of the degree of imperfect substitutability, all unions prefer to play sequentially rather than simultaneously in unionized-mixed duopolies. Thus, multiple subgame-perfect Nash equilibria can be sustained: {S, F}, {F, S}. Thus, we have the following proposition.

Proposition 1: *Suppose that under Bertrand competition, the decision of the timing of wage setting is delegated to unions. Then, there can be sustained multiple endogenous orders of moves. The orders are* $\{S, F\}$, *and* $\{F, S\}$.

The intuition behind Proposition 1 is as follows. Although the leader firm hires fewer employees in the sequential case (i.e., $x_i^l < x_i^b < x_i^f$), the wage is higher when each union prefers to play sequentially (i.e., $w_i^b < w_i^f < w_i^l$)[10]. Moreover, in the sequential case, the follower union secures a higher wage and greater employment. Regardless of the degree of substitutability, the utilities of unions in the sequential case are higher than those in the simultaneous case. This result is standard in the literature on wage bargaining (see Barcena-Ruiz and Casado-Izaga, 2008)[11].

Similar to the timing of wage setting between unions, using each private firm's profit and social welfare, the reduced endogenous-timing game between firms can be represented by the following payoff Table 2.

Table 2: Timing of Wage Setting between Firms

		Private Firm 1	
		F	S
Public Firm 0	F	SW^b, π_1^b	SW^l, π_1^f
	S	SW^f, π_1^l	SW^b, π_1^b

Before finding the subgame-perfect Nash equilibrium of the timing of wage setting between the private and public firms, we obtain the following result by comparing the level of the private firm's profit and the social welfare across the various fixed-timing games[12]:

Proposition 2: *Suppose that under Bertrand competition, the decision of the timing of wage setting is not delegated to the unions. Then, a comparison of the private firm's profit and the social welfare across equilibria yields that*

$$\pi_1^f > \pi_1^b > \pi_1^l, \quad SW^f > SW^l, \qquad SW^b > SW^f > SW^l \quad \text{if} \quad c \in (0, 0.19];$$
$$SW^f > SW^b > SW^l \quad \text{if} \quad c \in [0.2, 0.27]; \qquad SW^f > SW^l > SW^b \quad \text{if} \quad c \in [0.28, 1).$$

Proposition 2 tells us that the public firm prefers to play simultaneously if $c \in (0, 0.19]$, while the private firm has a dominant strategy for the second opportunity. Moreover, if $c \in [0.2, 0.27]$, then each firm has a strictly dominant strategy for the second opportunity. Finally, the public firm prefers to play sequentially if $c \in [0.28, 1)$, while the private firm has a strictly dominant strategy for the second opportunity. Thus, we have the following proposition.

Proposition 3: *Suppose that under Bertrand competition, the decision of the timing of wage setting is not delegated to the unions. Then, there are two possible timings in endogenous wage*

[10] Comparing each wage and output is here omitted because it can be easily verified by simple calculations.

[11] As Barcena-Ruiz and Campo (2000) suggested, the leader chooses a higher wages than the follower. This result is due to the fact that wages are strategic complements.

[12] All calculations are in the Appendix A.

setting. If $0 < c \leq 0.26$*, the order is the second opportunity, i.e.,* $\{S,\ S\}$*. If* $0.27 \leq c < 1$*, the order is public leader-private follower, viz.,* $\{F,\ S\}$*.*

The intuition underlying Proposition 3 is as follows. From Proposition 1, we understand that from the viewpoint of the private firm, the outputs obtained by the leader firm are lower than those in the simultaneous case which means that $\pi_1^b > \pi_1^l$, while the outputs obtained by the follower firm in the sequential case are higher than those in the simultaneous case, which means that $\pi_1^f > \pi_1^b$. This case is the same for the public firm if $c \in [0.20.27]$. Moreover, when $c \in (0, 0.19]$, although the public firm gets lower outputs in the simultaneous case than in the follower case, it pays lower wages in the former case. This is because the public firm's price[13] in simultaneous wage-setting is lower than that in the follower case, which means that $SW^b > SW^f$. On the other hand, if $c \in (0, 0.19]$, the outputs obtained by the public firm in the leader case are lower than those in the simultaneous case, while the price reached by the public firm in the leader case is higher than that in the simultaneous case, which leads to the result, $SW^b > SW^l$.

However, in Proposition 1, we have considered that the workers employed in the public firm in the follower case are greater than those in the simultaneous case and that the unions get higher wages in the sequential case if $c \in [0.28, 1)$. From the viewpoint of the public firm, given that the private firm prefer to play at the second opportunity, the public firm in the leader case pays higher wages, produces less outputs, and realizes less social welfare, which leads to the results, $SW^f > SW^l$. This is because the public firm in the follower case produces more output and consumer pays a lesser price for the public firm's goods. On the other hand, although the public firm's price in the leader case is higher than that in the simultaneous case and the unions get higher wages in the sequential case, the public firm wishes to bargain at the first opportunity. That is, when $c \in [0.28, 1)$, the public firm in the leader case understands that decreasing the output is not desirable for maximizing the social welfare. Hence, it considers a profit increase as well as the union's utility in both public and private firms. In other words, when $c \in [0.28, 1)$, the public firm appropriately controls its wages and prices as the latter effect dominates the former, which means that $SW^l > SW^b$.

Hence, given that there is a dominant strategy for the private firm, which is to bargain at the second opportunity, the social welfare is determined by depending upon the degree of imperfect substitutability: if the degree of imperfect substitutability is in the range of $c \in [0.28, 1)$, firms in the sequential case get either greater profits or greater social welfare than those in the simultaneous case, while it is desirable in terms of improving welfare to force the all the firms to choose the simultaneous case if c is sufficiently small.

Given Proposition 1, 2, and 3, we obtain the following result.

Proposition 4: *(1) If* $0.27 \leq c < 1$*, bargaining over wages is sequential, regardless of who decides the timing of endogenous wage-setting. (2) If either* $0 < c \leq 0.19$ *or* $0.2 < c \leq 0.26$*, bargaining over wages is simultaneous if and only if all the firms decide on the timing of endogenous wage-setting. Otherwise, wage setting takes place sequentially.*

Proof: The part of the proof when $0.2 < c \leq 0.22$ is the same as that provided by Barcena-Ruiz and Casado-Izaga (2008, pp. 155-157) (see also the supplementary proof in Appendix A). Hence, we examine the first case ($0.2 < c \leq 0.26$) and the second case when $0 < c \leq 0.19$.

Case (1) $0.27 \leq c < 1$.

First, consider the case that both unions decide the timing of wage setting. Then, the public

[13] We can easily calculate that $p_i^l > p_i^f > p_i^b$.

and private firms' unions do not have incentives to play simultaneously, as noted in Proposition 1.

Second, consider that the private and public firms decide the timing of wage setting. As shown in Proposition 1 and 2, both firms prefer to play sequentially when $0.27 \leq c < 1$, i.e., the public firm wishes to bargain at either the first or the second opportunity and the private firm has a dominant strategy for the second opportunity. This case shows that all the firms prefer to play sequentially even though there is no dominant strategy for the public firm.

Third, consider that the private (respectively, public) firm and the public (respectively, private) firm's union decide on the timing of endogenous wage-setting. The public and private firms' unions do not have incentives to play simultaneously. Given this, the private (respectively, public) firm gets a higher profit (respectively, social welfare)) under the second (respectively, first) opportunity. Then, we have a sequential game, {F, S}, at the bargaining stage.

Finally, suppose that the game is played by both firms and unions. In this case, the public firm's union does not deviate from the first opportunity and the private firm's union does not deviate from the second opportunity because both unions prefer to play sequentially. On the other hand, if one firm prefers simultaneous wage bargaining, then both unions will prefer to play sequentially. Given this, neither firm deviates from the sequential case because both unions always prefer to play sequentially. The same result can be obtained if the game is played by one firm and its union and vice versa.

Case (2) $0 < c \leq 0.19$.

First, consider the case when both unions decide the timing of wage setting. Then, the public and private firms' unions do not have incentives to play simultaneously, as noted in Proposition 1.

Second, consider that the private and public firms decide the timing of wage setting. As shown in Proposition 1 and 2, there is a strictly dominant strategies for the private firm, which is to bargain at the second opportunity when $0 < c \leq 0.19$. However, the public firm prefers to play simultaneously since $SW^b > SW^f > SW^l$ in the range of $c \in (0, 0.19]$. This case shows that the public firm prefers to play simultaneous even though there is no dominant strategy.

Third, consider that the private (respectively, public) firm and the public (respectively, private) firm's union decide on the timing of endogenous wage-setting. The public and private firms' unions have incentives to play sequentially. Given this, the private (respectively, public) firm gets a higher profit (respectively, social welfare) under the second (respectively, first) opportunity.

Finally, suppose that the game is played by both firms and unions. In this case, both unions do not have an incentive to deviate from the sequential case. Given this, the public and private firms always prefer to play simultaneously. ■

Proposition 4 suggests that sequential wage setting is an equilibrium outcome if the degree of imperfect substitutability is sufficiently large, which is in contrast to the finding of prior literature – but this result does not hold if the degree of imperfect substitutability is sufficiently small. The results of Proposition 4 are directly dependent on the degree of imperfect substitutability under Bertrand competition in a unionized-mixed duopolistic market. That is, Proposition 4 implies that as long as the degree of imperfect substitutability is sufficiently large, the equilibratory nature of sequential wage setting does not depend on who establishes the timing of wage setting, i.e., the two unions, the unions and only one firm, or one union and both firms. In contrast, the explanation for the case when imperfect substitutability is sufficiently small is largely analogous to the arguments of Barcena-Ruiz and Casado-Izaga (2008), De Fraja (1993a), and Corneo (1995) in the literature on both spatial and non-spatial competitions.

Given Proposition 4, it is instructive to compare the social welfare in a simultaneous case with that in a sequential case. From Proposition 1, 2 and 3, the social welfare when $c \in (0, 0.27]$ is always determined by SW^b under Bertrand competition in a unionized-mixed duopoly if and if only all the firms decide on the timing of endogenous wage-setting. However, if the decision on wage setting is delegated to unions when $c \in (0, 0.27]$, the social welfare is determined by either SW^l or SW^f. Thus, in the first stage, the public firm (or government) might make a decision regarding wage setting because simultaneous wage setting is more desirable in terms of improving welfare than sequential wage setting when $c \in (0, 0.19]$. On the other hand, if the degree of imperfect substitutability is sufficiently large, sequential wage bargaining arises as an equilibrium outcome. Therefore, if the degree of imperfect substitutability is sufficiently large, a lower level of the social welfare, SW^l, can be obtained regardless of whether the timing of wage setting is established only by the two unions, by the unions and only one firm, or by one union and all firms. Proposition 4 also suggests that differences in the implementation of leadership depend on the degree of imperfect substitutability. That is, the outcome of sequential wage setting on the equilibrium path when $0.27 \leq c < 1$ is lower in terms of improving welfare than the other outcome of sequential wage setting.

On the other hand, given the union's utility, the consumer surplus, CS, in the cases of simultaneous and sequential wage-setting is represented with the same superscripts as in Proposition 5.

Proposition 5: *Each level of the consumer surplus is determined by*

$$\begin{aligned} &CS^b > CS^f > CS^l \quad \text{if} \quad c \in (0, 0.13]; \\ &CS^f > CS^b > CS^l \quad \text{if} \quad c \in [0.14, 0.29]; \\ &CS^f > CS^l > CS^b \quad \text{if} \quad c \in [0.3, 1); \quad \text{and} \\ &CS^f > CS^l. \end{aligned}$$

Proposition 5 suggests that the explanations for the consumer surplus are largely analogous to the arguments for the social welfare. Compared to Proposition 3, Proposition 5 gives us the situation that is the best in terms of consumer surplus if and only if bargaining over wages is simultaneous and the degree of imperfect substitutability falls into the interval, $c \in (0, 0.13]$. On the other hand, if the degree of imperfect substitutability is sufficiently large, sequential wage bargaining arises as an equilibrium outcome: a lower level of the consumer surplus, CS^l, can be obtained than that of CS^f, regardless of who decides upon the timing of endogenous wage setting.

5 Concluding Remarks

Concerning Bertrand competition, this paper has investigated changes in the social welfare according to the timing of endogenous wage setting in a unionized-mixed duopolistic market. Unlike extant literature on mixed duopolies that is based on the timing of exogenous wage-setting, we have found that the choice of the timing of endogenous wage-setting potentially differs from De Fraja (1993a), Corneo (1995), and Barcena-Ruiz and Casado-Izaga (2008), which focus on private firms.

We have found that bargaining over wages under Bertrand competition in a unionized mixed duopoly is always sequential if the degree of imperfect substitutability is sufficiently large. Otherwise, the standard results hold if the degree of imperfect substitutability is sufficiently small. The intuition of this result is as follows: on the one hand, the public firm in the sequential case understands that increasing both the output and wage are desirable for maximizing the social welfare. On the other hand, the public firm in the simultaneous case also considers price decreases for the consumer surplus. Hence, if the degree of imperfect substitutability is sufficiently small, the public firm appropriately controls its outputs, wages, and prices as the effect of lowered prices on the consumer surplus dominates the effect of raising both the output and wage, and vice versa. Moreover, simultaneous wage setting is more likely to improve the welfare if the degree of imperfect substitutability is sufficiently small, while sequential wage setting (public firm being the leader) as an equilibrium outcome is inferior in terms of improving welfare than the other outcome of sequential wage setting (private firm leader) if the degree of imperfect substitutability is sufficiently large.

Finally, we did not extend our results by considering a model where the public firm competes with n private firms or both domestic and foreign private firms. Also, in this paper, we have used the simplifying assumption that each firm decides the optimal price policy by facing its trade union. However, in the real world, public and private firms endogenously determine their vertically differentiated product quality. Moreover, it is interesting and useful to compare the timing of endogenous wage setting under privatization with that under either nationalization or partial privatization. The extension of our model in these directions remains an agendum for future research.

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Appendix A: Proofs

Proof of Proposition 2

(i) Comparison of each social welfare under the unionized mixed duopoly

$$
\begin{aligned}
SW^f > SW^l \Leftrightarrow & -64c^2 - 32c^3 - 796c^4 + 104c^5 - 2772c^6 - 1240c^7 \\
& + 1160c^8 - 292c^9 + 64c^{10} - 10c^{11} + 52c^{12} > 0. \\
SW^b > SW^l \Leftrightarrow & \ 4096c^2 - 3584c^3 - 48512c^4 + 11328c^5 + 63584c^6 - 13336c^7 \\
& - 28975c^8 + 6942c^9 + 6000c^{10} - 1350c^{11} - 995c^{12} > 0 \quad \text{if} \quad c \in (0, 0.27]; \\
& \text{otherwise,} \ \ SW^b < SW^l \quad \text{if} \quad c \in [0.28, 1). \\
SW^b > SW^f \Leftrightarrow & \ 12288c^2 - 16384c^3 - 256256c^4 + 68864c^5 + 337152c^6 - 119296c^7 \\
& - 142648c^8 + 109256c^9 + 22696c^{10} - 55492c^{11} - 1067c^{12} + 14902c^{13} \\
& - 1680c^{14} - 1650c^{15} + 355c^{16} > 0, \ \text{if} \quad c \in (0, 19]; \\
& \text{otherwise,} \ \ SW^b < SW^l \quad \text{if} \quad c \in [0.2, 1).
\end{aligned}
$$

(ii) Comparison of each private firm's profit under unionized mixed duopoly

$$
\begin{aligned}
\pi_1^b > \pi_1^l \Leftrightarrow & \ 4096c^2 - 4096c^3 + 32512c^4 + 13568c^5 + 17600c^6 - 17856c^7 - 10880c^8 \\
& + 11664c^9 + 2892c^{10} - 3780c^{11} - 45c^{12} + 486 - 81c^{14} > 0, \\
\pi_1^b < \pi_1^f \Leftrightarrow & - 2048 + 2048c + 5632c^2 - 6144c^3 - 5248c^4 + 6848c^5 + 1552c^6 - 3368c^7 \\
& + 263c^8 + 618c^9 - 153c^{10} < 0.
\end{aligned}
$$

Proof of Proposition 4

We provide the case of the range of $c \in [0.2, 0.27]$ under the unionized mixed duopoly as follows.

First, consider that both unions decide the timing of wage setting. Then the both unions do not have incentives to play simultaneously. Given the preference of the rival's union, each union prefer to play sequentially.

Second, consider that the public firm and the private firm's union decide on the timing of endogenous wage setting. Then, the public firm has a strictly dominant strategy for the second opportunity since $SW^f > SW^b > SW^l$ in the range of $c \in [0.2, 0.27]$. However, the private firm's union prefers to play sequentially, which prefers to be a follower or a leader union.

Third, consider that the private and public firms decide the timing of wage setting. As shown in Proposition 2, the both firms prefer to play simultaneously regardless of c, which is to bargain in the second opportunity.

Fourth, consider that the private firm and the public firm's union decide on the timing of endogenous wage setting. Then, the public firm's union prefers to play sequentially, which prefers to be a follower or a leader union. However, the private firm has a strictly dominant strategy for the second opportunity $\pi_1^f > \pi_1^b > \pi_1^l$ in the range of $c \in [0.2, 0.27]$.

Finally, suppose that the game is played by both firms and unions under each market. In this case, both unions do not have incentives to deviate from the sequential case. Given this, the public and private firms always prefer to play simultaneously.

Proof of Proposition 5

Comparison of each consumer surplus under the unionized mixed duopoly.

Using Lemma 1-5, we get the consumer surplus in both simultaneous and sequential cases. These calculations are as follows:

$$CS^b = \frac{80 + 16c - 224c^2 - 44c^3 + 225c^4 - 10c^5 - 72^6 + 26^7 + 3c^8}{2(1-c^2)^2(8-5c^2)^2},$$
$$CS^l = \frac{320 + 64c - 1040c^2 - 184c^3 + 1785c^4 + 174c^5 - 672c^6 - 54c^7 + 135c^8}{128(1-c^2)^2(4-3c^2)^2},$$
$$CS^f = \frac{1280 + 256c - 5248c^2 - 1024c^3 + 11992c^4 + 1600c^5 - 7610c^6 - 1220c^7 + 3613c^8 - 896^{10} - 66^{11} + 91c^{12}}{128(1-c^2)^2(2-c^2)^2(4-3c^2)^2}.$$

Comparing each consumer surplus under the unionized mixed duopoly shows that

$$\begin{aligned}
CS^f > CS^l \Leftrightarrow & -192c^2 + 32c^3 - 372c^4 - 104c^5 - 838c^6 + 124c^7 \\
& - 1070c^8 - 64c^9 + 289c^{10} + 12c^{11} + 44c^{12} > 0. \\
CS^b > CS^l \Leftrightarrow & \, 4096c^2 - 512c^3 - 50304c^4 - 10816c^5 + 74720c^6 + 26136c^7 \\
& - 46209c^8 - 20094c^9 + 16080c^{10} + 5094c^{11} - 2943c^{12} > 0 \quad \text{if} \quad c \in (0, 0.29]; \\
& \text{otherwise, } \; CS^b < CS^l \quad \text{if} \quad c \in [0.3, 1). \\
CS^b > CS^f \Leftrightarrow & \, 4096c^2 - 226048c^4 - 50432c^5 + 412800c^6 + 182784c^7 \\
& - 451128c^8 - 263040c^9 + 369402c^{10} + 124356c^{11} - 142149c^{12} + 13078c^{13} \\
& + 19888c^{14} + 5394c^{15} - 1846c^{16} > 0, \; \text{if} \quad c \in (0, 13]; \\
& \text{otherwise, } \; CS^b < CS^l \quad \text{if} \quad c \in [0.14, 1).
\end{aligned}$$

It is available from author upon request

Appendix B: Proof of Proposition 1

For the reviewers and editor, this appendix B will not be included in the main paper. However, this is only available for the reviewers and editor: proof of Proposition 1. In this case where we have been abbreviated, we present on separate page.

(a) Comparison of each union's utility under unionized mixed duopoly

$$
\begin{aligned}
&u_0^b < u_0^l \Leftrightarrow -64c^4 + 32c^5 + 140c^6 - 48c^7 - 105c^8 + 18c^9 + 27c^{10} < 0,\\
&u_0^b < u_0^f \Leftrightarrow -2048c^3 + 1536c^4 + 6400c^5 - 4834c^6 - 7968c^7 + 4888c^8 + 4936c^9 - 2564c^{10}\\
&\qquad - 1521c^{11} + 699c^{12} + 186c^{13} - 71c^{14} < 0,\\
&u_0^l < u_0^f \Leftrightarrow -32c^3 + 32c^4 + 56c^5 - 14c^6 - 32c^7 + 29c^8 + 6c^9 - 5c^{10} < 0, \text{ if } c \in (0, 0.56];\\
&\qquad \text{otherwise, } u_0^l > u_0^f \quad \text{if} \quad c \in [0.57, 1).\\
&u_1^b < u_1^l \Leftrightarrow -64c^4 + 64c^5 - 5632c^6 - 128c^7 - 170c^8 - 1212c^9 + 11c^{10} - 18c^{11} + 3c^{12} < 0,\\
&u_1^b < u_1^f \Leftrightarrow -512c^3 + 384c^4 + 1088c^5 - 720c^6 - 776c^7 + 431c^8 + 186c^9 - 81c^{10} > 0,\\
&u_1^l < u_1^f \Leftrightarrow -16c^3 + 14c^4 + 28c^5 - 15c^6 - 6c^7 + 3c^8 < 0, \text{ if } c \in (0, 0.64]; \text{ otherwise, } u_1^l > u_1^f \text{ if } c \in [0.65, 1).
\end{aligned}
$$

(b) Comparison of each wage and output under unionized mixed duopoly

$$
\begin{aligned}
&w_1^b < w_1^f \Leftrightarrow -4 + c + 3c^2 < 0, \quad w_1^b < w_1^l \Leftrightarrow 0 < c^2, \quad w_1^l > w_1^f \Leftrightarrow 1 - c > 0,\\
&w_0^b < w_0^l \Leftrightarrow 0 < c^2, \quad w_0^b < w_0^f \Leftrightarrow -4c^3 + 2c^4 + 3c^5 - c^6 < 0, \quad w_0^l > w_0^f \Leftrightarrow 2 - c - c^2 > 0,\\
&x_0^b > x_0^l \Leftrightarrow 16 - 4c - 24c^2 + 3c^3 + 9c^4 > 0, \quad x_0^b < x_0^f \Leftrightarrow -8 + 4c + 10c^2 - 4c^3 - 3c^4 + c^5 < 0,\\
&x_1^b > x_1^l \Leftrightarrow 16c^2 - 8c^3 - 24c^4 + 10c^5 + 9c^6 - 3c^7 > 0, \quad x_1^b < x_1^f \Leftrightarrow -4 + c + 3c^2 < 0.
\end{aligned}
$$