Exposure at Default Model for Contingent Credit Line

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By

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Abstract

In-spite of large volume of Contingent Credit Lines (CCL) in all commercial banks paucity of Exposure at Default (EAD) models, unsuitability of external data and inconsistent internal data with partial draw-down, has been a major challenge for risk managers as well as regulators for managing CCL portfolios. Current paper is an attempt to build an easy to implement, pragmatic and parsimonious yet accurate model to determine exposure distribution of a CCL portfolio. Each of the credit line in a portfolio is modeled as a portfolio of large number of option instrument which can be exercised by the borrower determining the level of usage. Using an algorithm similar to basic CreditRisk+ and Fourier Transforms we arrive at a portfolio level probability distribution of usage.

I Introduction

Bank for International Settlement (BIS) in its Basel II guidelines[1] describes capital as a function of Probability of Default (PD), Loss Given Default (LGD) and Exposure at Default (EAD) with all playing an equally vital role, but a simple Google search on each term returns 5.7 Million, 65 Thousand and 29 Thousand[2] hits respectively. Google search may not be concrete enough to conclude but it does indicate the trends in terms of importance given to each component by academicians and practitioners. Similar assertions are made by Financial Services Authority (FSA)[3], UK regarding EAD models as well.

Contingent Credit Lines (CCL) or Loan Commitments[4] are contractual promises by the bank to specific obligors to lend up to specified limits on pre-determined rates and terms. They are generally accompanied with different fees which must be paid over the life of the commitment, and the material adverse change (MAC) clause which states that bank may cancel the line if the credit quality deteriorates of the specific obligor.

According to Federal Deposit Insurance Corporation survey[5] close to 80% of all commercial and industrial loans are done using commitment contracts and as of

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1 Refer BCBS (2006) revised document on Basel II guideline

2 Searched on 24 Feb 2010 from www.google.com

3   Please refer EAD Expectation Note (2007)

4 Contingent Credit Line, Loan/Credit Commitment, Line of Credit has been used interchangeably in the paper

5 For details see Federal Deposit Insurance Corporation, Statistics on Banking, table RC-6 (November 2009)

September 2009, the outstanding (unused) CCLs of U.S. firms were close to $1.9 trillion. Avery and Berger (1991) states that main reason for using credit commitment is to provide flexibility during slowdown and as noted by Kanatas (1987) these can be seen as hedging instruments. Hawkins (1982) comments that credit lines help borrowers manage fluctuations in working capital but Sufi (2008) reports that firm with low cash flow or high cash flow volatility rely more heavily on cash rather than credit line.

Basel II guidelines calculate regulatory capital charge of contingent credit commitments based on credit conversion factor (0% to 50%) and the Risk Weight (0% to 100%). As noted by Hull (1989), credit conversion factor (CCF) for a small bank underestimates the capital requirement as "fat tails" effect increases the capital requirement proportionately more for off-balance sheet items. In its Advanced Internal Rating Based (AIRB) methodology Basel II does allow banks to compute its own estimates of CCF and henceforth its own estimate of Exposure at Default (EAD) for CCL.

Apart from negligence of EAD models by consultants and academics leading to a paucity of external data and models, other issues identified by FSA, has been scarcity of usable data regarding draw-downs in each bank and unsuitability of external data. When external data is available the suitability is always questioned as the estimates will be strongly influenced by lender's behavior.

The current paper is an attempt to build an easy to implement, parsimonious yet

accurate model for estimation of exposure for CCL portfolio. Each CCL is modeled as portfolio of options with the obligors which they can exercise with the bank at pre-specified terms & conditions. Modeling the exercise of options as a Poisson process, a stochastic distribution of exposure at different segment of portfolio level has been constructed. A standard Fast Fourier Transform algorithm is used to convolute these portfolio segments and generate the exposure probability distribution of the complete portfolio.

This model with help from internal research of the bank can be used for the estimation of EAD for banks as mandated by Basel II for CCL portfolios. Apart from regulatory requirement, stochastic exposure distribution generated can form as an input for different economic capital model and stress testing procedures to capture accurate risk profile of the portfolio. This will also contribute in providing better insights in the problem of managing liquidity risk for portfolio of CCL e.g. credit card portfolio or Home Equity Line of Credit (HELC) portfolio.

In the following pages we will see in section II about past studies related to CCL, section III will detail on how we can use options executions to link the draw-downs in a portfolio, section IV provides an implementation with hypothetical portfolios and finally section V concludes with potential future research areas and implications.

II CCL, Options & Partial Draw-down

To estimate EAD for CCL there are two standard form of equation used. In the

first form an appropriate conversion factor is multiplied with the total limit (TL) of the facility.

$$EAD=CF \ .TL \tag{1}$$

In the second form an appropriate conversion factor (α)[6] is multiplied with the unused part of the limit (L) of the facility.

$$EAD=CurrentExposure+\alpha \ \cdot L \tag{2}$$

As shown by Moral (2006) both will yield same results for EAD except when full utilization is in place. As discussed by Miu and Ozdemir (2008) Basel CCF is equivalent to α in equation (2). We will use equation (2) for all our subsequent analysis. As current exposure and L are known modeling α i.e. partial draw-down of the unused limit will give us clear insight into the problem of EAD estimation of CCL.

Studies related to CCL are concentrated around pricing of CCL[7] and level of partial draw-down in each credit line. In the current endeavor we will concentrate on latter.

Thakor et al. (1981) utilize a put option[8] approach to price the loan commitments and measure the sensitivity of these values to changes in interest rates. Partial draw-down on CCL is explained through, interest elasticity of demand for borrowed funds and

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6 This is also referred as Loan Equivalent amount (LEQ) in many literatures.
7 Pricing of CCL option theoretic approach has been used by Loukoianova et al. (2006), Chateau (1990), Chateau and Wu (2003).
8 In this case the Bank is buying the put option from the obligor as the obligor is selling its debt to the bank by availing the credit lines on pre-specified terms and conditions.

bank customer relationship dynamics. A firm with infinite opportunities for investment and no restriction on capital structure or leverage, the interest elasticity will be perfectly elastic and vice-versa. Alternatively if we look into the bank customer relationship framework, customer will try to minimize the expected cost of renewal of the line in next year and opportunity loss of not utilizing the full facility this year when availing the line.

Kaplan and Zingales (1997) find that the un-drawn portion of credit lines decreases when firms are more liquidity constrained. Gatev and Strahan (2003)[9] report that partial draw-downs increase when the commercial paper - T bill rate spread rises. Both of these studies indicate presence of interest incentive framework for partial draw-down which inspired Jones and Wu (2009) for using the same for modeling partial draw-down. They models credit quality as a jump-diffusion process while partial draw-down and pricing of CCL is done as a function of dynamic credit state. The proportion of the credit line drawn is modeled as function of the difference between alternative opportunity rate and the marginal cost of line borrowings. Opportunity rate is defined as the rate of interest charged to the borrower if she borrows outside the purview of the defined credit line. To incorporate linking of loan spread to the credit default swap spread of the borrower, marginal cost of borrowing is defined as function of reference rate, contractual spread over reference rate and proportion of the excess that is added to current period loan. Apart from the interest rate differential, sensitivity of drawdown with interest rate differential is included to model amount of partial draw-down. This

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9 They used a sample from the set of all commercial paper backup lines of credit on for large US Corporation in period 1Q 1991 - 1Q 2002, for a total of 2,695 commitments.

sensitivity is similar to the *interest elasticity* proposed by Thakor et al. (1981).

Both of the approach (Thakor et al. (1981) and Jones et al. (2009)) looks intuitive and convincing but implementing the same in banks where most of the CCL are extended to unrated obligors whose market spread may not be easily available, might pose problems of parameterization. Some parameter like interest elasticity may be affected by firm specific behavior as well as present macro economic variables. Another approach of estimating usage of limits under continuous-time model where the credit provider and the credit taker interact within a game-theoretic framework has been attempted by Leippold et al (2003).

Attempts to directly estimate partial draw-down has also been undertaken in past. Asarnow and James (1995) present partial draw-down estimates based on credit lines issued by Citibank to publicly-rated North American firms over the five-year period from 1987 to 1992. They found a downward sloping of usage level from high rated obligors to low rated obligors i.e. lower rated firms would have already consumed their credit lines earlier than when it approaches default. Similar trend is also noted by Araten and Jacobs (2001) where partial draw-down decreases as the firm approaches default. Their estimates of partial draw-down is based on 1,021 observations (408 facilities of 399 borrowers) at a quarterly frequency in the period 1Q 1995 - 4Q 2000 for Chase borrowers. They also report that level of usage has been affected by risk rating but not by commitment size and borrower industry. Jacobs (2008) empirical study is based on dataset encompassing 281 defaulted instruments from 720 U.S. borrowers with public

credit ratings over the period from 1985 to 2006. He finds that similar trends in terms of usage and points out statistically significant affect of obligor profits in the level of usage.

Agarwal et. al. (2005) examined utilization of HELC in US market and confirms that borrowers with deteriorating credit quality increase their utilization. Jiménez et al. (2009) reports a credit line usage of different firms granted by banks in Spain between 1984 and 2005. The final dataset used consisted of 696,445 credit lines granted to 334,442 firms by 404 banks. They reported variety of factors such as commitment size; collateralization and maturity of the CCLs affect the usage level. They also report a statistically significant higher usage rate for firms that are defaulting at least 3 years prior to default and the usage monotonically increases as the default approaches for these firms.

Level of usage is mainly affected by two distinct forces, the lender might realize the deteriorating credit quality of the borrower and cut back the limits thereby increasing the utilization ratio or the borrower may actually use up the line before the lender realizes deteriorating credit quality. As indicated by Qi (2009) with credit card usage in US, borrowers are more active than lenders in this game of "race to default." Martin and Santomero (1997) analyzes the pricing of CCL from demand side of firms and show that credit line usage depends on business growth potential of the firm as well as the uncertainty involved in those investment opportunities. External macro economic variables, size of credit line, collateralization etc. may also determine level of partial draw-down of obligors.

III Partial Draw-down at Portfolio Level

To put ourselves on firmer basis lets define a bank B with a CCL portfolio of N obligors each having one facility of CCL each. Each of these credit lines with the bank can be used before the expiry of the contract. For obligor A with CCL size of L_A we can safely assume that A has infinite number of put options which she can choose to exercise and the number of instrument she will exercise will decide the level of partial draw-down. If she exercises all the puts, than she will have consumed her whole limit. We assume she has n number of puts at her disposal, where n is sufficiently large. Also size of each put can be given as

$$Q_A = \frac{L_A}{n} \tag{5}$$

So the amount of partial draw-down can be given as r X Q_A where r is the number of puts exercised by A in the time frame of consideration. So the probability generating function (PGF) of the option exercise can be defined as follows:

$$F_A(z) = P(r=0)Z^0 + P(r=1)Z^1 + P(r=2)Z^2 + P(r=3)Z^3 + P(r=4)Z^4 \ldots \ldots \tag{6}$$

Let's assume expected usage of the CCL is α_A so average number of puts used

by A is
$$\lambda_A = \frac{\alpha_A L_A}{Q_A} \tag{7}$$

Using a Poisson process of exercise of each option we have the PGF given by equation (6) reduces to

$$F_A(z) = \frac{e^{-\lambda_A}\lambda^0}{0!}Z^0 + \frac{e^{-\lambda_A}\lambda^1}{1!}Z^1 + \frac{e^{-\lambda_A}\lambda^2}{2!}Z^2 + \dots \quad = \quad F_A(z) = e^{-\lambda_A}e^{\lambda_A Z} \tag{8}$$

For all obligors, m (m<N) in the portfolio having put size equal to Q we have the PGF for r number of puts being used and assuming independence of obligors in exercising of each option

$$\prod F_A(z) = \prod e^{-\lambda_A}e^{\lambda_A Z} = e^{\sum_{i=1}^{i=m}-\lambda_i + \sum_{i=1}^{i=m}\lambda_i Z} \tag{9}$$

Now lets assume the overall average usage in the portfolio is α and unused limits in this portfolio of m obligor be L_i hence we have

$$\alpha = \frac{\sum_{i=1}^{i=m}\alpha_i L_i}{\sum_{i=1}^{i=m}L_i} \tag{10}$$

Let $S_i = \sum_{i=1}^{i=m}\lambda_i$ and as we have assumed all the size of put is Q_i henceforth and let

$$\beta_i = \sum_{i=1}^{i=m}S_i \cdot Q_i \quad \text{so} \quad S_i = \sum_{i=1}^{i=m}\lambda_i = \frac{\sum_{i=1}^{i=m}\alpha_i L_i}{Q_i} = \frac{\alpha}{Q_i} \cdot \sum_{i=1}^{i=m}L_i \, hence \beta_i = \alpha \cdot \sum_{i=1}^{i=m}L_i \tag{11}$$

So the PGF of usage being equal to r X Q will be given by

$$F_{QI}(z) = e^{(Z-1)S_I} = e^{-S_I}\left[\frac{(S_1 z)^0}{0!} + \frac{(S_1 z)^1}{1!} + \frac{(S_1 z)^2}{2!} + \dots\right] \tag{12}$$

Now for any real portfolio, Q will not be equal for all obligors. To simplify our calculation we will segregate the whole portfolio in different sub-portfolios, such that

obligors in each sub-portfolio will have same size of put as Q_i. Since in each sub-portfolio Q_i is same whenever there is exercise of 1 put there is usage of $1 \times Q_i$ and if there is exercise of 2 puts there is usage of $2 \times Q_i$ so we can write:

$$P\left(usage = r \cdot Q_i\right) = P\left(r \text{ puts being used}\right) \tag{13}$$

So the PGF of the sub-portfolio usage can be written as:

$$F_{Qi}(Z) = \sum_{r=0}^{\infty} P(Usage = r.Q_i) Z^{r \cdot Q_i} = \sum_{r=0}^{\infty} P(Usage = r) Z^{r \cdot Q_i} \tag{14}$$

$$F_{Qi}(z) = \sum_{r=0}^{\infty} e^{-S_i} \frac{S_i^r}{r!} Z^{r \cdot Q_i} = e^{-S_i} e^{S_i Z^{Q_i}} \tag{15}$$

Hence to find the usage distribution of the whole portfolio we must convolute each of these sub-portfolios. Hence form equation (11) for the whole portfolio with t sub-portfolios PGF can be written as (assuming independence of each sub-portfolio):

$$F_Q(z) = \prod_{i=1}^{t} F_{Q_i}(z) = \prod_{i=1}^{t} e^{-S_i} e^{S_i Z^{Q_i}} = e^{-\sum_{i=1}^{t} S_i + \sum_{i=1}^{t} S_i Z^{Q_i}} \tag{16}$$

Now to determine the exposure distribution of the portfolio we can have from Taylor's theorem

$$P(Usage = r.Q_i) = \frac{1}{r!} \frac{d^r(F_Q(z))}{dz^r} \Big|_{Z=0} \quad \text{for r=0,1,2,...} \tag{17}$$

Let $$W_r = \frac{1}{r!} \frac{d^r F_Q(Z)}{dz^r} \Big|_{Z=0} = \frac{1}{r!} \frac{d^{r-1}}{dz^{r-1}} \frac{d(F_Q(Z))}{dz} \Big|_{Z=0} \tag{18}$$

As $\sum_{i=1}^{t} S_i$ is constant hence:

$$W_r = \frac{1}{r!} \frac{d^{r-1}}{dz^{r-1}} \left(F_Q(Z) \cdot \frac{d\left(\sum\limits_{i=1}^{t} S_i Z^{Q_i}\right)}{dz} \right) \Big|_{z=0}$$ (19)

By Lebinitz's formula for nth order differentiation we can have:

$$W_r = \frac{1}{r!} \sum_{k=0}^{r-1} {}^{r-1}C_k \cdot \frac{d^{r-1-k} F_Q(Z)}{dz^{r-1-k}} \cdot \frac{d^{k+1}\left(\sum\limits_{i=1}^{t} S_i Z^{Q_i}\right)}{dz^{k+1}} \Big|_{Z=0}$$ (20)

If $Q_i = K+1$ & Z =0 then

$$\frac{d^{k+1}\left(\sum\limits_{i=1}^{t} S_i Z^{Q_i}\right)}{dz^{k+1}} = (k+1)! S_i$$ (21)

else if $Q_i \neq K+1$ $\qquad \dfrac{d^{k+1}\left(\sum\limits_{i=1}^{t} S_i Z^{Q_i}\right)}{dz^{k+1}} = 0$ (22)

And we have from equation (18)

$$W_{r-1-k} = \frac{1}{(r-1-k)!} \frac{d^{r-1-k} F_Q(Z)}{dZ^{r-1-k}} \Big|_{Z=0}$$ (23)

Hence combining equation (20), (21), (22) and (23) at Z=0 we have

$$W_r = \frac{1}{r!} \sum_{k=Q_i-1}^{r-1} {}^{r-1}C_k \cdot (k+1)! S_i \cdot (r-1-k)! W_{r-1-k}\big|_{Z=0} = \frac{1}{r} \sum_{i; Q_i \leq r} Q_i \cdot S_i \cdot W_{r-Q_i}$$ (24)

Hence from (24) and (10)

$$W_r = \frac{1}{r} \sum_{i; Q_i \leq r} \beta_i \cdot W_{r-Q_i}$$ (25)

Now if we have Q$_i$ with integers starting from 1 to a large enough integer we can

get the complete probability distribution of portfolio usage, starting from $W_0 = \sum_{i=1}^{t} S_i$.

If we let Q_i vary from 1 we will have exposure distribution from 1 \$ as seen from equation (3) Q_i represents the size of each put.

Till now we have used a constant α for the whole portfolio, in reality we will have a portfolio where α will be different for each segment of the portfolio depending upon product type, and other factors etc. As noted in a survey[10] banks generally prefer to use segment wise α for its EAD estimation. Hence after performing the computation till equation 25 we will be left with probability distribution of usage for different segments of the portfolio. To finally arrive at a portfolio level exposure distribution we will follow a standard convolution procedure using Fourier Transforms.

For simplicity lets assume we have only two distinct segment of the CCL portfolio and from (25) we can have two distinct vectors (F =$f_0,f_1,\ldots f_{l-1}$& G=$g_0,g_1,\ldots g_{m-1}$) representing the probabilities of usage for each segment. Let R represent the vector formed by convolution of F and G. To perform Fast Fourier Transform (FFT)[11] we will pad each of the vector such that length of each vector is s, where s=>l+m and s is of the form 2^x where x is an integer. We know from convolution theorem that

$$R = F \otimes G = Inverse\,FFT \quad (FFT\,(F).FFT\,(G)) \tag{26}$$

Hence R will give us the overall portfolio usage probability distribution. The procedure can easily be replicated if we have more than two segments in our portfolio.

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10 See RMA Survey (2004) on estimation of EAD & LGD
11 For details on Convolution using FFT please refer Mario (2004) and Robertson (1992)

IV Numerical Experiment

For a typical CCL portfolio, ΣS_i for the whole portfolio may be quite large and we are trying to assign probability to each dollar of usage in the algorithm so it may finally turn out to be daunting task to achieve the full distribution. For the calculation of negative exponential of a very large number (ΣS_i) for initiation of the calculation (as W_0) we will soon be confronted with precision issues under double-precision[12] regime of most common software applications. Most applications including Matlab, Octave or Microsoft Excel under default settings will approximate W_0 as zero and as the distribution depends on W_0 for derivation of full distribution, the full distribution would be evaluated incorrectly.

There may be many alternative to circumvent the problem in standard applications. One solution to this problem may be use of libraries which can handle very high precision calculation[13]. This may also require higher computational power in terms of hardware as well, the specification of which is beyond the scope of the current paper.

For illustration[14] we have chosen two sample segments[15] of 20 obligors each with of $\alpha = 10\%$ and $\alpha = 40\%$ respectively with limits[16] of each obligor varying from \$2,266 to \$96,330. Each of the obligor limit is divided into 1,000 puts. The figure in appendix B illustrates that the usage distribution of each segment and final convoluted portfolio; the

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12 For details see Monniaux (2008)
13 See http://gmplib.org/ for details
14 The calculations are done using Linux based Genius 1.0.7 as arbitrary precision calculator and Linux based Octave for FFT and final distribution evaluation.
15 Details of the portfolio presented in appendix A
16 The limits are chosen by using Microsoft Excel's random number generation function between \$500 and \$100,000

descriptive statistics of each of the distributions are presented in table 3 of appendix B for reference.

To explore the affect of choice of n in the exposure distribution we use first five obligors of portfolio A. The results of the experiment are summarized in Table 3 in appendix C. As noted the standard deviation of the usage distribution decreases as we increase the number of puts used. This may be explained by the fact that we are implicitly assuming a known value of α in our modeling i.e. a zero volatility of α and this fact is becoming more prominent once we start increasing the number of puts (n) i.e. more like real life scenario. The mean value remain relatively stable but the extreme points converge towards the mean to produce a shrinkage in the distribution shape.

Another prime variable in the algorithm is the value set for α, we vary the value of α to find its affect on the final distribution. The results of the same are summarized in Table 4 in appendix C. For our 5 obligor portfolio we see increase in the usage level also increase the volatility associated.

To incorporate volatility of α explicitly in the model we can also use a mixed Poisson process where we chose different values of λ from an assumed distribution. More commonly used mixture distribution has distribution of λ as Gamma distribution, resulting in negative binomial; this has the advantage of analytically tractable two parameter distribution. Similar combination[17] can be used to incorporate volatility of α explicitly in each segment of the portfolio.

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17 See Karlis and Xekalaki (2005) for various mixed Poisson distributions

An argument against using any mixed distribution may be that this will induces a second set of assumptions in our model and will require the banks to calculate usage volatility of each segment. In the current model the volatility of the final distribution will depend how spread out the expected usage is between each the segment. This may be more pragmatic approach considering that it will have minimum data requirement at portfolio-segment level and segmentation of the portfolio can be decided with internal research and expert judgment on usage.

One of the segmentation possibilities of the portfolio may be based on commitment fee[18] and service fee[19]. As shown by Thakor and Udell (1987) when the bank is uncertain about the level of partial draw-down, it may segregate borrowers by keeping high commitment fees and low service fees in one contract and low commitment fees and high service fees in other contract. Former would be attractive to borrowers with higher probability of draw-down as they are more probable to pay a service fee and more interested in having an active credit line, this will not be true for borrowers who are less confident about draw-downs. Contract choice may not be always that simple as discussed by Maksimovic (1990) it may also depend on structure of the borrower's industry, as in imperfect competition presence of predetermined rates of financing in borrowers armory provides enhanced strategic position.

V Conclusion

This paper formulated a parsimonious model for estimation of exposure at

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18 commitment fee is an up-front fee paid when the commitment is made
19 an service or annual fee is paid on the borrowed amount

portfolio level for a typical CCL portfolio by modeling each CCL with a borrower as a portfolio of option instruments. The exercise of each put has been modeled as standard Poisson process where average usage (α) of CCL at the portfolio level is assumed to be known. This value as indicated by Agarwal et. al. (2005), Qi (2009), Jiménez et al. (2009) Gatev et al. (2006) etc. depends on change in credit quality, difference between contractual rate and actual market rate and other factors. Both empirical research and theory suggest a correlation between credit quality and usage of credit line. The algorithm presented here can accommodate different values of α, to handle mentioned correlation the portfolio may be segmented in terms of credit quality along with any other criterion decided by the bank. Some of the methods of estimating α has been outlined by Moral (2006) and similar research needs to done on the area of finding expected usage at a given type of portfolio and as indicated earlier, they can work best if the bank themselves does the research on internal data, as this will be highly influenced by Bank's behavior in catching early signs of deterioration in credit quality. Further work may also be needed so that stable distribution parameters can be determined which will not be affected by choice of number of puts used.

Most of the current credit risk model viz. JP Morgan's CreditMetrics, KMV Portfolio Manager, CSFB CreditRisk+[20] has constant exposure as an input to calculate credit Value-at-Risk. Rosen and Marina (2002) describes that stochastic exposures make a notable difference in credit economic capital calculation. Akkaya and Wagner

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[20] For details see Gordy (2000) or Crouhy, Galai and Mark (2000) for a comparison of some credit risk models.

(2003) describe stochastic exposure in CreditRisk+ modeling framework. Stochastic Exposure generated from the discussed model can be useful in improvement of credit economic capital modeling for CCL portfolio.

Accurate exposure calculation is fundamental for liquidity risk management as well, e.g. credit card portfolio or HELC where all the accounts has a un-drawn but committed line pose a challenge to risk managers in terms of expected usage and henceforth liquidity positions; the algorithm presented here may prove helpful in providing meaningful insight into the problem.

The other implication of the algorithm is in the area of EAD estimation for Basel II. As pointed out earlier compared to PD estimation, limited research has gone in estimating EAD. This can provide a good starting point for the banks under AIRB approach of Basel II. Finally this may also be used in terms of stress testing tool to determine worst case liquidity scenarios for the portfolio. As we have the complete distribution of the usage value we can get a good estimate of our worst case scenarios from 99th or 99.9th percentile depending upon the risk appetite of the bank.

Further work needs to be done to improve the algorithm so as to use it in standard software applications with minimized hardware requirements. This will greatly help in quick and smooth implementation of the otherwise intuitive model.

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Appendix A

Table 1: **Portfolio A**

α=10%, n=1000			
Obligor	**Unused Limit ($)**	**Actual Put Size**	**Rounded Put Size**
A	81,289	81.29	82
B	13,626	13.63	14
C	10,941	10.94	11
D	20,709	20.71	21
E	20,786	20.79	21
F	70,371	70.37	71
G	63,514	63.51	64
H	65,435	65.44	66
J	96,330	96.33	97
I	87,555	87.56	88
J	33,906	33.91	34
K	40,567	40.57	41
L	23,650	23.65	24
M	10,779	10.78	11
N	20,838	20.84	21
O	7,602	7.60	8
M	16,695	16.70	17
Q	48,049	48.05	49
R	81,403	81.40	82
S	26,139	26.14	27

Table 2: **Portfolio B**

α=40%, n=1000			
Obligor	**Unused Limit ($)**	**Actual Put Size**	**Rounded Put Size**
A	7,129	7.12	8
B	29,253	29.25	30
C	27,353	27.35	28
D	41,340	41.34	42
E	2,266	2.26	3
F	51,946	51.94	52
G	23,439	23.43	24
H	70,214	70.21	71
J	6,191	6.19	7
I	55,709	55.7	56
J	45,242	45.24	46
K	5,861	5.86	6
L	80,444	80.44	81
M	44,273	44.27	45
N	19,605	19.6	20
O	34,828	34.82	35
M	68,265	68.27	69
Q	18,571	18.57	19
R	36,324	36.32	37
S	23,826	23.82	24

Appendix B

Table 3: Descriptive Statistics of Sample Portfolio [21] Limits

Portfolio	A	B	Convoluted Portfolio (C)
α	10.00%	40.00%	
Mean ($)	84,019	276,831	360,850
Standard Deviation ($)	2,287	3,691	4,342
Skewness	0.0324	0.0156	0.0144
Kurtosis	3.0011	3.0002	3.0005

Figure 1: Chart for Probability Distribution of sample portfolio A and B

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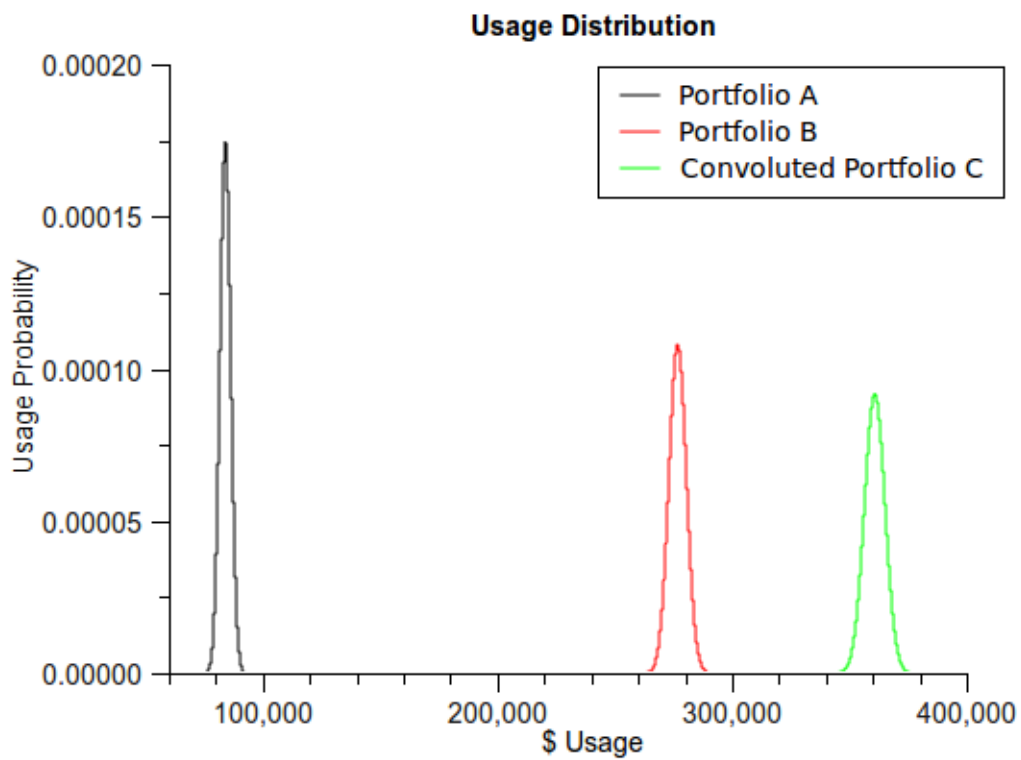
21   Here n is set at 1000

Appendix C

Table 4: Variation of Usage Distribution ($) parameters with n[22]

Percentile	n=700	n=800	n=900	n=1000	n=1100	n=1200	n=1300	n=1400	n=1500
50.00%	14,718	14,720	14,722	14,723	14,724	14,726	14,726	14,726	14,727
99.00%	17,272	17,100	16,970	16,849	16,741	16,662	16,583	16,520	16,460
99.50%	17,557	17,364	17,219	17,084	16,965	16,874	16,788	16,718	16,651
99.75%	17,823	17,612	17,452	17,304	17,173	17,074	16,978	16,903	16,829
99.90%	18,151	17,916	17,739	17,574	17,429	17,320	17,214	17,130	17,048
Standard Deviation	1,058	988	936	886	842	809	777	751	726

Table 5: Variation of Usage Distribution ($) parameters with α[23]

Percentile	α=0.10	α=0.20	α=0.40	α=0.50	α=0.60	α=0.70	α=0.80
50.00%	14,724	29,459	58,929	73,664	88,400	103,135	117,870
95.00%	16,214	31,552	61,876	76,955	92,001	107,023	122,024
99.00%	16,850	32,439	63,116	78,338	93,513	108,652	123,764
99.90%	17,575	33,445	64,519	79,901	95,220	110,492	125,728
99.97%	17,904	33,899	65,151	80,603	95,987	111,319	126,610
Standard Deviation	886	1,253	1,722	1,981	2,170	2,344	2,506

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22 Here α is kept constant at 10%
23 Here n is kept constant at 1000. Lower numbers for n and α is chosen to reduce computational time taken.