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Information Acquisition and Price Discrimination

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We consider a Hotelling model of price competition where firms may acquire costly information regarding the preferences (i.e. "location") of customers. By purchasing additional information, a firm has a finer partition regarding customer preferences, and its pricing decisions must be measurable with respect to this partition. If information acquisition decisions are common knowledge at the point where firms compete via prices, we show that a pure strategy subgame perfect equilibrium exists, and that there is "excess information acquisition" from the point of view of the firms. If information acquisition decisions are private information, a pure strategy equilibrium fails to exist. We compute a mixed strategy equilibrium for a range of parameter values.

1   Introduction

Usually, any type of price discrimination requires customer-specific information[1]. In general, it is costly to acquire information regarding customers. Recent developments in information technology allow firms to acquire more information on their customers, which may be used to practise price discrimination. Loyalty cards issued by supermarkets and customer data collected by specialist companies are just two examples of information acquisition.

Consider a model of competition between firms who are able to charge different prices if they can distinguish customer characteristics. Most research on discriminatory pricing assumes that the information regarding consumers is exogenously given. The price discrimination literature concentrates on monopolistic price discrimination (Pigou, 1920;

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[1]   The exceptions for this claim are the case in which the firm practices price discrimination through setting a uniform price when the cost of supply is different and when firm uses a non-linear pricing strategy.

Robinson, 1933; Schmalensee, 1981; Varian, 1985; Varian, 1989; and Hamilton & Slutsky, 2004). Such discrimination always leads to higher profits for the monopolist, since she solves her profit maximisation problem with fewer constraints.

Some of the more recent work on competitive price discrimination concentrates on efficiency from society's point of view, the firm's profit, and the number of the firms in a free entry and exit case[2] (Borenstein, 1985; Corts, 1998; Armstrong & Vickers, 2001; and Bhaskar & To, 2004), but still the information regarding consumers is exogenously given.

Bhaskar & To (2004) prove that without free entry, perfect price discrimination is socially optimal, but in free entry case, the number of firms is always excessive.

Liu & Serfes (2004, 2005) study the relation between of the exogenously given quality of firm information and market outcomes in oligopoly. They show that when the information quality is low, unilateral commitments not to price discriminate arise in equilibrium. However, once information quality is sufficiently high, firms discriminate. Equilibrium profits are lower, the game effectively becoming a prisoners' dilemma.

Shaffer & Zhang (2002) investigate one-to-one promotions. They assume that customers can be contacted individually, and firms know something about each customer's preferences. They find that one-to-one promotions always lead to an increase in price competition and average prices will decrease. However, they show that if one of the firms has a cost advantage or higher quality product, the increase in its market share may outweigh the effect of lower prices..

Corts (1998) investigates price discrimination by imperfectly competitive firms. He shows that the intensified competition, leading to lower prices, may make firms worse off and as a result firms may wish to avoid the discriminatory outcome. Unilateral commitments not to price discriminate may raise firm profits by softening price competition.

In this paper, we endogenize the information firms acquire by introducing an information acquisition technology. We assume that firms decide on how many units of information to acquire. Then each firm can charge different prices for different customers based on the information she acquired. We study a Hotelling type model where two firms are located at the ends of the unit interval. Each unit of information gives a firm a finer partition over the set of customers. Specifically, a firm's information consists of a partition of the unit interval, and an extra unit of information allows the firm to split one of the subintervals into two equal-sized segments. In our benchmark model, the information acquisition decisions of firms are common knowledge at the point where firms compete via prices.

Our main result is that the equilibrium outcome is partial information acquisition, even if information costs are arbitrarily small. Quite naturally, a firm has no incentive to acquire information on customers who are firmly in its rival's turf, i.e. those that it will never serve in equilibrium. But more interestingly, we find that a firm has an incentive not to fully acquire information on customers it competes for with the other firm. This allows it to commit to

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[2] For a more detailed survey on recent literature in price discrimination see Armstrong (2006).

higher prices, and thereby softens price competition. Finally, as in the existing literature, we find that there is "excess information acquisition" from the point of view of the firms, in the sense that profits are lower as compared to the no information case.

Information acquisition results in tougher competition, and lower prices. After information acquisition stage when firms compete via prices, if two firms share the market over a given set of customers, a decrease in the price of one firm over this interval decreases the marginal revenue of the other firm by decreasing its market share on this interval. As a result this reinforces the other firm to decrease its price over that interval. We can interpret this result in context of strategic complementarity as defined by Bulow et al (1985). In our benchmark model, pricing decisions are strategic complements. Since a firm's optimal price is an increasing function of her opponent's price. The literature on strategic complementarity finds similar results to our results when firms' actions are strategically complements. Fudenberg & Tirole (1984) show that in a two stage entry game of investment, the incumbent might decide to underinvest in order to deter entry. d'Aspremont et al (1979) consider a Hotelling framework, with quadratic transportation costs, when firms should choose their location. They show that in the equilibrium in order to avoid tougher competition, firms locate themselves at the two extremes (maximum differentiation). Similarly, in our model, a firm acquires less information in order to commit to pricing high, thereby increases the price of her rival.

We also analyse a game where a firm does not know its rival's information acquisition decision at the point that they compete in prices. We show, quite generally that there is no pure strategy equilibrium in this game. We compute a mixed strategy equilibrium for a specific example.

Section 2 presents the basic model. Section 3 analyzes the extensive form game; where each firm observes her rival's information partition so that the information acquisition decisions are common knowledge. Section 4 studies the game where information acquisition decisions are private. Section 5 summarizes and concludes.

2   The Model

The model is based on a simple linear city (Hotelling model) where two firms (A and B) compete to sell their product to customers located between them. Both firms have identical marginal costs, normalized to zero. The distance between two firms is normalized to one; firm A is located at 0 and firm B at 1. The customers are uniformly distributed on the interval $[0,1]$ and the total mass of them is normalized to one. Each customer, depending on her location and the prices charged by firms, decides to buy one unit from any of the firms or does not buy at all. The utility of buying for each customer has a linear representation $U = V - P - TC$ where P stands for price, and TC represents the transportation cost to buy from each firm that is a linear function of distance and t is the transport cost per unit distance. Assume that V is sufficiently high to guarantee that all the market will be served. Then the utility of the customer who is located at $x \in [0,1]$ is

$$U(x) = \begin{cases} V - P_A(x) - tx & \text{if she buys from } A \\ V - P_B(x) - t(1-x) & \text{if she buys from } B \end{cases}. \qquad (1)$$

A unit of information enables the firm to split an interval segment of her already recognized customers to two equal-sized sub-segments. The information about the customers, below and above the mid-point of [0,1] interval, is revealed to the firm if it pays a cost $\tau(0)$. Every unit of more information enables the firm to split an already recognized interval, $[a , b]$ to two equal-sized sub-intervals. The cost to the firm is $\tau(k)$ where $b - a = \left(\frac{1}{2}\right)^k$ (where $k \in \aleph \cup \{0\}$). The information cost function can be represented by the infinite sequence $\langle \tau(k) \rangle_0^\infty$. It seems reasonable to assume that $\tau(k)$ is decreasing in k. Intuitively the smaller the interval, the fewer consumers on whom information is needed. Then a reasonable assumption for the information cost function is that τ is a decreasing function.

We assume a decreasing information cost function when the cost of acquiring information on an interval $[a,b]$ is:

$$\tau(k) = \frac{\tau_0}{2^k}, \qquad \text{where} \qquad b - a = \left(\frac{1}{2}\right)^k, \qquad (2)$$

and τ_0 is a constant. Note that because of our information acquisition technology k is always an integer.

By buying every extra unit of information, a firm is acquiring more specific information with less information content in terms of the mass of customers.

The general results of the paper, i.e. the excessive information acquisition, the trade-off between information acquisition and tougher competition, and the characteristics of equilibrium are consistent for a wide range of information cost functions. In appendix B, we extend our results to two other functional forms.

We analyse two alternative extensive form games. In the first game, each firm observes its rival's information acquisition decision. That is, the information partitions become common knowledge before firms choose prices.

The first game is defined as follows:

- **Stage 1:** *Information acquisition*: Each firm ($f \in \{A, B\}$) chooses a partition I_f of [0,1] from a set of possible information partitions Ω

- *Observation*: Each firm observes the partition choice made by the other firm, e.g. firm A observes I_B. Note that firm f's information partition remains I_f .

- **Stage 2:** *Price decision*: Each firm chooses $P_f : [0,1] \to \Re^+ \cup \{0\}$ which is measurable with respect to I_f. Once prices have been chosen, customers decide whether to buy from firm A or firm B or not to buy at all.

The vector of prices chosen by each firm in stage 2 is segment specific. In fact a firm's ability to price discriminate depends on the information partition that she acquires in stage 1.

Acquiring information enables the firm to set different prices for different segments of partition.

In the second game firms do not observe their rival's information partition. It means that the firms simultaneously choose a partition and a vector of prices measurable with their chosen partition. In order to make it simpler, the two games are called *the two-stage game* and *the simultaneous move game* respectively.

Following we formally define the information acquisition technology. Intuitively, in this setting when a firm decides to acquire some information about customers, it is done by assuming binary characteristics for customers. Revealing any characteristics divides known segments customers to two sub-segments. We assume that these two sub-segments have equal lengths.

> **Definition:** The information acquisition decision for player f is the choice of I_f from the set of feasible partitions Ω on $[0,1]$. Ω is defined using our specific information acquisition technology:
>
> Suppose I is an arbitrary partition of $[0,1]$ of the form $[0,a_1), [a_1,a_2), \ldots, [a_{n-1},1]$ if and only if:[3]

$$s_0 = [a_0, a_1] \qquad \& \qquad s_i = [a_{i-1}, a_i] \qquad i \in \{2, 3, \cdots, n\}$$

$$\forall i \in \{1, 2, \cdots, n-1\} \qquad \exists l, k \in \{0, 1, \cdots, n\} \qquad l \neq k \qquad \therefore \qquad a_i = \frac{a_l + a_k}{2}$$

$$s_k \cap s_l = \emptyset \qquad \forall l, k \in \{0, 1, \cdots, n\} \qquad l \neq k \qquad\qquad (3)$$

$$\bigcup_{k=1}^{n} s_k = [0, 1].$$

A firm's action in stage 1 is the choice of an information partition from the set of possible information partitions. This choice can be represented by a sequence of {Yes, No} choices on a decision tree (figure 1). The firm begins with no information so that any customer belongs to the interval $[0,1]$. If the firm acquires one unit of information, the unit interval is partitioned into the sets $[0,0.5]$ and $(0.5,1]$. That is, for any customer with location x, the firm knows whether x belongs to $[0,0.5]$ or $(0.5,1]$, but has no further information. If the firm chooses No at this initial node, there are no further choices to be made. However, if the firm chooses Yes, then it has two further decisions to make. She must decide whether to partition $[0,0.5]$ into the subintervals $[0,0.25]$ and $(0.25,0,5]$. Similarly, she must also decide whether to partition $(0.5,1]$ into $(0.5,0.75]$ and $(0.75,1]$. Once again, if she says *No* at any decision node, then there are no further decisions to be made along that node, whereas if she says *Yes*, then it needs to make two further choices. The cost associated with each *Yes* answer is $\tau(k)$ (see equation (2) and figure 1). A *No* answer has no cost.

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[3] The set of equations in (3) are the technical definition of our information acquisition technology. Defining each element of the partition as a half-closed interval is without loss of generality, since customers have uniform distribution and each point is of measure zero.
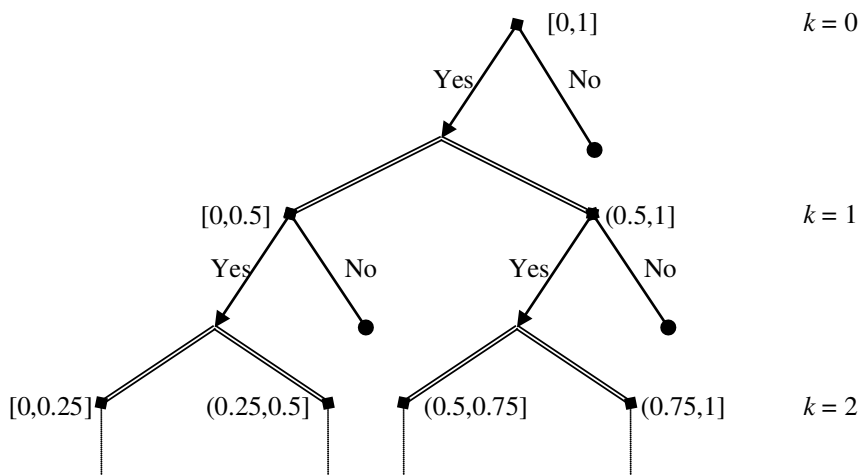
Figure 1: The decision tree for each firm regarding the information acquisition

Acquiring information enables a firm to price discriminate. The prices are segment-specific. The price component of any strategy (P_f) is a non-negative step function measurable with respect to I_f :

$$P_f: [0,1] \rightarrow \Re^+ \cup \{0\}$$

$$x, y \in s_i \quad \Rightarrow \quad P_f(x) = P_f(y). \qquad (4)$$

Then a feasible strategy for player f can be written as:

$$S_f = (I_f, P_f) \text{ where } P_f \text{ is measurable with respect to } I_f.$$

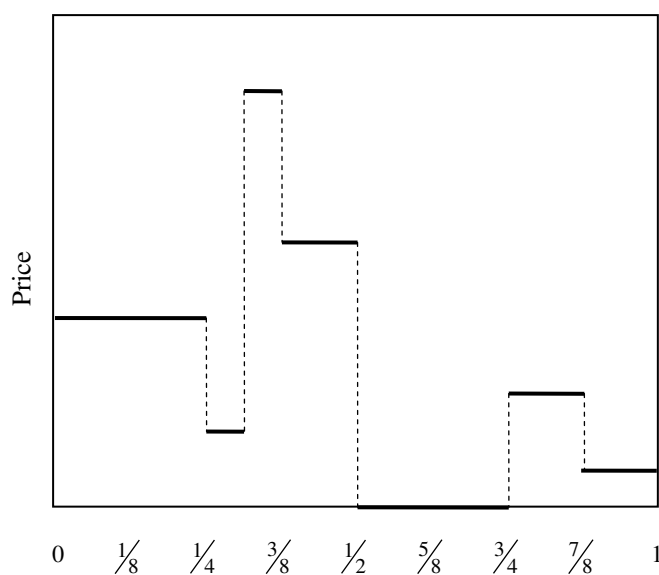Figure 2 shows a possible choice of strategy for one of the players.

Figure 2: A price function consistent with the information acquisition definition

Firm f 's payoff can be written as:

$$\pi_f = \int_{x \in Z_f} P_f(x).dx - \Gamma \qquad \text{where} \qquad Z_f = \{x | x \in [0,1], \qquad U_f(x) > U_{-f}(x)\},$$

and Γ is the total information cost for the firm, $U_f(x)$ represents the utility of customer located at x if she buys from firm f with the general form of (1), $-f$ stands for the other firm, and Z_f represents the set of customers who buy from f.

Lemma 1: Suppose $s_i \in I_A$ and $s_j \in I_B$, where $s_i \cap s_j = \emptyset$, Then either s_i is a subset of s_j or s_j is a subset of s_i.

Proof: By acquiring any information unit a firm can divide one of her existing intervals into two equal-sized sub-intervals. Given s_i is an element of A's information partition, three possible distinct cases may arise: i) s_i is an element of firm B's information partition. ii) s_i is a strict subset of an element of firm B's information partition. iii) s_i is the union of several elements from B's information partition.[4]

Figure 3 shows these three possibilities where i) s_i and s_j are equal (case 1), ii) s_i is a proper subset of s_j (case 2), and iii) s_j is a proper subset of s_i (case 3).
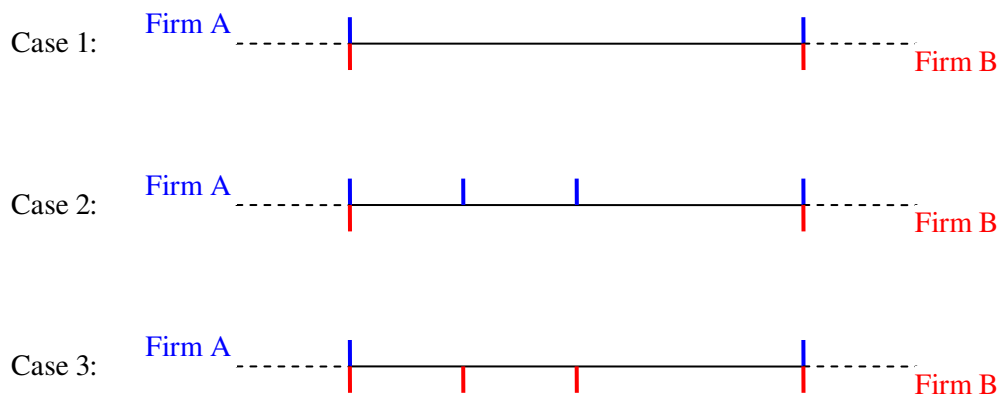
The blue and red lines show the partitions for s_i and s_j respectively (the information partitions chosen by firm A and firm B).

Figure 3: Three possible segmentation scenarios

3   The Two-Stage Game

This game can be broken down into four different scenarios (Figure 4). The first scenario relates to the case when neither of the firms acquires information. The second scenario represents the case where both firms acquire information. The third and fourth scenarios represent the situation where only one of the firms decides to acquire information.

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[4]   It is expected that in each firm's turf the preferred segmentation scenario of the firm contains smaller segments compared with her rival's preferred segments, but all cases are solved.

		Firm B	
		NI	I
Firm A	NI	1st Scenario	4th Scenario
	I	3rd Scenario	2nd Scenario

Note: 3rd and 4th scenarios are symmetric

Figure 4: Four different scenarios for the two-stage game

Scenario One: Neither firm acquires information

The first scenario (the case of acquiring no information and therefore no price discrimination) is easily solvable. In the equilibrium both firms charge uniform prices ($P_A = P_B = t$), they share the market equally, and each firm's profit is $\frac{t}{2}$.

Scenario Two: Both firms acquire information

In this scenario each firm acquires at least one unit of information that splits the interval [0,1] into subintervals ([0,0.5] and (0.5,1]. Let us consider competition on an interval that is a subset of [0,0.5] (given the symmetry of the problem, our results also extend to the case where the interval is a subset of (0,5,1]).

Let $\hat{s} \subseteq [0, 0.5]$ and $\hat{s} \in I_A$, i.e. assume that $\hat{s}$ is an element of firm A's information partition. Consider first the case where $\hat{s}$ is the union of several elements of B's information partition, i.e. $\hat{s} = \bigcup_{i=1}^{n} s_i$, for $i = 1,2,...n$; This situation corresponds to case 3 in Figure 3.

Since firm A's profits on the rest of the interval do not depend upon $P_{\hat{s}}$, she must choose $P_{\hat{s}}$ aiming to maximize her profit on $\hat{s}$. By lemma 1, firm B's profits on the components of $\hat{s}$ only depend upon her prices on this interval. Therefore, a necessary condition for the Nash equilibrium is that:

a) A chooses $P_{\hat{s}}$ to maximize her profits on $\hat{s}$,

b) B chooses $P_1, \cdots, P_n$ to maximize her profits on $\hat{s}$.

An analogous argument also applies in cases 1 and 2.

From the utility function (1), the indifferent customer x_i in each $s_i = (a_{i-1}, a_i]$ is located at:

$$\text{Case 1：} \quad x_i = \frac{1}{2} + \frac{P_B - P_A}{2t}, \qquad \text{for } i = 1; \qquad (5)$$

$$\text{Case 2：} \quad x_i = \frac{1}{2} + \frac{P_B - P_{iA}}{2t}, \qquad \text{for } i = 1,2,\cdots,n; \qquad (6)$$

$$\text{Case 3：} \quad x_i = \frac{1}{2} + \frac{P_{iB} - P_A}{2t}, \qquad \text{for } i = 1,2,\cdots,n. \qquad (7)$$

In addition, these values for x_i must lie in within the interval, i.e. the following inequality should be satisfied for each x_i:

$$a_{i-1} \leq x_i \leq a_i, \qquad \text{for } i = 1,2,\cdots,n \text{; and} \qquad (8)$$

Where a_{i-1} and a_i are respectively the lower and upper borders of the segment s_i and $0 \leq a_{i-1} < a_i \leq 0.5$.

In each segment s_i, the customers who are located to the left of x_i buy from firm A, and the customers to the right of x_i buy from firm B. If x_i (calculated in (5) or (6) or (7)) is larger than the upper border (a_i) all customers on s_i buy from firm A. In this situation to maximize her profit, firm A will set her price for s_i to make the customer on the right border indifferent. Similarly, if $x_i < a_{i-1}$, firm B is a constrained monopolist on s_i and will set her price to make the customer on the left border indifferent.

Profits for the firms in each section of case 1 are:

$$\pi_A = P_A \left(\frac{1}{2} + \frac{P_B - P_A}{2t} - a_0 \right), \text{and} \qquad (9)$$

$$\pi_B = P_B \left(a_1 - \frac{1}{2} - \frac{P_B - P_A}{2t} \right). \qquad (10)$$

In cases 2 and 3, as mentioned before since the profit for each firm over s_i ($i = 1,2,...,n$) can be presented only as a function of the prices over $\hat{s}$, the maximization problem is solvable for $\hat{s}$ independently. In case 2, firms' profits on $\hat{s}$ are:

$$\pi_{iA} = P_{iA} \left(\frac{1}{2} + \frac{P_B - P_{iA}}{2t} - a_{i-1} \right), \qquad \text{for } i = 1,2,\cdots,n \text{; and} \qquad (11)$$

$$\pi_B = P_B \sum_{i=1}^{n} \left(a_i - \frac{1}{2} - \frac{P_B - P_{iA}}{2t} \right) = P_B \left(\sum_{i=1}^{n} a_i - \frac{n}{2} - \frac{n P_B - \sum_{i=1}^{n} P_{iA}}{2t} \right). \qquad (12)$$

Similarly for case 3, the profits can be written as:

$$\pi_A = P_A \sum_{i=1}^{n} \left(\frac{1}{2} + \frac{P_{iB} - P_A}{2t} - a_{i-1} \right) = P_A \left(\frac{n}{2} + \frac{\sum_{i=1}^{n} P_{iB} - n P_A}{2t} - \sum_{i=1}^{n} a_{i-1} \right); \text{and} \qquad (13)$$

$$\pi_{iB} = P_{iB} \left(a_i - \frac{1}{2} - \frac{P_{iB} - P_A}{2t} \right), \qquad \text{for } i = 1,2,\cdots,n. \qquad (14)$$

So for cases 2 and 3, there are $n+1$ maximization problems on each $\hat{s}$ which should be solved simultaneously.

Let I_A and I_B be two feasible information partitions for A and B respectively. Let I^* be the join of I_A and I_B, i.e. the coarest partition of $[0,1]$ that is finer than either I_A or I_B. Let s^* be the element of I^* which is of the form $[a,0.5)$, i.e. s^* is the element that lies on the left and is closest to the midpoint.

Lemma 2: s^* is the only element of I^* which lies to the left of 0.5 such that both firms share the market. On every other element of I^* which lies to the left of the midpoint, all customers buy from firm A.[5]

Proof: See appendix A.

So both firms sell positive quantities only in the most right hand segment of firm A's turf and the most left hand segment of firm B's turf.

This lemma has the following important implication. A firm has no incentive to acquire information in its rival's turf. For example if firm A acquires some information on interval $(0.5,1]$; then firm B can choose a set of profit maximizing prices where she shares the market with firm A only on the very first segment of this interval. So acquiring information on the interval $(0.5,1]$ makes no difference on firm A's ability to attract more customers.

As a result of lemma 2, each firm sets a uniform price for all customers located on her rival's turf. Let us call these prices P_{RA} and P_{LB}. P_{RA} is the price firm A sets for $[0.5,1]$ and P_{LB} is the price firm B sets for $[0,0.5)$. This price is set to maximize firm's profit in the only segment in the opposite turf that firm sells positive quantity in it. This uniform price affects the rival's price in her constrained monopoly segments. Thus the pricing behavior of firm A can be explained by these rules:

– In all segments on $[0,0.5]$ except the very last one, s^*, firm A is a constrained monopolist. She sets her prices to make the customer on the right hand border of each segment indifferent.

– On s^*, the last segment to the right hand side of $[0,0.5]$, firm A competes against the uniform price set by firm B for the $[0,0.5]$ interval.

– On $(0.5,1]$ she only can sell on the very first segment then sets her uniform price for $(0.5,1]$ in order to maximize her profit on that very first segment.

So, firm A's partition divides $[0,0.5]$ into n segments and she acquires no information on $(0.5,1]$. Equivalently, firm B acquires no information on $[0,0.5]$ and has m segments in her partition for $(0.5,1]$.

We now solve for equilibrium prices. The prices for loyal customers in each side would come from:

$$P_{iA} + t\, a_i = P_{LB} + t\,(1 - a_i), \qquad \text{for } a_{i-1} \le x_i \le a_i\,; \qquad i = 1, \cdots, n-1, \text{and}$$

$$P_{RA} + t\, a_{i-1} = P_{iB} + t\,(1 - a_{i-1}), \qquad \text{for } a_{i-1} \le x_i \le a_i\,; \qquad i = n+2, \cdots, n+m\,.$$

The prices for two shared market segments are represented by (recall from (A1) and (A2)):

$$P_{nA} = \frac{2t}{3}\,(1 - 2a_{n-1}) \qquad \text{and} \qquad P_{n+1,B} = \frac{2t}{3}\,(2a_{n+1} - 1).$$

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[5] The solution for the interval $[0,0.5]$ can be extended to interval $(0.5,1]$ where the solution is the mirror image of the result on $[0,0.5]$.

And the uniform prices for the opposite side could be written as:

$$P_{RA} = \frac{1}{2} \ P_{n+1,B} \qquad \text{and} \qquad P_{LB} = \frac{1}{2} \ P_{nA}.$$

Then given the prices for these two segments prices for other segments can easily be calculated as:

$$P_A(x) = \begin{cases} 2t\left(\frac{2}{3} - a_i - \frac{1}{3}a_{n-1}\right) & a_{i-1} < x < a_i; \qquad i = 1, \cdots, n-1 \\ \frac{2t}{3}(1 - 2a_{n-1}) & a_{n-1} < x < \frac{1}{2} \\ \frac{t}{3}(2a_{n+1} - 1) & \frac{1}{2} < x < 1 \end{cases} \qquad ; \qquad (15)$$

and

$$P_B(x) = \begin{cases} \frac{t}{3}(1 - 2a_{n-1}) & 0 < x < \frac{1}{2} \\ \frac{2t}{3}(2a_{n+1} - 1) & \frac{1}{2} < x < a_{n+1} \\ 2t\left(\frac{1}{3}a_{n+1} + a_{i-1} - \frac{2}{3}\right) & a_{i-1} < x < a_i; \qquad i = n+2, \cdots, n+m \end{cases} \qquad . \qquad (16)$$

The associated gross profits are (market shares for border segments are calculated using the prices by (5)):

$$\pi_A = 2t\sum_{i=1}^{n-1}(a_i - a_{i-1})\left(\frac{2}{3} - a_i - \frac{1}{3}a_{n-1}\right) + \frac{2t}{3}(1 - 2a_{n-1}).\frac{2}{3}\left(\frac{1}{2} - a_{n-1}\right)$$

$$+ \frac{t}{3}(2a_{n+1} - 1).\frac{1}{3}\left(a_{n+1} - \frac{1}{2}\right), \qquad \text{and}$$

$$\pi_B = \frac{t}{3}(1 - 2a_{n-1}).\frac{1}{3}\left(\frac{1}{2} - a_{n-1}\right) + \frac{2t}{3}(2a_{n+1} - 1).\frac{2}{3}\left(a_{n+1} - \frac{1}{2}\right)$$

$$+ 2t\sum_{i=n+1}^{n+m}(a_i - a_{i-1})\left(\frac{1}{3}a_{n+1} + a_{i-1} - \frac{2}{3}\right).$$

After simplifying, the net profits are represented by:

$$\pi_A = t\left(2a_{n-1}\left(\frac{2}{3} - \frac{1}{3}a_{n-1}\right) - 2\sum_{i=1}^{n-1}a_i(a_i - a_{i-1}) + \frac{8}{9}\left(\frac{1}{2} - a_{n-1}\right)^2 + \frac{2}{9}\left(a_{n+1} - \frac{1}{2}\right)^2\right) - \Gamma_A, \text{and} \quad (17)$$

$$\pi_B = t\left(-\frac{4}{3} + 2a_{n+1} - \frac{2}{3}a_{n+1}^2 + 2\sum_{i=n+2}^{n+m}a_{i-1}(a_i - a_{i-1}) + \frac{2}{9}\left(\frac{1}{2} - a_{n-1}\right)^2 + \frac{8}{9}\left(a_{n+1} - \frac{1}{2}\right)^2\right) - \Gamma_B. \quad (18)$$

where Γ_A, Γ_B are the information costs paid by firm A and firm B in order to acquire information.

If we want to follow the firm's decision making process we can suppose that the firm starts with only one unit of information and splitting [0.1] interval to [0,0.5] and (0.5,1]. This

first unit enables the firm to start discriminating on her half. Paying for one more unit of information on her own half means that firm is now a constrained monopolist on one part and should share the market on the other (i.e. for firm *A*, the customers on [0,0.25] are her loyal customers and she shares the market on (0.25,0.5] with firm *B*). In the loyal segment the only concern for the information acquisition would be the cost of the information. Firms fully discriminate the customers depending on the cost of information.

But there is a trade off in acquiring information and reducing the length of shared segment. On one hand, this decision increases the profit of the firm through more loyal customers. On the other hand, since her rival charges a uniform price for all customers in the firm's turf, the firm should lower the price for all of her loyal segments. Therefore the second effect reduces the firm's profit. These two opposite forces affect the firm's decision for acquiring a finer partition in the border segment. Proposition 1 shows how each firm decides on the volume of the customer-specific information she is going to acquire.

Proposition 1: Firm *A* uses these three rules to acquire information:

1-1. if $\frac{\tau_0}{t} \leq \frac{1}{16}$, then firm *A* fully discriminates on [0,0.25]. The equal-size of the segments on this interval in the equilibrium partition is determined by the information cost.

1-2. Firm *A* acquires no further information on (0.25,0.5].

1-3. Firm *A* acquires no information on (0.5,1].

Proof: In order to avoid unnecessary complications, the first part of the proof has been discussed in appendix A. It shows that firm *A* should make a series of decisions regarding to split the border segment (see Figure 1). Starting from the point of acquiring no information on the left hand side, firm *A* acquires information in her own turf as long as this expression is non-negative:

$$\Delta \pi_A = \frac{t}{2^k} \left[\frac{1}{2^k} \left(\frac{7}{12} - \frac{1}{2^{r+2}} \right) - \frac{1}{6} \right] - \Gamma, \qquad (19)$$

where $\frac{1}{2} - a_{n-1} = \left(\frac{1}{2} \right)^k$ is the length of the border segment,

$\left(\frac{1}{2} \right)^r$ is the preferred length of loyal segments (where $r = \left\lfloor \log_2 \frac{t}{2\tau_0} \right\rfloor$ [6]), and

$\Gamma = \frac{\tau_0}{2^k} \left(\frac{r-k+1}{2} \right)$ is the total information cost.

We follow this chain of decision makings, starting with $n = 1$ (i.e. no initial loyal segment for firm *A*). The procedure is that firm *A* starts with $k = 1$, if equation (19) is non-negative then she decides to acquire information on [0, 0.5], splitting this interval to [0, 0.25] and (0.25,0.5]. After this he is the constrained monopolist on [0, 0.25] and the preferred length for all loyal segments is:

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[6] $\lfloor \quad \rfloor$ notation represents the floor function (or the greatest integer).

$$\left(\frac{1}{2}\right)^r, \qquad \text{where} \qquad r = \left\lfloor \log_2 \frac{t}{2\tau_0} \right\rfloor. \qquad (20)$$

After buying the first unit of information on [0, 0.5] (and consequently the preferred units of information on [0, 0.25]) then firm A checks non-negativity of (19) for $k = 2$ and so forth.

Table 1 shows the chain of the first two decision statements. As it is clear, the value of the decision statement on the second row (and also for every $k > 1$) is always negative.

Table 1: Firm A's chain of decision statements

a_{n-1}	a_n	k	Decision statement		
0	$\frac{1}{2}$	1	$\frac{t}{16}\left(1 - \frac{1}{2^r}\right) - \frac{r\,\tau_0}{4} \geq 0$	where	$r = \left\lfloor \log_2 \frac{t}{2\tau_0} \right\rfloor$
$\frac{1}{4}$	$\frac{1}{2}$	2	$\frac{t}{32}\left(-\frac{1}{6} - \frac{1}{2^r}\right) - \frac{(r-2)\,\tau_0}{8} \geq 0$	where	$r = \left\lfloor \log_2 \frac{t}{2\tau_0} \right\rfloor$

The minimum value for r is 2 (the biggest possible length for a loyal segment is $\frac{1}{4}$). It can be shown that the value of decision statement (equation (19) for $k = 1$) is non-negative if and only if $\frac{\tau_0}{t} \leq \frac{1}{16}$. It is clear the second row's decision statement is always negative. That means the length of the shared segment, regardless to the information cost, equals 0.25. Assuming $\frac{\tau_0}{t} \leq \frac{1}{16}$, then the preferred segmentation scenario for firm A is to fully discriminate between [0,0.25] (the preferred segment length in this interval is a function of information and transportation cost) and acquiring no information for (0.25,0.5]. ♦ *QED*

Each firm prefers to just pay for information in her own turf, and the segmentation in the constrained monopoly part depends on the transportation and information cost. The only segment in each turf that may have a different length is the border segment and firms prefer to buy no information on their rival's turf.

One of the findings in the proof of proposition 1 is the functional form of firms' marginal profit of information in loyal segment. Equation (A.16) shows that the marginal profit of information in a loyal segment (dividing a loyal segment to two) is

$$\frac{t}{4}(a_i - a_{i-1})^2 - \tau(k) = \frac{1}{2^k}\left(\frac{t}{4 \times 2^k} - \tau_0\right)$$

where a_i and a_{i-1} are the boundaries of the loyal segment and $a_i - a_{i-1} = \frac{1}{2^k}$. As it is clear the marginal profit of information is decreasing. Decreasing marginal profit guarantees that if the firm decides not to split a loyal segment, there is no need to worry about the profitability of acquiring a finer partition.

When the information cost is insignificant ($\tau_0 = 0$) the preferred length of a loyal segment goes to zero. In other words, firms acquire information for every individual customer on [0,0.25] and charge a different price for each individual based on her location. In this case,

in equation (19) $k \to \infty$, starting with $a_{n-1} = 0$ and $a_n = 0.5$ and based on the proposition 1 the chain of decisions for $\tau = 0$ is:

i) On [0,0.5], $a_{n-1} = 0$ then equation (19) turns into $\frac{t}{16} > 0$ and the result is to acquire the first unit of information, and consequently acquire full information on [0,0.25].

ii) On (0.25,0.5], $a_{n-1} = 0.25$ then $-\frac{t}{192} < 0$ and the result is to acquire no further information on (0.25, 0.5]. That means even when the information cost is insignificant, the positive effect of acquiring more information on the interval of (0.25,0.5] is dominated by the negative effect of falling the constrained monopolistic prices on segments on [0,0.25].

Figure 5 shows an example for an equilibrium strategy for firm A. Firm B's preferred strategy will be a mirror image of this example.
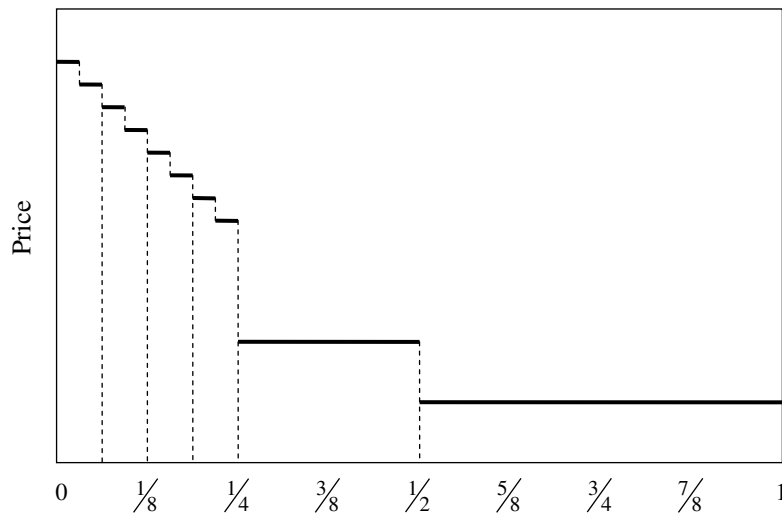

Figure 5: An example of the equilibrium strategy for firm A in the second scenario

Figure 6 shows the preferred number of segments by firm A in her turf against the ratio of information cost to transportation cost. This figure also shows the net profit of the firm in her own turf as a multiplication of transportation cost.
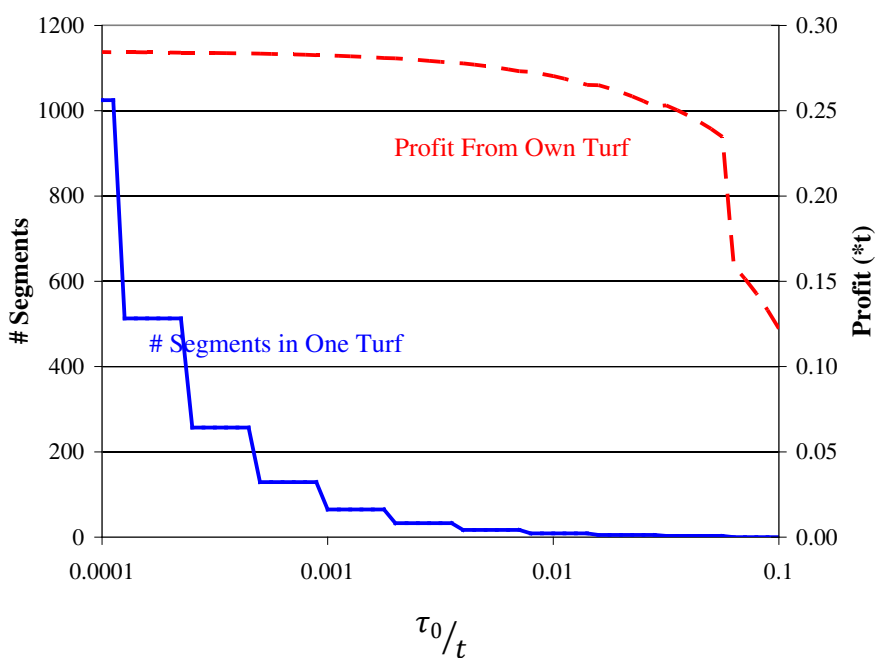
**Figure 6: The preferred number of segments and the profit of each firm
as a function of the ratio of information cost over transportation cost**

Figure 7 represents each firm's preferred length for shared and loyal segments as a function of the information and transportation costs.
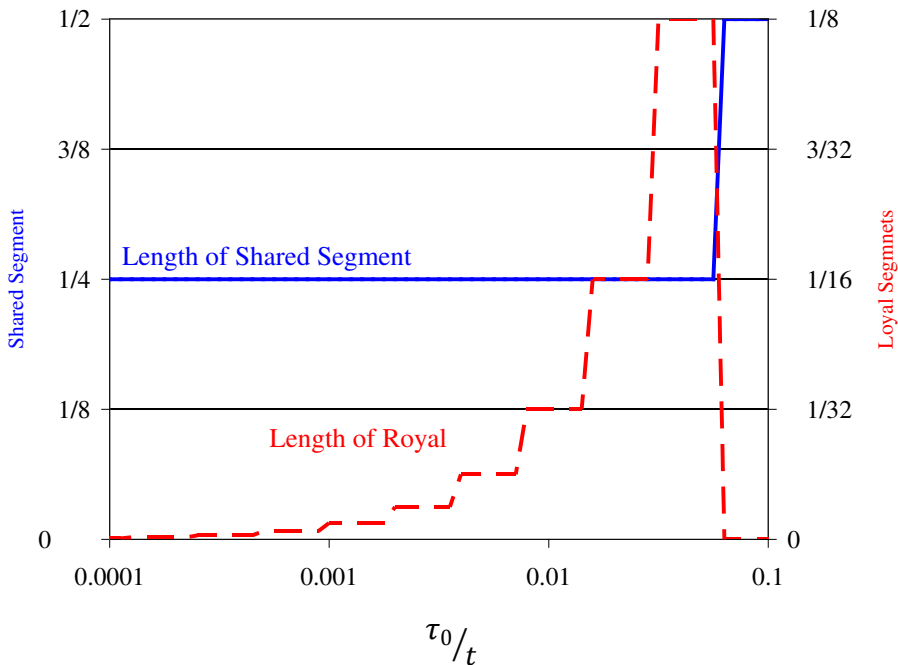

**Figure 7: The length of shared and loyal segments
as a function of the ratio of information cost over transportation cost**

Figure 7 also shows that if the cost of the first information unit[7] is less than $\frac{t}{16}$, then firm A would be better off by discriminatory pricing in her own turf.

In appendix B, we show that proposition 1 is also true for two other information cost functional forms. However, the upper limit on τ_0 and the preferred length of loyal segments are different for each case.

Scenario Three: Only firm A acquires information

This scenario is symmetric with scenario 4 (only firm B acquires information).

Proposition 2: When firm B acquires no information, firm A uses these two rules to acquire information:

2-1. <u>In her own turf:</u> prefers to fully discriminate (subject to information cost).

2-2. <u>In her rival's turf:</u> acquires no information.

Proof: See appendix A.

In this case firm B is not able of any discrimination and charges a uniform price for all customers. If $\frac{1}{16} \leq \frac{\tau_0}{t} \leq \frac{1}{8}$, then firm A would prefer to acquire just one unit of information in the left hand side $[0,0.5]$ and the unique equilibrium prices are:

$$P_A(x) = \begin{cases} t & x \in [0,0.25] \\ \dfrac{3t}{4} & x \in (0.25,0.5] \\ \dfrac{t}{4} & x \in (0.5,1] \end{cases} \qquad \text{and} \qquad P_B(x) = \frac{t}{2}.$$

If $\frac{\tau_0}{t} \leq \frac{1}{16}$, then the proof of proposition 2 shows that firm B has no share from the left hand side (even in the one next to 0.5 point) and only chooses her unique price to maximize her profit on $(0.5,1]$ interval. On the other hand, firm A would prefer to fully discriminate the left hand side $[0,0.5]$ and the preferred length of the segments are determined by equation (20):

$$P_A(x) = \begin{cases} \left(\dfrac{7}{6} - a_i\right)t & x \in s_i = (a_{i-1}, a_i] \subset [0,0.5] \\ \dfrac{t}{3} & x \in (0.5,1] \end{cases} \qquad \text{and} \qquad P_B(x) = \frac{2t}{3}.$$

Figure 8 shows a possible solution for this sub-game. As it is clear the indifferent customer on $[0,0.5]$ is the customer who is located exactly on 0.5. Then all the left hand side customers buy from firm A. On the right hand side, firm A's market share is $\frac{1}{3}$ and the rest buy from firm B.

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[7]   This condition comes from (19) and (20). If the information cost is higher than this upper limit, then firm A decides to acquire no information in her own turf at all.
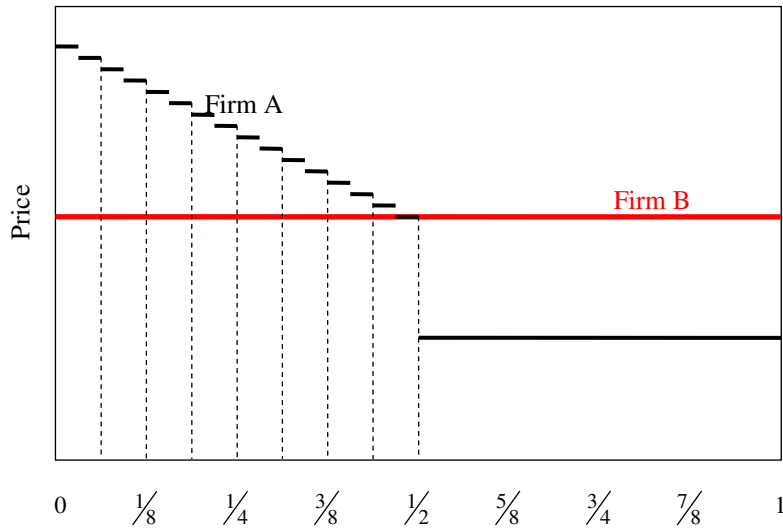
Figure 8: An example of the equilibrium strategy for the firms in the third scenario

Outcome of the Two-Stage Game

Figure 9 shows the strategic representation of the game when the information cost is insignificant ($\tau_0 \to 0$). As it can be seen the game is a prisoners' dilemma.

Firm B

		NI	I
Firm A	NI	$\dfrac{t}{2}, \dfrac{t}{2}$	$\dfrac{2t}{9}, \dfrac{25t}{36}$
	I	$\dfrac{25t}{36}, \dfrac{2t}{9}$	$\dfrac{43t}{144}, \dfrac{43t}{144}$

Figure 9: The outcome of the game when $\tau_0 \to 0$

Figure 10 represents firm A's profit as a function of $\dfrac{\tau_0}{t}$. In each pair of strategies the first notation refers to firm A's strategy and the second one to firm B's. If firm B acquires information, firm A's best response is to do so, irrespective of the information cost. If the other firm acquires no information, the best response is to acquire information if the information cost is sufficiently low. So if the information cost is sufficiently low, the game becomes a prisoners' dilemma and both firms would have a dominant strategy to acquire information. This threshold is $\dfrac{\tau_0}{t} \approx 0.039$.
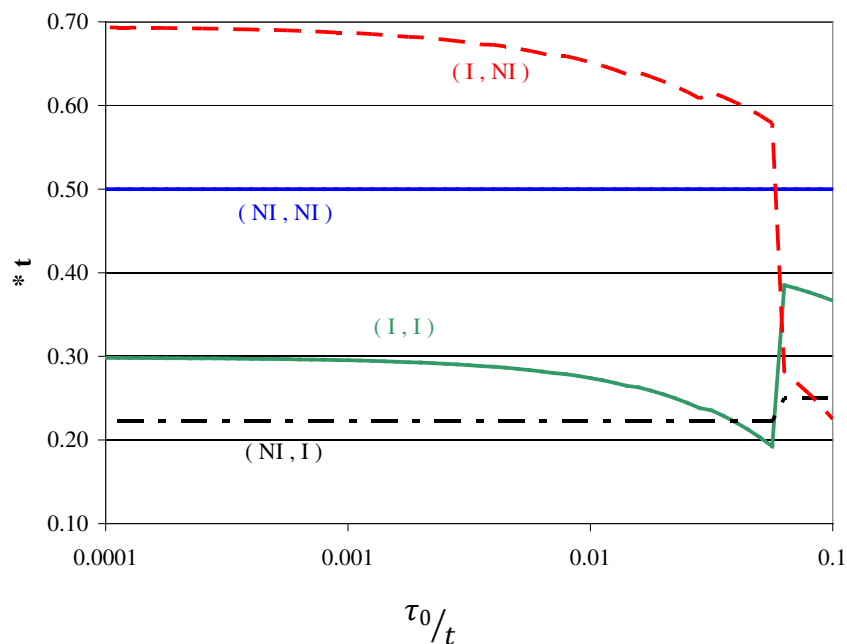
**Figure 10: Firm *A*'s profit for four different scenarios versus
the information / transportation cost ratio**

Then if $\frac{\tau_0}{t} > 0.039$, the game has two Nash equilibria: i) both firms acquire information
and ii) neither of the firms acquire information. If $\frac{\tau_0}{t} \leq 0.039$, the game is a prisoners'
dilemma where information acquisition is the dominant strategy for both firms. In this case,
we have excess information acquisition from the firm point of view. Acquiring more
information will lead to tougher competition and even in the limit, when $\tau_0 \rightarrow 0$, will lead to
about 40% decrease in firms' profits.

Acquiring information has two opposite effects on the firm's profit. It enables the firm to
price discriminate and on the other hand toughens the competition. The latter effect dominates
the former and when both firms acquire information, they both worsen off. Fixing the
partitions for both firms, then pricings are strategically complement.

Given the outcome of this game, one might ask why do the firms not freely give each
other information about customers on their own turfs? The issue of collusion in sharing the
information in this game can be looked at from two different points of view.

Firstly, in the real world situation that our setting might be applied to sharing the
customer information with a third party is usually illegal. For example, Tescos –the biggest
supermarket in the UK with almost one third of the market share- has a huge pool of specific
information about its customers via its club-card scheme. However, it is illegal for Tescos to
share this information with other supermarkets.

Secondly, as it is showed in this section, information about the customers in the other side
of market has no strategic importance for the firm. Then even if the information is available, it
makes no vital part in pricing decision. Since the outcome of the game shows excessive
information acquisition, one possibility for collusion is to collude and not acquire

information. However, as it was showed, firms have incentive to deviate from this agreement and acquire information.

4 The Simultaneous Move Game

In this game, firms cannot observe their rival's information partition. It seems that the two-stage game is able to offer a better explanation of the information acquisition decision in a competitive market. Firms (especially in retailer market) closely monitor their rival's behavior. Then it seems a reasonable assumption to consider that while competing via prices, they are aware of the information partition chosen by their rival.

We will show that there is no pure strategy equilibrium in this game. Remember that every strategy has two parts, the segmentation scenario and the prices for each segment. To prove non-existence of pure strategy equilibrium, we show that for different cases (regarding the information acquisition decision), at least one of the firms has incentive to deviate from any assumed pure strategy equilibrium.

Case 1: None of the firms acquire information

The proof for the situation that none of the firms acquire information is trivial. When both firms decide to buy no information, the outcome would be charging a uniform price of t for both firms. It is clear that a firm has incentive to deviate from this strategy and acquire some information when τ_0 is sufficiently small.

Case 2: Both firms acquire information

Suppose there is a pure strategy equilibrium where both firms acquire some information. Firstly we will show that in this equilibrium, firms acquire no information on their rival's turf. Assume firm B acquires some information on [0,0.5]. In the equilibrium, every firm can predict her rival's partition accurately. Therefore, based on lemma 2, firm B makes no sale on every interval except the final right segment on [0,0.5]. Then the information on this interval is redundant for firm B. She can profitably deviate, acquire no information on [0,0.5], and charge the same price for the entire interval. Then in any pure strategy equilibrium firm B acquires no information on firm A's turf. We will therefore consider all different possibilities for firm A to acquire information on [0,0.5]. Then we will show that in any candidate equilibrium at least one of the firms has incentive to deviate (the fact that in the equilibrium each firm can predict her rival's strategy is accurately used).

i) *No further information in [0,0.5]*: The corresponding equilibrium prices at this interval are

$$P_A(x) = \frac{2t}{3} \qquad \text{and} \qquad P_B(x) = \frac{t}{3}; \qquad \text{for } \forall x \in [0,0.5].$$

It is trivial that firm A has incentive to deviate and acquire some information on the left hand side. The information cost is not a binding constraint here. It has been shown in the proof of proposition 1 that the constraint on whether to acquire some information on [0,0.5) is

more relaxed than acquiring any information in the first place (acquiring information on [0,1]).

ii) *Partial discrimination on [0,0.25]*: This means firm *A* acquires the information which splits [0,0.5] interval to [0,0.25] and (0.25,0.5] and some information (but not fully discrimination) on [0,0.25] (and possibly some information on (0.25,0.5]). In the equilibrium, firm *A* knows that firm *B* sets a uniform price on the left hand side to maximize her payoff from the very last segment on the right hand side of (0.25,0.5] interval. Responding to this, as shown in proof of proposition 1, firm *A* has incentive to fully discriminate on [0,0.25]. So a strategy profile like this cannot be an equilibrium.

iii) *Full discrimination on [0,0.25] and no further information on (0.25,0.5]*: As the results of lemma 2 and proposition 1 show, if the information cost is sufficiently low in equilibrium, firm *A* fully discriminates customers between [0,0.25] (subject to information cost) and charges a uniform price for the section (0.25,0.5], and firm *B* charges a uniform price for all customers on the left hand side in order to gain the most possible profit from the customers on (0.25,0.5]. Now we want to investigate the players' incentive to deviate from this strategy profile.

Firm *A* has incentive to deviate from this strategy and acquire more information in the information acquisition stage. Unlike the two-stage game, deviation from this equilibrium and acquiring more information in the very last segment of the left hand side (shared segment) doesn't affect firm *B*'s price for the left hand side. Recalling (A17) from the proof of proposition 1, firm *A* decides to acquire more information in (0.25,0.5] if $\frac{1}{2} - \frac{1}{4} \geq 2\sqrt{\frac{\tau}{t}}$ or $\frac{\tau}{t} \leq \frac{1}{64}$. This is exactly the same upper bound for information cost that satisfies firm *A*'s decision to acquire any information in the left hand side in the first instance. That means if information cost is small enough to encourage firm *A* to acquire some information in [0,0.5] interval, then firm *A* also has incentive to deviate from the proposed strategy profile.

iv) *Full discrimination on [0,0.5]*: The corresponding equilibrium prices for the left hand side are ($(a_{n-1},0.5]$ is the very last segment on the right where firm *A* acquires information):

$$P_A(x) = \begin{cases} 2t\left(\frac{2}{3} - a_i - \frac{1}{3}a_{i-1}\right) & x \in (a_{i-1}, a_i] \\ \frac{2t}{3}(1 - 2a_{n-1}) & x \in (a_{n-1}, 1] \end{cases} \quad \text{and} \quad P_B(x) = \frac{t}{3}(1 - 2a_{n-1}).$$

Firm *B* has incentive to deviate and acquire some information on the left hand side. If firm *B* buys one unit of information on the left hand side, then she can charge a different price ($\hat{P}_B$) for [0,0.25]. The extra profit which she can achieve will be:

$$\Delta \pi_B = \left[\frac{1}{4} - \frac{1}{2t}\left(\hat{P}_B - 2t\left(\frac{5}{12} - \frac{a_{n-1}}{3}\right)\right)\right]\hat{P}_B - \tau = \left(\frac{1}{6} - \frac{a_{n-1}}{3} - \frac{\hat{P}_B}{2t}\right)\hat{P}_B - \tau.$$

The first order condition results $\hat{P}_B = \left(\frac{1}{6} - \frac{a_{n-1}}{3}\right)t$ and the corresponding extra profit of $\Delta\pi_B = \frac{t}{8}(0.5 - a_{n-1})^2 - \tau$, which (considering the upper bound on information cost for acquiring information in a firm's own turf) gives firm B incentive to acquire at least one unit of information on the left hand side.

Then the game has no equilibrium when both firms acquire information.

Case 3: Only firm A acquires information

Suppose this case has an equilibrium. In the equilibrium each firm can predict her rival's strategy including preferred partition; so firm A knows that in the equilibrium, her rival can predict her chosen partition. We try to construct the characteristics of this equilibrium. Since in the equilibrium firm B can predict her rival's partition accurately, then we can use some of the results that we had from the first game.

As seen in lemma 2, firm A knows if she acquires information on the right hand side, firm B can prevent her of selling to any customer in the right hand segments, except the very first segment. Then firm A has no incentive to acquire information in the right hand side.

As for the left hand side, proposition 2 shows that firm B knows that firm A can gain the most possible profit by fully discriminating. So firm B sets her price to just maximize her profit from the only segment in her turf and firm A fully discriminate the left hand segment.

An equilibrium for this case should have these two characteristics:

1- In the right hand side, firm A (the only firm who acquires information in this scenario) buys no information. Then there is only one segment $(0.5,1]$ and the prices would be

$$P_A^{RHS} = \frac{2t}{3} \qquad \text{and} \qquad P_B = \frac{4t}{3}.$$

2- In the left hand side, firm A fully discriminates subject to information cost given the firm B's uniform price.

Now we want to investigate firm A's incentive to deviate from this equilibrium. Given firm B's uniform price, if firm A deviates and acquires just one unit of information in the right hand side his marginal profit would be the difference between his equilibrium profit over $(0.5,1]$ and the deviation strategy profit over $(0.5,0.75]$ and $(0.75,1]$. Then the deviation profit can be written as (P_A^L the price charged for the left sub-segment and P_A^R the price for the right sub-segment):

$$\pi_A^{RHS} = P_A^L\left(\frac{\frac{4t}{3} - P_A^L}{2t}\right) + P_A^R\left(\frac{\frac{4t}{3} - P_A^R}{2t} - \frac{1}{4}\right) - \tau \qquad (21).$$

Solving the FOCs, the first part of (21) exactly gives firm A the same profit as the supposed equilibrium. If the second part of the profit is greater than the information cost, then firm A has incentive for deviation. From the FOC $P_A^R = \frac{5t}{12}$, the marginal profit of deviation is:

$$\Delta\pi_A^{RHS} = \frac{5t}{12}\left(\frac{11}{24} - \frac{1}{4}\right) - \tau = \frac{25t}{288} - \tau \;.$$

If $\frac{\tau}{t} \leq \frac{25}{288} \approx 0.087$, firm A has incentive to deviate and acquire at least one unit of information in the right hand side. This condition is more relaxed than the condition calculated in section 3.2 for acquiring information in her turf at all. That means if the information cost is low enough that firm A decides to acquire information in the left hand side in the first place, she has incentive to deviate from any equilibrium strategy that constructed for this case.

Outcome of the Simultaneous Move Game

The major result of studying the simultaneous move game is the non-existence of equilibrium in pure strategies. Therefore, the only possible equilibrium of this game would be in mixed strategies. Considering that each pure strategy consists of an information partition and a pricing function measurable with the chosen information partition, one can imagine that in general there are many possible pure strategies. This makes finding the mixed equilibrium of the game a difficult task. In appendix C, we investigate the existence of a mixed strategy equilibrium through a simple example where the number of possibilities are exogenously restricted.

5   Summary and Conclusion

This paper has analysed a model of information acquisition by firms, where information allows firms to price discriminate. Our benchmark model is one where information acquisition decisions are common knowledge at the time that firms compete via prices. We show that information acquisition increases price competition and reduces profits, so that we have an outcome similar to a prisoners' dilemma. Our second main finding is that firms acquire less information as compared to a monopoly situation, since this softens price competition.

By introducing information cost (and as a consequence segmentation scenario), the third-degree price discrimination problem can end up neither on fully discrimination policy nor on non-discrimination decision. Depending on the cost of every unit of information, every firm needs to answer two questions: how should I discriminate (what is the preferred length of every segment) and what is the best price to charge for every specified customer? The result would be a partial discrimination policy.

The two-stage game is a prisoners' dilemma which in equilibrium, firms acquire excessive information. Firms also prefer to discriminate partially. Our results show that there is a trade-off in acquiring more information. It improves firms' performances in terms of profit by enabling them to price discriminate. On the other hand, acquiring more information tends to make the competition between the firms more fierce. Tougher competition drives the prices down and ultimately decreases firms' profits.

Our results demonstrate how decreasing marginal profit of information limits firms' willingness to acquire more information. Furthermore, in equilibrium –regardless of the cost of information- firms do not have incentive to acquire information on their rival's turf. Acquiring information in a firm's own turf is also restricted. Extra information in this area makes firms profit to fall as result of tougher competition.

We have also analysed a model where information acquisition decisions are not observed by the rival firm. In this game, there is no pure strategy equilibrium. We solve for a mixed strategy equilibrium for a simple example, where firms have restricted information acquisition possibilities.

Appendix A

Proof of lemma 2: Since the firms' profit on $\hat{s}$ can be written as a function of the firms' prices on $\hat{s}$ and the segments associated with that $(s_i\ ;\ i=1,2,\cdots,n)$, firms solve the maximization problem for $\hat{s}$ independently of its compliment. We solve the problem for each possible segmentation situation

<u>Case 1:</u>

By solving the first order conditions $(\frac{\partial \pi_A}{\partial P_A}=0\ ,\frac{\partial \pi_B}{\partial P_B}=0)$, market shares, prices, and profit of firms is calculated. By applying the FOC to (9) and (10) and solving them simultaneously, firms' prices are:

$$P_A = \frac{t}{3}(2a_1 - 4a_0 + 1)\text{ , and} \qquad \text{(A1)}$$

$$P_B = \frac{t}{3}(4a_1 - 2a_0 - 1)\ . \qquad \text{(A2)}$$

By substitution the prices from (A1) and (A2) into (5), the location of marginal customer in every segment is derived:

$$x = \frac{1}{6}(2a_1 + 2a_0 + 1)\ . \qquad \text{(A3)}$$

In the segments that both firms sell positive quantities, the prices should be non-negative ($P_A \geq 0$, $P_B \geq 0$) and the marginal customer should be located within the segment $(a_0 \leq x \leq a_1)$[8]. From the information acquisition technology at the definition of the model, every two consecutive breaking points (the borders of each segment) have the following form:

$$a_0 = \frac{2k-1}{2^p}\ , a_1 = \frac{2l-1}{2^q} \qquad \text{where} \qquad k,l,p,q \in \aleph \qquad \text{(A4)}$$

And also the length of segment is represented as[9]:

$$a_1 - a_0 = \frac{1}{2^r} \qquad \text{where} \qquad r = \max\{p,q\}\ . \qquad \text{(A5)}$$

---

[8] Obviously, the price restrictions and the location restrictions are equivalent.

[9] This definition covers all amounts of a_0 except $a_0 = 0$. That can be solved as: $0 < 2a_1 + 1 < 6a_1$ then $\frac{1}{4} < a_1$. Since $a_1 - a_0 = \frac{1}{2} > \frac{1}{4}$ and $a_0 = 0$ then $a_1 = \frac{1}{2}$ is the only possible upper border of a segment with positive demand for both firms.

Solving the restrictions, the results are:

i) If $p > q$ then the restrictions hold only for $a_1 = \frac{1}{2}$; both firms sell positive quantities in the segment located exactly on the left hand side of the middle point[10].

ii) If $p < q$ then the restrictions hold only for $a_0 = \frac{1}{2}$; both firms sell positive quantities in the segment located on the right side of the middle point.

iii) $p = q$ is impossible, it is equivalent with a segment of length zero.

In this case, market sharing takes place only for two segments located around $\frac{1}{2}$.

Case 2:

In case 2 by solving the first order conditions on $\hat{s}$ ($\frac{\partial \pi_{iA}}{\partial P_{iA}} = 0, \frac{\partial \pi_B}{\partial P_B} = 0; i = 1,2, \cdots n$), market share, prices, and profit of firms is calculated. By applying the FOC to (11) and (12) and solving them simultaneously the firms' prices are:

$$P_{iA} = \frac{t}{3} \left[\frac{1}{n} \left(2a_n + \sum_{i=1}^{n-1} a_i - a_0 \right) - 3a_{i-1} + 1 \right] \qquad \text{for } i = 1,2, \cdots, n; \text{ and} \qquad \text{(A6)}$$

$$P_B = \frac{t}{3n} \left(4a_n - 2 \sum_{i=1}^{n-1} a_i - 2a_0 - n \right). \qquad \text{(A7)}$$

The solution is started from the first segment (s_1 which is the first segment in the very left hand side of $\hat{s}$) and shows that the location for indifferent customer does not satisfy $a_0 \leq x_1 \leq a_1$, then the maximization problems are reduced to n. This procedure continues to the most right hand side segment with a couple of FOCs and problem reduces to the problem solved in case 1.

$$x = \frac{1}{2} + \frac{P_B - P_{1A}}{2t} = \frac{1}{2} + \frac{1}{6n} \left(a_n - a_0 + \sum_{i=1}^{n} a_i + 3na_0 - 2n \right). \qquad \text{(A3)}$$

Define $\lambda_i = a_i - a_0$ then:

$$x = \frac{1}{6} + \frac{2}{3}a_0 + \frac{1}{6n} \left(\lambda_n + \sum_{i=1}^{n} \lambda_i \right).$$

To be credible we should have $x_1 \leq a_0 + \lambda_1$ and simultaneously $P_B > 0$ then:

$$\frac{1}{6} + \frac{2}{3}a_0 + \frac{1}{6n} \left(\lambda_n + \sum_{i=1}^{n} \lambda_i \right) < a_0 + \lambda_1, \qquad \text{(A8)}$$

and from (A7):

$$\lambda_n + \sum_{i=1}^{n} \lambda_i + na_0 - \frac{n}{2} > 0 \qquad \text{(A9)}$$

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[10] Proof: $a_1 = a_0 + \frac{1}{2^p} = \frac{2k}{2^p}$. Substitute in (A1) and (A2), and apply in the price restrictions: $k > 2^{p-2} - \frac{1}{2}$, and $k < 2^{p-2} + 1$. Since $k \in \aleph$ then $k = 2^{p-2}$, and $a_1 = \frac{1}{2}$. The proofs of other two cases are quite similar.

After simplifying (A8):

$$\frac{1}{2} - a_0 + \frac{1}{2n}\left(\lambda_n + \sum_{i=1}^{n} \lambda_i\right) - 3\lambda_1 < 0, \qquad \text{(A10)}$$

And from (A9):

$$\lambda_n + \sum_{i=1}^{n} \lambda_i > n\left(\frac{1}{2} - a_0\right).$$

Define $\frac{1}{2} - a_n = \gamma \geq 0$:

$$\text{The RHS of (A10)} > \frac{1}{2} - a_0 + \frac{n}{2n}\left(\frac{1}{2} - a_0\right) - 3\lambda_1 = \frac{3}{2}(\lambda_n + \gamma) - 3\lambda_1.$$

For every $n > 1$; $\lambda_n \geq 2\lambda_1$ then (A8) and (A9) cannot be held simultaneously.

Then we showed for every $n > 1$ in the first segment, firm A is a constrained monopolist. For the ith step if $n > i$ it is easy to show that these two constraints turn to this form (with $n + 2 - 1$ FOCs):

$$x_i = \frac{1}{2} + \frac{P_B - P_{iA}}{2t} = \frac{1}{2} + \frac{1}{6(n+1-i)}\left(a_n - a_{i-1} + \sum_{j=i}^{n} a_j + 3(n+1-i)a_{i-1} - 2(n+1-i)\right) < a_i,$$

$$\text{and} \quad P_B > 0 \quad \Rightarrow \quad a_n + \sum_{j=i}^{n} a_j - a_{i-1} - \frac{n+1-i}{2} > 0.$$

After simplifying:

$$\frac{1}{2} - a_0 + \frac{3}{2}\lambda_i - 3\lambda_{i-1} + \frac{1}{2(n+1-i)}\left(\lambda_n - \lambda_{i-1} + \sum_{i=1}^{n} \lambda_i\right) < 0, \qquad \text{(A11)}$$

$$\text{and} \quad \lambda_n - \lambda_{i-1} + \sum_{i=1}^{n} \lambda_i > (n+1-i)\left(\frac{1}{2} - a_0\right). \qquad \text{(A12)}$$

Then

$$\text{The RHS of (A11)} > \frac{3}{2}\left(\frac{1}{2} - a_0\right) - \frac{3}{2}(2\lambda_i - \lambda_{i-1})\lambda_{i-1} = \frac{3}{2}(\lambda_n - 2\lambda_i + \lambda_{i-1} + \gamma).$$

For $i < n$; $\lambda_i \leq \frac{1}{2}(\lambda_n + \lambda_{i-1})$ Then (A11) cannot be hold, supposing (A12) holds.

This procedure continues until the FOCs reduce to 2 conditions and the problem transforms to the problem has been solved in case 1.

Case 3:

In this case by solving the first order conditions for each segment ($\frac{\partial \pi_A}{\partial P_A} = 0, \frac{\partial \pi_{iB}}{\partial P_{iB}} = 0$; $i = 1, 2, \cdots n$) market shares, prices, and profit of firms is calculated. By applying the FOC to (13) and (14) and solving them simultaneously the firms' prices are:

$$P_A = \frac{t}{3n}\left(n + 4a_n - 2\sum_{i=1}^{n} a_i - 4a_0\right); \text{ and} \qquad (A13)$$

$$P_{iB} = \frac{t}{3}\left[3a_i - 1 + \frac{1}{n}\left(2a_n - \sum_{i=1}^{n} a_i - 2a_0\right)\right] \qquad \text{for } i = 1, 2, \cdots, n. \qquad (A14)$$

Again for the first segment:

$$x_1 = \frac{1}{2} + \frac{P_{1B} - P_A}{2t} = \frac{1}{2} + \frac{1}{6n}\left(-2a_n + 2a_0 + \sum_{i=1}^{n} a_i + 3na_1 - 2n\right).$$

Then we should have:

$$x_1 = \frac{1}{6} + \frac{a_1}{2} + \frac{1}{6n}\left(2a_0 - 2a_n + \sum_{i=1}^{n} a_i\right) < a_1.$$

Or equivalently:

$$\frac{1}{6} - \frac{a_1}{2} + \frac{1}{6n}\left(2a_0 - 2a_n + \sum_{i=1}^{n} a_i\right) < 0.$$

Again consider $\lambda_i = a_i - a_0$ then:

$$\frac{1}{2} - \frac{3}{2}a_0 - \frac{3}{2}\lambda_1 + \frac{1}{2n}\left(-2\lambda_n + \sum_{i=1}^{n} \lambda_i + na_0\right) < 0,$$

or:

$$\frac{1}{2} - a_0 - \frac{3}{2}\lambda_1 + \frac{1}{2n}\left(-2\lambda_n + \sum_{i=1}^{n} \lambda_i\right) < 0. \qquad (A15)$$

Now, we show that the minimum amount of the left hand side is non-negative. It is easy to show that the minimum of $\left(-2\lambda_n + \sum_{i=1}^{n} \lambda_i\right)$ is $-\frac{1}{2^{n-1}}\lambda_n$ and the maximum amount of λ_1 is $\frac{1}{2}\lambda_n$ then (recall $\frac{1}{2} - a_n = \gamma \geq 0$):

$$\text{The RHS of (A15)} > \frac{1}{2} - a_0 - \frac{3\lambda_1}{2} - \frac{\lambda_n}{n2^n} = \gamma + \lambda_n\left(\frac{1}{4} - \frac{1}{n2^n}\right).$$

Then as long as $n > 1$, (A15) is not valid and in the first segment firm A is a constrained monopolist. For the ith step, if $n > i$ it is easy to show that firms' preferred price for each section turn to this form:

$$P_A = \frac{t}{3(n - i + 1)}\left(n - i + 1 + 4a_n - 2\sum_{k=i}^{n} a_k - 4a_0\right); \text{ and}$$

$$P_{jB} = \frac{t}{3}\left[3a_j - 1 + \frac{1}{n}\left(2a_n - \sum_{k=i}^{n} a_k - 2a_0\right)\right] \qquad \text{for } j = 1, 2, \cdots, i.$$

The location of indifferent customer for the last segment could be calculated as following (with $n + 2 - i$ FOCs):

$$x_i = \frac{1}{2} + \frac{P_{iB} - P_A}{2t} = \frac{1}{2} + \frac{1}{6(n+1-i)}\left(-2a_n + 2a_0 + \sum_{k=i}^{n} a_k + 3(n+1-i)a_i - 2(n+1-i)\right).$$

By following a procedure as before it could be shown that $x_i \geq a_i$ as long as $n > i$ and $\gamma \geq 0$. It means in this case we again end up with a problem similar to case 1 and firms share the market only in the very two extreme segments in the middle. ♦ *QED*

Proof of proposition 1: Firm A's decision to acquire one more unit of information to split the segment between any two already known consequent points can be considered as one of these two cases (note it is already proved that firm A has no incentive to acquire information on the right hand side).

i) Acquisition of one more information unit for dividing one of the first $n-1$ segments to two equal sub-segments. Since firm A splits one of her loyal segments to two loyal sub-segments, then the marginal profit of this segmentation for the firm A is $(a_i - a_{i-1} = \left(\frac{1}{2}\right)^k$ where $k > 1)$:

$$\Delta \pi_A = 2t \left[-a_i\left(a_i - \frac{a_i + a_{i-1}}{2}\right) - \left(\frac{a_i + a_{i-1}}{2}\right)\left(\frac{a_i + a_{i-1}}{2} - a_{i-1}\right) + a_i(a_i - a_{i-1}) \right] - \tau(k).$$

Where the first two parts are the amounts of extra profit that firm gets from the two sub-segments and the third part represents the similar amount for the original segment that should be subtracted. After simplifying:

$$\Delta \pi_A = \frac{t}{4}(a_i - a_{i-1})^2 - \tau(k). \qquad (A16)$$

This shows that firm A's demand for more information and consequently more precise price discrimination in this part (the first *n-1* segments) continues as far as the length of pre-final segments satisfies:

$$a_i - a_{i-1} \geq 2\sqrt{\frac{\tau(k)}{t}}.$$

or equivalently (by substituting $a_i - a_{i-1} = \left(\frac{1}{2}\right)^k$ and $\tau(k) = \frac{\tau_0}{2^k}$):

$$k \leq \log_2 \frac{t}{4\tau_0}. \qquad (A17)$$

This result means the firm has incentive to split a loyal segment, if k satisfies this inequality. This is equivalent of minimum length which the firm has incentive to split the interval if the loyal interval is bigger than this minimum length. Therefore, the preferred length of a loyal segment is $\left(\frac{1}{2}\right)^{k+1}$.

We also can conclude that the preferred length for every firm's loyal segment in her turf does not depend on the location of the segment and depends only on the transportation and information costs and considering (A17) and the fact that $k \in \aleph \cup \{0\}$ then the preferred length for a loyal segment is:

$$\left(\frac{1}{2}\right)^r \qquad where \qquad r = \left\lfloor \log_2 \frac{t}{2\tau_0} \right\rfloor. \qquad (A18)$$

$\lfloor \ \ \rfloor$ notation represents the floor function (or the greatest integer). It is clear since it is about the length of a loyal segment then the minimum acceptable value for r is 3.

ii) Acquisition of one more information unit for dividing the nth segment to two equal sub-segments. Since firm A splits one shared segment to two sub-segments which the left one will be a loyal segment and the right one is a shared segment with her rival, then the marginal profit of such segmentation for the firm A is $(\frac{1}{2} - a_{n-1} = (\frac{1}{2})^k)$:

$$\Delta\pi_A = \frac{4t}{3}\left(\frac{\frac{1}{2} + a_{n-1}}{2} - a_{n-1}\right) - \frac{2t}{3}\left[\left(\frac{\frac{1}{2} + a_{n-1}}{2}\right)^2 - a_{n-1}^2\right] - 2t\left(\frac{\frac{1}{2} + a_{n-1}}{2} - a_{n-1}\right)\left(\frac{1}{4} + \frac{a_{n-1}}{2}\right)$$

$$+ \frac{8t}{9}\left(\frac{1}{2} - \frac{\frac{1}{2} + a_{n-1}}{2}\right)^2 - \frac{8t}{9}\left(\frac{1}{2} - a_{n-1}\right)^2 - \tau(k) ,$$

$$\text{or} \quad \Delta\pi_A = \frac{t}{3}\left(\frac{1}{2} - a_{n-1}\right)^2 - \frac{t}{6}\left(\frac{1}{2} - a_{n-1}\right) - \tau(k) = \frac{t}{3 \times 2^k}\left(\frac{1}{2^k} - \frac{1}{2}\right) - \tau(k) .$$

For $\forall k$, this marginal profit is negative and shows the firm A's profit reduces by acquiring one more unit of information in this part regardless of information cost. But we should consider that the left sub-segment created after this information acquisition all are loyal customers now and the possibility of making extra profit by using constrained monopoly power on this sub-segment should be considered. This possibility can be investigated. Consider (A18), assume the preferred length of loyal segment is $(\frac{1}{2})^r$ where $r = \left\lfloor \log_2 \frac{t}{2\tau_0} \right\rfloor \geq 3$. It is clear acquiring information in this interval is only profitable if $r > k$; however the following result is true for any value of r. By adding the profit of this chain of segmentation the net profit of segmentation in the nth segment, $\left(a_{n-1}, \frac{1}{2}\right]$, is:

$$\Delta\pi_A = \frac{4t}{3}\left(\frac{\frac{1}{2} + a_{n-1}}{2} - a_{n-1}\right) - \frac{2t}{3}\left[\left(\frac{\frac{1}{2} + a_{n-1}}{2}\right)^2 - a_{n-1}^2\right]$$

$$- 2t\sum_{i=1}^{2}\left(\frac{1}{2}\right)^r\left(\frac{1}{4} - \frac{a_{n-1}}{2}\right)\left[a_{n-1} + \frac{i}{2^r}\left(\frac{1}{4} - \frac{a_{n-1}}{2}\right)\right] + \frac{8t}{9}\left(\frac{1}{2} - \frac{\frac{1}{2} + a_{n-1}}{2}\right)^2$$

$$- \frac{8t}{9}\left(\frac{1}{2} - a_{n-1}\right)^2 - \Gamma ,$$

or

$$\Delta\pi_A = t\left[\left(\frac{7}{12} - \frac{1}{2^{r+2}}\right)\left(\frac{1}{2} - a_{n-1}\right)^2 - \frac{1}{6}\left(\frac{1}{2} - a_{n-1}\right)\right] - \Gamma ;$$

and finally

$$\Delta\pi_A = \frac{t}{2^k}\left[\frac{1}{2^k}\left(\frac{7}{12} - \frac{1}{2^{r+2}}\right) - \frac{1}{6}\right] - \Gamma \qquad (19)$$

Where $\frac{1}{2} - a_{n-1} = \left(\frac{1}{2}\right)^k$, and $r = \left\lfloor \log_2 \frac{t}{2\tau_0}\right\rfloor$ and the information cost is

$$\Gamma = \tau(k) + \sum_{i=k+1}^{r-1} 2^{i-(k+1)}\tau(i) = \frac{\tau_0}{2^k} + \sum_{i=k+1}^{r-1} 2^{i-(k+1)}\frac{\tau_0}{2^i} = \frac{\tau_0}{2^k} \cdot \frac{r - k + 1}{2} .$$

If the net profit calculated by (19) is negative then the segmentation in the nth segment is not profitable for firm A. Then the segmentation in the shared segment in firm A's turf is preferred by her as far as (19) is non-negative which because of it is importance is discussed in the main body of paper.

♦ *QED*

Proof of proposition 2:

We know that the firms only share the customers on the two border segments. Suppose Firm A acquires information in her own side such that the last left hand segment is $(a, 0.5]$, and as it has been proved in proposition 1 she has no incentive to pay for information in the right hand side. Then the maximization problems that should be solved simultaneously are:

Firm A's profit for $(a, 0.5]$: $\qquad \pi_{LA} = P_{LA}\left(\frac{1}{2} + \frac{P_B - P_{LA}}{2t} - a\right).$

Firm A's profit for $(0.5, 1]$: $\qquad \pi_{RA} = P_{RA}\left(\frac{P_B - P_{RA}}{2t}\right).$

Firm B's profit for $(a, 1]$: $\qquad \pi_B = P_B\left(\frac{1}{2} - \frac{2P_B - P_{LA} - P_{RA}}{2t}\right).$

Solving the FOCs and the results are:

$$P_B = \left(\frac{1}{2} - \frac{a}{3}\right)t, \qquad P_{LA} = \left(\frac{3}{4} - \frac{7a}{6}\right)t, \text{ and } \qquad P_{RA} = \left(\frac{1}{4} - \frac{a}{6}\right)t.$$

And the locations of indifferent customers in these two segments are:

$$x_L = \frac{3}{8} + \frac{5a}{12} \qquad \text{and} \qquad x_R = \frac{5}{8} - \frac{a}{12}.$$

These values should satisfy $P_i > 0$, $a < x_L < 0.5$, and $0.5 < x_L < 1$.

Since if $a > 0.3$, then x_L will not satisfy its condition. Therefore, for $a > 0.3$, firm B cannot gain from the left hand side. That means if firm A decides to split up $(0.25, 0.5]$ then firm B just set her price to gain the most possible profit from the right hand side.

Then we compare the profit of firm A for different possible decisions (the profits are easily calculated similar to the result of lemma 2)[11]:

1- No information in the left hand side: $\qquad \pi_A = \frac{5t}{16} - \tau_0,$

2- Fully discriminate $[0, 0.25]$: $\qquad \pi_A = \left(\frac{241}{576} - \frac{1}{2^{r+2}}\right)t - \frac{r+3}{2}\tau_0,$ and

3- Fully discriminate $[0, 0.5]$: $\qquad \pi_A = \left(\frac{25}{36} - \frac{1}{2^{r+2}}\right)t - \frac{2r+3}{2}\tau_0$;

where $r = \left\lceil \log_2 \frac{t}{2\tau_0} \right\rceil$.

If $\frac{\tau_0}{t} \leq \frac{53}{96r}$ then the third one provides the largest profit for firm A (the profit on the first case exceeds the third one only when $\frac{\tau_0}{t} \geq \frac{101}{576}$ which is larger than $\frac{1}{16}$ the upper bound of information acquisition decision calculated in proposition 1); then she prefers to fully discriminate all the left hand side. In this case firm B's profit equals to $\frac{2t}{9}$. ♦ *QED*

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[11] The length of the loyal segments is calculated with the same rule as equation (20).

Appendix B

In this appendix we replace our assumed form of information cost function with two alternative functional forms. Then we represent the changes to the outcomes of the model as a result of these changes. Note that lemma 1 and lemma 2 are true regardless of the information cost sequence. We focus on how the changes to equation (2) changes proposition 1, proposition 2, and the outcome of two-stage game. The result is that firms strategically behave in the same way. Only the length of the loyal segments and the thresholds on information cost calculated in the proofs are different.

Our two alternative information cost functions are:

i) constant information cost: $\tau(k) = \bar{\tau}$ for $\forall k$.

ii) information cost with general functional form of:

$$\tau(k) = \alpha^k \tau_0, \quad \text{for} \quad \forall k. \quad \text{(B1)}$$

The other two information cost functions (equation (2) and the constant information cost) are specific cases of (B1).[12]

In order to emphasis the changes, we add notations c and g to each equation number that changes for *constant* and *general* information cost functions respectively.

Constant information cost:

Assume equation (2) is replaced by:

$$\tau(k) = \bar{\tau} \qquad \text{for} \qquad \forall k.$$

Proposition 1:

The proposition 1 is true for this case. Only because of the change in information cost function to a linear cost function the limit on the first rule is different:

Recall equation (A16) the marginal profit of acquiring a unit of information in a loyal segment:

$$\Delta \pi_A = \frac{t}{4}(a_i - a_{i-1})^2 - \tau(k). \qquad (A16)$$

This equation gives us the limit to the which is now (by substituting $a_i - a_{i-1} = \left(\frac{1}{2}\right)^k$ and $\tau(k) = \bar{\tau}$) equation (A17) changes to:

$$\left(\frac{1}{2}\right)^k \qquad \text{where} \qquad k \leq \frac{1}{2}\log_2 \frac{t}{4\bar{\tau}}. \qquad (A17.c)$$

Then the preferred length for a loyal segment is:

$$\left(\frac{1}{2}\right)^r \qquad \text{where} \qquad r = \left\lfloor \frac{1}{2}\log_2 \frac{t}{\bar{\tau}} \right\rfloor. \qquad (20.c)$$

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[12] We decided to have the special case of $\alpha = 0.5$ in the main body of paper, since calculating the closed form of some equations which helps to demonstrate our results is more complicated for the general functional form of (3).

The marginal profit of a unit of information on the shared border segment with the length of $\left(\frac{1}{2}\right)^k$ is (note the marginal profit has the same format and only the information cost and the preferred length of the loyal segment are different):

$$\Delta\pi_A = \frac{t}{2^k}\left[\frac{1}{2^k}\left(\frac{7}{12} - \frac{1}{2^{r+2}}\right) - \frac{1}{6}\right] - 2^{r-k}\,\bar{\tau} \qquad (19.\text{c})$$

where $\frac{1}{2} - a_{n-1} = \left(\frac{1}{2}\right)^k$, and $r = \left\lfloor \frac{1}{2}\log_2\frac{t}{\bar{\tau}}\right\rfloor$.

Then the chain of decision statements in table 1 changes to table 2.

Table 2: Firm A's chain of decision statements with a constant information cost

a_{n-1}	a_n	k	Decision statement		
0	$\frac{1}{2}$	1	$\frac{t}{16}\left(1 - \frac{1}{2^r}\right) - 2^r\bar{\tau} \geq 0$	where	$r = \left\lfloor\frac{1}{2}\log_2\frac{t}{\bar{\tau}}\right\rfloor$
$\frac{1}{4}$	$\frac{1}{2}$	2	$\frac{t}{32}\left(-\frac{1}{6} - \frac{1}{2^r}\right) - 2^r\bar{\tau} \geq 0$	where	$r = \left\lfloor\frac{1}{2}\log_2\frac{t}{\bar{\tau}}\right\rfloor$

As it can be seen the information acquisition rules set out in proposition 1 is still true and only the upper limit on the information cost changes to $\frac{\bar{\tau}}{t} \leq \frac{1}{64}$.

Proposition 2:

The first part of the proof to proposition 2 is the same. We have to continue the proof form the point where the profits should be compared. We compare the profit of firm A for different possible decisions:

1- No information in left hand side: $\pi_A = \frac{5t}{16} - \bar{\tau}$

2- Fully discriminate [0,0.25]: $\pi_A = \left(\frac{241}{576} - \frac{1}{2^{k+4}}\right)t - \left(2^k + 1\right)\bar{\tau}$

3- Fully discriminate [0,0.5]: $\pi_A = \left(\frac{25}{36} - \frac{1}{2^{k+3}}\right)t - \left(2^{k+1} + 1\right)\bar{\tau}$

where $k = \max\left[z, 0 | z \in \mathcal{Z}, \frac{1}{2}\log_2\frac{t}{\bar{\tau}} - 3 < z \leq \frac{1}{2}\log_2\frac{t}{\bar{\tau}} - 2\right]$

Obviously the third one provides the largest profit for firm A (the profit on first case exceeds the third one only when $\frac{\bar{\tau}}{t} \geq 0.14$ that it is larger than the upper bound of information acquisition decision calculated on 3.2); then she prefers to fully discriminate all the left hand side. In this case firm B's profit would be equal to $\frac{2t}{9}$.

If the constant information cost is considered to be equal to the cost of first unit of information in the equation (2) ($\bar{\tau} = \tau_0$) then switching to a constant information cost increases the information cost and as a result the limit on the information would be tighter to get the same equilibrium outcome.

Outcome of the game:

Again similar to the benchmark functional from in the paper; If firm B acquires information, firm A's best response is to do so, irrespective of the information cost. If the other firm acquires no information, the best response is to acquire information if the information cost is sufficiently low. So if

the information cost is sufficiently low, the game becomes a prisoners' dilemma and both firms would prefer to acquire information. This threshold is $\frac{\bar{\tau}}{t} = 0.024$.

Then if $\frac{\bar{\tau}}{t} > 0.024$ the game has two Nash equilibria; i) both firms acquire information and ii) neither of the firms acquire information. If $\frac{\bar{\tau}}{t} \leq 0.024$, the game becomes a prisoners' dilemma where information acquisition is the dominant strategy for both firms. It is clear form the profits that in this case we have excess information acquisition from the firm point of view.

General information cost function:

Assume equation (2) is replaced by:

$$\tau(k) = \alpha^k \tau_0 \qquad \text{for} \qquad \forall k. \qquad (3)$$

The only restriction that we need to impose on α is $\alpha > \frac{1}{4}$ which we will discuss this shortly. Note that if $\alpha > \frac{1}{2}$ then this case is equivalent to (2); and if $\alpha = 1$, then this functional form is equivalent to constant information cost.

Proposition 1:

The proposition 1 is true and only because of the change in information cost function to a general cost function the limit on the first rule is different. Recall equation (A16) the marginal profit of acquiring a unit of information in a loyal segment:

$$\Delta \pi_A = \frac{t}{4}(a_i - a_{i-1})^2 - \tau(k). \qquad (A16)$$

Before we go any further, we impose the restriction on this marginal profit to make it a decreasing function on k. It is necessary, because it guarantees that when the firm discovered that the marginal profit of splitting a loyal segment gets to zero, there is no need to investigate the profitability of any finer partition. It can be shown that the sufficient condition for decreasing marginal cost is $\alpha > \frac{1}{4}$.

This equation gives us the limit to the which is now (by substituting $a_i - a_{i-1} = \left(\frac{1}{2}\right)^k$ and $\tau(k) = \alpha^k \tau_0$) equation (A17) changes to

$$\left(\frac{1}{2}\right)^k \qquad \text{where} \qquad \mathrm{k} \leq \log_{4\alpha} \frac{t}{4\tau_0}. \qquad (A17.\,\mathrm{g})$$

Then the preferred length for a loyal segment is

$$\left(\frac{1}{2}\right)^r \qquad \text{where} \qquad r = \left\lceil \log_{4\alpha} \frac{\alpha t}{\tau_0} \right\rceil. \qquad (20.\,\mathrm{g})$$

The marginal profit of a unit of information on the shared border segment with the length of $\left(\frac{1}{2}\right)^k$ is (note the marginal profit has the same format and only the information cost and the preferred length of the loyal segment are different)

$$\Delta \pi_A = \frac{t}{2^k}\left[\frac{1}{2^k}\left(\frac{7}{12} - \frac{1}{2^{r+2}}\right) - \frac{1}{6}\right] - \Gamma. \qquad (19.\,\mathrm{g})$$

Where $\frac{1}{2} - a_{i-1} = \left(\frac{1}{2}\right)^k$, $r = \left\lceil \log_{4\alpha} \frac{\alpha t}{\tau_0} \right\rceil$, and the information cost is

$$\Gamma = \tau(k) + \sum_{i=k+1}^{r-1} 2^{i-(k+1)}\tau(i) = \frac{\tau_0}{2^k} + \sum_{i=k+1}^{r-1} 2^{i-(k+1)}\alpha^i\tau_0 = \frac{\tau_0}{2^k}\left(1 + \sum_{i=k+1}^{r-1} 2^{i-1}\alpha^i\right).$$

Then the chain of decision statements in table 1 changes as it is demonstrated in table 3.

Table 3: Firm A's chain of decision statements with a general information cost function

a_{n-1}	a_n	k	Decision statement		
0	$\frac{1}{2}$	1	$\frac{t}{16}\left(1 - \frac{1}{2^r}\right) - \tau_0\left(1 + \sum_{i=1}^{r-1} 2^{i-1}\alpha^i\right) \geq 0$	where	$r = \left\lfloor \log_{4\alpha}\frac{\alpha t}{\tau_0}\right\rfloor$
$\frac{1}{4}$	$\frac{1}{2}$	2	$\frac{t}{32}\left(-\frac{1}{6} - \frac{1}{2^r}\right) - \frac{\tau_0}{4}\left(1 + \sum_{i=2}^{r-1} 2^{i-2}\alpha^i\right) \geq 0$	where	$r = \left\lfloor \log_{4\alpha}\frac{\alpha t}{\tau_0}\right\rfloor$

As it can be seen the information acquisition rules set out in proposition 1 is still true and only the upper limit on the information cost depends on the value of α.

Proposition 2:

The first part of the proof to proposition 2 is the same. We have to continue the proof form the point where the profits should be compared. We compare the profit of firm A for different possible decisions:

1- No information in left hand side: $\pi_A = \frac{5t}{16} - \tau_0$

2- Fully discriminate [0,0.25]: $\pi_A = \left(\frac{241}{576} - \frac{1}{2^{r+4}}\right)t - \tau_0\left(\frac{3}{2} + \sum_{i=2}^{r-1} 2^{i-2}\alpha^i\right)$

3- Fully discriminate [0,0.5]: $\pi_A = \left(\frac{25}{36} - \frac{1}{2^{r+3}}\right)t - \tau_0\left(\frac{3}{2} + \sum_{i=2}^{r-1} 2^{i-1}\alpha^i\right)$

where $r = \left\lfloor \log_{4\alpha}\frac{\alpha t}{\tau_0}\right\rfloor$

In order to have a result similar to proposition 2 the following inequality must hold:

$$\frac{\tau_0}{t} \leq \frac{\frac{2}{7} - \frac{1}{2^{r+4}}}{\sum_{i=2}^{r-1} 2^{i-2}\alpha^i} \qquad \text{where} \qquad r = \left\lfloor \log_{4\alpha}\frac{\alpha t}{\tau_0}\right\rfloor$$

It showed that this inequality holds as long as information cost is small enough that firm acquires some information in the first place.

Appendix C

Example: Suppose that only one unit of information is available for the firms which costs τ. This means the only possible strategies for firms are:

σ_1: No information acquisition and charging a uniform price of P_{i1} , $i = A, B$

σ_2: Acquiring one unit of information and charging P_{i2}^L for [0,0.5] and P_{i2}^R for (0.5,1] , $i = A, B$

We showed that there is no pure strategy equilibrium in general when there was no exogenous limit on the number of information units a firm can acquire. But since in this example we restrict available information units to one, we need to investigate this matter again. Three different cases should be considered.

Case 1: Both firms choose σ_1: In this case (no information acquisition) by solving the first order condition the equilibrium candidate is $P_{A1} = P_{B1} = t$ and the profits are $\pi_A = \pi_B = {}^t/_2$. Firm A has incentive to deviate and acquire information if

$$\pi_A^D = P_{A2}^L \left(\frac{1}{2} + \frac{t - P_{A2}^L}{2t} \right) + P_{A2}^R \left(\frac{t - P_{A2}^R}{2t} \right) - \tau \geq \frac{t}{2} \; . \qquad \text{(C1)}$$

The deviation strategy for firm A would be to choose σ_2 and charge $P_{A2}^L = t$ and $P_{A2}^R = {}^t/_2$ which gives her the deviation profit of $\pi_A^D = {}^{5t}/_8 - \tau$. Therefore firm A has incentive to deviate in this case if

$$\tau \leq \frac{t}{8} \; . \qquad \text{(C2)}$$

Case 2: One firm (say A) chooses σ_1 and the other chooses σ_2: In this case, by solving the first order condition the equilibrium candidate is $P_{A1} = {}^t/_2$ and $P_{B2}^L = {}^t/_4$, $P_{B2}^R = {}^{3t}/_4$ and the profits are $\pi_A = {}^t/_4$ and $\pi_B = {}^{5t}/_{16} - \tau$. We will investigate both firms incentive to deviate in this case.

Firm A has incentive to deviate and acquire information if

$$\pi_A^D = P_{A2}^L \left(\frac{1}{2} + \frac{{}^t/_4 - P_{A2}^L}{2t} \right) + P_{A2}^R \left(\frac{{}^{3t}/_4 - P_{A2}^R}{2t} \right) - \tau \geq \frac{t}{4} \; . \qquad \text{(C3)}$$

The deviation strategy for firm A would be choosing σ_2 and, by solving (C3) for the fist order condition, the deviation prices are $P_{A2}^L = {}^{5t}/_8$ and $P_{A2}^R = {}^{3t}/_8$ (the boundary conditions will be held for these values) that gives her the deviation profit of $\pi_A^D = {}^{17t}/_{64} - \tau$. Therefore firm A has incentive to deviate in this case if:

$$\tau \leq \frac{t}{64} \; . \qquad \text{(C4)}$$

Firm B's incentive to deviate and choosing σ_1 depends on whether the following inequality holds or not:

$$\pi_B^D = P_{B1} \left(\frac{1}{2} - \frac{P_{B1} - {}^t/_2}{2t} \right) \geq \frac{5t}{16} - \tau \; . \qquad \text{(C5)}$$

By solving (A23) for the first order condition, the deviation price is $P_{B1} = {}^{3t}/_4$ which gives her the deviation profit of $\pi_A^D = {}^{9t}/_{32}$. Therefore firm B has incentive to deviate in this case if:

$$\tau \geq \frac{t}{32} \; . \qquad \text{(C6)}$$

So if ${}^t/_{64} \leq \tau \leq {}^t/_{32}$ this case has a pure strategy equilibrium and for every other value of τ at least one of the firms has incentive to deviate. It worth mentioning that in general when there is no external limits on information acquisition (despite this example that only one unit of information is available) these two boundaries move towards each other and there will be no pure strategy equilibrium.

Case 3: Both firms choose σ_2: in this case by solving the first order conditions the equilibrium candidate is $P_{A2}^L = {2t}/{3}$, $P_{A2}^R = {t}/{3}$ and $P_{B2}^L = {t}/{3}$, $P_{B2}^R = {2t}/{3}$ and the profits are $\pi_A = \pi_B = {5t}/{18} - \tau$. Firm A has incentive to deviate, acquire no information and charge a uniform price if

$$\pi_A^D = P_{A1}\left(\frac{1}{2} + \frac{{t}/{3} - P_{A1}}{2t} + \frac{{2t}/{3} - P_{A1}}{2t}\right) \geq \frac{5t}{18} - \tau \ . \qquad (C7)$$

By solving the first order condition for (C7), the deviation price would be calculated as $P_{A1} = {t}/{2}$ (the boundary conditions will be satisfied for this value) that gives her the deviation profit of $\pi_A^D = {t}/{4}$; therefore firm A (or firm B) has incentive to deviate in this case if:

$$\tau \geq \frac{t}{36} \ . \qquad (C8)$$

Summarizing our finding ((C2), (C3), (C4), and (C8)) from these three cases, we can claim that for this example:

i) If $\tau > {t}/{8}$ then there is a pure strategy equilibrium of acquiring no information. In other words in this case information is too expensive to acquire. The similar condition has been shown for the two-stage game in general.

ii) If ${t}/{36} \leq \tau \leq {t}/{8}$ then there is no pure strategy equilibrium.

iii) If $\tau < {t}/{36}$ then there is a pure strategy equilibrium of acquiring the only unit of information available.

If ${t}/{36} \leq \tau \leq {t}/{8}$ then there is no pure strategy equilibrium. The lower limit on this condition is a result of having an exogenous limit on the number of information unit available. As it has been shown earlier having this limit removed in general case this lower limit will vanish.

Mixed Strategy Equilibrium: If firm B randomizes between two strategies with probability β_1 and β_2 respectively then

$$\beta_1 + \beta_2 = 1 \ . \qquad (C9)$$

So firm A's profit related to strategies σ_1 and σ_2 respectively are:

$$\pi_{A1} = \beta_1 P_{A1}\left(\frac{1}{2} + \frac{P_{B1} - P_{A1}}{2t}\right) + \beta_2 P_{A1}\left(\frac{1}{2} + \frac{P_{B2}^L - P_{A1}}{2t} + \frac{P_{B2}^R - P_{A1}}{2t}\right) \ , \text{and}$$

$$\pi_{A2} = \beta_1\left[P_{A2}^L\left(\frac{1}{2} + \frac{P_{B1} - P_{A2}^L}{2t}\right) + P_{A2}^R\left(\frac{1}{2} + \frac{P_{B1} - P_{A2}^R}{2t}\right)\right]$$
$$+ \beta_2\left[P_{A2}^L\left(\frac{1}{2} + \frac{P_{B2}^L - P_{A2}^L}{2t}\right) + P_{A2}^R\left(\frac{1}{2} + \frac{P_{B2}^R - P_{A2}^R}{2t}\right)\right] - \tau \ .$$

So the FOCs are:

$$\frac{\partial \pi_{A1}}{\partial P_{A1}} = \beta_1\left(\frac{1}{2} + \frac{P_{B1}}{2t} - \frac{P_{A1}}{t}\right) + \beta_2\left(\frac{1}{2} + \frac{P_{B2}^L}{2t} + \frac{P_{B2}^R}{2t} - \frac{2P_{A1}}{t}\right) = 0 \ , \qquad (C10)$$

$$\frac{\partial \pi_{A2}}{\partial P_{A2}^L} = \beta_1 \left(\frac{1}{2} + \frac{P_{B1}}{2t} - \frac{P_{A2}^L}{t} \right) + \beta_2 \left(\frac{1}{2} + \frac{P_{B2}^L}{2t} - \frac{P_{A2}^L}{t} \right) = 0 \text{ , and} \qquad \text{(C11)}$$

$$\frac{\partial \pi_{A2}}{\partial P_{A2}^R} = \beta_1 \left(\frac{P_{B1}}{2t} - \frac{P_{A2}^R}{t} \right) + \beta_2 \left(\frac{P_{B2}^R}{2t} - \frac{P_{A2}^R}{t} \right) = 0 \text{ .} \qquad \text{(C12)}$$

If we concentrate on the symmetric mixed strategy equilibrium then we have[13]:

$$P_{A1} = P_{B1} \text{ ,} \qquad P_{A2}^L = P_{B2}^R \text{ , and} \qquad P_{A2}^R = P_{B2}^L \text{ .}$$

Considering these, after some simplifications and solving (C10) to (C12) simultaneously we will get:

$$P_{A1} = P_{B1} = \frac{2t}{4 - \beta_1 - \beta_1^2} \text{ ,} \qquad P_{A2}^L = P_{B2}^R = \frac{4 - 2\beta_1}{3 - \beta_1} P_{A1} \text{ , and} \qquad P_{A2}^R = P_{B2}^L = \frac{4 - 2\beta_1}{3 - \beta_1} P_{A1} - \frac{t}{3 - \beta_1}.$$

Also the mixed strategy equilibrium should make firm A indifferent between two strategies, this means:

$$\pi_{A1} = \pi_{A2} \text{ .} \qquad \text{(C13)}$$

By solving these four equations, the mixed strategy of $(\beta_1, P_{A1}, P_{A2}^L, P_{A2}^R)$ can be calculated.[14]

Figure 15 shows the value of β_1 for different ratios of information cost over transportation cost. The figure shows that when the information cost is higher, it is significantly more likely for the firms to acquire the information in the mixed strategy equilibrium.
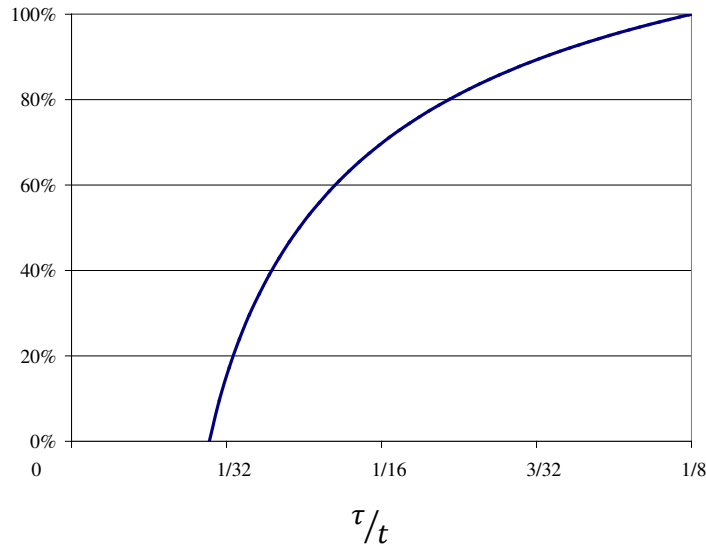

Figure 15: The probability of choosing the non-information acquisition strategy

in the mixed strategy equilibrium

Figure 16 plots the trend of prices and profit as a multiplication of *t*. As it can be seen, the prices are stable for a wide range of information costs; it could be because in this simple example only one

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[13] If we want to investigate the existence of asymmetric mixed strategy equilibrium, different probabilities for choosing S_1 and S_2 should be considered for firm *A* and for this simple example we will end up with 10 equations and 10 unknowns.

[14] To solve for these four equations to find the four unknowns, we use numerical methods and these results are the unique possible outcomes.

unit of information is available. By increasing the information cost all prices tend to increase, this means the lower the information cost, the more intense the competition.
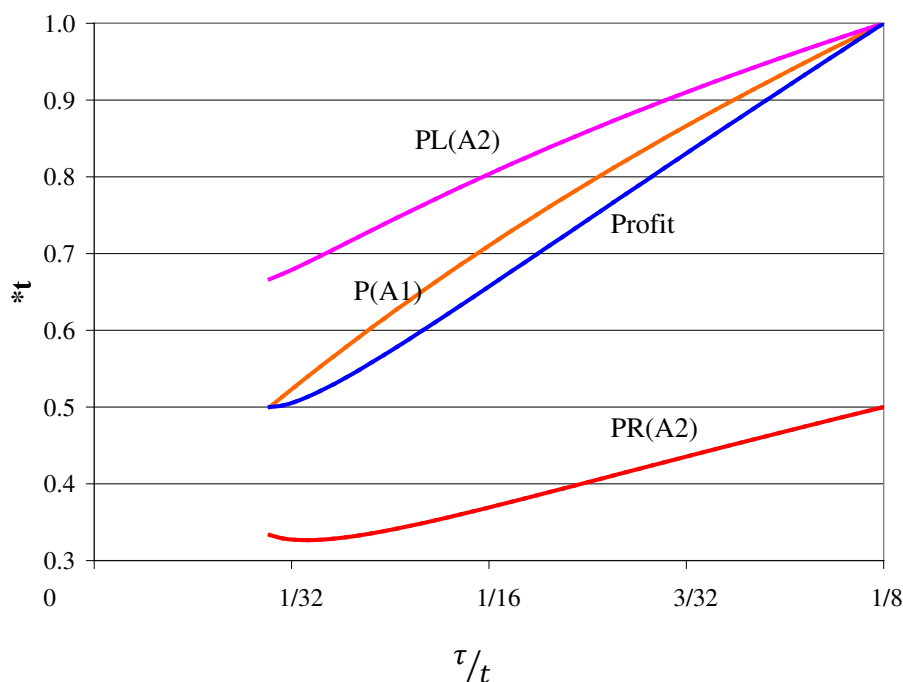

Figure 16: The prices and profit in the mixed strategy equilibrium

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