Hyperbolic discounting and the sustainability of rotational savings arrangements

Basu, Karna

2008

Hyperbolic Discounting and the Sustainability of Rotational Savings Arrangements

Karna Basu*

Department of Economics, University of Chicago

May 12, 2008

Abstract

People across the developing world join rotational savings and credit associations (roscas) to fund repeated purchases of nondivisible goods. When the scope for punishment is weak, there is a natural question about why agents do not defect from these groups. I model a rosca as a commitment savings device for hyperbolic discounters. Roscas are attractive for two reasons: the possibility of getting the nondivisible good early (the standard reason), and the fixed saving requirement (valued only by time-inconsistent agents). I find explicit conditions under which an agent strictly prefers to remain in a rosca, *even in the absence of formal contracting or social punishment.* I show why, unlike with standard commitment products, a hyperbolic discounter will not postpone entry into a rosca. Finally, this paper makes predictions about the relative survival of random and fixed roscas. Random roscas are more resilient and beneficial than fixed roscas when information is limited and matching for new roscas is fast.

*Contact: karna@uchicago.edu; I am indebted to my advisors Abhijit Banerjee, Xavier Gabaix and Sendhil Mullainathan for advice, comments, and support. I thank Jean-Marie Baland, Sylvain Chassang, Antara Dutta, Dean Karlan, and participants at the MIT Development Workshop for discussions and comments.

1 Introduction

Roscas (rotating savings and credit associations) are a prominent form of saving across much of the developing world. This paper studies roscas from the perspective of hyperbolic discounting, and models them as equilibrium phenomena in environments with no scope for formal contracting or social punishment.

A rosca consists of a group of individuals who meet at regular intervals and contribute a fixed amount to a collective "pot," which is then granted to one of the members. The existing literature on roscas frequently stresses the need for social sanctions to ensure that participants do not defect after receiving the pot. One might expect that these sanctions need to be stronger when individuals have present-biased preferences. I find that this is not the case. In fact, in contrast to settings with exponential discounters, roscas with hyperbolic discounters might survive even if no social punishment is possible. A sophisticated hyperbolic discounter has two objectives: to indulge today, and to discourage her future selves from doing the same. Since a rosca appeals to the second objective by improving future savings, the individual will continue to participate even if it involves a sacrifice of immediate consumption. The model below allows us to solve some puzzles about the high observed survival of roscas, and to derive implications for existence and sustainability under varying underlying conditions.

This paper is organized as follows. Section 2 outlines the arguments of this paper in some detail. Section 3 provides a literature review. Section 4 characterizes the autarky equilibrium. Section 5 discusses roscas as commitment devices, focusing on sustainability in a benchmark case and the entry problem. Section 6 models roscas as equilibria in a decentralized setting. Section 7 discusses empirical implications of the model. Section 8 concludes.

2 Outline of Arguments

2.1 Rosca Basics

A rosca "round" consists of exactly as many meetings as there are members. Within a rosca round, each member gets to take the pot home exactly once. The order in which members are given the pot can be determined in several ways. In this paper, I focus on "fixed" and "random" roscas. In a fixed rosca, the order is randomly determined at the first meeting of the rosca, and then repeated indefinitely through future rounds.[1] In a random rosca, the order is randomly determined at the start of each new round. Often, rosca members who leave at the end of a round are free to rejoin a rosca at a later date. However, people who choose to leave *during* a round are punished–they are not allowed to rejoin any future rosca and, if possible, punished with other forms of social sanctions.

Roscas are widespread across developing countries (and immigrant communities in developed countries), and often survive in environments with poor contracting and limited or nonexistent formal banking. Members of the Tidiane community in Senegal use them as a means to save for annual pilgrimages to Mecca. In Philadelphia, women from the Ivory Coast join roscas to pay for childbirths and funerals. Roscas can be found in several parts of Kenya, where the money is put to various uses, from home repairs to the purchase of food and clothing. Levenson and Besley (1996) point out that, in any given year, one-fifth of all households in Taiwan participate in a rosca. Bouman (1995) cites several studies of African roscas where participation rates are even higher, and are a significant saving vehicle.[2]

2.2 Theoretical Foundations

Following the model proposed by Besley, Coate, and Loury (1993), I assume that agents would like to save for a nondivisible good. If they were to save alone, each would have to wait a certain number of periods before she could consume the good. If agents join a rosca instead, they pool their savings and allow some members to get the nondivisible good sooner. The likelihood of an early nondivisible gives agents an incentive to join a rosca. This expected benefit, however, does not persist after they have actually joined. As Anderson, Baland, and Moene (2003) show, when agents are exponential discounters, they will have an ex-post incentive to leave any rosca. Consider an agent in a fixed rosca who has just received the nondivisible good. If she now leaves the rosca, she can replicate the rosca outcome by saving alone (there is no longer a positive probability of getting the good sooner through the rosca). Furthermore, if she prefers a declining pattern of savings, she can do strictly better by saving alone. In a random rosca, some agents have an even greater incentive to leave. Consider the first-ranked agent in a round.

[1] In practice, other considerations (such as age) sometimes come into play in determining the initial ordering. However, my results about an arbitrary individual's incentive to defect from a fixed rosca would continue to be relevant.

[2] Roscas and other informal savings groups are the source of half the national savings in Cameroon. Savings through roscas amount to 8-10% of Ethiopia's gross national product.

If she leaves the rosca after receiving the nondivisible, she can ensure that she continues to be "first-ranked" by walking out and saving alone. If she stays in the rosca, there is no guarantee that she will again be ranked first in the next round. By staying on in the rosca, the agent is effectively continuing to save at a negative interest rate. The authors conclude that, in the absence of contracting, the threat of social punishment must be severe enough that an agent who would otherwise choose to leave a rosca will now choose to continue participating.

2.3 Questions to be Answered

The papers described above provide a compelling framework for understanding the ex-ante appeal and subsequent sustainability of roscas, but they also lead to some interesting questions. First, there is some evidence that roscas survive even without much threat of social punishment (Gugerty, 2005). Why do agents not leave after receiving the nondivisible? Second, for any given outside option, agents in random roscas have greater incentives to leave than agents in fixed roscas do (the first-ranked member's expected value of staying on in a random rosca is lower than anyone's expected value of staying on in a fixed rosca). Why do random roscas exist, given that they provide the same ex-ante utility as fixed roscas? Third, even if the threat of social sanctions can be used to ensure participation within a round, it is unlikely that the threats apply to people who leave after completing a round. Then, why does the last-ranked member in a fixed rosca stay on? If she were an exponential discounter, she should leave at the end of a round if there is any probability that she will find a rosca that ranks her higher (since, even if she does not find a rosca, she can save alone and do no worse than in her original rosca).

2.4 Results

This paper is built on two key assumptions. First, agents in my model have time-inconsistent preferences and are aware of it. There is empirical evidence that this is indeed the case–Gugerty (2005) shows that members of her dataset most often cite self-control problems as the reason for joining a rosca.[3] Second, I take seriously the fact that roscas are informal institutions that sometimes exist even in environments where contracting and social networks are weak. I assume that participation cannot be contracted upon and that social punishment is infeasible. In this context, roscas can be viewed as commitment savings devices with particular advantages. Not

[3] Some quotes: "You can't save alone–it is easy to misuse money;" "Saving money at home can make you extravagant in using it."

only do they improve savings behavior, but they can survive without social sanctions, and will be adopted without postponement. In contrast with exponential discounters, roscas generate two complementary benefits for hyperbolic discounters–value from high expected rank, and value from commitment. The value from high expected rank ensures that agents will actually enter a rosca, and the value of future commitment gives agents who receive the pot early a reason to repay the "debt." The arguments are developed in four broad parts.

First, I describe a sophisticated quasi-hyperbolic discounter who values a nondivisible good, and solve for her autarky Markov Perfect Equilibrium across the β-parameter space[4]. As β gets smaller (she puts more weight on the present), her ideal outcome involves consuming in the present but saving rapidly for the nondivisible in future periods. However, such behavior does not constitute an equilibrium. I show that the speed of saving in equilibrium must drop as β drops.

Second, I show how roscas can be sustainable commitment devices. Roscas provide commitment in the following sense: if, when the agent is furthest away from the nondivisible, she prefers to remain in the rosca, then she will always prefer to remain in the rosca. This is because, as she gets closer to the nondivisible, the rosca locks in more of her savings and thus reduces her incentive to leave. I assume there is a single provider of roscas who cannot make credible threats of punishment. The only enforceable rule is that an agent who leaves a rosca can never rejoin. I find that there is always a parameter region where the agent will never leave a rosca.

My argument rests on a key trade-off that a rosca participant faces, especially when her next nondivisible is far away. If she leaves the rosca, she gets the pleasure of consuming her current income but faces the unwelcome prospect of saving slowly in autarky in the future. On the other hand, if she stays in the rosca, she has to sacrifice current consumption, but knows that her discounted utility from future periods will be higher than in autarky. She will remain in the rosca if the promise of good future behavior (induced by the rosca) outweighs her desire to over-consume in the present. I also find here that the parameter region that supports random roscas is a strict subset of the region that supports fixed roscas.

Roscas also have the particularly appealing property of enticing agents to join without delay. With standard commitment devices, costs are incurred in the present while benefits (in the form of matured savings) arrive in the future. This can cause hyperbolic discounters to delay take-up

[4] The "β" refers to the agent's hyperbolic discount factor, by which she discounts her entire future in any given period.

of such devices, even if they value the commitment. However, in the case of roscas, an agent knows that there is a likelihood of her being an instant winner (she might get ranked first). I show that this ensures that the agent will not postpone entry into a rosca.

Fourth, I model roscas as decentralized equilibria by lifting two assumptions of the benchmark case–I vary the available information about agents' past rosca behavior, and allow the generation of new roscas in any period (there is some exogenous probability with which people get matched into new roscas). Agents play a dynamic game and strategies can be conditioned on reputation, to the extent that it is available. The objective is to find equilibria that prevent frivolous defection (which would involve leaving one rosca in search of a higher rank in another).

This setup provides two further insights. First, even in completely anonymous environments, roscas can survive if the probability of matching into new roscas is sufficiently low. Second, I find conditions under which random roscas can be preferred to fixed roscas. To do so, I model what is perhaps the most realistic reputation environment–"partial reputation." This is the case where only the agents who leave without completing a round develop a bad reputation–these agents can be barred from future roscas. This allows us to restrict our focus to agents' desires to leave *between* rounds. The last-ranked member in a fixed rosca is permanently last ranked, so she might have an incentive to leave after a round, in anticipation of a better rank in a new rosca. Now, random roscas have a particular advantage over fixed roscas–by re-randomizing at the start of every round, they internalize the attractiveness of the outside option. There is no incentive for a member of a random rosca to leave between rounds. I find that, under certain conditions, random roscas survive longer and increase welfare relative to fixed roscas.

3 Related Literature

Standard explanations of roscas focus on the individual's desire to save for a nondivisible good. In addition to the theoretical models described above, there is a wide range of empirical papers on roscas in several parts of the world.[5] Most rely on an informal notion of social punishment to explain why agents don't defect.

Anderson and Baland (2002) find evidence that roscas are used by women to restrict their husbands' access to their savings. In their model, women have a greater preference for the nondivisible than men, but have limited power over expenditures within the household. If

[5] See Bouman (1994), Handa and Kirton (1999), and Kimuyu (1999).

women were to save at home, their husbands would direct too much of their savings towards immediate consumption. On the other hand, if women save in a rosca, husbands have no access to their savings until the pot is received. At this stage, assuming the woman has sufficient bargaining power to purchase the nondivisible good, it is in fact purchased. In this setting, a rosca can be viewed as another kind of commitment savings device. A woman would like to save for the nondivisible, but she knows that if she saves at home, she will not be able to save as fast as she would like to. A rosca, by locking in savings, allows her to prevent over-consumption by her household in future periods.

Gugerty (2005) finds direct evidence that individuals use roscas to overcome their own time-inconsistency.[6] In her dataset, self-control problems are cited as the most common reason for joining a rosca (36% of the members say it is the primary reason). Based on anecdotal evidence, there appears to be very limited scope for credible social sanctions. Gugerty's study is set in a rural community in Kenya where banks, if available, are very far away. The average rosca in her dataset is 6.5 years old, with the average round lasting a little under a year. She finds that only 6% of members left a rosca is the last round studied. 37% of the roscas are fixed , 58% are random, and the rest use other forms of negotiation/randomization. On the other hand, Anderson, Baland, and Moene study a poor urban neighborhood near Nairobi, where 71% of the roscas are fixed and 29% are random. Funds generated through roscas are more often spent on nondivisible goods with immediate benefits than on large durables.[7]

Quasi-hyperbolic discounting has been used to describe time-inconsistent preferences in several papers.[8] The fact these preferences result in a need for commitment has also been widely studied. For examples, see Ashraf, Gons, Karlan and Yin (2003), Thaler and Bernartzi (2004), and Ashraf, Karlan and Yin (2005). In the context of this literature, the point of this paper is to show that roscas are effective commitment devices even without "commitment" in the standard sense (in settings where agents cannot pre-commit to join and cannot be forced, through contracts or social punishment, to continue participating).

Finally, there are two aspects of roscas that are worth mentioning even though my results do

[6] In a paper on Benin, Dagnelie and LeMay (2005) provide further evidence of roscas being used as commitment devices.

[7] Gugerty (2005) finds that the largest proportions of rosca funds are spent on household cooking items, school fees, and food. In Dagnelie and LeMay's (2005) dataset, only 19% of funds are spent on durable goods.

[8] Originally proposed by Phelps and Pollack (1968), it has been developed in papers by Laibson, Harris, Rabin, and others. More recently, Krusell and Smith (2003b) characterize the mixed-strategy equilibria in a Ramsey-style consumption-savings problem with lump-sum investment.

not directly relate to them. First, the allocation of pots can also be determined by an auction. Though these "bidding" roscas are not common in the empirical papers that motivate my model, they nevertheless feature prominently in many developing countries. It is possible to conceive of them as risk-sharing arrangements under limited commitment (see Ligon, Thomas, and Worrall (2002)). However, this approach is less plausible with fixed and random roscas, where the allocation order is generally inflexible. Recent papers on bidding roscas include Calomiris and Rajaraman (1998), Klonner (2005), and Klonner and Rai (2006).

Second, there is the question of efficiency. In a companion paper to the one previously described, Besley, Coate, and Loury (1994) argue that random (and, by extension, fixed) roscas do not result in efficient allocations in general. In a recent paper, Ambec and Treich (2005) show how roscas can be the best possible institutions, ex ante, when agents value commitment (but their paper assumes contracts are binding and that agents can commit to joining a rosca at a future date). This leaves open the question of whether, in an environment with hyperbolic discounting and limited contracting, an institution more efficient or sustainable than roscas exists.[9] However, since roscas do enjoy widespread success, I hope this paper takes a useful step towards understanding when and why they survive.

4 Autarky Model

In this section, I assume the individual does not have access to a rosca, and study her behavior in terms of equilibria played across her per-period selves.

4.1 Assumptions

The agent is an infinitely lived, sophisticated quasi-hyperbolic discounter. She has a per-period non-stochastic income y, and no initial endowment. Borrowing is not possible, and no interest is earned on savings. There are two types of goods: a consumption good (denoted c; price 1) and a nondivisible good (denoted d; price ky, where k is a positive integer). Saving has to be lump-sum, in multiples of y. (This assumption allows us to model autarky equilibria using mixed strategies). The agent's per-period utility function is $u(c + bd)$ (where u is strictly concave and

[9] One possibility is a lottery, which retains the wealth pooling property of a rosca without the problem of sustainability. This is a concern that is as relevant to existing theories of fixed and random roscas as it is to mine. Since roscas do exist in the face of this alternative, two informal explanations (beyond the scope of my model) come to mind: perhaps people don't trust the fairness of lotteries, or they dislike the uncertainty about how often they will win.

defined over the domain $[0, \infty)$, with $u(0) = 0$; b is a positive constant).

Given this per-period utility function, intertemporal utility at time t is:

$$U_t = u(c_t + bd_t) + \beta \sum_{i=1}^{\infty} \delta^i u(c_{t+i} + bd_{t+i}), \text{ where } \beta \in (0,1) \text{ and } \delta \in (0,1)$$

Finally, I assume that the nondivisible good is "desirable":

$$\delta^{k-1} u(b) > \sum_{i=0}^{k-1} \delta^i u(y) \tag{1}$$

This ensures that, if the agent were an exponential discounter, she would repeatedly save all her wealth for the nondivisible good (any other saving rule would violate either time consistency or the condition above).

4.2 Equilibrium concept

I decompose the individual into a series of time-indexed independent selves with utility functions $\{U_t\}$, and assume they play a Markov Perfect Equilibrium. In any period, the agent observes her total wealth, w_t, and makes a decision about how much to save, s_t (s_t is a gross saving decision). Wealth and savings are related in the following way: $w_t = s_{t-1} + y$. Since saving is lump-sum and initial wealth is 0, it must be that $w_t = iy$, where $i \in \{1, 2, 3, ...\}$. In any state iy the action set, $\{0, y, 2y, ..., iy\}$, includes all feasible levels of saving. A strategy associates every state $w = iy$ with a sequence of positive probabilities, $\{\pi_i(0), \pi_i(y), \pi_i(2y), ..., \pi_i(iy)\}$, that sums to 1 and denotes a probability distribution over all feasible actions.

We can immediately restrict our set of possible states to: $\{y, 2y, 3y, ..., (k-1)y\}$. Since the nondivisible provides the only incentive to save, and since initial wealth is 0, there will never be an equilibrium where the agent encounters wealth higher than ky. The agent with wealth ky will always save 0, regardless of future behavior.

A strategy is an equilibrium if and only if, for any state $w \in \{y, 2y, 3y, ..., (k-1)y\}$, every action s that is played with positive probability satisfies:

$$s \in \max_{s' \in \{0,1,2,...,w\}} \left[u\left(w - s'\right) + \beta\delta V\left(s' + y\right)\right]$$

Here, $V(\cdot)$ is the value function of the exponential discounter, defined recursively:

$$V(ay) = \sum_{j=0}^{a} \pi_a(j) \left[f(ay - jy) + \delta V((j+1)y) \right]$$

where

$$f(x) = \begin{cases} u(x), & \text{if } x < ky \\ u(b), & \text{if } x = ky \end{cases}$$

4.3 Predicting Equilibrium Choice

Multiple-self models with quasi-hyperbolic discounting can lead to a multiplicity of equilibria. In this section, I present some results that allow us to restrict the set of strategies that are candidates for equilibrium, and predict which equilibrium will be chosen in the case of multiplicity. I find that there is always an equilibrium that weakly dominates all others in every state. The proofs of the following lemmas and proposition are in Appendix A.

Consider any strategy in which some saving occurs. For this to be an equilibrium, it must be the case that all deviations (in terms of lower saving) at all levels of wealth are dominated. It follows directly from concavity that if an agent with low wealth chooses to save a certain amount, an agent with higher wealth cannot possibly wish to save any less. The stock must always weakly rise until ky is reached.

Lemma 1 *If at wealth w the agent (weakly) prefers to save s, then at any wealth $w' > w$ the agent will never save any $s' < s$ with positive probability.*

This means that in any equilibrium in which the nondivisible is purchased with positive probability, an agent with wealth w will either save w or mix between w and $w - y$.

If there are multiple equilibria at any values of β, we need to predict which among them the agent will actually play. This is relatively straightforward if we establish that the best equilibrium in any one state is also the best equilibrium in all other states. The following proposition shows that this is indeed the case.

Proposition 1 *There is always an equilibrium that is "optimal" in the sense that, at any level of wealth that is reached in equilibrium, the agent does not strictly prefer to play any other equilibrium.*

This proposition is proved with the help of the following lemmas.

Lemma 2 *Consider two equilibria, denoted A and B. Suppose* $1 \geq \pi_y^A(y) \geq \pi_y^B(y) \geq 0$, *and that* $\pi_y^A(y) > 0$ *and* $\pi_y^B(y) < 1$. *Then, in any state* w, *where* $y \leq w \leq (k-1)y$, $V_A(w) \geq V_B(w)$.

Lemma 3 *Consider two equilibria, A and B, such that* $\pi_y^A(y) = \pi_y^B(y) = 1$. *Let* $\bar{w}$ *be the lowest state at which* $\pi_{\bar{w}}^A(\bar{w}) > \pi_{\bar{w}}^B(\bar{w})$. *Then,* $V_A(w) \geq V_B(w)$ *at all states* w.

The proposition and lemmas above tell us the following: given the set of all possible equilibria in which the nondivisible is purchased with some probability, the optimal equilibrium is the one with the highest probability of saving at wealth y. (If there are multiple equilibria with full saving at y, we repeat the comparison at higher levels of wealth.) Furthermore, any equilibrium in which the nondivisible is purchased is always preferred to any equilibrium in which it is not purchased. The agent always wants her future selves to save more; and the more they save, the more she is willing to save in the current period.

4.4 Optimal Autarky Equilibria Across Parameter Regions

I now construct the optimal autarky equilibrium for all possible values of β. While it might not be the case that the agent actually plays the optimal equilibrium, it serves as a reasonable benchmark, especially as it stacks the odds against rosca survival (the better the autarky option, the lower the agent's incentive to stay on in a rosca). Clearly, if $\beta = 1$, the agent behaves exactly like an exponential discounter, and if $\beta = 0$, she never wishes to save any amount. To map out the equilibria between these boundaries, I focus on the equilibrium with the highest possible level of saving at wealth y, for every value of β (starting at $\beta = 1$ and dropping down to $\beta = 0$). I find that there is always a β-region in which the agent behaves like an exponential discounter. However, once β gets sufficiently small, she no longer wishes to save today if she knows she would start saving tomorrow anyway. Once this occurs, to re-induce saving, the agent plays a mixed strategy equilibrium. Finally, there is a region that cannot support any saving equilibrium.

A full-saving strategy is defined as the following: at any $w \in \{0, 1, 2, ..., (k-1)y\}$, the agent saves w.

Lemma 4 *Consider a strategy with full saving at all* $w > w'$. *If the agent at wealth* w' *(weakly) prefers to save* w' *than to save any lower amount, agents at all higher levels of wealth strictly prefer to save fully.*

Proof. The strategy determines some continuation value $V(\cdot)$. If the agent at w' weakly prefers to save fully, this means:

$$\beta\delta V\left(w' + y\right) \geq u(y) + \beta\delta V\left(w'\right)$$

To prove the lemma, we need to show that $V(x)$ is strictly convex for $x \geq w'$. Note that $V(x) = \delta^{k-x}u(b) + \delta^{k-x+1}V(y)$. Since $V(y) > 0$, V is strictly convex. ■

Proposition 2 *Consider $\bar{\beta}$ as defined in Equation 2 below. When $\beta \in [\bar{\beta}, 1)$, the full-saving strategy is an equilibrium.*

Proof. A set of necessary and sufficient no-deviation conditions must be satisfied for the full-saving strategy to be an equilibrium. Specifically, at every wealth level $w \in \{0, 1, 2, ..., (k-1)\,y\}$, the agent must prefer to save her entire wealth relative to any lower level of saving. If we can show that, at each w, the agent prefers to save w over $w - y$, Lemma 1 ensures that all other conditions will be satisfied. Thus, a full-saving equilibrium exists if and only if, for each w:

$$\beta\delta V(w + y) \geq u(y) + \beta\delta V(w)$$

By Lemma 4, a necessary and sufficient condition for all the above conditions is:

$$\begin{aligned} \beta\delta V(2y) &\geq u(y) + \beta\delta V(y) \\ \Leftrightarrow \beta &\geq \frac{u(y)}{\delta\left[V(2y) - V(y)\right]} = \frac{1-\delta^k}{\delta^{k-1}\left[u(b)\right]} \cdot \frac{u(y)}{1-\delta} \\ \bar{\beta} &= \frac{1-\delta^k}{\delta^{k-1}\left[u(b)\right]} \cdot \frac{u(y)}{1-\delta} \end{aligned} \tag{2}$$

Since $u(y) > 0$ and $V(2y) > V(y)$, $\bar{\beta}$ is above 0. Also, expanding the term, we see that it must be less than 1 if the nondivisible is desirable (Condition 1). ■

The proposition above also tells us that at $\bar{\beta}$, agents at wealth greater than y strictly prefer to save fully if all future selves save. Therefore, when the full-saving equilibrium can no longer be supported, we expect to find a region in which the agent with y plays a mixed strategy, but all others save fully. Below $\bar{\beta}$, to create incentives to save, the agent with wealth y must play a strategy where she saves with some probability $\pi_y(y) < 1$. Then, she knows that if she does not save today, there is a possibility that she will not even save tomorrow. By worsening the consequences of not saving today, she again becomes willing to save.

The next proposition establishes that there is always a β-region in which a mixed strategy is played–one with full saving above y, and mixing at y. As β drops below $\bar{\beta}$, the agent at y will save with decreasing probability down to some $\underline{\underline{\beta}}$ (defined in the proof of the proposition) at which $\pi_y(y) = 0$. This strategy will be an equilibrium at any $\beta \in \left(\bar{\beta}, \underline{\underline{\beta}}\right)$ only if agents at wealth levels above y continue to strictly prefer to save. If that condition holds, then $\underline{\underline{\beta}}$ is the cutoff below which saving equilibria can no longer be supported.

However, if δ is reasonably high, and $u(b)$ sufficiently close to $u(y)$, this will not be the case.[10] Then, there will be a region in which the optimal equilibrium will involve mixing between y and 0 at wealth y, mixing between $2y$ and y at wealth $2y$, and full saving at all higher levels of wealth. While explicit solutions for mixed strategy equilibria depend on actual parameter values (see Appendix D for an example with $k = 2$), they can always be constructed using the following rule: Moving down from $\bar{\beta}$, solve for $\pi_y(y)$ at each β so that the agent at y is indifferent between saving and consuming. Continue until some β_1 where either (a) the agent at $2y$ no longer strictly prefers to save, or (b) $\pi_y(y) = 0$. If (b), then define $\underline{\beta} = \beta_1$. If (a), continue below β_1, now solving for $\pi_y(y)$ and $\pi_{2y}(2y)$ to get indifference at y and $2y$, respectively. Continue until some β_2 where either (a) the agent at $3y$ is indifferent, or (b) $\pi_y(y) = 0$. If (b), $\underline{\beta} = \beta_2$. If (a), continue. By repeating these steps until (b) is satisfied, there will be some $i \in \{1, 2, ..., k-1\}$ such that $\beta_i = \underline{\beta}$. This is formalized in Proposition 3 (the proof is in Appendix A).

Proposition 3 *There exist $\{\beta_1, \beta_2, ..., \beta_{k-1}\}$ satisfying $\bar{\beta} > \beta_1 \geq \beta_2 \geq ... \geq \beta_{k-1} = \underline{\beta}$, such that the optimal equilibrium at any β has the following properties: for $\beta \in [\beta_1, \bar{\beta})$, wealth y plays a mixed strategy while all others save fully; for $\beta \in [\beta_2, \beta_1)$, wealth y and $2y$ play mixed strategies while all others save fully; and so on.*

Ultimately, $\underline{\beta}$ is the point at which the agent at y no longer wishes to save ($\pi_y(y) = 0$). It is clear that $\pi_y(y)$ must go to 0 before other probabilities, $\pi_w(w)$, do. If this were not the case, it would mean that the agent at y would be saving some $\pi_y(y) > 0$ while an agent at a higher level of wealth, $\bar{w}$, saved with $\pi_{\bar{w}}(\bar{w}) = 0$. But then it would not be rational for agent y to save, since the nondivisible would never be bought.

Proposition 4 *There is some $\underline{\beta} > 0$ in $[\underline{\underline{\beta}}, \bar{\beta})$ such that, for $\beta > \underline{\beta}$, there is always an equilibrium in which the nondivisible is purchased, and for $\beta' \leq \underline{\beta}$, the nondivisible can never be*

[10] As δ gets high, the incentives to defect at $2y$ start to get almost as strong as the incentive to defect at y, because the fact that we are one period closer to the durable good becomes less significant. See Appendix B for an example.

purchased in equilibrium.($\underline{\underline{\beta}}$ is defined Equation 11 in Appendix A, and $\bar{\beta}$ is defined in Equation 2.)

The proof is in Appendix A. In Appendix C, I also show that for $\beta \in [\underline{\beta}, \bar{\beta}]$, the lifetime autarky utility for the agent at wealth y is weakly concave in β (see Figure 2).

5 Roscas as Commitment Devices

When agents are hyperbolic discounters, it is natural to think of roscas as commitment savings devices. I describe a rosca as a group of k people (k as defined in Section 3), where one rosca round lasts k periods. The per-period contribution is y. This rosca can be either fixed or random.

Consider an agent in a fixed rosca, in the period after which she has received the nondivisible. Suppose she values the commitment provided by the rosca and chooses to stay. Then, she knows that in all future periods she will continue to stay. This is because, for every additional period that she participates in the rosca, more of her savings get locked in (they are consumed by someone else, so there is no way for her to access them). This argument can be similarly applied to the first-ranked agent in a random rosca. If she chooses to stay in the rosca in the second period, the lock-in property ensures that she will always choose to stay.

This value of commitment that a rosca provides also allows for an effective punishment mechanism to ensure continued participation even without the threat of formal or social punishment. If an agent knows that once she leaves a rosca she can never return, she might strictly prefer to stay even when she is furthest away from her next nondivisible. This insight is formalized in the following subsection.

5.1 Sustainability

I first analyze a rosca that operates according to the following rule: any agent who leaves can never rejoin any rosca, but cannot be punished in any other way. This can be interpreted as a case where roscas can only form at some central location, which allows agents' past behavior to be monitored.[11] While this is admittedly a stylized environment, it is one in which roscas with exponential discounters cannot survive (and hence allows us to isolate the arguments that lead to such starkly different results for hyperbolic discounters).

[11] This assumption in relaxed in Section 5.

5.1.1 Fixed Roscas

Consider the last-ranked agent in a fixed rosca. How strong is her temptation to defect in the first period? If she has a strict incentive to stay on in the rosca, it follows that she will always have a strict incentive to stay on. If the agent were an exponential discounter, she would not have this strict incentive since there are no social sanctions and she could replicate the rosca outcomes in autarky.

Suppose the last-ranked agent is a hyperbolic discounter who is unable to save continuously in autarky. She would ideally like to pause the rosca for one period (so she could consume her income today) and continue with rosca participation from tomorrow (since continuous saving in the future maximizes her discounted utility). However, pausing the rosca is not an option. She now faces a trade-off between autarky (high instantaneous utility, low future utility) and staying in the rosca (low instantaneous utility, high future utility). If she values the commitment provided by the rosca highly enough that she is willing to forego current consumption, she will strictly prefer to stay on.

Proposition 5 *Consider β^* as defined in Equation 3 below, and $\bar{\beta}$ as defined in Equation 2. Suppose a member of a rosca knows that the other members of the rosca will never defect. Then, when the alternative is autarky, she strictly prefers to never defect if $\beta \in \left(\beta^*, \bar{\beta}\right)$. If $\beta < \beta^*$, there will be periods when she strictly prefers to leave. If $\beta \geq \bar{\beta}$, there will be periods when she weakly prefers to leave.*

Proof. Consider an agent's decision when she is k periods away from the next nondivisible. For simplicity, denote the continuation value from autarky equilibrium at any level of wealth, $V_A(\cdot)$. Denote the continuation value from a strategy in which all future selves save fully, $V_F(\cdot)$. First consider the region, $\beta \in [\underline{\beta}, \bar{\beta})$. If the individual defects from the rosca, she will have to play autarky forever (and forego any contributions she has made to the rosca so far). Since in autarky she is indifferent between saving and not saving at wealth y, her autarky utility will be $\beta\delta V_A(2y)$, where $V_A(2y)$ involves probabilistic saving in at least some future periods. If the agent remains in the rosca, her utility is $\beta\delta V_F(2y)$. Since $V_F(2y)$ involves optimal saving by all future selves, it must be greater than $V_A(2y)$. Therefore, if the agent chooses to save with positive probability in autarky, she will strictly prefer to remain in the rosca.

Second, I show that there is a region below $\underline{\beta}$ in which the agent still strictly prefers to remain in the rosca. Consider the indifference condition at $\underline{\beta}$. The agent at y is made indifferent with $\pi = 0$:

$$\underline{\beta}\delta V_A\left(2y\right) = u\left(y\right) + \frac{\underline{\beta}\delta u\left(y\right)}{1-\delta}$$

The utility from a rosca, $\underline{\beta}\delta V_F\left(2y\right)$, still strictly dominates the utility from autarky. The agent continues to strictly prefer a rosca down to β^* that satisfies:

$$\begin{aligned}
\beta^*\delta V_F\left(2y\right) &= u\left(y\right) + \frac{\beta^*\delta u\left(y\right)}{1-\delta} \\
&\Leftrightarrow \quad \beta^* = \frac{u\left(y\right)}{\delta\left[V_F\left(2y\right) - \frac{u(y)}{1-\delta}\right]} \\
&\Leftrightarrow \quad \beta^* = \frac{u\left(y\right)}{\delta\left[\frac{\delta^{k-2}\{u(b)\}}{1-\delta^k} - \frac{u(y)}{1-\delta}\right]} \qquad (3)
\end{aligned}$$

Since $u\left(y\right) > 0$, we know that $\beta^* > 0$. ■

It is useful to note here that $\beta^* < \underline{\underline{\beta}}$. This means that, even if the agent's preferences are so present-biased that she would never save in autarky, she might be willing to sacrifice current consumption for the sake of future benefits from a rosca.

Source of Commitment A natural question here is: what exactly about the rosca provides commitment to the hyperbolic discounter? A rosca comes with (1) the threat of being barred for non-payment and (2) illiquidity of savings. We can consider two alternative commitment savings devices that separately perform these functions: a friend who promises to monitor your saving and credibly threatens to stop helping if you under-save, and a fixed-deposit that locks up your savings until you reach a target amount (in this case, ky).

In the $\left(\underline{\beta}, \bar{\beta}\right)$ range, illiquidity plays no role, since even in autarky the agent never dips into her savings. As long as we are in a region where some saving occurs in equilibrium, access to a fixed deposit cannot improve savings behavior. Here, a fixed rosca is very similar to a friend who offers to "help". The fact that the rosca can offer a credible threat to deny access to defectors creates a large enough utility gap between the rosca and autarky equilibria to ensure participation.[12]

[12] In this case, if the agent were able to play history dependent (instead of Markov) equilibria with herself, she should be able to replicate rosca behavior. However, an individual's ability to play such equilibria on her own is limited by the fact that the punishment strategy would be dominated by the strategy along the equilibrium path. If she deviates under a history-dependent equilibrium, she can easily renegotiate with herself to not play the punishment strategy.

At lower levels of β, the illiquidity provided by the rosca can play a role. Consider the region in which an agent stays in a rosca but would not save in autarky ($\beta < \underline{\beta}$):

$$\frac{\beta\delta^{k-1}u(b)}{1-\delta^{k}} \geq u(y) + \frac{\beta\delta u(y)}{1-\delta}$$

Now, suppose the agent had the helpful friend instead of the rosca. The above condition might no longer be sufficient to ensure cooperation. She would also need to ensure that at wealth $2y$ she did not have an incentive to consume everything:

$$\frac{\beta\delta^{k-2}u(b)}{1-\delta^{k}} \geq u(2y) + \frac{\beta\delta u(y)}{1-\delta}$$

If $u(\cdot)$ is not very concave, and δ is sufficiently high, then the second condition can fail even if the first is satisfied. With δ high, the agent does not benefit as much from being one period closer to the nondivisible. On the other hand, if u is almost linear, the benefit of consuming $2y$ can be enough to outweigh the fact that she will no longer save in the future.[13] However, even when the illiquidity plays a role, this does not make the contracting aspect of a rosca irrelevant. With a fixed-deposit instead of a rosca, the agent might simply not deposit any money in subsequent periods.

While this is by no means an exhaustive list of alternative commitment savings devices, the examples above allow us to decompose the commitment provided by roscas.[14]

5.1.2 Random Roscas

The only difference between a random and a fixed rosca is that, in a random rosca, the ordering is re-randomized at the start of each round. Consider an agent who has received the nondivisible in period 1. In period 2, her expected value from staying in the rosca is $\frac{\beta\delta^{k-1}u(b)}{(1-\delta)k}$. If, in this period, she stays in the rosca, she will always stay in the rosca, since this is the furthest away from the nondivisible she can ever be.

Proposition 6 *If δ is sufficiently large, there is a β-region, bounded by $\left(\beta^{*}_{ran}, \bar{\beta}_{ran}\right)$, within which an agent will always strictly prefer to remain in a random rosca (when the alternative is*

[13] One can expand the region in which illiquidity plays a role if there is either an intermediate "temptation" good or if income is stochastic. In each of these cases, the agent will have a greater incentive to dip into her savings in autarky, and she might therefore further value the fact that a rosca will prevent her from doing so.

[14] As Section 4.2 demonstrates, roscas have another attractive property not shared by these devices.

autarky). The region will be a strict subset of the region in which an agent stays in a fixed rosca: $\beta^*_{ran} > \beta^*$, $\bar{\beta}_{ran} < \bar{\beta}$.

Proof. First, I show that the β-region that supports full participation in a random rosca must be smaller than the equivalent region for a fixed rosca. The lowest expected value from staying in a random rosca is smaller than the lowest expected value from staying in a fixed rosca:

$$\frac{\beta\delta^{k-1}u(b)}{(1-\delta)k} < \frac{\beta\delta^{k-1}u(b)}{1-\delta^k}$$

(This is always true for $\delta < 1$ and $k > 1$). In each case, the outside option (autarky) is identical. Therefore, if an agent weakly prefers to stay in a random rosca, she will strictly prefer to stay in a fixed rosca.

Second, consider the first-ranked agent's decision in the second period of a rosca round. She will remain in the rosca if $\frac{\beta\delta^{k-1}u(b)}{(1-\delta)k}$ is higher than her autarky equilibrium. If there is a region in which this is true, the lower bound, β^*_{ran}, must lie in $(0, \underline{\beta})$and the upper bound, $\bar{\beta}_{ran}$, must lie in $(\underline{\beta}, \bar{\beta})$. This is because, at $\beta = 0$ and $\beta = \bar{\beta}$, autarky is strictly preferred, and for $\beta \in \left(0, \underline{\beta}\right)$, autarky utility is linear and increasing in β, and for $\beta \in \left[\underline{\beta}, \bar{\beta}\right]$, autarky utility is concave and increasing in β. β^*_{ran} is given by:

$$\begin{aligned} \frac{\beta^*_{ran}\delta^{k-1}u(b)}{(1-\delta)k} &= u(y) + \frac{\beta^*_{ran}\delta u(y)}{1-\delta} \\ \beta^*_{ran} &= \frac{(1-\delta)u(y)}{\delta^{k-1}u(b) - k\delta u(y)} \end{aligned} \tag{4}$$

In Appendix E, I show that, for $\delta \to 1$, $\underline{\underline{\beta}} \to \frac{u(y)}{u(b)-(k-1)u(y)} > 0$. So, for any δ, however large, there will be no autarky saving for $\beta < \frac{u(y)}{u(b)-(k-1)u(y)}$. As $\delta \to 1$, $\beta^*_{ran} \to 0$. This ensures that, for sufficiently high δ, there will be a region in which an agent always prefers to remain in a random rosca. ■

The intuition for the above result is the following: as δ gets large, the agent does not mind the fact that her rank in the next round is uncertain (her expected utility converges to the expected utility of staying on in a fixed rosca). However, when δ is small, she cares about the fact that she has to wait particularly long for the next nondivisible. This limits the range of β-values for which she will choose to remain in the rosca. The graphs below provide a summary of the results from the sections above.

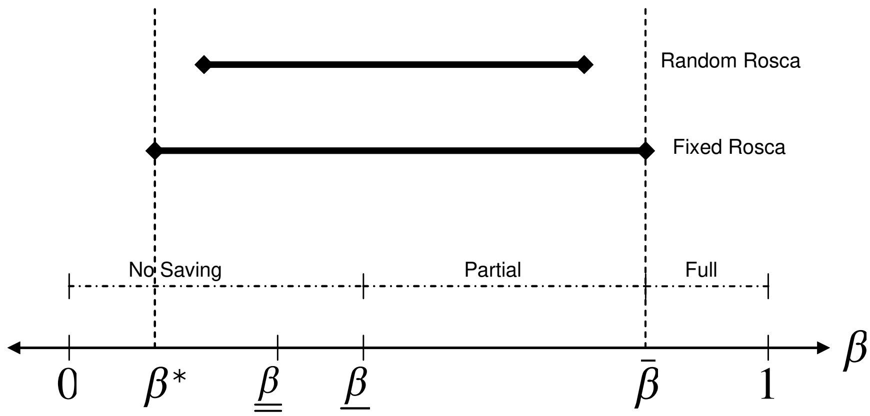

Figure 1: Comparison of β-regions that support random and fixed roscas. The lowest (broken) line indicates the type of equilibrium played in autarky.

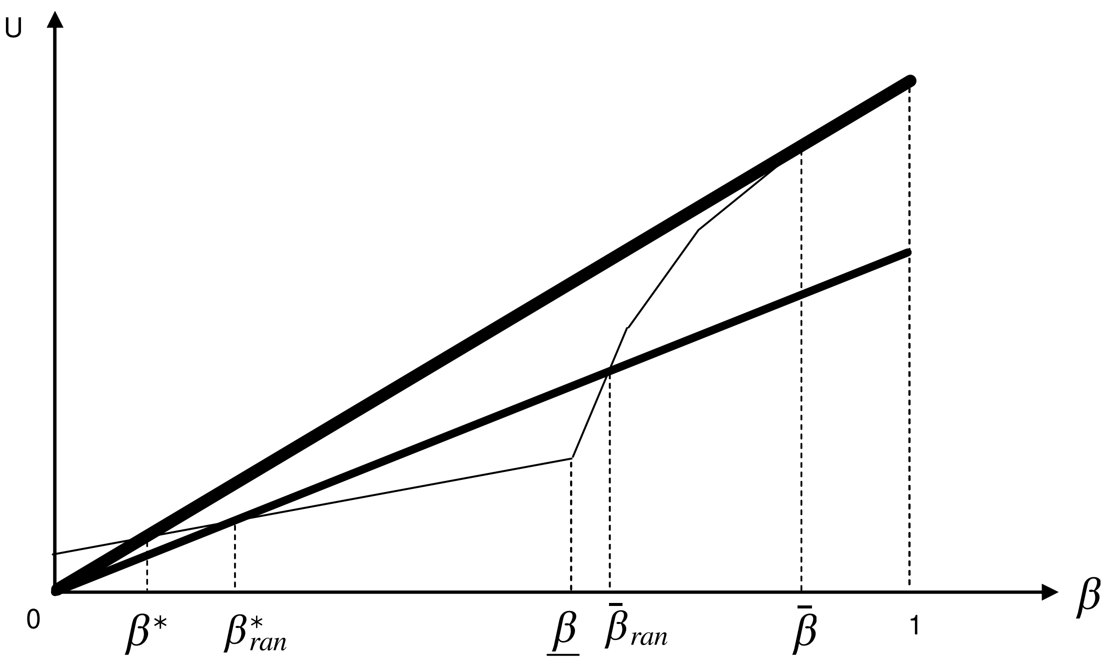

Figure 2: Comparison of lifetime utilities across β. The thickest line is the utility of a fixed rosca member who is k periods away from her next nondivisible. The lower line of medium thickness is the utility of a random rosca member in period 2, conditional on having received the nondivisible in period 1. The thinnest (crooked) line is the autarky utility at wealth y. In $(\beta^*, \bar{\beta})$, fixed roscas are preferred to autarky. In $(\beta^*_{ran}, \bar{\beta}_{ran})$, random roscas are preferred to autarky.

5.2 Entry and Welfare

A potential problem with commitment savings devices, especially when start dates cannot be contracted upon, is that the agent might have an incentive to postpone entry.[15] As O'Donoghue and Rabin (1999) show, when tasks are costly in the present and have delayed benefits, a quasi-hyperbolic discounter will procrastinate even though the welfare-maximizing outcome involves completing the task immediately.

Since commitment saving typically entails saving today for future benefits, the agent might wait to join even if she values the commitment. This problem disappears with roscas because of the initial randomization in ranking. I show below that the possibility of getting the nondivisible in the current period ensures that the hyperbolic discounter will not want to postpone entry.[16]

Suppose there is no defection from a rosca once it forms. In this section, I study a single agent's entry decision. An agent faces identical expected values from joining a random rosca or a fixed rosca. In each case, she expects to get the nondivisible once every k periods, with some uncertainty about her exact rank. The expected value is $\frac{u(b)}{k}\left[1+\frac{\beta\delta}{1-\delta}\right]$.

Proposition 7 *If an agent knows that she will always stay in a rosca once she joins, she will always join in the first period of her life.*

Proof. Suppose an agent can join a rosca in any period, and knows she will remain in it forever once she enters. She will join in the first period of her life only if she would rather not postpone by one period:

$$\begin{aligned} \frac{u(b)}{k}\left[1+\frac{\beta\delta}{1-\delta}\right] \quad &> \quad u(y)+\frac{u(b)}{k}\left[\frac{\beta\delta}{1-\delta}\right] \\ &\Rightarrow \quad \frac{u(b)}{u(y)} > k \end{aligned}$$

It is important to note that this condition does not depend on β. We can see that, as long as the nondivisible is desirable, the condition is always satisfied. The desirability condition (from

[15] If the agent's true welfare is measured by taking the discounted sum of her utility with $\beta = 1$, the welfare maximizing outcome is the one where she starts saving immediately.

[16] This result relies on the assumption that goods yield immediate benefits (as supported by many empirical studies). However, even when goods are more durable, agents will be less likely to postpone entry into roscas than other commitment devices.

Equation 1) is:

$$\begin{aligned} \delta^{k-1} u(b) &> \sum_{i=0}^{k-1} \delta^{i} u(y) \\ &\Rightarrow \frac{u(b)}{u(y)} > \frac{\sum_{i=0}^{k-1} \delta^{i}}{\delta^{k-1}} \end{aligned}$$

For $\delta \in (0,1)$, it is always true that $\frac{\sum_{i=0}^{k-1} \delta^{i}}{\delta^{k-1}} > k$. Therefore, if the nondivisible is desirable to exponential discounters, then hyperbolic discounters will never choose to postpone entry into a rosca. ■

Since the individual has present-biased preferences, she is tempted by the possibility of an immediate reward in the same way that she is tempted to over-consume in the present. This shows that, if the conditions for remaining in the rosca are met, the expected value of the rosca is sufficiently high that she will choose to join immediately.

6 Roscas as Decentralized Equilibria

In the previous section, I have shown that it is possible for a hyperbolic discounter to strictly prefer to remain in a rosca at all times if departure results in being banned from future roscas. It follows that if all agents in a rosca satisfy the parameter conditions, then no individual has an ex-post incentive to leave if others choose to stay. In this section, I loosen two assumptions. I limit the available information about an agent's past rosca behavior. This places a restriction on what rosca rules can be conditioned on. I also allow the formation of new roscas in any period. The rate at which this happens is pinned down by an exogenous probability with which rosca aspirants get matched into groups of size k.

This gives us two key results. First, while we have seen that neither contracts nor social sanctions are needed to prevent defection from a rosca, we would like to know how roscas might survive if there is limited information about agents' past behavior. I find that, even under complete anonymity, roscas can survive if the exogenous probability of finding new roscas is sufficiently low. This follows directly from the commitment value for hyperbolic discounters. If all agents were exponential discounters, even an infinitesimal probability of finding a higher rank in a new rosca would create a strict incentive to leave.

Second, the previous section does not provide a reason for random roscas to exist (the greatest incentive to defect from random roscas is stronger than for fixed roscas, while ex-ante utilities are the same). In this section I show that under limited reputation, when matching is fast, random roscas are more resilient than fixed roscas. Suppose we have "partial reputation"–this is a case where agents who have left roscas in the middle of past rounds are remembered as defectors, but agents who leave after completing a round are indistinguishable from all others (this is reasonable–we are most likely to remember the people who owe us money). This limits the problem of defection during rounds, but leaves open the possibility that an agent might wish to leave after completing a round. Now, a particular advantage of random roscas becomes salient. Since the ordering is re-randomized at the end of each round, no agent has an incentive to leave a random rosca between rounds. On the other hand, the last-ranked member of a fixed rosca knows that she will be last-ranked forever. Therefore, she might still wish to leave at the end of a round, which would keep her reputation intact and allow her to rejoin a new rosca with a higher expected rank. This gives us conditions under which random roscas are both more sustainable and more welfare-generating than fixed roscas.[17]

6.1 Assumptions

I study three reputation environments. Under *anonymity*, agents' past behavior is completely invisible. Under *partial reputation*, agent's histories become public to the extent that others know if they have ever defected from a rosca *during* a round. This can be thought of as a black mark that defectors acquire (this is a natural outcome if memory is costly). Finally, under *full reputation*, an agent's entire past rosca behavior is publicly known (strategies can be conditioned on whether the agent has ever left a rosca). The better the reputation environment is, the easier it is for roscas to condition strategies on agents' past behavior, thus increasing the sustainability and benefits of roscas.

I assume a large, growing population of identical hyperbolic discounters. For analytical convenience, I restrict the agents to $\beta \in \left(\beta^*, \underline{\beta}\right)$. In this region, agents value a rosca and would not save in autarky (so their actions are limited to rosca-related decisions). Finally, I assume that in any period, an infinitesimal proportion of the population experiences a shock that leaves them unable to save from the current period onwards. For the purposes of this exercise, we can

[17] This distinction between fixed and random roscas survives even if agents are exponential discounters. It should be possible to extend the results of this section to settings without time-inconsistency (of course, we would then have to also introduce some exogenous punishment for defection).

think of them as becoming fully myopic ($\beta = 0$).

6.1.1 Timing and Strategies

I assume there are two rosca "pools"–the pool of agents looking for a new rosca (pool New), and the pool of agents who wish to fill an open slot in an existing rosca (pool *Old*). The timing of the game is as follows. Each period is divided into 5 sub-periods:

a) Agents in existing roscas choose whether to stay (Y) or leave (N). Agents who are not in a rosca choose whether to move to *Old* (by default, they are in New).

b) Each existing rosca (defined as a rosca with at least 1 remaining member) with vacated slots makes rank-specific offers to agents in pool *Old*. In pool New, some proportion, p, of agents are randomly matched into groups of size k. (p is exogenously determined. I assume this is an index of how easily people are able to find each other and form groups).

c) Agents accept (Y) or reject (N) offers of membership in roscas (in either pool).

d) New roscas randomly determine the ordering. Existing random roscas randomly determine the ordering if a new round is starting.

e) Agents in roscas decide whether to stay (Y) or leave (N).

If any agent leaves a rosca or rejects an offer, she can only re-enter the pool in the following period. Any rosca that is unable to fill its slots breaks up and agents re-enter the pool in the following period.

In any period, the following actions are available to agents: those in an existing rosca observe their state (profile of other members and distance to the next nondivisible) and must choose Y or N in sub-periods (a) and (e). Agents who are not in a rosca choose whether to move to *Old* in sub-period (a). If they receive an offer, they choose Y or N in sub-period (c). Finally, in sub-period (e), agents can again choose Y or N after learning their rank.

A rosca round starts in sub-period (d) of period 1 and continues for k periods. Then, for example, under partial reputation, if an agent leaves in subperiod (c) of round 1 (before the ordering is determined), she does not acquire a reputation as a defector.

A rosca strategy is a decision about how to choose members from pool *Old* if there is an opening in the rosca (all members are aware of their rosca strategy). An equilibrium is an action associated with each information set (for individuals) and a rosca strategy for each rosca configuration, such that no agent has an incentive to deviate from her strategy at any information set.

6.2 Outside Option

The problem of sustainability is directly affected by an agent's outside option. This is defined as the expected lifetime utility from leaving a rosca in any period. Especially when available information is low, an agent might know that she has a realistic chance of leaving a rosca in which she has a low rank, and re-entering one with a higher expected rank.

Assume all roscas survive forever. Consider an agent in a fixed rosca who is k periods away from the next nondivisible. Suppose she is free to leave the rosca and re-enter any other rosca starting in the next period. If she is certain to get a new rosca ($p = 1$), clearly she prefers to leave (if not, it would violate the assumption that $\beta > \beta^*$). We would like to find conditions under which she will not leave her rosca. The agent will have a strict incentive to stay if $p < p^*$, where p^* is defined by:

$$\begin{aligned} \frac{\beta\delta^{k-1}u(b)}{1-\delta^k} &= u(y) + \beta\delta\left[\frac{p^*\left(\frac{u(b)}{k(1-\delta)}\right) + (1-p^*)\,u(y)}{1-(1-p^*)\,\delta}\right] \\ \Rightarrow \quad p^* &= \frac{(1-\delta)\left(\frac{\beta\delta^{k-1}u(b)}{1-\delta^k}\right) - (1-\delta+\beta\delta)\,u(y)}{\frac{\beta\delta u(b)}{k(1-\delta)} - \frac{\beta\delta^k u(b)}{1-\delta^k} + (\delta-\beta\delta)\,u(y)} \end{aligned} \tag{5}$$

Similarly, consider the first-ranked agent in a random rosca that survives forever. If, by leaving, she can find a new rosca with some probability p, she will only stay on in her current rosca for $p \leq p^*_{ran}$, where p^*_{ran} is given by:

$$\beta\delta^{k-1}\left(\frac{u(b)}{k(1-\delta)}\right) = u(y) + \beta\delta\left[\frac{p^*_{ran}\left(\frac{u(b)}{k(1-\delta)}\right) + (1-p^*_{ran})\,u(y)}{1-(1-p^*_{ran})\,\delta}\right] \tag{6a}$$

$$\Rightarrow \quad p^*_{ran} = \frac{(1-\delta)\left(\frac{\beta\delta^{k-1}u(b)}{k(1-\delta)}\right) - (1-\delta+\beta\delta)\,u(y)}{\frac{\beta\delta u(b)}{k(1-\delta)} - \frac{\beta\delta^k u(b)}{k(1-\delta)} + (\delta-\beta\delta)\,u(y)}$$

It follows that $p^*_{ran} < p^*$. In other words, to ensure that no one has an incentive to leave, the probability of forming new roscas needs to be relatively lower in a setting with random roscas.

6.3 Fixed Roscas

6.3.1 Anonymity

Here, I assume that strategies cannot be conditioned on any aspect of an agent's past behavior. If there is an opening in a rosca, the rosca simply decides whether to make an arbitrary offer to a person in the pool. Similarly, an agent has no information about the other members' past rosca experience.

First, consider fixed roscas. We can see that if p is low enough, there are equilibria with roscas surviving over time. Without such limits on p, every agent will have an incentive to leave her rosca immediately after receiving the nondivisible–she can consume her income today and join a new rosca in the next period.

Proposition 8 *Assume complete anonymity. Consider p^* as defined in Equation 5. Then, strategies in which agents always stay in fixed roscas constitute an equilibrium only when $p \leq p^*$.*

Proof. Suppose $p \leq p^*$. Consider the following strategy: Agents in roscas, or with rosca offers, always play Y; agents outside roscas always enter New; roscas with openings randomly make offers from Old. Then, by definition of p^*, the outside option from defection is sufficiently small that the agent who is k periods away from the next nondivisible prefers to remain in the rosca. Therefore, every group of k agents who are lucky enough to form a rosca will never separate.

Suppose $p > p^*$. By the definition of p^*, a rosca cannot survive forever in equilibrium: if all agents have a strategy of never leaving a rosca, then any individual who has just received a nondivisible has a strict incentive to deviate. ■

6.3.2 Partial Reputation

Under partial reputation, an agent develops a bad reputation if she has left during any round, which is defined as starting in the sub-period in which the ordering is randomized in period 1, and ending after round k (and continuing every k periods after that).

Now, the problems associated with anonymity are alleviated to some extent. It is possible for agents to have strategies where they refuse to join roscas with people who have left during a past round (believing that anyone who has done so is now a $\beta = 0$ type). However, such strategies cannot stop the last-ranked member of a fixed rosca from leaving at the end of a round. Since she cannot be distinguished from those who have never been in a rosca, she will have an incentive to join a new rosca if p is high enough.

Proposition 9 *Assume partial reputation. Consider p^* as defined in Equation 5. Then, strategies in which agents always stay in fixed roscas constitute an equilibrium only when $p \leq p^*$.*

Proof. (1) Assume $p \leq p^*$. Then, agents can play the same equilibrium strategy as under anonymity.

(2) Assume $p > p^*$. Roscas cannot last forever with the same membership. Since there is no strategy that can be conditioned on whether an agent left at the end of a round, the last ranked player has a strict incentive to leave after a round if she knows that, in the future, all agents will stay in a rosca forever. ■

6.3.3 Full Reputation

In this section, I assume that strategies can be conditioned on whether an agent has left a rosca in the past, and if so, at what stage of a round she left. Here, fixed roscas can be sustained even with $p = 1$. Consider the following beliefs: Any agent who has ever left a past rosca is now a $\beta = 0$ type. Consider the following strategies: a rosca with open slots accepts any people who have never left a rosca before; agents outside roscas remain in New; agents in roscas or with rosca offers always play Y unless there is a $\beta = 0$ type in the group. Since any agent who leaves a rosca must play autarky, we know that no agent will leave. The strategies described are an equilibrium, and the beliefs are justified.

6.4 Random Roscas

Following the arguments above, we can see that under full reputation and anonymity, the set of parameter values (β, p) at which there is no defection from random roscas is a subset of the values at which there is no defection from fixed roscas. Under full reputation, both random and fixed roscas will survive at any p, but this happens for a larger set of β-values in the case of fixed roscas. Under anonymity, since $p^*_{ran} < p^*$ and $\beta^*_{ran} > \beta^*$, random roscas will survive over both a larger set of β-values and a larger set of matching probabilities.

However, with partial reputation, when p is high, I find that random roscas can survive forever even when fixed roscas cannot. The intuition for this result is the following. When

$p > p^*$, there cannot be a fixed rosca in which all agents stay forever, because the last ranked member would have an incentive to leave. However, there is no such incentive to leave a random rosca between rounds. Therefore, if β is high enough so that agents *within* the rosca prefer to stay rather than play autarky, then random roscas will survive forever, and any agent's expected value from a rosca will be higher than it would be under fixed roscas (see Figure 3 in Section 7).

Proposition 10 *Assume partial reputation. If $\beta > \beta^*_{ran}$ (as defined in Equation 4), then random roscas can survive forever in equilibrium. If, in addition, $p > p^*$, then random roscas are strictly welfare improving relative to fixed roscas.*

Proof. Consider the following beliefs: any agent who has left a rosca during a past round has $\beta = 0$. Consider the following strategies: a rosca with open slots accepts any people who have never left a past rosca; individuals outside roscas enter New; agents in roscas or with rosca offers play Y only if the other members have never left a past rosca.

Suppose there is an equilibrium with random roscas. It must be the case that the first-ranked agent always prefers to remain in the rosca:

$$\begin{aligned} \frac{\beta\delta^{k-1}u(b)}{(1-\delta)k} &> u(y) + \frac{\beta\delta u(y)}{1-\delta} \\ &\Rightarrow \beta > \beta^*_{ran} \end{aligned}$$

If this condition is satisfied, then random roscas will exist forever for any value of p.

Now suppose $p > p^*$. Then, the expected value for any agent entering a rosca is: $\frac{u(b)}{k}\left(1 + \frac{\beta\delta}{1-\delta}\right)$. If fixed roscas survived forever, this would be identical to the expected value from fixed roscas. Since there is no such equilibrium, the expected value from fixed roscas must be lower. ■

These results are discussed further in the next section.

7 Empirical Implications

I now study how the results in previous sections can be related to our empirical understanding of roscas. My focus is on implications that directly related to quasi-hyperbolic discounting, including (1) comparative statics generated by the model, (2) predictions about the survival of random and fixed roscas, and (3) comparisons between durable and nondurable goods.

If hyperbolic discounting is indeed a primary explanation of rosca participation, then this model predicts that members of long-lasting roscas will exhibit intermediate levels of time-inconsistency. The actual size of the β-region within which roscas survive depends on several parameter values. All else equal, roscas are more likely to survive as the nondivisible gets more expensive relative to income. Also, roscas are more likely to survive as the utility from small units of consumption increases relative to the utility from the nondivisible (up to the point where the nondivisible is no longer valued even by exponential discounters).

In the previous section, we have seen how it is possible for roscas to survive in decentralized settings. The defection incentive in these cases is created by the option value of a higher rank in a new rosca. In environments that are completely anonymous, fixed roscas can survive forever only if matching for new roscas is sufficiently low. When there is some reputation, fixed roscas again survive when matching is sufficiently slow, but random roscas can be welfare-improving when matching probabilities are high. When roscas can access more information about an agent's past rosca behavior, equilibria with repeating fixed roscas can exist even under perfect matching (see Figure 3).

This gives us some testable predictions. If we conjecture that the likelihood of matching is positively correlated with population density, then p rises as communities get urbanized. When reputation is informal, the availability of information is likely to be inversely correlated with urbanization (full reputation is a feature of very small rural communities, while urban areas are more anonymous). Then, the model predicts that fixed roscas are more likely than random roscas at fully rural and fully urban extremes. In semi-rural communities, we are more likely to encounter conditions suited to the survival of random roscas.[18]

The Anderson, Baland (2002) study, set in an urban neighborhood, finds that a majority of roscas are fixed. The Gugerty (2005) study, set in rural Kenya, finds that a majority of roscas are random. These patterns appear consistent with my predictions, but we would require more information for an accurate analysis.

In this model, as in some earlier models, I describe the nondivisible good as yielding a one-

[18] Anderson, Baland, and Moene (2003) make predictions about participation in random and fixed roscas under individual heterogeneity. An approach that integrates their predictions with mine is probably feasible and potentially useful.

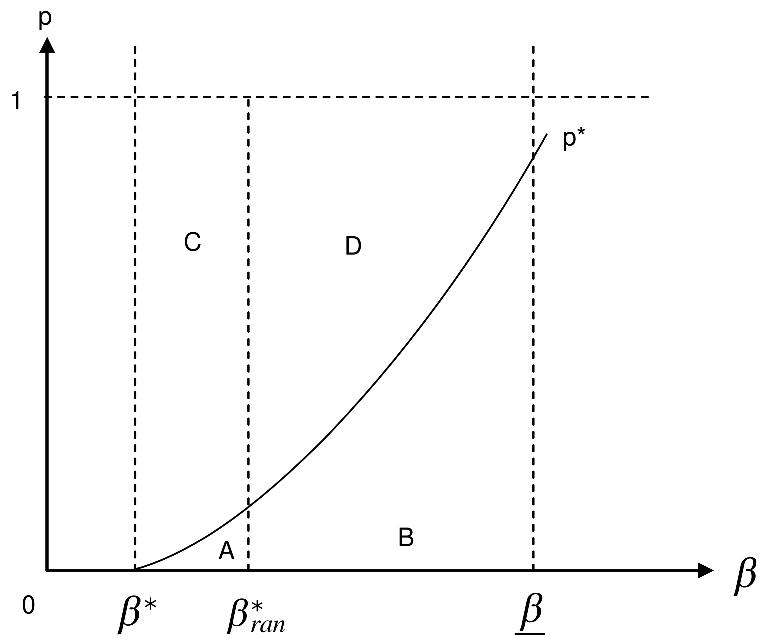

Figure 3: Partial Reputation (p is the exogenous probability of matching into new rosca groups; assume p^*_{ran} is close to p^*): In region A, only fixed roscas can survive forever. In region B, fixed and random roscas can survive forever. In region C, no roscas survive forever (if any roscas exist, they must be fixed). In region D, random roscas are more likely to survive, and provide higher welfare than any fixed rosca equilibrium.

period benefit. This is not an unreasonable assumption, since there is a range of empirical evidence suggesting that rosca members do not spend the money on durable goods. However, it is useful to identify implications for rosca survival if they save for durable goods instead. When benefits are spread across multiple periods, agents might place less value on immediate consumption. This has two implications. First, as the good becomes more durable, agents are more likely to postpone entry into roscas. Second, once an agent is actually in a rosca, the conditions under which roscas survive will expand. Since the benefit from a potentially higher rank in a new rosca is dampened, agents are more willing to remain in an existing rosca than they would if they were purchasing nondurables. This implies that, as goods get more durable, individuals may choose to delay entry into roscas but existing roscas are more likely to survive over time.

8 Conclusion

I have shown that roscas can be effective commitment savings devices even without the threat of social sanctions. Agents with self-control problems derive benefits from staying in a rosca and improving their savings behavior. A particularly useful feature of roscas is that hyperbolic discounters have no incentive to postpone take-up. Unlike other commitment savings devices that yield delayed benefits, roscas always come with the possibility that an agent might be an instant winner. The randomization of rank and subsequent commitment play complementary roles–the first draws an agent into a rosca, and the second gives her a reason to stay.

In this paper, I also highlight the relative advantages of fixed and random roscas. Within a round, agents in random roscas have a weaker incentive to stay than they would in fixed roscas. However, between rounds, agents in random roscas never have an incentive to leave, while late ranked agents in fixed roscas might prefer to leave if they can join a new rosca. Empirically, both random and fixed roscas exist in large numbers. I make predictions about the survival of each, based on the depth of reputation and the speed of matching in a community.

This paper gives us several directions for further research. Models that allow for heterogenous populations (with varying time preferences and saving objectives) are likely to alter the size and structure of roscas and conditions under which they survive in the absence of contracting. Additional insights might also be gained from endogenizing the matching speed (p) as described in Section 6. Also, there is scope for testing the predictions of the model, especially if convincing measures of time-inconsistency, reputation, and matching speed can be constructed.

However, the overall idea of roscas as commitment savings devices in environments with poor contracting seems to be realistic, and helps explain their resilience across the developing world.

9 Appendix

A Proofs for Section 4

Proof of Lemma 1. Suppose not. If at wealth w the agent saves s with positive probability, it means:

$$u(w-s)+\beta\delta V(s+y)\geq u\left(w-s'\right)+\beta\delta V\left(s'+y\right)$$

Now, if at wealth $w' > w$ the agent saves s' with positive probability, it means:

$$u\left(w'-s\right)+\beta\delta V\left(s+y\right)\leq u\left(w'-s'\right)+\beta\delta V\left(s'+y\right)$$

This implies that:

$$u\left(w-s'\right)-u\left(w-s\right)\leq u\left(w'-s'\right)-u\left(w'-s\right)$$

This violates strict concavity of u. ■

Proof of Lemma 2. Step 1:

Since $\pi_y^A(y) > 0$, we know that:

$$u\left(y\right)+\beta\delta V^A\left(y\right)\leq\beta\delta V^A\left(2y\right) \tag{7}$$

Since $\pi_y^B(y) < 1$, we know that:

$$u\left(y\right)+\beta\delta V^B\left(y\right)\geq\beta\delta V^B\left(2y\right) \tag{8}$$

By definition, we know the following is true for $\alpha = A, B$:

$$\begin{aligned} V_\alpha\left(y\right) &= \pi_y^\alpha\left(y\right)\left[\delta V_\alpha\left(2y\right)\right]+\left[1-\pi_y^\alpha\left(y\right)\right]\left[u\left(y\right)+\delta V_\alpha\left(y\right)\right] \\ V_\alpha\left(y\right) &= \frac{\pi_y^\alpha\left(y\right)\left[\delta V_\alpha\left(2y\right)\right]+\left[1-\pi_y^\alpha\left(y\right)\right]u\left(y\right)}{1-\left[1-\pi_y^\alpha\left(y\right)\right]\delta} \end{aligned} \tag{9}$$

Combining Equation 9 with Inequalities 7 and 8, respectively, we get:

$$\beta\delta\left(1-\delta\right)V_A\left(2y\right)\geq u\left(y\right)\left[1-\delta+\delta\pi_y^A\left(y\right)\left(1-\beta\right)\right]+\beta u\left(y\right)$$

$$\beta\delta\left(1-\delta\right)V_B\left(2y\right)\leq u\left(y\right)\left[1-\delta+\delta\pi_y^B\left(y\right)\left(1-\beta\right)\right]+\beta u\left(y\right)$$

Therefore:

$$V_A\left(2y\right)\geq V_B\left(2y\right) \tag{10}$$

Step 2:

(a) Suppose $\pi_y^B(y) = 0$. Then, $V_B(y) = \frac{u(y)}{1-\delta}$. Also, by Inequality 7, we know that $\delta V^A(2y) > u(y) + \delta V^A(y)$. Then, by Equation 9, we know that $V_A(y) > u(y) + \delta V_\alpha(y) \Rightarrow V_A(y) > \frac{u(y)}{1-\delta} =$

$V_B(y)$.

(b) Suppose $1 > \pi_y^A(y) \geq \pi_y^B(y) > 0$. Then, by Inequalities 7 and 8 and Inequality 10, we know that $V_A(y) \geq V_B(y)$.

(c) Suppose $\pi_y^A(y) = 1$ and $\pi_y^B(y) > 0$. Then, $V_A(y) = \delta V_A(2y)$. By Inequality, 8 we know that $\delta V_B(2y) > u(y) + \delta V_B(y)$. Then, by Equation 9, $V_B(y) < \delta V_B(2y)$. So, by 10, $V_A(y) > V_B(y)$.

Therefore:

$$V_A(y) \geq V_B(y)$$

Step 3:

Suppose there is some $\bar{w}$ such that, for all $w \leq \bar{w} < ky$, $V_A(w) \geq V_B(w)$. Then, I show that $V_A(\bar{w}+y) \geq V_B(\bar{w}+y)$.

Suppose not. Then, $V_B(\bar{w}+y) > V_A(\bar{w}+y)$. Since $\pi_y^A(y) > 0$, it must be true that $\pi_i^A(i) > 0$ for all $y \leq i \leq (k-1)y$. Then, $\beta\delta V_A(\bar{w}+y) \geq u(j) + \beta\delta V_A(\bar{w}+y-j)$ for all $y \leq j \leq \bar{w}$. It follows from our assumptions that $\beta\delta V_B(\bar{w}+y) > u(j) + \beta\delta V_B(\bar{w}+y-j)$ for all $y \leq j \leq \bar{w}$. This means that at state $\bar{w}$, the agent playing equilibrium B strictly prefers to save $\bar{w}$ over all other options. So $V_B(\bar{w}) = \delta V_B(\bar{w}+y)$. But $V_A(\bar{w}) \leq \delta V_A(\bar{w}+y)$. So $V_A(\bar{w}) < V_B(\bar{w})$. This contradicts our assumption.

The lemma is proved. ■

Proof of Lemma 3. Following Step 1 of the previous lemma, this means that $V_A(\bar{w}+y) > V_B(\bar{w}+y)$. Following Step 2 of the previous lemma, $V_A(\bar{w}) > V_B(\bar{w})$. By equation 9 and given that $\pi_w^A(w) = \pi_w^B(w)$ for $w < \bar{w}$, we know that $V_A(\bar{w}-y) > V_B(\bar{w}-y)$. Repeating this argument, we see that $V_A(w) > V_B(w)$ for all $w \leq \bar{w}+y$. Following Step 3 of the previous lemma, $V_A(w) > V_B(w)$ for all w. ■

Proof of Proposition 1. At least one Markov Perfect Equilibrium exists (Fudenberg and Tirole, 1991). Lemmas 2 and 3 give us a complete and transitive ordering over any set of equilibria. ■

For Proposition 3, I simplify the notation in the following manner: for any wealth level w, $\pi_w(w)$ is rewritten as π_w. This implicitly implies that $\pi_w(w-y) = 1 - \pi_w$ (since agents playing mixed strategy equilibria will not save less).

Proof of Proposition 3. This can be proved by applying the following algorithm to construct $\{\beta_1, \beta_2, ..., \beta_{k-1}\}$.

Step 1:

By construction:

$$u(y) + \bar{\beta}\delta V(y; 1, 1, ..., 1) = \bar{\beta}\delta V(2y; 1, 1, ..., 1)$$

And by concavity of V (Lemma 4):

$$u(y) + \bar{\beta}\delta V(iy; 1, 1, ..., 1) > \bar{\beta}\delta V((i+1)y; 1, 1, ..., 1), \text{ for } i \geq 2$$

Consider strategies $\pi = (\pi_1(\beta), 1, 1, .., 1)$ for $\beta < \bar{\beta}$, where $0 \leq \pi_1 < 1$. Then, by definition:

$$\begin{aligned} V(y; \pi) &= \frac{\pi_1 \delta V(2y; \pi) + (1 - \pi_1) u(y)}{1 - (1 - \pi_1)\delta} \\ V(2y; \pi) &= \delta^{k-2} u(b) + \delta^{k-1} V(y; \pi) \end{aligned}$$

So:

$$V(y; \pi) = \frac{\pi_1 \delta^{k-1} u(b) + (1 - \pi_1) u(y)}{1 - (1 - \pi_1)\delta - \pi_1 \delta^k}$$

It can be verified that, if the nondivisible is good, $\frac{\partial V(y;\pi)}{\partial \pi_1} > \frac{\partial V(2y;\pi)}{\partial \pi_1} > 0$.

For this strategy to be an equilibrium, the following must be true: (a) wealth y is indifferent between saving and not saving, and (b) wealth $2y$ and above strictly prefer to save. Wealth y's indifference condition is:

$$\begin{aligned} u(y) + \beta\delta V(y; \pi) &= \beta\delta V(2y; \pi) \\ &\Rightarrow u(y) + \beta\delta V(y; \pi) = \beta\delta^{k-1} u(b) + \beta\delta^k V(y; \pi) \\ &\Rightarrow V(y; \pi) = \frac{\delta^{k-1} u(b)}{\delta - \delta^k} - \frac{u(y)}{\beta(\delta - \delta^k)} \end{aligned}$$

Clearly, $\frac{\partial V(y;\pi)}{\partial \beta} > 0$. Then, for $\beta < \bar{\beta}$, the indifference condition and the definition of $V(y; \pi)$

yield a probability $\pi_1(\beta)$. Define $\underline{\underline{\beta}}$ as the lowest value of β at which indifference holds:

$$\begin{aligned}
u(y) + \underline{\underline{\beta}}\delta V(y; 0, 1, 1, ..., 1) &= \underline{\underline{\beta}}\delta V(2y; 0, 1, 1, ..., 1) \\
&\Rightarrow \underline{\underline{\beta}}\left(\delta^{k-1}u(b) + \frac{\delta^k u(y)}{1-\delta}\right) = u(y) + \underline{\underline{\beta}}\delta\frac{u(y)}{1-\delta} \\
&\Rightarrow \underline{\underline{\beta}} = \frac{u(y)}{\delta^{k-1}u(b) - \frac{(\delta-\delta^k)u(y)}{1-\delta}} \qquad (11)
\end{aligned}$$

Now, we must verify that this strategy is indeed optimal at higher levels of wealth. By concavity of V, we know that $2y$ strictly prefers to save if $\beta = \bar{\beta}$. For lower levels of β, $\pi_1(\beta)$ determines $V(2y; \pi)$ and $V(3y; \pi)$, each of which is continuous and increasing in π_1. Suppose, at $\beta = \underline{\underline{\beta}}$,

$$u(y) + \underline{\underline{\beta}}\delta V(2y; \pi) \leq \underline{\underline{\beta}}\delta V(3y; \pi)$$

Then, $\beta_1 = \beta_2 = ... = \beta_{k-1} = \underline{\underline{\beta}}$. Optimal equilibria across the entire parameter region have been characterized.

Suppose, at $\beta = \underline{\underline{\beta}}$,

$$u(y) + \underline{\underline{\beta}}\delta V(2y; \pi) > \underline{\underline{\beta}}\delta V(3y; \pi)$$

Then, since $\frac{\partial V(3y;\pi)}{\partial \pi_1} > \frac{\partial V(2y;\pi)}{\partial \pi_1} > 0$ and $V(3y; \pi) > V(2y; \pi)$, there must be a unique β_1, $\underline{\underline{\beta}} < \beta_1 < \bar{\beta}$, such that

$$u(y) + \beta_1\delta V(2y; \pi) = \beta_1\delta V(3y; \pi)$$

Below β_1, wealth 2 is no longer willing to save fully.

We have now fully characterized the optimal equilibrium for $\beta_1 \leq \beta \leq \bar{\beta}$. In this region, wealth levels 2 and above save fully. This maximizes the incentive of wealth y to save, so the equilibrium described above is indeed optimal.

Step 2:

Now, consider the case where $\underline{\underline{\beta}} < \beta_1 < \bar{\beta}$. Then, there will be some β_2 such that, for $\beta \in [\beta_2, \beta_1)$, wealth y and $2y$ will play mixed strategies while all higher levels of wealth will save. So we are considering strategies of the following form: $\pi = (\pi_1(\beta), \pi_2(\beta), 1, 1, ..., 1)$.

The indifference conditions are:

$$\begin{aligned} u(y) + \beta\delta V(y;\pi) &= \beta\delta V(2y;\pi) \\ u(y) + \beta\delta V(2y;\pi) &= \beta\delta V(3y;\pi) \end{aligned}$$

This yields:

$$V(y;\pi) = \frac{\delta^{k-2}u(b)}{\delta - \delta^{k-1}} - \frac{2u(y)}{\beta\left(\delta - \delta^{k-1}\right)} \tag{12}$$

$$V(2y;\pi) = \frac{\delta^{k-2}u(b)}{\delta - \delta^{k-1}} - \frac{u(y)}{\beta}\left[\frac{2}{\left(\delta - \delta^{k-1}\right)} - \frac{1}{\delta}\right] \tag{13}$$

Again, $\frac{\partial V(y;\pi)}{\partial \beta}, \frac{\partial V(2y;\pi)}{\partial \beta} > 0$

By definition:

$$V(y;\pi) = \frac{\pi_1 \delta V(2y;\pi) + (1-\pi_1)u(y)}{1-(1-\pi_1)\delta} \tag{14}$$

$$V(2y;\pi) = \frac{\pi_2 \delta V(3y;\pi) + (1-\pi_1)u(y)}{1-(1-\pi_1)\delta} \tag{15}$$

$$V(3y) = \delta^{k-3}u(b) + \delta^{k-2}V(y;\pi) \tag{16}$$

Substituting from equation 13 in equation 14, we get the function $\pi_1(\beta)$ that satisfies $\pi_1'(\beta) > 0$. Substituting from equation 12 in equation 16, and further substituting in equation 15, we get the function $\pi_2(\beta)$ with $\pi_2'(\beta) > 0$. Note that wealth y's indifference condition implies that it can never be the case that $\pi_1 > 0$ and $\pi_2 = 0$.

Again, following the arguments of step 1, β_2 is defined by the higher of $\pi_1(\beta) = 0$, and:

$$u(y) + \beta\delta V(3y;\pi) = \beta\delta V(4y;\pi)$$

If the first is true, then $\beta_2 = \beta_3 = ... = \beta_{k-1}$. If not, repeat the steps above as described in general below.

Step 3:

Suppose we have found β_n. For $\beta < \beta_n$, consider $\pi = (\pi_1(\beta), \pi_2(\beta), ..., \pi_{n+1}(\beta), 1, 1, ..., 1)$.

Indifference conditions, for all $i \leq n+1$, are:

$$u(y) + \beta\delta V(iy;\pi) = \beta\delta V((1+1)\,y;\pi)$$

By definition, for all $i \leq n+1$:

$$V(iy;\pi) = \frac{\pi_i \delta V((i+1)\,y;\pi) + (1-\pi_i)\,u(y)}{1-(1-\pi_i)\,\delta} \tag{17}$$

And:

$$V((n+2)\,y;\pi) = \delta^{k-n-2} u(b) + \delta^{k-n-1} V(y;\pi)$$

Substituting from the indifference conditions into Equations 17 yields functions $\pi_i(\beta)$ for all $i \leq n+1$. β_{n+1} is defined by the higher of $\pi_1(\beta) = 0$ and:

$$u(y) + \beta\delta V((n+2)\,y;\pi) = \beta\delta V((n+3)\,y;\pi)$$

If the former is true, then $\beta_{n+1} = \beta_{n+2} = ... = \beta_{k-1}$. If not, repeat this step. ■

Proof of Proposition 4. (1) By Step 1 of Proposition 3, we know that if full saving is sustainable at wealth higher than y, then $\underline{\beta} = \underline{\underline{\beta}}$. Therefore, $\underline{\underline{\beta}}$ must be a lower bound on $\underline{\beta}$ because, at $\beta < \underline{\beta}$, the agent at y will never save.

(2) Take any $\beta' \in [\underline{\underline{\beta}}, \bar{\beta})$ such that there is a saving equilibrium at β'. Then, there must be a saving equilibrium at all $\beta'' > \beta'$, since, given strategies at other states, each agent has a strictly greater incentive to save at β'' than at β' (from Proposition 3). ■

B Example for Section 4.4

Suppose the nondivisible costs $3y$. Then a full saving equilibrium exists as long as the agent at y strictly prefers to save fully:

$$\frac{\beta\delta^2 u(b)}{1-\delta^3} > u(y) + \frac{\beta\delta^3 u(b)}{1-\delta^3}$$

Once β falls low enough so that the above condition fails, the agent at y can start playing a mixed strategy, which must satisfy:

$$\beta\delta^2 u(b) + \beta\delta^3 V(y) = u(y) + \beta\delta V(y)$$

The RHS responds faster to $\pi_y(y)$ than the LHS does. If the agent at $2y$ continues to save fully, the agent at y can be kept indifferent down to some $\left(\hat{\beta}, 0\right)$, which in this case corresponds to:

$$\begin{aligned} \hat{\beta}\delta^2 u(b) + \frac{\hat{\beta}\delta^3 u(y)}{1-\delta} &= u(y) + \frac{\hat{\beta}\delta u(y)}{1-\delta} \\ &\Rightarrow \hat{\beta} = \frac{u(y)}{\delta\left[\delta u(b) - (1+\delta) u(y)\right]} \end{aligned}$$

If the agent at $2y$ is still willing to save fully at the $\left(\hat{\beta}, 0\right)$ combination, then the following condition must hold:

$$\begin{aligned} \hat{\beta}\delta u(b) + \frac{\hat{\beta}\delta^2 u(y)}{1-\delta} &\geq u(y) + \hat{\beta}\delta^2 u(b) + \frac{\hat{\beta}\delta^3 u(y)}{1-\delta} \\ &\Rightarrow \hat{\beta} \geq \frac{u(y)}{\delta\left[(1-\delta) u(b) + \delta u(y)\right]} \end{aligned}$$

For both of these conditions to be simultaneously satisfied, we must satisfy:

$$\begin{aligned} (1-\delta) u(b) + \delta u(y) &\geq \delta u(b) - (1+\delta) u(y) \\ &\Rightarrow (1+2\delta) u(y) \geq (2\delta - 1) u(b) \end{aligned}$$

which is only true if δ is sufficiently small, or if $u(y)$ is sufficiently large relative to $u(b)$.

C Mixed Strategy Equilibria

I show that the utility from autarky (from the point of view of the agent at wealth y) is weakly concave over $[\underline{\beta}, \bar{\beta}]$. Take $\beta \in \left(\beta_i, \beta_{i-1}\right)$. Recall that this is a region in which agents at wealth iy and below play a mixed strategy. Since they are all indifferent, we know that for $j \leq i$:

$$u(y) + \beta\delta V(jy) = \beta\delta V(jy + y)$$

and for $j > i$:

$$V(jy) = \delta^{k-j} u(b) + \delta^{k-j+1} V(y)$$

Combining these, we get:

$$V(y) = \frac{\delta^{k-i} u(b)}{1 - \delta^{k-i+1}}$$

which means that the lifetime utility at wealth y is: $\beta \frac{\delta^{k-i} u(b)}{1-\delta^{k-i}} - \frac{iu(y)}{1-\delta^{k-i}}$. Since i is weakly decreasing in β and $\frac{\delta^{k-i} u(b)}{1-\delta^{k-i}}$ is strictly increasing in i, lifetime utility is less responsive to β as β rises; i.e. it is weakly concave.

D The $k = 2$ Case

I solve an example in which $k = 2$. First, I provide explicit solutions for all key terms. Then, I make specific assumptions about δ, $u(y)$, and $u(b)$, and graph the resulting values.

Here, $\bar{\beta} = \left(\frac{1+\delta}{\delta}\right) \frac{u(y)}{u(b)}$ and $\underline{\beta} = \underline{\underline{\beta}} = \frac{1}{\delta} \frac{u(y)}{u(b)-u(y)}$. For $\beta \in \left(\underline{\beta}, \bar{\beta}\right)$, the agent plays a mixed strategy, where she saves with probability π at wealth y. The mixed strategy satisfies the indifference condition:

$$\beta\delta u(b) + \beta\delta^2 V = u(y) + \beta\delta V$$

$$\Rightarrow V = \frac{\beta\delta u(b) - u(y)}{\beta\delta - \beta\delta^2}$$

V is also given by,

$$V = \frac{\pi\delta u(b) + (1-\pi) u(y)}{(1-\delta)(1+\pi\delta)}$$

where

$$\pi = \frac{\beta}{1+\beta} \frac{u(b)}{u(y)} - \frac{1+\beta\delta}{\delta+\beta\delta}$$

Then, in autarky, the agent's utilities from equilibrium (at wealth y) are given by:

$$\frac{\beta\delta u(b)}{1-\delta^2}, \text{ for } \beta \geq \bar{\beta}$$
$$u(y) + \left[\frac{\beta\delta u(b) - u(y)}{1-\delta}\right], \text{ for } \beta \in \left(\underline{\beta}, \bar{\beta}\right)$$
$$u(y) + \frac{\beta\delta u(y)}{1-\delta}, \text{ for } \beta \leq \underline{\beta}$$

Now I look at roscas. The lower bound of the region in which the agent always prefers a fixed rosca is:

$$\beta^* = \frac{\left(1-\delta^2\right) u(y)}{\delta u(b) - (1+\delta)\,\delta u(y)}$$

In the region with no autarky saving, the agent always strictly prefers a random rosca if:

$$\frac{\beta\delta u(b)}{2(1-\delta)} > u(y) + \frac{\beta\delta u(y)}{1-\delta}$$

This determines β^*_{ran} (assuming δ is high enough):

$$\beta^*_{ran} = \frac{2(1-\delta)\,u(y)}{\delta\,(u(b) - 2u(y))}$$

In the region with partial saving in autarky, the agent always strictly prefers a random rosca if:

$$\frac{\beta\delta u(b)}{2(1-\delta)} > u(y) + \left[\frac{\beta\delta u(b) - u(y)}{1-\delta}\right]$$

This determines $\bar{\beta}_{ran}$:

$$\bar{\beta}_{ran} = 2\left(\frac{u(y)}{u(b)}\right)$$

Finally, under anonymity and partial reputation, fixed roscas survive forever if $p < p^*$, where:

$$p^* = \frac{(1-\delta)\left(\frac{\beta\delta u(b)}{1-\delta^2}\right) - (1-\delta+\beta\delta)\,u(y)}{\frac{\beta\delta u(b)}{2(1-\delta)} - \frac{\beta\delta^2 u(b)}{1-\delta^2} + (\delta-\beta\delta)\,u(y)}$$

Let us assume that $\delta = .95$, $u(y) = 1$, $u(b) = 2.5$. This ensures that the nondivisible is desirable to an exponential discounter. Then, $\bar{\beta} = 0.821$, $\underline{\beta} = 0.702$, $\beta^* = 0.187$. The lifetime utilities from equilibrium (at wealth y) are:

$$24.359\beta,\text{ for }\beta\geq\bar{\beta}$$

$$47.5\beta-19,\text{ for }\beta\in\left(\underline{\beta},\bar{\beta}\right)$$

$$19\beta+1,\text{ for }\beta\leq\underline{\beta}$$

The agent will always strictly prefer to stay in a random rosca if $\beta\in\left(\beta^*_{ran},\bar{\beta}_{ran}\right)=(0.211,0.8)$. (See Figure 4 for a graphical summary of these results.)

Under the assumptions above, $p^*=\frac{.05-.268\beta}{.341\beta-.95}$. In Figure 5, I plot p^* as a function of β.

E Approximations of Critical Values

To see how the critical β values relate to each other, I take first-order approximations for δ close to 1. Let $\delta=1-\varepsilon$, where ε is very close to 0. The comparative statics suggested by these approximations are discussed in Section 7.

$$\begin{aligned}\bar{\beta} &= \frac{1-\delta^k}{\delta^{k-1}-\delta^k}\left(\frac{u(y)}{u(b)}\right)=\frac{1-(1-\varepsilon)^k}{(1-\varepsilon)^{k-1}-(1-\varepsilon)^k}\left(\frac{u(y)}{u(b)}\right)\\ &\rightarrow k\left(\frac{u(y)}{u(b)}\right)\end{aligned}$$

$$\begin{aligned}\underline{\underline{\beta}} &= \frac{u(y)(1-\delta)}{(1-\delta)\delta^{k-1}u(b)-\delta u(y)\left(1-\delta^{k-1}\right)}\\ &= \frac{u(y)(1-(1-\varepsilon))}{(1-(1-\varepsilon))(1-\varepsilon)^{k-1}u(b)-(1-\varepsilon)u(y)\left(1-(1-\varepsilon)^{k-1}\right)}\\ &\rightarrow \frac{u(y)}{u(b)-(k-1)u(y)}>0\end{aligned}$$

$$\begin{aligned}\beta^* &= \frac{\left(1-\delta^k\right)(1-\delta)u(y)}{\left(\delta^{k-1}-\delta^k\right)u(b)-\left(\delta-\delta^{k+1}\right)u(y)}\\ &= \frac{\left(1-(1-\varepsilon)^k\right)(1-(1-\varepsilon))u(y)}{\left((1-\varepsilon)^{k-1}-(1-\varepsilon)^k\right)u(b)-\left((1-\varepsilon)-(1-\varepsilon)^{k+1}\right)u(y)}\\ &\rightarrow 0\end{aligned}$$

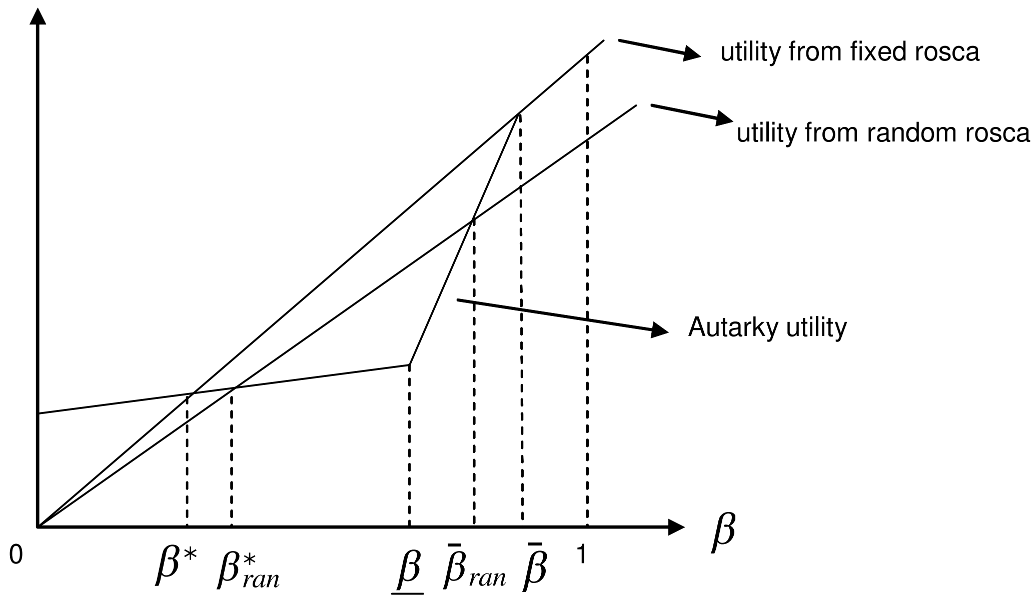

Figure 4: The lowest utility from a random rosca is lower than the lowest utility from a fixed rosca. The region in which a random rosca survives is smaller than the region in which a fixed rosca survives.

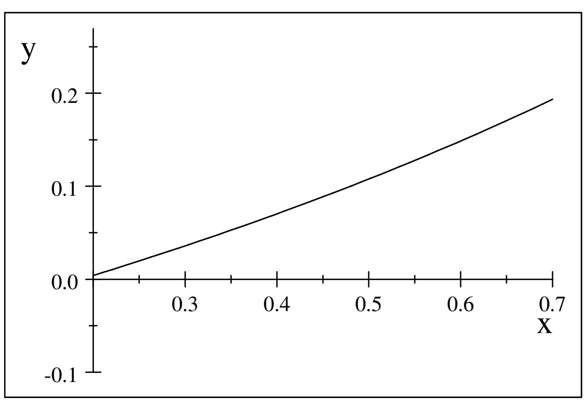

Figure 5: The x axis spans the relevant β-range. The y axis shows the corresponding value of p^*.

References

[1] AMBEC, S., and TREICH, N. (2007), "Roscas as Financial Agreements to Cope with Self-Control Problems", *Journal of Development Economics*, 82 (1), 120-137.

[2] ANDERSON, S., and BALAND, J. (2002), "The Economics of Roscas and Intrahousehold Resource Allocation", *Quarterly Journal of Economics*, 117 (3), 963-995.

[3] ANDERSON, S., BALAND, J., and MOENE, K. (2003), "Enforcement and Organizational Design in Informal Saving Groups" (working paper).

[4] ASHRAF, N., GONS, N., KARLAN, D., and YIN, W. (2003), "A Review of Commitment Savings Products in Developing Countries", *Asian Development Bank Economics and Research Department Working Paper #45*.

[5] ASHRAF, N., KARLAN, D., and YIN, W. (2006), "Tying Odysseus to the Mast: Evidence from a Commitment Savings Product in the Philippines", *Quarterly Journal of Economics* 121(2), 635-672.

[6] BERTRAND, M., KARLAN, D., MULLAINATHAN, S., SHAFIR, E., and ZINMAN, J. (2006), "What's Psychology Worth? A Field Experiment in the Consumer Credit Market", (working paper).

[7] BERTRAND, M., MULLAINATHAN, S., and SHAFIR, E. (2004), "A Behavioral Economics View of Poverty", *American Economic Review Papers and Proceedings*, 419-423.

[8] BESLEY, T. and COATE, S. (1995), "Group Lending, Repayment Incentives, and Social Collateral", *Journal of Development Economics*, 46, 1-18.

[9] BESLEY, T., COATE, S., and LOURY, G. (1993), "The Economics of Rotating Savings and Credit Associations", *American Economic Review*, 83, 792-810.

[10] BESLEY, T., COATE, S., and LOURY, G. (1994), "Rotating Savings and Credit Associations, Credit Markets and Efficiency", *Review of Economic Studies*, 61, 701-719.

[11] BOUMAN, F. (1994), "ROSCA and ASCRA: Beyond the Financial Landscape", in: F.J.A. Bouman and O. Hospes (eds.),

Financial Landscapes Reconstructed: The Fine Art of Mapping Development, Westview Press, Boulder, 375-394.

[12] BOUMAN, F. (1995), "Rotating and Accumulating Savings and Credit Associations: A Development Perspective", *World Development*, 23, 371-384.

[13] CALOMIRIS, C. and RAJARAMAN, I. (1998), "The Role of ROSCAS: Lumpy Durables or Event Insurance?", *Journal of Development Economics*, 56, 207-216.

[14] DAGNELIE, O., and LeMAY, P. (2007), "Rosca Participation in Benin: a Commitment Issue", (working paper).

[15] FUDENBERG, D., and TIROLE, J. (1991), Game Theory, The MIT Press, Cambridge, MA.

[16] GUGERTY, M. (2007), "You Can't Save Alone: Commitment in Rotating Savings and Credit Associations in Kenya", *Economic Development and Cultural Change*, 55, 251-282.

[17] HANDA, S. and KIRTON, C. (1999), "The Economics of Rotating Savings and Credit Associations: Evidence from the Jamaican 'Partner'", *Journal of Development Economics*, 60, 173-194.

[18] HARRIS, C. and LAIBSON, D. (2001), "Dynamic Choices of Hyperbolic Consumers", *Econometrica*, 69(4), 935-957.

[19] KARLAN, D. (2007), "Social Connections and Group Banking", *Economic Journal*, 117, 52-84.

[20] KIMUYU, P. (1999), "Rotating Saving and Credit Associations in Rural East Africa", *World Development*, 27, 1299-1308.

[21] KLONNER, S. (2003), "Rotating Savings and Credit Associations when Participants are Risk Averse", *International Economic Review*, 44(3), 979-1005.

[22] KLONNER, S. and RAI, A. (2006), "Adverse Selection in Credit Markets: Evidence from Bidding Roscas", (working paper).

[23] KRUSELL, P., and SMITH, A. (2003a), "Consumption-Savings Decisions with Quasi-Geometric Discounting", *Econometrica*, 71, 365-375.

[24] KRUSELL, P., and SMITH, A. (2003b), "Consumption-Savings Decisions with Quasi-Geometric Discounting: The Case with a Discrete Domain", (working paper).

[25] LAIBSON, D. (1997), "Golden Eggs and Hyperbolic Discounting", *Quarterly Journal of Economics*, 62, 443-479.

[26] LEVENSON, A., and BESLEY, T. (1996), "The Anatomy of an Informal Financial Market: Rosca Participation in Taiwan", *Journal of Development Economics*, 51, 45-68.

[27] LIGON, E., THOMAS, J., and WORRALL, T. (2002), "Informal Insurance Arrangements with Limited Commitment: Theory and Evidence from Village Economies", *Review of Economic Studies*, 69, 209-244.

[28] O'DONOGHUE, T., and RABIN, M. (1999), "Doing it Now or Later", *American Economic Review*, 89, 103-124

[29] PHELPS, E., and POLLACK, R. (1968), "On Second-Best National Saving and Game-Equilibrium Growth", *Review of Economic Studies*, 35, 185-199.

[30] THALER, R. and BERNARTZI, S. (2004), "Save More Tomorrow: Using Behavorial Economics in Increase Employee Savings." *Journal of Political Economy*, 112, 164-187.