A behavioral model of simultaneous borrowing and saving

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A Behavioral Model of Simultaneous Borrowing and Saving

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Abstract

Why do individuals borrow and save money at the same time? I present a model in which sophisticated time-inconsistent agents, when faced with a future investment opportunity, rationally choose to save their wealth and then borrow to fund the investment. The combination of savings and a loan generates incentives for future selves to invest optimally by punishing over-consumption. This paper contains two main results. First, I show that agents who simultaneously save and borrow can have higher lifetime welfare than those who don't. Second, I show that agents who have access to a non-secure savings technology can be better off than those who only have access to secure savings.

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1 Introduction

Why would an individual simultaneously save and borrow when the interest rate on saving is no higher than on borrowing? There are a number of economic explanations for such behavior . Traditional theories usually rely on the option value of savings – under risky conditions, an agent might maintain savings for use in case of an emergency (if, for example, there are transaction costs of taking a loan on short notice, or if bankruptcy laws don't require the agent to repay a loan even if there are assets in the bank). Behavioral explanations, most notably Laibson, Repetto and Tobacman (2001), focus on illiquid savings as a self-control device. Agents lock assets for future consumption while smoothing short-term consumption with high-interest credit card debt.

This paper is motivated by the environment and experience of microfinance institutions (MFIs). Morduch and Armendariz (2005) describe how one of the principle innovations of modern microfinance has been the introduction of savings technologies. Clients of microfinance are commonly encouraged to maintain savings accounts with the MFI, which can serve as a source of secondary loans for other clients. There is evidence that some borrowers save more than is required by the MFI (or necessary to secure access to future loans). Examples of this phenomenon can be found in Peru (Karlan, 2007), Bangladesh (Dehejia, Montgomery, and Morduch, 2005), and Cameroon (Baland, Guirkinger, and Mali, 2007). In Baland, Guirkinger, and Mali's (2007) data from Cameroon, savings exceed loan size for 20% of the loans. They present a model in which individuals save and borrow to disguise their wealth in the face of social pressures to donate.

In this paper, I propose an alternative explanation for simultaneous saving and borrowing. While this model is not intended to capture every motivation behind an individual's decision to save with and borrow from an MFI, it highlights an unexplored mechanism that can help us understand some of this behavior. As in Laibson, Repetto and Tobacman (2001), agents have time-inconsistent preferences, but savings serve a different purpose. I exploit the possibility that savings accounts are not entirely secure. This is a reasonable assumption in settings where clients' savings are distributed as secondary loans.[1] I show that individuals might value the uncertainty associated with savings, as this allows them to generate threats that improve the behavior of future selves. When faced with a future investment opportunity, an agent can create

[1] This is indeed the case with FINCA in Peru (Karlan, 2007).

incentives to invest by simultaneously saving and borrowing. The risk associated with saving can no longer be interpreted as an effective reduction in the interest rate; rather, it serves to magnify the penalties associated with failing to invest. This also suggests that the presence of a few unreliable borrowers can prove beneficial to others.

The implications of this model are not limited to microfinance, and I hope to make some points about commitment devices in general. A commitment device is a technology that allows an individual to restrict her future choice set or alter rewards and punishments associated with future decisions. This could, for example, take the form of a contract that stipulates future investment or a deadline for the completion of a costly task. Such technologies might be valued by a time-inconsistent agent because they encourage her future selves to make those decisions that maximize her long-run welfare.[2] An interesting implication of the model is that, even when explicit commitment is unavailable, its effects can be replicated if individuals have access to a non-secure savings instrument. In this context, what appears to be an irrational destruction of wealth is in fact the price of the commitment device.

This paper is outlined as follows. Section 2 provides an intuitive description of the model in the context of related literature. Section 3 covers the main results. Section 4 contains a discussion of comparative statics generated by the model. Section 5 concludes.

2 Description of the Model

In this section, I describe some of the intuition of the model. The goal of the model is to explain an individual's decision to make a savings and borrowing decision at the same time and in the same bank. I consider a banking structure that is broadly consistent with microfinance. Individuals can borrow money, which must be repaid by the end of a cycle. During this cycle, they can also save with the bank (at the same interest rate as borrowing). Savings remain illiquid until the end of the cycle, but can be used to repay the loan at the end. However, there is a small risk that agents who save with the bank will fail to get their deposits back.[3]

The possibility that savings are maintained for their option value is unlikely for two reasons. First, since the lender organization is the same as the borrower, it is not possible for an agent to

[2] "Welfare" is not a well-defined concept when agents have time-inconsistent preferences. I follow O'Donoghue and Rabin (1999), who evaluate welfare as the utility the agent would enjoy *if* she were a time-consistent agent.

[3] To be more precise, the agent *perceives* a risk that her savings will not mature. Alternatively, we could assume that there is a risk of delay in repayment (instead of complete default).

default on a loan while still having access to her savings in the case of a negative shock. Second, savings are illiquid during a loan cycle. Then, if an agent has a sudden need for liquidity, it is not clear that it is easier for her to access her savings than it is to simply take out a fresh loan from the bank. Since alternative banking services are very limited, it is unlikely that a savings account with an MFI would be useful as collateral for other banks.

I also suggest that, unlike in Laibson, Repetto and Tobacman (2001), agents are not using savings for their illiquidity. In their model, agents save to ensure adequate consumption in the distant future, and take out high-interest loans if they suffer negative income shocks. However, in our context, while savings are not liquid within a loan cycle, they can nevertheless be used to pay back loans at the end of a cycle. Also, agents make the decision to maintain debt and savings *simultaneously*, so the loans are not a response to an unanticipated shock.

In the following model, the agent is a quasi-hyperbolic discounter who has the opportunity to make an investment in the middle of her life.[4] While her early selves would like her to invest, she is unwilling to make the sacrifice when the opportunity presents itself. If this was simply a problem of over-consumption due to time-inconsistent preferences, the young agent would try to lock some of her wealth in an illiquid account (which would mature later in life). However, in this case, such an action can have adverse effects: first, this reduces future incentives to invest; and second, it reduces future ability to invest (if investment requires a large amount of liquid cash). The young agent's goal is to leave enough liquid assets for future investment while also creating an incentive to invest. I find that, in some cases, this can be done optimally by saving in the bank while also borrowing from it. Borrowing ensures that the agent has enough money to invest in the future. Saving creates the incentive to do so.

Saving, apart from the potential interest earned, is also a source of uncertainty. If the agent saves her assets in the bank while leaving borrowed money for her future self, that self must decide whether to (a) indulge her present-biased preferences and consume, or (b) consume less and invest. If the money was not borrowed (or if savings were entirely secure), she might choose to indulge. Now, however, indulgence becomes more costly. Since it is possible that her savings will not mature in the future, she risks defaulting on her own loans if she over-consumes today. On the other hand, if she invests, she is insuring herself against the small possibility that her savings will not mature in the future. If the punishment for default is sufficiently high, she will

[4] In particular, I focus on forms of investment that can plausibly be construed as safe, such as inventory or working capital.

choose to invest.

The young agent is effectively paying (by simultaneously saving and borrowing) for a commitment device. This commitment device generates costly punishments for "bad" behavior, thus ensuring that her future selves invest optimally. As long as savings are sufficiently (but not fully) secure, the cost of this commitment device is low enough to justify its use.

3 Model

3.1 Assumptions

There is one individual who lives for 3 periods. As shown in the timeline below, there is no consumption in period 0 but banking decisions must be made at this time. In period 1, the agent can consume but also has the opportunity to invest. In period 2, savings mature, loans are repaid, and the agent consumes her remaining assets.

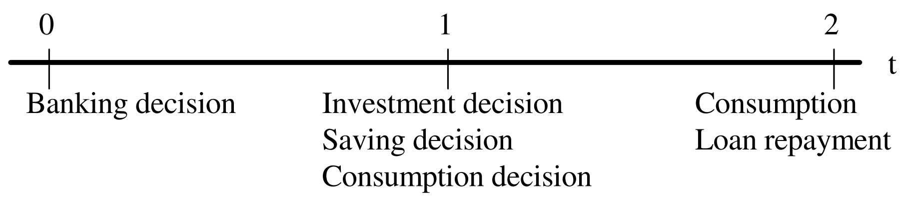

Figure 1: Timeline

The agent has an endowment w in period 0. The price of investment is $p < w$, and the monetary benefit of investment is $b > p$. Let $g = b - p$ denote the net benefit of investment. The agent has a strictly concave per-period utility function $u(\cdot)$. She is a quasi-hyperbolic discounter with $\delta = 1$ (the exponential discount factor) and $0 < \beta \leq 1$ (the hyperbolic discount factor). Following convention, I treat the individual as three independent time-indexed agents: agent 0, agent 1, and agent 2. Then, agent 0's discounted utility is $V = u(c_1) + u(c_2)$ while agent 1's discounted utility is $U = u(c_1) + \beta u(c_2)$. This automatically implies that the optimal plan from agent 1's perspective deviates from agent 0's optimal plan.

If banking services are used, savings takes place at an exogenous interest rate r ($R \equiv 1 + r$), such that x in period 0 yields Rx in period 2. If an agent saves with the bank, the savings disappear with some probability $\varepsilon > 0$. This can be interpreted as a natural outcome of intra-group saving – if information is imperfect and there are some unreliable borrowers in a group, there is a possibility that my savings will never mature. She also has the option of saving at home (these savings are fully secure, fully liquid, and earn no interest).

The agent can borrow from the bank at the same interest rate, r. The largest allowed loan is denoted $l^{\max}$.[5] If she does not repay her loan in full, her assets (amounts saved privately and in the bank) are seized and she faces an additional utility loss of F. This can be interpreted as social sanctions or the cost of restricted access to future banking services. I assume that F is large enough that it is always worth reducing consumption to avoid a risk of future default.[6] Formally, for a given ε: $u\left(Rl^{\max}\right) - u\left(0\right) < \beta\varepsilon F$.[7]

I use the following notation in the paper: c is the amount consumed in period 1, s is the amount saved with the bank in period 0, and l is the amount borrowed from the bank in period 0. Any amount neither saved in the bank nor consumed is automatically sent to the next period. Finally, I apply the following tie-breaking rule: when the agent is indifferent between investing and not investing, she always chooses to invest.

In the following sections, I use backward induction to solve for the Subgame Perfect Nash Equilibrium under different settings. In each period, the agent observes state variables (assets in each account), and makes an investment/consumption/savings decision based on the options available to her.

3.2 No Investment Opportunity

It is useful to first study the agent's behavior in the absence of the investment opportunity. If she does not have access to the bank, there is no action available to agent 0. Therefore, agent 1 will simply choose her consumption level to equalize discounted marginal utilities.

$$\max_{0 \le c \le w} u\left(c\right) + \beta u\left(w - c\right)$$

Agent 1 will choose c to satisfy $u'\left(c\right) = \beta u'\left(w - c\right)$ (or the corner solution, $c = w$).

[5] Let $l^{\max} > 2w + g$. This ensures that the maximum loan size will not pose a constraint in the analysis that follows.

[6] F does not have to constitute an explicit punishment. Basu (2008) shows that the threat of being banned from access to future loans can be sufficient to ensure that an agent always repays.

[7] I also conjecture that, with appropriate assumptions on the concavity of the utility function, the results of this paper can be derived in the absence of any punishment for default.

3.2.1 Loan

Now suppose banking is available. Agent 0 would like to improve period 2 consumption. She would therefore borrow money only if it encouraged agent 1 to raise agent 2's consumption. However, since a loan effectively destroys wealth, agent 1 will not save more than in the no-banking case. Any outcome achieved by a combination of borrowing and saving can be achieved (or improved on) by simply saving less and borrowing 0.

Proposition 1 *When there is no investment to be made, the agent will never borrow in period 0.*

The proof of this proposition is in the appendix.

3.2.2 Saving

How much should the agent save in period 0? Given that saving is risky, she might choose to save only a fraction of wealth in the bank, allowing the rest to pass through agent 1.[8] For this section, agent 0's discounted utility and agent 1's discounted utility are respectively described as the following:

- $V(s,c) = u(c) + (1-\varepsilon)\, u(w-c+rs) + (\varepsilon)\, u(w-c-s)$
- $U(s,c) = u(c) + \beta\left[(1-\varepsilon)\, u(w-c+rs) + \varepsilon u(w-c-s)\right]$

Exponential Discounter As a benchmark, I solve for the optimal savings level for an exponential discounter.

Agent 1 takes s as given and solves:

$$\max_{0\le c\le w-s} V(s,c)$$

If there is an interior solution, it is given by the first-order condition:

$$u'(c) = (1-\varepsilon)\, u'(w-c+rs) + (\varepsilon)\, u'(w-c-s) \tag{1}$$

Let $\bar{c}_{\exp}(s)$ denote the consumption level that satisfies agent 1's maximization problem..

[8] If $u'''(\cdot) > 0$, a precautionary saving motive will discourage her from saving all her wealth in the bank. If not, risk aversion can directly limit the amount to be saved in the bank, even if the interest rate is attractive.

Agent 0's problem is the following:

$$\max_{0 \le s \le w} V(s, c(s))$$

Again, for an interior solution, the first-order condition is:

$$u'(c)\,\bar{c}'_{\exp}(s) + (1-\varepsilon)\,u'(w-c+rs)\left(r - \bar{c}'_{\exp}(s)\right) + (\varepsilon)\,u'(w-c-s)\left(-1 - \bar{c}'_{\exp}(s)\right)$$

Applying the envelope theorem, this can be simplified to:

$$(1-\varepsilon)\,ru'(w-c+rs) - (\varepsilon)\,u'(w-c-s) = 0 \tag{2}$$

Conditions 1 and 2, along with the corner restrictions, provide a solution to the optimal saving problem. Let the utility-maximizing saving level be denoted $\bar{s}_{\exp}$. Observe that, if $(1-\varepsilon)\,r \le \varepsilon$, then $\bar{s}_{\exp} = 0$ and if $(1-\varepsilon)\,r > \varepsilon$, then $\bar{s}_{\exp} > 0$.[9]

Let the maximized utilities be $\bar{U}_s^{\exp} = U(\bar{s}_{\exp}, \bar{c}_{\exp}(\bar{s}_{\exp}))$ and $\bar{V}_s^{\exp} = V(\bar{s}_{\exp}, \bar{c}_{\exp}(\bar{s}_{\exp}))$.

Quasi-Hyperbolic Discounter Now, agent 1 solves:

$$\max_{0 \le s \le w} U(s, c)$$

The first-order condition for an interior solutions (if it exists) is:

$$u'(c) = \beta(1-\varepsilon)\,u'(w-c+rs) + \beta(\varepsilon)\,u'(w-c-s) \tag{3}$$

Let $\bar{c}(s)$ denote the solution to agent 1's maximization problem.

Agent 0's problem is the following:

$$\max_{0 \le s \le w} V(s, c(s))$$

Again, for an interior solution, the first-order condition is:

$$u'(c)\,\bar{c}'(s) + (1-\varepsilon)\,u'(w-c+rs)\left(r - \bar{c}'(s)\right) + (\varepsilon)\,u'(w-c-s)\left(-1 - \bar{c}'(s)\right)$$

[9] Starting at $s = 0$, agent 0 would increase saving only if this raised agent 2's utility. If $(1-\varepsilon)\,r \le \varepsilon$, then the marginal effect on agent 2's utility from a rise in s cannot be positive.

By substituting from Condition 3, this can be simplified to:

$$(1-\varepsilon)\, ru'\,(w-c+rs)-(\varepsilon)\, u'\,(w-c-s)=\bar{c}'\,(s)\, u'\,(c)\,\frac{(1-\beta)}{\beta} \tag{4}$$

Conditions 3 and 4, along with the corner restrictions, provide a solution to the optimal saving problem. Let the utility-maximizing saving level be denoted $\bar{s}$. As for the exponential discounter, $\bar{s}>0$ if $(1-\varepsilon)\, r>\varepsilon$. However, in this case, agent 0 might choose to save even if $(1-\varepsilon)\, r\leq\varepsilon$. This is because the negative effect of "burning" money (by saving) might be outweighed by reduced consumption by agent 1. Since agent 1 initially consumes too much from agent 0's perspective, this could bring marginal utilities across periods closer to each other. Let the maximized utilities be $\bar{U}_s=U\,(\bar{s},\bar{c}\,(\bar{s}))$ and $\bar{V}_s=V\,(\bar{s},\bar{c}\,(\bar{s}))$.

For the purposes of this exercise, it is convenient to restrict the interest rate so that the agent has no incentive to save if she knows the investment will be made: $R<\frac{u'(w-p-s)}{u'(b+Rs)}$. This eliminates any direct savings motive from the problem.

3.3 The Role of Investment

Suppose agent 0 wants the investment to be made in period 1. Then, for agent 1 to respond appropriately, she should have the necessary liquid cash, and sufficient incentives to invest. First, consider the case where agent 0 does not engage in banking. The only decision that remains to be made is a saving amount in period 1. If this amount exceeds p, then the investment is made. Figures 2 and 3 depict period 1 utility and period 2 utility, each as a function of period 1 consumption, c. In Figure 3, the jump occurs at the minimum savings required for investment.

The optimal outcome from agent 0's perspective will fall into one of the following three categories:

(a) $u'\,(c)=u'\,(w-c)$ (if $c>w-p$)

(b) $c=w-p$

(c) $u'\,(c)=u'\,(w-c+g)$ (if $c<w-p$)

The first outcome involves no investment, the second is a corner solution involving investment (invest, but consume as much as possible in period 1), and in the third case agent 1 not only invests but also transfers some additional cash to agent 2. If there is indeed a value of c that

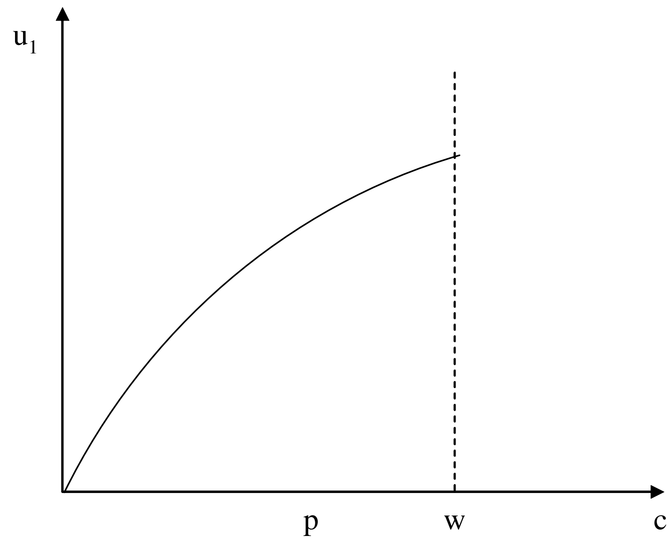

Figure 2: Period 1 utility as a function of period 1 consumption, assuming period 0 did not use banking.

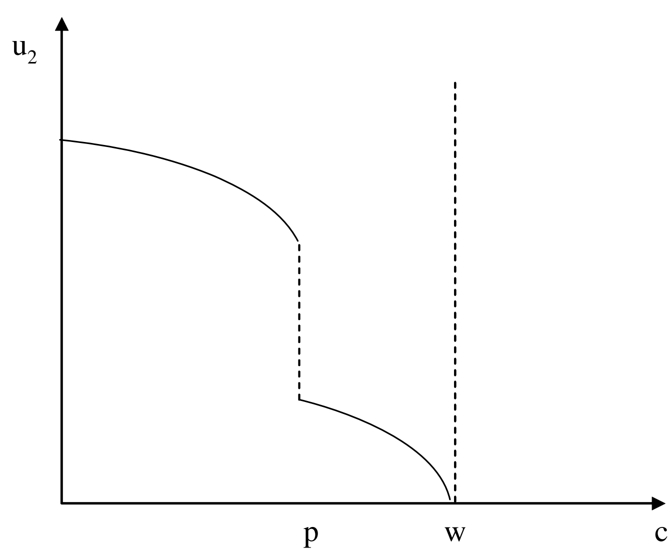

Figure 3: Period 2 utility as function of period 1 saving, assuming period 0 did not use banking.

satisfies condition (c), that must be the solution. If instead, condition (a) is satisfied, then agent 0 will compare the outcomes from condition (a) and condition (b), and choose the one that provides higher discounted utility.

Now consider the same problem from agent 1's perspective. Her analysis will be similar to the one above, except that the RHS of conditions (a) and (c) will be multiplied by β. In other words, she will have a tendency to consume more than is optimal (from the period 0 perspective). It is easy to identify the set of possible decisions in period 1 based on the optimal outcome. If the optimal category is (a), agent 1's choice must also lie in (a). If the optimal is (b), then agent 1's choice can lie in (a) or (b). Finally, if the optimal is (c), agent 1's decision can fall into any category (which will be determined by the magnitude of β).

3.3.1 Outcomes of Interest

In the following table, I divide possible outcomes into 9 categories based on the following: (1) the optimal outcome without banking (Opt); (2) agent 0's optimal banking solution, assuming agent 1 plays like an exponential discounter (Opt Bank); (3) agent 1's decision in the absence of banking; (4) agent 1's reaction to agent 0's optimal banking solution.

#	Opt	Opt Bank	Hyp	Hyperbolic's Reaction to Optimal Banking
1	(c)	No banking	(c)	Invest as before (but consume more than ideal)
2			(b)	Invest as before (but consume more than ideal)
3			(a)	Don't invest (much worse than ideal)
4	(b)	Take a loan	(b)	Invest as before, and consume the loan (ideal)
5			(a)	Invest, and consume the loan (ideal)
6				Don't invest, but consume less (much worse than ideal)
7	(a)	Take a loan	(a)	Invest, and consume the loan (ideal)
8				Don't invest, but consume less (much worse than ideal)
9		Save some	(a)	Don't invest (but consume more than ideal)

In regions 4, 5, 7, the time-inconsistency poses no problem since agent 1 behaves exactly as agent 0 would like her to. In regions 1, 2, and 9, agent 1 does not behave in an ideal manner, but there is no disagreement on the major decision of investment. I will focus on regions 3, 6 and 8. In regions 6 and 8, in particular, agent 0 faces the following problem: the ideal outcome is one in which a loan is taken and the investment made. However, at the optimal level of loan,

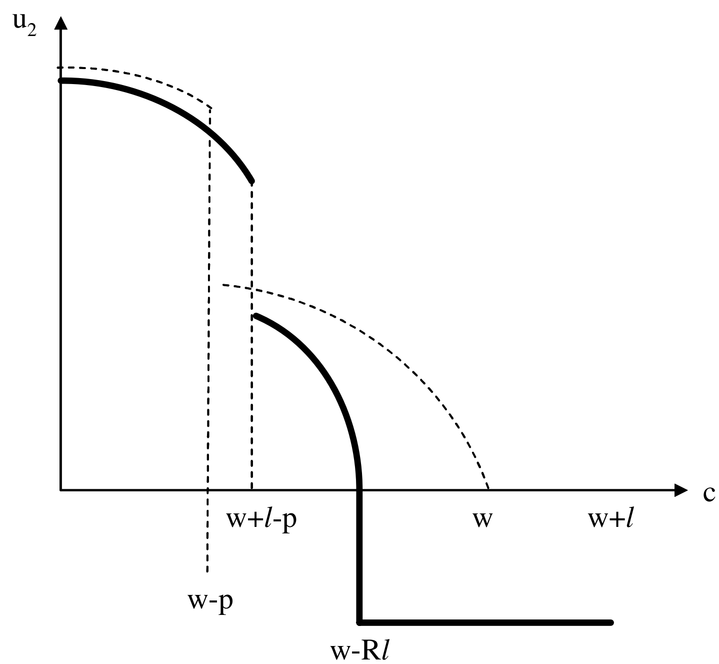

Figure 4: Period 2 utility if period 0 takes a loan. If agent 1 consumes more than $w - Rl^*$, she will default. Also, note that agent 1 can now make the investment at a higher level of c

the quasi-hyperbolic agent will not actually invest. Even though there is enough liquid cash for the investment, agent 1 does not have enough incentives to forgo current consumption for the investment.

Suppose agent 0's optimal outcome involves investment with a loan (investing is sufficiently expensive that it is worth compensating agent 1 with a loan. The optimal loan, l^*, will satisfy:

$$u'(w - p + l^*) = u'(b - Rl^*) \tag{5}$$

However, given that agent 1 is free to make her investment decision, if she is sufficiently time-inconsistent she might nevertheless not invest.

Consider the decision in period 1. The agent has wealth $w + l^*$ and must consume some amount c to maximize her discounted utility. Observe that she will reduce her consumption in response to the loan. Since a loan is effectively a cost imposed on agent 2, agent 1 will, at the very least, transfer some additional money to agent 2 to equalize discounted marginal utilities. This increases the relative attractiveness of the investment. However, if β is sufficiently low, she will still choose $c > w + l^* - p$ (This is formally described in the sections below). This is the type of agent we are going to study.

Figure 4 shows how agent 2's utility function changes (from the broken line to the unbroken line) as a result of a loan. Since a loan needs to be repaid, note that the new utility function has a steeper slope (relative to the old function) everywhere.

3.4 Inducing Investment

As l rises, agent 1's marginal benefit of saving increases (relative to the marginal cost of saving) with loan size. However, as we have seen, a loan of l^* might not be sufficient to induce investment. Then, the agent in period 0 has access to three types of actions:

1. Give up on investing and save some amount s in the bank. The maximized utilities from saving are $\bar{U}_s$ and $\bar{V}_s$.

2. Increase l so that the agent in period 1 has an incentive to invest.

3. Simultaneously save and borrow. This creates a threat for the period 1 agent – if she does not invest, there is a possibility that she will be unable to pay her loan in period 2. If F is large, this threat can create incentives to invest.

3.4.1 Pure Loan

Consider agent 1's decision as the loan size increases. She will focus on two effects. First, investment requires a lower sacrifice of current consumption (since the loan transfers consumption from period 2 to period 1). Second, the marginal utility loss for agent 2 (as a function of c) becomes higher, thus creating incentives to consume less. These effects together lead to a reduction in c, possibly up to a point where the investment becomes worthwhile.

The proposition in the next section establishes the following: if some l is large enough for the investment to be made, it will continue to be made at all larger values of l. Also, given the assumption on F, it is clear that agent 1 will choose to invest at $l \geq \frac{w}{r}$ (if not, agent 2 would default regardless of agent 1's consumption level). Therefore, agent 0 can always induce investment with a sufficiently large loan. Let the smallest such loan be denoted $\bar{l}$, which will satisfy $l^* < \bar{l} \leq \frac{w}{r}$. Agent 0's discounted utility from this loan is:

$$\bar{V}_l = u\left(w - p + \bar{l}\right) + u\left(b - R\bar{l}\right)$$

Agent 1's discounted utility is:

$$\bar{U}_l = u\left(w - p + \bar{l}\right) + \beta u\left(b - R\bar{l}\right)$$

3.4.2 Simultaneous Saving and Borrowing

In this section, I show how a combination of loans and savings can be used to induce investment. The following two propositions show that, for any target consumption c (and corresponding loan level, l), there is a function that pins down the level of "simultaneous saving and borrowing" needed to ensure investment. I also establish some convenient properties of this function. Section 3.5 provides a graphical intuition of the results below.

Let x be the amount of simultaneous borrowing and saving and l be the level of pure borrowing (so that the total loan is $l + x$ and the total saved in the bank is x). Assume $l + x \leq \frac{w+g}{r}$ (this ensures that loan repayment is feasible). Let $\hat{c}(l, x)$ be agent 1's optimal consumption level. The following two points are established in Proposition 2: As long as no investment is made, agent 1's consumption is decreasing in loan size and level of simultaneous saving and borrowing. If agent 1 decides to invest, she will continue to invest for larger levels of l or x.

Proposition 2 *Assume $l + x \leq \frac{w+g}{r}$. (a) When $\hat{c} > w + l - p$, $\hat{c}(l, x)$ decreases strictly in x, and when $\hat{c} \leq w + l - p$, $\hat{c}(l, x)$ decreases weakly in x. (b) When $\hat{c} > w + l - p$, $\hat{c}(l, x)$ decreases strictly in l, and when $\hat{c} \leq w + l - p$, an increase in l can never lead to $c > w + l - p$.*

Proof. At any level of l and x, agent 1's optimal choice of c can fall into one of the following categories: (i) $c = w - rl - Rx$ and satisfies (corner, no investment)

$$u'(c) > \beta\left[(1-\varepsilon)\, u'(w - c - rl) + (\varepsilon)\, u'(w - c - rl - Rx)\right]$$

(ii) $c \in (w + l - p, w - rl - Rx)$ and satisfies (interior, no investment)

$$u'(c) = \beta\left[(1-\varepsilon)\, u'(w - c - rl) + (\varepsilon)\, u'(w - c - rl - Rx)\right]$$

(iii) $c = w + l - p$ and (corner, investment)

$$u'(c) > \beta\left[(1-\varepsilon)\, u'(w - c + b - rl) + (\varepsilon)\, u'(w - c + b - rl - Rx)\right]$$

(iv) $c \in [0, w + l - p)$ and satisfies (interior, investment)

$$u'(c) = \beta\left[(1-\varepsilon)\, u'(w - c + b - rl) + (\varepsilon)\, u'(w - c + b - rl - Rx)\right]$$

(a) Consider the change in c in response to an increase in x in each of the categories.

(i) An increase in x must lead to a decrease in c (this is necessitated by the shift in the corner).

(ii) An increase in x raises the RHS of the equality condition. c must decrease to re-establish equality (or move to category (iii)).

(iii) An increase in x raises the RHS of the inequality. If the agent continues to invest, c must remain the same or decrease. Also, agent 1 will certainly not choose to forgo investment. Suppose she does. Then,

$$\begin{aligned} & u(c) + \beta\left[(1-\varepsilon)\, u(w - c + b - rl) + (\varepsilon)\, u(w - c + b - rl - Rx)\right] \\ < \;& u(\tilde{c}) + \beta\left[(1-\varepsilon)\, u(w - \tilde{c} - rl) + (\varepsilon)\, u(w - \tilde{c} - rl - Rx)\right] \end{aligned}$$

where $c = w + l - p$ and $\tilde{c}$ satisfies the condition described in category (i) or (ii). Now consider a marginal reduction in x. The LHS rises by less than the RHS. Then, it was not optimal to be in category (iii) in the first place. Contradiction.

(iv) Using the reasoning in category (iii), an increase in x leads to an decrease in c.

(b) Consider the change in c in response to an increase in l in each of these categories.

(i) An increase in l must lead to a decrease in c because of the shift in the corner.

(ii) An increase in l raises the RHS of the equality condition. c must decrease to re-establish equality (or move to category (iii)).

(iii) First, note that an increase in l cannot lead agent 1 to forgo investment. Suppose she now forgoes investment and consumes some $\tilde{c} > w + l - p$. Then,

$$\begin{aligned} & u(\tilde{c}) + \beta\left[(1-\varepsilon)\, u(w - c - rl) + (\varepsilon)\, u(w - c - rl - Rx)\right] \\ > \;& u(c) + \beta\left[(1-\varepsilon)\, u(w - c + b - rl) + (\varepsilon)\, u(w - c + b - rl - Rx)\right] \end{aligned}$$

where $c = w + l - p > \tilde{c}$. A marginal decrease in l leads to a larger rise in the LHS of this inequality. So it would not have been optimal for the agent to be in category (iii) in the first place. Contradiction. Now, as long as $u'(c) > \beta[(1-\varepsilon)u'(w-c+b-rl) + (\varepsilon)u'(w-c+b-rl-Rx)]$, an increase in l will lead to an increase in c. After equality is achieved, c will move inversely with l and the agent will be in category (iv).

(iv) Using the reasoning above, we know that c will move inversely with l (as far as it is feasible).

■

Proposition 3 shows that the level of simultaneous saving and borrowing required to induce investment is decreasing in loan size. Since a larger loan size makes investment less painful for agent 1, this reduces agent 0's need to provide additional inducement through simultaneous saving and borrowing.

Proposition 3 *Let $\hat{x}(l)$ be the minimum level of x required to ensure investment. Assume $l \leq \frac{w}{r}$. Then, (a) $\hat{x}(l) \leq \frac{w+rl}{r}$ and (b) $\hat{x}(l)$ is weakly decreasing in l.*

Proof. (a) By the assumption on F, agent 1 will not risk default. Therefore, if $x + l \geq \frac{w}{r}$, or $x \geq \frac{w-rl}{r}$, she will certainly invest.

(b) Consider some l and the corresponding $x(l)$. By the definition of $x(l)$,

$$\begin{aligned} & u(w-p+l) + \beta[(1-\varepsilon)u(b-Rl) + (\varepsilon)u(b-Rl-Rx)] \\ \geq \;& u(c) + \beta[(1-\varepsilon)u(w-c-rl) + (\varepsilon)u(w-c-rl-Rx)] \end{aligned} \quad (6)$$

for any $c < w - p + l$. Now, consider $\tilde{l} > l$. Suppose there is some $\tilde{c} < w - p + \tilde{l}$ such that

$$\begin{aligned} & u\left(w-p+\tilde{l}\right) + \beta\left[(1-\varepsilon)u\left(b-R\tilde{l}\right) + (\varepsilon)u\left(b-R\tilde{l}-Rx\right)\right] \\ < \;& u(\tilde{c}) + \beta\left[(1-\varepsilon)u\left(w-\tilde{c}-r\tilde{l}\right) + (\varepsilon)u\left(w-\tilde{c}-r\tilde{l}-Rx\right)\right] \end{aligned}$$

Let $\tilde{l} - l = k$. Then, by the concavity of u,

$$\begin{aligned} & u\left(w-p+\tilde{l}-k\right) + \beta\left[(1-\varepsilon)u\left(b-R\tilde{l}+Rk\right) + (\varepsilon)u\left(b-R\tilde{l}-Rx+Rk\right)\right] \\ < \;& u(\tilde{c}-k) + \beta\left[(1-\varepsilon)u\left(w-\tilde{c}-r\tilde{l}+Rk\right) + (\varepsilon)u\left(w-\tilde{c}-r\tilde{l}-Rx+Rk\right)\right] \end{aligned}$$

This contradicts Condition 6. Therefore, $x\left(\tilde{l}\right) \leq x(l)$. ■

Note that $\bar{l}$ is the lowest loan level for which $\hat{x}(l) = 0$.

3.5 Simultaneous Saving & Borrowing: Existence

The following thought experiment illustrates the role of simultaneous saving and borrowing. Suppose, starting at $s = 0$ and $l = l^*$ (the optimal level), agent 0 was to increase loan and bank saving simultaneously and by identical amounts. The period 1 utility (as a function of c) will remain unchanged. However, period 2 utility as a function of c will drop. In Figure 5, the broken line represents the base case ($s = 0$ and $l = l^*$). The solid line, depicting period 2 utility after some simultaneous borrowing an saving, is broken into four segments. The lowest segment is the region in which the agent in period 2 will certainly default. The second-lowest segment is the region in which agent 2 might default (if the savings do not mature). The second-highest segment is the case where there is no chance of default, and the investment is not made. The uppermost segment describes the case where the investment is made in period 1.

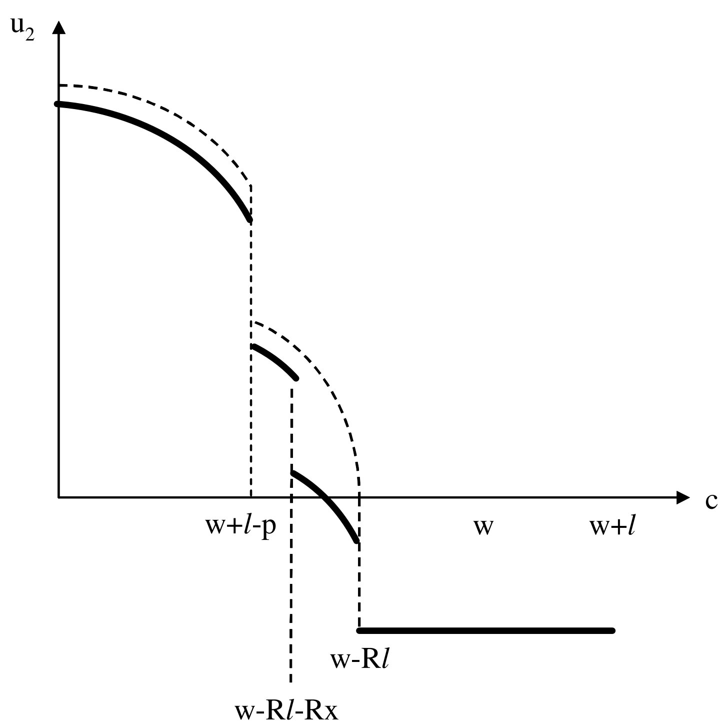

Figure 5: Period 2 utility as agent 0 raises loan and savings simultaneously

Suppose, at $s = 0$ and $l = l^*$, agent 1 chooses some amount c that satisfies $w + l - p < c < w - Rl - Rx$. In this case, she is avoiding default but not making the investment. Suppose agent

0 saves more *and* borrows more. Now, the region in which default is possible expands. If agent 1 continues to save her original amount, she might face punishment in period 2. To prevent this, she must raise the amount she saves. Here, it can become optimal for the agent to invest. Note that if ε is low, the final outcome is close to the optimal outcome (from the perspective of period 0).

In this case, saving creates the incentive to invest, while borrowing is used to actually fund the investment.

Proposition 4 *There is a parameter region over $(r, \beta, F, \varepsilon)$ in which the agent will save $s > 0$ and borrow $l > 0$ in period 0.*

Proof. If r is sufficiently low, the optimal outcome involves a loan, $l^* > 0$. Let agent 0's optimal discounted utility be denoted V^*:

$$V^* = u\left(w - p + l^*\right) + u\left(b - Rl^*\right)$$

Recall that the optimal no-investment outcome is denoted $\bar{V}_s$. For any r, an upper bound on $\bar{V}_s$ exists and can be found by setting $\varepsilon = 0$. Therefore, if r is sufficiently low (*regardless* of ε), the optimal investment outcome is strictly preferred to the best achievable no-investment outcome.

$$V^* > \bar{V}_s$$

Given r, if β is sufficiently low, the optimal investment outcome is strictly preferred to the best achievable investment outcome.

$$V^* > \bar{V}_l$$

Let D be the difference between the optimal outcome and the best possible outcome from a pure loan or pure saving:

$$D = V^* - \max\left\{\bar{V}_l, \bar{V}_s\right\} > 0$$

Since there is an upper bound on $\bar{V}_s$ that does not depend on ε, there must be a lower bound on D that does not depend on ε. Let this value be denoted $D_{\min}$. If r is sufficiently low, and if given r, β is sufficiently low, $D_{\min} > 0$.

For any target l, let the period 0 discounted utility from simultaneous saving and borrowing be denoted $V_{sb}(l)$. By construction,

$$V_{sb}(l) = [u(w - p + l) + (1 - \varepsilon) u(b - Rl) + (\varepsilon) u(b - Rl - R\hat{x}(l))]$$

Let the optimal value of $V_{sb}(l)$ be $\bar{V}_{sb}$.

$$\bar{V}_{sb} = \max_{l \geq 0} V_{sb}(l)$$

Since $\hat{x}(l)$ is bounded above (Proposition 6), $\lim_{\varepsilon \to 0} V_{sb}(l) = u(w - p + l) + u(b - Rl)$. Therefore, if ε is sufficiently low, $V^* - \bar{V}_{sb} < D_{\min}$.

Finally, given ε, assume F is large enough (re-apply the previous assumption on F):

$$u(Rl^{\max}) - u(0) < \beta\varepsilon F$$

Under these conditions, agent 0 strictly prefers simultaneous saving and borrowing to pure saving and pure borrowing. ■

The actual optimal point will not involve pinning agent 1 to the original optimal consumption. Suppose agent 0 chose $l = l^*$ and the corresponding $\hat{l}(x^*)$. The marginal utility benefit (to agent 1) and marginal utility cost (to agent 2) from raising l are, respectively:

$$u'(w - p + l^*)$$

$$(1 - \varepsilon) Ru'(b - Rl^*) + (\varepsilon) R \left(1 + \hat{x}'(l^*)\right) u'(b - Rl^* - R\hat{x}(l^*))$$

If $\hat{x}$ did not vary with l, agent 0 would clearly like to reduce the loan size and transfer some consumption to agent 2 (this follows from the definition of l^*). Whether she actually does so depends on the magnitude of $\hat{x}'(l)$. If $\hat{x}'(l)$ has a large magnitude (i.e. it is costly to induce investment – $\hat{x}'(l)$ is close to -1), then agent 0 will in fact choose to raise l. If $\hat{x}'(l)$ has a small magnitude, agent 0 might lower l.

If agent 0 lowers l, she must raise x to maintain incentives for agent 1 to invest. This lowers the wealth available in period 1, which results in a lowering of c and raising of agent 2's consumption level. While this brings the marginal utilities of consumption between the two periods closer to each other, it also lowers the total amount of wealth to be shared. Alternatively, consider a rise in l and a drop in x. While this raises c and lowers agent 2's consumption (thus pushing marginal utilities further apart), the change in c is greater than the change in agent 2's consumption. The relative strengths of this trade-off will determine the direction in which the actual l will deviate from l^*.

It is also useful to note here that if, instead, the individual only had access to secure savings (at a lower interest rate), then she would never choose to borrow and save simultaneously. In this case, it is impossible for agent 0 to create a discontinuity in period 2 utility that gets exacerbated if agent 1 over-consumes. To induce her future self to save, she will have to create incentives by lowering the relative marginal cost of saving in period 1. Rather than use the costly device of saving *and* borrowing, she will simply borrow to the point where agent 1 is willing to save. This is because, in either case, agent 0 must appeal to the period 1 agent's incentive to invest without a new threat being created. To see this, consider any loan-savings combination that induces investment. As shown in the proposition below, agent 0 can reduce both loan and savings in such a way that total wealth rises (less money is burned), and the benefits accrue to agent 1. If agent1 has money at her disposal, her incentive to invest remains intact. Thus, investment continues to happen with less money wasted due to simultaneous saving and borrowing.

Proposition 5 *Suppose the agent can borrow at interest rate r ($R \equiv 1 + r$) and can save at interest rate t ($T = 1 + t$), where $r > t$. Then she will never save and borrow simultaneously.*

The proof of this proposition is in the appendix.

4 Comparative Statics and Welfare

The propositions of the previous section suggest some natural comparative statics. I look at variation in the four (potentially) observable exogenous variables – r, β, ε, F – and examine their impact on the values associated with pure saving, pure borrowing, and simultaneous borrowing and saving. To recap Proposition 4: (a) If r is sufficiently low, then agent 0 prefers the optimal investment outcome (V^*) to the pure saving (no-investment) outcome ($\bar{V}_s$); (b) If β is sufficiently low, then agent 0 prefers V^* to the pure-loan (investment) outcome ($\bar{V}_l$); (c) If ε is sufficiently low (but above 0), then agent 0 prefers the simultaneous borrowing and saving (investment) outcome ($\bar{V}_{sb}$) to $\bar{V}_s$ and $\bar{V}_l$; (d) If F is sufficiently high, then the assumption (in parts a, b, and c) that the agent will always avoid the possibility of future default (if possible) indeed holds true.

We can think of F varying both at the level of the bank and at the level of the individual. As F drops, agent 0's ability to use simultaneous saving and borrowing as a threat initially stays constant and then abruptly declines. Once F drops sufficiently low, it is not even in agent 0's interest to have a loan repaid. Hence, we expect that clients of banks with weaker default penalties or individuals that are less susceptible to punishment will be less likely to engage in simultaneous saving and borrowing.

It is common for microfinance institutions to assign clients to groups before issuing loans. If internal savings takes place within such groups, the probability of savings not maturing, ε, is likely to vary across groups. When $\varepsilon = 0$, simultaneous saving and borrowing cannot happen. As ε rises, the initial effect is ambiguous: the condition on F becomes easier to satisfy, thus increasing the likelihood of simultaneous behavior; but the relative attractiveness of $\bar{V}_l$ compared to $\bar{V}_{sb}$ rises. Ultimately, ε becomes too high to justify the utility cost of saving any money.

As β rises, $\bar{V}_l$ rises and approaches $\bar{V}_{sb}$. However, this also makes it easier for the threat of punishment to have bite. So, for low values of β, the effect of a rise is ambiguous, but ultimately the pure loan option dominates and simultaneous saving and borrowing disappears.

Finally, consider the effect of changes in the interest rate. As we have seen, an increase in r from very low levels leads to a rise in $\bar{V}_s$ and a drop in $\bar{V}_l$ and $\bar{V}_{sb}$. Also, this leads to a tightening on the conditions for ε and β, with no change in the condition for F. Therefore, the likelihood of simultaneous saving and borrowing declines in r.

The model also provides some interesting welfare implications that are directly linked to the time inconsistent preferences of the agent. Following convention, we can think of welfare as agent 0's discounted lifetime utility. We can see that rises in ε and F, both of which are associated with utility reductions for an exponential discounter, can raise the quasi-hyperbolic agent's utility. This is because a positive value of ε and a high value of F allow agent 0 to construct a commitment device for agent 1. This is impossible to do if $\varepsilon = 0$ or F very low.

5 Conclusion

I have attempted to solve a puzzle of simultaneous borrowing and saving by providing a new rationale for the phenomenon. When agents are sophisticated quasi-hyperbolic discounters, access to a non-secure source of saving can be useful – by creating the threat of a large punishment in the event of default, the agent can induce her future selves to invest. Actual utility loss in equilibrium is limited if the probability of default is low.

I have shown that, in this setting, simultaneous borrowing and saving cannot be optimal if agents have time-consistent preferences. I have also shown that, if savings are secure, an interest rate differential cannot explain this behavior. The agent is always better off when she simply borrows to fund investment. Furthermore, when there is a small chance that savings will disappear, the agent can find herself better off than if savings mature with certainty.

6 Appendix

A Additional Proofs

Statement of Proposition 1: When there is no investment to be made, the agent will never borrow in period 0.

Proof. Suppose agent 0 saves any amount s and borrows $l > 0$. The period 1 wealth is $w - s + l$. Agent 1 will consume some $0 \leq c^* \leq w - s - rl$ (since she will avoid default in the bad state in period 2). From the period 0 perspective, lifetime utility is:

$$u(c^*) + (1 - \varepsilon) u(w - c^* + rs - rl) + \varepsilon u(w - c^* - s - rl) \tag{7}$$

Now, consider the same s as above, but change the loan to $\hat{l} = 0$. The period 1 wealth is now $w - s$. Consider the following consumption period 1: $\hat{c} = c^* + rl$. This is an outcome where the agent in 1 consumes what was previously interest on the loan, while leaving period 2 consumption unchanged. This gives us a lower bound on welfare from agent 0's perspective (if the agent in 1 deviates from this plan, it will be to transfer more consumption to period 2).

Agent 0's discounted utility is bounded below by:

$$u(c + rl) + (1 - \varepsilon) u(w - c^* + rs - rl) + \varepsilon u(w - c^* - s - rl) \tag{8}$$

The utility in 8 is strictly higher than the utility in 7. ■

Statement of Proposition 5: Suppose the agent can borrow at interest rate r ($R \equiv 1 + r$) and can save at interest rate t ($T = 1 + t$), where $r > t$. Then she will never save and borrow simultaneously.

Proof. Agent 0 will either plan for the investment to be made, or not. If the investment is not made, clearly the optimal strategy is to save some amount s such that:

$$u'(w-s) = u'(Ts)$$

In this case, no loan will be taken.

If the agent 0 takes a loan, it must be to induce investment in period 1. Suppose the agent borrows $l > 0$ and saves $s > 0$. It must be the case that the investment is made in period 1. The utility from period 0 perspective (where s_1 is the amount saved in period 1) is:

$$u(w - s + l - s_1) + u(b + s_1 - p + Ts - Rl)$$

Now suppose we lower l and s such that $\Delta s = \frac{R}{T}\Delta l$. The period 1 incentive to save rises. Utility from the period 0 perspective must go up. ∎

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