Liquidity, Financial Intermediation, and Monetary Policy in a New Monetarist Model

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Abstract

A model of monetary exchange with private financial intermediation is constructed. Claims on financial intermedaries of two types are traded in transactions: circulating notes and deposits. There can be a role for the government in supplying liqudity, and level changes in the money supply accomplished through open market operations can be nonneutral. A Friedman rule is suboptimal, due to costs of maintaining the stock of currency. The model is used to address some issues related to current monetary policy in the United States.

1 Introduction

Liquidity consists of a class of assets that are somehow useful in exchange. Some of these liquid assets are government liabilities. In the United States, for example, Federal Reserve notes serve as a medium of exchange in retail transactions, deposits with the Federal Reserve are used as a medium of exchange in interbank transactions, and Treasury bills play an important role as collateral in financial transactions. As well, there are liquid assets that are the liabilities of private financial intermediaries, or the products of these intermediaries. Banks issue deposit liabilities which can be traded using debit cards and checks, and securitized loans can be traded on financial markets or can serve as collateral in financial transactions. Further, in the past, for example in the United States prior to the Civil War, in Scotland in the early 19th century, and in Canada before 1935, private banks could issue small-denomination liabilities that circulated, sometimes alongside government-issued currency. To further complicate matters, private financial intermediaries that issue liquid liabilities also hold liquid government liabilities as assets.

Conventional wisdom holds that the main role of a central bank is to manage public liquidity in a manner that controls inflation, and influences the provision of private liquidity and credit. However, the mechanism by which central bank actions affect prices and quantities still appears to be poorly understood. As evidence for this, consider the dramatic actions taken by the Federal Reserve System during 2008 and 2009, and the reaction of economists to these actions. There is considerable disagreement about the implications and appropriateness of these actions, both inside and outside of the Federal Reserve System. How do these dramatic interventions matter for inflation and for real activity?

The main purpose of this paper is to build a model of public and private liquidity to answer some basic questions in monetary economics. What is the role of a central bank? What is a liquid asset, and what roles do privately-provided and publicly-provided liquid assets play in exchange? Do open market operations matter, and if so, why? How should we evaluate the recent actions of the Federal Reserve System?

The model here builds on the basic New Monetarist framework of Lagos and Wright (2005) (see also Williamson and Wright 2009). The model adds a financial intermediation sector, based on a costly state verification friction, as in Williamson (1987), and on risk-sharing, as in Diamond-Dybvig (1983). One attraction of this type of intermediation model is that it yields a rich structure of financial arrangements, including debt contracts and endogenous default premia. As well, the model yields coexistence of deposit claims and currency, and determines liquidity premia on tradeable assets, along the lines of Lagos (2008), Lester, Postlewaite, and Wright (2009), and Lagos and Rocheteau (2008).

The first step in the analysis is to consider equilibria without a government. In such an environment, private financial intermediaries issue two types of tradeable claims: circulating notes and deposits. These two types of intermediary liabilities exploit the available information technologies for making transactions.

Under some conditions, private financial intermediaries will not provide sufficient liquid assets in equilibrium to achieve efficiency in exchange, and the economy will have too much private financial intermediation in equilibrium. These are circumstances where the government can do better.

We model the government as having the ability to tax, to ban private circulating notes, and to issue two types of liabilities: nominal bonds and outside money. In the policy regime we study, fiscal policy governs the rate of growth in the total quantity of nominal government liabilities, while monetary policy determines the composition of the nominal debt. Under conditions such that the private sector fails to supply sufficient liquidity, there always exists a government policy which achieves an optimal allocation of resources. The government's outside assets and power to tax permit it to correct the "insufficient liquidity" problem, but the critical source of liquidity turns out to be bonds, rather than money.

In a regime where there is insufficient liquidity, a one-time open market purchase is not neutral. Holding constant the rate of growth in total nominal government debt, if there is a one-time increase in the ratio of money to bonds, this permanently reduces the real interest rate, and increases the quantity of lending - there is a type of "credit channel" effect of monetary policy. Thus, in this regime the central bank may view itself as a powerful institution with the ability to manipulate GDP at will. However, if the government were setting policy optimally, the central bank would not have this power - monetary policy is essentially neutral at the optimum.

There are at least two ways for the government to achieve optimality. First, it could ban circulating private notes, increase the total nominal debt at the appropriate rate, and provide a sufficiently large ratio of bonds to money to achieve efficiency. Second, it could issue sufficient government debt to achieve efficiency, and permit unrestricted issue of private circulating notes. Thus, in the model, central banking is not critical to achieving efficiency.

An important result is that a Friedman rule is not optimal. This follows from the fact that it is costly to maintain the stock of currency. That is, there are costs associated with replacing worn-out currency and designing the currency to prevent counterfeiting, that result in a deviation from the Friedman rule. Just as in Sanches and Williamson (2008), where we consider the role of theft as a cost of operating a monetary system, the higher are the costs of operating the monetary system, the larger will be the deviation from the Friedman rule.

An interesting application of the model is to current monetary policy in the United States, as the model permits us to study a wide range of central bank interventions. We first consider a monetary policy under which the nominal interest rate on government debt is zero temporarily, with the expectation that the policy will be "unwound" in the future. Given the zero nominal interest rate, if the central bank injects more outside money in the present than is necessary to achieve a zero nominal interest rate, then this outside money will be hoarded as bank reserves, with no effect on any prices or quantities. Essentially, there is a liquidity trap. However, the government could act to issue outside money in the present to finance private lending, using the returns on its portfolio to retire the money in the future. If the government lends on the same terms as do private sector intermediaries, then its lending will simply displace an equal quantity of private lending, and the stock of outside money can increase by a large amount in the present with no effects on quantities and prices. However, if the government lends on better terms than does the private sector, this will reduce nominal interest rates on loans, and expand lending. In this case, however, the returns on the central bank portfolio are insufficient to unwind the monetary injection, and taxes must be levied to retire the additional outside money in the future. This causes a redistribution from taxpayers to borrowers. If the central bank lends only in a segment of the credit market, on favorable terms, then this serves to reallocate credit. Borrowers who are not on the receiving end of central bank lending will face higher interest rates, and private lending contracts.

The results here cast the current "quantitative easing" policies of the Fed in both a positive and a negative light. On the positive side, provided that large current monetary injections are expected to be undone in the future, it is possible to have no inflationary consequences from a large expansion in the Federal Reserve balance sheet. On the negative side, once the nominal interest rate is zero, central bank intervention is at best of no consequence, and at worst has important implications for the distribution of credit and wealth.

The remainder of the paper is organized as follows. The second section is a description of the model, while the third section contains the details of the intermediary structure. Then, Section 4 contains an analysis of equilibrium without government, while the Section 5 analysis concerns equilibrium with a government. Finally, Section 6 applies the model to issues concerning recent monetary policy in the United States, and Section 7 is a conclusion.

2 The Model

The basic model builds on Lagos-Wright (2005), with an information structure related to Sanches and Williamson (2009), and a financial intermediation sector similar to Williamson (1987). Time is indexed by $t = 0, 1, 2, ...$, and there are two subperiods within each period that we denote day and night.

The population consists of three types of economic agents: buyers, sellers, and entrepreneurs. There is a continuum of buyers with mass one, and each buyer has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t [-H_t + u(x_t)],$$

Here, $0 < \beta < 1$, H_t denotes the difference between labor supply and consumption during the day, x_t is consumption in the night, and $u(\cdot)$ is a strictly increasing, strictly concave, and twice continuously differentiable function with $u(0) = 0$, $u'(0) = \infty$, and with the property that there exists some $\hat{q} > 0$ such that $u(\hat{q}) - \hat{q} = 0$. Define q^* by $u'(q^*) = 1$. There is a continuum of sellers with unit mass, and each seller has preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t [X_t - h_t],$$

where X_t is consumption in the day and h_t is labor supply in the night. The production technology potentially available to buyers and sellers allows the production of one unit of the perishable consumption good for each unit of labor supply. Buyers can produce only in the day, and sellers only in the night, so we have one of the necessary ingredients for monetary exchange - a double coincidence problem.

During the day of each period, a continuum of entrepreneurs with mass α is born, and each lives until the day of the following period. An entrepreneur has no endowment during his or her life. An entrepreneur born in the day of period t consumes only in the day of period $t+1$ and is risk neutral. An entrepreneur has access to an investment project. This project is indivisible and requires one unit of the consumption good in the day of period t to operate, and yields a return of w in the day of period $t+1$, where w is distributed according to the distribution function $F(w)$, with associated density function $f(w)$, which is strictly positive on $[0, \bar{w}]$, where $\bar{w} > 0$. Assume also that $f(\cdot)$ is continuously differentiable. Investment project returns are independent across entrepreneurs. The return w is private information to the entrepreneur, but subject to costly state verification, whereby any other individual can bear a fixed cost and observe w ex post. The verification cost γ is entrepreneur-specific, and $G(\gamma)$ denotes the distribution of verification costs across entrepreneurs, with $\gamma \geq 0$.

During the night, each buyer is matched at random with a seller. The seller in a match is not able to observe the buyer's history, and the seller will never have an opportunity to signal default on a credit arrangement, so the seller will not accept a personal IOU in exchange for goods. A fraction ρ of nighttime bilateral meetings are *not monitored*, in the sense that, if the buyer wants to acquire goods from the seller, he or she must have a claim to goods in the next day, where the claim is somehow documented in an object that the buyer carries. In what follows, these objects may be circulating liabilities issued by financial intermediaries or fiat money issued by the government. In each case, the issuer of the object must incur a cost to render the object recognizable for what it is. We will specify this in more detail in what follows. A fraction $1-\rho$ of buyers and sellers are in *monitored* meetings at night. In these meetings, though a credit transaction cannot take place between the buyer and the seller, a communication technology is costlessly available which permits the buyer to transfer ownership of a claim on a financial intermediary to the seller. Again, we will spell out more details in what follows.

An important feature of the environment is that, when production and consumption decisions are made during the day, buyers do not know whether they will be in a non-monitored or a monitored meeting with a seller in the following night. Each buyer learns this information, which then becomes public, at the end of the day. This will give rise to a Diamond-Dybvig (1983) risk-sharing role for financial intermediaries, in addition to the costly-state-verification/delegated-monitoring role for intermediation that also exists here.

3 Financial Intermediation

We will assume that stochastic verification is not feasible. Assume that entrepreneurs are economic agents who are subject to full commitment. Then, as in Williamson (1987), an efficient lending arrangement is for individual entrepreneurs to act as perfectly-diversified financial intermediaries. Efficient and incentive-compatible loan contracts with entrepreneurs take the form of non-contingent debt. That is, the financial intermediary observes the verification cost γ associated with the entrepreneur in the daytime of period t and offers him or her a contract that specifies a non-contingent payment $R_t(\gamma)$ that the entrepreneur must make to the intermediary in the day of period $t+1$. If the entrepreneur cannot make the loan payment, then default occurs, the intermediary incurs the verification cost γ, observes the return w, and seizes it. As shown in Williamson (1987), the expected payoff to the intermediary from the loan contract, as a function of the non-contingent payment R and the verification cost γ, is then given by

$$\pi(R,\gamma) = R - \gamma F(R) - \int_0^R F(w)dw \tag{1}$$

Then, letting $R_t(\gamma)$ denote the gross real loan interest rate on a loan to an entrepreneur of type γ, equation (1) allows us to define the default premium faced by an entrepreneur of type γ, which is

$$D_t(\gamma) = \gamma F[R_t(\gamma)] + \int_0^{R_t(\gamma)} F(w)dw. \tag{2}$$

Since the financial intermediary is perfectly diversified (this requires only that it hold a positive mass of loans to entrepreneurs), it can guarantee a certain return per unit invested, which we denote r_t. In equilibrium, the payoff per unit invested will be the same for each loan made by the financial intermediary, so

$$r_t = R_t(\gamma) - \gamma F[R_t(\gamma)] - \int_0^{R_t(\gamma)} F(w)dw \tag{3}$$

for each entrepreneur who receives a loan. Differentiating the intermediary's expected payoff function in (1), we obtain

$$\pi_1(R,\gamma) = 1 - \gamma f(R) - F(R),$$
$$\pi_{11}(R,\gamma) = -\gamma f'(R) - f(R)$$

and assume that $-\gamma f'(w) - f(w) < 0$ for all $w \in [0, \bar{w}]$ and for all $\gamma \geq 0$. Then $\pi(R,\gamma)$ is strictly concave in R for $R \in [0,\bar{w}]$ and attains a maximum for $R = \hat{R}(\gamma) < \bar{w}$, where

$$1 - \gamma f\left[\hat{R}(\gamma)\right] - F\left[\hat{R}(\gamma)\right] = 0. \tag{4}$$

In equilibrium, there is a marginal entrepreneur in each period t, with verification cost γ_t^* and facing the gross loan interest rate R_t^*, where, from (4),

$$1 - \gamma_t^* f\left(R_t^*\right) - F\left(R_t^*\right) = 0, \tag{5}$$

so that the gross loan interest rate faced by the marginal entrepreneur maximizes the expected return to the financial intermediary given the marginal entrepreneur's verification cost γ_t^*. Further, the financial intermediary earns an expected return r_t from lending to the marginal borrower, or from (3),

$$r_t = R_t^* - \gamma_t^* F\left(R_t^*\right) - \int_0^{R_t^*} F(w)dw \tag{6}$$

Then, each entrepreneur who receives a loan has $\gamma \leq \gamma_t^*$, and if $\gamma < \gamma_t^*$ then $R_t(\gamma) < \hat{R}(\gamma)$. Entrepreneurs with $\gamma > \gamma_t^*$ do not receive loans as, even if $R_t(\gamma) = \hat{R}(\gamma)$ for one of these agents, so that the expected return to the intermediary is maximized, the intermediary will have an expected loss from the loan. That is, verification costs for the set of entrepreneurs with $\gamma > \gamma_t^*$ are too high for lending to be profitable.

Then, the total quantity of loans extended by financial intermediaries during the day of period t is given by

$$L_t = \alpha G(\gamma_t^*). \tag{7}$$

Therefore, given the certain return on investment r_t, the loan quantity L_t, the verification cost of the marginal borrower, γ_t^*, and the gross loan interest rate faced by the marginal borrower R_t^*, are determined by (5), (6), and (7). It is straightforward to show that

$$\frac{dL_t}{dr_t} = -\frac{\alpha G'(\gamma_t^*)}{F\left(R_t^*\right)} < 0 \tag{8}$$

Thus, given an increase in the payoff per unit of lending by the financial intermediary, the quantity of lending by financial intermediaries must decline. This is because, from (3), the loan interest rate for each entrepreneur receiving a loan will increase, and it will be unprofitable for an intermediary to lend to a formerly marginal entrepreneur, once the payoff per unit of loans increases. Further, since the loan interest rate will be higher for each creditworthy entrepreneur when the deposit rate increases, from (2) each of these creditworthy entrepreneurs will be faced with a higher default premium.

Given (8), we can write $L_t = L(r_t)$, where $L(\cdot)$ is a decreasing function. There is some rich detail in how entrepreneurs' investment projects are funded and the structure of loan interest rates and default premia across projects, which will be particularly useful in the last section of the paper. However the mechanics of investment are fundamentally similar to what we would obtain under standard production technology with a decreasing marginal product of capital and 100% depreciation.

4 Equilibrium Without Government Activity

To understand the role of a central bank in our model, it helps to see how this economy will perform in the absence of a government. Suppose then, that there is no central bank or fiscal authority. During the daytime of period t, buyers make production decisions and entrepreneurs make investment decisions, before buyers learn whether they are making non-monitored or monitored transactions during the night of period t. During the day, buyers produce, make deposits with financial intermediaries, and the intermediaries lend to entrepreneurs. The deposit contracts written by buyers with financial intermediaries provide for particular withdrawal privileges, to be specified below. After these actions are taken, each buyer learns whether he or she will be engaged in a non-monitored or a monitored transaction during the night.

If the buyer will meet a seller in the night in a non-monitored transaction, then (as part of the deposit contract previously written with the financial intermediary) the buyer withdraws *circulating notes* from the financial intermediary. Circulating notes are claims on the financial intermediary that are non-replicable, and any seller can verify that these claims are what they purport to be. For the intermediary, circulating notes are costly to issue, requiring σ units of goods for each unit of consumption goods the notes will trade for in nighttime exchange. This cost is incurred to encode on these notes a non-replicable message that indicates that the noteholder has a claim on the financial intermediary. If the buyer learns at the end of the day that he or she will be in a monitored meeting, then the buyer does not withdraw notes. When a buyer meets a seller in a monitored meeting, the buyer and seller are able to contact the bank and transfer deposit claims from the buyer to the seller. Note that this communication technology is not available in a non-monitored meeting.

Now, let x_t denote the quantity of goods deposited with the financial intermediary by each buyer in the daytime of period t, y_{t+1}^n the payoff to each noteholder in the financial intermediary in the daytime of period $t+1$, and y_{t+1}^d the payoff for each depositor in period $t+1$ who did not withdraw circulating notes. Assume that, in a bilateral meeting between a buyer and seller in the night, that the buyer makes a take-it-or-leave-it offer. Then, if all circulating notes are traded away by buyers in the night (as we will show is equilibrium behavior), the cost to the financial intermediary of issuing circulating notes is $\sigma\beta y_{t+1}^n$. This then implies that the zero-profit condition for the financial intermediary is

$$\left(x_t - \rho\sigma\beta y_{t+1}^n\right) r_t = \rho y_{t+1}^n + (1-\rho) y_{t+1}^d. \tag{9}$$

In equation (9), the left-hand side is the payoff per unit invested in loans to entrepreneurs by the financial intermediary, which is total deposits minus the cost of note issue, all multiplied by the gross expected return per loan. On the right hand side are the total payouts by the intermediary to noteholders and deposit-holders respectively.

An equilibrium deposit contract $(x_t, y_{t+1}^n, y_{t+1}^d)$ maximizes the expected utility of the buyer, and satisfies the zero-profit condition (9). Therefore, an equilibrium deposit contract solves

$$\max_{x_t, y^n_{t+1}, y^d_{t+1}} \left(-x_t + \rho u(\beta y^n_{t+1}) + (1-\rho) \max_{\hat{y}^d_{t+1} \leq y^d_{t+1}} \left\{ u[\beta(y^d_{t+1} - \hat{y}^d_{t+1})] + \beta \hat{y}^d_{t+1} \right\} \right)$$

subject to (9). In the optimization problem above, $\hat{y}^d_{t+1}$ is the quantity of deposits not traded away by deposit-holders in monitored transactions during the night. As the demand for intermediary deposits would be infinite if $r_t > \frac{1}{\beta}$, in equilibrium we must have

$$r_t \leq \frac{1}{\beta},$$

which implies that in equilibrium $(x_t, y^n_{t+1}, y^d_{t+1})$ satisfies

$$u'(\beta y^n_{t+1}) = \frac{1 + \sigma r_t \beta}{r_t \beta}, \tag{10}$$

$$u'(\beta y^d_{t+1}) = \frac{1}{r_t \beta}, \text{ if } r_t < \frac{1}{\beta}, \tag{11}$$

$$y^d_{t+1} \geq \frac{q^*}{\beta}, \text{ if } r_t = \frac{1}{\beta}, \tag{12}$$

and (9).

Note that the solutions for y^n_{t+1} and y^d_{t+1} from (10)-(12) give the same consumption for buyers in monitored and non-monitored meetings as we would obtain in equilibrium if buyers knew in the daytime whether they would be in monitored or non-monitored meetings before consumption and production decisions are made. However, if that were the case, then production would be different in the daytime for monitored and non-monitored buyers. In the model we are considering here, the deposit contract implies that each buyer makes the same deposit, so that there is an element of Diamond/Dybvig-type insurance in the financial intermediation arrangement.

4.1 Case 1: Sufficient Liquidity

We will first construct an equilibrium where some buyers will choose to carry deposit claims from one day to the next. Confine attention to stationary equilibria where $r_t = r$ for all t. Buyers will want to carry over deposit claims from a monitored nighttime trade to the next day if and only if $r = \frac{1}{\beta}$ and consumption for buyers in a monitored trade is q^*, the surplus-maximizing quantity. In this equilibrium, $y^n_{t+1} = \tilde{y}^n$ where, from (10), $\tilde{y}^n$ is determined by

$$u'(\beta \tilde{y}^n) = 1 + \sigma. \tag{13}$$

Therefore $\beta \tilde{y}^n < q^*$, so that the quantity traded in non-monitored meetings during the night is less than the surplus-maximizing quantity, due to the cost of note issue. As σ increases, consumption by buyers in nonmonitored trades falls, from (13). The total quantity of loans will be $L\left(\frac{1}{\beta}\right)$. Therefore, given the zero-profit condition for the financial intermediary, (9), we have

$$L\left(\frac{1}{\beta}\right)\frac{1}{\beta}=\rho\tilde{y}^{n}+(1-\rho)\tilde{y}^{d}, \tag{14}$$

where $y_{t+1}^{d}=\tilde{y}^{d}$ for all t. Then, from (12) and (14), in equilibrium we must have

$$L\left(\frac{1}{\beta}\right)\geq\beta\rho\tilde{y}^{n}+(1-\rho)q^{*}. \tag{15}$$

Inequality (15) is what we mean by "sufficient liquidity." This condition states that, when the real interest rate is equal to the rate of time preference, there is a sufficiently large quantity of liquid assets to support an efficient level of transactions in monitored trades, along with the associated equilibrium level of transactions in non-monitored trades.

We can restate inequality (15) in terms of the underlying loan contracts between financial intermediaries and entrepreneurs, using (5), (6), and (7). Inequality (15) becomes

$$\alpha G(\gamma^{*})\geq\beta\rho\tilde{y}^{n}+(1-\rho)q^{*},$$

where the verification cost for the marginal entrepreneur, γ^{*}, and the gross loan interest rate faced by the marginal entrepreneur, R^{*} solve

$$1-\gamma^{*}f(R^{*})-F(R^{*})=0,$$

and

$$\frac{1}{\beta}=R^{*}-\gamma^{*}F\left(R^{*}\right)-\int_{0}^{R^{*}}F(w)dw.$$

Thus, whether there is sufficient liquidity or not will depend on the functions $F(\cdot)$ and $G(\cdot)$. In general, if returns on investment projects are higher, say in the sense of first-order stochastic dominance for the distribution $F(\cdot)$, then the economy's capacity to produce private liquidity is greater. Also, if verification costs are lower across the population of entrepreneurs, i.e. if $G(\cdot)$ shifts upward, then this will tend to increase private liquidity capacity.

4.2 Case 2: Insufficient Liquidity

Next, the case with insufficient liquidity arises if and only if

$$L\left(\frac{1}{\beta}\right)<\beta\rho\tilde{y}^{n}+(1-\rho)q^{*}. \tag{16}$$

Here, buyers will trade away all notes and deposit claims in monitored trades and $r<\frac{1}{\beta}$. Then, from (11) and (12) we get

$$u'(\beta y^{n})=\frac{1+\sigma r\beta}{r\beta}, \tag{17}$$

$$u'(\beta y^d) = \frac{1}{r\beta}. \tag{18}$$

and the zero-profit condition for financial intermediaries (9) gives

$$L(r)r = \rho y^n + (1-\rho)y^d \tag{19}$$

Then, equations (17)-(19) solve for y^n, y^d, and r. Assume that

$$-\frac{cu''(c)}{u'(c)} \leq \frac{1}{1+\sigma}, \text{ for } c > 0. \tag{20}$$

Proposition 1 *If (20) and (16) hold, then there exists a unique equilibrium with insufficient liquidity with* $r < \frac{1}{\beta}$.

Proof. Define $z^n \equiv \frac{y^n}{r}$ and $z^d \equiv \frac{y^d}{r}$, and rewrite (17)-(19) as

$$\frac{r}{1+\sigma r\beta} u'(\beta r z^n) = \frac{1}{\beta}, \tag{21}$$

$$ru'(\beta r z^d) = \frac{1}{\beta}, \tag{22}$$

$$L(r) = \rho z^n + (1-\rho)z^d. \tag{23}$$

Then, (21), (22) and (20) imply that, for $r \leq \frac{1}{\beta}$, we can write $z^n = \psi^n(r)$ and $z^d = \psi^d(r)$ with $\psi^{n\prime}(r) > 0$, $\psi^{d\prime}(r) > 0$, $\psi^n(0) = \psi^d(0) = 0$, $\psi^n(\frac{1}{\beta}) = \beta\tilde{y}^n$, and $\psi^d(\frac{1}{\beta}) = q^*$. From (23), the equilibrium level of r is determined by

$$L(r) = \rho\psi^n(r) + (1-\rho)\psi^d(r). \tag{24}$$

Since the left-hand side of (24) is continuous and strictly decreasing and the right hand side of (24) is continuous and strictly increasing for $r \in [0, \frac{1}{\beta}]$, with $L(0) > \rho\psi^n(0) + (1-\rho)\psi^d(0)$, and $L(\frac{1}{\beta}) < \rho\psi^n(\frac{1}{\beta}) + (1-\rho)\psi^d(\frac{1}{\beta}) = \beta\rho\tilde{y}^n + (1-\rho)q^*$ (from (16)), therefore there is a unique solution for $r \in (0, \frac{1}{\beta})$, and we also solve for unique z^n and z^d, and in turn for unique y^n and y^d. ■

In an equilibrium with insufficient liquidity, the economy does not have enough intertemporal productive capacity to support an efficient quantity of exchange in monitored transactions. From (17)-(19), and (20), an increase in the cost of note issue, σ, causes y^n to fall, so that buyers consume less in non-monitored exchanges during the night. As well, when σ increases, r increases in equilibrium, the quantity of lending falls, and y^d increases, so that buyers consume more in monitored exchanges in the night.

5 Equilibrium with a Government

Thus far, we have shown how this economy performs without a government. Financial intermediaries act to efficiently channel investment funds to entrepreneurs, and the liabilities of financial intermediaries, which can be interpreted as circulating currency and bank deposits, are exchanged in decentralized transactions. Financial intermediaries perform a delegated monitoring role, and they also act to provide insurance against the need for liquid assets in different types of decentralized transactions. Without a government, there is a limit to the quantity of liquid assets that the private sector can supply for use in transactions. This limit is determined by the quantity of investment projects, verification costs, and the costs of providing circulating notes.

In this section, we introduce a government, so that we can address questions related to public vs. private provision of liquidity. As we will see, both interest-bearing and non-interest-bearing government liabilities will play important roles, with fiscal policy determining the total stock of government liabilities and monetary policy determining the mix of interest-bearing vs. non-interest-bearing liabilities.

We will assume that the government in our model has the power to tax, and the power to prohibit the private issue of circulating notes, but otherwise has available the same technologies as does the private sector. Each buyer receives a lump-sum transfer τ_t in real terms, from the government during the daytime of period t. The government issues two liabilities. The first is fiat currency, which is perfectly divisible and, just as is the case for privately issued circulating notes, is costly to maintain. To prevent counterfeiting and to encode the information that makes it clear that the currency was issued by the government requires that σ units of goods be absorbed in the daytime of period t for each unit of goods the currency trades for at night. The second government liability is a one-period nominal bond. Nominal bonds are issued during the day of period t, with each selling for one unit of money and paying off z_{t+1} units of money in the day of period $t+1$. Nominal bonds are accounting balances held with the government that can be transferred using the same communications technology discussed previously in the context of nighttime monitored transactions. Thus, government bonds cannot be traded in non-monitored transactions, for the same reasons that deposits cannot be traded in these transactions.

Letting ϕ_t denote the price of currency in terms of goods in the daytime of period t, we can then write the government budget constraint as

$$\phi_t \left[M_t - M_{t-1} \right] + \phi_t B_t = \sigma q_t + \phi_t z_t B_{t-1} + \tau_t, \tag{25}$$

for $t = 1, 2, \ldots$. Here, M_t denotes the money stock in period t, B_t is the stock of nominal bonds, and q_t is the real value of the money stock in exchange during the night of period t. The left-hand side of (25) is the revenue from money creation and bond issue, while the right-hand side is the cost of maintaining the stock of money, plus total payouts by the government to retire outstanding bonds, plus transfers. In period 0,

$$\phi_0 M_0 + \phi_0 B_0 = \sigma q_0 + \tau_0, \tag{26}$$

so that private agents are endowed with no outside assets at the first date.

At this stage, we will assume that private sector intermediaries are prohibited from issuing circulating notes in this regime, so that it is not possible to use private note issue to finance lending to entrepreneurs or the purchase of government bonds. However, private intermediaries can hold government bonds and issue deposits. In equilibrium, it will be irrelevant whether buyers hold government bonds directly or the deposits of institutions that intermediate government bonds, so for convenience we will assume that all government bonds intermediated, with intermediary deposits backed by loans to entrepreneurs and nominal government bonds.

As in the economy without a government, during the day each buyer produces and makes a deposit with the financial intermediary. Here, the intermediary holds a portfolio of nominal government bonds, loans to entrepreneurs, and currency. Buyers who learn that they will meet sellers during the night in non-monitored trades withdraw currency from the financial intermediary at the end of the day. In equilibrium, the intermediary must be indifferent between holding nominal government bonds and loans to entrepreneurs, so

$$r_t = \frac{\phi_{t+1} z_{t+1}}{\phi_t}.$$

Further, in equilibrium bonds and loans to entrepreneurs will dominate currency in rate of return, since deposit claims cannot be traded in non-monitored exchanges at night, but currency can be exchanged in all transactions. As well, no gross rate of return can exceed $\frac{1}{\beta}$, since this would represent an arbitrage opportunity. Therefore, in equilibrium,

$$\frac{\phi_{t+1}}{\phi_t} \leq r_t \leq \frac{1}{\beta}. \tag{27}$$

Now, given (27), and since there is no aggregate uncertainty, the financial intermediary will hold only enough currency to satisfy the withdrawal demands of buyers in non-monitored trades. Let m_t denote the real quantity of money balances acquired in the daytime of period t by the financial intermediary, per depositor who will actually withdraw money at the end of the day. As well, let d_t denote the real quantity of government bonds and loans to entrepreneurs acquired by the intermediary, per depositor who will not withdraw. It is then straightforward to show, in a manner similar to the analysis in the previous section, that m_t solves

$$\left(\frac{\beta\phi_{t+1}}{\phi_t}\right) u'\left(\frac{\beta\phi_{t+1}}{\phi_t} m_t\right) = 1, \tag{28}$$

and d_t solves

$$\beta r_t u'(\beta r_t d_t) = 1, \text{ if } r_t < \frac{1}{\beta}, \tag{29}$$

or

$$d_t \geq q^*, \text{ if } r_t = \frac{1}{\beta}. \tag{30}$$

In equilibrium each buyer deposits $\rho m_t + (1-\rho) d_t$ with the financial intermediary in the daytime of period t. Thus, as in the regime without a government, the financial intermediary provides Diamond-Dybvig-type insurance. However, in contrast to the Diamond-Dybvig (1983) setup, the bank here insures against the state where the buyer is in a monitored trade.

In equilibrium, all assets must be willingly held. Therefore,

$$(1-\rho)\,d_t = L(r_t) + \phi_t B_t, \tag{31}$$

and

$$\rho m_t = \phi_t M_t. \tag{32}$$

Finally, the government budget constraints (25) and (26) must hold.

5.1 Government Policy

In this subsection, we set up a policy regime in which the total nominal government debt grows at a constant rate, determined by fiscal policy, and monetary policy determines the composition of the total outstanding debt. In the daytime of period 0, the government begins by transferring M_0 units of fiat money and B_0 nominal bonds in equal amounts to buyers, with the real value of the transfer, from (26), determined by the equilibrium price of money, ϕ_0. Then, assume that in the daytime of each period $t = 1, 2, 3, ...$, the government levies taxes on buyers to pay the interest on the government bonds that come due, and to pay the costs of maintaining the currency stock. Further, during the daytime of periods $t = 1, 2, 3, ...$, the government gives a nominal transfer of $M_t - M_{t-1}$ units of money and $B_t - B_{t-1}$ nominal bonds to each buyer. Then, it is straightforward to verify that the government budget constraint (25) holds for each $t = 1, 2, 3, ...$.

Now, let μ denote the constant gross rate of growth of currency outstanding and nominal bonds, with $M_t = \mu M_{t-1}$ and $B_t = \mu B_{t-1}$ for $t = 1, 2, 3, ...$. This implies that the ratio of the currency stock to the stock of bonds is constant over time. For convenience, let δ denote the fraction of nominal debt outstanding that consists of currency, i.e.

$$M_t = \delta(M_t + B_t) \tag{33}$$

for all t, where $0 \leq \delta \leq 1$. We can then interpret μ as being determined by fiscal policy, and δ by monetary policy, though in the model there is nothing that distinguishes the fiscal authority from the monetary authority.

5.1.1 Equilibrium with Sufficient Liquidity

We will confine attention to stationary equilibria with valued fiat currency, which implies that

$$\frac{\phi_{t+1}}{\phi_t} = \frac{1}{\mu} \tag{34}$$

for all t. As in the case with no government, if there is sufficient liquidity available, then in monitored exchange during the night an efficient quantity q^* is traded, and the gross return on loans and bonds is $r = \frac{1}{\beta}$. From (28) and (34), we get

$$\frac{\beta}{\mu}u'\left(\frac{\beta}{\mu}\hat{m}\right) = 1, \tag{35}$$

which solves for the real quantity of currency per buyer in non-monitored trades, $\hat{m}$. Then, from (31), (32), and (33), in equilibrium,

$$d = \frac{L(\frac{1}{\beta})}{1-\rho} + \frac{(1-\delta)\rho\hat{m}}{\delta(1-\rho)} \tag{36}$$

Therefore, from (30) and (36), there is sufficient liquidity if and only if

$$L(\frac{1}{\beta}) + \frac{(1-\delta)\,\rho\hat{m}}{\delta} \geq (1-\rho)q^*,$$

where the left-hand side is the quantity of lending to entrepreneurs at the gross real return $\frac{1}{\beta}$, plus the equilibrium real quantity of government debt, and the right-hand side is the total quantity of goods traded in efficient exchanges in monitored meetings at night. Here, $\hat{m}$ is determined by (35). Thus, the real quantity of bonds is determined by the demand for currency in non-monitored transactions, from (35), and the government policy rule (33). We can rewrite the above inequality as

$$\frac{1-\delta}{\delta} \geq \frac{(1-\rho)q^* - L(\frac{1}{\beta})}{\rho\hat{m}}. \tag{37}$$

Proposition 2 *For any $\mu \geq \beta$ there exists a $\delta^* > 0$ such that, if $\delta \leq \delta^*$, then there exists a government equilibrium with sufficient liquidity.*

Proof. Given any $\mu \geq \beta$, (35) determines $\hat{m}$. Then there exists an equilibrium with sufficient liquidity if and only if (37) holds. But (37) holds if and only if $\delta \leq \delta^*$, where

$$\delta^* = 1, \text{ if } q^*(1-\rho) - L(\frac{1}{\beta}) \leq 0,$$

$$\delta^* = \frac{\rho\hat{m}}{\rho\hat{m} + (1-\rho)q^* - L(\frac{1}{\beta})}, \text{ if } (1-\rho)q^* - L(\frac{1}{\beta}) > 0.$$

■

Thus, for any feasible rate of growth for the total nominal government debt, there exists a critical ratio of money to nominal government bonds such that, if the ratio of money to bonds is less than this critical level, an equilibrium with sufficient liquidity exists. From (35), the real quantity of currency is determined by the rate of growth in total government liabilities, which also determines the inflation rate. Then, monetary policy, which has no effect on the real quantity of money, determines the real quantity of bonds, which in part backs financial intermediary deposits. Thus, it is the real quantity of bonds that is the key source of publicly provided liquidity influenced by monetary policy. The result above states that it is always possible to find a monetary policy that results in sufficient provision of public liquidity that there is efficient trade in monitored transactions.

5.1.2 Equilibrium with Insufficient Liquidity

Next, look for an equilibrium in which there is insufficient liquidity to support efficient exchange in monitored meetings during the night. In this equilibrium, $r < \frac{1}{\beta}$, and from (29) the quantity of deposit claims held at the end of the day by each buyer who will trade in a monitored meeting at night, d, is determined by

$$\beta r u'(\beta r d) = 1. \tag{38}$$

As in the case with sufficient liquidity, the real quantity of currency held by each buyer at the end of the day who will trade in a non-monitored meeting, m, is determined by

$$\frac{\beta}{\mu} u'\left(\frac{\beta}{\mu} m\right) = 1. \tag{39}$$

Also, from (31), (32), and (33), we have

$$d = \frac{L(r)}{1-\rho} + \frac{(1-\delta)\rho m}{\delta(1-\rho)} \tag{40}$$

Equations (38)-(40) solve for d, m, and r.

Proposition 3 *If*

$$-\frac{cu''(c)}{u'(c)} \leq 1, \text{ for } c > 0, \tag{41}$$

and

$$\frac{1-\delta}{\delta} < \frac{(1-\rho)q^* - L(\frac{1}{\beta})}{\rho \tilde{m}}, \tag{42}$$

where $\tilde{m}$ solves

$$\frac{\beta}{\mu} u'\left(\frac{\beta}{\mu} \tilde{m}\right) = 1, \tag{43}$$

then an equilibrium with a government and insufficient liquidity exists and it is unique.

Proof. Given μ, equation (39) solves for the real quantity of currency which we denote $\tilde{m}$. Then, from (38), if (41) holds then $d = \omega(r)$, where $\omega(\cdot)$ is an increasing and continuous function. Then from (40), we have

$$\omega(r) = \frac{L(r)}{1-\rho} + \frac{(1-\delta)\rho \tilde{m}}{\delta(1-\rho)}. \tag{44}$$

Now, let $\psi(r)$ denote the right-hand side of (44), which is a decreasing and continuous function of r.We then have $\omega(0) = 0$, $\omega(\frac{1}{\beta}) = q^*$, $\psi(0) = \alpha$, and $\psi(\frac{1}{\beta}) = \frac{L(\frac{1}{\beta})}{1-\rho} + \frac{(1-\delta)\rho\tilde{m}}{\delta(1-\rho)}$. Then, since $\psi(0) > \omega(0)$, if $\psi(\frac{1}{\beta}) < \omega(\frac{1}{\beta})$, then a unique solution for r exists with $r < \frac{1}{\beta}$. Thus, the condition we need is

$$\frac{L(\frac{1}{\beta})}{1-\rho} + \frac{(1-\delta)\rho\tilde{m}}{\delta(1-\rho)} < q^*,$$

which can be manipulated to give (42). Once we have a unique solution for r, we solve uniquely for d from $d = \omega(r)$. ■

Now, given (41), an increase in μ results in a decrease in m, from (39). Then, equation (44) implies that r decreases, and so the quantity of lending to entrepreneurs, $L(r)$, increases. Then, (38) implies that d falls. Thus, consumption falls for all buyers during the night. Thus, an increase in the growth rate in the total nominal stock of government liabilities, when there is insufficient liquidity, acts to reduce the real quantity of money, and by implication the real quantity of government bonds in equilibrium, thus reducing the real stock of liquid assets. The real interest rate falls, and lending increases, thus increasing the stock of privately-supplied liquid assets, but this does not completely mitigate the loss of publicly-supplied liquid assets.

An increase in δ is interpreted as a monetary policy action, in that this increases the ratio of money to bonds, holding constant the growth rate in the total stock of government liabilities. This is essentially a one-time open-market purchase of bonds. From (44), this reduces r, so that $L(r)$, the quantity of lending to entrepreneurs, increases. From (39), m remains unaffected, so that the price level increases in proportion to the increase in the money stock. Consumption for buyers in non-monitored trades in the night is unaffected, but consumption for buyers in monitored trades must fall.

Thus, monetary policy is not neutral, given insufficient liquidity. A one-time open market purchase, which produces a level increase in the money stock, reduces the real quantity of government-supplied liquid assets, the real interest rate falls, and there is an increase in lending. All of these effects are permanent. This is a key result, and appears to be a unique property that this model does not share with alternative frameworks for analyzing monetary policy. In the insufficient liquidity case, monetary policy is not neutral, but the nonneutrality does not work through conventional means, as what we typically consider to be "liquidity" - the stock of money - is invariant in real terms to the change in monetary policy. Again, the key publicly-provided liquid asset is the stock of government bonds, and monetary policy actions affect the real value of this stock, causing changes in the privately-provided liquidity and the quantity of private lending.

5.1.3 Optimal Government Policy

In this section we want to determine, within the class of policies under consideration, which ones are optimal. Suppose first that we look for a policy that maximizes the total net surplus in nighttime trading, minus the total cost per period of maintaining the money stock. Then, letting q and s denote, respectively, the quantity of output traded during the night in nonmonitored and monitored trades, this quantity is given by

$$W = \rho\,[u(q) - q] + (1 - \rho)[u(s) - s] - \sigma\rho q$$

Then, suppose that we choose q and s to maximize W. This gives $q = \hat{q}$ and $s = q^*$ where $\hat{q}$ solves

$$u'(\hat{q}) = 1 + \sigma \tag{45}$$

From (35) and (37), this outcome can be supported as an equilibrium with

$$\mu = \beta(1 + \sigma) \tag{46}$$

and with δ set to satisfy

$$\frac{1-\delta}{\delta} \geq \frac{(1-\rho)q^* - L\left(\frac{1}{\beta}\right)}{\rho(1+\sigma)\hat{q}}, \tag{47}$$

so that there is sufficient liquidity. Thus, the above outcome can be achieved with a unique growth rate for total nominal government debt. This optimal growth rate is larger than the Friedman-rule money growth rate, with the gap between the optimal money growth rate and the Friedman rule rate increasing with the cost of maintaining the stock of currency. The optimal nominal interest rate on bonds is σ, and equation (46) implies that, in equilibrium at the optimum, the inflation tax is just sufficient to finance the costs of maintaining the stock of currency. Given the optimal money growth rate, (47) states that δ, the ratio of money to total nominal government debt, must be sufficiently small so that the stock of liquid assets backing bank deposits is sufficiently large, in real terms.

Given the government policy specified by (46) and (47), the financial intermediation arrangement we have specified allocates resources efficiently between entrepreneurs and other agents (under the restriction of pure verification strategies). Therefore, this policy is one that maximizes the sum of utilities across agents in a stationary equilibrium, within the class of policies under consideration.

Now, note that, if (15) holds, so that there is sufficient liquidity in the absence of government, that the optimal government policy achieves the same equilibrium allocation as in the case without a government, where all liquidity is supplied by private financial intermediaries. Government policy can improve on what the private sector can accomplish, only if private intermediation supplies an insufficient quantity of liquidity to support efficient exchange. Thus, in this environment, government-provided liquidity need not be essential.

However, if (16) holds, then the ability of the private sector to provide liquidity is sufficiently limited that the government can improve matters. It is important to note, though, that even in these circumstances a central bank is not necessary. An optimum could either be achieved with a prohibition on the issue of private circulating notes and an appropriate monetary policy with central banking, or by having the government supply the appropriate quantity of government debt while permitting unrestricted issue of private circulating notes. As well, if private circulating notes are banned and the government is behaving optimally, so that (47) holds, then at the margin monetary policy is irrelevant, in that changes in δ serve only to change the price level and affect no real quantities. Monetary policy can matter in a way that appears consistent with conventional central-banking wisdom, in that an open market purchase reduces the real interest rate and increases private lending. However, monetary policy matters in this fashion only when the government is behaving suboptimally.

Since the departure from the Friedman rule dictated by (46), and the optimal inflation rate depend on σ, the cost of maintaining the money stock, one obvious question is how large σ might be in practice. If σ represents only the costs of wear-and-tear on the currency stock, and the costs of designing the currency stock to deter counterfeiting, then these costs are likely negligible, as a fraction of the value of transactions supported by the currency stock over a given period of time. However, if we take σ to represent all of the costs of operating a central bank, then clearly these costs are significant, and will matter in an important way for determining the optimal inflation rate. A case can certainly be made for the latter interpretation. Further, if we seriously address the costs of counterfeiting and theft that arise from economic activities that are a pure deadweight loss, then this would add to the costs of operating a currency system, and would give us additional reasons for departures from the Friedman rule (see for example Sanches and Williamson 2008).

6 U.S. Monetary Policy and the Financial Crisis

There are two unusual features associated with recent monetary policy in the United States. First, the nominal interest rate on short-term government debt has been essentially zero since late 2008, which has never been the case in the history of the Federal Reserve System. For monetary economists, there is nothing unusual about a zero nominal market interest rate. Indeed, we are accustomed to working with models in which a Friedman rule achieves efficiency, and does this by driving the nominal interest rate to zero in all states of the world. However, it seems clear that the U.S. economy is not currently in a permanent Friedman-rule monetary regime. The Federal Reserve System appears intent on increasing the nominal interest rate on Treasury bills above zero sometime in the future, though the Fed has made no commitment as to when this will happen. Second, the Fed has engaged in some unusual types of lending, for example to non-bank financial intermediaries, and has accumulated some unusual assets (for a central bank), including commercial paper and mortgage-backed securities. These types of interventions are certainly not common currency for monetary economists, and are not the types of experiments we typically consider in the context of monetary models.

The approach we will take in this section is to consider a world in which, as of $t = 0$, whatever shock or shocks led to the financial crisis have been realized, with all features of the environment and fiscal policy fixed from $t = 0$ forward. Then, we take monetary policy for $t = 1, 2, 3, ...$, as given, and consider how alternative monetary policies at $t = 0$ matter. We first consider the effects of open market purchases at $t = 0$ when the nominal interest rate is positive. Then, we show what level of open market purchases at $t = 0$ will support a zero nominal interest rate at the first date, and determine the effects of further open market purchases when the nominal interest rate is zero. Then, we consider how an expanded role for central bank financial intermediation at $t = 0$ matters, conditional on the nominal interest rate being zero.

6.1 Open Market Purchases at $t = 0$: Positive Nominal Interest Rate

First, fix fiscal policy for $t = 0, 1, 2, ...$, with $\mu = \hat{\mu}$, and assume that the intention of the central bank is to pursue a policy in periods $t = 1, 2, 3, ...$, with $\delta = \hat{\delta}$. This policy may be suboptimal, but for the experiments we consider here we will treat it as fixed, and think about the effects of central bank intervention in period 0. Given the policy in periods $t = 1, 2, 3, ...$, whether the economy has sufficient liquidity or not, at $t = 1, 2, 3, ...$, we have $m = \hat{m}$ determined by (35) given $\mu = \hat{\mu}$. Therefore, the price of money in period 1 is given by

$$\phi_1 = \frac{\rho \hat{m}}{M_1}, \tag{48}$$

given take-it-or-leave-it offers by the buyer and market clearing during the day. Further, so long as $\hat{\mu} > \beta$, we have $\hat{m} < q^*$.

Now, consider the effects at $t = 0$ of monetary policy, characterized by δ_0, the ratio of outside money to total nominal government debt in period 0. The stocks of outside money and nominal bonds at $t = 0$ are then

$$M_0 = \frac{M_1 \delta_0}{\hat{\delta} \hat{\mu}}, \tag{49}$$

and

$$B_0 = \frac{M_1 (1 - \delta_0)}{\hat{\delta} \hat{\mu}} \tag{50}$$

Then, let m_0 and d_0 denote, respectively, the real quantity of outside money held by each buyer who will be trading with currency in the night and the real quantity of interest-bearing assets backing the deposits of each buyer who will be trading deposits at night, as of the end of the day in period 0. If there is sufficient liquidity in period 0, the period 0 real interest rate is $r_0 = \frac{1}{\beta}$. Analogous to (35) we then have

$$\frac{\beta \phi_1}{\phi_0} u' \left(\frac{\beta \phi_1}{\phi_0} m_0 \right) = 1, \tag{51}$$

and sufficient liquidity for efficient trading in monitored meetings in the night of period 0 requires that

$$d_0 \geq q^*. \tag{52}$$

Further, in equilibrium the daytime markets in period 0 for money and interest-bearing assets, respectively, must clear, so

$$m_0 = \frac{\phi_0 M_0}{\rho}, \tag{53}$$

$$d_0 = \frac{L(\frac{1}{\beta})}{1-\rho} + \frac{\phi_0 B_0}{(1-\rho)}. \tag{54}$$

If the nominal interest rate is positive at $t = 0$, then

$$\frac{\phi_1}{\phi_0} < \frac{1}{\beta}. \tag{55}$$

Therefore, from (48)-(55), given δ_0, we solve for ϕ_0 according to

$$\phi_0 = \frac{\beta\rho\hat{m}}{M_1} u'\left(\frac{\beta\hat{m}\delta_0}{\hat{\delta}\hat{\mu}}\right), \tag{56}$$

and for sufficient liquidity δ_0 must satisfy

$$\frac{L(\frac{1}{\beta})}{1-\rho} + \frac{\beta\rho\hat{m}\,(1-\delta_0)}{(1-\rho)\hat{\delta}\hat{\mu}} u'\left(\frac{\beta\hat{m}\delta_0}{\hat{\delta}\hat{\mu}}\right) \geq q^*. \tag{57}$$

Now, with sufficient liquidity, an increase in δ_0 (an open market purchase at $t = 0$) has no effect on the real interest rate r_0, as it only reduces the left-hand side of (57). Buyers in monitored meetings during the night of period 0 will have sufficient deposit holdings to purchase the efficient quantity q^*. Since there is no effect on r_0, the quantity of lending in period 0, $L(r_0)$, does not change. However, from (56), ϕ_0 falls, and so the gross nominal interest rate, $\frac{r_0\phi_0}{\phi_1}$, must fall, since r_0 and ϕ_1 are unaffected by the change in δ_0. Consumption by buyers in non-monitored exchanges during the night must rise.

Next, consider the case where there is insufficient liquidity at $t = 0$, so that the quantity traded in each monitored trade is less than q^*. Then, as in the sufficient liquidity case, given δ_0, the price of money in period 0 is determined by (56), but for insufficient liquidity we require

$$\frac{L(\frac{1}{\beta})}{1-\rho} + \frac{\beta\rho\hat{m}\,(1-\delta_0)}{(1-\rho)\hat{\delta}\hat{\mu}} u'\left(\frac{\beta\hat{m}\delta_0}{\hat{\delta}\hat{\mu}}\right) < q^*. \tag{58}$$

Further, analogous to (38) we have

$$\beta r_0 u'(\beta r_0 d_0) = 1, \tag{59}$$

and, similar to (54) we have

$$d_0 = \frac{L(r_0)}{1-\rho} + \frac{\phi_0 B_0}{(1-\rho)},$$

or, using (50) and (56),

$$d_0 = \frac{L(r_0)}{1-\rho} + \frac{\beta\rho\hat{m}\,(1-\delta_0)}{(1-\rho)\hat{\delta}\hat{\mu}} u'\left(\frac{\beta\hat{m}\delta_0}{\hat{\delta}\hat{\mu}}\right). \tag{60}$$

In this case, from (56), (59), and (60), an increase in δ_0 reduces r_0 and ϕ_0, and reduces the nominal interest rate in period 0. As a result, lending increases in period 0, and the quantity of goods traded in each monitored and nonmonitored transaction during the night falls and increases, respectively.

Therefore, whether there is sufficient or insufficient liquidity at $t = 0$, an open market purchase acts to reduce the nominal interest rate and increase the consumption of buyers trading in non-monitored meetings in the night of period 0. The real interest rate will fall, and lending will increase, only if there is insufficient liquidity.

6.2 Open Market Purchases at $t = 0$: A Zero Nominal Interest Rate and the Liquidity Trap

Thus far, we have determined that, given an arbitrary fiscal policy, and fixing future monetary policy, open market purchases of nominal government bonds at $t = 0$ will reduce the nominal interest rate. The next issue concerns whether it is feasible to reduce the nominal interest rate to zero at $t = 0$. If so, then we wish to determine the consequences of open market purchases at the zero lower bound.

First, suppose that $\delta_0 = \tilde{\delta}_0$, where $\tilde{\delta}_0$ is sufficiently large that the nominal interest rate is zero, but buyers in non-monitored exchanges during the night of $t = 0$ have just enough currency to purchase q^* goods. This implies, from (56), that

$$\tilde{\delta}_0 = \frac{q^* \hat{\delta} \hat{\mu}}{\beta \hat{m}}. \tag{61}$$

Then, for there to be sufficient liquidity, from (61) and (57), we must have

$$L(\frac{1}{\beta}) + \frac{\rho \beta \hat{m}}{\hat{\delta} \hat{\mu}} \geq q^*. \tag{62}$$

Further, for the zero nominal interest rate policy to be feasible, we require $\tilde{\delta}_0 \leq 1$ or, from (61),

$$\frac{\beta \hat{m}}{\hat{\delta} \hat{\mu}} \geq q^*. \tag{63}$$

If (62) and (63) hold, and $\delta_0 = \tilde{\delta}_0$, then an equilibrium exists where the nominal interest rate is zero at $t = 0$, implying that q^* is exchanged in all trades during the nighttime of $t = 0$. Now, suppose that (62) and (63) hold, but $\delta_0 \geq \tilde{\delta}_0$. What happens in this case if δ_0 increases? Essentially, there is a liquidity trap. When $\delta_0 \geq \tilde{\delta}_0$, there is sufficient liquidity to support exchange of q^* units of goods in each nighttime transaction, and for a financial intermediary nominal government bonds and outside money are equivalent, as they yield the same return. Thus, an open market purchase changes no quantities and prices, and financial intermediaries are content to hold the additional outside money as reserves until $t = 1$.

Next, consider the case where the nominal interest rate is zero and there is insufficient liquidity. In contrast to (62), here we must have

$$L(\frac{1}{\beta}) + \frac{\rho \beta \hat{m}}{\hat{\delta} \hat{\mu}} < q^*. \tag{64}$$

We know from the previous analysis that, when the nominal interest rate is greater than zero and there is insufficient liquidity, an increase in δ_0 will reduce the nominal interest rate. We then need to determine that, when (64) is satisfied, there exists some $\tilde{\delta}_0$, with $0 < \tilde{\delta}_0 \leq 1$, such that $r_0 = \frac{\phi_1}{\phi_0}$ when $\delta_0 \geq \tilde{\delta}_0$. From (56), (59), and (60), the critical value $\tilde{\delta}_0$, the real interest rate given that critical value, $\tilde{r}_0$, and the quantity of deposits per buyer in monitored trades, $\tilde{d}_0$, are jointly determined by

$$\beta \tilde{r}_0 u' \left(\frac{\beta \hat{m} \tilde{\delta}_0}{\hat{\delta} \hat{\mu}} \right) = 1, \tag{65}$$

$$\beta \tilde{r}_0 u'(\beta \tilde{r}_0 \tilde{d}_0) = 1, \tag{66}$$

$$\tilde{d}_0 = \frac{L(\tilde{r}_0)}{1-\rho} + \frac{\beta \rho \hat{m} \left(1 - \tilde{\delta}_0\right)}{(1-\rho)\hat{\delta}\hat{\mu}} u' \left(\frac{\beta \hat{m} \tilde{\delta}_0}{\hat{\delta} \hat{\mu}} \right). \tag{67}$$

From (65)-(67), $\tilde{\delta}_0$ is uniquely determined, but in addition to (64) we require that $\tilde{\delta}_0 \leq 1$. If $\alpha = 0$, so that there are no entrepreneurs in the model and the only liquidity is publicly provided, then $L(\tilde{r}_0) = 0$ for all $\tilde{r}_0$. Then, by continuity, as long as α is sufficiently small, if (64) holds, then (65)-(67) solves for $\tilde{\delta}_0 \leq 1$, and there is a sufficiently large open market purchase in period 0 that will drive the nominal interest rate to zero.

Now, suppose that (64) holds and it is feasible to set δ_0 so that the nominal interest rate is zero in period 0. What happens if $\delta_0 \geq \tilde{\delta}_0$ and δ_0 increases? Just as in the case with sufficient liquidity, there is a liquidity trap. However, with insufficient liquidity, the effect of the open market purchase in period 0 is to simply ask financial intermediaries to exchange nominal government bonds for outside money, which they are happy to do. The intermediaries then hold the money as reserves backing deposits, which all change hands in transactions made during the night of period 0. There is no effect of the increase in δ_0 on any quantities or prices.

6.3 Direct Lending By the Central Bank

Suppose that, at $t = 0$, the central bank has conducted sufficient open market purchases of nominal government debt to drive the nominal interest rate to zero. However, the central bank (for whatever reason) would like private financial intermediaries to lend more. Conducting further open market purchases does not accomplish this, as financial intermediaries will hold the extra outside money that is injected as reserves, until $t = 1$. Frustrated in its attempts to get private intermediaries to lend more, the central bank decides to intermediate private lending directly, rather than working through private intermediaries. For simplicity, suppose that the central bank sets $\delta_0 = \tilde{\delta}_0$ so as to just achieve a zero nominal interest rate, and then issues $\tilde{M}$ additional units of outside money in order to finance loans to entrepreneurs. These loans are made on the same terms that private intermediaries are willing to offer. That is, we assume that the central bank is as efficient as private intermediaries, facing the same verification costs as the private sector, and that it writes efficient debt contracts. If the central bank takes on as much loans as it can with an expected payoff of r_0, then it will have issued a total nominal quantity of money in period 0 equal to

$$M_0 + \tilde{M} = M_0 + L\left(r_0\right) \frac{1}{\phi_0}.$$

Lending to entrepreneurs will now have become unprofitable for private financial intermediaries, the stock of money will have expanded by $\tilde{M}$ in period 0, but all prices and quantities (at all dates) will be identical to what they were in the absence of central bank lending to entrepreneurs. In period 1, the extra outside money $\tilde{M}$ is retired using the returns on the central bank's portfolio. This government lending program accomplished nothing, other than to displace an equal quantity of private lending. However, the program also had no effect on prices, in spite of a potentially large increase in the stock of outside money. The key to this result is that the extra money that was injected was fully backed by private loans, and was retired in the future.

Now, note that, even in the equilibrium with a zero nominal interest rate on deposits and government bonds, there are some interest rates that are positive. In particular, from (3), an entrepreneur in period 0 with verification cost γ will face a nominal interest rate

$$\frac{\phi_0}{\phi_1}\left\{\gamma F[R_0(\gamma)] + \int_0^{R_0(\gamma)} F(w)dw\right\} > 0,$$

which is equal to the gross inflation rate multiplied by the entrepreneur's default premium, given his or her gross real loan interest rate $R_0(\gamma)$. Now, suppose that the central bank is even more ambitious, and decides it will lend to entrepreneurs on better terms than would private intermediaries, with the goal of reducing nominal loan interest rates and increasing lending to entrepreneurs. The central bank writes efficient loan contracts with entrepreneurs, but extends loans with an expected payoff of $\bar{r}_0 < r_0$, where r_0 is the real return on nominal government bonds, and on outside money, given that the nominal interest rate is zero in period 0.

If there is sufficient liquidity, i.e. (62) holds, then since $L(\bar{r}_0) > L(r_0)$, there is now a larger real supply of liquidity at $t = 0$, with the extra injection of outside money, in nominal terms, equal to

$$\tilde{M} = L\left(\bar{r}_0\right) \frac{1}{\phi_0}.$$

With a positive nominal interest rate and insufficient liquidity, d_0, r_0, and ϕ_0 were determined by (56), (59), and (60). Now, with a zero nominal interest rate, insufficient liquidity, and central bank lending, d_0 and r_0 are determined by

$$\beta r_0 u'(\beta r_0 d_0) = 1, \tag{68}$$

and

$$d_0 = L(\bar{r}_0) + \frac{\rho \hat{m}}{\hat{\delta}\hat{\mu} r_0}, \tag{69}$$

where $L(r_0)$ is the real value of the outside money ($\tilde{M}$) issued to finance lending to entrepreneurs, and $\frac{\rho \hat{m}}{\hat{\delta}\hat{\mu} r_0} = \phi_0(M_0 + B_0)$ is the real value of the unbacked money and bonds issued at $t = 0$. The price of money at $t = 0$ is

$$\phi_0 = \frac{\phi_1}{r_0} = \frac{\rho \hat{m}}{M_1 r_0}. \tag{70}$$

As with the sufficient liquidity case, assume that lump-sum taxes are levied on buyers at $t = 1$ to make up for the shortfall in the returns on the central bank portfolio, as required to retire $\tilde{M}$ units of money at $t = 1$.

Now, what happens if the central bank reduces $\bar{r}_0$, lending on more generous terms to entrepreneurs? Clearly, the quantity of lending will increase. Also, given the assumption (41), from (68) and (69) r_0 and d_0 increase, so that the real interest rate on bonds rises, and buyers consume more in meetings during the night, since there is a greater quantity of liquidity backing deposits. As well, since r_0 increases, ϕ_0 falls, so the price level rises at the first date. but private agents are willing to hold the extra outside money injection until $t = 1$ facing the same equilibrium prices as when there was no central bank lending program. However, at $t = 1$, the returns on the central bank loan portfolio are now insufficient to retire the extra $\tilde{M}$ units of outside money. The money can be retired through lump-sum taxation of buyers at $t = 1$. Thus, the central bank will have reduced the nominal interest rates faced by entrepreneurs at $t = 0$, lending will have increased, and entrepreneurs will be better off. However, buyers will be worse off due to the higher lump-sum taxes they face at $t = 1$. The central bank has achieved its goal, but there has been a redistribution of wealth. Some inefficient investment projects are being undertaken at public expense.

Now, suppose that central bank lending takes a somewhat different form. In particular, assume that the central bank makes loans with an expected payoff of $\bar{r}_0$ in period 0, but only to entrepreneurs who would not obtain loans from private intermediaries. In this case, our results are not much different in the sufficient liquidity case, except that the tax imposed at $t = 1$ does not need to be as large, since the central bank makes a loss only on marginal loans. In the case with insufficient liquidity, the quantity of outside money that must be issued at $t = 0$ to finance central bank lending is

$$\tilde{M} = [L(\bar{r}_0) - L(r_0)] \frac{1}{\phi_0}.$$

However, d_0 and r_0 are determined exactly as in the case where the central bank lends to all entrepreneurs, by equations (68) and (69). The difference here is that r_0 increases when $\bar{r}_0$ decreases. Therefore, when the central bank lends on more generous terms to marginal borrowers, it reduces the amount of private credit, and raises the interest rates faced by entrepreneurs who borrow from private financial intermediaries.

6.4 Discussion

The policy experiments studied in this section illustrate some key features of the current policy dilemma for the Federal Reserve System in the US. There is both good news and bad news. On the positive side, it is possible for there to be large increases in the stock of outside money without inflationary consequences. Once the nominal interest rate is zero in the present, additional money injections conducted by way of conventional open market purchases of government debt are held in equilibrium as reserves by financial intermediaries. Given that it is anticipated that the outside money will be retired in the future, the path for prices does not change. As well, unconventional purchases of assets by the central bank, which are essentially direct lending by the central bank in our model that expand the stock of outside money, are irrelevant for prices, provided the central bank lends on the same terms as does the private sector. The price level can increase in the present only if there is insufficient liquidity and the central bank lends on better terms than the private sector, thus expanding the supply of credit. However, this has no implications for future prices if the outside money issued to finance central bank lending is retired in the future.

On the negative side, our analysis casts "quantitative easing" in a bad light. Conventional open market purchases at a zero nominal interest rate are irrelevant. There is a liquidity trap phenomenon, whereby the money injection is held by financial intermediaries until it is retired in the future, with no effect on any prices or quantities. We could modify our model to include long-term government debt, and open market purchases conducted in this long-term debt, and our results would not change. If the central bank purchases private debt instruments, effectively acting much like a private financial intermediary, then this is also irrelevant, if the central bank lends on the same terms as does the private sector. However, if the central bank lends on better terms than the private sector, then this has redistributional implications. Borrowers clearly gain in the present, but someone must suffer in the future if today's money injection is to be retired, since the return on the central bank's loan portfolio will be insufficient for the task. Further, if the central bank confines its subsidized lending to one segment of the credit market (the mortgage market, for example), then this serves to reallocate credit. Borrowers who are not on the receiving end of central bank lending borrow at higher interest rates as a result of the central bank intervention.

In acquiring private debt instruments, such as mortgage-backed securities, the central bank may feel that it is somehow taking up the slack for private financial intermediaries that are reluctant to lend. However, this is an endogenous outcome. In our model, in the absence of the central bank lending, the private sector would be lending more. Further, to the extent that the central bank is lending on generous terms, or investing in assets of questionable quality, its ability to retire money in the future is impaired, and the Fed could find itself unable to keep the nominal interest rate at zero or to curb the resulting inflation.

7 Conclusion

The model constructed here contains an explicit role for liquid assets in retail transactions. Under some circumstances, given the available information technology, what is required for making transactions is an asset like currency. However, currency is costly to produce, as counterfeiting must be deterred, the currency stock wears out over time, and information must be encoded in the currency concerning what it is a claim to. In our model, the private sector and the government possess the same currency technology, and so circulating notes can in principle be issued by the private sector. There may be circumstances in the model where a sophisticated information technology is available, allowing decentralized transactions to be executed using deposit claims on financial intermediaries. These transactions correspond to retail payments using debit cards and checks. Deposit claims on financial intermediaries are backed by loans to entrepreneurs and by government bonds, which are account balances with the government. Thus, the underlying assets which are the source of liquidity for deposit claims are private loans and government bonds.

It may be the case that the economy has sufficient intertemporal productive capacity for supplying liquid assets. In this case, efficiency can be achieved without a central bank. Otherwise, there is a role for the government in augmenting the supply of liquid assets. One way to achieve efficiency when the private sector cannot supply sufficient liquidity is for the government to ban the issue of private circulating notes, and then conduct monetary and fiscal policy appropriately. Alternatively, the government could supply the appropriate quantity of government bonds, and permit unrestricted issue of private circulating notes by financial intermediaries.

Monetary policy can be non-neutral, in that a one-time open market purchase can reduce the real interest rate permanently and increase private lending. However, this nonneutrality occurs at the margin only if the government is behaving suboptimally. At the optimum, the costs of currency issue imply a deviation from the Friedman rule at the optimum.

Our model delivers results that are favorable to a private role for the issue of circulating currency. The consensus view appears to be that it is appropriate for the government to have a monopoly in the issue of small-denomination circulating liabilities. Indeed, even Milton Friedman (Friedman 1960) reasoned that there are market failures unique to the market for currency that make government monopolization welfare-improving. However, there is ample theoretical and empirical support for the relative efficiency of private money systems. On the theoretical side, Cavalcanti and Wallace (1999), Champ, Smith, and Williamson (1996), Cavalcanti, Erosa, and Temzelides (1999), and Williamson (1999), have all constructed models showing how private money systems can work well. On the empirical side, Champ, Smith, and Williamson (1996) show how the Canadian private money system that existed before 1935 acted to provide an elastic currency, and Smith and Weber (1999) study the similar behavior of the Suffolk banking system during the free banking era in the United States. Thus, it is not far-fetched to propose that a private money system could be efficient.

Our results concerning recent monetary policy in the United States are both encouraging and discouraging. On the encouraging side, it is certainly possible to have large increases in the stock of outside money which do not produce inflation. This can be due either to a liquidity trap phenomenon or because the increase in the money stock is backed by private lending. In either case, prices do not change as a result of the monetary expansion because the additional outside money is retired in the future, either through taxation or by using the returns on the central bank's portfolio. On the discouraging side, when the nominal interest rate is zero, it is not possible to engineer additional increases in private lending through "quantitative easing," unless the central bank lends on better terms than does the private sector. But in that case, this results in a redistribution of wealth and can change the allocation of credit.

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