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Abstract: In this paper, we introduce a volatility-based method for clustering analysis of financial time series. Using the generalized autoregressive conditional heteroskedasticity (GARCH) models we estimate the distances between the stock return volatilities. The proposed method uses the volatility behavior of the time series and solves the problem of different lengths. As an illustrative example, we investigate the similarities among major international stock markets using daily return series with different sample sizes from 1966 to 2006. From cluster analysis, most European markets countries, United States and Canada appear close together, and most Asian/Pacific markets and the South/Middle American markets appear in a distinct cluster. After the terrorist attack on September 11, 2001, the European stock markets have become more homogenous, and North American markets, Japan and Australia seem to come closer.

Keywords: Cluster analysis; GARCH; International stock markets; Volatility.

1. Introduction

The general problem in clustering financial time series is the separation of a set of time series data into groups or clusters, with the property that series in the same group have a similar stochastic dependence structure and series in other groups are quite distinct. To perform cluster analysis of time series, we have to define a relevant measure of distance between the time series in a data set. The stochastic behavior of most financial time series renders the usual methodologies used to measure the distance between different stock returns inappropriate. Mantegna (1999), Bonanno, Lillo and Mantegna (2001), among others, used the Pearson correlation coefficient as similarity measure of a pair of stock returns. They computed a $k \times k$ matrix, where k is the number of stocks, with the $k(k-1)/2$ different pairs of correlation coefficients, and used the metric

$$d_{COR}(i, j) = \sqrt{2(1 - \widehat{\rho}_{xy})}, \tag{1}$$

where $\widehat{\rho}_{xy}$ is the correlation coefficient between the returns series i and j. Although this metric can be useful to ascertain the structure of stock returns movements, it has three important limitations: (i) it does not use the information about the autocorrelation structure of each stock return; (ii) it does not take into account the information about the return volatilities; and (iii) it cannot be used for comparison and grouping stocks with unequal sample sizes.

In this paper, we present a method for clustering analysis of financial time series without these drawbacks. First, we introduce a distance measure based on the generalized autoregressive conditional heteroskedasticity (GARCH) estimated models of the stock returns. We then investigate whether major international stock markets have similar volatility behavior. Previous studies have investigated the comovements of international equity returns by using mean correlations (see Longin and Solnik, 1995, Karolyi and Stulz, 1996, Mei and Ammer, 1996, Ramchmand and Susmel, 1998, Ball and Torous, 2000, and Morana and Beltratti, 2006), cointegration (see Arshanapalli and Doukas, 1993, Bessler and Yang, 2003, Syriopoulos, 2004, and Tahai, Rutledge and Karim, 2004), common factor analysis (see Engle and Susmel, 1993, and Hui, 2005), and other approaches. However, the problem of identifying similarities or dissimilarities in stock returns seems not to be enough explored in the empirical finance literature using cluster analysis.

The remainder of the paper is organized as follows. In Section 2, we introduce the parametric distance-based method for clustering of financial time series. In Section 3, we describe the data set used in this paper. In Section 4, we present the cluster analysis evidence for the empirical results. The final section summarizes the paper.

2. GARCH-feature based distance

We know that many of the recent finance time series theories are concerned with the conditional variance, or volatility, of a process. The volatility is a measure of the intensity of unpredictable changes in asset returns, so we can think of volatility as a random variable that follows a stochastic process. The task of any volatility model is to describe the historical pattern of volatility and possibly use this to forecast future volatility. Engle (1982) introduced the autoregressive conditional heteroskedasticity or ARCH(q) model assuming that the conditional variance depends on past volatility measured as a linear function of past squared values. The need of a long lag length q and the non-negativity conditions imposed in ARCH parameters led Bollerslev (1986) to propose a more parsimonious parameter structure model, the GARCH(p,q) model, defined by $\sigma_t^2 = c + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2$. In most applications, the simple GARCH(1,1) model has been found to provide a good representation of a wide variety of volatility processes as discussed in Bollerslev, Chou and Kroner (1992).

We now introduce a parametric approach for clustering of financial time series using the information about the estimated GARCH parameters. Suppose we fit a GARCH(1,1) model to both return series r_x and r_y. Let $L_x = (\widehat{\alpha}_x, \widehat{\beta}_x)$ and $L_y = (\widehat{\alpha}_y, \widehat{\beta}_y)$ be the vectors of the estimated ARCH and GARCH parameters and V_x and V_y the estimated covariance matrices, respectively. Building upon the work of Caiado, Crato and Peña (2007), a measure of distance between the volatilities of the return series $r_{t,x}$ and $r_{t,y}$ can be defined by

$$d_{GARCH}(x,y) = (L_x - L_y)' V^{-1} (L_x - L_y), \tag{2}$$

where $V = V_x + V_y$. It is straightforward to show that this measure satisfies all the usual properties of a metric (except the triangle inequality): (i) $d(x,y) \geq 0$; (ii) $d(x,y) = 0$ if $x = y$; and (iii) $d(x,y) = d(y,x)$. The advantages of this measure over other distance-based methods are that it conveys all the stochastic structure of the conditional variance of a process and it solves the problem of comparison of time series with unequal length. We also should note that the proposed distance measure can be easily extended to larger GARCH models and to other type of volatility models.

3. Data description

We consider data of daily index returns for 27 international stock markets from Americas (Brazil, Argentina, United States and Canada), from Asia/Pacific (India, Hong-Kong, Indonesia, Malaysia, Korea, Japan, Singapore, Taiwan, and Australia), from Europe (Netherlands, Austria, Belgium, France, Germany, United Kingdom, Spain, Italy, Sweden, Norway, and Switzerland), and from Middle East (Egypt and Israel), as reported in Table 1. These data were obtained from Yahoo Finance (http://finance.yahoo.com) and correspond to the adjusted close prices.

Table 2 contains the GARCH(1,1) estimates used to compute the volatility-based metric defined in (2). The sum of the ARCH and GARCH coefficients quantifies the shock persistence to volatility. A value of unity indicates a unit root in the conditional variance (see Engle and Bollerslev, 1986). The ARCH test is the Lagranger multiplier test for ARCH effects in the residuals (see Engle, 1982). The Q (Q^2) is the Ljung-Box test statistic for serial correlation in the residuals (squared residuals). In the GARCH models, all estimated coefficients are significant at conventional levels and have the appropriate signs. The shock persistences to volatility are close to one for all the markets. For Malaysia and Egypt, the summation of ARCH and GARCH estimates is slightly higher than 1. The diagnostic tests show that the models for all the stock markets perform well in terms of the variance equation except Brasil, United Kingdom, Hong-Kong, and Mexico, which show evidence of ARCH effects in the fitted residuals.

4. Cluster analysis

To investigate the affinity between the major international stock markets, we perform a cluster analysis of the time series of daily stock-market indices using all available data (different sample sizes from 1966 to 2006) and using the sample period after the terrorist attack on Septemper 11, 2001 (equal sample sizes from 2001 to 2006). For each data set, we compute a distance matrix with $k(k-1)/2$ different pairs using the GARCH-based method. Then, by using dendrogram and multimidensional scaling techniques (see for instance, Johnson and Wichern, 1992) based on the computed distances, we display clusters for the return series.

Table 1
Daily indices of international stock markets

Stock market	Country	Period	Sample size
New York Stock Exchange	United States (US)	1966 - 2006	10259
TXS Venture Exchange	Canada (CAN)	2003 - 2006	1716
Sao Paolo Stock Exchange	Brazil (BRA)	1993 - 2006	3329
Buenos Aires Stock Exchange	Argentina (ARG)	1996 - 2006	2473
Mexico Stock Exchange	Mexico (MEX)	1991 - 2006	3721
Bombay Stock Exchange	India (IND)	1997 - 2006	2293
Hong Kong Stock Exchange	Hong-Kong (HK)	1987 - 2006	4890
Jakarta Stock Exchange	Indonesia (IND)	1997 - 2006	2233
Kuala Lumpur Stock Exchange	Malaysia (MAL)	1993 - 2006	3165
Korea Stock Exchange	Korea (KOR)	1997 - 2006	2277
Japan Stock Exchange	Japan (JAP)	1984 - 2006	5602
Singapore Stock Exchange	Singapore (SING)	1987 - 2006	4692
Taiwan Stock Exchange	Taiwan (TAI)	1997 - 2006	2277
Australian Stock Exchange	Australia (AUST)	1984 - 2006	5607
Amsterdam Stock Exchange	Netherlands (NET)	1992 - 2006	3557
Vienna Stock Exchange	Austria (AUS)	1992 - 2006	3437
Brussels Stock Exchange	Belgium (BEL)	1991 - 2006	3899
Paris Stock Exchange	France (FRA)	1990 - 2006	4185
Xetra Stock Exchange	Germany (GER)	1990 - 2006	4000
London Stock Exchange	United Kingdom (UK)	1984 - 2006	5687
Madrid Stock Exchange	Spain (SPA)	1993 - 2006	3321
Milan Stock Exchange	Italy (ITA)	2000 - 2006	1752
Stockholm Stock Exchange	Sweden (SWE)	2001 - 2006	1452
Oslo Stock Exchange	Norway (NOR)	2001 - 2006	1429
Swiss Stock Exchange	Switzerland (SWI)	1990 - 2006	4001
Egypt Stock Exchange	Egypt (EGY)	1997 - 2006	1815
Tel Aviv Stock Exchange	Israel (ISR)	1997 - 2006	1853

Table 2
Estimates for the international stock-market volatilities based on the GARCH(1,1) model

Market	ARCH	GARCH	Persistence	$Q(20)$	$Q^2(20)$	$LM(20)$	Size
United States	0.08017	0.90451	0.98468	230.40*	16.86	16.84	10258
Canada	0.05254	0.94309	0.99563	19.83	10.30	9.47	1715
Brazil	0.11041	0.87385	0.98426	79.23*	32.56**	32.01**	3328
Argentina	0.11909	0.85635	0.97544	44.47*	17.06	17.43	2472
Mexico	0.11416	0.86893	0.98309	122.55*	39.34*	38.88*	3721
India	0.11926	0.84775	0.96701	69.44*	15.99	15.72	2292
Hong-Kong	0.13476	0.84615	0.98091	95.12*	170.64*	181.59*	4889
Indonesia	0.13141	0.84919	0.98060	117.62*	18.37	17.73	2232
Malaysia	0.11713	0.88651	1.00364	130.43*	21.39	21.92	3164
Korea	0.07183	0.92705	0.99888	32.91**	8.42	7.94	2276
Japan	0.12237	0.87537	0.99774	32.57**	15.80	15.85	5601
Singapore	0.15544	0.80763	0.96307	104.46*	3.46	3.50	4691
Taiwan	0.07212	0.92181	0.99393	22.86	28.96	27.91	2276
Australia	0.22299	0.69768	0.92067	91.10*	7.79	7.80	5606
Netherlands	0.09020	0.90293	0.99313	31.95**	28.36	29.59	3556
Austria	0.09491	0.86437	0.95928	72.41*	16.23	16.55	3436
Belgium	0.10615	0.86154	0.96769	72.74*	4.30	4.24	3898
France	0.07682	0.90647	0.98329	24.58	15.40	15.69	4184
Germany	0.07827	0.90359	0.98186	27.53	3.10	3.02	3999
United Kingdom	0.09030	0.89146	0.98176	35.20**	58.11*	56.61*	5686
Spain	0.09379	0.89372	0.98751	30.24	14.16	13.35	3320
Italy	0.08154	0.91051	0.99205	17.83	25.34	24.59	1751
Sweden	0.09499	0.89237	0.98736	18.39	18.24	18.37	1451
Norway	0.12810	0.80014	0.92824	29.03	19.02	19.09	1428
Switzerland	0.12178	0.83409	0.95587	28.78	4.19	4.17	4000
Egypt	0.18812	0.85540	1.03352	101.34*	10.84	10.37	1814
Israel	0.09684	0.81474	0.91158	25.23	16.57	16.19	1852

* (**) Significant at the 1% (5%) level.

Table 3
Stock-markets cluster features (all data available)

Cluster	Mean×100	SD×100	Skewness	Kurtosis	Size	ARCH	GARCH	Persistence
AUST	0.014	0.29	-0.59	6.98	5606	0.22299	0.69768	0.92067
2	0.022	0.58	-0.25	7.71	3348	0.12832	0.85099	0.97854
1	0.007	0.56	-0.18	6.35	3687	0.09249	0.88538	0.97787

4.1. All available data from 1966 to 2006

Figure 1 represents the map of distances across international stock markets using the 2-dimensional GARCH scaling. We found that all the markets are nearly at the same first coordinate except Australia, United States, and Canada. Looking at the second coordinate, we appear to have the major European markets close together, the South/Middle American markets are at the same position, and some Asian/Pacific markets are at the same location. Figure 2 shows the dendrogram for the stock markets by complete linkage algorithm from which the clusters of markets can be identified. The GARCH-based metric can split the indices returns into two very reasonably clusters: Cluster 1 = (SPA, SWE, UK, FRA, GER, US, AUS, ISR, NET, ITA, KOR, TAI, CAN), Cluster 2 = (BEL, IND, ARG, NOR, SWI, , BRA, MEX,HK, INDO, SING, EGY, MAL, JAP), and separate from the others the AUST index return. Cluster 1 includes eight of the major European markets (France, Germany, Italy, United Kingdom, Netherlands, Spain, Austria, and Sweden), the North American countries (United States and Canada), Israel, Korea and Taiwan. Cluster 2 includes the South/Middle American markets (Brazil, Mexico, and Argentina), six of the major Asian/Pacific markets (Japan, Malaysia, Hong-Kong, India, Indonesia, and Singapore), three European countries (Switzerland, Belgium, and Norway), and Egypt. Australia is not grouped.

Table 3 provides the relevant statistics (mean, standard deviations, skewness, kurtosis coefficients, and sample size of the return series) and volatility features (ARCH, GARCH and persistence) for the identified stock-markets clusters. Clusters are ordered by increasing shocks persistences to volatility. We can see that ARCH and GARCH average estimates are negatively correlated across the three clusters. Clusters 1 and 2 have similar univariate statistics in terms of the level returns, including sample size.

4.2. Sample period from September 11, 2001 to 2006

Figure 3 shows the map of distances across stock markets in the sample period from September 11, 2001 to 2006. We appear to have most developed countries United States, Canada, Australia, Germany and Japan close to each other, and close to European countries United Kingdom, France, Spain, Netherlands and Italy. Looking at the dendrogram presented in Figure 4, we found three distinct clusters: Cluster 1 includes eight European countries (Germany, France, Spain, Netherlands, United Kingdom, Switzerland, Belgium and Sweden), Japan, Singapore, Korea, Israel and Argentina; Cluster 2 includes United States, Canada, Australia, Italy, Taiwan, Hong-Kong, Egypt and Brazil; and Cluster 3 includes Austria, Norway, Malaysia, India, Indonesia and Mexico.

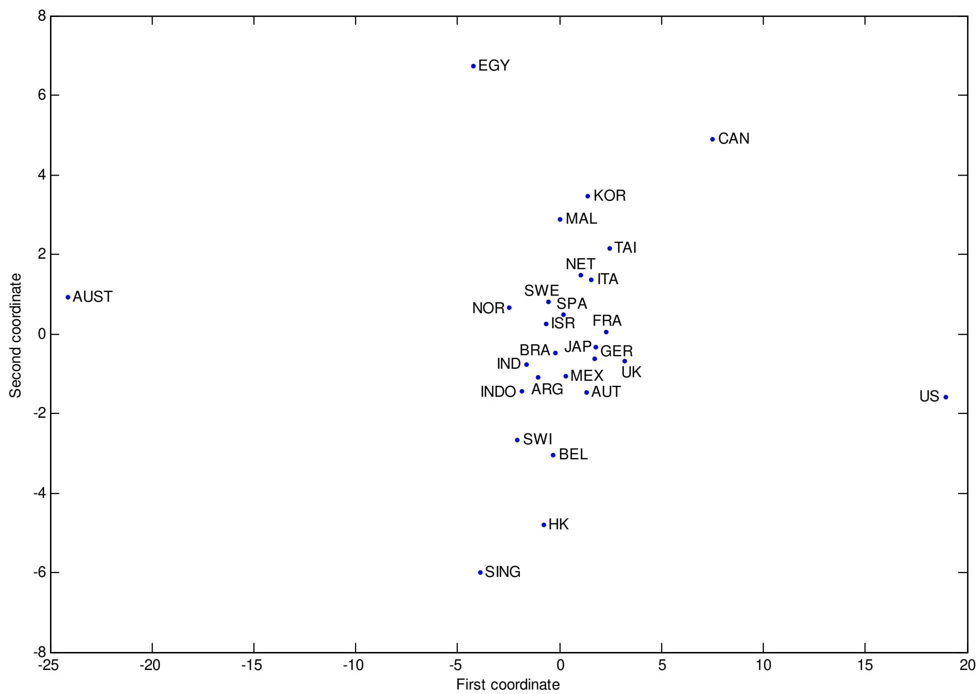

Figure 1. Plot of 2-dimensional scaling of the major international stock markets using the GARCH method (all available data)

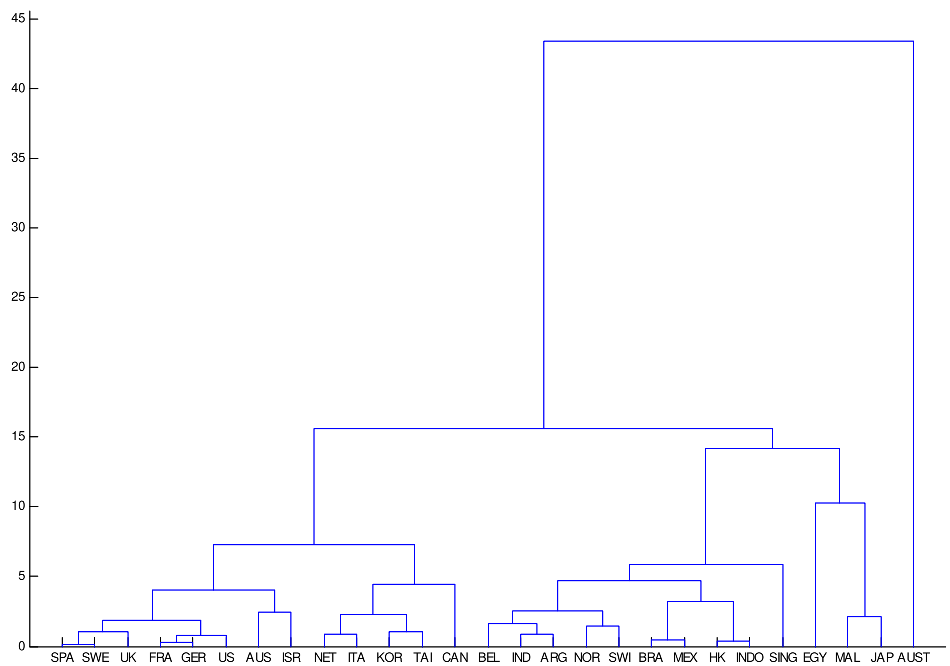

Figure 2. Dendrogram of the major international stock markets using the GARCH method (all available data)

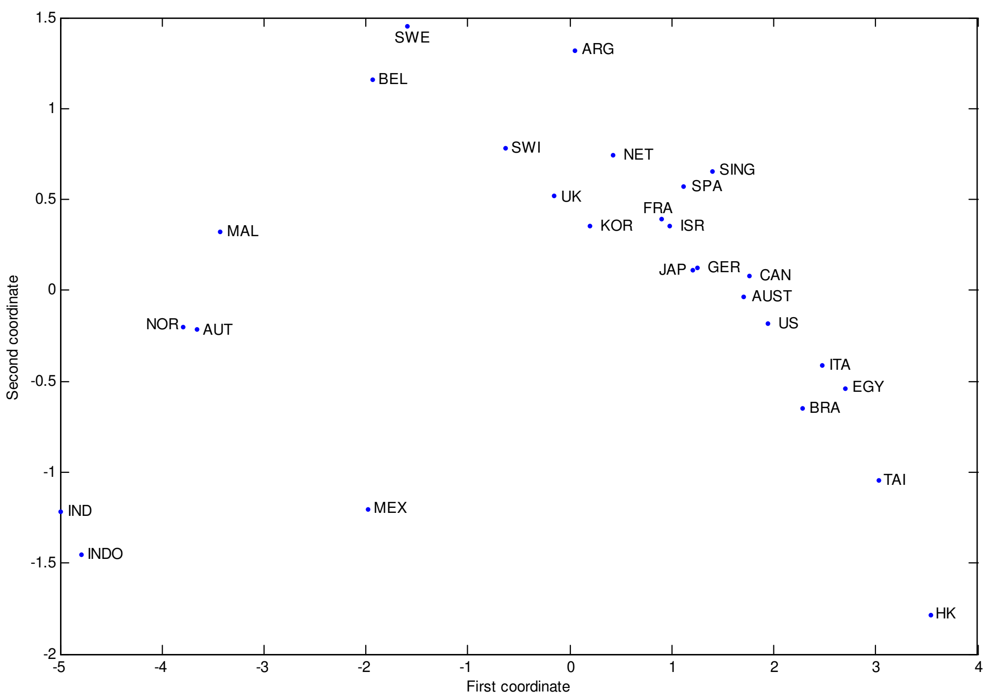

Figure 3. Plot of 2-dimensional scaling of the major international stock markets using the GARCH method (after September 11, 2001)

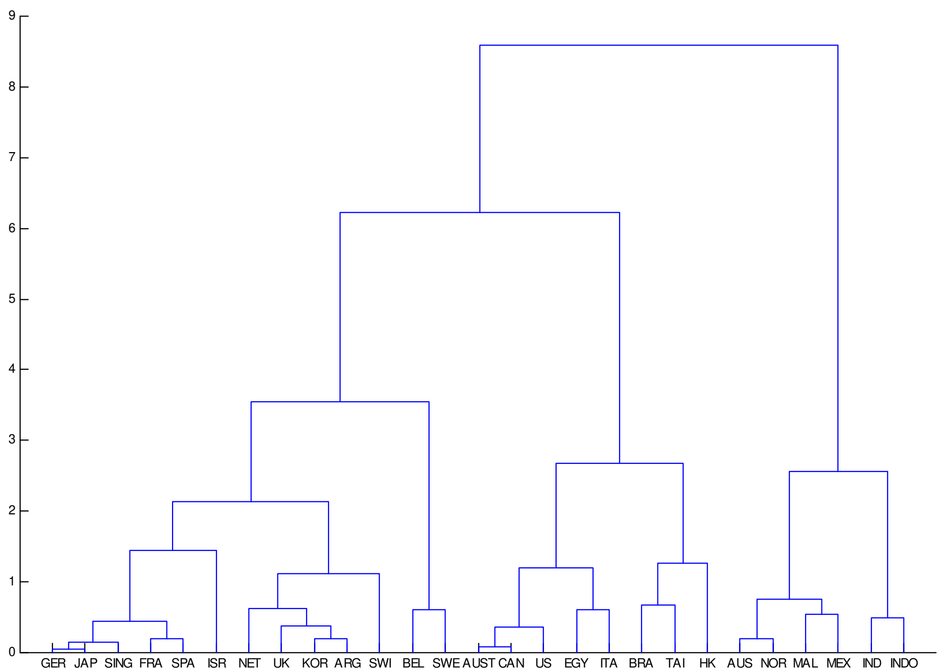

Figure 4. Dendrogram of the major international stock markets using the GARCH method (after September 11, 2001)

5. Conclusions

In this paper, we introduced a volatility-based method for comparison of financial time series, and we investigated the similarities among major international and stock-markets returns. The proposed method takes into account the stochastic volatility dependence of the processes and solves the problem of classification of time series with unequal length.

We performed a cluster analysis for daily stock indices returns with unequal sample sizes from 1966 to 2006. In our empirical study, we found that the persistence estimates are very similar for all stock markets except Australia, which makes it hard to identify dissimilarities among the stock market volatilities. However, using the GARCH-feature based method, we found two clear clusters. One cluster is formed by most European markets (France, Germany, Italy, United Kingdom, Netherlands, Spain, Austria, and Sweden), United States, Canada, Israel, Korea and Taiwan. The other is formed by South/Middle American markets (Brazil, Argentina, and Mexico), the major Asian/Pacific markets (Japan, Hong-Kong, India, Malaysia, Indonesia, and Singapore), Egypt and some European countries. The results are slightly different in the sample period from 2001 to 2006. The European countries seem to become more homogenous after the terrorist attacks on September 11, in part due to the euro area markets integration, and the United States, Canada, Australia and Japan markets tend to cluster together.

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