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Naito, Katsuyuki

Graduate School of Economics, Kyoto University, Japan

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Katsuyuki Naito†

Graduate School of Economics, Kyoto University

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Abstract

We consider an overlapping generations model with public education and social security where the overall size of these policies is determined in a repeated voting game. We investigate the interaction between the politically determined policies and economic development in a Markov perfect equilibrium. The following results are obtained. First, the level of human capital determines whether these policies are sustained in the Markov perfect equilibrium. Second, if the level of initial human capital is sufficiently high, human capital grows forever. In contrast, if the level of initial human capital is low, the economy might be caught in a poverty trap.

JEL classification : H28; H55; O16

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†E-mail address: k.naito.71@gmail.com

1 Introduction

In nearly every country, the government provides public education. Public education contributes to the accumulation of human capital, and thus, a large body of literature investigates its effects on economic development [1]. In addition to its impact on economic development, public education has another important feature: it is considered to be a redistribution policy from old generations to young ones, because it imposes upon parents expenditure on their children's education.

In democratic countries, the size of public policies is determined through political processes. Whether public education is politically sustained depends on the degree of parents' altruism toward their children. If individuals are non-altruistic and do not care about their children, they have no incentive to bear the cost of education, and public education is unlikely to be supported. In such a situation, how can we make public education politically sustainable? How far does its political sustainability depend on the economic environment? Furthermore, how does politically determined public education affect the process of economic development? This paper is motivated by these questions.

In this paper, we adopt a pay-as-you-go social security system that redistributes income from young generations to old ones as a political means to sustain public education. If the government implements pay-as-you-go social security combines with public education, parents might support public education, as it raises their children's productivity and would increase parents' social security benefits. In this paper, we consider a situation where the overall size of public education and pay-as-you-go social security is determined in a political process. Furthermore, to investigate in a simple manner the interaction between politically determined public policies and economic development, we focus on a Markov perfect equilibrium as a politico-economic equilibrium.

The importance of the interaction between public education and pay-as-you-go social security is recognized, and hence, many studies analyze situations where these policies coexist. Kaganovich and Zilcha (1999) consider a situation where the government allocates tax revenue between public education and social security, and they investigate the effects of the allocation rules

[1] de la Croix and Michel (2002) survey the recent literature investigating the effects of public education in overlapping generations economies.

on economic development. Boldrin and Montes (2005) investigate the role of these policies from the viewpoint of economic efficiency. In both studies, however, the size of these policies is given exogenously, and thus, it remains to be considered how the size of these policies is determined.

Some authors investigate the size of these policies in political economies. Kemnitz (2000) considers a situation where individuals affect the decisions of the government through voting and lobbying activities. In his paper, however, it is assumed that the government does not take into consideration any effect that current policies might have on future policies. Rangel (2003) analyzes the interaction between forward intergenerational goods (for instance, public education) and backward intergenerational goods (such as social security) in a repeated voting game. He shows that the backward intergenerational goods are necessary to sustain the forward intergenerational goods in politico-economic equilibria.

Rangel (2003) adopts trigger strategies as individuals' voting strategies, and thus there are numerous subgame perfect equilibria. Therefore it is difficult to predict the degree to which public policies are politically implemented [2]. In contrast to his study, in this paper we focus on a Markov perfect equilibrium. In the Markov perfect equilibrium, the size of public policies is represented as a function of the state variable of the economy, and hence, we can explicitly investigate the following questions:

1. How does the size of public policies depend on the economic environment in the politico-economic equilibrium?

2. How do the politically determined public policies affect the process of economic development?

We consider a simple overlapping generations economy where individuals with non-altruistic preferences live for three periods (young, middle and old). They receive public education when young, pay lump-sum tax when middle, and receive social security benefits when old. The population size of each generation grows at a constant and positive rate. We assume a small open economy, and focus on the accumulation of human capital as an engine of

[2] Assuming heterogeneity in each generation, Poutvaara (2006) analyzes the characteristics of public education and social security in the policico-economic equilibria. He also adopts trigger strategies, and thus, in his study there are numerous subgame perfect equilibria.

economic development. A child's human capital depends on public education and parental human capital. Furthermore, we assume that there exists complementarity between these factors. That is, the marginal productivity of public education expenditure increases with parental human capital. The government allocates tax revenue between public education expenditure for the young and social security benefits for the old. The size of lump-sum tax is determined in a repeated majority voting game where only the middle and the old are given voting rights. Since the rate of population growth is assumed to be positive, the majority in the voting game are the middle in each period. When voting, the middle takes into account the fact that current lump-sum tax will affect social security benefits in the next period. In our model, human capital is the only state variable, and hence, the overall size of public policies is represented as a function of human capital in the Markov perfect equilibrium. In such a set-up, we obtain the following results.

1. When the level of human capital is sufficiently high, public education and social security are sustained in the Markov perfect equilibrium, because the marginal productivity of public education is sufficiently high and hence, there exists a benefit to support these policies. In contrast, when the level of human capital is low, these policies cannot be sustained because of the low marginal productivity of education.

2. When the level of the initial human capital is sufficiently high, these policies continue to be sustained, and thus, human capital grows forever. In contrast, when the level of the initial human capital is low, these policies cannot be sustained, and hence, the economy might be caught in a poverty trap.

Lastly, we briefly analyze the effects of a decline in the rate of population growth on the sustainability of public policies and the pattern of economic development. This would raise the productivity of public education and increase the likelihood that public policies would continue to be politically sustained, implying that a declining birth rate might be a necessary condition for long-run economic development.

This paper is organized as follows. In section 2, we describe the basic structure of the model and then characterize the competitive equilibrium. In section 3, we consider the repeated voting game and analyze the characteristics of the politico-economic equilibrium. We present our conclusions in section 4. Appendix gives the proof of proposition 1.

2 The competitive equilibrium

The basic environment

We consider a small open overlapping generations economy where individuals live for three periods (young, middle and old). Within each generation, individuals are homogeneous. We denote by N_t the size of population born in period t. The population of each generation grows at a constant and positive rate $n > 0$. This implies that $N_{t+1} = (1+n)N_t$.

Firms

Production take place according to neoclassical production technology using physical and human capital. The production technology is represented as a constant returns to scale production function

$$Y = F(K, H) = f(\kappa)H; \quad \kappa \equiv \frac{K}{H},$$

where Y is output, K and H are inputs of physical and human capital respectively. The function f is strictly increasing, strictly concave, and satisfies the Inada conditions. All markets are competitive. The competitive equilibrium conditions imply that

$$R = f'(\kappa_t), \tag{1}$$

$$w_t = f(\kappa_t) - f'(\kappa_t)\kappa_t, \tag{2}$$

where R is the world interest rate and w_t is the wage rate per unit of human capital. From (1) and (2), we obtain,

$$\kappa_t = (f')^{-1}(R) \equiv \kappa,$$

$$w_t = f(\kappa) - f'(\kappa)\kappa \equiv w.$$

Therefore, the wage rate is constant over time.

Individuals

Individuals are non-altruistic and draw utility from consumption when middle and old. Each individual born in period $t-1$ has the following utility function:

$$U(c_{1t}, c_{2t+1}) = u(c_{1t}) + \beta u(c_{2t+1}); \quad \beta \in (0, 1),$$

where c_{1t} and c_{2t+1} are consumption when middle and old, respectively, and β is the discount factor. Consumption when young is ignored for simplicity. The instantaneous utility function u is strictly increasing, strictly concave, and satisfies the Inada conditions.

When young, individuals are educated in the public education system and accumulate human capital. In this period, they make no decisions. When middle, they supply human capital inelastically and earn the wage. In this period, lump-sum tax is levied on them. They allocate their disposable income between consumption and savings:

$$c_{1t} + s_t = wh_t - \tau_t,$$

where s_t is savings, h_t is human capital, and τ_t is lump-sum tax. When old, they retire and consume the proceeds of their savings and social security benefits:

$$c_{2t+1} = Rs_t + b_{t+1},$$

where b_{t+1} is the social security benefit. Their intertemporal budget constraint is

$$c_{1t} + \frac{c_{2t+1}}{R} = wh_t - \tau_t + \frac{b_{t+1}}{R}. \tag{3}$$

The right-hand side of (3) represents their lifetime income. Individuals choose consumption and savings in order to maximize their utility subject to their intertemporal budget constraint, taking h_t, w, R, τ_t and b_{t+1} as given. Since they can save or borrow freely, their indirect utility is a strictly increasing function of their lifetime income.

Government

In each period, the government levies lump-sum tax on the middle. The government allocates tax revenue between public education and social security. Because individuals are homogeneous within each generation, neither the social security nor the public education redistributes income intragenerationally. We denote by $x \in [0, 1]$ the fraction of tax revenue allocated to public education. The budget constraints of the government are

$$N_t e_t = xN_{t-1}\tau_t,$$

$$N_{t-2}b_t = (1 - x)N_{t-1}\tau_t,$$

where e_t is per capita public education expenditure. From the budget constraints, we obtain

$$e_t = \frac{x}{1+n}\tau_t, \tag{4}$$

$$b_t = (1-x)(1+n)\tau_t. \tag{5}$$

Throughout this paper, we assume that the fraction x is fixed over time [3].

Human Capital Production

A child's human capital depends on public education and parental human capital. We assume a Stone-Geary type human capital production function represented as

$$h_{t+1} = G(e_t, h_t) = (e_t+\mu)^\theta h_t^{1-\theta}; \quad \mu > 0; \quad \theta \in (0,1). \tag{6}$$

This specification has the following features:

$$G(0,h) = \mu^\theta h^{1-\theta} > 0, \tag{7}$$

$$\frac{\partial^2 G}{\partial h \partial e}(e,h) = \theta(1-\theta)\frac{G(e,h)}{(e+\mu)h} > 0, \tag{8}$$

$$\lim_{e\to 0}\frac{\partial G}{\partial e}(e,h) = \theta\left(\frac{h}{\mu}\right)^{1-\theta} < \infty. \tag{9}$$

Equation (7) implies that even if there is no public education, a child's human capital cannot be zero. Equation (8) implies complementarity between public education and parental human capital. That is, the marginal productivity of public education is increasing in parental human capital. Equation (9) implies that the production function does not satisfy the Inada condition. That is, even if public education expenditure is close to zero, the marginal productivity of public education remains finite.

From (4) and (6), a child's human capital is represented as a function of the lump-sum tax levied on parental generation and parental human capital:

$$h_{t+1} = \left(\frac{x}{1+n}\tau_t + \mu\right)^\theta h_t^{1-\theta} \equiv \hat{G}(\tau_t, h_t). \tag{10}$$

[3] The effect of a change in the fraction x on the politico-economic equilibrium is investigated in Section 3.

3 The politico-economic equilibrium

In this section, we investigate characteristics of the politico-economic equilibrium. We are interested in the political sustainability of public policies, and hence assume that the size of lump-sum tax, that is, the overall size of public policies, is determined in a majority voting game in each period. In many countries, young educated individuals do not participate in elections, and thus, we assume that voting rights are applicable only to the middle and the old.

In contrast to previous literature, we focus on a Markov perfect equilibrium. This enables us to investigate explicitly how the size of public policies depends on the economic environment in the politico-economic equilibrium, and furthermore, how public policies affect the process of economic development. As human capital is the only state variable, in the Markov perfect equilibrium, lump-sum tax in period t depends on human capital in period t:

$$\tau_t = \tau(h_t).$$

Individuals vote on the size of current lump-sum tax to maximize their lifetime income. When voting, they take into account the fact that the current lump-sum tax affects the level of human capital of their children, and thus, might affect the social security benefits in the next period.

The rate of population growth is assumed to be positive, and hence, the majority in the voting game are the middle in each period. Therefore, we analyze the voting behavior of the middle. The lifetime income of the middle in period t is

$$\begin{aligned} V(\tau_t, h_t) &= wh_t - \tau_t + \frac{1}{\theta g}\tau(h_{t+1}) \\ &= wh_t - \tau_t + \frac{1}{\theta g}\tau[\hat{G}(\tau_t, h_t)], \end{aligned} \tag{11}$$

where $g \equiv R/[\theta(1-x)(1+n)]$. The Markov perfect equilibrium tax function $\tau(h)$ must satisfy the following functional equation

$$\tau(h) = \underset{0\le\tau\le wh}{\text{argmax}} \left\{ wh - \tau + \frac{1}{\theta g}\tau[\hat{G}(\tau, h)] \right\}. \tag{12}$$

The first order condition of the optimization problem is

$$-1 + \frac{1}{\theta g}\tau'[\hat{G}(\tau, h)]\frac{\partial \hat{G}}{\partial \tau}(\tau, h) \le 0. \tag{13}$$

The first term of the left-hand side of (13) represents the cost of increasing current lump-sum tax, which results from the reduction of the disposable income of the middle. In contrast, the second term represents the benefit of increasing current lump-sum tax. An increase in current lump-sum tax implies an increase in children's human capital, which implies an increase in social security benefits in the next period.

In our set-up, there exist multiple Markov perfect equilibria. For instance, if lump-sum-tax and social security benefits did not exist in the next period, the middle would not have any incentive to support public policies in the current period. Therefore the situation where lump-sum tax is represented as

$$\forall h > 0 \;:\; \tau(h) = 0$$

is a Markov perfect equilibrium. However, this equilibrium is obviously uninteresting. In this paper, we focus on a situation where lump-sum tax is represented as a quasi linear function of human capital in the Markov perfect equilibrium, to make the model tractable and obtain some interesting results. On the Markov perfect equilibrium, we obtain the following proposition.

Proposition 1. *If the parameters satisfy*

$$0 < x < 1; \quad 1 < g < \frac{1}{\theta}; \quad \frac{(1+n)g^{\frac{1}{\theta}}}{x} < w,$$

there exists a Markov perfect equilibrium where lump-sum tax is represented as

$$\tau(h) = \begin{cases} 0 & \text{if } 0 \leq h \leq \phi \\ \hat{\tau}(h) \equiv \frac{1+n}{x}\left(g^{\frac{1}{\theta}}h - \mu\right) & \text{if } h > \phi \end{cases}, \tag{14}$$

where

$$\phi \equiv \frac{\mu}{g^{\frac{1}{\theta}}}\frac{\theta}{1-\theta}\left(\frac{1}{\theta g} - 1\right) > 0.$$

Proof. See Appendix. □

The equilibrium tax function given by (14) has the following features. First, when the level of human capital is sufficiently high, the equilibrium tax is positive and is represented as an increasing linear function of human capital. The intuition is as follows. Suppose that the equilibrium tax function in the next period is a linear increasing function of human capital:

$$\tau' = \alpha_1 + \alpha_2 \hat{G}(\tau, h),$$

where α_1 and $\alpha_2 > 0$ are constant variables. From (13), the cost and the benefit of increasing current lump-sum tax are respectively

$$C(\tau, h) = 1,$$

$$B(\tau, h) = \frac{\alpha_2}{\theta g} \frac{\partial \hat{G}}{\partial \tau}(\tau, h).$$

The cost is independent of the level of human capital. In contrast, because of the complementarity between public education and parental human capital, the benefit is increasing in human capital:

$$\frac{\partial B}{\partial h}(\tau, h) = \frac{\alpha_2}{\theta g} \frac{\partial^2 \hat{G}}{\partial h \partial \tau}(\tau, h) > 0.$$

The higher the level of human capital, the larger the current lump-sum tax equalizing the cost with the benefit. In fact, by solving the equation $C(\tau, h) = B(\tau, h)$ with respect to τ, we obtain a linear increasing function of human capital (see (10)). Therefore the equilibrium lump-sum tax increases linearly with human capital.

Second, when the level of human capital is low, the equilibrium lump-sum tax is zero, and thus public policies cannot be sustained. If the level of human capital is low, the benefit of increasing current lump-sum tax is extremely low, and there is no positive current lump-sum tax to equalize the cost with the benefit.

Proposition 1 implies that if the government allocates tax revenue to both public education and pay-as-you-go social security (i.e., $0 < x < 1$), these policies can be politically sustained. However, can public education or pay-as-you-go social security be politically sustained in isolation? Here we consider two situations. The first case is one in which the government allocates all the tax revenue to social security benefits (i.e., $x = 0$). In this case, current lump-sum tax does not contribute to the accumulation of children's human capital, and thus, does not affect the size of social security benefits in the next period. As a result, there is no benefit from increasing the lump-sum tax. Hence, the social security system, in isolation, cannot be sustained in any Markov perfect equilibrium [4]. The second case is one

[4] We assume a small open economy, and thus, the size of public policies does not affect the return of savings. In contrast to this paper, Forni (2005) assumes a closed economy and analyzes characteristics of social security in Markov perfect equilibria. In the closed

in which the government allocates all the tax revenue to public education expenditure (i.e., $x = 1$). In this case, there is apparently no benefit from increasing current lump-sum tax. Thus the public education system, in isolation, cannot be sustained in any Markov perfect equilibrium [5]. These results are summarized in Corollary 1.

Corollary 1. *If the government allocates all the tax revenue to social security benefits, social security cannot be sustained in any Markov perfect equilibrium. If it allocates all the tax revenue to public education, public education cannot be sustained in any Markov equilibrium.*

4 Patterns of Economic Development in the Markov Perfect Equilibrium

In this section, we analyze the patterns of economic development in the Markov perfect equilibrium described in Proposition 1. By substituting $\tau(h)$ of (14) into (10), we obtain the dynamic equation of the equilibrium human capital:

$$h_{t+1} = \begin{cases} J_1(h_t) \equiv \mu^\theta h_t^{1-\theta} & \text{if } 0 \leq h_t \leq \phi \\ J_2(h_t) \equiv g h_t & \text{if } h_t > \phi \end{cases} . \tag{15}$$

When the level of human capital is below ϕ, public policies cannot be sustained, and human capital is accumulated according to the function J_1. In contrast, when the level of human capital is above ϕ, these policies are sustained, and human capital is accumulated according to the function J_2.

Patterns of economic development depend on exogenous parameters. Figure 1(a) depicts the pattern of human capital accumulation when $\mu < \phi$. Note that ϕ is the threshold of human capital whether or not public policies are politically sustained. In this case, there exists a locally stable steady state $h^* = \mu$, in which public policies do not exist.

If the level of the initial human capital is below ϕ, human capital monotonically converges to the steady state. On the transition path, public policies cannot be sustained in the political process, and the economy is caught in a poverty trap. If the level of the initial human capital is above ϕ, public

economy, an increase in the level of social security reduces the level of savings, raising the return of savings (general equilibrium effect). This effect makes social security politically sustainable.

[5]This result is similar to that of Rangel (2003).

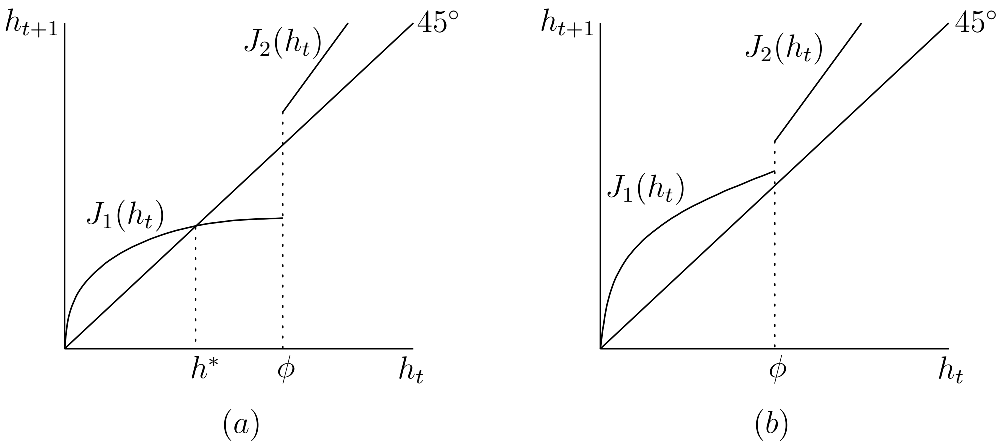

Figure 1: (a) and (b) depict the pattern of the equilibrium human capital accumulation when $\mu < \phi$ and when $\mu > \phi$ respectively.

policies continue to be sustained, and human capital grows forever at the rate of g.

Figure 1(b) depicts the pattern of human capital accumulation when $\mu > \phi$. In this case, there is no steady state and the process of human capital accumulation is different from that of the above case. Even if the level of the initial human capital is below ϕ, human capital grows without public education, and at some period, the level of human capital becomes higher than ϕ. After that period, public policies are sustained, and human capital grows forever.

These results are summarized in Proposition 2.

Proposition 2. *The process of human capital accumulation in the Markov perfect equilibrium described in proposition 1 is given by (15). Patterns of economic development depend on the exogenous parameters.*

- *When $\mu < \phi$, the economy with $h_0 < \phi$ is caught in poverty trap, whereas the economy with $h_0 > \phi$ grows forever.*
- *When $\mu > \phi$, even if the level of the initial human capital is below ϕ, the economy grows forever.*

Lastly, we analyze the effects of exogenous parameters, especially the rate of population growth n and the fraction of tax revenue allocated to public education x, on the pattern of human capital accumulation. To this end, we first investigate how these parameters affect the threshold ϕ. By a simple calculation, the threshold ϕ is shown to be increasing in the rate of population growth n:

$$\frac{\partial \phi}{\partial n} = \frac{\partial \phi}{\partial g}\frac{\partial g}{\partial n} = \frac{\mu R}{\theta(1-\theta)(1-x)(1+n)^2 g^{\frac{1+\theta}{\theta}}}\left(\frac{1}{g}+\frac{1}{\theta g}-1\right) > 0.$$

A decline in population growth has a positive effect on public education as it allows each child to obtain more education. Hence, the benefit of increasing the current lump-sum tax becomes higher, and the threshold decreases. When the rate of population growth is high, the realm where public policies are sustained in the Markov perfect equilibrium shrinks, and the threshold ϕ is likely to be larger than μ. Therefore, the economy with a low level of initial human capital and high population growth rate results in a poverty trap. In contrast, when the population growth rate is low, the threshold ϕ is likely to be less than μ. In this case, regardless of initial human capital, the economy will grow forever. This implies that a decline in population growth can protect the economy from a poverty trap.

Next, we analyze the effects of the fraction x on the pattern of economic development. The threshold ϕ is shown to be decreasing in the fraction x:

$$\frac{\partial \phi}{\partial x} = \frac{\partial \phi}{\partial g}\frac{\partial g}{\partial x} = -\frac{\mu R}{\theta(1-\theta)(1-x)^2(1+n) g^{\frac{1+\theta}{\theta}}}\left(\frac{1}{g}+\frac{1}{\theta g}-1\right) < 0.$$

The intuition is similar to that of the comparative statics with respect to the rate of population growth. A rise in the fraction allows each child to obtain more public education without increasing the lump-sum tax. As a result, the benefit of increasing current lump-sum tax becomes higher, and the threshold ϕ becomes smaller. From this comparative statics, we obtain a policy implication: even though the rate of population is high, if the government is capable of setting the fraction x high, it can prevent the economy from being caught in a poverty trap.

5 Conclusion

We consider an overlapping generations economy where the overall size of social security and public education are determined in a repeated voting game, and investigate the interaction between the politically determined public policies and economic development. In contrast to the previous literature, we focus on a Markov perfect equilibrium as a politico-economic equilibrium. Unlike previous literature, we can explicitly investigate how the overall size of public policies depends on economic environment, and how these policies affect the process of economic development.

In this paper, we obtain the following results. First, while public policies are sustained in the Markov perfect equilibrium when the level of human capital is sufficiently high, these policies cannot be sustained when the level of human capital is low. Second, there are certain patterns to the process of human capital accumulation. The economy with a high level of initial human capital grows forever, whereas that with a low level of initial human capital might be caught in a poverty trap.

We conclude this paper with the areas for possible further study. In this paper, we assume that individuals are homogenous within each generation, and thus public policies do not redistribute income intragenerationally. However, when considering the roles of these policies, it might be important to investigate the intragenerational redistributive effects of these policies. Fernandez and Rogerson (1995) analyze the interaction between public education subsidies and income inequality in a political economy. Casamatta et al. (2000) consider an overlapping generations model in which individuals are heterogeneous with respect to their productivity and investigate the effects of social security as an intergenerational and intragenerational income redistribution policy. Furthermore, we assume a small open economy, and thus, do not consider the effects of public policies on the process of physical capital accumulation. As mentioned in standard textbooks such as Blanchard and Fischer (1989), social security affects saving behaviors of individuals and the process of physical capital accumulation. Lastly, we assume that the government maintains a balanced budget. If it can issue public debt, it can prevent the economy from being caught in a poverty trap by issuing public debt and investing in public education.

6 Appendix : The Proof of Proposition 1

First, we guess that the equilibrium tax function in the next period is linear with respect to the human capital in the next period:

$$\tau' = \alpha_1 + \alpha_2 \hat{G}(\tau, h), \tag{16}$$

where α_1 and α_2 is unknown variables. Given (16), the function V is represented as

$$V(\tau, h) = wh - \tau + \alpha_1 \frac{1}{\theta g} + \alpha_2 \frac{1}{\theta g} \hat{G}(\tau, h),$$

and the first order condition is

$$-1 + \alpha_2 \frac{1}{\theta g} \frac{\partial \hat{G}}{\partial \tau}(\tau, h) = 0. \tag{17}$$

Solving (17) with respect to τ, we obtain

$$\tau = \frac{1+n}{x} \left[\left(\frac{\alpha_2}{g} \frac{x}{1+n} \right)^{\frac{1}{1-\theta}} h - \mu \right] = \alpha_1 + \alpha_2 h.$$

Comparing the coefficients, we obtain

$$\alpha_1 = -\frac{1+n}{x}\mu \ , \ \alpha_2 = \frac{1+n}{x} g^{\frac{1}{\theta}}. \tag{18}$$

Here we set a candidate of an equilibrium tax function such that

$$\hat{\tau}(h) \equiv \frac{1+n}{x} \left(g^{\frac{1}{\theta}} h - \mu \right). \tag{19}$$

However, it is obvious that the guess given by (19) is incorrect because the value of $\hat{\tau}(h)$ is negative when $0 \leq h < \mu / g^{\frac{1}{\theta}}$.

Next, we guess that the equilibrium tax in the next period is represented as the following function

$$\tau' = \begin{cases} 0 & \text{if } 0 \leq \hat{G}(\tau, h) \leq \phi \\ \hat{\tau}[\hat{G}(\tau, h)] & \text{if } \hat{G}(\tau, h) > \phi \end{cases}, \tag{20}$$

where ϕ is a unknown variable. Given equation (20), the function V is represented as

$$V(\tau, h) = \begin{cases} V_1(\tau, h) \equiv wh - \tau & \text{if } 0 \leq \tau \leq \bar{\tau}(h) \\ V_2(\tau, h) \equiv wh - \tau + \frac{1}{\theta g} \frac{1+n}{x} \left[g^{\frac{1}{\theta}} \hat{G}(\tau, h) - \mu \right] & \text{if } \bar{\tau}(h) < \tau \leq wh \end{cases},$$

where

$$\bar{\tau}(h) \equiv \frac{1+n}{x} \left[\left(\frac{\phi}{h^{1-\theta}} \right)^{\frac{1}{\theta}} - \mu \right].$$

Note that, by solving the following equation $\partial V_2(\tau, h)/\partial\tau = 0$ with respect to τ, we obtain $\tau = \hat{\tau}(h)$.

From the properties of the function $\hat{\tau}$ and $\bar{\tau}$, we obtain the following equivalence relation

$$\hat{\tau}(h) > \bar{\tau}(h) \iff h > \frac{\phi}{g}.$$

Furthermore, from the properties of the function V_1 and V_2, we obtain the following equivalence relation

$$V_2[\hat{\tau}(h), h] > V_1(0, h) \iff h > \frac{\mu}{g^{\frac{1}{\theta}}} \frac{\theta}{1-\theta} \left(\frac{1}{\theta g} - 1 \right).$$

We set a candidate for ϕ such that

$$\phi = \frac{\mu}{g^{\frac{1}{\theta}}} \frac{\theta}{1-\theta} \left(\frac{1}{\theta g} - 1 \right).$$

From the properties of the function $\hat{\tau}$, $\bar{\tau}, V_1, V_2$ and the candidate ϕ, we obtain that:

$$\begin{array}{lll} \bar{\tau}(h) \leq \hat{\tau}(h) \ , & V_1(0,h) > V_2[\hat{\tau}(h), h] & \text{if } 0 \leq h \leq \frac{\phi}{g}, \\ \bar{\tau}(h) < \hat{\tau}(h) \ , & V_1(0,h) \geq V_2[\hat{\tau}(h), h] & \text{if } \frac{\phi}{g} < h \leq \phi, \\ \bar{\tau}(h) < \hat{\tau}(h) \ , & V_1(0,h) < V_2[\hat{\tau}(h), h] & \text{if } h > \phi. \end{array}$$

Figure 2(a), 2(b) and 2(c) depict the shape of the function V when $0 \leq h \leq \phi/g$, $\phi/g < h \leq \phi$, and $h > \phi$, respectively. From these figures, it is shown that:

- $\tau = 0$ is optimal when $0 \leq h \leq \phi/g$.
- $\tau = 0$ is optimal when $\phi/g < h \leq \phi$ [6].
- $\tau = \hat{\tau}(h)$ is optimal when $h > \phi$.

Therefore, if the equilibrium tax in the next period is given as equation (20), the optimal tax is represented as

$$\tau = \begin{cases} 0 & \text{if } 0 \leq h \leq \phi \\ \hat{\tau}(h) & \text{if } h > \phi \end{cases}.$$

This functional form is the same as (20), and thus (20) is shown to be a Markov perfect equilibrium tax function.

[6] When $h = \phi$, $\tau = 0$ and $\tau = \hat{\tau}(h)$ are indifferent. For simplicity, we assume that $\tau = 0$ is chosen in this case.

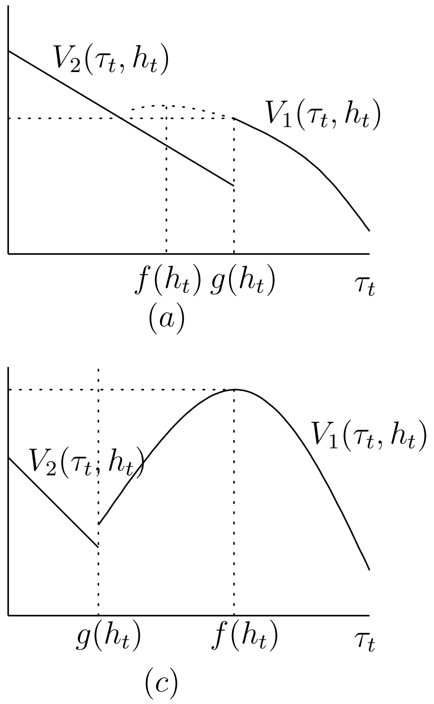

Figure 2: (a), (b) and (c) depict the shape of the function V when $0 \leq h_t \leq \phi/g$, when $\phi/g < h_t \leq \phi$ and when $h_t > \phi$, respectively.

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