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DIVERGENCES IN THE RESULTS OF STOCHASTIC AND DETERMINISTIC SIMULATION OF AN ITALIAN NON LINEAR ECONOMETRIC MODEL

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The importance of the simulation (both deterministic and stochastic) in the validation process of a non linear econometric model is underlined. Synthetic results of a large set of simulations on a non linear model of the Italian economy are presented. The benefits and the risks of the stochastic simulation are discussed, with particular emphasis on the problem of the existence of divergences in the results of the two methods of simulation.

1. INTRODUCTION

In econometric model building, many different steps can be identified, which are, in sequence, specification, identification, estimation, validation, experimental design and analysis of the results [11], [15]. Our attention is mainly focussed on particular aspects of validation, the step in which one must decide if the model has good predictive performances, both within and beyond the estimation period.

We recall that the validation process of a model usually involves two main steps: the first is based on the traditional statistics of the estimation phase (t test, R^2, Durbin-Watson test, etc.), all generally referring to the single equation; the second, which should be based on an analysis of the behaviour of the whole model as a simultaneous equation system, is usually performed in different ways according to the characteristics of the model. In fact, if the model is linear both in the parameters and in the variables, the validation can be accomplished by means of analytical methods, so that simulation techniques would add no further information about the validity of the model. If the model is non linear in the variables (but linear in the parameters, as most of the models we usually consider), an analytical solution cannot generally be obtained: numerical solutions (or simulations) are requested and validation techniques are based on the analysis of the simulation results [6]. We want to point out that, in this context, we intend as simulation "the synthetic representation of reality by sequential solution of a mathematical model, conditional on estimates of the model's parameters and on actual or supposed values of the exogenous variables" [15].

In this paper, we do not want to enter into details about the different types of simulation (one-step vs dynamic; control vs experimental; etc.); our attention will concentrate on a particular aspect of the use of stochastic simulation as an alternative to deterministic simulation in the validation process of a non linear model; this aspect, that will be discussed in details in section 5, is the possible existence of systematic divergences in the results of the two types of simulation [5].

2. METHODOLOGIES AND PURPOSES OF THE STOCHASTIC SIMULATION

A few considerations about deterministic and stochastic simulation are necessary at this point. The difference between these two types of simulation is related to the stochastic element which is inserted (for the reasons of this insertion, see Johnston [7]), generally in an additive way, in the behavioural equations in the specification and estimation phases. In the deterministic simulation, which is generally performed for convenience reasons, each disturbance is set equal to its expected value, which is zero. In the stochastic simulation, a random shock, drawn from a multivariate distribution which should reflect the stochastic properties of the true model as much as possible (in terms of equation variance, inter equation covariance and within equation serial correlation), is added to each behavioural equation each time the model is solved. As pointed out in [15], the principal characteristic of the stochastic simulation, likewise all the "Monte Carlo methods", is that the solution with random shocks can be replicated a number of times for each period in such a way as to produce a distribution of outcomes. This results, of course, in a major cost, but also in a greater deal of information about the behaviour of the model, that the deterministic simulation is not able to produce. To summarize, there are many different reasons to perform stochastic simulation:

(1) "The incorporation of random disturbances into a stable economic model could reproduce cyclical behaviour in the endogenous variables of the model" [14].
(2) "Abandoning the stochastic component in simulation experiments is certainly a methodological inconsistency" [3].
(3) Howrey and Kelejian [6] have analytically shown that deterministic simulations of non linear models can lead to results that are systematically different from the real properties of the model, in terms of reduced form equations.
(4) With more emphasis, Howrey states: "a deterministic simulation of a non linear model can deviate uniformly from a stochastic simulation of the same model" [5].
(5) By means of the empirical distribution of the results of stochastic simulations, it is possible to draw statistical inferences and to test some hypotheses [15].
(6) In economic policy experiments, it could be necessary to investigate the consequences of shocks attributed to the system under various policy regimes [3].

For all these reasons, it seems convenient to perform this kind of simulation even if its costs are much higher than the cost of the deterministic simulation.

3. THE MAIN FEATURES OF THE ANALYZED MODEL

The results presented in this paper refer to the stochastic simulation of a macro model of the Italian economy. The model, which has been originally developed at the University of Ancona by a team leaded by G Fua' [4], has been revised, updated and reestimated during the year 1974. The results of this estimation, presented in [1], have been used in this research.

The model consists of 38 equations, 16 of which are behavioural. As most of the post-keynesian models, it is mainly focussed on the real and fiscal sectors. Its main endogenous variables are: private consumption expenditure, imports, degree of capacity utilization in the industrial sector, added value in industry, changes in inventories, employment, wage rates, wholesale and retail prices, cost of living, etc. It has been estimated by means of the ordinary least squares method. The sample period, due to lack of data, is not the same for all the behavioural equations. The final year is always 1973, but the initial year varies from 1953 to 1959.

The model is based on yearly data and is dynamic for the presence of a large number of lagged (one and two periods) endogenous variables. Therefore, on this model both a one step (static) and a dynamic solution can be accomplished: in the former, the lagged values of the endogenous variables are set equal to the observed values; in the latter, the lagged values of the endogenous variables are set equal to the previously computed values of the same variables (except the first one or two years).

To finish up with the description of the main features of the model, we must recall that it is linear in the parameters, but non linear in the endogenous variables; the involved non linearities are logarithms, products of at least two endogenous variables, ratios in which at least one endogenous variable appears at the denominator.

4. UTILIZED ALGORITHMS

The generation of the random shocks to be added to the behavioural equations has been performed in the following way.

(1) Independent pseudo - random numbers uniformly distributed in the open interval (0,1) have been generated. The power residue method has been adopted, with prime modulus $2^{31}-1$ and primitive root 7^5 as a multiplier [8].

(2) After an intermediate phase of shuffling, in order to avoid the problems of non independency described by Neave in [12], the logarithmic - trigonometric transformation of Box and Müller [2] has been applied to generate standard normal deviates.

(3) A procedure developed by McCarthy has been used to transform the standard normal deviates into the required pseudorandom disturbances, under the constraint that their variance-covariance matrix must be equal to the

variance-covariance matrix of the model, as it is estimated from the regression residuals [9].

The output vector of the last step is used to perform a run (replication) of stochastic simulation.

Another procedure, developed by Nagar, could not be utilized, because its applicability is subject to the condition that the sample period length should be greater than the number of behavioural equations [9]. In our model this was not the case, because, as already mentioned, the sample period is, in a few equations, only of 15 years, while the number of behavioural equations is 16.

For the solution of the model, the Gauss-Seidel algorithm [11] has been used. As it requires the model to be normalized (i.e. each endogenous variable must be equated to a function of the remaining variables), several definition equations have been added to the original model.

The tolerance to be satisfied in the iterative solution procedure has been varied in different trials, in order to see if some significant differences could be individuated in the results. This was not the case, so that, in general, a tolerance of 10^{-6} has been imposed. Other different trials have been performed as far as the starting vector, assumed as initial guess of the solution, is concerned. In general, the observed values of the endogenous variables for the same year for which the solution is performed are assumed as starting point. The convergence is reached in approximately 20 iterations. No significant differences in the number of iterations, nor in the computed results have been found, when the observed values of the endogenous variables of the previous year or those of two years before have been assumed as starting point.

The programs have been written mainly in FORTRAN G language and tested by means of different sample models whose results were known a-priori. All the computations have been performed on an IBM/370 model 168 installed at C.N.U.C.E. (National University Center for Electronic Computing) of C.N.R. (National Council for Research) in Pisa. To give an idea of the machine time, we can say that a stochastic solution of the whole non linear model (approximately 60 equations, included the definition ones inserted for normalization and for improvement of computational speed) for 13 years (from 1961 to 1973) with 200 replications each year took approximately 90 seconds of CPU time.

5. EXPERIMENTAL RESULTS

As already mentioned, the purpose of this work was to have an empirical confirmation of the divergence in the results of stochastic and deterministic simulation of a non linear model. This divergence is well known from a theoretical point of view ("the solution of a nonlinear system obtained by setting all disturbances equal to their expected values is not, in general, equal to the expected value of the stochastic solution of the model" [5]), and can be easily shown with ad hoc examples [6]. On the other hand, in the applications of stochastic simulation techniques to models of national economic systems which have been examined ([3], [10], [15]), divergences are not empirically appreciable; for this reason we decided to perform our experiments trying to put in evidence:

(1) If there are or not systematic divergences.
(2) By what means these divergences could be detected.
(3) The involved cost of computation.
(4) Cases in which there is accordance of experimental with theoretical results.

Due to the complexity of the model and the big volume of numerical results, we shall fix our attention only on two equations that are:

Imports of manufacturing and agricultural products

$$\log MMA = -16.30915+1.56021\log K+ +1.68391\log(CPR+CAPA+I+X)+ +0.51339\log(PNOIL/PMMA) \quad (5.1)$$

Private consumption expenditure

$$CPR = 178.975+1.00049CPR_{-1}+0.42528 (\Delta(YDC-YDWC)+LEDC_{-1}.\Delta(YDWC/LEDC))+ +1.32114(YDWC/LEDC.\Delta LEDC)) \quad (5.2)$$

(the subscript -1 means value lagged of one year, and the symbol Δ means first difference)

where the variables:

CPR = private consumption expenditure
K = degree of capacity utilization in the industrial sector
LEDC = total non-agricultural employment

MMA = imports of manufacturing and agricultural products
PMMA = implicit price deflator for MMA
PNOIL= general index of wholesale prices excepted fuels and lubricants
YDC = personal disposable income
YDWC = disposable income of contractual workers

are endogenous variables, and

CAPA = consumptions of the government sector
I = total gross investments in plant and equipment
X = exports of goods and services

are exogenous variables.

5.1. Up to 200 replications

The number of replications we shall adopt is always greater than the number normally used (20÷30) in the different applications we know; their purpose is quite different from ours. A first experiment of one step solution with 40 replications per year has been performed. The results have been summarized in tables 1, 2, for the variable MMA, 3 and 4 for the variable CPR.

In the different tables the following information is displayed for the years 1961-1973: observed values, results of deterministic simulation, mean value of stochastic simulation across the replications, minimum and maximum value across the replications, annual percentage changes for the observed, the deterministic and the mean stochastic value, the standard deviation of the stochastic solution and, finally, the standard deviation of the mean value.

table 1

MMA. One step sol. with 40 repl.

Year	Obsrv. Value	Detrm. Solut.	Mean Stoch.	Min. Value	Max. Value
1961	2762.9	2792.0	2776.7	2254.1	3217.2
1962	3207.4	3177.9	3197.0	2840.2	3765.8
1963	4023.6	3659.2	3641.7	3019.5	3996.0
1964	3606.0	3602.5	3635.1	3133.4	4243.2
1965	3485.9	3572.0	3571.7	3276.3	4181.5
1966	4060.3	3783.9	3763.6	3209.5	4317.6
1967	4608.4	4428.7	4398.9	3891.0	5063.4
1968	4834.6	5504.7	5496.0	4964.2	6111.0
1969	5861.9	5982.1	5985.9	5165.6	6462.2
1970	6835.3	6352.6	6375.8	5554.0	7308.6
1971	6701.5	7017.7	7010.8	6300.9	7915.7
1972	7619.3	7548.7	7549.1	6893.1	8651.3
1973	8822.6	8464.9	8555.5	7841.3	9077.8

table 2

MMA. One step sol. with 40 repl.

Year	Obsrv. %Chng.	Detrm. %Chng.	Stoch. %Chng.	Stnd. Dev.	St.Dev. of Mean
1961				205.59	32.507
1962	16.09	13.82	15.14	202.08	31.952
1963	25.45	15.15	13.91	200.47	31.698
1964	-10.38	-1.55	-0.18	242.29	38.309
1965	-3.33	-0.85	-1.75	202.99	32.095
1966	16.48	5.93	5.37	239.18	37.818
1967	13.50	17.04	16.88	273.68	43.272
1968	4.91	24.30	24.94	256.62	40.575
1969	21.25	8.67	8.91	267.66	42.320
1970	16.61	6.19	6.51	341.62	54.015
1971	-1.96	10.47	9.96	373.71	59.088
1972	13.70	7.57	7.68	392.62	62.078
1973	15.79	12.14	13.33	310.76	49.136

table 3

CPR. One step sol. with 40 repl.

Year	Obsrv. Value	Detrm. Solut.	Mean Stoch.	Min. Value	Max. Value
1961	17330.	17475.	17465.	16392.	18698.
1962	18450.	18472.	18518.	17646.	19769.
1963	20090.	19547.	19583.	18648.	20379.
1964	20683.	20827.	20849.	20039.	21864.
1965	21251.	21719.	21757.	20882.	22908.
1966	22688.	22054.	21978.	20887.	23018.
1967	24310.	23468.	23473.	22656.	24799.
1968	25492.	25872.	25867.	25028.	26901.
1969	27036.	26920.	26944.	25866.	27748.
1970	29144.	27944.	27880.	26768.	28604.
1971	29861.	30351.	30295.	29379.	31586.
1972	30842.	31111.	31076.	30182.	32696.
1973	32755.	32682.	32851.	31561.	33883.

table 4

CPR. One step sol. with 40 repl.

Year	Obsrv. %Chng.	Detrm. %Chng.	Stoch. %Chng.	Stnd. Dev.	St.Dev. of Mean
1961				455.47	72.016
1962	6.46	5.70	6.03	464.39	73.427
1963	8.89	5.82	5.75	360.05	56.930
1964	2.95	6.55	6.47	470.81	74.442
1965	2.75	4.28	4.35	457.79	72.384
1966	6.76	1.54	1.01	493.33	78.003
1967	7.15	6.41	6.80	473.06	74.797
1968	4.86	10.24	10.20	430.37	68.047
1969	6.06	4.05	4.16	416.96	65.927
1970	7.80	3.80	3.47	482.66	76.315
1971	2.46	8.62	8.66	549.28	86.849
1972	3.29	2.50	2.58	528.26	83.526
1973	6.20	5.05	5.71	491.19	77.664

As one can notice, the observed values lie almost always between the minimum and the maximum of the stochastic solutions; this can suggest the usual considerations about the usefulness of stochastic simulation to give suggestions on the possible paths of the historical data (for example for forecast); on the other hand, it is clear that there are no significant differences between the deterministic and the mean stochastic solutions neither in values, nor in annual percentage changes. Also the comparison between solutions (both deterministic and mean stochastic) and observed data, not correct from a statistical point of view [6], but very useful for practical economic purposes, do not show particular differences. These comparisons are made in terms of Root Mean Square Errors and of two Theil's inequality coefficients (table 5), that are computed as follows [16],[15]).

$$RMSE = \sqrt{\frac{\Sigma\,(C_t - O_t)^2}{\Sigma\,O_t^2}}$$

$$TH1^2 = \frac{\Sigma\,(c_t - o_t)^2}{\Sigma\,o_t^2}$$

$$TH2^2 = \frac{\Sigma\,(c_t - o_t)^2}{\Sigma\,(o_t - o_{t-1})^2}$$

where the sums run over all the simulation period (from 1961 to 1973), C_t is, in turn, the computed deterministic or mean stochastic value, O_t the observed value and c_t and o_t the corresponding annual percentage changes.

table 5

Root Mean Square Errors and Theil's inequality coefficients

One step solution with 40 repl.

	MMA	CPR
RMSE detrm.	0.05556	0.02094
RMSE stoch.	0.05418	0.02147
TH1 detrm.	0.66098	0.58800
TH1 stoch.	0.67437	0.60090
TH2 detrm.	0.64732	1.16148
TH2 stoch.	0.66044	1.18697

Almost the same considerations could be done for all the other endogenous variables of the model.

One step simulation was then performed with 100 and 200 replications without getting any particular improvement towards our thesis. Some results of the experiment with 200 replications per year, always concerning the same two variables, are presented in tables 6 and 7.

table 6

MMA. One step sol. with 200 repl.

Year	Mean Stoch.	Min. Value	Max. Value	Stnd. Dev.	St.Dev. of Mean
1961	2802.2	2246.7	3457.2	190.38	13.462
1962	3176.2	2562.4	3898.1	206.87	14.628
1963	3667.3	3002.3	4329.7	223.66	15.815
1964	3602.6	3036.7	4184.8	218.39	15.443
1965	3585.0	3173.3	4191.2	211.79	14.976
1966	3799.2	3214.7	4429.8	226.83	16.039
1967	4410.8	3836.3	5053.9	233.75	16.528
1968	5496.3	4669.2	6211.0	273.49	19.339
1969	5986.8	5211.4	7096.6	304.47	21.529
1970	6313.9	5696.5	6993.5	275.04	19.449
1971	6981.3	6138.5	7918.7	325.70	23.030
1972	7547.5	6841.6	8615.5	336.03	23.761
1973	8471.9	7697.1	9380.7	328.82	23.251

table 7

CPR. One step sol. with 200 repl.

Year	Mean Stoch.	Min. Value	Max. Value	Stnd. Dev.	St.Dev. of Mean
1961	17477.	15793.	18837.	463.36	32.766
1962	18449.	17123.	19684.	469.60	33.205
1963	19556.	18556.	20985.	449.81	31.806
1964	20818.	19711.	21989.	444.04	31.399
1965	21739.	20570.	23220.	455.85	32.234
1966	22076.	20701.	23411.	481.83	34.071
1967	23459.	22249.	24701.	456.00	32.244
1968	25849.	24570.	27026.	448.87	31.740
1969	26912.	25642.	28387.	507.71	35.900
1970	27881.	26694.	29187.	435.40	30.787
1971	30316.	28888.	31706.	522.92	36.976
1972	31065.	29437.	32736.	535.41	37.860
1973	32640.	31243.	34024.	507.46	35.883

If we stopped our experiments at this point, bearing in mind the crucial aim of this work, as pointed out at the beginning of this section, an analysis of the obtained results would not confirm the existence of systematic biases between deterministic and stochastic simulation, when dealing with non linear models. In fact, by means of appropriate tests, we could not reject the null hypothesis that the expected value of the stochastic simulation (in the tables, the mean stochastic value is its estimate) is equal to the deterministic simulation value. Furthermore, the dispersion of the stochastic values around the deterministic ones, do not show the presence of underestimation or overestimation effects in a systematic way. Obviously, remarks of this kind are not made as a counter argument to Howrey's statement, but for operational purposes in order to have some indications about the dimensions (i.e. number of replications) necessary to reach the expected results. We proceed as in the following section.

5.2. Up to 10000 replications

Let us have a look at the equation of imports of manufacturing and agricultural products. Instead of considering the values of MMA, as it appears in the behavioural equation in the logarithmic form (eq.(5.1)), we look at the tables of the results of logMMA (experiment with 100 replications per year, tables 8 and 9).

table 8

logMMA. One step sol. with 100 repl.

Year	Obsrv. Value	Detrm. Solut.	Mean Stoch.	Min. Value	Max. Value
1961	7.9240	7.9345	7.9365	7.7452	8.1021
1962	8.0732	8.0640	8.0588	7.8989	8.2772
1963	8.3000	8.2050	8.2018	8.0721	8.3189
1964	8.1904	8.1894	8.1815	8.0707	8.3345
1965	8.1565	8.1809	8.1896	8.0618	8.3296
1966	8.3090	8.2385	8.2329	8.0658	8.4139
1967	8.4356	8.3958	8.3875	8.2301	8.5262
1968	8.4836	8.6134	8.6054	8.4343	8.7085
1969	8.6762	8.6965	8.7004	8.5304	8.8116
1970	8.8299	8.7566	8.7612	8.6394	8.8658
1971	8.8101	8.8562	8.8534	8.7425	8.9743
1972	8.9384	8.9291	8.9184	8.8002	8.9891
1973	9.0851	9.0437	9.0455	8.9269	9.1388

table 9

logMMA. One step sol. with 100 repl.

Year	Obsrv. %Chng.	Detrm. %Chng.	Stoch. %Chng.	Stnd. Dev.	St.Dev. of Mean
1961				.07360	.00736
1962	1.88	1.63	1.54	.06804	.00680
1963	2.81	1.75	1.77	.05380	.00538
1964	-1.32	-0.19	-0.25	.06498	.00650
1965	-0.41	-0.10	0.10	.06298	.00630
1966	1.87	0.70	0.53	.06237	.00624
1967	1.52	1.91	1.88	.06125	.00613
1968	0.57	2.59	2.60	.05736	.00574
1969	2.27	0.97	1.10	.04785	.00479
1970	1.77	0.69	0.70	.04540	.00454
1971	-0.22	1.14	1.05	.04695	.00470
1972	1.46	0.82	0.73	.03682	.00368
1973	1.64	1.28	1.42	.04320	.00432

As we do not observe any particular difference between the deterministic and the mean stochastic value of logMMA, we formulate the hypothesis that, each year, the stochastic values have a normal distribution with mean equal to the deterministic solution and standard deviation equal to the estimated one (as we shall perform some very rough computations, the values have interest as order of magnitude). We must remark that this hypothesis of normality, even if not exact due to the non linearity of the equation system, is of great help for interpreting the results.

In other words we suppose that for each year:

$$(\log MMA)_{stoc} = (\log MMA)_{det} + u \qquad (5.3)$$

where u is normally distributed with zero mean and standard deviation σ.

From this follows that:

$$MMA_{stoc} = MMA_{det} \cdot e^{u} \qquad (5.4)$$

where e^u has a log-normal distribution with mean value $e^{\frac{\sigma^2}{2}}$ (= 1.001 ÷ 1.0025 according to our values of σ). As this value is greater than 1, it is clear from eq.(5.4) that we should expect, for MMA_{stoc}, a mean value greater than the deterministic one of about 0.1% ÷ 0.25%; a very small difference indeed, that could be put in evidence only by means of a very large sample, that is, in our case, a very high number of replications.

In fact, if we consider, besides the standard deviation of the stochastic solution, also the standard deviation of the mean, obtained by the former divided by the square root of the number of replications, and if we fix a confidence interval around the mean of ± 1.96 times its standard deviation (corresponding to a 95% of probability in the case of normal distribution), we should expect that the deterministic solution of MMA systematically falls outside this interval, on the left, as soon as the number of replications makes the confidence interval narrower than 0.1% ÷ 0.25% of the value of MMA.

A rough computation indicates that, in this case, a number of replications of about 4000 ÷ 7000 should be sufficient. To be sure of putting in evidence these conclusions, we have performed one step simulations with 10000 replications (CPU time a bit more than one hour). The results are presented in tables 10, 11, 12 and 13.

table 10

MMA. One step sol. with 10000 repl.

Year	Obsrv. Value	Detrm. Solut.	Mean Stoch.	Min. Value	Max. Value
1961	2762.9	2792.0	2800.1	2044.9	3557.4
1962	3207.4	3177.9	3185.5	2501.5	4079.5
1963	4023.6	3659.2	3659.9	2880.5	4698.4
1964	3606.0	3602.5	3607.5	2645.4	4472.1
1965	3485.9	3572.0	3573.8	2859.8	4391.7
1966	4060.3	3783.9	3788.0	2976.1	4626.4
1967	4608.4	4428.7	4427.5	3674.1	5308.4
1968	4834.6	5504.7	5509.2	4461.0	6577.2
1969	5861.9	5982.1	5987.8	4916.2	7187.1
1970	6835.3	6352.6	6360.6	5302.2	7462.2
1971	6701.5	7017.7	7020.6	5808.6	8231.4
1972	7619.3	7548.7	7557.3	6241.1	8872.1
1973	8822.6	8464.9	8471.3	7010.8	10007.

table 11

MMA. One step sol. with 10000 repl.

Year	Obsrv. %Chng.	Detrm. %Chng.	Stoch. %Chng.	Stnd. Dev.	St.Dev. of Mean
1961				197.99	1.9799
1962	16.09	13.82	13.76	212.44	2.1244
1963	25.45	15.15	14.89	225.43	2.2543
1964	-10.38	-1.55	-1.43	222.98	2.2298
1965	-3.33	-0.85	-0.94	220.11	2.2011
1966	16.48	5.93	5.99	224.00	2.2400
1967	13.50	17.04	16.88	242.53	2.4253
1968	4.91	24.30	24.43	277.33	2.7733
1969	21.25	8.67	8.69	291.80	2.9180
1970	16.61	6.19	6.23	298.69	2.9869
1971	-1.96	10.47	10.38	323.64	3.2364
1972	13.70	7.57	7.64	342.31	3.4231
1973	15.79	12.14	12.09	364.37	3.6437

table 12

CPR. One step sol. with 10000 repl.

Year	Obsrv. Value	Detrm. Solut.	Mean Stoch.	Min. Value	Max. Value
1961	17330.	17475.	17481.	15614.	19160.
1962	18450.	18472.	18477.	16858.	20363.
1963	20090.	19547.	19538.	17857.	21234.
1964	20683.	20827.	20832.	18734.	22632.
1965	21251.	21719.	21712.	20121.	23390.
1966	22688.	22054.	22056.	20295.	23796.
1967	24310.	23468.	23457.	21610.	25291.
1968	25492.	25872.	25870.	23830.	27474.
1969	27036.	26920.	26918.	25039.	28910.
1970	29144.	27944.	27946.	26234.	29905.
1971	29861.	30351.	30354.	28694.	32342.
1972	30842.	31111.	31107.	29236.	33278.
1973	32755.	32682.	32680.	30572.	34744.

table 13

CPR. One step sol. with 10000 repl.

Year	Obsrv. %Chng.	Detrm. %Chng.	Stoch. %Chng.	Stnd. Dev.	St.Dev. of Mean
1961				457.65	4.5765
1962	6.46	5.70	5.70	456.44	4.5644
1963	8.89	5.82	5.74	449.88	4.4988
1964	2.95	6.55	6.62	457.25	4.5725
1965	2.75	4.28	4.22	472.19	4.7219
1966	6.76	1.54	1.59	466.02	4.6602
1967	7.15	6.41	6.35	462.59	4.6259
1968	4.86	10.24	10.29	465.02	4.6502
1969	6.06	4.05	4.05	479.45	4.7945
1970	7.80	3.80	3.82	477.55	4.7755
1971	2.46	8.62	8.62	498.89	4.9889
1972	3.29	2.50	2.48	519.76	5.1976
1973	6.20	5.05	5.06	521.06	5.2106

An analysis of the results shows that the deterministic solution of MMA lies outside the confidence interval around the mean (on the left) only in the years 1961, 1962, 1964, 1970 and 1972, and not in more than the 95% of the cases, as expected, even if, as expected, it is always on the left of the stochastic mean. An explanation can be found considering the action of the variable CPR on MMA. If we suppose, in fact, that CPR_{stoc} has a normal distribution with mean value equal to the deterministic solution (hypothesis that can be confirmed by the results of tables 12 and 13), on estimating the mean of the variable $logCPR_{stoc}$ we have a value sistematically smaller than $logCPR_{det}$, due to the well known property that geometric mean is always less than (or equal to) arithmetic mean. LogCPR is on the right hand side of the equation of logMMA, with positive sign, and this causes on MMA an effect of opposite sign to the one discussed above, so that 10000 replications of one step simulation are not yet sufficient to reach our objective.

As a last experiment, we performed also a dynamic stochastic simulation with 10000 replications. Even if dynamic simulation should be considered with particular care, due to asymmetric "treatment given to lagged endogenous variables during the estimation process and during the simulation process" [13], the results, that are presented in tables 14, 15, 16 and 17, can suggest the following considerations:

(1) The standard deviation increases with time both for MMA and CPR, and is much higher than in the case of one step simulation; that is an obvious consequence of the error accumulation

implicit in the dynamic solution.
(2) The value of $e^{\frac{\sigma^2}{2}}$ we discussed above is higher than in the case of one step simulation; a rough computation shows that 1000 ÷ 4000 replications should push the deterministic value of MMA outside the confidence interval of the stochastic mean; in spite of the above explained effect of CPR, 10000 replications are largely sufficient to put in evidence the existence of systematic divergences between deterministic and stochastic simulation.
(3) We observe from table 16 that the deterministic solution of CPR is always less than the stochastic mean after 1964, with increasing differences, while in the first years (from 1961 to 1963) it is greater, with decreasing differences. There is, in this case, an instability effect due to the coefficient of the lagged value of CPR, that is greater than 1 (see eq.(5.2)).

table 14

MMA. Dynamic sol. with 10000 repl.

Year	Obsrv. Value	Detrm. Solut.	Mean Stoch.	Min. Value	Max. Value
1961	2762.9	2792.0	2793.5	2135.7	3566.6
1962	3207.4	3243.4	3250.3	2401.9	4301.5
1963	4023.6	3745.7	3756.1	2687.3	5175.5
1964	3606.0	3441.9	3453.5	2370.3	4756.1
1965	3485.9	3375.2	3388.9	2205.9	4837.0
1966	4060.3	3737.2	3751.8	2274.9	5542.5
1967	4608.4	4173.4	4194.5	2617.5	6383.9
1968	4834.6	4937.3	4958.6	2998.0	7413.0
1969	5861.9	5593.5	5622.7	3402.5	8265.9
1970	6835.3	5907.3	5940.9	3777.8	9031.8
1971	6701.5	6149.5	6185.1	3657.6	9142.7
1972	7619.3	6784.8	6823.1	4284.2	10588.
1973	8822.6	7846.4	7892.1	4761.9	11879.

table 15

MMA. Dynamic sol. with 10000 repl.

Year	Obsrv. %Chng.	Detrm. %Chng.	Stoch. %Chng.	Stnd. Dev.	St.Dev. of Mean
1961				197.05	1.9705
1962	16.09	16.17	16.35	258.79	2.5879
1963	25.45	15.49	15.56	329.40	3.2940
1964	-10.38	-8.11	-8.06	344.78	3.4478
1965	-3.33	-1.94	-1.87	372.09	3.7209
1966	16.48	10.73	10.71	428.18	4.2818
1967	13.50	11.67	11.80	496.13	4.9613
1968	4.91	18.30	18.22	575.18	5.7518
1969	21.25	13.29	13.39	643.08	6.4308
1970	16.61	5.61	5.66	696.64	6.9664
1971	-1.96	4.10	4.11	740.38	7.4038
1972	13.70	10.33	10.32	802.67	8.0267
1973	15.79	15.65	15.67	890.34	8.9034

table 16

CPR. Dynamic sol. with 10000 repl.

Year	Obsrv. Value	Detrm. Solut.	Mean Stoch.	Min. Value	Max. Value
1961	17330.	17475.	17465.	15553.	19033.
1962	18450.	18642.	18638.	16255.	21095.
1963	20090.	19799.	19798.	16802.	22949.
1964	20683.	20319.	20319.	16787.	23725.
1965	21251.	21101.	21106.	17096.	25217.
1966	22688.	21897.	21902.	17070.	26144.
1967	24310.	22746.	22757.	17753.	28407.
1968	25492.	24413.	24429.	19016.	30009.
1969	27036.	26066.	26089.	20292.	31988.
1970	29144.	27054.	27079.	20955.	33326.
1971	29861.	28516.	28550.	21766.	34879.
1972	30842.	29674.	29716.	23385.	36256.
1973	32755.	31542.	31581.	24653.	39145.

table 17

CPR. Dynamic sol. with 10000 repl.

Year	Obsrv. %Chng.	Detrm. %Chng.	Stoch. %Chng.	Stnd. Dev.	St.Dev. of Mean
1961				452.58	4.5258
1962	6.46	6.68	6.72	636.33	6.3633
1963	8.89	6.21	6.22	793.52	7.9352
1964	2.95	2.62	2.63	941.36	9.4136
1965	2.75	3.85	3.88	1080.9	10.809
1966	6.76	3.78	3.77	1192.8	11.928
1967	7.15	3.88	3.90	1316.9	13.169
1968	4.86	7.33	7.35	1414.2	14.142
1969	6.06	6.77	6.80	1515.5	15.155
1970	7.80	3.79	3.80	1602.5	16.025
1971	2.46	5.40	5.43	1682.1	16.821
1972	3.29	4.06	4.08	1730.5	17.305
1973	6.20	6.29	6.28	1802.5	18.025

6. CONCLUSIONS

The stochastic simulation of a nonlinear econometric model is really able to show, when existing, systematic divergences between deterministic and stochastic solutions. However, in the model we have considered, as in other models on which stochastic simulation has been performed, non linearities do not seem to have a strong effect, so that the number of replications required in order to show some divergences is too high and the divergences themselves too small for practical purposes. We should conclude that, unless a model presents such kinds of strong non linearities, as the two equations example model in [6], the main object of stochastic simulation should not be the investigation of systematic divergences between deterministic and stochastic solutions, but rather, as already mentioned in section 2, the analysis of the results of experiments of economic policy, the study of cyclical behaviour, of turning points, and in general of the statistical properties of the endogenous variables; that is, the same purposes for which stochastic simulation is performed also on linear models.

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