

MPRA

Munich Personal RePEc Archive

Levy Subordinator Model of Default Dependency

Balakrishna, B S

14 March 2010

Online at <https://mpra.ub.uni-muenchen.de/21386/>
MPRA Paper No. 21386, posted 14 Mar 2010 20:58 UTC

Lévy Subordinator Model of Default Dependency

B. S. BALAKRISHNA*

March 14, 2010

Abstract

The article presents a model of default dependency based on Lévy subordinator. It is a tractable one-factor model with an architecture similar to that of the standard Gaussian copula model, providing easy calibration to individual hazard rate curves and efficient pricing with Fast Fourier Transform techniques. The subordinator is a stable Lévy process with a probability distribution function known as the Lévy distribution. The model provides a reasonable fit to market data with two parameters necessary to assess dependency risk, a measure of correlation and a measure of catastrophe.

Correlation products are derivatives sensitive to default correlation among a collection of credit names. Pricing of these involves either directly or indirectly modeling default dependency among the credit names. Market standard among such models is still the Gaussian copula model, a one-parameter model that enables easy quotation of market prices. But, it is well-known that the model is inadequate to price nonstandard products.

Major attraction of the Gaussian copula model is its simplicity and tractability. It can easily be calibrated to individual hazard rate curves. It can be formulated in closed form providing a semi-analytical framework for pricing. It can be priced efficiently by employing Fast Fourier Transform techniques. As it turns out, there exists a model similar in architecture that also enjoys these properties. Unlike the Gaussian copula model, it is a two-parameter model, but is able to offer a reasonable explanation of the correlation smile. The two parameters provide the two ingredients necessary to quantify dependency risk, a measure of correlation and a measure of catastrophe. The model is driven by $\alpha=1/2$ stable Lévy process for which the probability distribution function is available in closed form and is known as the Lévy distribution. Though it is inevitable that, with a model of such few parameters, there is bound to exist a residual smile, the ability to capture the smile will be helpful in sensitivity analysis and stress testing.

Issues with the Gaussian copula model have been addressed before. Brigo, Pallavicini and Torresetti [2010] provide an account of the developments in this field. Hull and White [2006] introduce an implied copula model that can calibrate consistently across all CDO tranches for a given maturity. Brigo, Pallavicini and Torresetti [2006] provide a fit across tranches as well as maturities in a generalized Poisson cluster loss model. Balakrishna [2009] models the jump distribution in an intensity based model. A doubly stochastic framework for intensity

*Email: balak_bs@yahoo.co.in

modeling is introduced in Duffie and Garleanu [2001]. Joshi and Stacey [2005] consider an intensity based modeling of business time as a gamma process. Chapovsky, Rennie and Tavares [2007] model default intensity as a jump-diffusion process. Errais, Giesecke and Goldberg [2006] present a model based on affine point processes.

An approach using Markov chains is presented in Di Graziano and Rogers [2005]. Putyatin, Prioul and Maslova [2005] use a Markovian approach with a Poissonian mixing distribution. Bennani [2005] presents a Markov functional approach to instantaneous loss rate. Sidenius, Piterbarg and Andersen [2005] consider a no-arbitrage approach to modeling future loss distribution. Overbeck and Schmidt [2005] propose a threshold model based on time-scaled Wiener processes. Albanese, Chen, Dalessandro and Vidler (2006) present a structural model based on spectral analysis. Baxter (2007) presents a structural framework using Lévy processes. Albrecher, Ladoucette and Schoutens (2007) present a unified approach to generic one-factor Lévy processes.

Section 1 formulates one-factor models independent of the factor dynamics. Section 2 realizes this framework with Lévy processes called subordinators, and presents a specific model based on the Lévy subordinator. Section 3 discusses large homogeneous pool approximation. Section 4 presents a semi-analytical framework to compute expected loss for a finite number of names with Fast Fourier Transform techniques. Section 5 discusses CDO pricing semi-analytically and via a Monte Carlo algorithm. Section 6 concludes with a brief summary. Table 1 presents the results of a numerical investigation.

1 One-Factor Formulation

Consider a model wherein common variables determine default dependency. Factors governing default dependency can be considered to be functions of the sample paths followed by the common variables. In a simplified version of the model, only one such path-function would be relevant. Let $F_t()$ be the distribution function of this path-function at time t . Given a value for $F_t()$, the defaults are considered to be independent of each other so that, for the joint default probability during the time interval $(0, t)$, we may write

$$P_{ij\dots}(t) = \int_0^1 dF [p_i(F, t)p_j(F, t)\dots], \quad (1)$$

where $p_i(F, t)$ is the conditional default probability, probability that i^{th} credit name has defaulted during $(0, t)$ given that the distribution $F_t()$ has value F at time t . To see that such a common factor should exist, at least in principle, note that, in a homogeneous collection of credit names, $p_i(F, t)$ itself can be viewed upon as a path-function implying that a homogeneous collection should in general be describable by a one-factor model. Similarly, a heterogeneous collection of names should be describable by not more than the number of heterogeneous name types in the collection.

The above formulation of one-factor models, though appears rather simple and straightforward, has the attractive feature that the time-dependence of the common factor has disappeared into the integration variable F . As we will see, this helps us define the model independent of the dynamics governing the common factor. Note however that the above

does not fully specify the model dynamics, and that would require specifying the factor dynamics as well, but for our purpose the above formulation is sufficient.

Consider $p_i(F, t)$ that is an increasing function of F . It is further expected to be increasing as a function of t . These properties let us relate it to the distribution function of a stochastic process at t so that, we may write for $p_i(F, t)$, or equivalently for the conditional survival probability $q_i(F, t) \equiv 1 - p_i(F, t)$,

$$q_i(F, t) = e^{-\kappa_i(t)} \mathbb{E} \{ \mathbf{1}_{\Phi_i(t) \geq F} \}, \quad (2)$$

where $\mathbf{1}_{\{\dots\}}$ denotes the indicator function. $\kappa_i(t)$ (with $\kappa_i(0) = 0$) is deterministic and increasing as a function of t . $\Phi_i(t), i = 1, \dots, n$ are independent stochastic processes taking values in $[0, 1]$ with $\Phi_i(0) = 1$ and $\Phi_i(\infty) = 0$, having only non-increasing sample paths. For the individual credit name, its default probability $P_i(t)$, or equivalently its survival probability $Q_i(t) \equiv 1 - P_i(t)$, can be written as

$$\begin{aligned} Q_i(t) &= \int_0^1 dF q_i(F, t) = e^{-\kappa_i(t)} \int_0^1 dF \mathbb{E} \{ \mathbf{1}_{\Phi_i(t) \geq F} \} \\ &= e^{-\kappa_i(t)} \mathbb{E} \left\{ \int_0^1 dF \mathbf{1}_{\Phi_i(t) \geq F} \right\} = e^{-\kappa_i(t)} \mathbb{E} \{ \Phi_i(t) \}. \end{aligned} \quad (3)$$

Note that this formulation of one-factor models covers the well-known one-factor models. The standard one-factor Gaussian copula model is recovered with $\kappa_i(t) = 0$ and

$$\Phi_i(t) = N \left(\frac{1}{\sqrt{\rho}} \left(\sqrt{1 - \rho} Z_i - K_i(t) \right) \right), \quad (4)$$

where $N()$ is the cumulative normal distribution function, Z_i is a standard normal random variable associated with the i^{th} credit name, ρ is the correlation parameter and $K_i(t) = N^{-1}(P_i(t))$. This follows after writing $F = N(-Y)$ where Y is a standard normal random variable associated with the common factor, and assumes a straightforward extension of expression (1) into a copula.

It is more convenient to work with $\Lambda_i(t) = -\ln \Phi_i(t)$, a non-decreasing stochastic process, taking values in $[0, \infty]$ with $\Lambda_i(0) = 0$ and $\Lambda_i(\infty) = \infty$. The conditional survival probability $q_i(F, t)$ then reads

$$q_i(F, t) = e^{-\kappa_i(t)} \mathbb{E} \{ \mathbf{1}_{\Lambda_i(t) \leq -\ln F} \}. \quad (5)$$

For the individual survival probability $Q_i(t)$, this gives

$$Q_i(t) = e^{-\kappa_i(t)} \mathbb{E} \{ e^{-\Lambda_i(t)} \}. \quad (6)$$

We may also express the joint survival probability $Q_\Omega(t)$ for a list of names in Ω as

$$Q_\Omega(t) = e^{-\sum_{i \in \Omega} \kappa_i(t)} \mathbb{E} \{ e^{-\text{Max}_{i \in \Omega} \Lambda_i(t)} \}, \quad (7)$$

where $\text{Max}_{i \in \Omega}$ picks up the largest $\Lambda_i(t)$ in the list Ω . This follows from the fact that $\Lambda_i(t)$ are independent stochastic processes. The above is not needed for our discussion to follow, but it is interesting to note that it defines the model independent of the common factor that has been integrated away.

2 Lévy Subordinator Model

The above formulation of one-factor models left us with individual stochastic processes that are a priori expected to be quite complicated. Fortunately, as we will see below, they can be realized neatly with a class of Lévy processes with non-decreasing sample paths known as subordinators. Of these, a Lévy process called the Lévy subordinator, with its distribution known in closed form, turns out to be an appropriate one to choose.

Let $X_i(t), i = 1, \dots, n$ be n independent subordinators. For a parsimonious model, let us assume that they are identically distributed. Let $\eta(u)$ be their Laplace exponent given by

$$\mathbb{E} \{ e^{-uX_i(t)} \} = e^{-t\eta(u)}. \quad (8)$$

Given such subordinators, let us set

$$\kappa_i(t) = (1 - \sigma\eta(1))\theta_i(t), \quad \Lambda_i(t) = X_i(\sigma\theta_i(t)), \quad (9)$$

which introduces σ , a measure of correlation, as one of the parameters of the model. Here $\theta_i(t)$, that may be referred to as intrinsic time, is derived from the individual survival probability $Q_i(t)$ as

$$\theta_i(t) \equiv -\ln Q_i(t). \quad (10)$$

Now, individual hazard rate curves are automatically calibrated to, since

$$Q_i(t) = e^{-\kappa_i(t)} \mathbb{E} \{ e^{-\Lambda_i(t)} \} = e^{-(1-\sigma\eta(1))\theta_i(t)} \mathbb{E} \{ e^{-X_i(\sigma\theta_i(t))} \} = e^{-\theta_i(t)}. \quad (11)$$

As for the conditional survival probability, we get

$$q_i(F, t) = e^{-(1-\sigma\eta(1))\theta_i(t)} \mathbb{E} \{ 1_{X_i(\sigma\theta_i(t)) \leq -\ln F} \} = e^{-(1-\sigma\eta(1))\theta_i(t)} g(-\ln F, \sigma\theta_i(t)). \quad (12)$$

where $g(x, t) = \mathbb{E} \{ 1_{X_i(t) \leq x} \}$ is the cumulative distribution function of $X_i(t)$. Note that, due to the introduction of parameter σ , one may standardize parameters of $X_i(t)$ so as to yield a fixed value for $\eta(1)$.

A subclass of subordinators are α -stable processes having $0 < \alpha < 1$ and maximally skewed to the right (skew parameter $\beta = 1$). Their distributions are not known in closed form except for the $\alpha = 1/2$ -stable subordinator known as the Lévy subordinator. The cumulative distribution function of the Lévy subordinator is the Lévy distribution (standardized to $\eta(1) = 1 + \mu$)

$$g(x, t) = 2N \left(-t / \sqrt{2(x - \mu t)} \right), \quad (13)$$

where $N()$ is the cumulative normal distribution function. It has a positive drift component μt , introducing μ as the second of our model parameters. It has support only to the right of μt . The above distribution gives us for the conditional survival probability,

$$q_i(F, t) = 2e^{-(1-\sigma(1+\mu))\theta_i(t)} N \left(-\sigma\theta_i(t) / \sqrt{-2(\ln F + \mu\sigma\theta_i(t))} \right). \quad (14)$$

Positive drift forces $q_i(F, t)$ to zero above $F = e^{-\mu\sigma\theta_i(t)}$. This F -threshold is name-dependent, but, in the large homogeneous pool approximation to be discussed next, it implies that there

is a probability mass at $\nu = 1$, suggesting a finite probability for all the names in the pool defaulting, μ being the measure of such a catastrophe.

We modeled the individual process $\Lambda_i(t)$ as a subordinator (with time scaling). Variations are possible, for instance, it could be the time-integral of a nonnegative stochastic process, perhaps mean-reverting, that in some sense could be interpreted as stochastic default intensity. Such variations result in individual survival probabilities that are not as easily calibrated to. Though it is simpler and to some extent equivalent when the stochastic process is a stable process, the model presented above is the simplest and still results in a reasonable fit to market data.

The formulation bears some resemblance to barrier models, but the resemblance does not go too far since the barrier here is random and common to all the names, and the processes have non-decreasing sample paths. It is known that the distribution of default times in the simplest version of a barrier diffusion model follows the Lévy distribution and the fact that the same distribution is relevant here is also intriguing. However, the roles of time and other variables in the distribution appear to be of a different nature.

Result (14) defines our two-factor model. Though, in general, the two parameters could be different for different names, and time-dependent as well, they are considered uniform and constant for ease of calibration. The model's viability can only be examined by calibration to market data. Hence, let us next consider various approaches to pricing.

3 Large Homogeneous Pool

Large homogeneous pool approximation can be a useful tool since it admits an explicit expression for the loss distribution. Consider a homogeneous collection of n credit names. The joint default probability for k or less number of defaults during $(0, t)$ then reads

$$P_{\{k\}}(t) = \sum_{j=0}^k \binom{n}{j} \int_0^1 dF [p_t(F)]^j [1 - p_t(F)]^{n-j}, \quad (15)$$

where $p(F, t)$ has been written as $p_t(F)$. For a large homogeneous pool of names, as is well-known, the above simplifies to

$$G_t(\nu) \equiv P_{\{\nu n\}}(t) = \int_0^1 dF 1_{p_t(F) \leq \nu}, \quad (16)$$

where $\nu = k/n$ is the fraction of names defaulted during $(0, t)$. The above indicates that $G_t(\nu)$ can be obtained by summing up the region of F over which $p_t(F) \leq \nu$.

We have considered $p_t(F)$ to be an increasing function of F . Hence, $G_t(\nu)$ can be obtained by solving $p_t(F) = \nu$ for $F = G_t(\nu)$. If $\sigma < 1/(1 + \mu)$, as is found to be the case upon calibration, there is a $\nu_{\min}(t)$ below which $G_t(\nu) = 0$,

$$\nu_{\min}(t) = 1 - e^{-(1-\sigma(1+\mu))\theta(t)} \quad \text{if } \sigma < 1/(1 + \mu), \quad \text{else } 0. \quad (17)$$

Above $\nu_{\min}(t)$, $G_t(\nu)$ is

$$G_t(\nu) = \exp \left\{ -\mu\sigma\theta(t) - \frac{1}{2}(\sigma\theta(t))^2 \left[N^{-1} \left(\frac{1}{2}(1 - \nu)e^{(1-\sigma(1+\mu))\theta(t)} \right) \right]^{-2} \right\}, \quad \nu \geq \nu_{\min}(t). \quad (18)$$

Note that $G_t(\nu) \rightarrow e^{-\mu\sigma\theta(t)}$ as $\nu \rightarrow 1$ so that there is a probability mass at $\nu = 1$, suggesting a finite probability $1 - e^{-\mu\sigma\theta(t)}$ of total collapse.

In this approximation, the expected loss per tranche size for a tranche with attachment point a and detachment point b is

$$\bar{L}(t)_{[a,b]} = 1 - \frac{1}{\nu_b - \nu_a} \int_{\nu_a}^{\nu_b} d\nu G_t(\nu). \quad (19)$$

where $\nu_a = a/(1 - R)$, $\nu_b = b/(1 - R)$ and R is the recovery rate (this known result follows from the usual expressions). Note that $\int_0^1 d\nu G_t(\nu) = Q_i(t)$ as expected. The above shows that, in this approximation, the expected loss is 100% of the tranche size if the detachment point is not above $(1 - R)\nu_{\min}$. This leads to overpricing of the equity tranches. Finite n and heterogeneity is expected to smoothen out the small ν -behavior.

4 Finite n with FFT

Large homogeneous pool approximation yields fast results, but at the expense of accuracy. As is well-known, many of the factor models can be accurately computed for finite n by employing Fast Fourier Transform (FFT) techniques. This is true with the present model as well. To obtain the loss distribution for finite n , consider the loss process conditional on F ,

$$\mathcal{L}(F, t) = \sum_{j=1}^n L_j 1_{\tau_j(F) < t}. \quad (20)$$

Here $\tau_j(F)$ is its conditional default time random variable and $L_j = (1 - R_j)w_j$ is the normalized loss-given-default, R_j being the recovery rate and w_j the fraction of the pool notional associated with the j^{th} name. L_j is assumed to be deterministic and time-independent. Defaults being independent conditional on F , the above process has the characteristic

$$\mathbb{E} \{ e^{iu\mathcal{L}(F,t)} \} = \prod_{m=1}^n [q_m(F, t) + p_m(F, t)e^{iuL_m}], \quad (21)$$

where $i = \sqrt{-1}$. This characteristic is the Fourier transform of the loss distribution (conditional on F unless mentioned otherwise). Hence, the loss distribution can be obtained by inverting the above using FFT techniques. The result can be used to compute the expected loss per tranche size for a tranche with attachment point a and detachment point b ,

$$\bar{L}(F, t)_{[a,b]} = 1 - \frac{1}{b - a} \int_a^b dx H_t(F, x), \quad (22)$$

where $H_t(F, \cdot)$ is the cumulative distribution function of the loss distribution.

FFT requires discretization of u . Discretization is straightforward if L_i 's are uniform at L across the collection ($L = (1 - R)/n$ if R_i s are uniform as well). Inversion then yields the loss distribution at loss-points $j = 0, \dots, n$ in units of L . This gives the default probability density $P_{[j]}(F, t)$, the sum of products of various combinations of j of the $p_i(F, t)$ s and $n - j$

of the $q_i(F, t)$ s. Consider it extended up to $j = N - 1 \geq n$ by padding with zeros where N is a power of 2, as is usually done for an efficient FFT. In this case, (21) reads

$$\sum_{j=0}^{N-1} P_{[j]}(F, t) e^{i\omega jk} = \prod_{m=1}^n [q_m(F, t) + p_m(F, t) e^{i\omega k}], \quad k = 0, \dots, N - 1, \quad (23)$$

where $\omega = 2\pi/N$. This can easily be computed and inverted using FFT techniques to obtain $P_{[j]}(F, t)$. This gives the cumulative loss distribution $H_t(F, jL)$, $j = 0, \dots, n$, or equivalently, the cumulative default probability distribution $G_t(F, \nu)$ where $\nu = j/n$ is the fraction of names defaulted. The normalized expected loss is then

$$\bar{L}(F, t)_{[a,b]} = 1 - \frac{1}{\nu_b - \nu_a} \int_{\nu_a}^{\nu_b} d\nu G_t(F, \nu). \quad (24)$$

where $\nu_a = a/(nL)$, $\nu_b = b/(nL)$, and $G_t(F, \nu)$ is flat in-between successive ν -points. Integration of $P_{[j]}(F, t)$ over F gives $P_{[j]}(t)$, the unconditional default probability density. This integration is deferred to the end of computations for efficiency reasons.

5 CDO Pricing

The analytical results for the expected loss can be used to price the CDO tranches. The default leg of a tranche per tranche size can be priced as

$$DL_{[a,b]} = \int_0^1 dF \left\{ \int_0^T D(t) d\bar{L}(F, t)_{[a,b]} \right\}, \quad (25)$$

where T is the maturity and $D(t)$ is the discount factor for the time period $(0, t)$. Similarly the premium leg per tranche size and per unit spread can be priced as

$$PL_{[a,b]} = \int_0^1 dF \left\{ \sum_{i=1}^{N_\delta} \delta_i D(t_i) [1 - \bar{L}(F, t_i)_{[a,b]}] \right\}. \quad (26)$$

where δ_i is the accrual factor for the period (t_{i-1}, t_i) and N_δ is the number of periods. To this premium leg, we need to add a contribution from accrued interest payments made upon default,

$$PL'_{[a,b]} = \int_0^1 dF \left\{ \sum_{i=1}^{N_\delta} \int_{t_{i-1}}^{t_i} \delta'_i(t) D(t) d\bar{L}(F, t)_{[a,b]} \right\}, \quad (27)$$

where $\delta'_i(t)$ is the accrual factor for the partial period covering (t_{i-1}, t) . Given the leg values, the fair spread can be obtained by dividing the default leg by the premium leg, after taking care of any upfront payments.

It is found to be efficient to perform the numerical integration over F after the expressions within the curly brackets are computed over a sufficiently fine time-grid. The super senior tranche can be priced like an ordinary tranche along with an additional premium leg of notional that is a fraction R of the total notional of the underlying credit default swaps,

or, if recovery rates are nonuniform, sum of fractions R_i of the individual notionals of the underlying credit default swaps.

Though the model can be handled semi-analytically as detailed above, a Monte Carlo algorithm could be a useful tool to price more exotic products. It can also be useful for pricing standard tranches as it is found to be efficient, accurate, easily implementable and does not involve discretization of time. Note that the following is not to be viewed as a simulation of the model dynamics, rather just a method of computing the above integrals. The algorithm reads as follows.

1. Draw a uniformly distributed random number F and n independent exponentially distributed unit mean random numbers $u_i, i = 1, \dots, n$.

2. For each credit name i , first determine whether it defaults before the time horizon T by checking if $q_i(F, T) < u_i$ where $q_i(F, \cdot)$ is as given in equation (14). If so, solve the equation $q_i(F, t) = u_i$ for $\theta_i(t)$. Determine default time t_i by a table look up into the hazard rate curve of credit name i .

3. Given the default times before the time horizon, price the instrument. For the next scenario, go to step 1.

4. Average all the prices thus obtained to get a price for the instrument.

Given a scenario of default times, it is straightforward to price the CDOs. To improve efficiency, quasi random sequences such as Sobol sequences can be used to generate each of the independent random numbers.

6 Conclusions

The article has presented a one-factor model of default dependency to capture the correlation smile. It is driven by alpha=1/2 stable Lévy subordinator process for which the probability distribution function is known in closed form as the Lévy distribution. The attractive feature of the model is its tractability, at par with that of the Gaussian copula model. It gets automatically calibrated to individual hazard rate curves. It can be used for pricing both semi-analytically and via a Monte Carlo algorithm. As can be seen from Table 1, despite having only two parameters at its disposal, it is able to capture the smile reasonably well. Market quotes are as on October 2, 2006 (source: Brigo, Pallavicini and Torresetti [2006]), and it remains to see how the model performs during distressed market conditions (when parameter μ is expected to play a significant role). Calibration is done for a homogeneous collection, with constant interest rates and recovery rates, and with hazard rates flat in-between maturities. Relaxing these assumptions could improve the fit. A better fit could also be attempted by having parameters σ and μ in $G_t(\nu)$ to be ν -dependent, that implies a certain distribution for $p_t(F)$ to be used in pricing.

Though the model has been developed with an application to CDOs in mind, it could be useful in other disciplines. Tail distributions have turned out to be important under different contexts as in, for instance, the computation of value-at-risk. The model provides two minimum ingredients necessary to assess such risks, a measure of correlation and of catastrophe. Simplicity and tractability with its large homogeneous pool approximation and an efficient semi-analytical framework makes the model an attractive choice.

References

- [1] Albanese, C., O. Chen, A. Dalessandro, and A. Vidler (2006), “Dynamic Credit Correlation Modeling”, Available at http://www.defaultrisk.com/pp_corr_75.htm.
- [2] Albrecher, H., S. Ladoucette, W. Schoutens (2007), “A Generic One-factor Lévy model for Pricing Synthetic CDOs”, in “Advances in Mathematical Finance”, Birkhauser, 259-277.
- [3] Balakrishna, B. S. (2009), “Lévy Density Based Intensity Modeling of the Correlation Smile”, Available at http://www.defaultrisk.com/pp_crdrv128.htm.
- [4] Baxter, M. (2007). “Lévy Simple Structural Models”, in “Credit Correlation - Life after Copulas”, Lipton and Rennie (Editors), World Scientific.
- [5] Bennani, N. (2005), “The Forward Loss Model: A Dynamic Term Structure Approach for the Pricing of Portfolio Credit Derivatives”, http://www.defaultrisk.com/pp_crdrv_95.htm.
- [6] Brigo, D., A. Pallavicini and R. Torresetti (2006), “Default Correlation, Cluster Dynamics and Single Names: The GPCL dynamical loss model”, <http://ssrn.com/abstract=956827>.
- [7] Brigo, D., A. Pallavicini and R. Torresetti. (2010) “Credit Models and the Crisis, or: How I learned to stop worrying and love the CDOs”, <http://ssrn.com/abstract=1529498>.
- [8] Chapovsky, A., A. Rennie and P. A. C. Tavares (2007), “Stochastic Intensity Modeling for Structured Credit Exotics”, *Int. Jnl. of Theoretical and Applied Finance*, 10(4), pp. 633-652.
- [9] Di Graziano, G. and C. Rogers (2009), “A Dynamic Approach to the Modeling of Correlation Credit Derivatives Using Markov Chains”, *Int. Jnl. of Theoretical and Applied Finance*, 12(1), pp. 45-62.
- [10] Duffie, D. and N. Garleanu (2001), “Risk and the Valuation of Collateralized Debt Obligations”, *Financial Analysts Journal*, 57, pp. 41-59.
- [11] Errais, E., K. Giesecke and L. Goldberg (2006), “Affine Point Processes and Portfolio Credit Risk”, Available at <http://ssrn.com/abstract=908045>.
- [12] Hull, J. and A. White (2006), “Valuing Credit Derivatives Using an Implied Copula Approach”, *Journal of Derivatives*, 14(2), pp. 8-28.
- [13] Joshi, M. and A. Stacey (2005), “Intensity Gamma: A New Approach to Pricing Credit Derivatives”, *Risk Magazine*, July 2006.
- [14] Overbeck, L. and W. Schmidt (2005), “Modeling Default Dependence with Threshold Models”, *Journal of Derivatives*, 12(4), pp. 1019.
- [15] Putyatin, V., D. Prieul and S. Maslova (2005), “A Markovian approach to Modeling Correlated Defaults”, *Risk Magazine*, May 2005.
- [16] Sidenius, J., V. Piterbarg and L. Andersen (2005), “A New Framework for Dynamic Credit Portfolio Loss Modeling”, http://www.defaultrisk.com/pp_model_83.htm.

Table 1: Best fits to the five tranches of CDX.NA.IG and iTraxx Europe CDOs for each maturity, obtained for a homogeneous collection using semi-analytical pricing with FFT. Hazard rate is taken to be flat in-between maturities. Interest rate is constant at 5.0% and 3.5% respectively, and recovery rate at 40%. Equity tranche is quoted as an upfront fee in percent (plus 500bp per year running) and the other tranches are quoted as spreads per year in bp (with bid-ask spreads in brackets).

CDX.NA.IG7								
	0-3%	3-7%	7-10%	10-15%	15-30%	30-100%	0-100%	σ, μ
3y	9.75(2.0)	7.9(1.6)	1.20(0.2)	0.50(0.1)	0.20(0.1)		24.0(0.5)	0.38,0.0
	10.15	7.45	1.51	0.54	0.28	0.12	24.1	
5y	30.50(1.0)	102.0(6.1)	22.50(1.4)	10.25(0.6)	5.00(0.3)		40.0(0.5)	0.62,0.06
	32.00	102.3	22.04	10.51	4.80	1.32	40.0	
7y	45.63(2.0)	240.0(48.0)	53.00(10.6)	23.00(4.6)	7.20(1.4)		49.0(0.5)	0.68,0.03
	47.05	257.6	55.42	22.99	7.74	1.19	49.0	
10y	55.00(1.0)	535.0(21.4)	123.00(7.4)	59.00(3.5)	15.50(0.9)		61.0(0.5)	0.63,0.06
	56.72	530.0	139.50	52.38	16.41	2.60	61.0	
iTraxx Europe 6								
	0-3%	3-6%	6-9%	9-12%	12-22%	22-100%	0-100%	σ, μ
3y	3.50(1.5)	5.5(4.0)	2.25(3.0)				18.0(0.5)	0.45,0.0
	4.08	6.1	1.54	0.61	0.28	0.13	18.1	
5y	19.75(0.25)	75.0(1.0)	22.25(1.0)	10.50(1.0)	4.00(0.5)	1.50(0.5)	30.0(0.5)	0.76,0.03
	20.55	77.4	21.19	10.31	4.60	0.85	30.0	
7y	37.12(0.25)	189.0(2.0)	54.25(1.5)	26.75(1.5)	9.00(1.0)	2.85(0.5)	40.0(0.5)	0.73,0.03
	38.70	204.7	53.12	24.11	9.73	1.32	40.0	
10y	49.75(0.25)	474.0(4.0)	125.50(3.0)	56.50(2.0)	19.50(1.0)	3.95(0.5)	51.0(0.5)	0.68,0.03
	53.95	470.1	136.48	56.10	20.18	2.15	51.0	

Market quotes as on October 2, 2006. Source: Brigo, Pallavicini and Torresetti [2006].