Quantum mechanism helps agents combat Pareto-inefficient social choice rules

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18 February 2010

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Abstract

Quantum strategies have been successfully applied in game theory for years. However, as a reverse problem of game theory, the theory of mechanism design is ignored by physicists. In this paper, we generalize the classical theory of mechanism design to a quantum domain and obtain two results: 1) We find that the mechanism in the proof of Maskin's sufficiency theorem is built on the Prisoners' Dilemma. 2) By virtue of a quantum mechanism, agents who satisfy a certain condition can combat Pareto-inefficient social choice rules instead of being restricted by the traditional mechanism design theory.

1 Introduction

Game theory is a very useful tool for investigating rational decision making in conflict situations. It was first founded by von Neumann and Morgenstern [1]. Since its beginning, game theory has been widely applied to many disciplines, such as economics, politics, biologies and so on. Compared with game theory, the theory of mechanism design just concerns the *reverse* question: given some desirable outcomes, can we design a game that produces it? The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2007 was awarded jointly to Hurwicz, Maskin and Myerson for having laid the foundations of mechanism design theory.

As Serrano [2] has described, we suppose that the goals of a group of self-interested agents (or a society) can be summarized in a social choice rule (SCR). An SCR is a mapping that prescribes the social outcome (or outcomes) on the basis of agents' preferences over the set of all social outcomes [3]. The theory of mechanism design answers the important question of whether and how it is possible to implement different SCRs. According to Maskin and Sjöström [4], whether or not an SCR is implementable depend on which game theoretic solution concept is used (e.g., dominant strategies and Nash equilibrium). Ref. [3] is a fundamental work in the field of mechanism design. It provides an almost complete characterization of social choice rules that are Nash implementable.

In 1999, some pioneering breakthroughs were made in the field of quantum games [5,6]. The game proposed by Eisert *et al* [5] showed a fascinating "quantum advantages" as a result of a novel quantum Nash equilibrium. Benjamin and Hayden [7], Du *et al* [8], Flitney and Hollenberg [9] investigated multiplayer quantum Prisoners' Dilemma. As a comparison, so far the theory of mechanism design is still investigated only by economists. To our best knowledge, up to now, there is no research in the cross field between quantum mechanics and mechanism design. Motivated by quantum games, in this paper, we will investigate what happens if agents can use quantum strategies in the theory of mechanism design.

Section 2 of this paper recalls some preliminaries of mechanism design published in Ref. [2], while Section 3 reformulates the well-known Maskin mechanism as a physical mechanism and proves they are equivalent to each other. Section 4 generalizes the physical mechanism to a quantum domain and proves that under a certain condition, an original Nash implementable social choice rule will no longer be implemented. Section 5 draws the conclusions.

2 Preliminaries

Let $N = \{1, \cdots, n\}$ be a finite set of *agents* with $n \geq 2$, $A = \{a_1, \cdots, a_k\}$ be a finite set of social *outcomes*. Let T_i be the finite set of agent i's types, and the *private information* possessed by agent i is denoted as $t_i \in T_i$. We refer to a profile of types $t = (t_1, \cdots, t_n)$ as a *state*. Let $\mathcal{T} = \prod_{i \in N} T_i$ be the set of states. At state $t \in \mathcal{T}$, each agent $i \in N$ is assumed to have a complete and transitive *preference relation* $\succeq_i^t$ over the set A. We denote by $\succeq^t = (\succeq_1^t, \cdots, \succeq_n^t)$ the profile of preferences in state t. The utility of agent i for outcome a in state t is $u_i(a, t) : A \times \mathcal{T} \to R$, i.e., $u_i(a, t) \geq u_i(b, t)$ if and only if $a \succeq_i^t b$. We denote by $\succ_i^t$ the strict preference part of $\succeq_i^t$. Fix a state t, we refer to the collection $E =< N, A, (\succeq_i^t)_{i \in N} >$ as an *environment*. Let ε be the class of possible environments. A *social choice rule* (SCR) F is a mapping $F : \varepsilon \to 2^A \backslash \{\emptyset\}$. A *mechanism* $\Gamma = ((M_i)_{i \in N}, g)$ describes a message or strategy set M_i for agent i, and an outcome function $g : \prod_{i \in N} M_i \to A$.

An SCR F satisfies *no-veto* if, whenever $a \succeq_i^t b$ for all $b \in A$ and for all agents i but perhaps one j, then $a \in F(E)$. An SCR F is *monotonic* if for every pair of environments E and E', and for every $a \in F(E)$, whenever $a \succeq_i^t b$ implies that $a \succeq_i^{t'} b$, there holds $a \in F(E')$. We assume that there is *complete information* among the agents, i.e., the true state t is common knowledge among them. Given a mechanism $\Gamma = ((M_i)_{i \in N}, g)$ played in state t, a *Nash equilibrium* of Γ in state t is a strategy profile m^* such that: $\forall i \in N, g(m^*(t)) \succeq_i^t g(m_i, m_{-i}^*(t)), \forall m_i \in M_i$. Let $\mathcal{N}(\Gamma, t)$ denote the set of Nash equilibria of the game induced by Γ in state t, and $g(\mathcal{N}(\Gamma, t))$ denote the corresponding set of Nash equilibrium outcomes. An SCR F is *Nash implementable* if there exists a mechanism $\Gamma = ((M_i)_{i \in N}, g)$ such that for every $t \in \mathcal{T}$, $g(\mathcal{N}(\Gamma, t)) = F(t)$.

Maskin [3] provided an almost complete characterization of social choice rules that were Nash implementable. The main results of Ref. [3] are two theorems: 1) (Necessity) If an SCR F is Nash implementable, then it is monotonic. 2) (Sufficiency) Let $n \geq 3$, if an SCR F is monotonic and satisfies no-veto, then it is Nash implementable. In order to facilitate the following investigation on quantum mechanism, we briefly recall the Maskin mechanism as follows [2]:

Consider the following mechanism $\Gamma = ((M_i)_{i \in N}, g)$, where agent i's message set is $M_i = A \times \mathcal{T} \times \mathbb{Z}_+$. A typical message sent by agent i is described as $m_i = (a_i, t_i, z_i)$. The outcome function g is defined in the following three rules: (1) If for every agent $i \in N$, $m_i = (a, t, 0)$ and $a \in F(t)$, then $g(m) = a$. (2) If $(n-1)$ agents $i \neq j$ send $m_i = (a, t, 0)$ and $a \in F(t)$, but agent j sends $m_j = (a_j, t_j, z_j) \neq (a, t, 0)$, then $g(m) = a$ if $a_j \succ_j^t a$, and $g(m) = a_j$ otherwise. (3) In all other cases, $g(m) = a'$, where a' is the outcome chosen by the agent with the lowest index among those who announce the highest integer.

3 Physical mechanism

It can be seen that in the Maskin mechanism, a message is an abstract mathematical notion. People usually neglect how it is realized physically. However, the world is a physical world. Any information must be related to a physical entity. Here we assume:

1) Each agent has a coin and a card. The state of a coin can be head up or tail up (denoted as H and T respectively).

2) Each agent i independently chooses a strategic action ω_i whether to flip his/her coin. The set of agent i's action is $\Omega_i = \{Not\ flip, Flip\}$. An action $\omega_i \in \Omega_i$ chosen by agent i is defined as $\omega_i : \{H, T\} \to \{H, T\}$. If $\omega_i = Not\ flip$, then $\omega_i(H) = H$, $\omega_i(T) = T$; If $\omega_i = Flip$, then $\omega_i(H) = T$, $\omega_i(T) = H$.

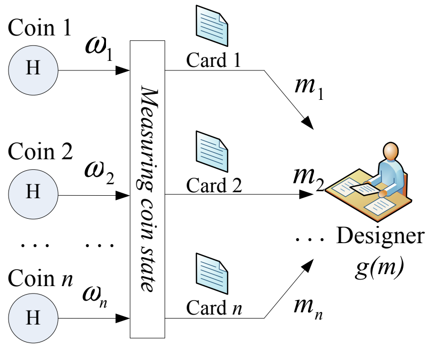

Fig. 1 The setup of a physical mechanism. Each agent has a coin and a card. The state of a coin can be head up or tail up. A message is written on one side of a card. Each agent independently chooses a strategy whether to flip his/her coin.

3) The two sides of a card are denoted as Side 0 and Side 1. A message is written on one side of a card. The message written on the Side 0 (or Side 1) of card i is denoted as $card(i,0)$ (or $card(i,1)$).

Based on aforementioned assumptions, we reformulate the Maskin mechanism $\Gamma = ((M_i)_{i\in N}, g)$ as a *physical mechanism* $\Gamma^P = ((S_i)_{i\in N}, G)$, where $S_i = \Omega_i \times C_i$, C_i is agent i's card set, $C_i = A \times \mathcal{T} \times \mathbb{Z}_+ \times A \times \mathcal{T} \times \mathbb{Z}_+$. A typical card written by agent i is described as $c_i = (card(i,0), card(i,1))$, where $card(i,0) = (a_i, t_i, z_i)$, $card(i,1) = (a'_i, t'_i, z'_i)$. A physical mechanism $\Gamma^P = ((S_i)_{i\in N}, G)$ describes a strategy set S_i for agent i and an outcome function $G : \prod_{i\in N} S_i \to A$. We shall use S_{-i} to express $\prod_{j\neq i} S_j$, and thus, a strategy profile is $s = (s_i, s_{-i})$, where $s_i = (\omega_i, c_i) \in S_i$ and $s_{-i} = (\omega_{-i}, c_{-i}) \in S_{-i}$. A *Nash equilibrium* of Γ^P played in state t is a strategy profile $s^* = (s^*_1, \cdots, s^*_n)$ such that for any agent $i \in N$, $s_i \in S_i$, $G(s^*_1, \cdots, s^*_n) \succeq^t_i G(s_i, s^*_{-i})$. Fig. 1 depicts the setup of a physical mechanism. From the viewpoint of the designer, the physical mechanism works in the same manner as the Maskin mechanism does. The working steps of the physical mechanism are as follows:

Step 1: Nature selects a state $t \in \mathcal{T}$ and assigns t to the agents. Each coin is set head up.

Step 2: In state t, if all agents agree that the social choice rule F

is Pareto-inefficient, i.e., there exists $\hat{t} \in \mathcal{T}$, $\hat{t} \neq t$, $\hat{a} \in F(\hat{t})$ such that $\hat{a} \succeq_i^t a \in F(t)$ for every $i \in N$, and $\hat{a} \succ_j^t a \in F(t)$ for at least one $j \in N$, then goto Step 4.

Step 3: Each agent i sets $c_i = ((a_i, t_i, z_i), (a_i, t_i, z_i))$ (where $a_i \in A$, $t_i \in \mathcal{T}$, $z_i \in Z_+$ the set of nonnegative integers), $\omega_i = Not\ flip$, $m_i = card(i, 0)$. Goto step 5.

Step 4: Each agent i sets $c_i = ((\hat{a}, \hat{t}, 0), (a_i, t_i, z_i))$, then chooses a strategic action $\omega_i \in \Omega_i$ whether to flip coin i, and sends $card(i, 0)$ (or $card(i, 1)$) as m_i to the designer if coin i is head up (or tail up).

Step 5: The designer receives the overall message $m = (m_1, \cdots, m_n)$ and let the final outcome $G(s) = g(m)$ using rule 1, 2 and 3. END.

Proposition 1: Given an SCR F and a state $t \in \mathcal{T}$, $\mathcal{N}(\Gamma^P, t)$ is equivalent to $\mathcal{N}(\Gamma, t)$.

Proof: First, for $s^* = (s_1^*, \cdots, s_n^*) \in \mathcal{N}(\Gamma^P, t)$ and $a = G(s^*)$. Define a function $R : \{H, T\} \to \{0, 1\}$, $R(H) = 0$, $R(T) = 1$. If a is generated by step 4 and 5, then for each agent i, let $m_i^* = card(i, R(\omega_i^*(H)))$; if a is generated by step 3 and 5, then for each agent i, let $m_i^* = card(i, 0)$. Obviously, $m^* = (m_1^*, \cdots, m_n^*) \in \mathcal{N}(\Gamma, t)$.

Next, for $m^* = (m_1^*, \cdots, m_n^*) \in \mathcal{N}(\Gamma, t)$. For each agent i, let $s_i^* = (\omega_i^*, c_i^*)$, where $\omega_i^* = Not\ flip$, $c_i^* = (m_i^*, m_i^*)$, then $s^* = (s_1^*, \cdots, s_n^*) \in \mathcal{N}(\Gamma^P, t)$. Q.E.D.

Example 1: Let $N = \{Apple, Lily, Cindy\}$, $\mathcal{T} = \{t_1, t_2\}$, $A = \{a_1, a_2, a_3, a_4\}$. In each state $t \in \mathcal{T}$, the preference relations $(\succeq_i^t)_{i \in N}$ over the outcome set A and the corresponding SCR F are given in Table I. Obviously, F is monotonic and satisfies no-veto. By Maskin's theorem, F is Nash implementable. The SCR F is Pareto-inefficient from the viewpoint of the agents because in state $t = t_2$, all agents unanimously prefer a Pareto optimal outcome $a_1 \in F(t_1)$: for each agent i, $a_1 \succ_i^{t_2} a_2 \in F(t_2)$. Therefore

Table 1
An example of a Pareto-inefficient SCR.

State t_1			State t_2		
Apple	*Lily*	*Cindy*	*Apple*	*Lily*	*Cindy*
a_3	a_2	a_1	a_4	a_3	a_1
a_1	a_1	a_3	a_1	a_1	a_2
a_2	a_4	a_2	a_2	a_2	a_3
a_4	a_3	a_4	a_3	a_4	a_4
$F(t_1) = \{a_1\}$			$F(t_2) = \{a_2\}$		

when the true state is t_2, the physical mechanism enters Step 4.

Since $\hat{a} = a_1$ is Pareto optimal in state t_2, it seems that $(\hat{a}, \hat{t}, 0) = (a_1, t_1, 0)$ and "Not flip" should be a unanimous $card(i, 0)$ and an strategic action chosen by each agent i. However, *Apple* has an incentive to unilaterally deviate from $(a_1, t_1, 0)$ to $(a_4, *, *)$ by flipping her coin, since $a_1 \succ^{\hat{t}}_{Apple} a_4$, $a_4 \succ^{t}_{Apple} a_1$; *Lily* also has an incentive to unilaterally deviate from $(a_1, t_1, 0)$ to $(a_3, *, *)$ by flipping her coin, since $a_1 \succ^{\hat{t}}_{Lily} a_3$, $a_3 \succ^{t}_{Lily} a_1$. *Cindy* has no incentive to deviate from $(\hat{a}, \hat{t}, 0)$ because a_1 is her top-ranked outcome in two states. Therefore, $c_{Apple} = ((a_1, t_1, 0), (a_4, *, *))$, $c_{Lily} = ((a_1, t_1, 0), (a_3, *, *))$, $c_{Cindy} = ((a_1, t_1, 0), (a_1, t_1, 0))$.

Note that either *Apple* or *Lily* can certainly obtain her expected outcome only if just one of them flips her coin and deviates from $(\hat{a}, \hat{t}, 0)$ (If this case happens, rule 2 would be triggered). But this assumption is unreasonable, because all agents are rational, nobody is willing to give up and let the others benefit. Therefore, both *Apple* and *Lily* will flip their coins and deviate from $(\hat{a}, \hat{t}, 0)$. As a result, rule 3 will be triggered. Since *Apple* and *Lily* both have a chance to win the integer game, the winner is uncertain. Consequently, the final outcome is uncertain between a_3 and a_4, denoted as a_3/a_4.

To sum up, in state $t = t_2$, the dominant strategic action for *Apple* and *Lily* is *Flip*, which results in an uncertain outcome a_3/a_4. Even if the uncertain outcome is not preferred by each agent, it will always happen according to the mechanism in the proof of Maskin's sufficiency theorem. The underlying reason is just the same as what we have seen in the famous Prisoners' Dilemma, i.e., the individual rationality is in conflict with the group rationality. In this sense, the agents cannot combat a Pareto-inefficient SCR under classical physical circumstances.

4 Quantum mechanism

In 2007, Flitney and Hollenberg [9] investigated Nash equilibria in n-player quantum Prisoners' Dilemma. Following their procedures, we define:

$$\hat{\omega}(\theta,\phi) \equiv \begin{bmatrix} e^{i\phi}\cos(\theta/2) & i\sin(\theta/2) \\ i\sin(\theta/2) & e^{-i\phi}\cos(\theta/2) \end{bmatrix},$$

$\hat{\Omega} \equiv \{\hat{\omega}(\theta,\phi) : \theta \in [0,\pi], \phi \in [0,\pi/2]\}$, $\hat{J} \equiv \cos(\gamma/2)\hat{I}^{\otimes n} + i\sin(\gamma/2)\hat{\sigma_x}^{\otimes n}$, where γ is an entanglement measure. $\hat{I} \equiv \hat{\omega}(0,0)$, $\hat{D}_n \equiv \hat{\omega}(\pi,\pi/n)$, $\hat{C}_n \equiv \hat{\omega}(0,\pi/n)$.

In order to generalize the physical mechanism to a quantum domain, we revise the assumption 1 and 2 of the physical mechanism as follows:

1) Each agent i has a quantum coin i (qubit) and a classical card i. The basis vectors $|C\rangle \equiv (1,0)^T$, $|D\rangle \equiv (0,1)^T$ of a quantum coin denote head up and tail up respectively.

2) Each agent i independently performs a local unitary operation on his/her own quantum coin. The set of agent i's operation is $\hat{\Omega}_i = \hat{\Omega}$. A strategic operation chosen by agent i is denoted as $\hat{\omega}_i \in \hat{\Omega}_i$. If $\hat{\omega}_i = \hat{I}$, then $\hat{\omega}_i(|C\rangle) = |C\rangle$, $\hat{\omega}_i(|D\rangle) = |D\rangle$; If $\hat{\omega}_i = \hat{D}_n$, then $\hat{\omega}_i(|C\rangle) = |D\rangle$, $\hat{\omega}_i(|D\rangle) = |C\rangle$. $\hat{I}$ denotes "Not flip", $\hat{D}_n$ denotes "Flip".

In addition, we assume there is a device that can measure the

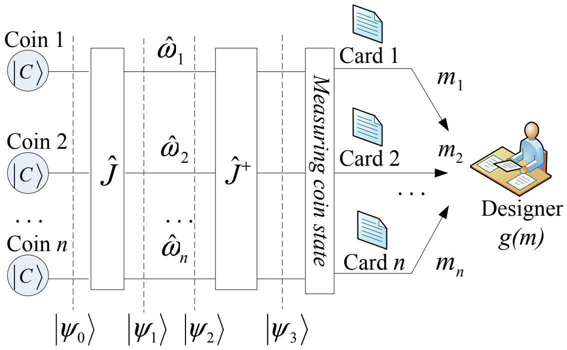

Fig. 2 The setup of a quantum mechanism. Each agent has a quantum coin and a card. A message is written on one side of a card. Each agent independently performs a local unitary operation on his/her own quantum coin.

state of n quantum coins and return the collapsed state to the agents. Based on aforementioned assumptions, we generalize the physical mechanism $\Gamma^P = ((S_i)_{i\in N}, G)$ to a *quantum mechanism* $\Gamma^Q = ((\hat{S}_i)_{i\in N}, \hat{G})$, where $\hat{S}_i = \hat{\Omega}_i \times C_i$. A quantum mechanism $\Gamma^Q = ((\hat{S}_i)_{i\in N}, \hat{G})$ describes a strategy set $\hat{S}_i$ for agent i and an outcome function $\hat{G} : \otimes_{i\in N}\hat{\Omega}_i \times \prod_{i\in N} C_i \to A$. We shall use $\hat{S}_{-i}$ to express $\otimes_{j\neq i}\hat{\Omega}_j \times \prod_{j\neq i} C_j$, and thus, a strategy profile is $\hat{s} = (\hat{s}_i, \hat{s}_{-i})$, where $\hat{s}_i \in \hat{S}_i$ and $\hat{s}_{-i} \in \hat{S}_{-i}$. A *Nash equilibrium* of a quantum mechanism Γ^Q played in state t is a strategy profile $\hat{s}^* = (\hat{s}^*_1, \cdots, \hat{s}^*_n)$ such that for any agent $i \in N$, $\hat{s}_i \in \hat{S}_i$, $\hat{G}(\hat{s}^*_1, \cdots, \hat{s}^*_n) \succeq^t_i \hat{G}(\hat{s}_i, \hat{s}^*_{-i})$. Fig. 2 depicts the setup of a quantum mechanism. Its working steps are as follows:

Step 1: Nature selects a state $t \in \mathcal{T}$ and assigns t to the agents. The state of every quantum coin is set as $|C\rangle$. $|\psi_0\rangle = |C \cdots CC\rangle$.

Step 2: In state t, if all agents agree that the social choice rule F is Pareto-inefficient, i.e., there exists $\hat{t} \in \mathcal{T}$, $\hat{t} \neq t$, $\hat{a} \in F(\hat{t})$ such that $\hat{a} \succeq^t_i a \in F(t)$ for every $i \in N$, and $\hat{a} \succ^t_j a \in F(t)$ for at least one $j \in N$, then goto step 4.

Step 3: Each agent i sets $c_i = ((a_i, t_i, z_i), (a_i, t_i, z_i))$ (where $a_i \in A$, $t_i \in \mathcal{T}$, $z_i \in Z_+$), $\hat{\omega}_i = \hat{I}$, and sends $card(i, 0)$ as m_i to the designer. Goto step 8.

Step 4: Each agent i sets $c_i = ((\hat{a}, \hat{t}, 0), (a_i, t_i, z_i))$. Let n quantum coins be entangled by $\hat{J}$. $|\psi_1\rangle = \hat{J}|C \cdots CC\rangle$.

Step 5: Each agent i independently performs a local unitary operation $\hat{\omega}_i$ on his/her own quantum coin. $|\psi_2\rangle = [\hat{\omega}_1 \otimes \cdots \otimes \hat{\omega}_n]\hat{J}|C \cdots CC\rangle$.

Step 6: Let n quantum coins be disentangled by $\hat{J}^+$. $|\psi_3\rangle = \hat{J}^+[\hat{\omega}_1 \otimes \cdots \otimes \hat{\omega}_n]\hat{J}|C \cdots CC\rangle$.

Step 7: The device measures the state of n quantum coins and returns the collapsed state to the agents. Each agent i sends $card(i, 0)$ (or $card(i, 1)$) as m_i to the designer if the state of quantum coin i is $|C\rangle$ (or $|D\rangle$).

Step 8: The designer receives the overall message $m = (m_1, \cdots, m_n)$ and let the final outcome $\hat{G}(\hat{s}) = g(m)$ using rule 1, 2 and 3. END.

Note that if $\hat{\Omega}_i$ is restricted to be $\{\hat{I}, \hat{D}_n\}$, then $\hat{\Omega}_i$ is equivalent to $\{Not\ flip, Flip\}$. In this way, a quantum mechanism is degenerated to a physical mechanism.

Given n ($n \geq 3$) agents, consider the payoff to the n-th agent, we denote by $\$_{C\cdots CC}$ the expected payoff when all agents choose $\hat{I}$ (the corresponding collapsed state is $|C \cdots CC\rangle$), and denote by $\$_{C\cdots CD}$ the expected payoff when the n-th agent chooses $\hat{D}_n$ and the first $n-1$ agents choose $\hat{I}$ (the corresponding collapsed state is $|C \cdots CD\rangle$). $\$_{D\cdots DD}$ and $\$_{D\cdots DC}$ are defined similarly. Different from Flitney and Hollenberg's requirements on the payoffs, for the case of quantum mechanism, the requirements on the payoffs are described as condition λ:

1) λ_1: Given an SCR F and a state t, if there exists $\hat{t} \in \mathcal{T}$, $\hat{t} \neq t$, $\hat{a} \in F(\hat{t})$ such that $\hat{a} \succeq_i^t a \in F(t)$ for every $i \in N$, and $\hat{a} \succ_j^t a \in F(t)$ for at least one $j \in N$, then in going from state $\hat{t}$ to t, there exist at least two agents that encounter a preference change around $\hat{a}$. Denote by l the number of these agents. Without loss of generality, let these l agents be the last l agents among n agents.

2) λ_2: Consider the payoff to the n-th agent, $\$_{C\cdots CC} > \$_{D\cdots DD}$, i.e., he/she prefers the expected payoff of a certain outcome (gen-

erated by rule 1) to the expected payoff of an uncertain outcome (generated by rule 3).

3) λ_3: Consider the payoff to the n-th agent, $\$_{C\cdots CC} > \$_{C\cdots CD}[1-\sin^2\gamma\sin^2(\pi/l)] + \$_{D\cdots DC}\sin^2\gamma\sin^2(\pi/l)$.

Proposition 2: For $n \geq 3$, given a state $t \in \mathcal{T}$ and a Pareto-inefficient SCR F (from the viewpoint of agents) that is monotonic and satisfies no-veto, by virtue of a quantum mechanism $\Gamma^Q = ((\hat{S}_i)_{i\in N}, \hat{G})$, agents satisfying condition λ can combat the Pareto-inefficient SCR F.

Proof: Given a state $t \in \mathcal{T}$, if an SCR F specified by the designer is Pareto-inefficient, then there exists $\hat{t} \in \mathcal{T}$, $\hat{t} \neq t$, $\hat{a} \in F(\hat{t})$ such that $\hat{a} \succeq_i^t a \in F(t)$ for every $i \in N$, and $\hat{a} \succ_j^t a \in F(t)$ for at least one $j \in N$. Hence, the quantum mechanism enters step 4. Since condition λ_1 is satisfied, according to Ref. [9], consider the payoff to the n-th agent (denoted as *Laura*), when she plays $\hat{\omega}(\theta,\phi)$ while the first $n-l$ agents play $\hat{I}$ and the middle $l-1$ agents play $\hat{C}_l$:

$$\begin{aligned}\langle\$_{Laura}\rangle =&\$_{C\cdots CC}\cos^2(\theta/2)[1-\sin^2\gamma\sin^2(\phi-\pi/l)]\\ &+\$_{C\cdots CD}\sin^2(\theta/2)[1-\sin^2\gamma\sin^2(\pi/l)]\\ &+\$_{D\cdots DC}\sin^2(\theta/2)\sin^2\gamma\sin^2(\pi/l)\\ &+\$_{D\cdots DD}\cos^2(\theta/2)\sin^2\gamma\sin^2(\phi-\pi/l)\end{aligned}$$

Since condition λ_2 is satisfied, then $\$_{C\cdots CC} > \$_{D\cdots DD}$, *Laura* chooses $\phi = \pi/l$ to minimize $\sin^2(\phi-\pi/l)$. As a result,

$$\begin{aligned}\langle\$_{Laura}\rangle =&\$_{C\cdots CC}\cos^2(\theta/2)\\ &+\$_{C\cdots CD}\sin^2(\theta/2)[1-\sin^2\gamma\sin^2(\pi/l)]\\ &+\$_{D\cdots DC}\sin^2(\theta/2)\sin^2\gamma\sin^2(\pi/l)\end{aligned}$$

Since condition λ_3 is satisfied, then *Laura* prefers $\theta = 0$, which leads to $\langle\$_{Laura}\rangle = \$_{C\cdots CC}$. In this case, $\hat{\omega}_{Laura}(\theta,\phi) = \hat{\omega}(0,\pi/l) = \hat{C}_l$.

By symmetry, let $\hat{s} = (\hat{\omega}, c)$, where $\hat{\omega} = (\hat{I},\cdots,\hat{I},\hat{C}_l,\cdots,\hat{C}_l)$

(the first $n-l$ agents choose $\hat{I}$, the rest l agents choose $\hat{C}_l$), and $c=(c_1,\cdots,c_n)$, where $c_i=((\hat{a},\hat{t},0),(a_i,t_i,z_i))$ $(i\in N)$, then $\hat{s}\in\mathcal{N}(\Gamma^Q,t)$. In step 7, the corresponding collapsed state of n quantum coins is $|C\cdots CC\rangle$, therefore $m_i=(\hat{a},\hat{t},0)$ $(i\in N)$, $\hat{G}(\hat{s})=g(m)=\hat{a}\notin F(t)$. Q.E.D.

Let us reconsider Example 1. Since F is Pareto-inefficient, the quantum mechanism enters step 4. $l=2$, condition λ_1 is satisfied.

$$\begin{aligned}
c_{Apple} &= ((a_1,t_1,0),(a_4,*,*)),\\
c_{Lily} &= ((a_1,t_1,0),(a_3,*,*)),\\
c_{Cindy} &= ((a_1,t_1,0),(a_1,t_1,0)).
\end{aligned}$$

Let *Cindy* be the first agent, for any agent $i\in\{Apple, Lily\}$, let her be the last agent. Consider the payoff to the n-th agent, suppose $\$_{CCC}=3$ (the corresponding outcome is a_1), $\$_{CCD}=5$ (the corresponding outcome is a_4 if $i=Apple$, and a_3 if $i=Lily$), $\$_{DDC}=0$ (the corresponding outcome is a_3 if $i=Apple$, and a_4 if $i=Lily$), $\$_{DDD}=1$ (the corresponding outcome is a_3/a_4). Hence, condition λ_2 is satisfied and condition λ_3 becomes: $3\geq 5[1-\sin^2\gamma\sin^2(\pi/2)]$. If $\sin^2\gamma\geq 0.4$, condition λ_3 is satisfied. According to Proposition 2, the message corresponding to $\hat{s}^*\in\mathcal{N}(\Gamma^Q,t)$ is $m^*=(m_1^*,m_2^*,m_3^*)$, where $m_1^*=m_2^*=m_3^*=(a_1,t_1,0)$. Consequently, $\hat{G}(\hat{s}^*)=g(m^*)=a_1\notin F(t)=\{a_2\}$.

To help the reader understand the aforementioned result, let the SCR in Table 1 be "No smoking". Let a_1 and a_2 denote "Smoking" and "No smoking" respectively. Suppose everybody likes smoking very much in state t_2, then the SCR is Pareto-inefficient to the smoker group. According to the traditional theory of mechanism design, the "No smoking" SCR can be Nash implemented because "No smoking" satisfies monotonicity and no-veto. But by virtue of quantum strategies, this smoker group can combat the "No smoking" SCR!

Remark: In Maskin and Sjöström [4], the authors used a modulo game instead of the integer game. The rule 3 is replaced by "3) In all other cases, $g(m) = a_j$, for $j \in N$ such that $j = (\sum_{i\in N} z_i)(\text{mod } n)$". Similar to aforementioned analysis, it can be derived that the results of this paper still hold.

5 Conclusions

In conclusion, this paper considers what happens if the theory of mechanism design is quantized. The main results are two folds:

1) The paper proposes that the success of Maskin's mechanism is indeed built on an underlying Prisoners' Dilemma, which may not be awared clearly by the economic society.

2) Under the classical circumstance, if an SCR satisfies monotonicity and no-veto, then no matter whether it is Pareto-efficient or not (from the viewpoint of the agents), it can be certainly Nash implemented. But now, when the additional condition λ is satisfied, an original Nash implementable Pareto-inefficient SCR will no longer be Nash implementable in the context of quantum domain.

Ref. [10] pointed out that in quantum games, quantum strategies just constructed a new game and solved it, not the original game. However, from the viewpoint of the designer, the interface between agents and the designer in the quantum mechanism is the same as that in the Maskin mechanism. Therefore, from the viewpoint of agents, quantum mechanism helps them combat Pareto-inefficient social choice rules specified by the designer.

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