The Risk of Operational Incidents in Banking Institutions

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ABSTRACT

Banking-financial institutions are organizations which might be included in the category of complex systems. Consequently, they can be applied after adaptation and particularization, in the general description and assessment methods of the technical or organizational systems. The banking-financial system faces constrains regarding the functioning continuity. Interruptions in continuity as well as operational incidents represent risks which can lead to the interruption of financial flows generation and obviously of profit. Banking incidents include from false banknote, cloned cards, informatics attacks, false identity cards to ATM attacks. The functioning of banking institutions in an incident-free environment generates concern from both risk assessment and forecasting points of view.

Key words: operational risk, banking reliability, complex systems, incident probability, the risk of functioning interruption.

1 THE ISSUE STATEMENT

Usually, an incident-free performance is expressed by the reliability term. The reliability represents a qualitative characteristic of systems in the largest meaning of the term. Reliability is measured as probability of success. Specific to the numerical expressions of reliability is the fact that the main indicator is a probability, and therefore a positive number between 0 and 1.

In probabilistic expression, the risk of incidents has the following synthetic form:

$$I(t) = 1 - R(t) \tag{1}$$

This is a complementary value for the probability of performance without incidents, which is the operational reliability of the bank:

$$R(t) = P\{T \geq t\} \tag{2}$$

reliability which is expressed through a probability that measures the chance that the time T of functioning without incidents surpasses a previously established period t. We can say about $R(t)$ that:

- it is a decreasing function, and therefore reliability decreases in time;
- for $t=0$, $R(t)=1$, so at the starting point, the bank must be in perfect performance;
- for $t\to\infty$, $R(t)=0$, which means that for very long periods of time, reliability tends to become null;

Since reliability measures the probability of a system to fulfil its functions in a certain time period, time must be introduced as condition element in estimating reliability.

An important element is represented by the intensity or rate of banking incidents:

$$\lambda(t) = \frac{f(t)}{1 - F(t)} = \frac{f(t)}{R(t)} \tag{3}$$

where $f(t)$ is the probability density and $F(t)$ the distribution function (the probability of a banking incident), determined after testing the incident distribution law for the analysed banking system.

The probability density is related to an important indicator of the performance of a banking system, and that is the "distribution function of time without operational incidents" $F(t;\theta_i)$, which is defined as a probability of the operational malfunctioning in a bank:

$$F(t;\theta_i) = Prob\{T < t\} \tag{4}$$

The distribution function can be considered as "unreliability function" or "risk function" because there is an obvious relation:

$$Prob\{T < t\} + Prob\{T \geq t\} = 1 \tag{5}$$

The specific indicators of banking risk and reliability are presented in Table 1.

Table 1. The main indicators of risk and reliability

No.	Name	Calculus relation
1	The distribution function of performance non – incident time (*T*)	$F(t) = Prob\{T < t\}$
2	The probability density of performance non-incident time	$f(t) = F_t^{'}(t) = \left(\frac{dF(t)}{dt}\right)$
3	Reliability function (non-incident)	$R(t) = P\{T \geq t\}$
4	The average period of functioning without incidents	$E(T) = \int_0^\infty t \cdot f(t)dt$
5	The variance of functioning time without incidents	$Var(T) = E(T^2) - [E(T)]^2$
6	Variation coefficient	$CV(T) = \frac{\sqrt{Var(T)}}{E(T)}$

2 OPERATIONAL RISK INDICATORS

Risk measurement can be achieved through two major groups of indicators:

- Indicators based on descriptive statistics methods (non-parametric indicators)
- Indicators based on probabilistic models (parametric indicators).

2.1 Non-parametric indicators

They are indicators determined based on observation data, more precisely based on information collected on a period of time, for a bank portfolio.

Among these indicators we can find:

1. The relative frequency of "incidents" in banks:

$$\tilde{f}(t_i) = \frac{r_i}{\sum_{i=1}^{m} r_i} \tag{6}$$

where r_i – is the number of banking incidents.

Based on these relative frequencies we now calculate

2. Cumulated relative frequency of incidents

$$\hat{F}(t_i) = \frac{1}{N}\sum_{j=1}^{i} r_i \tag{7}$$

which expresses the weight of banking units with incidents until the end of interval *i*; its value is increasing and equals 1 at the last interval of the series.

3. The relative frequency of non-incident banking units, is determined as complement to 1 of the cumulative relative frequency of incidents:

$$\hat{R}(t_i) = 1 - \hat{F}(t_i) = \frac{N_i}{N} \tag{8}$$

4. The average frequency (number) of incidents in a period of observation is determined as the total number of incidents $N = \sum_{i=1}^{m} r_i$ divided to the amount of non-incidents time of all banking units in the sample $\sum_{i=1}^{m} t_i \cdot r_i$.

$$\bar{f} = \frac{\sum_{i=1}^{m} r_i}{\sum_{i=1}^{m} t_i \cdot r_i} \text{ or } \bar{r} = \frac{\sum_{i=1}^{m} t_i \cdot r_i}{\sum_{i=1}^{m} t_i} \tag{9}$$

5. The mean time between failures (mean time of functioning without operational incidents) – MBTF is :

$$\bar{t} = \frac{\sum_{i=1}^{m} t_i \cdot r_i}{\sum_{i=1}^{m} r_i} = \frac{\sum_{i=1}^{m} t_i \cdot r_i}{N} \tag{10}$$

Obviously, there is an inverse relation between MTGF and *f*, MTBF = *1/f*.

MTBF is a direct indicator, because its measurement is directly proportional to the degree of reliability: a higher degree of reliability means a higher MTBF, and the other way around.

6. Incidents density, determined by dividing the number of incidents registered in an observation interval to the length of that interval. In the case in which time intervals are equal during the entire length of the series, the indicator becomes analogous to the experimental distribution density.

7. Incident rate

This indicator shows the weight of incidents during an observation time period as against to the existing one at the beginning of that period,

$$\lambda(t) = \frac{r_i}{\Delta t \cdot (n - r_i)} \tag{11}$$

where $N_i = n - r_i$ is the number of units where no incidents occurred at the beginning of interval *i*. If the product functions in stationary regime, the incident rate for the entire sample equals the average frequency of incidents.

2.2 Parametric indicators. Statistic models generation for non-incidents operational time

In his monumental paper "Encyclopaedia of Statistical sciences" the well-known statisticians N.L. Johnson and S. Kotz (Johnson & Kotz 1983) state that one of the basic preoccupations of classical statistics was, and still is, the finding of statistic models generation mechanism, in other words finding distribution functions or probability densities.

Accordingly, they enumerate five systems of “frequency curves”: Pearson (as differential equation), Gram-Charlier-Edgeworth (as series development), Burr (as differential equation), Johnson (as normality transformation), and Turkey (special transformation).

Also worth mentioning are – in chronological order – the McKay system (1932), based on the use of Bessel functions, the Perks system (1932), which generates a very wide range of logistic distributions, the Darmois (1935) and Koopman (1936) systems regarding exponential distributions, the Gnedenko system (1943) using extreme value distributions, the Toranzos (1952) system, a generalization of the Pearson and Mathai-Saxena systems (1966) based on the generalized hypergeometric function.

In 1982, M. A. Savageau (Savageau 1982) proposes a system of differential equations in which the unknown variables are two densities f_1 and f_2:

$$\begin{cases} \dfrac{df_1}{dx} = a_1 \cdot f_1^{\alpha_{11}} \cdot f_2^{\alpha_{12}} - b_1 \cdot f_1^{\beta_{11}} \cdot f_2^{\beta_{12}} \\ \dfrac{df_2}{dx} = a_2 \cdot f_1^{\alpha_{21}} \cdot f_2^{\alpha_{22}} - b_2 \cdot f_1^{\beta_{21}} \cdot f_2^{\beta_{22}} \end{cases} \tag{12}$$

Where a_1, a_2, b_1, b_2 are real negative numbers and α_{11}, α_{12} etc. are random real numbers which provide known densities through various particularizations.

For example, $a_1 = 1,\ a_2 = 1,\ b_1 = 1,\ b_2 = 0,\ \alpha_{11} = 1,\ \alpha_{12} = -1,\ \beta_{11} = 1,\ \beta_{12} = -1,\ \alpha_{21} = 0,$ $\alpha_{22} = 0,\ \beta_{21} = 0,\ \beta_{22} = 0$ leads us to:

$$\frac{df_1}{dx} = \frac{f_1}{f_2} - f_1 \cdot f_2 \text{ and } \frac{df_2}{dx} = 1,$$

where $f_2 = x$ and therefore:

$$\frac{df_1}{dx} = \frac{1}{x} \cdot f_1 - x \cdot f_1 \text{ or } \frac{df_1}{f_1} = \left(\frac{1}{x} - x\right) \cdot dx,$$

which through integrations leads us to the density

$$f_1(x) = \exp\left\{\int \left(\frac{1}{x} - x\right) dx\right\} = \exp\left\{\ln x - \frac{x^2}{2}\right\} = x \cdot \exp\left\{-\frac{x^2}{2}\right\}, x \geq 0$$

which is precisely the reduced Rayleigh density.

The majority of the proposed systems are in fact differential equations in which the unknown variable is either F (the distribution function) or f (the probability function).

Karl Pearson’s classical system (Pearson 1895) is given by the separate values differential equation:

$$\frac{df}{dx} = \frac{(x - c_0) \cdot f}{c_1 + c_2 x + c_3 x^2}$$

or

$$\frac{df}{f} = \frac{(x - c_0)}{c_1 + c_2 x + c_3 x^2} \cdot dx \tag{13}$$

In which various particularizations of the c_i coefficients lead to the majority of the classical models.

Thus, if $c_0 = \mu, c_1 = -\sigma^2, c_2 = c_3 = 0,$ where $\mu \in R$, $\sigma > 0,$ for $x \in R$, we have:

$$\frac{df}{f} = \frac{x - \mu}{-\sigma^2} dx \quad \text{or} \quad f = A \cdot \exp\left\{-\frac{(x - \mu)^2}{2\sigma^2}\right\}$$

where A is the normalization factor $\left(1/\sigma\sqrt{2\pi}\right)$, and therefore we have the Gauss-Laplace density.

For $c_0 = k/\theta, c_1 = c_3 = 0, c_2 = 1/2, (k,\theta > 0)$ we have

$$f = \frac{1}{\theta^{k+1}\Gamma(k+1)} \cdot x^k \cdot \exp\left\{-\frac{x}{\theta}\right\}, x \geq 0, \theta, k > 0 \tag{14}$$

which is the well-known Gamma model.

In general, the density expression depends on the roots of the denominator $c_1 + c_2 x + c_3 x^2$ and as can be seen in the two examples presented, the c_i coefficients are various moments of the distribution. The complete list of the "Pearson curves" can be consulted in "Distributions in Statistics. Continuous Univariate Distributions" (Johnson & Kotz 1970).

The Gram-Charlier-Edgeworth system (Cohen 1998) is based on a theorem which assures the possibility of writing the $f(x)$ density of a random continuous variable X as an infinite series of terms depending on $f_0(x)$ - the density of the standard normal variable:

$$f_0(x) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{x}{2}\right\} \tag{15}$$

and certain coefficients. The Gram-Charlier-Edgewords decomposition uses Hermite polynoms. F. Y. Edgeworth (Perote & Del Brio 2006) simplified to a point the Gram-Charlier series, proposing the following expression for the density approximation:

$$\begin{aligned} f(x) = \frac{1}{\sqrt{2\pi}} \Big[1 - \frac{1}{6}\sqrt{\beta_1}\left(x^2 - 3x\right) + \frac{1}{24}(\beta_2 - 3) \cdot \left(x^4 - 6x^2 + 3\right) + \\ + \frac{1}{72}\beta_1\left(x^5 - 10x^3 + 14x\right) \Big] \cdot \exp\left\{-\frac{x^2}{2}\right\} \end{aligned} \tag{16}$$

where $\sqrt{\beta_1}$ and $\sqrt{\beta_2}$ are asymmetry and excess coefficients.

Irving W. Burr (Burr 1942) proposes a relatively malleable system through a differential equation, in which the unknown variable is given by this distribution function

$$dF = F(1/F)g(x)dx$$

where $0 \leq F \leq 1$, and $g(x)$ is a non-negative function, chosen conveniently.

Although the Burr system provides twelve explicitly expressed distribution functions, it seems that only one of these functions made a name for itself in statistics. It's the "Burr distribution":

$$F(x;c,k) = 1 - \left(1 + x^c\right)^{-k}, x \geq 0, c, k > 0 \tag{17}$$

1. A general model

In the followings, we will propose a general differential equation, which has the interesting property that depending on the particular choices of the implicated elements, it provides the distribution function or the probability density, which neither one of the other systems does.

The differential equation is:

$$\frac{d\varphi}{dx} = a(x) \cdot \varphi^\alpha + b(x) \cdot \varphi^\beta, \tag{18}$$

where φ is a strictly positive real function, for x usually between $(0, +\infty)$, $a(x)$ and $b(x)$ are continuous functions defined on R, and α, β two random numbers.

We will demonstrate that this equation provides distribution functions or density functions, by case. First of all, let us notice that for

$$b(x) = -a(x), \quad \alpha = 1 \text{ and } \beta = 2$$

we have

$$\frac{d\varphi}{dx} = \varphi(1 - \varphi) \cdot a(x), \tag{19}$$

which is precisely the Burr model; therefore, this equation can be considered a natural generalization of the Burr model.

In the international literature, there are procedures which generate functions for laws such as the exponential law, power, normal, log-normal etc.

In the followings, we develop a highly general law, the Weibull law, which involves other models as particular cases.

2. The Weibull Model

If we chose $a(x) = b(x) = \frac{k}{\theta} x^{k-1}, \theta, k > 0, \alpha = 0, \beta = 1$ and determine

$$\frac{d\varphi}{dx} = (1-\varphi)\frac{k}{\theta} x^{k-1} \quad \text{or} \quad \frac{d\varphi}{1-\varphi} = \frac{k}{\theta} x^{k-1},$$

meaning that:

$$-\ln(1-\phi) = \frac{x^k}{\theta}, \quad \text{so that} \quad 1-\phi = \exp\left\{-\frac{x}{\theta}\right\}$$

therefore

$$\varphi(x;\theta,k) = 1 - \exp\left\{\frac{-x^k}{\theta}\right\}, \quad x \geq 0, \quad k > 0 \tag{20}$$

which is the well-known Weibull distribution.

Let us look at some historic elements: in 1939, the Swedish engineer Waloddi Weibull, professor at the Royal Institute of Technology in Stockholm (KTH - Kungl Tekniska Högskolan), proposed a statistical model to describe the spread manner of the observed values during experiments regarding the resistance of materials.

He started from the idea that the "fatigue" of the materials cannot be described realistically by the exponential model, since its failure rate does not depend explicitly on time. The model can also be used in the case of banking incidents, due to its general virtues.

In order to avoid this fault of the exponential case, Weibull takes into consideration the following distribution:

$$X : F_x(x;\gamma,\theta,k) = 1 - \exp\left\{-\left(\frac{x-\gamma}{\theta}\right)^k\right\}, \tag{21}$$

where $x \geq \gamma > 0, \quad \theta, k > 0$. For the particular case of $\gamma = 0$ and $k = 1$, it contains the exponential model, while for $\gamma = 0$ and $k = 2$, Lord Rayleigh's model (1842-1919).

Let us observe that for Rayleigh's case

$$F_x(x;0,\theta,2) = 1 - \exp\left\{-\left(\frac{x}{\theta}\right)^2\right\}, x \geq 0, \quad \theta > 0 \tag{22}$$

the incident intensity is a linear time function $z(x) = \frac{2}{\theta} \cdot x$ and that it's obvious that if $x_1 < x_2$ than $z(x_1) < z(x_2)$, and therefore the respective indicator is directly proportional to time. This aspect makes the Rayleigh model be an efficient describer of usual incidents.

Weibull's papers were published in a small journal at the time (The Swedish Technical Academy Annals - Ingeniörs Vetenskaps Akademieus Handligar), and remained without immediat echo. Only after the Second World War, in 1951 more precisely, did Weibull re-start his research through the American magazine, Journal of Applied Mechanics, under the title "A statistical distribution of wide applicability", which started a real "Weibull euphoria".

Later on, in 1977, Weibull himself commenced a bibliographic research and identified a number of 1019 articles and 36 book titles which dealt with various aspects of the model. Weibull had already expressed his intentions of completing this study through the investigation of other publications besides the ones in English, but his death, in 1979, put an end to these plans.

An important explanation is imposed: Weibull has the merit of highlighting the wide range of applications of the model, but theoretically, it was deduced by the French mathematician Maurice Fréchet back in 1927 (Fréchet 1927).

The British statisticians R. A. Fisher and L. H. Tippet deduce it in a study concerning limiting forms of the extreme statistics (Fisher & Tippet 1928).

The Russian academician B. V. Gnedenko also studies the model thoroughly through these limiting distributions (Gnedenko 1943).

All these papers were purely mathematical, and they didn't have the necessary impact among practicians, while the Weibull Model entered and remained in the post-war scientific literature definitely related to the name of the Swedish engineer.

In Romanian translation, the first large paper to present a study in the field was due to the Russian authors Gnedenko B. V., Beleaev I.K., Soloviev A. D. – "Mathematical methods in the safety theory", together with spreading the term of "safety" (faith, trust, solidity, the Russian надежность) (Gnedenko & al. 1968).

In the francophone literature, also accessible in Romania, the monographic paper of two French engineers benefited from a certain spreading: "Fiabilité et Statistique Previsionelles: Méthodes de Weibull" (Pollard & Rivoire 1971). They were also the ones to introduce the expression "Weibull Method", taken by the Japanese under the term "Waiburumokei" and by the Russians as "Pravila Veibulla - Правила Veibulla", meaning Weibull's Rule.

Thus considering the bi-parametric distribution

$$X : F_x(x;\theta,k) = 1 - \exp\left\{-\left(\frac{x}{\theta}\right)^k\right\}, \quad x \geq 0, \quad \theta, k > 0 \tag{23}$$

to which the following density corresponds

$$X : f_x(x;\theta,k) = k\theta^{-k} x^{k-1} \exp\left\{-\left(\frac{x}{\theta}\right)^k\right\}, \quad x \geq 0, \quad \theta, k > 0 \tag{24}$$

For this form

$E(X) = \theta \cdot \Gamma(1 + 1/k)$ and $-Var(X) = \theta^2 \cdot \left[\Gamma(1 + 2/k) - \Gamma^2(1 + 1/k)\right]$

$X_{me} = \theta(\ln 2)^{1/k}$ - the median and $X_{mo} = \theta[(k/1)/k]^{1/k}$ - the mode $(k > 1)$.

If the *k* parameter is relatively large, than we can utilize the approximations:

$E(X) \approx 1 - 057722/\theta + 0{,}98905/\theta^2$ and $Var(X) \approx 1{,}64493/\theta^2$.

We also have $0{,}885 \cdot \theta < E(X < \theta)$ and $F_x(\theta) \approx 0{,}63$.

The variation coefficient $CV(X)$ only depends on the *k* parameter

$$CV(X) = \left[\frac{\Gamma(1 + 2/k)}{\Gamma^2(1 + 1/k)} - 1\right]^{1/2}$$

Estimating the model parameters through the ordinary least squares method, or the maximum likelihood method, are the most common procedures.

A synthesis of the Weibull Method was presented in Romanian (Isaic-Maniu 1983).

The Weibull Function Study. The frequency function or the Weibull operational risk function has a general form:

$$F(t) = 1 - R(t) = \exp\left[-\left(\frac{t-\gamma}{\theta}\right)^k\right] \tag{25}$$

Let us notice in figure 1, that if $t = \gamma$, then the value of this function is 1, and that the function is decreasing for $t > \gamma$. Also, if $t \to \infty$, then $R(t) \to 0$. $R(t)$ decreases monotonously and "abruptly" for $0 < k < 1$, it is convex and decreases less abruptly for the same *k*, but for a higher θ than in the previous case.

For *k* = 1 and θ established, $R(t)$ is monotonously decreasing and is convex (this is in fact the exponential case). For $k > 1$, $R(t)$ decreases if *t* decreases. The value of $R(t)$ for a mission of $t = \theta + \gamma$ (let's say, hours) is always 0,368, which is e^{-1} (the points marked with • in figure 1).

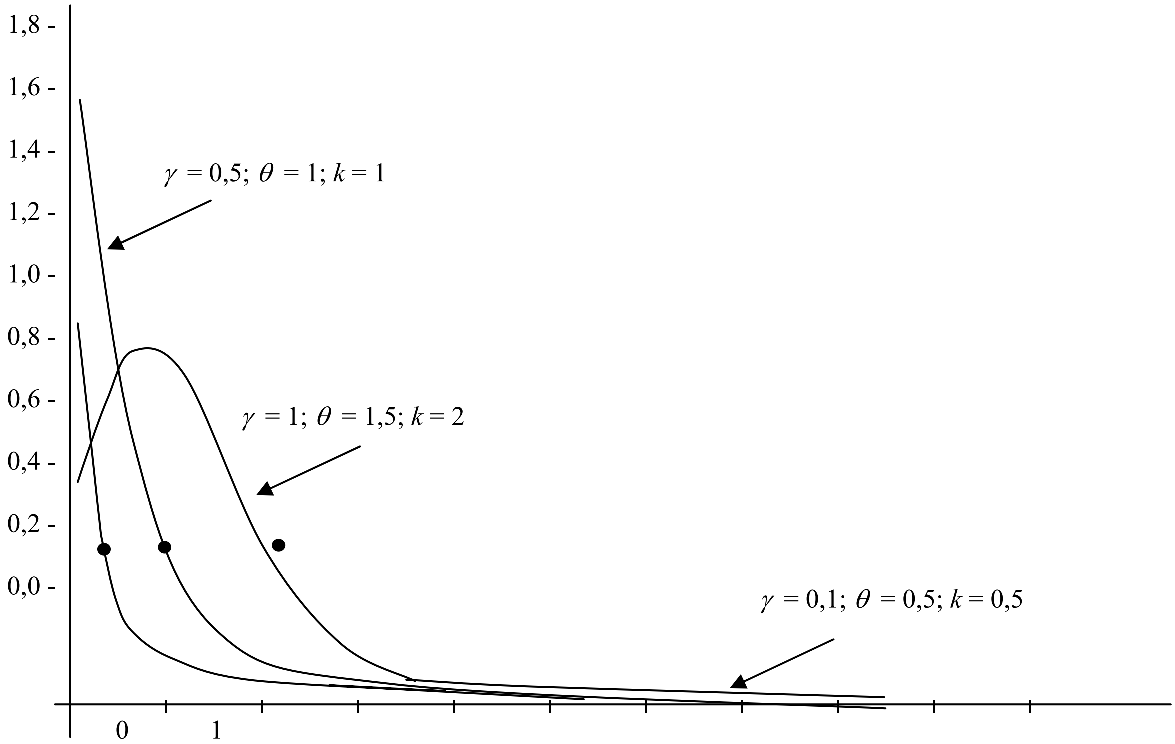

Figure 1. Weibull Reliability Function

An extremely useful indicator is the "conditional reliability function". Assuming that a banking system already functioned without incidents for *t* hours, then it is important to find out the reliability of a mission, another temporal interval of t_0 hours, which is $R(t;t_0)$.

It is known that

$$R(t;t_0) = \frac{R(t+t_0)}{R(t)},$$

where $R(t;t_0)$ is the reliability for a new mission (operation) of t_0 hours, being given the fact that the system functioned without incidents for *t* hours.

Therefore, in the Weibull case

$$R(t;t_0)=\frac{\exp\left[-\frac{(t+t_0-\gamma)^k}{\theta}\right]}{\exp\left[-\frac{(t-\gamma)^k}{\theta}\right]}=\exp\left\{-\left[\left(\frac{t+t_0-\gamma}{\theta}\right)^k-\left(\frac{t-\gamma}{\theta}\right)^k\right]\right\}$$

The t_R reliable life for a system with a given reliability $R(t_R)$, which begins functioning at moment zero, is obtained from:

$$R(t_R)=\exp\left[-\left(\frac{t_R-\gamma}{\theta}\right)^k\right] \tag{26}$$

through logarithmation

$$t_R=\gamma+\theta\{-\ln[R(t_R)]\}^{1/k}$$

This is given by the life period in which the system will operate without incidents with a given probability $R(t_R)$.

If $R(t_R)$ = 50%, then t_R is precisely the "median interval".

Estimating the model parameters based on truncated observations. Because of the fact that the most frequently encountered situation is not based on total observations but on partial ones, whose lengths are interrupted, this being a distinct situation from that of technical systems, observed till exhaustion, we shall consider this case, applicable to banking systems observable for a period of time, followed by assessment.

Thus, if we consider X_0 – the time when observations are interrupted, and $X_{(r)}$ the moment of the ***r***th incident, the likelihood function for this incident variant is described as:

$$L=f_{X_x}\left(X_{\gamma_0+1,\dots,}X_\gamma\right)$$

We obtain

$$\mathrm{L}=\frac{\mathrm{n}!}{(\mathrm{n}-\mathrm{r})!\mathrm{r}_0}\left(\frac{\beta}{\theta}\right)^{\mathrm{r}-\mathrm{r}_0}\cdot\prod_{\mathrm{i}=\mathrm{r}_0+1}^{\mathrm{r}}\left(\frac{\mathrm{x}_\mathrm{i}-\gamma}{\theta}\right)^{\beta-1}\cdot\exp\left[-\sum_{\mathrm{i}=\mathrm{r}_0+1}^{r}\left(\frac{\mathrm{x}_\mathrm{i}-\gamma}{\theta}\right)^{\beta}-(\mathrm{n}-\mathrm{r})\left(\frac{\mathrm{x}_\mathrm{i}-\gamma}{\theta}\right)^{\beta}\right]$$
$$\left\{1-\exp\left[-\left(\frac{\mathrm{t}_{\mathrm{r}_\mathrm{i}+1}-\gamma}{\theta}\right)^{\beta}\right]\right\}^{\mathrm{r}_0} \tag{27}$$

Calculating the derivatives of the logarithm for the previous function, relative to the three parameters, we have

$$\frac{\partial\ln L}{\partial\theta}=\frac{(\mathrm{r}-\mathrm{r}_0)\beta}{\theta}+\beta\frac{\sum_{\mathrm{i}=\mathrm{r}_0+1}^{r}(\mathrm{x}_\mathrm{i}-\gamma)^{\beta}}{\theta^{\beta+1}}+\frac{\beta(\mathrm{n}-\mathrm{r})(\mathrm{x}_\mathrm{r}-\gamma)^{\beta}}{\theta^{\beta+1}}-$$
$$-\frac{\beta\left(\mathrm{x}_{\mathrm{r}_0+1}-\gamma\right)^{\beta}}{\theta^{\beta+1}}\cdot\frac{\mathrm{r}_0\exp\left[-\left(\mathrm{x}^{\mathrm{r}_0+1}-\gamma\right)^{\beta}/\theta^{\beta}\right]}{1-\exp\left[-\left(\mathrm{x}^{\mathrm{r}_0+1}-\gamma\right)^{\beta}/\theta^{\beta}\right]} \tag{28}$$

for θ, and then for the form parameter

$$\frac{\partial\ln\mathrm{L}}{\partial\theta}=(r-r_0)\left(\frac{1}{\beta}-\ln\theta\right)+\sum_{i=r_0+1}^{\mathrm{r}}\ln(x_i-\gamma)-\sum_{i=r_0+1}^{\mathrm{r}}\left(\frac{x_i-\gamma}{\theta}\right)\ln\left(\frac{x_i-\gamma}{\theta}\right)-$$
$$-(\mathrm{n}-\mathrm{r})\left(\frac{x_r-\gamma}{\theta}\right)^{\beta}\ln\left(\frac{x_r-\gamma}{\theta}\right)+r_0\left(x_{r_0+1}-\gamma\right)^{\beta}\ln\left(\frac{x_{r_0+1}-\gamma}{\theta}\right) \tag{29}$$

and, finally, for the γ parameter

$$\frac{\partial \ln L}{\partial \gamma} = (1-\beta)\sum_{i=r_0+1}^{r}(x_i-\gamma)^{-1} + \frac{\beta}{\theta^\beta}\sum_{i=r_0+1}^{r}(x_i-\gamma)^{\beta-1} + (n-r)\frac{\beta}{\theta^\beta}(x_r-\gamma)^{\beta-1}$$
$$-\beta_{r_0}\left(x_{r_0+1}-\gamma\right)^{\beta-1}\cdot\frac{\exp\left[-\left(x_{r_0+1}-\gamma\right)^\beta/\theta^\beta\right]/\theta^\beta}{1-\exp\left[-\left(x_{r_0+1}-\gamma\right)^\beta/\theta^\beta\right]/\theta^\beta} \tag{30}$$

3 CHARACTERIZING OPERATIONAL BANKING INCIDENTS RISK

In the period July 1 – September 30 2009, the frequency of banking incidents was registered for a group of three banks with a total of 656 operative units.

The observation time values were transformed in standard work hours (8 hours), while the observations and results registered over the incidents were grouped in six day intervals (48 effective hours of operation), the final results being presented as a series in table 2. The banking incidents observations (false banknotes, cloned cards, false documents, informatics system hacking, the devastation of ATMs etc) concerned three banks in different categories of size:

1. Strong bank with 430 operative units.
2. Middle bank with 180 operative units
3. Small bank with 46 operative units.

Table 2. Banking incident distribution

No.	Intervals of observation	Operational Incidents by bank type			Total operational incidents
	(hours)	(k_1) 1	(k_2) 2	(k_3) 3	K Total
1	0 - 48	36	19	3	58
2	48 - 96	48	23	7	78
3	96 - 144	38	28	10	76
4	144 - 192	60	30	11	101
5	192 - 240	75	36	10	129
6	240 - 288	42	42	11	95
7	288 - 336	47	35	7	89
8	336 - 384	40	34	8	82
9	384 - 432	46	28	12	86
10	432 - 480	51	34	11	96
11	480 - 528	42	31	11	84
12	528 - 576	39	29	8	76
13	576 - 624	49	12	6	67
14	624 - 672	21	16	6	43
TOTAL		$\Sigma k_1 = 634$	$\Sigma k_2 = 397$	$\Sigma k_3 = 121$	$\Sigma k = 1160$

The total operational incidents, by hourly intervals, and by category of bank, are presented in table 2; both non-probabilistic indicators and indicators calculated based on a previously validated model shall be determined.

The mean of the overall operational incidents, by type of bank was:

$\overline{k} = 82,85$ - total mean

$\overline{k_1} = 45,29; \overline{k_2} = 28,36; \overline{k_3} = 82,85$

The average number of operational incidents by operational unit is:

$\overline{r} = 1,76$ - general rate

$\overline{r_1} = 1,47; \overline{r_2} = 2,21; \overline{r_3} = 2,63$

4 CONCLUSIONS

The main conclusion that can be drawn from interpreting the results concerns the high reliability of large banks, as the number of operational incidents is lower.

Establishing operational reliability indicators requires achieving a probabilistic model regarding the occurrence of incidents.

The relatively high frequency of the incidents excludes the Poisson model (for the incidents frequency) or the exponential model (for the time period when events occur) and indicates as viable the Weibull model, which includes as particular cases other models, through its generalizing qualities.

For the data in table 2 regarding the operational incidents distribution, the calculations lead to a mean of the operational time $\overline{t} = 323,35$ hours (in operational hours), with a standard deviation of $\hat{\sigma} = 289,3$ hours .

Applying the ordinary least squares method for the Weibull model parameters, in the tri-parametric variant, we have:

$\hat{\theta} = 357,65$ hours

$\gamma = 6,58$ hours

$\beta = 2,1$

The distribution function for the operational reliability model, for *t* = 240 hours, it is considered that the effective work time in a month is:

$$\hat{F}(t) = 1 - \exp\left[-\left(\frac{t - 6,58}{357,65}\right)^{2,1}\right] = 0,335$$

The reliability function for an operational monthly interval:

$$\hat{R}(t) = 1 - \exp\left[-\left(\frac{t - 6,58}{357,65}\right)^{2,1}\right] = 0,6648$$

The intensity of the banking operational incidents:

$$\hat{h}(t) = \frac{2,1(240 - 6,58)^{2,1-1,0}}{357,65^{2,1}} = 0,00367$$

The final conclusion concerns the low reliability of the system of the three banks under observation. With 66.5% chance of surpassing the 240 hour period of functioning without operational incidents, the system of the three banks proves to be extremely vulnerable.

The calculations were determined on the system of the three banks, and for smaller banks, reliability proves to be about 0.6 times lower as against to larger banks.

This fact demands increased preventive measures for this group of banks, parallel to a general preoccupation of reducing operational incidents.

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