Financial Development and Amplification

Hirano, Tomohiro

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Tomohiro Hirano*

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Abstract

Does financial development exacerbate or dampen financial amplification? This paper develops a macroeconomic model with the borrowing constraint and heterogeneous agents to answer this question. In our framework, financial development produces two competing forces. One is the effect which accelerates amplification by strengthening balance sheet effects. The other is the effect which reduces it, we call shock cushioning effects. Whether financial development exacerbates or dampens amplification depends on the balance of two effects. We find that the relation between financial development and amplification is non-monotone: amplification initially increases with financial development and later falls down.

Key Words: Non-Monotonicity, Balance sheet effects, Shock cushioning effects, the borrowing constraint, heterogeneous agents

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*Please address correspendence to: Tomohiro Hirano, Financial Services Agency, The Japanese Government, 3-2-1 Kasumigaseki Chiyodaku Tokyo, 100-8967 Japan. Email: tomohih@gmail.com The author thanks Shuhei Aoki, Julen Esteban-Pretel, Fumio Hayashi, Kikuo Iwata, Ryutaro Komiya, Hideaki Murase, Tamotsu Nakamura, Etsuro Shioji, Noriyuki Yanagawa, Naoyuki Yoshino, and seminar participants at Kobe University, Sophia University, and the University of Tokyo

1 Introduction

What are the effects of the development of financial markets on amplification over the business cycle? Traditional wisdom suggests that financial development stabilizes the economy by providing various channels for risk diversification. According to this view, financial innovation not only promotes long-run economic growth by enhancing efficiency in resource allocation, but also it helps to cushion consumers and producers from the effects of economic shocks.[1] This classical view seems to have been widely accepted. Indeed, several empirical and quantitative studies support the positive role of financial development in reducing volatility (See Cecchetti et al, 2006; Dynan et al, 2006; Jerman and Quadrini, 2008).

However, the situation has begun to change dramatically since the outbreak of the credit crisis of 2007-08. A new perspective has emerged: financial development destabilizes the economy by accelerating financial amplification. Before the crisis, it was often pointed out that thanks to financial innovation, the leverage of borrowers increased, and this high leverage generated economic booms. However, once the credit crisis occurred, people began to state that such a high leverage could lead to significant damages in borrowers' balance sheets, and eventually in the financial system as a whole. Financial development is suddenly blamed for increasing volatility. Indeed, IMF (2006, 2008) supports this new view by presenting empirical evidences that in more-advanced financial systems, the shock propagation effects become stronger.[2]

Motivated by these conflicting views, this paper theoretically investigates whether financial development accelerates or dampens financial amplification (macroeconomic volatility). To do so, we propose a macroeconomic model with the borrowing constraint and heterogeneous agents. In our model, financial development produces two competing forces. One is the effect which accelerates amplification by strengthening *balance sheet effects.*[3] The other is the effect which dampens amplification, we call *shock cushioning effects.*

[1] Levine (1997), Beck et al. (2000) show empirically that financial development causes long-run economic growth. Castro et al. (2004) and Khan and Ravikumar (2001) examine the impact of financial development including investor protection and risk-sharing on growth theoretically as well as empirically or numerically.

[2] IMF reports argue that the sensitivity of real GDP growth rate, corporate investment, household consumption, and residential investment response to equity busts, or business cycles, is increasing in more market-based financial systems.

[3] See Bernanke et al. (1996) for balance sheet effects.

Depending on which of these dominates, whether financial development exacerbates or weakens financial propagation is determined. Moreover, the balance between these two conflicting effects changes according to the degree of financial development.

Our main result shows that in a low level of financial development, while *shock cushioning effects* do not work well, financial development enhances *balance sheet effects* through raising leverage, thereby accelerating financial amplification. However, once the level of development passes a certain degree, *shock cushioning effects are generated* through an adjustment of the interest rate, which in turn weakens *balance sheet effects*, thereby dampening financial amplification. Hence, the relation between financial development and financial amplification is non-monotone: financial amplification initially increases with financial development and later falls down.

This paper is related to a number of researches on business cycle theory which emphasize the role of credit market imperfections. Following the seminal work by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), some researchers put financial factors a central role in accounting for business fluctuations (See Holmstrom and Tirole, 1997; Kiyotaki 1998, ; Bernanke et al., 1999; Kocherlakota, 2000; Cordoba and Ripoll, 2004). These studies demonstrate how shocks are amplified through *balance sheet effects*, assuming a fixed degree of financial development. The contribution of our paper is to examine how the sensitivity to the shocks changes as the degree of financial development changes.

Our paper is also related to Cooley et al. (2004), Rajan (2006), and Shin (2009) with regard to the effects of financial development on amplification (volatility). Cooley et al. emphasize a negative relation between the degree of contract enforceability, which corresponds to the degree of financial development in our paper, and aggregate volatility. They show that economies in which contracts are less enforceable display greater volatility of output than economies with stronger enforceability of contracts. The paper generates only a monotone relation. Our paper, however, generates a non-monotone dependence of volatility from financial development. Rajan argues that financial development has made the world better off, however it can accentuate real fluctuations, and economies may be more exposed to financial-sector-induced turmoil than in the past. However, Rajan does not necessarily propose a formal model of how financial development accelerates financial amplification. Shin presents a theoretical model where securitization by itself may not enhance financial stability. Our study shows within one framework that financial development initially accelerates amplification and later reduces it.

Concerning this non-monotone relation between financial development and amplification, Aghion et al. (1999) and Matsuyama (2007, 2008) are related to ours. Aghion et al. show that volatility is low when the development level is low or high. High volatility (cycles in their paper) occurs when the level has an intermediated value. Our paper also shows that volatility is high when financial development is an intermediated level. However, the source of high volatility is different from their paper. In their model, a change in the interest rate has a role in increasing volatility while in our model, it has a role in reducing volatility.[4] In our model, high volatility is caused by *balance sheet effects* with high leverage. Matsuyama develops a model of the borrowing constraint with various types of heterogeneities in an overlapping generations framework, and shows how it leads to a wide range of non-monotone phenomena. In Matsuyama's model, the source of non-monotonicity lies in the investment projects which do not produce capital goods. Matsuyama shows that a better credit market might be more prone to financing those investment projects, and such a change in credit allocation generates non-monotonicity. In our paper, the source of non-monotonicity lies in the adjustment of the interest rate which yields *shock cushioning effects*.

The paper is organized as follows. In the present paper, to demonstrate effectively how *shock cushioning effects* work, in section 1, we first present a model without the presence of a storage technology. In such a framework, we show that even though the borrowing constraint is binding, financial amplification does not occur through the adjustment in the interest rate. In section 2, we introduce the storage technology. We demonstrate that because of the presence of it, not only shocks get amplied through *balance sheet effects*, but also non-monotonicity emerges. Section 3 presents conclusion.

[4] In Aghion et al., a rise (decline) in the interest rate during booms (recessions) increases (reduces) debts repayment, which in turn produces recessions (booms). In this way, endogenous cycles with high volatility occur.

2 The Model

Consider a discrete-time economy with one homogenous goods and a continum of agents. At date t, a typical agent has expected discounted utility:

$$E_0\left[\sum_{t=0}^{\infty}\beta^t \log c_t^i\right], \tag{1}$$

where i is the index for each agent, and c_t^i is the consumption of him at date t. $\beta \in (0,1)$ is the subjective discount factor, and $E_0[x]$ is the expected value of x conditional on information at date 0.

There are two types of the agents. Some of them are called entrepreneurs, who have investment projects. The others are called investors, who do not have them. The investment technology of the entrepreneurs follows:

$$y_{t+1}^i = \alpha z_t^i, \tag{2}$$

where $z_t^i(\geq 0)$ is the investment of goods at date t. α is the marginal productivity of investment. y_{t+1}^i is the output at date $t+1$.

Each agent knows his own type at date t: whether he has investment or not. But he only knows his type with probability after date $t+1$. That is, each agent shifts stochastically between two states according to a Markov process: the state with or without investment. Specifically, an agent who has investment at date t may continue to have it at date $t+1$ with probability p. An agent who does not have it at date t may have it with probability $X(1-p)$. This switching probability is exogenous, and independent across entrepreneurs and over time. Assuming that the initial ratio of the entrepreneurs and the investors is $X:1$, the population ratio is constant over time. We assume that the switching probability is not too large:

$$p > X(1-p). \tag{3}$$

This assumption implies that there is a positive correlation between the present period and the next period.

In this economy, there are agency frictions in a credit market. The entrepreneur can pledge at most a fraction $\theta \in (0,1]$ of the future returns from his investment to creditors. In such a situation, in order for debt contracts to be credible, debts repayment does not exceed the pledgable value. That is, the borrowing constraint becomes

$$r_t b_t^i \leq \theta \alpha z_t^i, \tag{4}$$

where r_t is the gross interest rate from date t to $t+1$, and b_t^i is the amount of borrowing at date t. The parameter θ captures the degree of agency problems in the credit market (see Hart and Moore; 1994, and Tirole; 2006). In this sense, θ provides a simple measure of financial development. In this paper, we define an increase in θ as a financial development.

The agent's flow of funds constraint is given by

$$c_t^i + z_t^i = y_t^i - r_{t-1} b_{t-1}^i + b_t^i. \tag{5}$$

The left hand side of (5) is expenditure on consumption and investment. The right hand side is financing which comes from the returns from investment in the previous period minus debts repayment, which we call net worth in this paper, $y_t^i - r_{t-1} b_{t-1}^i$, and the amount of borrowing.

Each agent chooses consumption, investment, output, and borrowing $\left\{c_t^i, z_t^i, y_{t+1}^i, b_t^i\right\}$ to maximize the expected discounted utility (1) subject to (2), (4), and (5). From the optimal behavior of the entrepreneurs, we see that if $\alpha > r_t$, the entrepreneurs would borrow up to the limit. Hence the borrowing constraint binds. If $\alpha \leq r_t$, the constraint does not bind because the rate of return on investment is equal to or less than the interest rate.

Let us denote aggregate consumption of the entrepreneurs and the investors at date t as $\sum_{i \in m_t} c_t^i \equiv C_t$, and $\sum_{i \in n_t} c_t^i \equiv C_t'$, respectively, where m_t and n_t are a families of the entrepreneurs and the investors at date t. Similarly, let $\sum_{i \in m_t} z_t^i \equiv Z_t$, $\sum_{i \in m_t} b_t^i \equiv B_t$, and $\sum_{i \in n_t} b_t^i \equiv B_t'$ be aggregate investment, and the amount of borrowing of each type. Then, the market clearing for goods, and credit are

$$C_t + C_t' + Z_t = Y_t, \tag{6}$$

$$B_t + B_t' = 0, \tag{7}$$

where $\sum_{i \in m_{t-1}} y_t^i \equiv Y_t$ is the aggregate output at date t.

2.1 Equilibrium

The competitive equilibrium is defined as a set of prices $\{r_t\}_{t=0}^{\infty}$ and quantities $\left\{c_t^i, b_t^i, z_t^i, y_{t+1}^i, C_t, C_t', B_t, B_t', Z_t, Y_t\right\}_{t=0}^{\infty}$ which satisfies the conditions that

(i) each agent maximizes utility, and (ii) the market for goods, and credit all clear. Since there is no aggregate uncertainty, the agents have perfect foresight about aggregate quantities in the equilibrium.

We are now in a position to characterize equilibrium behavior of the entrepreneurs. For the moment, let us consider the case where the borrowing constraint is binding, which means that $\alpha > r_t$.

As is well known, since the utility function is log, both the entrepreneurs and the investors consume a fraction $(1-\beta)$ of their net worth, $c_t^i = (1-\beta)(y_t^i - r_t b_{t-1}^i)$. Then, by using (4), and (5), the investment function of the agents who have the investment projects at date t becomes

$$z_t^i = \frac{\beta(y_t^i - r_t b_{t-1}^i)}{1 - \dfrac{\theta\alpha}{r_t}}. \tag{8}$$

We see that the investment equals the leverage, $1/\left[1-(\theta\alpha/r_t)\right]$ times savings, $\beta(y_t^i - r_t b_{t-1}^i)$. The leverage increases with θ. This implies that when θ is large, the entrepreneurs can finance more investment with smaller net worth. We also see that the sensitivity of investment response to a change in the net worth becomes higher with θ, which suggests that when the leverage is high, even a small decline (increase) in the net worth can have a significant negative (positive) effect on the investment.

From (8), since investment is a linear function of the net worth, we can aggregate across the entrepreneurs to find the law of motion of the aggregate output:

$$Y_{t+1} = \alpha Z_t = \alpha \frac{\beta E_t}{1 - \dfrac{\theta\alpha}{r_t}}, \tag{9}$$

where $E_t \equiv \sum_{i \in m_t}(y_t^i - r_{t-1} b_{t-1}^i)$ is the aggregate net worth of the entrepreneurs at date t.

The movement of the aggregate net worth of the entrepreneurs evolves according to

$$E_t = p(\alpha Z_{t-1} - r_{t-1}B_{t-1}) + X(1-p)(r_{t-1}B_{t-1}). \tag{10}$$

The first term of (10) represents the aggregate net worth of the agents who continue to be entrepreneurs from the previous period. The second term represents the aggregate net worth of the agents who switch to the entrepreneurs from the investors.

When the borrowing constraint is binding, $r_{t-1}B_{t-1} = \theta Y_t$ holds. By substituting this relation into (10), we can derive the law of motion of the net worth share of the entrepreneurs, $s_t \equiv E_t/Y_t$:

$$s_t = p(1-\theta) + X(1-p)\theta. \tag{11}$$

The credit-market clearing, (7), can be written as

$$\frac{\theta\alpha}{r_t}\frac{\beta E_t}{1-\dfrac{\theta\alpha}{r_t}} = \beta E_t', \tag{12}$$

where $E_t' \equiv \sum_{i \in n_t}(y_t^i - r_{t-1}b_{t-1}^i)$ is the aggregate net worth of the investors at date t. In this economy, $Y_t = E_t + E_t'$ holds. The left hand side of (12) is the aggregate borrowing of the entrepreneurs, and the right hand side is the aggregate lending of the investors.

Rearranging (12), we have

$$\frac{\beta E_t}{1-\dfrac{\theta\alpha}{r_t}} = \beta Y_t. \tag{13}$$

(13) means that the aggregate investment of the entrepreneurs (the left hand side) equals the aggregate savings (the right hand side). (13) also suggests that given E_t and Y_t, the interest rate adjusts such that all the savings flow to the entrepreneurs' investment.

From (11) and (13), the equilibrium interest rate is

$$r_t = \frac{\alpha}{(1-p)/\theta + p - X(1-p)}. \tag{14}$$

We see that the interest rate increases with θ. This is because when θ rises, the borrowing constraint becomes relaxed, which results in a tighteness of the credit market. Moreover, if $\theta \in (0, 1/(1+X))$, $\alpha > r$. Thus, the borrowing constraint binds.

From (9) and (13), economic growth rate can be written as

$$g_t \equiv \frac{Y_{t+1}}{Y_t} = \beta\alpha. \tag{15}$$

We see that the growth rate is independent of wealth distribution or θ, even though the borrowing constraint is binding.

When $\theta = 1/(1+X)$, it is clear from (14) that the interest rate equals the rate of return on investment. Indeed, in $\theta \in [1/(1+X), 1]$,

$$r_t = \alpha. \tag{16}$$

Hence the borrowing constraint no longer binds.[5]

Also in this case, since the aggregate savings flow to the entrepreneurs' investment, the growth rate of the economy is described as (15). The net worth share of the entrepreneurs follow

$$s_t = ps_{t-1} + X(1-p)(1-s_{t-1}). \tag{17}$$

Thus, once an initial s_0 is given, the economy converges to the steady state.

The steady state of this economy is characterized by the value of θ.[6] We summarize the results in the following proposition.

Proposition 1 *Without the presence of the storage technology, there are two stages of financial development, corresponding to two different values of θ. The characteristics of each region are as follows:*

(a) Region 1: $\theta \in (0, \theta_1)$, where $\theta_1 \equiv 1/(1+X)$. The borrowing constraint binds. The steady state values of g^*, s^*, and r^* satisfy

$$g^* = \beta\alpha, \ \ s^* = p(1-\theta) + X(1-p)\theta, \ \ r^* = \frac{\alpha}{(1-p)/\theta + p - X(1-p)}. \tag{18}$$

(b) Region 2: $\theta \in [\theta_1, 1]$. The borrowing constraint does not bind. The steady state values satisfy

$$g^* = \beta\alpha, \ \ s^* = \frac{X}{1+X}, \ \ r^* = \alpha. \tag{19}$$

[5] $r_t > \alpha$ can not be an equilibrium because if $r_t > \alpha$, all the agents are willing to lend, and nobody would borrow.

[6] In $\theta \in (0, 1/(1+X))$, since the interest rate is lower than the rate of return on investment, income distribution is different between the entrepreneurs and the investors. In $\theta \in [1/(1+X), 1]$, since both the entrepreneurs and the investors earn the same rate of return, there is no difference in income distribution.

2.2 Dynamics

Now, let us examine how the growth rate of the economy responds to an unexpected shock to productivity to investment. Suppose that at date $\tau - 1$ the economy is in the steady state of region 1: $g_{\tau-1} = g^*, s_{\tau-1} = s^*$, and $r_{\tau-1} = r^*$. There is then an unexpected shock: The entrepreneurs find that the returns from their investment at date τ are $\alpha(1-\varepsilon)$. However, the shock is known to be temporary. The productivity at date $\tau + 1$ and thereafter returns to the normal level as in (2). Here we consider a negative shock (so ε is taken to be positive.). In this paper, we measure financial amplification (volatility) of a downward shock to be how far economic growth rate from τ to $\tau+1$ jumps down from the steady-state growth rate through the borrowing constraint.

From (15), we see that the economy's growth rate from τ to $\tau+1$ remains unchanged. In other words, no financial amplification occurs. The question is why is the growth rate not affected even if the shock hits the economy? In order to make it clear, let us investigate a response in the interest rate by this shock. The equilibrium in the credit market at date τ is

$$\frac{s_\tau}{1 - \dfrac{\theta\alpha}{r_\tau}} = 1. \tag{20}$$

(20) implies that the interest rate in period t depends on the share of the net worth of the entrepreneurs at date t.

By using (10), the net worth share of the entrepreneurs at date τ can be written as

$$s_\tau = \frac{p(1-\theta-\varepsilon) + X(1-p)\theta}{1-\varepsilon}. \tag{21}$$

Thus, we obtain an expression for the equilibrium interest rate at date τ:

$$r_\tau = \frac{\theta\alpha(1-\varepsilon)}{(1-p)(1-\varepsilon) + [p - X(1-p)]\,\theta}. \tag{22}$$

From (22), we see that the interest rate declines at the time of the shock. This results in dampening financial amplification. Intuition is that following the shock, the net worth share of the entrepreneurs declines. Because of this, the borrowing constraint becomes tightened, which causes investment to decrease. That is, balance sheet effects occur. On the other hand, together with the shock, the interest rate goes down in the credit market, which in turn relaxes the borrowing constraint. That is, the adjustment of the interest rate produces shock cushioning effects, which offsets balace sheet effects. As a result, financial amplification does not occur.

This result contrasts with conventional wisdom proposed by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), which suggests that when the borrowing constraint is binding, an unexpected productivity shock get amplified through balance sheet effects. This difference lies in the interest rate. In our model, the adjustment in the interest rate plays a crucial role in reducing amplification. On the other hand, in Bernanke and Gertler and Kiyotaki and Moore, the interest rate is fixed. Hence the adjustment does not operate.

In region 2, even if the economy is hit by the shock, since the financial system is well developed, it can transfer all the savings to the entrepreneurs without the adjustment of the interest rate. Note that in region 2, the interest rate is stick to α. Thus, the growth rate of the economy at date τ remain unchanged.

We summarize the above results in the following proposition.

Proposition 2 *Without the presence of the storage technology, no financial amplification occurs even if the borrowing constraint is binding, because shock cushioning effects offset balance sheet effects.*

3 Introducing a storage technology

In the model above, the investors, who do not have the investment technology, have no choice but to lend. Suppose now that the investors have an access to the storage technology, which earns a return equal to $\rho < \alpha$ per unit. This storage technology can be interpreted as the investment projects without agency fricitions, but with low returns. The model in this section is simillar to Bernanke and Gertler (1989) or especially Kiyotaki (1998).

Since the investors may invest in the storage technology, the goods market clearing, (6), changes into

$$C_t + C_t' + Z_t + X_t = Y_t, \tag{23}$$

where $\sum_{i \in n_t} x_t^i \equiv X_t$, and $\sum_{i \in m_{t-1}} \alpha z_t^i + \sum_{i \in n_{t-1}} \rho x_t^i \equiv Y_t$.

In describing the aggregate economy, let us consider the case where the borrowing constraint binds for the enterpreneurs, which implies

$$\rho \leq r_t < \alpha. \tag{24}$$

In equilibrium, the interest rate would be at least as high as ρ.[7] This implies that the presence of the storage technology creates a lower bound on the interest rate.

When we aggregate across agents, we derive the law of motion of the aggregate output as follows:

$$Y_{t+1} = \alpha \frac{\beta E_t}{1 - \dfrac{\theta\alpha}{r_t}} + \rho \left(\beta Y_t - \frac{\beta E_t}{1 - \dfrac{\theta\alpha}{r_t}} \right). \tag{25}$$

The first and second terms of (25) are the the returns from the investment of the entrepreneurs and the ones from the storage technology of the investors, respectively. As shown in the parenthesis of (25), the aggregate amount of the storage technology equals the aggregate savings minus the aggregate investment of the entrepreneurs. If $r_t = \rho$, the investors would use the storage technology, and hence the second term would be positive. If $r_t > \rho$, it would become zero because they do not have incentives to use the storage technology, which corresponds to region 1 in the previous section. In this case, the interest rate and the growth rate of the economy follow (14), and (15).

When the second term is positive, the growth rate of the economy can be written as

$$g_t \equiv \frac{Y_{t+1}}{Y_t} = \left[1 + \left(\frac{\alpha - \rho}{\rho - \theta\alpha} \right) s_t \right] \beta\rho. \tag{26}$$

(26) implies that economic growth rate increases with financial development. Intuitively, when financial development improves, the borrowing constraint becomes relaxed. In the credit market, more savings flow to the investment projects of the entrepreneurs from the storage technology. This change in the allocation of credit causes economic growth.

Aggregate TFP at date t is defined as follows:

[7] $r_t < \rho$ can not be an equilibrium because nobody would lend.

$$T_t = \frac{Y_{t+1}}{Z_t + X_t} = \left[1 + \left(\frac{\alpha - \rho}{\rho - \theta\alpha}\right) s_t\right] \rho. \tag{27}$$

By using (27), economic growth rate is rewritten as

$$g_t = \beta T_t. \tag{28}$$

From (28), we see that economic fluctuations are caused by the changes in the aggregate TFP. In this sense, our model seems to be simillar to standard real business cycle model. However, in the present model, the aggregate TFP is endogenously determined depending on saving allocations between the investment projects of the entrepreneurs and the storage technology.

The movement of the aggregate net worth of the entrepreneurs evolves according to

$$E_t = p(\alpha Z_{t-1} - r_{t-1} B_{t-1}) + X(1-p)(\rho X_t + r_{t-1} B_{t-1}). \tag{29}$$

Note that the second term includes the returns from the storage technology, which is different from the previous section.

By using (25) and (29), the net worth share of the entrepreneurs follow

$$s_{t+1} = \frac{p\dfrac{\alpha(1-\theta)}{\rho - \theta\alpha} s_t + X(1-p)(1-s_t)}{1 + \dfrac{\alpha - \rho}{\rho - \theta\alpha} s_t} \equiv \Phi(s_t, \theta). \tag{30}$$

If $r_t = \rho$, the dynamic evolution of the economy is characterized by the recursive equilibrium: $(Y_{t+1}, g_t, T_t, s_{t+1},)$ that satisfies (25), (26), (27), (28), and (30) as functions of the state variables (Y_t, s_t).

3.1 Steady State Equilibrium

The stationary equilibrium of this economy depends upon the degree of financial development. That is, we have the following proposition (Proof is in Appendix 1).

Proposition 3 *With the presence of the storage technology, there are three stages of financial development, corresponding to three different values of θ. The characteristics of each region are as follows:*

(3-a) Region 1-1: $\theta \in (0, \theta_2)$, *where* $\theta_2 \equiv (1-p)/\left[\alpha/\rho - p + X(1-p)\right]$. *The borrowing constraint is binding. The investors put some of their savings in the storage technology. The steady state values of* g^*, s^*, *and* r^* *satisfy*

$$g^* = \left[1 + \left(\frac{\alpha - \rho}{\rho - \theta\alpha}\right) s^*\right] \beta\rho, \;\; s^* = \Phi(s^*, \theta), \;\; r^* = \rho. \tag{31}$$

(3-b) Region 1-2: $\theta \in (\theta_2, \theta_1)$. *The borrowing constraint is binding. Only the entrepreneurs invest. The steady state values satisfy (18).*

(3-c) Region 2: $\theta \in [\theta_1, 1]$. *The borrowing constraint is not binding. Only the entrepreneurs invest. The steady state values satisfy (19).*

In region 1-1 where financial development is low, the financial system can not transfer all the savings to the investment projects of the entrepreneurs. In the credit market, some of the savings in the economy flow to the storage technology. In this sense, saving allocation is inefficient. As financial development improves, more savings are allocated to the investment projects with high returns. This improvement in the saving allocation boosts economic growth. However, the interest rate is unchanged in this region.[8] We should note here that this region is a newly created one due to the presence of the storage technology. Also this region corresponds to the economy Bernanke and Gertler (1989) or Kiyotaki (1998) analyze.

In region 1-2 where financial development is high, but not so high, the situation changes. As financial markets develop, the interest rate starts rising because of the tightness in the credit market. Since only the entrepreneurs produce goods, the growth rate of the economy becomes constant, and independent of θ, even though the borrowing constraint is still binding. This implies that once the financial system is developed to some degree, it can transfer enough savings to the entrepreneurs from the investors. Region 1-2 is similar to the property of region 1 in the previous section.

When financial markets grow further and reaches region 2, the borrowing constraint does not bind. All the savings flow to the investment projects of the entrepreneurs as in region 1-2.

[8] In this respect, our model is similar to Stiglitz and Weiss (1981). In their model, when information asymmetry is large, the interest rate is insensitive. Similarly, in our model, when financial development is low, the interest rate is sticky.

3.2 Dynamics

Now, let us look at how this economy responds to an unexpected shock. As in the previous case, suppose that at date $\tau - 1$ the economy is in the steady state: $g_{\tau-1} = g^*, s_{\tau-1} = s^*$ and $r_{\tau-1} = r^*$. There is then an unexpected shock to productivity: both the returns from the investment projects and the storage technology decline unexpectedly by ε. First, we consider region 1-1.

From (26), (29), and (30), we obtain

$$\text{Amplification} \equiv \frac{dg_{\tau+1}}{d\varepsilon}|_{\varepsilon=0} = \left(\frac{\alpha-\rho}{\rho-\theta\alpha}\right)\frac{ds_{\tau}}{d\varepsilon}|_{\varepsilon=0}\beta\rho < 0. \tag{32}$$

Since the entrepreneurs have a net debt in the aggregate, and debts repayment does not change by this shock, the net worth share of the entrepreneurs decreases at date τ, $ds_\tau/d\varepsilon < 0$ (See Appendix 2). Because the adjustment of the interest rate does not work well in region 1-1, their borrowing constraint becomes tightened. As a result, they are forced to cut back on their investment. That is, balance sheet effects occur. At the same time, more savings flow to the storage technology. That is, what is called flight to quality also occurs. Through these two effects, the aggregate TFP declines, so that economic growth rate from τ to $\tau+1$ jumps down from the steady state growth rate. This result is simillar to the one shown in Bernanke and Gertler (1989) or Kiyotaki (1998).[9] What we want to analyze in this paper is whether these propagation effects are exacerbated or dampened by financial development.

By differentiating (32) with respect to θ, we obtain

$$\frac{\partial^2 g_{\tau+1}}{\partial\theta\partial\varepsilon}|_{\varepsilon=0} = \underbrace{\frac{\partial}{\partial\theta}\left(\frac{\alpha-\rho}{\rho-\theta\alpha}\right)}_{\oplus}\underbrace{\frac{\partial s_\tau}{\partial\varepsilon}|_{\varepsilon=0}}_{\ominus}\beta\rho + \left(\frac{\alpha-\rho}{\rho-\theta\alpha}\right)\underbrace{\frac{\partial^2 s_\tau}{\partial\theta\partial\varepsilon}|_{\varepsilon=0}}_{\ominus}\beta\rho < 0. \tag{33}$$

The first term of (33) represents the sensitivity of the entrepreneurs' investment response to a change in the net worth share. Since it becomes higher with θ (high leverage), the entrepreneurs are forced to reduce their investment substantially by even a small decline in the net worth share. The

[9] We should note that the presence of the storage technology plays a crucial role in producing financial amplification, because it creates a new region (region 1-1) where shock cushioning effects are not generated.

second term represents the degree of a decline in the net worth share. It says that the decline by itself becomes larger with θ (See Appendix 2). This implies that when θ is high, the leverage also rises. In such a situation, even a small negative productivity shock can cause a large decline in the net worth share. Taken together, the entrepreneurs have to make deeper cuts in their investment. Moreover, this causes a substantial credit shift from the investment projects with high returns to the storage technology with low returns. That is, balance sheet effects and flight to quality are significant. Hence, in region 1-1, financial development accelerates financial amplification effects, thereby leading to increased macroeconomic volatility.

However, once the economy enters region 1-2, the situation changes dramatically. The adjustment of the interest rate starts operating, which generates shock cushioning effects as in region 1 of the previous section. As a result, financial amplification is dampened. This implies that once financial development passes a certain degree, the adjustment of the interest rate recovers, so that even if the economy is hit by the shock, the shock does not get amplified. Financial development leads to macroeconomic stability.

When financial development reaches region 2, no financial amplification occurs because the financial system can allocate all the savings to the investment projects with high returns without the adjustment of the interest rate.

We summarize the above results in the following proposition.

Proposition 4 *With the presence of the storage technology, the relation between financial development and financial amplification is non-monotone: financial amplification initially increases with financial development (in region 1-1) and later falls down (in region 1-2 and 2).*

This non-monotonicity is consistent with empirical studies. For example, Easterly et al. (2000) demonstrate that the relation between financial development and growth volatility is non-monotone. They show that while developed financial systems offer oppurtunities for stabilization, they may also imply higher leverage of firms and thus more risks and less stability. A recent study by Kunieda (2008) also show empirically that the relation is hump-shaped, i.e., in early stages of financial development, as the financial sector develops in an economy, it becomes highly volatile. However, as the financial sector matures further, the volatility starts to reduce once again.

Based on the above analysis, we might be able to explain why we observe two conflicting views. The traditional view might discuss region 1-2 or 2 where financial markets are well developed. Indeed, in Arrow-Debreu economy where there is no agency friction in the credit market, θ is equal to one. On the other hand, the new view might discuss region 1-1 where financial development is not so high, and there are agency frictions to some degree in financial markets. In this sense, the discrepancy between two views might arise from the difference in the degree of financial development.[10] We depict this situation in Figure 1. In the Figure, we take θ in horizontal axis, and in vertical axis, we take the magnitude of amplification. It is shown that the relation between θ and the magnitude is non-monotone.

This non-monotonicity has impliciations for the relation between growth and macroeconomic volatility. That is, in region 1-1, financial development causes economic growth. However, once negative productivity shocks hit the economy, downward amplification is significant since the economy is highly leveraged. In this sense, there is a trade-off between higher economic growth and macroeconomic stability. But, once financial development reaches region 1-2 or 2, both go together.

Moreover, our model may also have implications for asymmetric movements of business fluctuations. As Kocherlakota (2000) emphasizes, macroeconomics looks for an asymmetric amplification and propagation mechanism that can turn small shocks to the economy into the business cycle fluctuations. Our model might deliver this. For example, if the economy is around θ_2, to positive productivity shocks, even though the borrowing constraint is binding for the entrepreneurs, the economy will not respond upwardly because the interest rate will go up in the credit market. On the other hand, to negative productivity shocks, it will react downwardly because the interest rate does not adjust.[11]

[10] You may wonder why large downward amplification occurs repeatedly in the real economy where financial development keeps increasing over time, even though our model suggests that financial amplification eventually becomes small in high θ region. Here is one interpretation from this model. In this model, the important factor which affects the size of financial amplification is θ^H, which is put on high profitable investment, not on low profitable investment. Considering this point, think about the case where the existing projects with α^L disappper, and new investment opportunities with higher profitability than the existing α^H come into the economy. In such a situation, the θ which is put on those new investment projects matters. If the θ is low, the economy will get into region 1 again even if it was in region 2 or 3 before. In the real economy, this process might repeats itself.

[11] Here we consider small shocks. However, if we think about relatively large productivity shocks, business fluctuations may become asymmetric, even if the economy is far from θ_2.

We summarize this result in Proposition 5.

Proposition 5 *If the level of financial development is around θ_2, business fluctuations are asymmetric.*

4 Conclusion

This paper develops a macroeconomic model of credit market imperfections with heterogeneous agents in order to investigate whether financial development exacerbates or dampens financial amplification. In our framework, financial development produces two competing forces. One is the effect which accelerates amplification by strengthening *balance sheet effects.* The other is the effect which dampens amplification, we call *shock cushioning effects.* Depending on which of these dominates, whether financial development exacerbates or weakens financial propagation is determined. Moreover, the balance between these two conflicting effects changes according to the level of financial development.

We show that in a low level of financial development, while shock cushioning effects do not work well, financial development enhances balance sheet effects through raising leverage, thereby accelerating financial amplification. However, once the level of development passes a certain degree, financial development generates shock cushioning effects, which in turn weakens balance sheet effects, thereby dampening financial amplification. Hence, the relation between financial development and financial amplification is non-monotone: financial amplification initially increases with financial development and later falls down.

As future research, the next step would be that we want to develop quantitative assessment into the relation between the development of financial markets and volatility of the economy. Another step would be to consider the welfare cost of volatility in a heterogeneous agents model with aggregate uncertainty. These directions will be promising.

In the case with relatively large positive shocks, positive propagation occurs, but the degree of it is weakened because the adjustment of the interest rate works. However, to the negative shocks, because the adjustment does not work, the economy experiences large downward propagation.

Appendix 1

Proof of Proposition (3-a)

First, we derive θ_2. θ_2 is the value which satisfies $s_{t+1} = \Phi(s_t, \theta)$ when we put $s_t = s_{t+1} = 1 - \theta\alpha/\rho$.

Next, we prove that if $\theta \in (0, \theta_2)$, $r_t = \alpha$ and $Z'_t > 0$ hold in the neighborhood of the steady state. In order to prove this, we need to check that the investors invest positive amounts of goods. From the goods-market clearing condition, Z'_t becomes

$$Z'_t = \beta Y_t \left(1 - \frac{s_t}{1 - \dfrac{\theta\alpha}{\rho}} \right). \tag{34}$$

From (34), we observe that whether Z'_t is positive or zero depends upon the value of the right hand side. If $\theta = \theta_2$, $s^*(\theta) = 1 - \theta\alpha/\rho$ holds. Thus, we have $Z'_t = 0$. If $\theta < \theta_2$, from (30), $s^*(\theta) < 1 - \theta\alpha/\rho$ holds. Thus, we have $Z'_t > 0$.

Appendix 2

Since the returns from investment of both agents decrease by ϵ at date τ, (25) and (29) change into

$$Y_\tau = (1 - \varepsilon) \left[\alpha \frac{\beta E_{\tau-1}}{1 - \dfrac{\theta\alpha}{r_{\tau-1}}} + \rho \left(\beta Y_{\tau-1} - \frac{\beta E_{\tau-1}}{1 - \dfrac{\theta\alpha}{r_{\tau-1}}} \right) \right], \tag{35}$$

$$E_\tau = p \left[(1 - \varepsilon)\alpha Z_{\tau-1} - r_{\tau-1} B_{\tau-1} \right] + X(1 - p) \left[(1 - \varepsilon)\rho Z'_{\tau-1} + r_{\tau-1} B_{\tau-1} \right]. \tag{36}$$

Using (35) and (36), we obtain s_τ as follows:

$$s_\tau = \frac{p\dfrac{\alpha(1-\theta)}{\rho-\theta\alpha}s_{\tau-1} + X(1-p)(1-s_{\tau-1}) - \varepsilon\left[p\dfrac{\alpha}{\rho-\theta\alpha}s_{\tau-1} + X(1-p)(1-\dfrac{\rho}{\rho-\theta\alpha}s_{\tau-1})\right]}{(1-\varepsilon)\left[1+\dfrac{\alpha-\rho}{\rho-\theta\alpha}s_{\tau-1}\right]}. \tag{37}$$

From (37), differentiating s_τ with respect to ε, we have

$$\frac{\partial s_\tau}{\partial \epsilon}|_{\varepsilon=0} = [p - X(1-p)]\,\frac{-\theta\alpha s^*}{\rho-\theta\alpha+(\alpha-\rho)s^*} < 0. \tag{38}$$

And then, by using (38), we have

$$\frac{\partial^2 s_\tau}{\partial\theta\partial\epsilon}|_{\varepsilon=0} = [p - X(1-p)]\,\alpha\frac{-\theta\dfrac{\partial s^*}{\partial\theta}(\rho-\theta\alpha)-\rho s^*-(\alpha-\rho)s^{*2}}{\left[\rho-\theta\alpha+(\alpha-\rho)s^*\right]^2} < 0. \tag{39}$$

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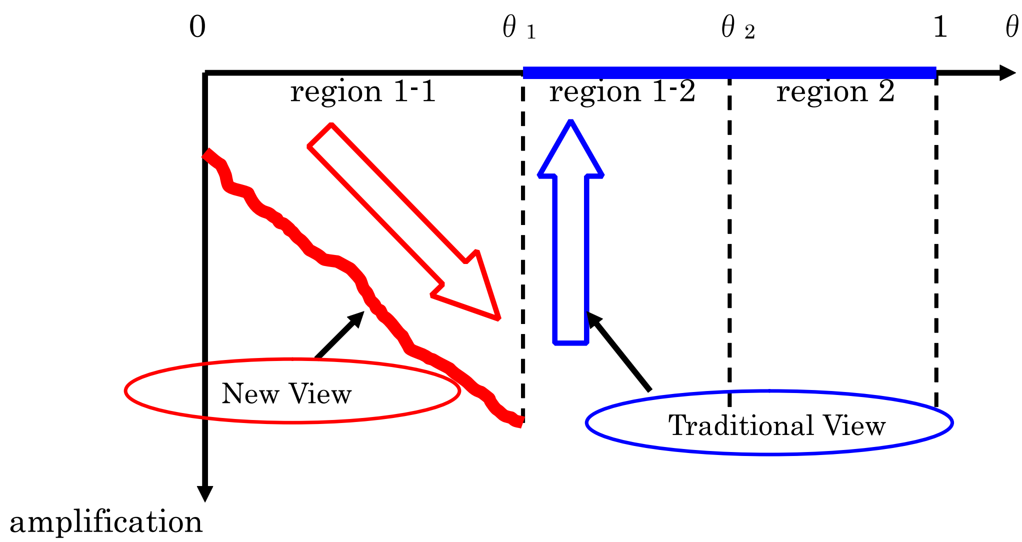

Figure 1: relation between θ and amplification