# Environmental Taxation under Productive Differentials: An Efficiency Analysis

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**Abstract**

In recent years, the pollution tax instrument has become a focus of the environmental policy debate. Many countries are presently considering implementing or increasing the rate of pollution taxes, while pollution abatement subsidies are used by local governments. However, a great part of the literature argues that environmental taxation fails to create a "double-dividend" outcome and leads to a trade off between pollution levels and unemployment. In this context, a simple search and matching model of labour market is developed, where workers are characterized by heterogeneous productive abilities, so as to examine the impact of a pollution tax on employment. Furthermore, an attemption is made in order to determine the efficient level of taxation in the short run, where the assumption of free entry of firms (zero profits) is dropped.

*Keywords*: pollution; search; taxes; unemployment.
*JEL classification*: H21; H23

# 1. Introduction

The main objective of this paper is the examination of the optimal environmental tax policy and its impact on unemployment and pollution discharges within the framework of a labour market in which workers are ex ante heterogeneous regarding their productivity (productive differentials). More specifically, our analysis is focused on the following two issues: i) Is there a trade off relation between environmental pollution and involuntary unemployment, when firms encounter an increase in the environmental tax? This proposition will be examined under two alternative regimes: with and without recycling of the collected tax revenues. In the former case, revenues are used for subsidizing firms. The recycling of tax revenues takes the form of either a hiring or an investment subsidy. ii) Which is the social efficient level of environmental tax in the above cases?

For this purpose, a simple matching model of a labour market has been developed, in which all firms are ex ante identical, but workers are characterized by heterogeneous skills. Our formulation is based on a model developed by Burdett (2001), which can be thought as an extension of the basic matching model literature introduced by Mortensen (1980), Diamond (1982), and Pissarides (1990) and captures the insights of the work presented by Lockwood (1986).

The first question posed in the first paragraph of this section was initially addressed in the mid 90s in the professional economics literature. Some representative studies are those by Bovenberg and de Mooij (1994), Bovenberg and van der Ploeg (1994a, b) and Goulder (1995). All these studies end up to the same conclusion; the second best efficient environmental tax should be lower than the Pigouvian tax and an increase in the pollution tax will decrease both employment and pollution levels. The research work which is closest to the present study is that of Strand (1999), who presented a model, where firms decide ex ante about the level of their pollution discharges in the context of profit maximization. However, the analysis of this paper differs from that of Strand in the following points: i) We do not have instantaneous matching in the sense that we do have vacant jobs ii) Workers are heterogeneous regarding their abilities and hence their productivity in our formulation. This assumption leads to the existence of productive and consequently wages differentials iii) We do not assume free entry for firms, examining in this way the operation of the economy in the short run[1].

One of the main findings of Strand's analysis is that, given that there is no tax recycling, the first best pollution tax is greater than the Pigouvian one and below the second best tax (which is still above the marginal social damage cost from pollution). However, he rules out the first best solution as infeasible, since it leads to full employment and implies an infinitely high tax. Moreover, he argues that an uncompensated increase in the pollution tax creates a trade off effect between pollution and unemployment (i.e. no double dividend can be achieved). In our analysis, we show that the existence of composition effects, working through reservation productivity, results to a double dividend effect as pollution tax increases. Moreover, the first best solution is feasible and implies lower unemployment than the market outcome and an environmental tax above the Pigouvian level (and also higher than that in Strand's exposition). Finally, the results

[1]The plausibility of free entry of vacancies is questioned in the short-run, since the decision of opening a vacancy by firms takes a lot of time and effort (see Ours and Ridder 1992).

we get, when tax revenues finance hiring subsidies or the investment costs of firms, are more or less similar to these of Strand's.

This paper is organized as follows. In Section 2, the benchmark model is presented. Section 3 examines the nature of the steady state equilibrium and presents the comparative statics analysis. Section 4 comments on the social efficiency. The case of tax revenues recycling is tackled in Section 5. Finally, our conclusion is presented in Section 6.

# 2. The Basic Model: Pollution Tax without Tax Revenues Recycling

## 2.1 Environment

Our economy is comprised by a large, fixed number of workers and the same fixed number of jobs (both normalized to 1). Time passes continuously. Each firm creates one job and can employ only one worker and vice versa (i.e. the same worker cannot be employed by different firms). Firms are ex ante identical. Workers are ex ante heterogeneous in the sense that they differ in the skill level they possess. More specifically, before entering labour market, each worker is endowed with a skill $y$, where $y$ is a random variable uniformly distributed over the interval [0, 1]. If an individual with skill $y$ is employed, then output $y$ is produced per unit of time. Firms and workers discount the future at the same rate $r$. At any moment of time, a worker is either employed or unemployed and a job is either filled or vacant. Let $a$ be the arrival rate of job offers for an unemployed individual, where $a$ is the parameter of a Poisson process. Moreover, unemployed workers obtain zero utility flow. If a worker with skill $y$ comes in contact with a vacant job, then the recruiting process is the following: First the firm decides whether to employ this worker or not. If the decision is positive, then the wage (per unit of time) paid to the worker, denoted as $w(y)$, is determined through a symmetric Nash bargaining process[2]. If the decision is negative, then the individual remains unemployed. We assume that there is no on-the-job search[3] (i.e. an employed individual does not contact other employers). The fact that all firms are identical by assumption implies that if a worker is acceptable to one employer, then he is acceptable to all employers with a vacancy. Filled jobs 'die' at an exogenous rate $\delta$. When an employer/worker match is destroyed, then the worker returns to unemployment and the firm leaves the market and is replaced by a new one, which offers a new vacancy.

[2] i.e. workers and firms share equally the product of their match.

The arrival rate of unemployed individuals for a vacant job is equal to $\gamma$. The fact that the number of workers is the same with the number of jobs, combined with the constant job-destruction rate, implies that $\gamma = a$. By dropping the free entry (endogeneity) assumption regarding job creation (i.e. vacancies are created up to the point where the expected profit of the marginal vacancy is equal to zero), we give a short-run character in our model. The cost of creating a vacancy is equal to $C$ and represents the capital investments necessary to establish a firm. We assume that this cost sunk when the job is filled. A filled job pollutes the environment at a constant rate $P$ per unit of output, where $0 < P < 1$. However, the firm can affect the level of $P$ through its establishment investment decision in the following way: $C'(P) < 0$, $C''(P) > 0$, i.e. as the establishment investment increases, pollution decreases but at a decreasing rate as $P$ decreases[4]. Finally, we assume that a linear tax $\tau$ is levied on pollution, where $0 < \tau < 1$.

For a worker with skill $y$, $U(y)$ is the value of unemployment, $W(y)$ is the value of employment, $J(y)$ is the value to the employer of filling a job and finally $V$ is the value of a vacancy.

## 2.2 Workers

### 2.2.1 Unemployed

The value function of an unemployed worker with skill $y$ acceptable to employers is equal to

$$rU(y) = a[W(y) - U(y)] \quad (1)$$

According to equation (1), the flow value of unemployment for a worker with a $y$ acceptable to employers is equal to the arrival rate of job offers times the capital gain by becoming employed.

### 2.2.2 Employed

The flow value of employment for a worker with skill $y$ is

$$rW(y) = w(y) + \delta[U(y) - W(y)] \quad (2)$$

[3]A model which will incorporate the assumption of on-the-job search can be considered as a topic for future research.
[4]Alternatively, we could have assumed that $P$ is a fuction of $C$, with $P'(C) < 0$, $P''(C) > 0$ and take the inverse function.

Equation (2) determines the flow value of employment as the sum of the flow return to employment (the wage) plus the instantaneous capital loss. It is obvious that workers with $y$ not acceptable to firms have $W(y) = 0$.

## 2.3 Firms

### 2.3.1 Vacant

The expected discounted profit from holding a vacancy can be written as

$$rV = -C(P) + a[E_y \max\{J(y) - V, 0\}] \tag{3}$$

Equation (3) incorporates the assumption that the exact value of $y$ is unknown to employers prior to their contact with workers. However, employers know the distribution of $y$ s'. Therefore, firms form expectations about their capital gain from having their vacancy filled. It is clear that given $w(y)$, a firm will hire a worker if $J(y) \geq V$.

### 2.3.2 Filled

The flow value to a job filled by a worker with skill $y$ is

$$rJ(y) = y - w(y) - \tau Py + \delta[V - J(y)] \tag{4}$$

where $Py$ is the total amount of pollution emitted by a firm producing output $y$.

From equations (1), (2) and (4) we get

$$U(y) = \frac{aw(y)}{r(r + a + \delta)} \tag{5}$$

$$W(y) = \frac{(r + a)w(y)}{r(r + a + \delta)} \tag{6}$$

$$J(y) = \frac{(1 - \tau P)y - w(y) + \delta V}{r + \delta} \tag{7}$$

## 2.4 Wage Formation and Reservation Skill

The surplus produced by the match between a worker with skill $y$ and a firm is

$$S(y) = J(y) + W(y) - V - U(y) \tag{8}$$

Equations (5), (6), (7) and (8) yield

$$(r + \delta)S(y) = (1 - \tau P)y - rV - rU(y) \tag{9}$$

Let's assume that there is a $y^R$ such that $S(y^R)=0$. This implies that if a worker has a $y \le y^R$, then he is never employed by a firm. Substituting $y^R$ in (9) implies

$$\frac{(1-\tau P)y^R}{r} = V + U(y^R) \tag{10}$$

Efficiency implies $V=J(y^R)$. Hence, by (7) and (10) we get

$$\frac{w(y^R)}{r} = U(y^R) \tag{11}$$

Substituting (11) into (10) gives

$$\frac{(1-\tau P)y^R - w(y^R)}{r} = V$$

From the above analysis follows that

$$W(y^R) = U(y^R) = w(y^R) = 0$$

Symmetric Nash bargaining implies that

$$\frac{1}{2}S(y) = W(y) - U(y) = J(y) - V \tag{12}$$

Using (5), (6), (7) and (12), we get that the wage earned by an individual with $y>y^R$ is

$$w(y) = \frac{[(1-\tau P)y - rV](r+a+\delta)}{2(r+\delta)+a}$$

But, as we showed, above $rV=(1-\tau P)y^R$ and thus

$$w(y) = \frac{(1-\tau P)(y-y^R)(r+a+\delta)}{2(r+\delta)+a} \tag{13}$$

Given the above analysis, we can derive $y^R$ using (3) as follows

$$rV = -C(P) + a[E_y \max\{J(y)-V,0\}] \Rightarrow$$

$$rV = -C(P) + \frac{a}{r+\delta}\int_{y^R}^{1}[(1-\tau P)y - w(y) - rV]dF(y) \Rightarrow$$

$$rV = -C(P) + \frac{a}{r+\delta}\int_{y^R}^{1}\{(1-\tau P)y - w(y) - [(1-\tau P)y^R - w(y^R)]\}dF(y) \Rightarrow$$

$$rV = -C(P) + \frac{a}{2(r+\delta)+a}\int_{y^R}^{1}(1-\tau P)(y-y^R)dF(y) \Rightarrow$$

$$y^R(1-\tau P) = -C(P) + \frac{a(1-\tau P)}{2(r+\delta)+a}\int_{y^R}^{1}(y-y^R)f(y)dy \tag{14}$$

where $F(y)$ and $f(y)$ are respectively the steady state cummulative and probability density distribution function of skills among those unemployed.

# 3. Steady State Equilibrium

In steady state the evolution of employed individuals is equal to zero, i.e. the flow of workers out of unemployment should be equal to the flow of workers back to unemployment. We showed previously that individuals with $y \leq y^R$ are never employed. Since $y$ is uniformly distributed between 0 and 1, the number of workers permanently unemployed is $y^R$. Among those with $y$ greater than $y^R$ some are unemployed in a steady state. Let $u(y^R)$ denote the number of workers with $y>y^R$, who are unemployed in a steady state. Steady state implies

$$au(y^R) = \delta[1 - y^R - u(y^R)] \Rightarrow$$
$$u(y^R) = \frac{\delta(1 - y^R)}{a + \delta} \tag{15}$$

Hence, the steady state unemployment will be

$$u = y^R + \frac{\delta(1 - y^R)}{a + \delta} = \frac{\delta + ay^R}{a + \delta} \tag{16}$$

Using (15) and (16), we derive $F(y)$

$$F(y) = \begin{cases} \dfrac{y(a+\delta)}{\delta + ay^R} & if \;\; y \leq y^R \\ \dfrac{ay^R + \delta y}{\delta + ay^R} & if \;\; y > y^R \end{cases}$$

Therefore, (14) becomes

$$y^R(1 - \tau P) = -C(P) + \frac{a\delta(1 - \tau P)(1 - y^R)^2}{2[2(r+\delta) + a](\delta + ay^R)} \tag{17}$$

$C$ is determined by the firm before the worker is hired and taking $\tau$ as exogenously given. Hence, the firm determines $C$ and thus $P$ by maximizing the expected discounted profit of its vacancy, i.e.

$$C'(P) = -\frac{a\delta\tau(1 - y^R)^2}{2[2(r+\delta) + a](\delta + ay^R)} \tag{18}$$

The equilibrium market values of $P$ and $y^R$ are given by the solution of the system of (17) and (18).

Differentiating (17) and (18) with respect to $\tau$ yields (see appendix)

$$\frac{dy^R}{d\tau}=\frac{-C'(P)\tau y^R(1-\tau P)+C''(P)C(P)P\tau}{D\tau(1-\tau P)} \tag{19}$$

$$\frac{dP}{d\tau}=\frac{-C'(P)(1-\tau P)^2(\frac{\Psi}{\tau}+1)+\Psi C(P)P\tau}{D\tau(1-\tau P)} \tag{20}$$

where $\Psi=\frac{-C'(P)[2\delta+a(1+y^R)]}{(1-y^R)(\delta+ay^R)}$ and $D=\Psi[y^R\tau-\frac{(1-\tau P)}{\tau}C''(P)]-(1-\tau P)C''(P)$

It can be shown that if the absolute value of the elasticity of $C'$ with respect to $P$ (denoted as $\varepsilon_{cc}$) is greater than the absolute value of the elasticity of $C$ with respect to $P$ (denoted as $\varepsilon_c$), then $D$<0 and $\frac{dy^R}{d\tau}$, $\frac{dP}{d\tau}$<0 (see appendix).

An increase in the environmental tax will decrease firm's net revenues (the cost of pollution is now higher for firms). This will force firms to become less picky in their selection process, namely to decrease their reservation productivity, in order to mitigate the tax burden[5] given the existing pollution rate. This reaction will lead to a decrease in unemployment. The inequality $\varepsilon_{cc}>\varepsilon_c$ implies that the additional investment cost burden, resulting from the reduction of pollution per unit of output, decreases at a faster rate than the increase in the establishment cost due to a lower pollution rate and this will give firms an incentive to reduce their pollution discharges by increasing their initial investment cost. The extra cost will be covered by the consequent decrease in the tax burden and the gains from the reduction of $y^R$. Hence, we conclude that if $\varepsilon_{cc}>\varepsilon_c$, then government does not face a trade off between pollution and unemployment, when the pollution tax is increased in isolation (i.e. tax revenues are not used for subsidizing firms and workers).

[5]Greater value for $y^R$ implies that the waiting time for a vacancy to become filled is greater, which in turn leads to greater firm costs [$C(P)$ sunk when a job is filled]. Moreover, the expected amount of pollution discharges, given the pollution per unit of output and consequently the expected amount of tax burden, increases with the reservation productivity.

# 4. Social Efficiency

The social planner has the following objective function:

$$H = \int_0^\infty e^{-rt}[(1-zP)\int_{y^R}^1 y\frac{(1-u)}{1-y^R}dy - C(P)u]dt \qquad (21)$$

where $z$ is the marginal social valuation of the pollution damage.

The expression inside the brackets is the current value of the net social surplus, which is equal to the social value of output (the first term) minus the total initial investment cost in the economy (the second term). Moreover, the social planner faces the following restriction, which determines the evolution of unemployment:

$$\dot{u} = \delta(1-u) - a(u - y^R) \qquad (22)$$

Let $\mu$ be a co-state variable. The optimal path of the reservation productivity $y^R$, the pollution rate $P$ and unemployment satisfies (22) and the following Euler conditions

$$\dot{\mu} - e^{-rt}\left\{\frac{(1-zP)[1+y^R]}{2} + C(P)\right\} - \mu(a+\delta) = 0 \qquad (23)$$

$$e^{-rt}(1-zP)\frac{(1-u)}{2} + a\mu = 0 \qquad (24)$$

$$e^{-rt}\left\{\frac{-z(1-u)(1+y^R)}{2} - C'(P)u\right\} = 0 \qquad (25)$$

To derive the conditions for the social efficient level of $y^R$ and $P$, we substitute $\mu$ from (24) into (23) and we evaluate the outcome and equation (25) in the steady state ($\dot{u}$ =0) to obtain

$$(1/2)r(1-zP)\frac{(1-y^R)}{a+\delta} = y^R - zPy^R + C(P) \qquad (26)$$

$$C'(P) = \frac{za(y^R-1)(y^R+1)}{2(\delta + ay^R)} \qquad (27)$$

Solving (26) with respect to $y^R$, yields

$$y^R = \frac{(1-zP)r - 2C(P)(a+\delta)}{(1-zP)[r+2(a+\delta)]} \qquad (28)$$

If $r(1\text{-}zP) < 2C(P)(a+\delta)$ [which can be true for any value of $P$ under certain functional forms of $C(P)$], then the social efficient value of reservation productivity ($y^{R*}$) is equal to zero[6] and the social efficient level of unemployment is less than that of the market level[7]. In order to find the efficient environmental tax, $\tau^*$, we substitute (27) into (18) and solve with respect to $\tau$:

$$\tau^* = \frac{[2(r+\delta)+a](y^{R*}+1)z}{(1-y^{R*})\delta} \tag{29}$$

where $y^{R*}$ is the social efficient value of $y^R$.

Equation (29) implies that the pollution tax rate required to implement the first-best level of $P$ is higher than the marginal damage cost and consequently higher than the traditional Pigouvian tax[8]. Moreover, our $\tau^*$ is higher than that of Strand (1999) (under 50/50 bargaining power). The former result is attributed to the same reason given in Strand (1999), i.e. a part of the environmental tax burden is transferred by firms to workers through the bargaining process, resulting to the reduction of firm's incentive to lower the pollution rate at the ex ante stage. The latter result can be ascribed to the fact that the aforementioned incentive is even lower in our analysis, since we examine the short run period where competition is less intense[9]. In order to get a better insight on the behavior of the model described above, a simulation is carried out. For our simulation, we assume the following functional forms and parameter values: $a = 0.9$, $\delta = 0.1$, $r = 0.1$, $C(P) = 10P^{-0.05}$, $z = 0.01$. All our parameter values were chosen to produce plausible results. The results are illustrated in Table I.

**Table I: Impact of environmental tax on pollution level and unemployment when there is no tax recycling**

| $\tau^* = 0.13$ | | $u^* = 0.10$ | |
|---|---|---|---|
| | $y^R$ | $P$ | $u$ |
| $\tau = 0.05$ | 0.073 | 0.574 | 0.166 |
| $\tau = 0.08$ | 0.072 | 0.364 | 0.165 |
| $\tau = 0.15$ | 0.071 | 0.198 | 0.164 |
| $\tau = 0.18$ | 0.070 | 0.166 | 0.163 |

[6]Actually, if $r(1\text{-}zP) < 2C(P)(a+\delta)$, the efficient value of $y^R$ is negative. However, since negative values of $y^R$ make no sense, we conclude that social efficiency requires firms to accept all workers, regardless of their productive abilities.

[7]Solving equation (18) with respect to $y^R$, we get two values. However, it can be easily shown that only one of them is less than one for the value of $P$ which verifies (18).

[8]Pigou (1920) showed that under competition, the efficient level of a unit pollution tax should be equal to the marginal social damage cost from pollution.

[9]The free entry assumption for firms is adopted in Strand's exposition.

where $u^*$ is the efficient level of unemployment.

## 5. Tax Recycling

In this section, our basic model will be appropriately extended so as to incorporate the case of tax recycling. More specifically, we will assume that in each period the government revenues from the pollution tax are transferred back to firms in their entirety. These transfers can take the following two forms:

- A lump-sum subsidy $M$ to firms' establishment costs, which can be thought as a subsidy to human capital formation or to hiring costs. In order to rule out meaningless solutions, we assume that $C(P) > M$.
- A proportional subsidy at rate $s$ (where $s < 1$) to firms' capital investments.

The government revenues from the pollution tax in each period are

$$T = \frac{\tau Pa(1-y^R)(1+y^R)}{2(a+\delta)}$$

Hence, the lump-sum subsidy and the proportional subsidy will be

$$M = \frac{\tau Pa(1-y^R)(1+y^R)}{2(ay^R+\delta)} \tag{30}$$

$$s = \frac{\tau Pa(1-y^R)(1+y^R)}{2(ay^R+\delta)C(P)} \tag{31}$$

We assume that the exact level of the corresponding subsidy cannot be observed by firms and hence, they maximize the expected discounted profit of their vacancies by taking $M$ and $s$ as exogenously given.

According to the preceding analysis, the equilibrium market values of $P$ and $y^R$ are given by solving

$$y^R(1-\tau P) = -C(P) + \frac{\tau Pa(1-y^R)(1+y^R)}{2(ay^R+\delta)} + \frac{a\delta(1-\tau P)(1-y^R)^2}{2[2(r+\delta)+a](\delta+ay^R)} \tag{32}$$

and (18), in the case of the lump-sum subsidy.

Moreover, in the case of the proportional subsidy, market outcome is defined by (32) and

$$C'(P) = -\frac{a\delta\tau(1-y^R)^2}{2[2(r+\delta)+a](\delta+ay^R)(1-s)} \tag{33}$$

Using the above equations, we can obtain, after tedious calculations, required expressions for $\frac{dy^R}{d\tau}$, $\frac{dP}{d\tau}$ under the two alternative subsidization regimes. More specifically, when a lump-sum subsidy is given, we get

$$\frac{\partial y^R}{\partial \tau} = \frac{1}{\Xi}\left\{\frac{C'(P)C(P)}{1-\tau P} + \frac{[C'(P)]^2}{\tau} - \frac{MC'(P)}{\tau P(1-\tau P)} + \frac{C''(P)C(P)P}{1-\tau P} - \frac{C''(P)M}{\tau(1-\tau P)}\right\} \tag{34}$$

$$\frac{\partial P}{\partial \tau} = \frac{1}{\Xi}\left\{\frac{-AC'(P)}{\tau} + \frac{-C'(P)}{\tau}(1-\tau P)(\frac{\Psi}{\tau}+1) + \frac{\Psi C(P)P}{1-\tau P} - \frac{\Psi M}{\tau(1-\tau P)}\right\} \tag{35}$$

where $\Xi = \Psi\left[\frac{-C(P)\tau}{1-\tau P} - C'(P) + \frac{M}{P(1-\tau P)} - \frac{C''(1-\tau P)}{\tau}\right] - C''(P)A - C''(P)(1-\tau P)$ and

$A = \frac{\tau a P[a(y^R)^2 + 2y^R\delta + a]}{2[ay^R+\delta]^2}$.

In the case of the proportional subsidy, we get

$$\frac{\partial y^R}{\partial \tau} = \frac{1}{\Omega}\left\{\frac{C'(P)C(P)}{1-\tau P} + \frac{[C'(P)]^2}{\tau} - \frac{MC'(P)}{\tau P(1-\tau P)} + \frac{C''(P)C(P)P}{1-\tau P} - \frac{C''(P)M}{\tau(1-\tau P)}\right\} \tag{36}$$

$$\frac{\partial P}{\partial \tau} = \frac{1}{\Omega}\left\{\frac{-AC'(P)}{\tau} + \frac{-C'(P)}{\tau}(1-\tau P)(\frac{\Psi(1-s)}{\tau}+1) + \frac{\Psi C(P)P}{1-\tau P} - \frac{\Psi M}{\tau(1-\tau P)}\right\} \tag{37}$$

where $\Omega = \Psi\left[\frac{-C(P)\tau}{1-\tau P} - C'(P) + \frac{M}{P(1-\tau P)} - \frac{C''(1-\tau P)(1-s)}{\tau}\right] - C''(P)A - C''(P)(1-\tau P)$

It can be easily shown that if $1+\varepsilon_c > \varepsilon_{cc}$, $\frac{\tau P}{(1-s)(1-\tau P)} < \varepsilon_c$ and $\frac{\varepsilon_{cc}\varepsilon_c}{1+\varepsilon_c} > \frac{\tau P}{(1-s)(1-\tau P)}$, then (34), (35), (36) and (37) are negative. Hence, a double-dividend policy is possible under both subsidization regimes. The above inequalities imply that both pollution level and unemployment decrease in tax rates, if the term which captures the proportional benefit for firm ($\frac{1-\tau P}{\tau P}$) from reducing pollution is greater than the one representing the net cost ($\frac{1+\varepsilon_c}{\varepsilon_{cc}\varepsilon_c(1-s)}$). Furthermore, the efficiency of environmental tax as an instrument for reducing pollution seems to be enhanced when a proportional subsidy is given. This conclusion accrues from the fact that under a proportional subsidization regime, the pollution tax which corresponds to the first-best solution, when a proportional subsidy is given, is lower than in the case of a lump-sum subsidy by a factor of (1-*s*).

## 6. Conclusion

In this paper, the authors examined the impact of the optimal environmental tax policy on unemployment and pollution discharges. It was shown that, in the short run, a more strict environmental policy expressed by higher pollution taxes can be compatible with both lower levels of pollution and involuntary unemployment, given that the tax revenues are not used for subsidizing firms or workers. Moreover, firms' pollution discharges are efficient, when firms face a tax rate which is greater than the marginal social damage cost from pollution. On the other hand, if the tax revenues are used for the financing of firms' investment costs, then again a double-dividend outcome can be achieved. A rather interesting result is that a proportional investment subsidy is preferred from a lump-sum one, since the former is related to a lower socially efficient tax rate.

A further study in this field could include the examination of the long run effects of the environmental tax policy on employment rate and pollution level. Furthermore, a topic for future research could be the investigation of the case in which pollution tax revenues are recycled back to workers in the form of subsidies.

## Appendix

The partial derivatives of $B = -\frac{a\tau\delta(1-y^R)^2}{2[2(r+\delta)+a][\delta+ay^R]} - C'(P)$, with respect to $y^R$, $P$ and $\tau$ are:

$$\frac{\partial B}{\partial y^R} = \frac{a\tau\delta(1-y^R)[2\delta+a(1+y^R)]}{2[2(r+\delta)+a][\delta+ay^R]^2} > 0$$

$$\frac{\partial B}{\partial P} = -C''(P) < 0$$

$$\frac{\partial B}{\partial \tau} = -\frac{a\delta(1-y^R)^2}{2[2(r+\delta)+a][\delta+ay^R]} < 0$$

Using equation (18), we get

$$\frac{\partial B}{\partial \tau} = \frac{C'(P)}{\tau}$$

The partial derivatives of $\Gamma = -C(P) + \frac{a\delta(1-\tau P)}{[2(r+\delta)+a][\delta + ay^R]}\int_{y^R}^{1}(y - y^R)dy - y^R(1-\tau P)$, with respect to $y^R$, $P$ and $\tau$ are:

$$\frac{\partial \Gamma}{\partial y^R} = -(1-\tau P)\left\{\frac{a\delta(1-y^R)[2\delta + a(1+y^R)]}{2[2(r+\delta)+a](\delta + ay^R)^2} + 1\right\} < 0$$

$$\frac{\partial \Gamma}{\partial P} = -C'(P) - \frac{a\tau\delta(1-y^R)^2}{2[2(r+\delta)+a](\delta + ay^R)} + \tau y^R$$

Using equation (18), we get

$$\frac{\partial \Gamma}{\partial P} = \tau y^R > 0$$

and finally

$$\frac{\partial \Gamma}{\partial \tau} = \left\{y^R - \frac{a\delta(1-y^R)^2}{2[2(r+\delta)+a](\delta + ay^R)}\right\}P$$

Using equation (17), we get

$$\frac{\partial \Gamma}{\partial \tau} = -\frac{C(P)P}{1-\tau P} < 0$$

In matrix form, we get

$$\begin{bmatrix} \dfrac{-C'(P)[2\delta + a(1+y^R)]}{(1-y^R)(\delta + ay^R)} & -C''(P) \\ -(1-\tau P)\left\{\dfrac{-C'(P)[2\delta + a(1+y^R)]}{(1-y^R)(\delta + ay^R)\tau} + 1\right\} & \tau y^R \end{bmatrix} \begin{bmatrix} \dfrac{dy^R}{d\tau} \\ \dfrac{dP}{d\tau} \end{bmatrix} = \begin{bmatrix} \dfrac{-C'(P)}{\tau} \\ \dfrac{C(P)P}{1-\tau P} \end{bmatrix} \qquad \text{(A.1)}$$

if $\Psi = \frac{a\tau\delta(1-y^R)[2\delta + a(1+y^R)]}{2[2(r+\delta)+a][\delta + ay^R]^2} = \frac{-C'(P)[2\delta + a(1+y^R)]}{(1-y^R)(\delta + ay^R)} > 0$, then the determinant of the first 2×2 matrix will be

$$D = \Psi[y^R\tau - \frac{(1-\tau P)}{\tau}C''(P)] - (1-\tau P)C''(P)$$

From equations (17) and (18) we get

$$y^R\tau = -C'(P) - \frac{C(P)\tau}{1-\tau P} \qquad \text{(A.2)}$$

Equation A.2 implies that

$$\varepsilon_c > \frac{\tau P}{1-\tau P}$$

where $\varepsilon_c$ is the absolute value of the elasticity of $C$ with respect to $P$.

In general, the sign of $D$ cannot be defined without giving specific values to the parameters and determining the exact functional form of $C(P)$. However, it can be easily shown that if $\varepsilon_{cc} > \varepsilon_c$, where $\varepsilon_{cc}$ is the absolute value of the elasticity of $C'$ with respect to $P$, then $y^R\tau - \frac{(1-\tau P)}{\tau}C''(P) < 0$ and hence $D$ will be always negative. Solving A.1 with respect to $dy^R/d\tau$ and $dP/d\tau$, we get (19) and (20).

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