The Allocation of Investment across Vintages of Technology

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Abstract

This paper proposes a new mechanism that explains continued investment in older-vintage technology which rests on complementarity between *long-lived* and *short-lived* vintage-specific capital. The main result is a threshold condition that relates the rate of vintage-specific technological progress ($\hat{q}$) to two investment patterns: if $\hat{q}$ is above the threshold, all investment is allocated to the newest vintage technology; otherwise, firms direct part of investment to older-vintage technologies. The evidence supports model's empirically testable implications: as $\hat{q}$ declines, investment is more allocated towards older-vintage technology; and equipment price-changes depend on capital's heterogeneous rates of depreciation.

1 Introduction

How much of investment in old-fashioned equipment should be allocated instead in state-of-the-art equipment? This study answers this question under the framework

*As his last student, I express my deep sorrow for the death of Gary Saxonhouse. This paper is based on a chapter of my Ph.D. dissertation submitted to the University of Michigan. I am grateful to my advisers, John Laitner, Dmitriy Stolyarov, Miles Kimball, Gary Saxonhouse, and Jan Svejnar for their guidance and encouragement. I also thank Hiroshi Ohashi and Tsuyoshi Nakamura for providing their data on steel furnaces, Kozo Kiyota, Yasuyuki Todo, Takanobu Nakajima, and participants in the seminars at the University of Michigan, Kansai University, Yokohama National University, the University of Tokyo, Aoyama Gakuin University, and Osaka University for their helpful comments.

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of conventional vintage growth models by assuming two types of complementary capital in each vintage production function. The model shows that the optimal allocation depends on the trade-offs between the magnitude of the remaining stocks of the old-fashioned complementary capital and the relative advantages of the frontier technology.

This paper's model has two key elements: (i) it is a vintage growth model in which a certain technology is built into each unit of capital; and (ii) each vintage production function consists of two kinds of complementary capital that have different rates of depreciation.[1] Both types of investment are irreversible. The main idea is intuitive; if one type of capital depreciates more slowly (*long-lived*) than the other (*short-lived*), then investing in short-lived capital with an old-fashioned technology is sometimes rationalized in order to exploit the existing stock of complementary long-lived capital with that vintage technology. Long-lived capital is possibly intangible capital such as knowledge, software, and system capital, structures or networks depending on context, while the short-lived capital is probably equipment in most instances.

The main result is a threshold condition that relates the rate of vintage-specific technological progress ($\hat{q}$) to two investment patterns: (i) if the rate of technological progress is above the threshold (the product of the long-lived capital's share and the difference in the rates of depreciation of the two capital types–which shows the relative importance of remaining stock of long-lived capital), then all new investment will concentrate on the two types of capital with the frontier technology; (ii) otherwise, a part of new investment will be allocated to short-lived capital with older-vintage technology to exploit the existing stock of old-fashioned long-lived capital. The result can be tested empirically. For example, the model predicts that the rate of productivity growth should be negatively correlated with the relative size of investment in old vintages. The evidence supports this; the lower the technological progress of industries, the higher the intensity of maintenance and repair which is a proxy for investment in vintage technology.

Another important result is that the price-changes (obsolescence) of vintage short-lived capital depend not only on the technological progress, but also on the difference in the rates of depreciation of the two capital types. In particular, if the rate of the technological progress is below the threshold, then the prices of old-fashioned

[1] In this study, "depreciation" solely refers to *physical depreciation*, and excludes *obsolescence* that is explicitly treated as endogenously determined price-changes in the following analysis.

short-lived capital remain unchanged even when the rate of technological progress is positive. A strand of literature (E.g., Gordon (1990), Hulten (1992), Greenwood, Hercowitz, and Krusell (1997), and Cummins and Violante (2002)) uses vintage models to measure the embodied technological progress from quality-adjusted equipment prices. My model implies that such estimate may be biased downwards about 1.5%, making the rate of technological progress even faster than previously thought.

Related literature describes how other types of mechanisms can make it optimal to invest in old capital: (i) vintage-specific human capital that is acquired in previous periods (Chari and Hopenhayn (1991), Parente (1994), and Jovanovic and Nyarko (1996)); and (ii) complementarity in production *across* vintages (Jovanovic (2009)). These papers describe the mechanisms by which investment in old capital can plausibly arise, but do not explore their empirically testable implications. The present paper, by contrast, puts the proposed explanation to empirical test.

Another strand of related literature is those analyze Solow-type vintage growth models with two capital types (Greenwood, Hercowitz, and Krusell (1997), Gort, Greenwood, and Rupert (1999) and Laitner and Stolyarov (2003)). These models incorporate capital heterogeneity, but do not provide the explanation for the investment in old vintage technology. The model presented here features investment allocation across and within vintages while comprehending the prevalent properties of Solow-type growth models.

The rest of the paper is organized as follows. Section 2 presents the model's framework and a characterization of a balanced growth path. Section 3 compares model's prediction with empirical data such as investment patterns and changes in prices. Section 4 discusses candidates of long-lived capital. Then Section 5 concludes the paper.

2 The Model

The model has two key elements: (i) each vintage of capital works with a separate production function that has a vintage-specific productivity level; and (ii) each vintage production function consists of two kinds of vintage compatible capital with different rates of depreciation. Apart from the second element, all assumptions are essentially identical to those of Solow (1960)'s.

I assume that the economy is competitive, and firms have perfect foresight and are rational. Each unit of capital is designed for a vintage-specific (v) technology that has an individual production function with a specific productivity level, q_v. At time $t \geq 0$, vintage $v \in [0, t]$ technology is available. Each vintage production technology requires complementary three inputs: two types of *vintage-specific* capital, A (*long-lived*) and B (*short-lived*), and *vintage-nonspecific* labor, L. A and B depreciate at the rates δ^A and δ^B where $\delta^A \leq \delta^B$. Let capital's subscript v denote a specific vintage v technology that is *embodied* in each type of capital, while L_v express the amount of labor that is *employed* for a vintage v. Each vintage-specific production function has the Cobb-Douglas form of

$$Y_v(t) = q_v A_v(t)^\alpha B_v(t)^\beta L_v(t)^{1-\alpha-\beta}, \tag{1}$$

where $Y_v(t)$ is output at the current time t produced by the vintage v technology, and α and β are constant shares of two capital types.[2] The frontier technology's productivity level (q_t) and labor supply (L) grow at constant rates, $\hat{q} > 0$ and $\hat{L} \geq 0$, where hat ($\hat{}$) denotes the time derivative of the natural log of the argument.

Assume that each output produced by vintage specific production function is homogeneous and keeps a constant physical unit over time. The aggregate homogeneous output can be defined as

$$Y(t) \equiv \int_0^t Y_v(t)\, dv. \tag{2}$$

The aggregate homogeneous output is divisible to consumption and two types of irreversible capital investment. Investment in a unit of capital with any vintage requires one unit of homogeneous output. A fixed portion ($\sigma \in (0, 1)$) of aggregate output is allocated to investment, and each type of capital is freely disposable. In the following analysis, the time index (t) is dropped to simplify the exposition.

The distinctive feature of the current model is that it uses different mechanism from existing models to explain the persistent use of old technology. The current model assumes complementarity of two types of capital within the same vintage technology, while existing models assume human capital that is associated with old technology acquired in previous periods (Chari and Hopenhayn (1991), Parente (1994), and Jovanovic and Nyarko (1996)) or complementarity of capital across vintages (Jo-

[2] In the model presented here, I omit Hicks-neutral technological progress that affects all vintages of production, since the omission does not change the main point of the results. Chapter 3 in Aruga (2006) shows the case when the neutral technological progress is also embedded.

vanovic (2009)).

2.1 Aggregation across Vintages

Model's simple structure makes it possible to aggregate the separated vintage production functions into a simple aggregate production function as Solow (1960). This subsection derives the aggregate production function that is the key in characterizing the balanced growth path (BGP) of the economy.

Under the competitive market assumption, firm's profit maximization conditions in terms of two capital types and labor are:

$$MPA_v = \alpha \frac{Y_v}{A_v} = P_v^A R_v^A, \tag{3}$$

$$MPB_v = \beta \frac{Y_v}{B_v} = P_v^B R_v^B, \text{ and} \tag{4}$$

$$MPL = (1-\alpha-\beta) \frac{Y_v}{L_v} = W, \tag{5}$$

where the MPX_v, P_v^X, and R_v^X are the marginal products, the prices in units of homogeneous output, and the rates of return of specific types of vintage capital, and $X \in \{A, B\}$. MPL and W are the marginal product of labor and the wage. MPL and W do not have vintage subscript because labor is vintage-nonspecific. Note that there is the relationship of

$$R_v^X - \delta^X + \hat{P_v^X} = r \; \forall \, v \in [0, t] \tag{6}$$

where r is the interest rate. This is because holding any type of capital with any vintage must be identical for investors after netting the depreciation (δ^X) and the obsolescence ($\hat{P_v^X}$). Note also that $P_v^X \in [0, 1]$ since each type of capital is freely disposable and investment in capital types with existing vintage technology is always possible.

Define the aggregate inputs to be the summation of marginal-product-weighted inputs relative to those of the frontier technology such that

$$A \equiv \int_0^t \frac{MPA_v}{MPA_t} A_v \, dv = \int_0^t \frac{Y_v/A_v}{Y_t/A_t} A_v \, dv = \frac{A_t}{Y_t} Y, \tag{7}$$

$$B \equiv \int_0^t \frac{MPB_v}{MPB_t} B_v \, dv = \int_0^t \frac{Y_v/B_v}{Y_t/B_t} B_v \, dv = \frac{B_t}{Y_t} Y, \text{ and} \tag{8}$$

$$L \equiv \int_0^t L_v \, dv = \int_0^t \frac{Y_v/L_v}{Y_t/L_t} L_v \, dv = \frac{L_t}{Y_t} Y. \quad (9)$$

Note that when rates of return (R_v^X) are unique across vintages, the defined aggregate input of that type simply show the total values of that type in units of the price of frontier capital of that type.

Using (1), (2) and (7) - (9), aggregate output can be expressed as

$$Y = \left[\frac{Y_t}{A_t} A\right]^\alpha \left[\frac{Y_t}{B_t} B\right]^\beta \left[\frac{Y_t}{L_t} L\right]^{1-\alpha-\beta} = q_t A^\alpha B^\beta L^{1-\alpha-\beta}. \quad (10)$$

Interestingly enough, the aggregate production function has the same form as (1) with frontier technology level q_t and the aggregate inputs.

Using (1), (5), and (9), aggregate consolidated capital defined as

$$J \equiv \int_0^t J_v \, dv \equiv \int_0^t \left[q_v A_v^\alpha B_v^\beta\right]^{\frac{1}{\alpha+\beta}} \, dv$$

can be expressed as

$$J = \left[q_t A^\alpha B^\beta\right]^{\frac{1}{\alpha+\beta}},$$

and the labor allocation across vintages is given by

$$L_v = \frac{J_v}{J} L.$$

The consolidated vintage capital (J_v) and aggregate labor (L) determine L_v, and thus Y_v, MPX_v, and Y without specifying prices of capital types.[3]

2.2 Balanced Growth Path (BGP)

This subsection identifies the balanced growth path (BGP) of the model. The economy's BGP of interest is where *(i) all the endogenous variables including the aggregate amounts defined by (7)-(10) grow at constant rates, and (ii) there is investment in both types of capital.*[4] The previous subsection characterized the state of an economy including the labor allocation and the output distribution across vintages provided

[3]The aggregate production function can be expressed as $Y = J^{\alpha+\beta} L^{1-\alpha-\beta}$, which has the same form as Solow (1960). J stands for Solow's *Jelly Capital*.

[4]If firms invest only in one type, the economy converges to the origin that is not a rational BGP as discussed in Shell and Stiglitz (1967).

the distribution of two types of vintage capital. Given the state, the next step is to determine investment patterns in two dimensions, across vintages and capital types in a BGP.

Although Solow (1960)'s vintage growth model with a single type of vintage capital presumes that all new investment concentrates on the capital that has the newest available vintage, there are other possibilities in the current model.[5] Suppose in the current model that, initially, the allocation of long-lived and short-lived capital with a specific vintage v is optimal such that the prices of two capital types are the same. Then, over time, the existing stock of the vintage long-lived capital becomes relatively abundant compared to that of the vintage short-lived capital without investment because of the difference in the rates of depreciation. This might result in the rise in the productivity and the price of the vintage short-lived capital. In a special case, investment in the vintage short-lived capital may become more attractive than that in the frontier technology. The existence of two types of vintage compatible capital complicates the characterization of investment patterns and price distribution across vintages and capital types.

In analyzing the investment patterns, the key is the relationships of capital prices across types and vintages in (3) and (4). Consider long-lived capital with two different vintages, A_v and $A_{v'}$ where $v, v' \in [0, t]$ and $v \neq v'$. Because the interest rate r must be the same across vintages, from (3) and (6) there is the relationship,

$$\frac{Y_v}{P_v^A A_v} - \frac{Y_{v'}}{P_{v'}^A A_{v'}} = \frac{\hat{P_{v'}^A} - \hat{P_v^A}}{\alpha}. \tag{11}$$

Since the both terms of the left hand side of (11) grow at constant rates and the right hand side is constant in a BGP, both sides must be zero. The same argument applies to short-lived capital (B). Therefore, using (6), for $X \in \{A, B\}$ and $\forall\, v, v' \in [0, t]$,

$$\hat{P_v^X} = \hat{P_{v'}^X} = \hat{P^X}, \text{ and} \tag{12}$$

$$R_v^X = R_{v'}^X = R^X. \tag{13}$$

[5]There is no investment in old technology in Solow (1960)'s model because the capital that embodies the newest available vintage technology always has the higher productivity than any other obsolete vintage capital, given that there is only one type of vintage-specific capital and the labor input is freely allocated across vintages.

(1), (3) - (5), and (13) provide the relationships of prices across vintages,

$$P_v^A = \left[\frac{q_v}{q_{v'}}\right]^{\frac{1}{\alpha+\beta}} \left[\frac{B_v/A_v}{B_{v'}/A_{v'}}\right]^{\frac{\beta}{\alpha+\beta}} P_{v'}^A, \text{ and} \tag{14}$$

$$P_v^B = \left[\frac{q_v}{q_{v'}}\right]^{\frac{1}{\alpha+\beta}} \left[\frac{B_v/A_v}{B_{v'}/A_{v'}}\right]^{-\frac{\alpha}{\alpha+\beta}} P_{v'}^B. \tag{15}$$

(14) and (15) provide

$$\frac{q_v}{q_{v'}} = \left[\frac{P_v^A}{P_{v'}^A}\right]^{\alpha} \left[\frac{P_v^B}{P_{v'}^B}\right]^{\beta}. \tag{16}$$

(14) and (15) show the relationships of prices across vintages. In both cases, the less the amount of a specific type of capital with a vintage, the higher the price of that type of capital with the vintage. Furthermore, the ratios of prices across vintages are proportional to the ratios of technological levels. (16) summarizes these relationships.

In a BGP, investment in a type of capital with an existing specific vintage must be continuous in order the growth rate of the stock of the type to be constant. Therefore, there are four possible investment schemes regarding the existing capital types in a BGP: *there is positive continuous investment (a) only in A_v; (b) only in B_v; (c) neither in A_v nor B_v; and (d) both in A_v and B_v.* Since (12) and (14) imply $\frac{B_v/A_v}{B_{v'}/A_{v'}}$ is constant $\forall\, v, v' \in [0,t]$ in a BGP, investment schemes must be unique across vintages $v \in [0,t]$. Using the classification of these investment schemes and (14) - (16), now I characterize investment patterns across vintages in a BGP as follows.

Proposition 1 (Investment patterns across vintages of technology)**.** *In a BGP there is investment in the two capital types with the frontier technology, A_t and B_t, and:*

(i) if $\hat{q} \geq \alpha(\delta^B - \delta^A)$, then the investment scheme is (c) $\forall\ v \in [0,t]$ where firms invest in neither A_v nor B_v (Fast Case)*; and*

(ii) otherwise, the investment scheme is (b) $\forall\ v \in [0,t]$ where firms continuously invest in B_v (Slow Case)*.*

Proof: See Appendix A.1.1.

In short, when technological progress is fast enough, there is no investment in capital types with old technologies. Otherwise, there will be investment in short-lived capital that embodies old technology in order to exploit existing long-lived capital. The threshold of the rate of technological progress is the product of long-lived capital's share (α) and the difference in the rates of depreciation of the two

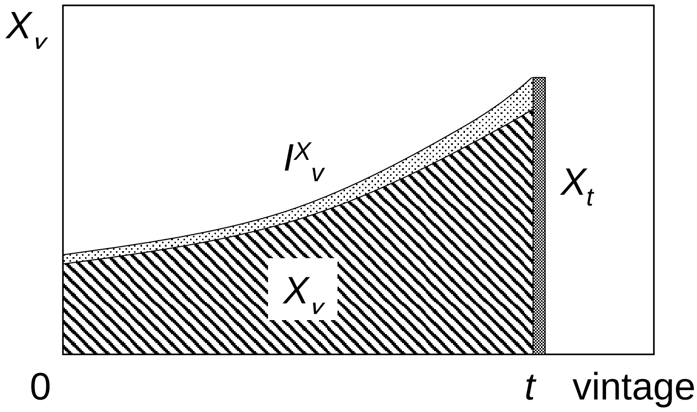

Figure 1: Allocation of investment in capital X.

capital types $(\delta^B - \delta^A)$. α shows the importance of long-lived capital in production function, and $(\delta^B - \delta^A)$ shows the rate of increase in relative size of long-lived capital compared to short-lived capital. Intuitively, investment in obsolete short-lived capital will be made when the increase in relative size of the compatible long-lived capital (along with the rise in the price of short-lived capital) is fast enough compared to the vintage-specific technological progress.[6]

Given the investment scheme in a BGP, now consider the allocation of the two capital types within vintages in a BGP. By assumption, the total amount of the investment is the fixed portion of the homogeneous output which can be expressed as

$$\sigma Y = I^A + I^B = \int_0^t I_v^A \, dv + A_t + \int_0^t I_v^B \, dv + B_t. \tag{17}$$

Note that the investment consists of the part of the distribution of existing technologies (I_v^X) and the part of the mass of the frontier technology (X_t) as illustrated in Figure 1. Now, to simplify the exposition of the following analysis, define the aggregate effective labor, $N \equiv q_t^{1/(1-\alpha-\beta)} L$, and use lower case letters to express the aggregate amounts in units of effective labor: $a \equiv A/N$ and $b \equiv B/N$. Then, the profit maximization conditions and the laws of motion of capital types provide the allocation of capital types in a BGP as follows.

[6]Note that short-lived capital's share (β) does not enter the threshold. This is because the inequality relation between prices of different vintages of short-lived capital is independent of β, although β affects relative levels of prices in Cobb-Douglas production technology. See equation (15) to confirm this point.

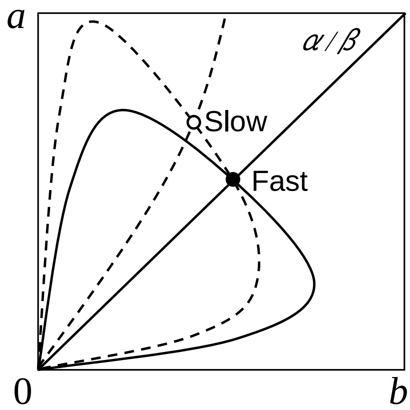

Figure 2: Relationship between a and b implied by (18)–upward sloping curves from profit maximization condition–and (19)–quasi-triangle curves from laws of motion. Solid and dashed lines correspond to the Fast and Slow Cases respectively. Black and white circles show the BGP equilibrium in the Fast and Slow cases.

Proposition 2 (Allocation of capital types in a BGP)**.** *In a BGP, a and b have a relationship from the profit maximization conditions,*

$$\beta a^{\alpha} b^{\beta-1} - \alpha a^{\alpha-1} b^{\beta} = \begin{cases} 0 & \text{when } \hat{q} \geq \alpha(\delta^B - \delta^A), \text{ and} \\ \delta^B - \left[\delta^A + \frac{\hat{q}}{\alpha}\right] & \text{otherwise,} \end{cases} \tag{18}$$

and a condition from the laws of motion,

$$\sigma a^{\alpha} b^{\beta} = \begin{cases} \left[\frac{\hat{q}+\alpha\delta^A+\beta\delta^B}{\alpha+\beta} + \hat{N}\right][a+b] & \text{when } \hat{q} \geq \alpha(\delta^B - \delta^A), \text{ and} \\ \left[\delta^A + \frac{\hat{q}}{\alpha}\right] a + \delta^B b + \hat{N}[a+b] & \text{otherwise,} \end{cases} \tag{19}$$

and there are unique, constant, and stable BGP values of $a = a^*$ and $b = b^*$ that satisfies conditions (18) and (19).

Proof: See Appendix A.1.2.

Figure 2 shows possible relationships of a and b implied by (18) and (19), and equilibrium (BGP). The black circle and solid lines correspond to the Fast Case, and white circle and dashed lines do to the Slow Case. (18) is a straight line from the origin with the slope of α/β in the Fast Case, while it is a convex curve above the straight line in the Slow Case. (19) is a quasi-triangle curve that goes through the origin. The curve of (19) in the Slow Case is more skewed to the a side than in the Fast Case because long-lived capital (A) is relatively more attractive when interest

Table 1: Properties of two cases of BGP.

BGP	(i) Fast Case	(ii) Slow Case
Technological progress ($\hat{q}$)	$\geq \alpha(\delta^B - \delta^A)$	$< \alpha(\delta^B - \delta^A)$
Investment scheme	(c)	(b)
Investment pattern	Frontier A/B only	Frontier A/B, and obsolete B
$\hat{P_v^A}^*$	$-[\hat{q} + \beta(\delta^B - \delta^A)]/(\alpha+\beta)$	$-\hat{q}/\alpha$
$\hat{P_v^B}^*$	$-[\hat{q} - \alpha(\delta^B - \delta^A)]/(\alpha+\beta)$	0 (Remains 1)
A_v/B_v	$> \alpha/\beta$	$> \alpha/\beta$
$[P_v^A A_v]/[P_v^B B_v]$	α/β	$> \alpha/\beta$
$A/B(= A_t/B_t)$	α/β	$> \alpha/\beta$

* Changes in vintage capital prices in a Fast Case are given by (14) and (15) applying $v' \to t$ with the condition $A_t/B_t = \alpha/\beta$ and $P_t^A = P_t^B = 1$. Those in a Slow Case are given by (16) with the condition $P_t^A = P_v^B = 1 \; \forall \, v$.

rate is low.

The allocation of capital types and investment is determined by the combination of Proposition 1 and 2. In the Fast Case, the total investment–the product of the aggregate output (Y) and the exogenous saving rate (σ)–is simply divided to the frontier two types of capital with the proportion of $\frac{\alpha}{\alpha+\beta}$ and $\frac{\beta}{\alpha+\beta}$. After the initial investment, the two types of capital with a specific vintage decline at the rates of depreciation. In the Slow case, the part of the total investment is allocated to the older short-lived capital such that the prices of them exactly remain the same level of homogeneous output. The remaining part of the total investment is allocated to the frontier capital with the proportion of the equilibrium values of $\frac{a^*}{a^*+b^*}$ and $\frac{b^*}{a^*+b^*}$.

2.3 Properties of the Two Types of BGP

Table 1 summarizes the properties of the two cases of BGP, which is obtained from the proofs of Proposition 1 and 2, and observed in Figure 2. In the Fast Case, the investment schemes of all the available vintages are (c), and all investment is allocated to the capital types with the frontier technology, A_t and B_t. Both prices of two capital types of a specific vintage decline exponentially over time as shown in Figure 3 (i). The decline in prices of short-lived capital are slower than that of long-lived capital since short-lived capital with a vintage becomes relatively scarce compared to long-lived capital of that vintage over time. This is because their depreciation rates differ.

The ratios of investment in the frontier capital types, A_t/B_t, of market values of

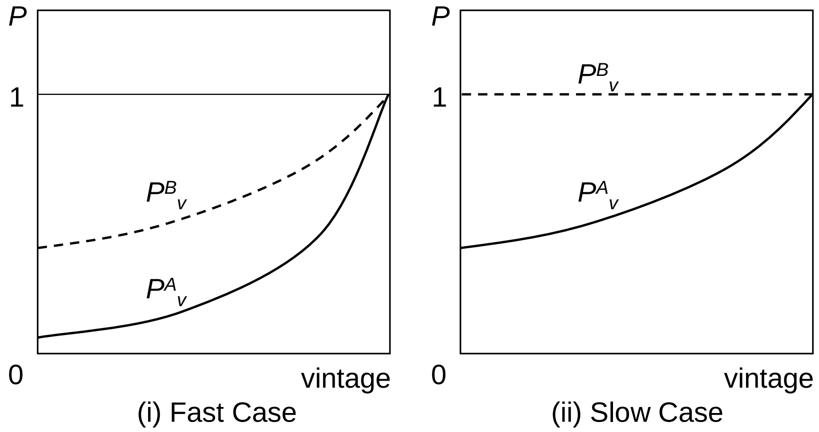

Figure 3: Prices of capital across vintages: (i) in a Fast Case, $\hat{q} \geq \alpha(\delta^B - \delta^A)$; and (ii) in a Slow Case $\hat{q} < \alpha(\delta^B - \delta^A)$.

their vintage, $[P_v^A A_v]/[P_v^B B_v]$, and of aggregate amounts of them, $A/B = a/b$, all keep α/β even when $\hat{q}$ changes. The reason is that prices of vintage capital types adjust such that they cancel the difference in their rates of depreciation. Indeed, the total depreciation–the sum of obsolescence $(\hat{P_v^X})$ and physical depreciation (δ^X)–is $[\hat{q} + \alpha\delta^A + \beta\delta^B]/[\alpha + \beta]$ for both capital types. The allocations of labor skew toward newer technology as $\hat{q}$ increases.

In the Slow Case, investment is not only allocated to the frontier technology capital types, A_t and B_t, but also to short-lived capital with obsolete vintages, $B_v \, \forall \, v \in [0, t]$. This is because marginal products of obsolete short-lived capital without investment are higher than those of the newest capital types, and thus there will be investment in short-lived capital with obsolete vintage technology. Therefore, while the prices of long-lived capital decline exponentially over time, the prices of short-lived capital across vintages remain the same level as new output as shown in Figure 3 (ii).

The ratios of investment in the frontier capital types, A_t/B_t, of market values of vintage capital, $[P_v^A A_v]/[P_v^B B_v]$, and of the aggregate amounts, $A/B = a/b$, are all the same and larger than α/β. Unlike in the Fast Case, the ratio A/B rises as $\hat{q}$ declines, because a decline in $\hat{q}$ lowers the interest rate r that makes long-lived capital relatively more attractive. The distributions of short-lived capital as well as labor skew toward older technology as $\hat{q}$ declines.

Solow type vintage growth models can be interpreted as specific cases of the current model. For example, BGP of Laitner and Stolyarov (2003)'s model is a special case of the Fast Case in the present model; they assume a single rate of depreciation, $\delta^A = \delta^B$, which assures the Fast Case ($\hat{q} \geq \alpha(\delta^B - \delta^A) = 0$) as long as the rate of technological progress is positive. The current model shows, however, that even when rates of depreciation differ, similar results to those in their model are observed in the Fast Case. The BGP of Shell and Stiglitz (1967)'s model is also a special case of the current model where there is no technological progress ($\hat{q} = 0$) and the rates of the depreciation of two capital types are the same ($\delta^A = \delta^B$).

3 Empirical Evidence and Relevancies

In the last section, the BGP analysis of the model reveals two distinct investment patterns depending on the relationships between the rate of technological progress and the threshold. This section shows how these results complements the existing literature by exploring two important empirically testable implications: proportion of investment in older-vintage technologies depends negatively on the rate of technological progress; and absolute rate of price-changes in short-lived capital with a specific vintage depends on the difference in the rates of depreciation of two capital types as well as the rate of technological progress. These two points are tested by using the variations in data across industries and equipment. Furthermore, it is shown that the model is consistent with other empirical relevancies such as heterogeneity of capital lives.

3.1 Investment in Obsolete Capital

Although counterintuitive, investment in old-fashioned equipment–which is less productive than cutting-edge equipment–is observed in the real economy. For example, production of steam locomotives had continued even after more efficient diesel locomotives had been introduced.[7] Existing literature (Chari and Hopenhayn (1991),

[7] See Figure 5 in Felli and Ortalo-Magne (1998). Other examples are found in production with cotton spinning (Saxonhouse and Wright (2000)), and with steel furnaces (Nakamura and Ohashi (2008)). Data in Nakamura and Ohashi (2008) show that the annual declining rate of the capacity size of open-hearth furnaces (OHFs) in Japan for 10 years after the introduction of more productive basic oxygen furnaces (BOFs) and for 5 years after the peak usage of OHFs were about 5% and 9% respectively, which are both much smaller than the rates of depreciation of "metalworking machines" in the U.S. official statistics, of 12%. This implies there had been investment in obsolete OHF technology after the new BOF technology became available.

Parente (1994), Jovanovic and Nyarko (1996), and Jovanovic (2009)) proposes mechanisms that could generate investment in old technology, but never explored the allocation of investment.

The current model provides quantitative predictions about the investment allocations of short-lived and long-lived capital across vintages. When the rate of technological progress is below the threshold, investment in short-lived capital with older-vintage is rationalized. In this case, the ratio of investment in short-lived capital with older-vintage to the total investment in short-lived capital (sum of older-vintage and newest vintage) will be[8]

$$\text{Ratio} = \frac{\int_0^t I_v^B dv}{\int_0^t I_v^B dv + B_t} = \frac{\delta^B - \delta^A - \frac{\hat{q}}{\alpha}}{\delta^B + \hat{N}}. \tag{20}$$

Figure 4 shows relationship between the ratio of the investment and vintage-specific technological progress given by (20) when the threshold ($\alpha(\delta^B - \delta^A)$) is 0.015.[9] As can be seen, when technological progress is below the threshold, the smaller the $\hat{q}$, the larger the allocation of investment in older-vintage capital. When technological progress is above the threshold, there is no investment in older-vintage technology.

The empirical analysis here focuses on varieties of production function at the industry level in order to compare Figure 4 and empirical data. I make five assumptions: (i) the economy is segregated at the industry level; (ii) all types of structures and equipment in an industry work as homogeneous short-lived capital, and there is a kind of complementary, vintage-specific, and longer-lived capital;[10] (iii) maintenance and repair (MR) expenditure is proportional to investment in older-vintage short-lived capital; (iv) production functions of industries are homogeneous except for the rate of vintage-specific technological progress; and (v) each industry's multifactor productivity (MFP) growth is proportional to its vintage specific technological progress.

I assume (iii) because there is no appropriate investment data that distinguishes

[8]In a Slow Case BGP, $\hat{A}_v = -\delta^A$ because there is no investment in vintage A_v capital. Since rates of return are constant in a steady state, using (3), (4), and the changes in prices in Table 1, growth rate of B_v is given by $\hat{B}_v = \hat{P_v^A} - \hat{P_v^B} + \hat{A}_v = -\frac{\hat{q}}{\alpha} - \delta^A$. Then, using (27) observe $I_v^B = [\hat{B}_v + \delta^B]B_v = [\delta^B - \delta^A - \frac{\hat{q}}{\alpha}]B_v$. Thus, $\int_0^t I_v^B\, dv = [\delta^B - \delta^A - \frac{\hat{q}}{\alpha}]B$. On the other hand, from (17), (31), and (33), we have $\int_0^t I_v^B\, dv + B_t = I^B = [\hat{B} + \delta^B - \hat{P^B}]B = [\delta^B + \hat{N}]B$.

[9]In the following analysis, all time units are year.

[10]I discuss the plausible long-lived capital in Section 4.

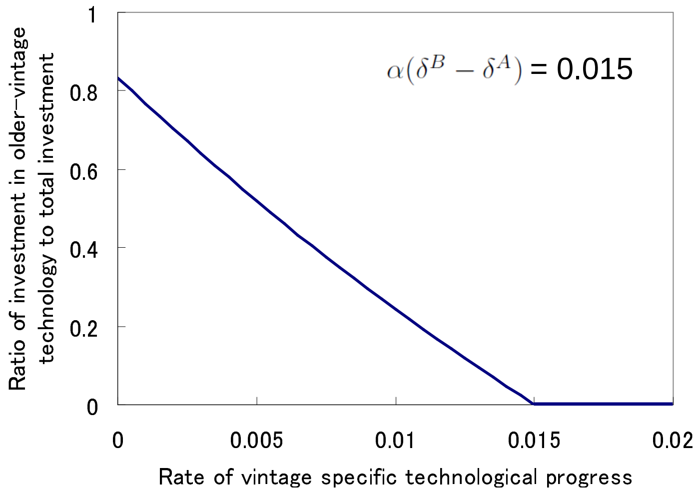

Figure 4: Relationship between the ratio of investment in short-lived capital that embodies older-vintage technology to the total investment in short-lived capital, and rate of vintage-specific technological progress. ($\alpha = 0.15, \beta = 0.25, \delta^A = 0, \delta^B = 0.1, \hat{L} = 0.02$)

investment's vintages. Firms try to keep using old technologies by MR and/or investment in new machinery with obsolete vintages. For example, replacement of tires or muffler of obsolete automobile may be considered as investment while that of wiper blades may be not. Whether this kind of expenditure is considered as MR or capital investment depends on its magnitude.[11] These assumptions should be plausible for the purpose of checking the consistency of the model with empirical data without undermining the main messages of the model.

Figure 5 shows the relationships between the MFP growth from 2005 to 2006 and the intensity of MR expenditure relative to capital investment in the U.S. 86 industries of manufacturing sector (NAICS four digit level) in 2006.[12] As can be seen, there

[11]U.S. Economic Census defines MR as "Included ... are payments made for all maintenance and repair work on buildings and equipment... Excluded from this item are extensive repairs or reconstruction that was capitalized, which is considered capital expenditures..."

McGrattan and Schmitz (1999) document that data from 1961 to 1993 in Canada shows that size of MR expenditure on equipment/structure reaches 50%/20% of the investment in equipment/structure respectively, and MR can be substitute of investment during recession. In an extreme case, when Canadian iron ore industry experiences severe downturn, even equipment investment reaches nearly zero, the industry still spent considerable expenditure on MR. Mullen and Williams (2004) develops a model that explains substitutability of MR and investment in newest type of capital, however, their model does not provide prediction about investment in older technology.

[12]Data on MFP growth is obtained form BLS. Total expenditure on capital and MR expenditure are obtained from 2007 U.S. Economic Census ("CEXTOT" and "PCHRPR").

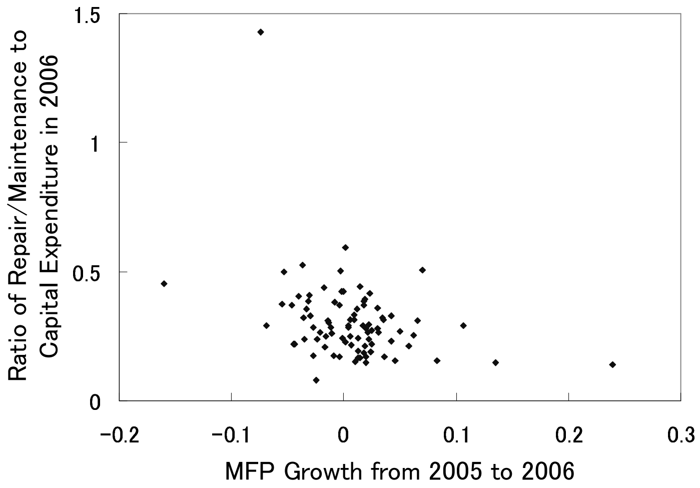

Figure 5: Negative relationships between the multifactor productivity (MFP) growth 2005-2006 and relative intensity of repair expenditure to capital investment in 2006 in the U.S. 86 industries (4-digit NAICS code). Source: BLS and 2007 Economic Census.

is statistically significant negative correlation between the relative MR expenditure and MFP growth.[13] This indicates that the less technological progress in a industry, the more investment towards older-vintage technology. The result is consistent with presented model's unique prediction which is not explored in existing models.

3.2 Changes in Capital Prices

The vintage growth models in existing literature derive direct proportional relationship between the rate of vintage-specific technological progress and changes in equipment prices.[14] However, in the current model, this is not necessarily the case when there is the difference in rates of depreciation of two capital types. From Table 1, we observe

$$\hat{P_v^B} = \begin{cases} -\frac{1}{\alpha+\beta}\hat{q} + \frac{\alpha}{\alpha+\beta}(\delta^B - \delta^A) & \text{when } \hat{q} \geq \alpha(\delta^B - \delta^A) \text{, and} \\ 0 & \text{otherwise.} \end{cases} \tag{21}$$

The current model predicts that, in a Fast Case, the changes in equipment prices

[13]The correlation and significance are -.34 and at the 1% level.

[14]E.g., Gordon (1990), Hulten (1992), Greenwood, Hercowitz, and Krusell (1997), and Cummins and Violante (2002).

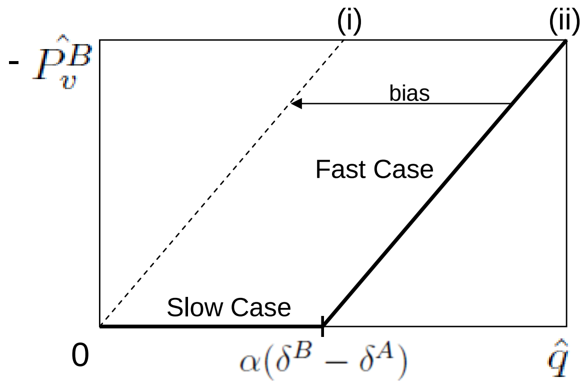

Figure 6: Changes in prices (i) when $\alpha(\delta^B - \delta^A) = 0$ (dashed-line), and (ii) when $\alpha(\delta^B - \delta^A) > 0$ (solid-line) given by equation (21).

depend not only on the rate of the technological progress, but also on the difference in the rates of depreciation between the short-lived and long-lived capital. In contrast, in a Slow Case, the price of short-lived capital remains at the output price, leaving the rates of technological progress and depreciation irrelevant.

Figure 6 shows the relationship between the changes in prices of short-lived capital ($\hat{P_v^B}$) and the rate of vintage-specific technological progress ($\hat{q}$) given by (21). When there is no long-lived capital (i.e., $\alpha = 0$ or $\delta^A = \delta^B$), the threshold value ($\alpha(\delta^B - \delta^A)$) is zero. In this case, as (i) shows, there is a direct proportional relationship between the price-changes in short-lived capital and the rate of technological progress as in the existing models listed in footnote 14. However, if there is long-lived capital (and thus the threshold value is positive, $\alpha(\delta^B - \delta^A) > 0$), the direct proportional relationship is biased downward to the size of the threshold as (ii) shows. This suggests that, in practice, the estimates of the rate of vintage-specific technological progress in previous literature may be biased downward up to the size of the threshold, making the true rate even faster than previously thought.

The size of bias may be considerably large. As will be discussed in Section 4, intangible capital is a good candidate for long-lived complementary capital to ordinary physical capital. Suppose that at the aggregate level the share of intangible capital is 15% as suggested by Corrado, Hulten, and Sichel (2006); and the difference in the rates of depreciation of physical and intangible capital is 10%.[15] Then, ad-hoc

[15] Ideas do not physically depreciate, while the average of physical rate of depreciation of private nonresidential equipment is about 11% that is obtained by using the average of total depreciation in Fraumeni (1997) and obsolescence from Gordon (1990).

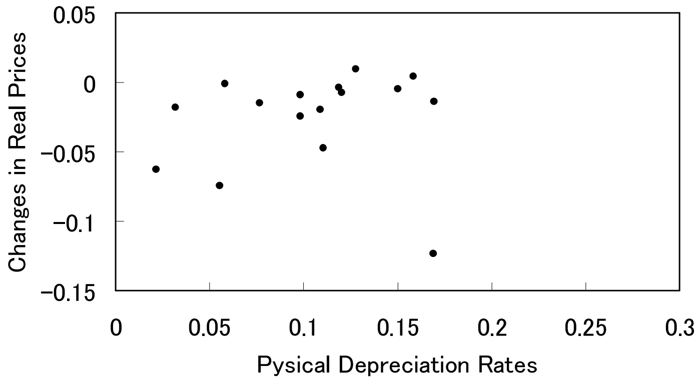

Figure 7: Relationships between the changes in equipment prices and physical rates of depreciation of 16 types of equipment in the U.S. 1947-1983. Source: Gordon (1990) and BEA.

threshold value is $\alpha(\delta^B - \delta^A) = 0.015$. That is, the actual rate of vintage-specific technological progress may be 1.5% higher than previous estimates.[16]

In order to examine the proposed relationships of (21), the following analysis focuses on well-documented price and depreciation data on various types of equipment.[17] I make four assumptions to utilize the data: (i) the economy is segregated at the equipment level; (ii) each type of equipment works as sole short-lived capital of equipment-specific production function for the corresponding economy; (iii) all production functions utilize a kind of complementary, vintage-specific, and longer-lived capital; and (iv) parameters of the all equipment-specific production functions are the same except for equipment-specific δ^B. Under these assumptions, the equation (21) predicts that price-changes and depreciation rate of equipment will have positive relationships when the production with the equipment is in a Fast Case.

Figure 7 shows the relationships between the changes in equipment prices and physical depreciation rates of 16 types of equipment in the U.S. 1947-1983.[18] As the model suggests, there is a statistically significant positive correlation between the price-changes and rates of depreciation when the outlier that has the largest price-

[16]E.g., this size is about one-half of Greenwood, Hercowitz, and Krusell (1997)'s estimate of vintage-specific technological progress, 3.2%.

[17]$\hat{q}$ cannot be used in addition to price of equipment in this analysis since there is no estimate on $\hat{q}$ from independent data sources.

[18]The physical depreciation rates are obtained by subtracting the rate of changes in prices (Gordon (1990)) from the total depreciation (sum of obsolescence and physical depreciation) in "BEA Depreciation Estimates."

changes during the period is excluded.[19] Furthermore, as can be seen in Figure 7, changes in equipment prices seem upper-bounded at zero as the Slow Case of (21) suggests. These relationships have never been explored and explained in existing literature.[20]

3.3 Heterogeneity of Capital Lives

The current model shows that the longevity–which varies inversely with total rates of depreciation (sum of physical depreciation and obsolescence)–of two capital types can be heterogeneous only in certain cases. As can be seen from Table 1, in a Fast Case where $\hat{q} \geq \alpha(\delta^B - \delta^A)$, the total rates of depreciation of short-lived and long-lived capital are the same,

$$\hat{P_v^A} - \delta^A = -\frac{\hat{q} + \alpha\delta^A + \beta\delta^B}{\alpha + \beta} = \hat{P_v^B} - \delta^B, \tag{22}$$

while otherwise (Slow Case) long-lived capital literally lives longer,

$$\hat{P_v^A} - \delta^A = -\frac{\hat{q}}{\alpha} - \delta^A > -\delta^B = \hat{P_v^B} - \delta^B. \tag{23}$$

Interestingly enough, heterogeneity of physical rates of depreciation yields the heterogeneity of lives only in the Slow Case.

The prediction in a Fast Case is consistent with technologies that develops very quickly. For example, although software must not physically depreciate while computer does, the estimates of total depreciation rates of computer and software are similar; they are .31 and .33 in Fraumeni (1997) and Corrado, Hulten, and Sichel (2006) respectively. The production technology consisting of computer (short-lived) and software (long-lived) has high rates of technological progress and they are complementary and vintage-specific. Therefore, even when the rates of physical depreciation of computer and software are different, the rates of their total depreciation can be the same as (22) predicts.

[19] The correlation and significance are 0.56 and at the 5% level respectively. The outlier is "office, computing, and accounting machinery" which will be associated with very high rate technological progress ($\hat{q}$), breaking the assumption (iv) but consistent with (21).

[20] There is no significant correlation if I use total depreciation instead of physical depreciation. This is consistent with the concern expressed by Fraumeni (1997) that there is double counting of obsolescence in the BEA's official estimation of capital stock.

Table 2: Service lives of systems and their components.

Technology	System	Component
Nuclear Power	Plants (60 years)[b]	Nuclear Fuels (4 years)[a]
Air Transportation	Airframes (15-25 years)[c]	Engines (6 years)[c]
Land Transportation	Trucks (14 years)[a]	Tires, etc. (3 years)[d]

[a] From Table 3 in Fraumeni (1997), Private, nonresidential equipment.
[b] From Office of Nuclear Reactor Regulation, U.S. Nulcear Regulatory Commission (2008).
[c] From Table 3 in Fraumeni (1997), Federal, National defense.
[d] From Table 3 in Fraumeni (1997), Durable goods owned by consumers.

The prediction in the Slow Case (23) is consistent with the behaviors of system and its components. While they are complementary and vintage-specific to some degree, data shows that the lives of systems are substantially longer than these of their components as presented in Table 2. This suggest that system can be considered as long-lived capital. Indeed, as the model predicts, firms invest in new components for old system in order to keep using the remaining system. For example, a large part of investment in nuclear fuel is for nuclear power plants with old generation technology that require different specifications from the newer type of plant.[21] A large size of a system corresponds to the large size of long-lived capital's share (α), which probably makes the technologies in Table 2 likely be in the Slow Case.

Previous vintage models do not provide consistent explanation for the difference in the lives of capital types and the investment in old technology. Most of existing vintage-growth literature assumes a single type of capital and no heterogeneity of its longevity. Although Laitner and Stolyarov (2003) study a model that assumes vintage-specific two types of capital, the behavior of these types including the realized lives are the same since they assume the same rates of physical depreciation of the types. Greenwood, Hercowitz, and Krusell (1997) and Gort, Greenwood, and Rupert (1999) assume difference in depreciation rates between two types of capital but no vintage-specific complementarity between the types, which results in investment only in the frontier technology at any time like Solow (1960).

[21]The second generation nuclear power plants built in the 1970s are still in operation although newer and more efficient generation III is introduced in the 1990s (U.S. DOE Nuclear Energy Research Advisory Committee and the Generation IV International Forum (2002)).

3.4 Other Empirical Relevancies

Two other empirical relevancies support the predictions of the model: high-tech investment patterns during boom and recession; and magnitude relation between the actual rate of technological progress and the proposed threshold. First, as shown in Figure 4, the model implies that acceleration in the rate of vintage-specific technological progress can cause reallocation of investment towards modern capital at the aggregate level. This is consistent with investment booms that are concentrated in certain "high-tech" equipment. There is a widely accepted observation that the economic boom in the late 1990s coincided with the diffusion of the information technology (IT).[22]

While typical growth models consider investment in IT equipment as a *source* of improvement in aggregate productivity, the current model provides a different viewpoint; the concentration of investment in IT equipment is a *result* of fast improvement of frontier productivity level. When technological progress of an aggregate economy is fast, firms should concentrate on investing in the capital with newest technology in order to benefit from the better technology. Otherwise, concentration of investment in the high-tech equipment is not necessarily the best decision since older technology with the stock of know-how may be more productive.

Second, it is of interest whether an economy possibly experiences both the Slow and the Fast Cases. The ad-hoc threshold value of $\alpha(\delta^B - \delta^A) = 0.015$ derived in the previous section is comparable in size to the growth rates of labor productivity and multifactor productivity in the postwar U.S. economy. It is likely that the rate of vintage-specific technological progress fluctuates around the threshold value, especially at industry/firm/equipment levels since productivity growths at the disaggregate level typically have wider variations than at the aggregate level.

4 What is Long-lived Capital?

The key assumption of the proposed model is the existence of two types of vintage compatible and complementary capital with different rates of depreciation. By assuming physical capital as short-lived and without specifying what is long-lived capital, empirical data confirms model's predictions–allocation of investment across vintages

[22]See Oliner and Sichel (2003), and Jorgenson, Ho, and Stiroh (2007) for example.

of technology, and changes in prices of equipment. These results indicate that the heterogeneity of capital depreciation cannot be negligible in economic analysis.

This section discusses what can be the long-lived capital. The model requires three main properties of the long-lived capital: smaller rate of physical depreciation, vintage-specificity, and complementarity to short-lived capital. I argue that intangible capital, such as know-how, software, and system capital is a promising candidate, while other possibilities such as structures, and networks should be also appropriate depending on context.

4.1 Intangible Capital

Although intangible expenditure had been simply regarded as intermediate input in the official economic statistics, recent literature has started considering its aspect as capital stock in production (Hall (2001), Atkeson and Kehoe (2005), and McGrattan and Prescott (2005)). While various types of intangible capital are proposed in the literature,[23] in practice, the Bureau Economic Analysis has recently started including software (1999) in the official statistics and releasing R&D satellite accounts (2006). Corrado, Hulten, and Sichel (2006)–by considering private sector's intangible expenditure that aims to increase future output of individual firms as investment–show in their growth accounting that intangible capital's income accounts for 15% of total income in the U.S. nonfarm business sector during the period of 2000-2003, while that of physical capital accounts for 25%. The importance of intangible capital rivals that of physical capital in the modern economy.

Several types of intangible capital possesses the properties of the long-lived capital. For example, suppose the CD drive (physical capital) of your PC crashes for some reason. Then, would you buy a new PC or merely replace the CD drive? If the change in the computer model develops quickly enough, you would purchase a new PC because it has much better features. Or you would replace the CD drive to keep using the existing PC with installed software (intangible capital) that is incompatible with newer types of PC. In this case, both PC and software are vintage-specific to

[23] Although its complete list is still under discussion, a tentative list should include: software (Corrado, Hulten, and Sichel (2006)), R&D (Prucha and Nadiri (1996) and Corrado, Hulten, and Sichel (2006)), brand (McGrattan and Prescott (2005), Corrado, Hulten, and Sichel (2006)), organization (Atkeson and Kehoe (2005), McGrattan and Prescott (2005), Corrado, Hulten, and Sichel (2006)), monopoly franchise (Hall (2001), McGrattan and Prescott (2005)), firm-specific human capital (Laitner and Stolyarov (2003)), and product designs (Laitner and Stolyarov (2003)).

some degree, they are complement within vintages, and software is longer-lived than PCs since software does not physically wear or tear.[24] Similar argument applies to combination of equipment and equipment specific know-how or configuration. Once accustomed to a specific type of equipment, it is sometimes difficult to switch to a new generation of that type.[25]

Additionally, production system that integrates various components can be considered as intangible capital. A production system consists of many components, and value of the whole system should be higher than the sum of raw value of each consisting component, since assembling components requires design and labor input.[26] This difference in the value of whole system and sum of the values of its raw components can be considered as *system capital*. The system capital should last longer than its components since the system keeps its original ability as long as components are properly maintained and repaired. This interpretation is consistent with the longevity data on several systems and their components as discussed in Section 3.[27]

4.2 Structures and Networks

Apart from intangible capital, structures are another possibility of long-lived capital. Suppose a railroad company operates railroad tracks (structures) designed for conventional trains across the country. If the company intends to introduce advanced bullet trains that require wider tracks for their speed, it has to invest in wider railroad tracks that are specifically designed for the new type of trains.[28] Since railroad tracks last longer than railroad equipment, they can be considered as long-lived and short-lived

[24]This type of hardware/software combination should apply to audio (analog record, cassette, compact disc, digital cassette, and iPod) and video (video cassette, laser-disc, DVD, and blue-ray disc) players.

[25]The roles of intangible and physical capital may reverse depending on context. For example, consider the *Coca-Cola Company* that produces and sells Coca-Cola using its factories (tangible capital) and brand name (intangible capital). Suppose the depreciation rate of its brand name is 60% as suggested by Corrado, Hulten, and Sichel (2006) that far exceeds that of their factories, and the rate of development of new beverages is slow. Then, advertisements for Coca-Cola can be interpreted as an investment in obsolete shorter-lived intangible capital to keep using the obsolete existing stock of longer-lived factories.

[26]If a component is not built in a system, the component alone has no productivity.

[27]Similar argument may apply to the organization/its human capital combination, and more broadly social system/its citizen combination.

[28]In Japan, *Shinkansen* network had been introduced in 1960's by constructing new tracks of its own in addition to the conventional train network.

capital.[29] In many cases, the old train network will be kept using because value of the stocks of the existing railroad tracks is large, which requires persistent investment with older types of trains. The similar reasoning may applies to the introduction of electric vehicles (equipment, short-lived) since they require new types of fuel station (structures, long-lived) that provides battery replacement service, plug-in charging, or hydrogen-fuel instead of conventional gasoline or diesel fuel. The production of conventional vehicles will be persistent for a while since the conventional fuel supply facilities will last longer than conventional vehicles.

Another example of long-lived capital is communication network–such as DSL or fiber-optic cable. Suppose you have Internet connection via 1 M bps DSL system that uses conventional metal line. When your DSL modem is broken, you have two options: replace it with new DSL modem; or invest in 100 M bps fiber-optic cable and modem in order to use the new broadband technology. Network cables have smaller physical rate of depreciation compared to modems, network is compatible with modems, and they are technology specific.[30] In this way, networks and communication equipment can be considered as long-lived and short-lived capital respectively.

5 Conclusion

This paper studies a model in which production function consists of long-lived and short-lived vintage-specific compatible capital. Both types of investment are irreversible. The model predicts two distinctive investment patterns: (i) if the rate of technological progress is above a threshold, then all new investment concentrates on the capital types that embody the frontier technology; otherwise, (ii) a part of the investment is allocated to obsolete short-lived capital to exploit existing obsolete long-lived capital. Intangible capital such as know-how, software, and system capital, structures and networks can be long-lived capital depending on context. The short-lived capital is probably equipment in many cases.

As a consequence of the neo-classical assumptions of the model, the model not only comprehends existing vintage growth models, but also provides original quantita-

[29]The lives of "railroad replacement tracks" and "other railroad structures" are 38 and 54 years respectively, which is substantially longer than that of "railroad equipment", 28 years in Fraumeni (1997).

[30]The lives of "communication" and "communication equipment" are 11 and 40 years respectively in Fraumeni (1997).

tive implications: relative intensity of investment in old technology; and relationship between depreciation rate and obsolescence of equipment. Two empirical analyses and other empirical relevancies support the model's predictions with some additional but reasonable assumptions. Model with capital heterogeneity provides a rich set of explanations for several economic observations that have not been well studied, suggesting that economists should pay closer attention to capital heterogeneity.

Avenues for future research will consist of both theoretical and empirical work. Theoretically important applications should include characterizing transition dynamics and generalizing production function. Transition dynamics of the model expands its applicability to broader practice in the real economy. Generalization of the production function (e.g., to CES) improves the promises of the model.

Empirical applications include econometric analyses of growth accounting, investment patterns across vintages, and obsolescence and depreciation, across countries, industries, firms, and types of equipment. For these empirical analysis, it is indispensable to properly separate physical depreciation from obsolescence, and to identify the long-lived capital. These analyses that explicitly consider capital heterogeneity between physical and intangible capital should provide better picture of policy implications of economic growth and investment patterns in modern knowledge economy.

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A Appendix

A.1 Proofs

A.1.1 Proposition 1 (Investment patterns across vintages of technology)

Suppose investment schemes are (d) $\forall\, v \in [0,t]$, which requires $P_v^A = P_v^B = 1\,\forall\, v \in [0,t]$. Then, the right hand side of (16) is unity, which cannot be true when technological progress is positive. Therefore, investment scheme cannot be (d) in a BGP.

Next, suppose investment schemes are (a) $\forall\, v \in [0,t]$, which requires $P_v^A = 1\,\forall\, v \in [0,t]$. Then from (3) and (4), $\widehat{\left[\frac{B_v}{A_v}\right]} = \widehat{\left[\frac{MPA_v}{MPB_v}\right]} = \hat{P_v^A} - \hat{P_v^B} = \hat{q}/\beta$ since R^Xs are constant in a BGP.[31] This requires disinvestment in A_v since $-[\delta^B - \delta^A] \leq 0 < \hat{q}/\beta$, which is not allowed by assumption of investment irreversibility.

Next, suppose investment scheme is (b) $\forall\, v \in [0,t]$. In this case, since A_t has the highest price among A capital with $P_t^A = 1$ and $\hat{P_v^A} = -\hat{q}/\alpha$ from (16), there will always be investment in the newest A_t and B_t. As the case of (a) above, $\widehat{\left[\frac{B_v}{A_v}\right]} = -\hat{q}/\alpha$. When $-[\delta^B - \delta^A] \geq -\hat{q}/\alpha$, there is no positive investment in B_v, which contradicts the definition of investment scheme (b). Therefore, in a BGP with $\hat{q} \geq \alpha(\delta^B - \delta^A)$, investment scheme must be (c) $\forall\, v \in [0,t]$.

Now, suppose investment scheme is (c) $\forall\, v \in [0,t]$. There is no investment in vintage capital and thus all investment should concentrate on the frontier capital types, A_t and B_t, which implies $P_t^A = P_t^B = 1$. Furthermore, observe that B_t/A_t is constant since $(\alpha/\beta)(B_t/A_t) = R_t^A/R_t^B$ from (3) and (4). But this is impossible when $\hat{q} < \alpha(\delta^B - \delta^A)$, because (15) implies that P_v^B exceeds one given $P_t^B = 1$ and constant A_t/B_t. Therefore, in a BGP with $\hat{q} < \alpha(\delta^B - \delta^A)$, investment scheme must be (b) $\forall\, v \in [0,t]$. ■

[31] Constant growth of r and $R_t^X = R^X$ and (6) impose constant r and R^X in a BGP.

A.1.2 Proposition 2 (Allocation of capital types in a BGP)

Relationship from Profit Maximizations Conditions: By canceling r from (3), (4), and (6), observe that

$$\left[\frac{\beta}{P_v^B B_v} - \frac{\alpha}{P_v^A A_v}\right] Y_v = [\delta^B - \hat{P_v^B}] - [\delta^A - \hat{P_v^A}]. \tag{24}$$

Using $Y/L = Y_t/L_t$, $A/L = A_t/L_t$, and $B/L = B_t/L_t$ from (7), (8), and (10), $P_t^A = P_t^B = 1$ and (12), and applying $v \to t$, rewrite (24) in units of effective labor as

$$\beta a^\alpha b^{\beta-1} - \alpha a^{\alpha-1} b^\beta = [\delta^B - \hat{P^B}] - [\delta^A - \hat{P^A}]. \tag{25}$$

Condition from Aggregate Laws of Motion: The laws of motion of the capital types of each vintage are

$$\begin{aligned} \dot{A}_v &= I_v^A - \delta^A A_v, \text{and} \quad (26)\\ \dot{B}_v &= I_v^B - \delta^B B_v. \quad (27)\end{aligned}$$

Since $P_t^A = P_t^B = 1$ in a BGP, using (13), (7) and (8) can be expressed as

$$\begin{aligned} A &= \int_0^t P_v^A A_v \, dv, \text{and} \quad (28)\\ B &= \int_0^t P_v^B B_v \, dv. \quad (29)\end{aligned}$$

Using (26) - (29), we obtain the laws of motion of aggregate capital,

$$\begin{aligned} \dot{A} &= \frac{\partial}{\partial t}\int_0^t P_v^A A_v \, dv \quad (30)\\ &= \int_0^t [P_v^A A_v][\hat{P_v^A} + \hat{A}_v] \, dv + A_t \\ &= \left[\hat{P^A} - \delta^A\right] A + \int_0^t I_v^A \, dv + A_t \\ &= \left[\hat{P^A} - \delta^A\right] A + I^A, \end{aligned}$$

and

$$\dot{B} = \left[\hat{P^B} - \delta^B\right] B + I^B. \tag{31}$$

The sum of the laws of motion, (30) and (31), in units of effective labor becomes

$$\dot{a} + \dot{b} = \sigma a^{\alpha} b^{\beta} - [\delta^A - \hat{P^A} + \hat{N}]a - [\delta^B - \hat{P^B} + \hat{N}]b. \tag{32}$$

Since A grows at a constant rate in a BGP by definition, (30) implies $\hat{I^A} = \hat{A}$. Similarly, $\hat{I^B} = \hat{B}$. Then, (17) implies $\hat{Y} = \hat{I^A} = \hat{I^B}$. Thus from (10),

$$\hat{A} = \hat{B} = \hat{Y} = \frac{\hat{q}}{1 - \alpha - \beta} + \hat{L} = \hat{N}. \tag{33}$$

Therefore, a and b are constant in a BGP.

Changes in Prices: When $\hat{q} < \alpha(\delta^B - \delta^A)$, because proposition 1 indicates that there always is investment in old B, (16) and proposition 1 provide

$$\hat{P^A} = -\frac{\hat{q}}{\alpha}, \text{ and } \hat{P^B} = 0. \tag{34}$$

When $\hat{q} \geq \alpha(\delta^B - \delta^A)$, since proposition 1 indicates that B_t/A_t is constant, and applying $v' \to t$, (14) and (15) provide

$$\hat{P^A} = -\frac{\hat{q} + \beta(\delta^B - \delta^A)}{\alpha + \beta}, \text{ and } \hat{P^B} = -\frac{\hat{q} - \alpha(\delta^B - \delta^A)}{\alpha + \beta}. \tag{35}$$

(25) and (32) can be expressed as (18) and (19) provided (34) and (35).

Uniqueness and Stability: In a Fast Case, the uniqueness and stability of the BGP can be easily confirmed by using the basic Solow model's approach with the relationship $a/b = \alpha/\beta$ from equation (18).

In a Slow Case, the relationship (25) can be expressed as

$$a = f(b). \tag{36}$$

Since (36) implies $\dot{a} = f'(b)\dot{b}$, (32) can be expressed as

$$\dot{b} = \frac{\sigma f(b)^{\alpha} b^{\beta} - [\delta^A - \hat{P^A} + \hat{N}]f(b) - [\delta^B - \hat{P^B} + \hat{N}]b}{f'(b) + 1}. \tag{37}$$

Clearly, $\dot{b} = 0$ when $b = 0$. Then, observe that the numerator of the right hand side of (37) can be expressed as $\frac{b^2}{a\beta - b\alpha}[\{\sigma(\delta^B - \hat{P^B} + \hat{P^A} - \delta^A) + (\delta^A - \hat{P^A} + \hat{N})\alpha - (\delta^B - \hat{P^B} + \hat{N})\beta\}\frac{a}{b} - (\delta^A - \hat{P^A} + \hat{N})(\frac{a}{b})^2\beta + (\delta^B - \hat{P^B} + \hat{N})\alpha]$. The inside of the square brackets is positive when $\left[\frac{a}{b}\right]_{b \to +0} = \frac{\alpha}{\beta}$ and negative when $\left[\frac{a}{b}\right]_{b \to \infty} \to \infty$. Since (37) is continuous and smooth, there is at least one set of $a^*$and b^* such that $\frac{a^*}{b^*} > \frac{\alpha}{\beta}$, $b^* > 0$ and the inside of the brackets is zero ($\dot{b} = 0$). At b^*, (18) implies $a^* > 0$ and $\dot{a} = 0$. Observe that the first series of the Taylor approximation of the summarized law of motion of capital (37) at b^* is $\dot{b} \approx \frac{(\alpha+\beta-1)\{\beta(\delta^A - \hat{P^A} + \hat{N})(a^*/b^*) + \alpha(\delta^B - \hat{P^B} + \hat{N})(b^*/a^*)\}}{2\alpha\beta + \beta(1-\beta)(a^*/b^*) + \alpha(1-\alpha)(b^*/a^*)}(b - b^*)$, where the coefficient is negative. Therefore, at a^* and b^*, the economy is stable and $b^* > 0$ will be a unique solution. ■