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Abstract

Many economic and financial time series show evidence of trending behavior or non stationarity in the mean. An important econometric goal is determining the most proper form of the trend in the data. The transformations of series depend on whether the series is trend stationary or difference stationary. In this paper, study is conducted to declare the nature of trend component in quarterly GDP of Pakistan whether it is trend stationary or difference stationary. It is necessary to know, because trend stationary and difference stationary models imply very different short run and long run dynamics. We have explored the type of trend in GDP series by ADF unit root test and also support our arguments by empirical distribution instead of asymptotical ones i.e., bootstrapping test. The purpose of the paper is not only to investigate the type of trend in the series by conventional methods but also to motivate small distribution theory like bootstrapping techniques that can helps ones in selection of advocate model for observed series.

Key words: Bootstrapping, Stationarity, Pivotal statistic, Unit root.

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1. Introduction

In time series analysis we do not confine ourselves to the analysis of stationary time series. In fact, most of the time series we encounter are non-stationary. Two commonly used methods for removing trend are detrending and differencing. The issue of whether the time series is trend stationary or difference stationary has both economic and statistical consequences. Since the seminal paper by Nelson and Plosser (1982) economists know that modeling the long run behavior by deterministic trend or by stochastic trend has far reaching consequences for the economic interpretation. Traditional econometrics assumes stationary variables (i.e., constant mean and time independent autocorrelations). This is one of the reasons why applied economists very often transform non stationary variables into stationary time series.

Eliminating the non stationarity in a trend stationary model by taking first difference one gets rid of the linear trend but the stationary stochastic part is over differenced and spurious short run cycles are introduced. If on the other hand, it is tried to eliminate the non-stationarity in difference stationary model by taking the residuals of a regression on a constant and on time as explanatory variable, spurious long run cycles are introduced. Moreover, regressing independent difference stationary processes on each other leads to the problem of spurious regression as Granger and Newbold (1974) have demonstrated in a simulation study. Engle and Granger (1987) offered a solution to the spurious regression problem by introducing the concept of co-integration.

The above discussion clearly indicates that the analysis of non-stationary time series requires a serious investigation of the trending behavior, so it is compulsory to recognize behavior of trend existing in a time series data.

Our objective in this paper is to recognize the behavior of GDP series for Pakistan because in almost all the macroeconomic issue one of the variables is the output of the economy which is measured by GDP. Most of the times researchers have used asymptotic theory based on Augmented Dickey Fuller (ADF) test and determine the status of the GDP series whether to detrend or difference it. It is well know that these asymptotic tests have very low power and often give biased results, therefore, we have used finite sample evidence to determine the nature of trend in the GDP series for Pakistan.

The contribution of our study is that we have used bootstrap simulation evidence to determine whether series is trend stationary or difference stationary. Bootstrap is a statistical technique that estimates the distribution of an estimator or test statistic by resampling the data. One reason to use the bootstrap instead of asymptotic inference is that often if a statistic is asymptotically pivotal, that is if the limiting distribution is free of nuisance parameters, the bootstrap offers asymptotic refinements, meaning that the gap between the true distribution of a statistics and the bootstrap distribution declines fasters as the sample size increases than the gap between the true distribution and asymptotic distribution. For hypothesis testing this means that size of bootstrap test will be closer to nominal level than that of asymptotic test.

GDP data in Pakistan are available on the annual basis. Arby and Kemal (2004) have got the quarterly estimates for GDP of Pakistan series on the basis of the direct observation of relevant accounting items. They have followed the same approach as FBS in Pakistan has recently used in preparation of quarterly estimates of nation income accounts. Annual data for year 1993-94 and 2002-03 is taken from the Statistical Year Book while some earlier values are also taken from 50 Years of Pakistan Statistics, both published by FBS. We have used quarterly GDP of Pakistan series made by Arby and Kemal (2004) in their work. All the following analysis on Quarterly GDP series ranges over 1971-III-2003-II period of time are run in R.2.4.1 Package.

In section two we have used usual asymptotic theory based on ADF test to find the nature of trend existing in the GDP data. We have carried out bootstrap simulation analysis in the next section and finally we draw conclusions.

2. Unit Root Testing Strategy.

Nelson and Plosser (1982) started with problem of discrimination between trend-stationary process (TSP) and difference stationary process (DSP) by ADF test. In ADF unit root test we can estimate the series in one of the following form.

$$\vartriangle y_t = \rho^* y_{t-1} + \sum_{j=2}^{p} \beta_j \vartriangle y_{t-j+1} + \varepsilon_t. \qquad (2.1)$$

$$\vartriangle y_t = a_0 + \rho^* y_{t-1} + \sum_{j=2}^{p} \beta_j \vartriangle y_{t-j+1} + \varepsilon_t. \qquad (2.2)$$

$$\vartriangle y_t = a_0 + \rho^* y_{t-1} + a_2 t + \sum_{j=2}^{p} \beta_j \vartriangle y_{t-j+1} + \varepsilon_t. \qquad (2.3)$$

As true data generating process is unknown so the question is whether we should estimate (2.1), (2.2) or (2.3).The purpose of this discussion is that the existence of additional estimated parameters reduces the power of the test. The second problem is that the appropriate test statistic distributions in ADF test depend upon regressors included in the model so point of discussion here is that it is important to use a regression equation that mimics the actual data generating process. Detail on this strategy is available in Enders (1995, p.217).

We analyze our data set ranges over the 1971:III-2003:II period of time. Let y_t denote the log of GDP of Pakistani series. We start with the least restrictive model which includes a trend and drift i.e.

$$\vartriangle y_t = a_0 + r y_{t-1} + a_2 t + \sum_{i=2}^{6} \beta_i \vartriangle y_{t-i+1} + \varepsilon_t \qquad (2.4)$$

and use the $\tau - statistic$ to test the following null hypothesis

$H_0 : r = 0 \;\; vs \;\; H_1 : r < 0$.

We have used 5 lags in order to overcome the problem of seasonality. The point estimates of (2.4) suggest that series is difference stationary. The $\tau - statistic$ for the hypothesis $r = 0$ is -0.0481 which is less in absolute term than the reported tabulated values at the conventional levels, hence we can not reject the null hypothesis of the unit root and conclude that series is difference stationary. As the power of the test may be reduced due to the presence of unnecessary components so we test the presence of trend component given that series has unit root. This can be done with ϕ_3.Since the calculated value of the ϕ_3 is 2.2704 which is less than the value reported at 1%, 5% and 10% so trend component is not significant. This means that restriction $a_2 = r = 0$ is not binding, so we can eliminate the trend component at this stage. Now we will run the equation (1.2) without trend component

$$\triangle y_t = a_0 + ry_{t-1} + \sum_{i=2}^{6} \beta_i \triangle y_{t-i+1} + \varepsilon_t \qquad (2.5)$$

Again the inclusion of the lags ensures that errors are white noise. We test the hypothesis

$H_0 : r = 0 \quad vs \quad H_1 : r < 0$.

Since in (2.5), the $\tau - statistic$ for the null hypothesis $r = 0$ is -2.1289 thus the null hypothesis of the unit root is not rejected at 1%,5% and 10% tabulated values .Again in order to check that either we should include the intercept term in the model or not. We have to check the significant of the intercept term in the model. For this purpose we have to test null hypothesis joint restriction $a_0 = r = 0$ and this can be done with ϕ_1-statistic. Since the sample value of ϕ_1-statistic for above describe restriction is 15.8582 which implies that restriction is binding and we can not eliminate intercept term from the model. Hence Pakistani GDP series is difference stationary.

3. Evidence from bootstrapping methodology

We can also investigate the type of trend component by bootstrap methodology There are two ways to bootstrap the time series models, one is based on resampling of residuals, called residual-based method and other called direct-method, base on resampling of pairs (x_i, y_i) in the model. Both methods have their own merits and demerits. We have computed a realization of $\hat{\tau}$ using the real data for model (2.4) under the null hypothesis of unit root is found to be -0.048.Now we compute B independent bootstrap test statistics $\tau_j^* : i = 1, 2 ... T$ (Where T in our case in 5000) using the real data and DGP given in (2.4) under H_0. As we wish to perform the test at conventional level of 5% so we will reject the null hypothesis when the value of $\hat{\tau}$ is unusually large. Given the actual and simulated test statistic we can compute the bootstrap p-value as

$$P^*\left(\hat{\tau}\right) = \frac{1}{T} \sum_{i=1}^{T} I\left(\tau_i^* < \hat{\tau}\right)$$

Where $I(.)$ an indicator function with value 1 if the condition is true and 0 otherwise. Actually $P^*(\hat{\tau})$ is just the fraction of bootstrap samples for which τ_j^* smaller than $\hat{\tau}$ .If this fraction is smaller than our chosen level then we will reject the null hypothesis of unit root.

This makes sense, since $\hat{\tau}$ is extreme relative to empirical distribution of τ_j^* when $P^*(\hat{\tau})$ is small. When we perform this bootstrap to our GDP series for DGP (2.4) under H_0 we observe that

$$P^*(\hat{\tau}) = 0.462$$

The shape of distribution of τ_j^* under H_0 is shown in Figure 1:

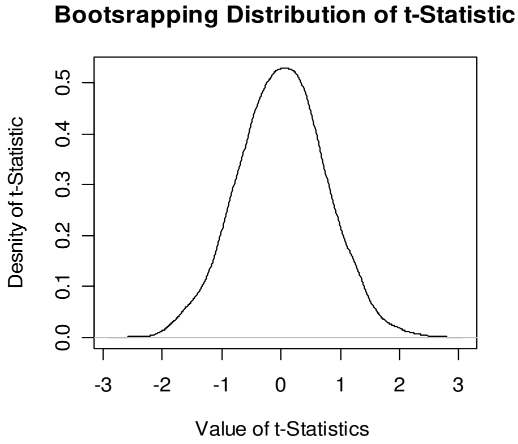

Figure 1

Since the value of $P^*(\hat{\tau})$ is large at our chosen conventional level of 5% which support our null hypothesis of unit root. Hence our series is difference stationary.

4.Multiple Unit Root Analysis.

We have also use Dickey and Pantula (1987) procedure which is simple and extension of basic procedure of unit root testing if more than one unit roots are suspected. From this analysis, it has been cleared that Pakistani GDP series is not integrated of higher order and it is integrated of order one.(For further detail contact to the authors.)

5. Concluding Remarks

In our analysis we have explored that our series belong to DSP and it is integrated of order one by formal testing Procedure called ADF unit root test. The use of critical values based on the strong assumptions of the simple Dickey Fuller model was also seen to be limitation when we consider the distribution of ADF test statistic. This shows that bootstrapping method will be more applicable when using the ADF test of unit root.

So in a gist we have analyzed that both ADF test (asymptotic theory) and finite sample theory (Bootstrap Simulation) indicate that the GDP series belong to DSP and it is integrated of order one. Our results have very strong implications in the sense that the GDP series is used almost in all macro econometric models e.g export-growth relationship, saving-growth relationship, FDI-growth relationship money-output relationship etc.

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