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12

Exogenous Growth Under International Capital and Labor Mobility

The world economy is characterized by diverse levels of income and patterns of growth across countries and across time. To understand the evolution of such diversity and whether there are tendencies toward cross-country equality, we have to study the mechanics of the growth process in each country individually and in the world equilibrium at large. In order to provide a systematic analysis of such process, we devote this chapter to a simplified world where the engine of growth is exogenously determined. As a natural extension, this growth engine will be endogenized in the next chapter.

Evidently, factor mobility could potentially influence the convergence/ divergence tendencies across countries within a growing world economy. We therefore focus our analysis on the roles that capital mobility and labor mobility can play in such a process. In addition, we examine the potential welfare gains that capital and labor mobility can generate. Also studied are the effects of factor mobility on the speed of adjustment to long run equilibria and on the rate of convergence (if any) across countries.

12.1 The Closed Economy

In this section, we provide an exposition of the standard growth model that emphasizes the role of saving and physical capital accumulation in driving short run output growth and that of (exogenous) human capital accumulation in driving long run growth.[1]

Consider a closed economy with a Cobb-Douglas technology that transforms physical capital (K_t) and human capital (H_t) into a single composite good (Y_t):

$$Y_t = A_t K_t^{1-\alpha} H_t^{\alpha} . \tag{12.1}$$

The productivity level A_t is, in general, time-varying and can be a source of growth and fluctuations. To make things simple at this stage, we assume that $A_t = A$, a deterministic constant. In Section 12.4 below, we will assume it to follow a stochastic process in order to analyze growth under uncertainty.[2] Human capital H_t is a product of the size of the labor force N_t (treated synonymously with the size of population, assuming constant labor force participation and inelastic work hours) and their skill level (h_t). An important feature of this model is constant returns to scale technology.[3] While increasing returns to scale will give rise to unbounded growth (which is thus inconsistent with the existence of a stable long run equilibrium), decreasing returns to scale cannot generate sustainable long run growth. [See Appendix A.]

Physical and human capital are accumulated according to the following laws of motion respectively:

$$K_{t+1} = I_t + (1-\delta_k) K_t . \tag{12.2}$$

where I_t is gross investment in, and δ_k the rate of depreciation of, physical capital.

$$H_{t+1} = (1+g_H) H_t . \tag{12.3}$$

where g_H is the exogenous rate of growth of human capital. Since $H_t = N_t h_t$ and both N_t and h_t are assumed to grow at exogenous constant rates g_N and g_h, we can decompose (13.3) into $N_{t+1} = (1+g_N)N_t$ and $h_{t+1} = (1+g_h)h_t$, with $(1+g_N)(1+g_h) = 1+g_H$.

The representative consumer is the owner of both physical and human capital. He/she accumulates these two forms of capital over time, and supplies them as factor inputs to the representative firm for production at the competitive wage (w_t) and rental (r_{kt}) rates, period by period. To derive its demand for physical capital (K_t^d) and human capital (H_t^d), the firm in turn solves a static profit maximization problem[4]:

$$Max_{\{K_t^d, H_t^d\}} \quad A(K_t^d)^{1-\alpha}(H_t^d)^{\alpha} - w_t H_t^d - r_{kt} K_t^d$$

This yields the standard marginal productivity-factor price relations:

$$w_t = \alpha A x_t^{1-\alpha} \quad \text{and} \quad r_{kt} = (1-\alpha) A x_t^{-\alpha}, \tag{12.4}$$

where x_t is the ratio of physical capital to human capital (K_t/H_t).

Preferences of the dynastic head of the family are given by an isoelastic utility function:

$$U = \sum_{t=0}^{\infty} \beta^t N_t \left(\frac{c_t^{1-\sigma} - 1}{1-\sigma} \right), \tag{12.5}$$

where $0 < \beta < 1$ is the subjective discount factor, and $\sigma > 0$ the reciprocal of the intertemporal elasticity of substitution in consumption (c_t). In the limiting case where $\sigma = 1$, the utility function is logarithmic. The household derives wage income from the supply of human capital ($w_t H_t$) and rental income from the supply of physical capital ($r_{kt} K_t$). He/she can either consume ($N_t c_t$) or save these incomes in the form of physical capital ($K_{t+1} - (1-\delta_k)K_t$). He/she can also borrow B_{t+1} at the rate of interest r_{t+1} in any period t, and repay $(1+r_t)B_t$ for his/her borrowing B_t from the previous period $t-1$.[5] The household's net saving is therefore given by $[K_{t+1} - (1-\delta_k)K_t] - [B_{t+1} - (1+r_t)B_t]$. His/her period-t budget constraint can thus be written as

$$[K_{t+1} - (1-\delta_k)K_t] = w_t H_t + r_{kt}K_t + [B_{t+1} - (1+r_t \tag{12.6}$$

The consumer chooses $\{c_t, K_{t+1}, B_{t+1}\}$ to maximize (12.5) subject to (12.2), (12.3), and (12.6). The first order conditions for this problem imply:

$$R_{B_{t+1}} = \frac{1}{\beta}\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} = R_{k_{t+1}}, \tag{12.7}$$

where $R_{Bt+1} = 1+r_{t+1}$ and $R_{kt+1} = 1+r_{kt+1} - \delta_k$. Recall that, as in Chapter 5, in making his borrowing decision, the consumer equates the gross rate of interest (R_{Bt+1}) to his/her intertemporal marginal rate of substitution (IMRS). In addition, as an owner of physical capital, he/she also equates the return on physical capital (R_{kt+1}) to the same IMRS. Arbitrage through capital market forces will drive equality between the interest rate and rental rate on capital (net of depreciation, i.e., $r_{t+1} = r_{kt+1} - \delta_k$.

The equilibrium values of the wage rate, rental rate, and interest rate are determined by the following market clearing conditions: $H_t = H_t^d$ in the labor market, $K_t = K_t^d$ in the physical capital market, and $B_{t+1} = 0$ in the financial capital market.[6] By the Walras law, the consumer budget constraint (12.6), the profit-maximizing conditions (12.4), and these market clearing conditions together imply the economy-wide resource constraint:

$$N_t c_t + K_{t+1} - (1-\delta_k)K_t = F(K_t, H_t). \tag{12.8}$$

That is, output is split between consumption and saving/investment.

The Mechanics of Economic Growth

The dynamics of our example economy are driven by the two fundamental laws of motion (12.2) and (12.3), which can be combined to yield a single difference equation in the physical capital-human capital ratio (or the capitals ratio, X_t):

$$X_{t+1} = \left(\frac{A}{1+g_H}\right) s_t X_t^{1-\alpha} + \left(\frac{1-\delta_k}{1+g_H}\right) X_t, \tag{12.9}$$

where s_t is the saving rate at time t, defined as the ratio between saving S_t (gross and net) and output Y_t (= I_t/Y_t since $S_t = I_t$ in the closed economy equilibrium). In general, the equilibrium saving rate is determined by solving the consumer's intertemporal optimization problem and imposing the market clearing conditions. We spell out the dynamics of the system in Appendix B, and illustrate how the saving rate is derived in an example economy in Appendix C.

We can express per capita output as y_t (= Y_t/N_t) = $A(X_t)^{1-\alpha}h_t$, where $h_t = h_0(1+g_h)^t$. Evidently, the dynamic path of y_t depends on that of X_t and h_t. In particular, the growth rate of y_t (g_{yt}) can be approximated (using $\ln(1+g) = g$) by

$$\begin{aligned}
\ln\left(\frac{y_{t+1}}{y_t}\right) &= (1-\alpha)\ln\left(\frac{X_{t+1}}{X_t}\right) + g_h \\
&= (1-\alpha)\ln\left[\left(\frac{A}{1+g_H}\right) s_t X_t^{-\alpha} + \frac{1-\delta_k}{1+g_H}\right] + g_h \\
&= (1-\alpha)\{\ln[A s_t X_t^{-\alpha} + (1-\delta_k)] - g_N\} + \alpha g_h .
\end{aligned} \tag{12.10}$$

Observe that g_{yt} depends positively on the saving rate (s_t) and the skill growth rate (g_h), and negatively on the capitals ratio (X_t) and the population growth rate (g_N). If, however, the economy converges to a long run time-invariant equilibrium where $X_{t+1} = X_t$ so that $\ln(X_{t+1}/X_t) = 0$, then g_y will converge to g_h.

Steady State Growth and Transitional Dynamics: Policy Implications

While equation (12.9) describes the entire growth path of the capitals ratio X_t, it can conveniently be divided into two components: the transition (to steady state) path and the steady state growth path. *Steady state* growth is defined as the particular pattern of growth where growth rates of all variables are constant over time, but possibly different from one another. (It is sometimes called `*balanced* growth' as well.) The constant returns to scale assumption, combined with isoelastic preferences (which implies equality between the average and marginal propensities to consume), are necessary for the existence of such long run equilibrium. With $X_t = X_{t+1} = \hat{X}$ and $s_t = s_{t+1} = \hat{s}$ in the steady state, the capitals ratio and the level of per capita output can be expressed as (positive) functions of the saving rate as follows:

$$\hat{X} = \left(\frac{sA}{g_H+\delta_k}\right)^{1/\alpha} \quad \text{and} \quad \hat{y}_t = \left(\frac{s}{g_H+\delta_k}\right)^{\frac{1-\alpha}{\alpha}} A^{\frac{1}{\alpha}} h_0 (1+g_h)^t .$$

The capital-(raw) labor ratio $\hat{k}_t$ (= K_t/N_t) is equal to $\hat{X}h_t = \hat{X}h_0(1+g_h)^t$. Observe that $\hat{y}_t$ and $\hat{k}_t$ both grow at the same rate g_h along the steady state growth path, implying that the capital-output ratio must be constant. Since the rate of return on capital ($\hat{r}_{kt}$) is given by $(1-\alpha)\hat{y}_t/\hat{k}_t$, it must also be constant. Indeed, these steady state properties of $\hat{y}_t$, $\hat{k}_t$, and $\hat{r}_{kt}$ are all consistent with a set of empirical regularities about patterns of growth established by Kaldor (1963).

Another component of the growth path exhibits transitional dynamics described by equation (12.9) for $X_t \neq X_{t+1} \neq \hat{X}$ and portrayed by Figure 12.1. For any initial stocks of physical and human capital (hence X_0), the capitals ratio in the next period (X_1) can be read off the concave curve for the corresponding equilibrium value of the saving rate (s_0, say s'). Given X_1, the capitals ratio in the period after (X_2) can be obtained from a point on the concave curve correspending to the equilibrium saving rate in period 1 (s_1, say, s"). Eventually, the capitals ratio will converge to the steady state at the intersection between the 45^0 line and the concave curve with constant saving rate $\hat{s}$ (point A). As shown in the example in Appendix C, the Solow (1956) model is a special case of our model (when $F = *_k = 1$) where the saving rate is time-invariant (say, $\hat{s}$). In this case, for $X_t < \hat{X}$, $X_{t+1} > X_t$ (as indicated by the arrows pointing to the right); and for $X_t > \hat{X}$, $X_{t+1} < X_t$ (as indicated by the leftward pointing arrows).

[insert Figure 12.1 here]

In Appendix B, we derive two measures of the speed of convergence to the long run equilibrium. The first one measures the half-life of the dynamical system, i.e., the time it takes to reach half of the distance between the initial position and the steady state. The second one, analogous to Barro and Sala-i-Martin (1992), measures the ratio between the deviations of output from its steady state value in two consecutive periods.

[Explain dependence of $(1+ g_h)8$ on underlying parameters.]

From the arguments just presented, it should now be evident that, in the exogenous growth paradigm, policies which target saving rates (such as taxation of capital income) can influence growth only along the transition path, but not in the long run steady state equilibrium. They can nonetheless affect the long run level of output per capita. In other words, these policies only have *level* effects, but no *growth* effects.

12.2 The Open Economy: Capital Mobility

From a global perspective, there is an issue as to whether countries with different levels of initial income will converge (with income gaps narrowing) over time. We have just shown that income convergence can occur even among isolated economies if the long run (capital account autarky) steady state positions they converge to are similar. This entails similar technologies (A, ", and $*_k$) and preferences ($ and F), and most importantly similar exogenous growth rates (g_H, or g_N and g_h).

When the economies are open, it is more likely that such converegence will take place because openness can potentially enhance technology transmission. Moreover, when capital markets are integrated into the world capital market, the process of convergence (its speed as well as the particular transition path) will likely be altered. International borrowing and lending can facilitate consumption smoothing and create additional investment opportunities, which will improve the efficiency in the allocation of consumption over time and the allocation of capital across countries. For these reasons, the capital market integration may generate welfare gains as well. In this section, therefore, we introduce capital mobility in this section to study its role for the growth process and for international convergence of income levels.

Perfect Capital Mobility

Consider integration of our example economy into the world capital market. Assume free trade in commodities. Since we have a single composite good here, although a country either exports or imports (but will not do both) within any given period, it can definitely engage in intertemporal trade in this single commodity (exporting in some periods and importing in others). This implies mobility in financial capital (in the form of various financial securities). Naturally, trade in goods can occur in the form of physical capital as well. In other words, physical capital also becomes internationally mobile.

Accordingly, we modify the consumer budget constraint (12.6) as follows:

$$
\begin{aligned}
& N_t c_t + (K^H_{t+1} + K^{H*}_{t+1}) - (1-\delta_k)(K^H_t + K^{H*}_t) \\
& {}^H + r^*_{kt} K^{H*}_t + (B^H_{t+1} + B^{H*}_{t+1} - B^{*H}_{t+1}) - (1+r_t)(B^H_t + B^{*H}_t) \ldots
\end{aligned}
\tag{12.11}
$$

K^H and B^H denote the domestic consumer's claims on physical capital residing in the home country and domestic debt respectively. Likewise, K^{H*} and B^{H*} stand for the domestic consumer's claims on physical capital residing in the foreign country and foreign debt. The domestic rate of interest and rental rate are denoted by r and r_k, and their foreign counterparts by r^* and r^*_k respectively.

Similarly, the resource constraint of the domestic economy (12.8) will have to be modified.

$$
\begin{aligned}
& N_t c_t + (K^H_{t+1} + K^{H*}_{t+1}) - (1-\delta_k)(K^H_t + K^{H*}_t) \\
& + r^*_{kt} K^{H*}_t - r_{kt} K^{*H}_t + (B^{H*}_{t+1} - B^{*H}_{t+1}) - [(1+r^*_t) B^{H*}_t - (\ldots
\end{aligned}
\tag{12.12}
$$

where K = K^H + K^{*H}, the latter being the stock of foreign direct investment in the domestic economy.

We can rearrange terms in (12.12) in order to derive the country's balance of payments accounts CA_t + KA_t = 0 as follows:

$$
\begin{aligned}
CA_t &= GNP_t - N_t C_t - I^H_t \\
& [F(K_t, H_t) + (r_t B^{*H} - r^*_t B^{H*}_t) + (r^*_{kt} K^{H*}_t - r_{kt} K^{*H}_t)] - N_t C_t - [K^H_{t+1} - (1-\delta_k) K^H_t], \ a\ldots \\
KA_t &= -(FDI_t + FPI_t) \\
&= -[K^{H*}_{t+1} - (1-\delta_k) K^{H*}_t] + [(B^{*H}_{t+1} - B^{*H}_t) - (B^{H*}_{t+1} - B^{H*}_t)].
\end{aligned}
$$

Here, CA stands for the current account balance, KA the capital account balance, FDI foreign direct investment, and FPI foreign portfolio investment.

In this deterministic setting, capital flows will be unidirectional, the direction being determined by the relative magnitudes of the domestic and foreign rates of interest. Furthermore, since $r = r_k - \delta_k$ and $r^* = r_k^* - \delta_k$ in equilibrium, physical capital and financial capital are perfect substitutes. Assume without loss of generality that $r > r^*$, so $r_k > r_k^*$. Then, $B^{*H} = 0$ and $K^{H*} = 0$.

To highlight the role of capital mobility for international convergence of per capita output levels, we assume that the rest of the world is initially in a long run steady state growth equilibrium of the sort studied in the previous section, whereas the home economy starts initially with a lower physical capital-human capital ratio ($X_0 < X_0^* = \hat{X}^*$).

Assume further that the rest of the world is large so that the home country is a price-taker in the world market. Capital market integration then ensures that $r_t = r^*$ (= $(1-\alpha)A(\hat{X}^*)^{-\alpha} - \delta_k$ from (13.4)). As a result, the capital inflow will generate an immediate convergence of the capitals ratio ($X_1 = \hat{X}^*$) and hence convergence of per capita GDPs ($y_t = \hat{y}_t^*$). From (13.7), one can verify that the consumption growth rate will converge at the same time to its long run value, g_h. Per capita GNPs and consumption will not converge, however, because the capital inflow effectively equalizes labor income between the home country and the rest of the world, while the former starts with a lower K_0. Evidently, the home economy reaps all the benefits from the intertemporal trade — since prices in the rest of the world remain unchanged while, for the domestic economy, the initial marginal productivity of capital net of depreciation exceeds the world rate of interest. Therefore, the consumption ratio c_t/c_t^* will initially rise, and then stay constant thereafter (given equality between their consumption growth rates).[7]

Free capital flows must generate welfare gains from intertemporal trade, akin to the standard gains from trade argument. Obviously, the magnitude of the gains is directly related to the difference in the initial capitals ratios between the home country and the rest of the world. (Recall that this is assumed to be the only source of heterogeneity in our analysis.) In Appendix D, we compute such gains and relate them to the initial cross-country difference. These gains are measured in utility terms. An alternative welfare concept ω, measured in terms of compensating variations in consumption, is defined implicitly by

$$\sum_{t=0}^{\infty} \beta^t U(c_t^{autarky}(1+\omega)) = \sum_{t=0}^{\infty} \beta^t U(c_t^{kmobility}) .$$

[calculate numerical values of ω as a function of $Y_0/\hat{Y}_0$]

Constrained Capital Mobility

Free capital mobility is an extreme situation based on ideal market structure with full information and absence of risk, default, and time inconsistent behavior (both for private agents and governments). When such elements are taken into account, borrowing and lending may be subject to significant constraints and regulations. To account for these possibilities in our aggregative model, we now introduce an upper bound on financial capital flows. No such bound is imposed, however, on foreign direct investment.

We assume that households (in their capacity as owners of the firms) can only borrow up to a certain limit. The limit is the collateral based on the ownership by the domestic households of the domestic firm's net worth, which is equal to K_t^H.[8] Recall, however, that an increase in domestic consumption can come from three sources: international borrowing, domestic output net of returns to FDI, and reduction in investment by domestic residents in domestic capital.

Therefore, if borrowing from abroad is constrained, domestic consumers can still offset the effect of this constraint by reducing their investment in domestic capital. Only when a lower bound on that form of investment is imposed will the consumption boom following the open-up of the international capital market become constrained. We therefore make the realistic assumption that gross investment of this kind be non-negative. (Effectively, this constraint will eliminate the possibility of using the domestic firm's capital stock, which is partly owned by foreigners, as an additional collateral for borrowing from abroad to finance domestic consumption.) If binding, these two constraints together with the resource constraint (12.12) will imply

$$ _t \;'\; F(K_t, H_t) \;\&\; r_{kt}K_t^{(H} \;\&\; r_{kt}^{(}K_t^{H} \;'\; w_tH_t \;\%\; (r_{kt}\&r_{kt}^{(})\,l \qquad (12.12)$$

At the same time, the domestic rate of interest will not be set equal to the world rate of interest because of the borrowing constraint. From the intertemporal condition (12.7), we have $c_{t+1}/c_t > [\$(1+r^*)]^{1/F}$. In comparison to the free capital mobility case, therefore, while the jump in consumption immediately following the open-up of international capital markets is smaller, the consumption growth rate is higher as long as the constraints remain binding. Thus, while the borrowing constraint lengthens the transitional dynamics, it generates a more accelerated rate of growth in consumption and income during that phase compared to the long run growth rate. Eventually, the constraints will become slack, and the growth rate will converge to the same long run value, g_h.

This is a stylized example of real world constraints that tend to slow down the adjustment process. The welfare gains from the opening up of credit markets will evidently be lower than those under free capital mobility.

12.3 Open Economy: Labor Mobility

Since both labor and physical capital are primary inputs in production, labor mobility is expected to play a similar role in the growth process as capital mobility. The skill levels are typically embodied in labor. Consequently, we have international transfer of skills and human capital as an integral part of labor mobility.

We now open up the home country to free labor flows. For simplicity, we assume that capital (financial as well as physical) is immobile internationally. As before, the rest of the world is assumed to be in the long run steady state initially, whereas the home economy starts with a lower physical capital-human capital ratio ($X_0 < X_0^* = \hat{X}^*$). This implies that $w_0 < w_0^* = \hat{w}^*$. We retain the assumption that the rest of the world is large so that the home country is a price-taker in the world market. Labor market integration then ensures that $w_t = \hat{w}^*$ ($= \alpha A(\hat{X}^*)^{1-\alpha}$ from (12.4)). As a result of the labor outflow, we obtain an immediate convergence of the capitals ratio ($X_1 = \hat{X}^*$) and hence convergence of per capita GDPs ($y_1 = \hat{y}^*$). Again, the consumption growth rate will converge at the same time to its long run value, g_h because wage rate equalization implies interest rate equalization. Per capita GNPs and consumption will not converge, however, because the home country starts out with a lower K_0, hence a lower initial wealth. Evidently, the home economy reap all the benefits from the intertemporal trade since prices in the rest of the world remain unchanged while, for the domestic economy, the initial marginal productivity of labor falls short of the world wage rate. Therefore, the consumption ratio c_t/c_t^* will initially rise, and then stay constant thereafter (given equality between their consumption growth rates). What we have just shown is that labor mobility is a perfect substitute for capital mobility, with the same welfare implications.

The assumption of free labor flows abstracts, however, from the significant costs of adjustment associated with such mobility (e.g., cultural, ethnic, family, and other differences). When such costs are accounted for, labor mobility becomes a less efficient convergence mechanism in comparison with capital mobility (even after incorporating the above-mentioned borrowing constraints).

12.4 Stochastic Growth

So far, we have been assuming that growth is deterministic. In order to highlight the effects of random shocks on the growth dynamics, we introduce in this section stochastic elements. To simplify the analysis, we revert to the closed economy setting developed in Section 12.1. The complexity in characterizing the stochastic dynamics excludes the possibility of an analytic inspection of the growth mechanism except for relatively low dimensional cases, one of which is elegantly analyzed by Campbell (1994) and is presented in this section. At the same time, however, we develop here tools of analysis that will be useful also for analyzing higher dimensional problems such as those in an open economy. These techniques will be further developed in Chapter 13.

To incorporate random disturbances into the model of Section 12.1, we assume that the productivity level a_t (= $\ln(A_t)$) follows a first order autoregressive process:

$$a_{t+1} = \rho a_t + \varepsilon_{t+1}, \tag{12.13}$$

where ε_{t+1} is an i.i.d. shock with zero mean and constant variance, and $0 \leq \rho \leq 1$ measures the persistence of the shock. In the presence of such shocks, the expectation operator will have to be inserted into equations (12.5) and (12.7).

Defining X = K/H and Z = c/h, we can rewrite the two dynamic equations (12.7) and (12.8) in K and c as (B.1) and (B.2) in Appendix B. Together with equation (12.13), these two laws of motion form a system of nonlinear expectational difference equations in (Z_t, X_t, A_t). To get a handle on the solution, we take a loglinear approximation of this nonlinear system around its deterministic steady state (with $\varepsilon_t = 0$) to obtain a linear system of expectation difference equations in x_t (= $\ln(X_t)$) and z_t = ($\ln(Z_t)$). Details of the solution procedure are laid out in Appendix E, from which we derive the approximate dynamic behavior of the economy as a function of the shock as follows:

$$a_t = \frac{1}{(1-\rho L)}\varepsilon_t . \tag{12.14}$$

$$x_{t+1} = \frac{\eta_{xa}}{(1-\eta_{xx}L)(1-\rho L)}\varepsilon_t . \tag{12.15}$$

$$z_t = \frac{\eta_{za} + (\eta_{zx}\eta_{xa} - \eta_{za}\eta_{xx})L}{(1-\eta_{xx}L)(1-\rho L)}\varepsilon_t . \tag{12.16}$$

$$\ln\left(\frac{y_t}{h_t}\right) = (1-\alpha)x_t + a_t = \frac{1 + [(1-\alpha)\eta_{xa} - \eta_{xx}]L}{(1-\eta_{xx}L)(1-\rho L)}\varepsilon_t . \tag{12.17}$$

Above, `L' denotes the lag operator, and η_{zx}, η_{za}, η_{xx}, and η_{xa} represent the partial elasticities of

z_t with respect to x_t, z_t with respect to a_t, x_{t+1} with respect to x_t, and x_{t+1} with respect to a_t respectively. The explicit expressions for these elasticities in terms of the fundamental parameters are spelled out in Appendix E. Observe from equations (12.15)–(12.17) that the logarithm of the ratio between physical capital and human capital (x_t) follows an AR(2) process, while the logarithm of the ratio between concumption and human capital (z_t) as well as the logaritm of the ratio between output and human capital ($\ln(y_t/h_t)$) follow an ARMA(2,1) process. Note also that they have identical autogressive roots η_{xx} and ρ.

We now flesh out some properties of the partial elasticities:

(1) η_{zx}, which measures the effect on current consumption of an increase in the capital stock for a fixed level of productivity, naturally does not depend on the persistence of the productivity shock ρ;

(2) η_{za}, which measures the effect on current consumption of a productivity increase with a fixed stock of physical capital, is increasing in ρ for low values of σ but decreasing for high values of σ. The inituition behind this is as follows. When substitution effects are weak (small σ), the consumer responds mostly to income effects, which are stronger the more persistent the shock. When substitution effects are strong (high σ), a persistent shock which raises the furture rate of interest will stimulate saving at the expense of current consumption, making the response in consumption small;

(3) η_{xx}, which measures the effect on the future level of capital stock of an increase in the current level of capital stock with a fixed level of productivity, does not depend on ρ but declines with σ. In fact, in our model of Section (12.1) where technology is time-invariant, $1-\eta_{xx}$ reflects the speed of convergence to the steady state, similar to our second measure of convergence on p.7.

We now turn to the general equilibrium implications of the persistence parameter D. Observe that if productivity follows a random walk ($D = 1$), we have $0_{zx} + 0_{za} = 1$ and $0_{xx} + 0_{xa} = 1$. Consequently, capital, consumption, and output will follow cointegrated random walk processes, so that the difference between any two of them is stationary.

Appendix A: Returns to Scale and Long Run Growth

Consider a Cobb-Douglas production function that allows for decreasing, increasing, and constant returns to scale.

$$Y_t = A K_t^{\beta} H_t^{\alpha} . \tag{A.1}$$

where $\beta + \alpha$ can be < 1, > 1, or $= 1$.

Following Solow (1956), assume for simplicity a constant saving rate s, so that investment (I_t) is equal to sY_t. The physical capital-human capital ratio (X_t) can be shown to evolve according to:

$$X_{t+1} = \left(\frac{s}{1+g_H} \right) A X_t^{\beta} H_t^{\alpha+\beta-1} + \left(\frac{1-\delta_K}{1+g_H} \right) X_t .$$

If $\alpha + \beta > 1$ (increasing returns), we will have unbounded growth in the long run. If $\alpha + \beta < 1$ (decreasing returns), the economy will vanish over time.

Appendix B: Local Dynamics of Our Example Economy

The dynamics of our example economy is governed by the system of difference equations (12.2), (12.3), and (12.7) in $\tilde{c}_t$, K_t, H_t, and A_t. For the sake of the analysis in Section 12.4, the productivity level A is allowed to be time-varying. Defining X_t as K_t/H_t as before and Z_t as c_t/h_t, we can combine (12.2) and (12.3) as one single difference equation in X_t.

$$(1+g_H)X_{t+1} = A_t X_t^{1-\alpha} + (1-\delta_k)X_t - Z_t. \tag{B.1}$$

Dividing (12.7) throughout by h_t yields a difference equation in Z_t.

$$Z_t^{-\sigma} = \beta E_t[(1+g_h)Z_{t+1}]^{-\sigma}[(1-\alpha)A_{t+1}X_{t+1}^{-\alpha} + (1-\delta_k)] \tag{B.2}$$

In the steady state, $A_t = A_{t+1} = A$, $X_t = X_{t+1} = \hat{X}$ and $Z_t = Z_{t+1} = \hat{Z}$. These steady state values are given by

$$\hat{X} = \left[\frac{(1-\alpha)A}{(1+g_h)^{\sigma}/\beta - (1-\delta_k)}\right]^{1/\alpha} \quad and \quad \hat{Z} = A\hat{X}^{1-\alpha} - (g_H+\delta_k)\hat{X}.$$

Log-linearizing (B.1) and (B.2) around the steady state $(\hat{Z},\hat{X})$ yields

$$\Rightarrow s_x(1+g_H)E_t\left(\frac{dX_{t+1}}{X_{t+1}}\right) - s_x(1-\delta_k)\left(\frac{dX_t}{X_t}\right) = (1-\alpha)\left(\frac{dX}{X}\right.$$
$$= \frac{\hat{X}^{\alpha}}{A} = \frac{1-\alpha}{r+\delta_k} \quad and \quad s_z = \frac{\hat{Z}}{\hat{Y}} = 1-(g_H+\delta_k)s_x = 1- \tag{B.3}$$

$$\left[E_t\left(\frac{dZ_{t+1}}{Z_{t+1}}\right) - \frac{dZ_t}{Z_t}\right] = \left(\frac{r+\delta_k}{1+r}\right) E_t\left[\left(\frac{dA_{t+1}}{A_{t+1}}\right) - \alpha\left(\frac{dX_{t+1}}{X_{t+1}}\right)\right] \tag{B.4}$$

We can group (B.3) and (B.4) into a matrix difference equation as follows:

$$A E_t\begin{pmatrix} \dfrac{dX_{t+1}}{X_{t+1}} \\ \dfrac{dZ_{t+1}}{Z_{t+1}} \end{pmatrix} = B\begin{pmatrix} \dfrac{dX_t}{X_t} \\ \dfrac{dZ_t}{Z_t} \end{pmatrix} + C E_t\left(\frac{dA_{t+1}}{A_{t+1}}\right) + D\left(\frac{dA_t}{A_t}\right), \tag{B.5}$$

where the matrices A, B, C, and D are given by

$$\begin{pmatrix} & 0 \\ \cdot & \Phi \end{pmatrix}, \quad B = \begin{pmatrix} (1-\alpha)+s_x(1-\delta_k) & -s_z \\ 0 & \Phi \end{pmatrix}, \quad C = \begin{pmatrix} 0 \\ \dfrac{r+\delta_k}{1+r} \end{pmatrix},$$

Equation (B.5) can be rewritten as:

$$E_t\begin{pmatrix} \dfrac{dX_{t+1}}{X_{t+1}} \\ \dfrac{dZ_{t+1}}{Z_{t+1}} \end{pmatrix} = W\begin{pmatrix} \dfrac{dX_t}{X_t} \\ \dfrac{dZ_t}{Z_t} \end{pmatrix} + R E_t\left(\frac{dA_{t+1}}{A_{t+1}}\right) + Q\left(\frac{dA_t}{A_t}\right), \tag{B.6}$$

where $W = A^{-1}B$, $R = A^{-1}C$, and $Q = A^{-1}D$.

To study the deterministic dynamics of this system, we revert to the case where A is time-invariant (hence the expectation operators and the terms dA_t/A_t and dA_{t+1}/A_{t+1} can be dropped from (B.6)). We can then compute the eigenvalues (call them λ_1 and λ_2) by solving the equation det(W − λI) = 0, which is a quadratic equation in λ. The values of the two roots λ_1 and λ_2 are complicated functions of the parameters $\alpha, \sigma, \beta, A, \delta_k, r$, and g_H. One can show that λ_1 and λ_2 satisfy the following conditions.

$$\lambda_1 \lambda_2 = \frac{1+r}{1+g_H}, \quad and \quad \lambda_1 + \lambda_2 = 1 + \left(\frac{1}{1+g_H}\right)\left\{(1+r) + \left(\frac{\alpha}{\sigma}\right)\left[\frac{r+\delta_k}{1-\alpha} - (g_H+\delta_k)\right]\left(\frac{r+\delta_k}{1+r}\right)\right\}.$$

Of the two eigenvalues of the fundamental matrix W, one exceeds unity (unstable root) and the other is less than unity (stable root, call it λ). Imposing the transversality condition: $\beta(1+g_H)^{1-\sigma} = (1+g_H)/(1+r) < 1$, we can eliminate the unstable root from the solution to obtain the following.

$$x_t - \hat{x} = (x_0 - \hat{x})\lambda^t, \quad and \tag{B.7}$$

$$z_t - \hat{z} = (z_0 - \hat{z})\lambda^t. \tag{B.8}$$

where x = ln(X) and z = ln(Z).

The dynamics in the vicinity of the steady state can be portrayed graphically by a `phase diagram'.[9]

[insert `phase diagram']

The stationary schedules corresponding to $x_t = x_{t+1}$ and $z_t = z_{t+1}$ are given by the vertical and the inverted-U curves respectively. The steady state $(\hat{z}, \hat{x})$ is attained at the intersection of these schedules. The unstable trajectories diverge from the vicinity of this steady state in the northwest and southeast directions, while the stable trajectories approach the steady from the northeast and southwest directions. The equilibrium transitional dynamics correspond to movements along the stable paths, described by equations (B.7) and (B.8).

As a measure of the speed of adjustment to the steady state, we use the concept of half life (t^*), defined as

$$\frac{x_{t^*} - \hat{x}}{x_0 - \hat{x}} = \frac{1}{2} = \frac{z_{t^*} - \hat{z}}{z_0 - \hat{z}} .$$

Substituting from (A2.3) and/or (A2.4) and solving for t^*, we get $t^* = -\ln(2)/\ln(\lambda)$.

Alternatively, we can measure the speed of convergence from the implied ratio of $y_{t+1} - \hat{y}_{t+1}$ to $y_t - \hat{y}_t$.

$$\frac{y_{t+1} - \hat{y}_{t+1}}{y_t - \hat{y}_t} = (1+g_h)\left(\frac{x_{t+1}^{1-\alpha} - \hat{x}^{1-\alpha}}{x_t^{1-\alpha} - \hat{x}^{1-\alpha}}\right) \approx (1+g_h)\left(\frac{[1+(1-\alpha)x_{t+1}] - [1+(1-\alpha)\hat{x}]}{[1+(1-\alpha)x_t] - [1+(1-\alpha)\hat{x}]}\right) = (1+g_h)\lambda .$$

where we have used $y_t = A(x_t)^{1-\alpha}h_0(1+g_h)^t$ in the first equality and the approximation $x^{1-\alpha} = 1+(1-\alpha)x$ in the second equality. This is the discrete time analogue of the convergence rate (β) in Barro and Sala-i-Martin (1992)'s continuous time setup, where

$$\ln\left(\frac{y_t}{\hat{y}_t}\right) = e^{-\beta t}\ln\left(\frac{y_0}{\hat{y}_0}\right) .$$

Appendix C: An Example Economy with Full-Fledged Solution

In order to highlight the mechanics of growth in this model, we specialize for simplicity by setting $\sigma = \delta_k = 1$. From equations (12.4) and (12.7), we get

$$\frac{c_{t+1}}{\beta c_t} = (1-\alpha) A X_{t+1}^{-\alpha} = (1-\alpha)\frac{Y_{t+1}}{K_{t+1}}. \tag{C.1}$$

We can write: $c_t = (1-s_t)Y_t$ and $K_{t+1} = s_t Y_t$. Substituting these into (12.7)' yields

$$\frac{1-s_{t+1}}{\beta(1-s_t)} = \frac{1-\alpha}{s_t}.$$

This can be restated as a first order difference equation in s_t, with an unstable root. Thus, the unique (economically plausible) solution is a constant saving rate $s = \beta(1-\alpha)$. Substituting this saving rate into (12.9) gives the following fundamental difference equation in X_t.

$$X_{t+1} = \left[\frac{\beta(1-\alpha)A}{1+g_H}\right] X_t^{1-\alpha}. \tag{C.2}$$

We display this dynamic equation graphically in Figure 12.1. The concave curve, depicting the right hand side of (B.2), intersects the 45° line at two points to produce two steady states, at the origin (unstable) and E (stable). For any given initial value $X_0 > 0$, the economy must converge to the long run equilibrium point E along the trajectories as indicated by the arrows. At E, the steady state value of the ratio between the two capital stocks is given by $\hat{X} = \{[\beta(1-\alpha)A]/(1+g_H)\}^{1/\alpha}$.

Given this closed form solution, it is straightforward to compute the consumer's utility along the entire growth path. Recursive substitution of (B.2) implies

$$x_t = \left[\frac{1-(1-\alpha)^t}{\alpha}\right]\ln\left[\frac{[1-\beta(1-\alpha)]A}{1+g_H}\right] + (1-\alpha)^t x_0 .$$

Substituting this into $c_t = (1-s_t)Y_t = [1-\beta(1-\alpha)]AX_t^{1-\alpha}h_0(1+g_h)^t$ and then the resulting expression into the utility function, we get

$$U = \sum_{t=0}^{\infty}\ln(c_t) = \left(\frac{1-\alpha}{1-\beta(1-\alpha)}\right)x_0$$
$$+\left(\frac{1}{1-\beta}\right)\left[\left(1+\frac{1-\alpha}{1-\beta(1-\alpha)}\right)\ln\{[1-\beta(1-\alpha)]A\}+\ln(h_0)+\beta g_h-\left(\frac{1-\alpha}{1-\beta(1-\alpha)}\right)g_H\right].$$

This welfare measure will be used in the capital mobility section to evaluate the gains from intertemporal trade.

Appendix D: Welfare Gains from Free Capital Flows

To evaluate the welfare gains from capital flows, we first compute the equilibrium consumption path under such regime. From the intertemporal condition (12.7), we have $c_t = c_0[\beta(1+r^*)]^t$, where c_0 can be solved from the consumer's present value budget constraint, obtainable by consolidating the flow budget constraint in (12.11).

$$\sum_{t=0}^{\infty} \frac{N_t c_t}{(1+r^*)^t} = \sum_{t=0}^{\infty} \frac{Y_t}{(1+r^*)^t} + K_0 .$$

As we make clear in the text, the capitals ratio will jump to its steady state in one period. Consequently, output per capita is given by

$$Y_t = AX_t^{1-\alpha}h_t = \begin{cases} AX_0^{1-\alpha}h_0 & t = 0 \\ A(\hat{X})^{1-\alpha}h_0(1+g_h)^t & t > 0 \end{cases}$$

Substituting this and $c_t = c_0[\beta(1+r^*)]^t$ into the present value budget constraint, we have

$$c_0 = [1-\beta(1+g_N)]\left[X_0h_0 + AX_0^{1-\alpha}h_0 + A\hat{X}^{1-\alpha}h_0\left(\frac{1+r^*}{r^*-g_h}\right)\right].$$

Substituting into the utility function yields

$$U = \sum_{t=0}^{\infty} \ln(c_t)$$

$$\left(\frac{1}{1-\beta}\right)\left\{\ln(1-\beta) + \ln\left[X_0h_0 + AX_0^{1-\alpha}h_0 + A\hat{X}^{1-\alpha}h_0\left(\frac{1+r^*}{r^*-g_h}\right)\right] + \left(\frac{\beta}{1-\beta}\right)\ln[\beta(1+r^*)]\right.$$

$$\text{where } r^* = (1-\alpha)A\left[\frac{[1-\beta(1-\alpha)]A}{1+g_H}\right]^{1/\alpha} - 1 .$$

Appendix E: Stochastic Dynamics

We follow Campbell (1994) by solving the equilibrium dynamics of the system of equations (B.3), (B.4), and (12.13) by the method of undetermined coefficients.

We guess a solution for z_t (= $\ln(Z_t)$) of the form:

$$z_t = \eta_{zx} x_t + \eta_{za} a_t, \tag{E.1}$$

where η_{zx} and η_{za} are unknown fixed coefficients, and $a_t = \ln(A_t)$. To verify this solution, substitute (E.1) into (B.3) and (B.4) and rearrange to get the following:

$$\begin{aligned} x_{t+1} &= \eta_{xx} x_t + \eta_{xa} a_t, \\ \text{where } \eta_{xx} &= \frac{1+r}{1+g_H} + \left[(g_H+\delta_k) - \frac{r+\delta_k}{1-\alpha} \right] \eta_{zx} \\ \text{and } \eta_{xa} &= \frac{r+\delta_k}{(1-\alpha)(1+g_H)} + \left[(g_H+\delta_k) - \frac{r+\delta_k}{1-\alpha} \right] \eta_{za}. \end{aligned} \tag{E.2}$$

$$E_t(x_{t+1}-x_t) + \eta_{za} E_t(a_{t+1}-a_t) = \sigma \left(\frac{r+\delta_k}{1+r} \right) E_t(a_{t+1}-x_{t+1} \tag{E.3}$$

Substituting (E.2) into (E.3) and using the fact that $E_t a_{t+1} = \rho a_t$ from (12.13), we arrive at one equation in x_t and a_t:

$$\eta_{zx} x_t + [\eta_{zx}\eta_{xa} + \eta_{za}(\rho-1)] a_t = \sigma \left(\frac{r+\delta_k}{1+r} \right) [(\rho-\eta_{xa}) a \tag{E.4}$$

Recall from (E.2) that η_{xx} is a linear function of η_{zx} and η_{xa} is a linear function of η_{za}. Comparing the coefficients on x_t on the two sides of equation (E.4), we can obtain a quadratic equation in 0_{zx} as follows:

$$
\begin{aligned}
& Q_2 0^2_{zx} \ \% \ Q_1 0_{zx} \ \% \ Q_0 \ \text{'} \ 0\,, \\
& \textit{where } Q_2 \ \text{'} \ (g_H \% \star_k) \ \& \frac{r\% \star_k}{1\&"}\,, \\
& Q_1 \ \text{'} \ \frac{r\& g_H}{1\% g_H} \% \mathrm{F}\left(\frac{r\% \star_k}{1\% r}\right)\left[(g_H \% \star_k) \ \& \frac{r\% \star_k}{1\&"}\right],\ \textit{and} \\
& Q_0 \ \text{'} \ \frac{\mathrm{F}\,(r\% \star_k)^2}{(1\&")\,(1\% r)}\,.
\end{aligned}
\tag{E.5}
$$

Imposing the transversality condition (which implies that r > g_H), we know that 0_{zx} must be positive. Taking only the positive root, we have

$$
0_{zx} \ \text{'} \ \& \frac{1}{2Q_2}\left(Q_1 \% \sqrt{Q_1^2 \& 4Q_0 Q_2}\right). \tag{E.6}
$$

Given this solution for 0_{zx}, we can compare the coefficients on a_t on the two sides of equation (E.4) to solve for 0_{za}:

$$
{}_a \ \text{'} \ \frac{\& \dfrac{0_{zx}(r\% \star_k)}{(1\&")\,(1\% g_H)} \ \% \ \mathrm{F}\left(\dfrac{r\% \star_k}{1\% r}\right)\left(\mathrm{D}\& \dfrac{r\% \star_k}{(1\&")\,(1\% g_H)}\right)}{(\mathrm{D}\& 1)\,\% \left[(g_H \% \star_k)\,\& \left(\dfrac{r\% \star_k}{1\&"}\right)\right]\left[0_{zx} \% \mathrm{F}\left(\dfrac{r\% \star_k}{1\% r}\right)\right]} \tag{E.7}
$$

The last two coefficients 0_{xx} and 0_{xa} can be backed out from (E.2), givem (E.6) and (E.7).

Problems

1. Consider the closed economy example in Appendix C (with $F = *_k = 1$). Suppose there is a time-invariant tax (J_k) levied on capital income ($r_{kt}K_t$) and the tax proceeds are rebated as lump sum transfers to the households. Show how this tax rate will affect the saving rate (s_t), the speed of adjustment to the long run equilibrium, and the levels and rates of growth of per capita output in that equilibrium.

2. In the previous problem, how will your answers be affected if the tax proceeds ($J_k r_{kt} K_t$) are used to finance government spending (g_t) in each period?

3. Consider the model with constrained capital mobility in Section 12.2. Calculate the jump in the initial level of consumption resulting from the opening up of the world capital market, in the presence of the two constraints described in the section. Compare this with the corresponding jump in initial consumption in the free capital mobility case.

4. Consider the case of labor mobility described in Section 12.3.
(a) Specify the consumer budget constraint and resource constraint, and derive the law of motion for the capitals ratio (x_t, redined as the ratio between physical capital and domestically employed labor).
(b) Consider a labor migration quota set by the receiving country (the rest of the world. What is the quota level (Q_t in each period t) which makes the constrained labor mobility regime equivalent to the constrained capital mobility regime?

References

Barro, Robert J., Mankiw, N. Gregory, and Sala-i-Martin, Xavier. 1995. Capital Mobility in Neoclassical Models of Growth. *American Economic Review* : ???.

Cass, David. 1965. Optimal Growth in an Aggregative Model of Capital Accumulation. *Review of Economic Studies* 32: 233-40.

Campbell, John Y. 1994. Inspecting the Mechanism: An Analytical Approach to the Stochastic Growth Model. *Journal of Monetary Economics* 33: 463-506.

Kaldor, Nicholas. 1963. Capital Accumulation and Economic Growth, in Friedrich A. Lutz and Douglas C. Hague, eds., Proceedings of a Conference Held by the International Economics Association. London: MacMillan.

Koopmans, Tjalling C. 1965. On the Concept of Optimal Economic Growth, in The Econometric Approach to Development Planning. Amsterdam: North Holland. 225-87.

Ramsey, Frank P., "A Mathematical Theory of Saving," *Economic Journal* 38 (1928): 543-59.

Solow, Robert M. 1956. A Contribution to the Theory of Economic Growth. *Quarterly Journal of Economics* 70: 65-94.

Endnotes

1. Solow (1956) interprets this factor as labor-augmenting technological progress.

2. On the other hand, we can also allow A_t to follow a deterministic growth trend, say, $A_{t+1} = (1+g_A)A_t$ (Hicks-neutral technological progress). We choose not to specify it this way because such process has already been subsumed under the accumulation process of human capital. In other words, one can interpret $1+g_H = (1+g_A)^{1/''}$, i.e., technological progress is labor-augmenting.

3. Indeed, in most of the discussions below, we need not specialize the production function to the Cobb-Douglas form. We choose this specific form in order to simplify the exposition.

4. An equivalent specification, as in Chapter 7, assumes that the firm is the owner of physical capital and makes its investment and production decision by solving an intertemporal profit maximization problem.

5. Note that the time convention in our notations in this chapter is slightly different from that used in Chapter 5. In this chapter, we denote current period (t) borrowing by B_{t+1} rather than B_t and the corresponding rate of interest by r_{t+1} instead of r_t. This change is done in order to be consistent with the time convention for factor accumulation.

6. In the closed economy, equilibrium in the financial capital market implies that gross saving (saving in the form of physical capital) amounts to consumer's net saving.

7. If the rest of the world starts from an off-steady-state position initially, the economies will immmediately converge and then grow together at the same rates towards a common long run steady state in terms of per capita GDP.

8. Barro, Mankiw, and Sala-i-Martin (1995) consider similar international borrowing constraints. Their model focuses on the impossibility of financing investment in human capital through borrowing.

9. Strictly speaking, the phase diagram aparatus applies only to continuous time dynmaics described in terms of a system of differential equations. The discrete time dynamics that we examine here are just approximations to their continuous time counterparts.

15

International Taxation and Endogenous Growth

Lucas (1988) posed the *problem of economic development* as the problem of accounting for *"... the observed pattern, across countries and across time, in levels and rates of growth of per capita income ..."*. Assuming symmetry in preferences and technology across countries, the growth literature has been successful in explaining income *level* differences in terms of asymmetry in initial factor endowments (as a transitory short run phenomenon in exogenous growth models and as a sustainable long run phenomenon in endogenous growth models). The explanation of growth *rate* differences is a much harder challenge—especially in exogenous growth models, where the natural growth rate is an unalterable given. In the context of recent models of endogenous growth, one explanation that has been widely explored lies in differences in national (especially tax) policies.[1] Such asymmetry can generate differential effects on the private agents' incentives to invest in growth-enhancing activities and hence the rates of productivity growth in different countries.[2]

With the increasing global integration of the world economy, factor mobility opens a room for cross-border spillovers of policy effects, with policy changes in one country exerting an impact on resource allocation and growth in another country through changes in factor price differentials. In this chapter, we examine whether the tax-driven diversity in income growth rates can be preserved when (a) factors of production are freely mobile across national borders, and (b) the factor incomes earned in the foreign country are potentially subject to double taxation by both the home and foreign governments and are thus affected by both domestic and foreign tax policies. In particular, is international income taxation a growth-diverging force? How do factor mobility and cross country tax structures interact to determine growth differentials?

15.1 Tax-Driven Divergence in a Closed Economy

Consider the closed economy endogenous growth model of Chapter 13. Suppose now that there is a fiscal authority that levies flat rate taxes on labor income (τ_{wt}) and capital income (τ_{rt}). We allow for tax-deductibility of depreciation expenses for physical capital ($\tau_{rt}\delta_k K_t$) and, possibly, human capital ($\phi\tau_{wt}\delta_h e_t N_t h_t$, with $\phi = 1$ if deductible and $\phi = 0$ otherwise) as well. If depreciation of human capital is tax-deductible and if income taxation is comprehensive and uniform so that the tax rates on labor and capital incomes are equal, the tax treatment of the two forms of capital becomes symmetric. As usual, in order to focus on the distortionary effects of taxation, we assume that the tax proceeds are rebated in a lump-sum fashion to the households. The consumer budget constraint, the counterpart of (13.2), is given by

$$\begin{aligned} & N_t c_t + [K_{t+1} - (1-\delta_k)K_t] - [B_{t+1} - (1+r_t)B_t] - T_t \\ & S_{wt} w_t (1-e_t) N_t h_t + \phi\tau_{wt}\delta_h w_t N_t h_t + S_{rt} r_{kt} K_t + \tau_{rt}\delta_k K \end{aligned} \tag{15.1}$$

where T_t is the lump-sum rebate, and the tax wedges are defined as $S_{rt} = 1 - \tau_{rt}$ and $S_{wt} = 1 - \tau_{wt}$.

The optimization problem facing the household is to choose $\{c_t, e_t, K_{t+1}, h_{t+1}, B_{t+1}\}_{t=0}^{\infty}$ to maximize utility (12.1) subject to the human capital accumulation equation (13.1) and the budget constraint (15.1), given $\{w_t, r_t, \tau_{wt}, \tau_{rt}\}_{t=0}^{\infty}$.

Following similar steps in Appendix A of Chapter 13, we can derive the following steady state equation in the time allocated to education (e).

$$1+g_h)^{1-\sigma}(1+g_N)\left\{1+\gamma\left(\frac{1-e}{e}\right)\left(\frac{g_h+\delta_h}{1+g_h}\right)\left[1+\phi\delta_h\left(\frac{\vartheta_w}{\Sigma_w}\right)\left(\frac{}{1}\right.\right.\right. \tag{15.2}$$

$$\text{where } g_h = Be^{\gamma}-\delta_h .$$

Given the solution for e from (15.2), the ratio of the stocks of physical capital to human capital $\tilde{k}$ (= K/Nh) can be solved from

$$(1+g_h)^{\sigma} = \beta\left\{1+\Sigma_r\left[(1-\alpha)A\left(\frac{\tilde{k}}{1-e}\right)^{-\alpha}-\delta_k\right]\right\}. \tag{15.3}$$

Direct inspection of (15.2) reveals that the intertemporal (capital) tax wedge (Σ_r) has no effect on the allocation of time between work and education, hence growth rates, in the steady state. The source of this neutrality lies in the fact that, in our model, there are no other time-consuming activities (such as leisure) and physical capital is not required for the production of human capital.

In the absence of depreciation allowance for human capital ($\phi = 0$), the intratemporal (labor) tax wedge (Σ_w) will have no effect on long run time allocation (hence, growth rate) either. This can be understood from Boskin's (1975) argument that increase in the *constant* tax rate on labor income along the balanced growth path will reduce both the returns (in terms of future wage earnings) and costs (in terms of forgone earnings) of investment in human capital equally at the margin.[3] When depreciation of human capital is tax-deductible ($\phi = 1$), however, the reduction in returns in terms of future wage earnings due to the wage tax are exactly offset by this allowance. Thus, since only the costs in terms of income forgone are reduced, the labor income tax is no longer neutral.

In reality, tax-deductibility of depreciation of human capital does not exist in the exact form as modelled here, but can be viewed as mimicking the effects of two common provisions of taxation: tax progressivity and subsidized health care.[4] This is proxied by $0 < N < 1$ in our model. On the other hand, our simplified setup has abstracted from modelling other time-consuming activities such as leisure or child-rearing (a driving force behind population growth), and the use of physical capital as an input in the production of human capital. Adding these features to our model will strengthen the effects of the labor income tax on the growth rate of human capital (and population) and introduce a channel through which the capital income tax may affect the growth rate as well.

In our model, countries with similar preferences and technology but different labor income tax rates will have different long run growth rates. Differences in capital income tax rates, however, will not lead to divergence in growth rates.

15.2 Tax Divergence in an Open Economy: Capital Mobility

We now integrate our economy into the world capital market. We continue to assume that tax rates are different across countries. If the residence principle of international taxation is adopted universally, then (as shown in Chapter 14) the pre-tax marginal products of physical capital will be equalized across countries. The after-tax marginal products will differ, however, if the capital income tax rates vary across countries. The interest-equalization arbitrage-based relation $(1\text{-}J_{rD})r = (1 \,!\, J_{rA} \,!\, J^{*}_{rN})r^{*}$ can be viewed as an additional condition (relative to the autarky case) in the set of world equilibrium conditions. Since the other equilibrium conditions remain the same as in the closed economy, the analogue of the time equation (15.2) indicates that the steady state growth rate of human capital also remains unchanged. This implies that countries with different (the same) labor income tax rates will grow at different (the same) rates [Implication 1].

If net capital flows are non-zero in the long run, the size of this flow can, in principle, be determined from the arbitrage-based condition. However, in the steady state, these flows should grow at the same rate as the outputs in both the home country and the rest of the world. Thus, if they exhibit identical (exogenous) rates of growth of population (as we have been assuming so far), they must also have the same growth rates in human capital (or consumption). But, this contradicts Implication 1, implying that either the steady state is non-existent or capital flows are zero.

Suppose, then, that net capital flows are zero in the steady state. In the absence of barriers to capital flows, the arbitrage-based condition will continue to hold even when no capital flows exist in equilibrium. From (15.3), we can write

$$\left(\frac{1+g_y}{1+g_y^*}\right)^{\sigma} = \frac{1+(1-\tau_{rD})(r_k-\delta_k)}{1+(1-\tau^*_{rA}-\tau_{rN})(r_k-\delta_k)}. \tag{15.4}$$

As explained in Chapter 14, the residence principle implies that $\tau_{rN} = 0$ and $\tau^*_{rA} = \tau^*_{rD}$. This implies that g_y and g^*_y are different (equal) as long as $\tau_{rD} \neq (=) \tau^*_{rD}$, thus ruling out zero capital flow as a steady state equilibrium phenomenon from Implication 1 (unless the two countries have strong similarities across the two tax bases). As a result, a steady state will not exist, in general, in the presence of tax differences. In other words, capital mobility will drive countries off the world steady state growth path.[5]

To restore the possibility of a steady state, we introduce an additional source of growth which, similar to human capital, involves time-consuming activities: endogenous population growth.

Given the close connection between population growth and economic growth in the development process and as a broadening of the definition of the *problem of development*, we shall devote equal emphasis to accounting for the observed diversity in the growth of (per capita and aggregate) income as well as population. When population growth is determined exogenously, taxes can only affect income growth through the growth engine (say, human capital), with indistinguishable effects on the growth of per capita income and aggregate income. Endogenizing population growth will introduce a new channel through which taxes can affect per capita income growth and aggregate income growth differently.

For the above reasons, we think that it is important to examine the interaction between taxation and (population and income) growth in the presence of factor mobility. To accomplish this, we need to extend the model in three dimensions: preferences for the quantity and quality of children, the time constraint, and the law of motion of population.

Consider the home country as a dynastic family with N_t identical members in each period ($t = 0,1,2,\ldots$) and two engines of growth (human capital and population). The typical household cares about his/her own consumption c_t and the other family members N_t. His/her preferences are given by:

$$\sum_{t=0}^{\infty} \beta^t N_t^{\xi} \left(\frac{c_t^{1-\sigma}}{1-\sigma} \right) \tag{15.5}$$

where $\xi \in (0,1)$ an altruism parameter.[6] As long as $\xi > 0$, altruism is reflected not only in preference for `quantity' but also `quality' of children (viz., consumption per capita, or standard of living)—since, with positive ξ, there is weight given to quantity, but the weight on the consumption term is magnified as well. Observe that if $\xi > 1-\sigma$, then there will be a relative bias in preference towards quantity; whereas if $\xi < 1-\sigma$, the bias will be in the opposite direction. When $\xi = 1-\sigma$, the representative agent is said to be `fairly altruistic' in the sense that he cares only about the size of the total pie ($N_t c_t$) to be shared among all family members, but is indifferent to the exact sharing arrangement.

In each period t, there are N_t members in the representative family (given N_0 at t= 0). As before, each household member is endowed with one unit of non-leisure time in each period t. But instead of splitting it between work and education, he/she can now divide the unit time among three time-consuming activities: work (n_t for number of work hours), learning in schools (e_t for education), and child-rearing (v_t for vitality). The child-rearing activity gives rise to population growth:

$$N_{t+1} = D(v_t^{\alpha})N_t + (1-\delta_N)N_t, \tag{15.6}$$

where $D > 0$ and $\alpha \in (0,1]$ are the fertility efficiency coefficient and productivity parameter respectively. One can think of N_{t+1}/N_t as one plus the number of children per family (when the number of parents is normalized to unity). Since the child-rearing cost (v) is increasing with the number of children, Dv^{α} can be thought of as the inverse function of this cost-quantity relation. This completes the description of the new elements of the model.

The Role of Capital Mobility in Growth Rate Convergence

As in Chapter 12, under free capital mobility, the law of diminishing returns implies that capital will move from capital-rich (low marginal product of capital, henceforth, MPK) countries to capital-poor (high MPK) countries. Over time, such cross-border capital flows will equalize the MPKs (pre-tax or post-tax, depending on the international tax principle) prevailing in all countries. In the long run, an empirically relevant steady state world equilibrium will involve positive net capital flows from some countries to some other countries.

Without further restrictions, two other situations are possible in the long-run: (a) all capital in the world resides in one single country; and (b) no cross-border capital flows (i.e., back to autarky). Both are unrealistic cases. We make sufficient assumptions to eliminate them even as theoretical possibilities. Case (a) will not occur if the MPK becomes infinitely high when the capital remaining in any capital-exporting country gets sufficiently small (i.e., the Inada conditions can rule out this corner solution). Case (b) will not occur as long as the countries are heterogeneous in some fundamentals. (If they were homogeneous in all respects, capital flows would not have taken place in the first place.) Since we want to investigate the role of taxes on growth, we shall assume that asymmetry in capital income tax rates is the factor that first induced cross-border capital flows. Suppose further that these countries were travelling along their steady state growth paths initially. Should these taxes remain different, the driving force that initiated capital movement to begin with will be reactivated if the countries revert to their long-run autarky growth paths. As such, (b) can also be ruled out. The only empirically interesting case that remains is the one that involves non-zero flows. In that case, we should expect the direction of capital flows to be from low after-tax MPK countries to high after-tax MPK countries.

Let us now try to understand how capital mobility may affect the convergence in long term growth rates across countries. In the world steady state equilibrium, the non-zero net capital flow of each country must be growing at the same rate as its total income. But since the capital inflow of one country is equal to the capital outflow of another country, the steady state (balanced growth) restriction forces the total income growth rates to be uniform across countries in the long run, i.e.,

Along the steady state growth path with nonzero net capital flows, the growth rates of (total, but not necessarily per capita) GDP must be equal across countries.

To prove this, suppose without loss of generality that capital flows from the rest of the world to the home country (since, in our full certainty model, equilibrium capital flows will be unidirectional.) Consider the resource constraint facing the home country with all the growing variables detrended by dividing the whole equation through by $N_t h_t$:

$$+g_N)(1+g_h)](K^H+K^{H*}) = F(K,n) - (1-\tau_{rN})\, rK^{*H}\Bigg(.$$

The steady state growth rate of per capita GDP, g_y, is equal to g_h, so that the steady state growth rate of GDP is $g_Y = (1+g_N)(1+g_h) - 1$; and similarly for the rest of the world. We can rewrite the last term as:

$$-(1-\tau_{rN})\, rK^{*H}\left(\frac{N^* h^*}{Nh}\right)\left(\frac{(1+g_N^*)(1+g_h^*)}{(1+g_N)(1+g_h)}\right)^t$$

where $N \equiv N_t/(1+g_N)^t$ and $h \equiv h_t/(1+g_h)^t$ are the detrended steady state levels of population and human capital respectively in the home country, and similarly in the rest of the world. Note that this term is time-varying unless $(1+g_N)(1+g_h) = (1+g_N^*)(1+g_h^*)$, implying equality of the growth rates of (total) GDP g_Y^i in the two countries along the steady state growth path.

We can decompose the *total* income growth rates into the *per capita* income growth rates and *population* growth rates: $(1+g_Y) = (1+g_N)(1+g_y)$. Together with the total growth equalization result, this decomposition implies that $(1+g_N)(1+g_y) = (1+g_N^*)(1+g_y^*)$. Two empirical implications follow:

(a) *Long-term rates of growth of population and per capita incomes should be negatively correlated across countries*;[7] and

(b) *Total income growth rates should exhibit less variation than per capita income growth rates across countries.*

Some empirical support for these and other related implications is provided by Razin and Yuen (1995d).

The Role of International Capital Taxation in Growth Rate Convergence

When capital is mobile, the choice of international tax principle and tax rates levied on capital incomes earned by residents and non-residents at home and abroad will affect the after-tax rates of return on capital and, indirectly, the rates of growth of per capita consumption and population (g_c and g_N) across countries through the intertemporal conditions:

$$(1+g_N)^{1-\eta}(1+g_c)^{\sigma} = 1 + (1-\tau_{rD})(r_k-\delta_k), \text{ and}$$
$$(1+g_N^*)^{1-\eta}(1+g_c^*)^{\sigma} = 1 + (1-\tau_{rD}^*)(r_k^*-\delta_k).$$

Along the steady state growth path, $g_c = g_y$ and $g_c^* = g_y^*$ and, as we have just shown,

$(1+ g_N)/(1+ g^*_N) = (1+ g^*_y)/(1+ g_y)$. The arbitrage-based condition implies that $(1 ! J^*_{rD})(r^*_k ! *_k) = (1 ! J^*_{rA} ! J_{rN})(r_k ! *_k)$. Substituting these conditions into the above equations, and dividing on equation by the other, we get

$$\left(\frac{1\%g_y}{1\%g_y^{(}}\right)^{>\&(1\&F)} \;'\; \frac{1\%(1\&J_{rD})(r_k\&*_k)}{1\%(1\&J^{(}_{rA}\&J_{rN})(r_k\&*_k)} . \tag{15.7}$$

This equation shows how the relative (per capita) income growth rates in the two countries depend on their capital tax rates and relative bias in preference towards quantity versus quality of children ($>$ versus $1 ! F$).

Recall that, under perfect capital mobility, the no-arbitrage restrictions will force the after-tax rates of return on capital ($\bar{r}$'s) to be equalized across countries under the source principle. Equation (15.7) therefore implies convergence in both the per capita and total income growth rates if the *source* principle prevails (when $J^*_{rA} = 0$ and $J_{rN} = J_{rD}$). Under the alternative *residence* principle (when $J_{rN} = 0$), since the after-tax interest rates are not equalized by capital mobility, asymmetry in $\bar{r}$'s (due to the asymmetry between J_{rD} and J^*_{rD}) implies, in turn, asymmetry in growth rates.

Equation (15.7) also indicates that under residence-based taxation, when $> \ldots 1 ! F$, asymmetric tax rates may have differential effects on the growth of per capita income and population. In particular, when people are more biased towards quality than quantity ($> < 1 ! F$), the country with a higher capital tax rate will exhibit faster growth in per capita income and slower growth in population. The reverse is true when people are more biased towards quantity than quality ($> > 1 ! F$).[8] Other things equal, the country with a higher capital tax rate will have less incentive to invest in physical capital and more to invest in child quality if > < 1 ! F or in child quantity if > > 1 ! F. We summarize these results as follows.

Under capital mobility and international capital income taxation:

(1) **When both countries adopt the source principle, they will exhibit identical rates of growth of per capita income and population if > … 1 ! F irrespective of international tax differences;**

(2) **When both countries adopt the residence principle, they will exhibit different rates of growth of per capita income and population in general. In particular, $g_y \gtreqless g_y^*$ and $g_N \lesseqgtr g_N^*$ as $J_{rD} \lesseqgtr J_{rD}^*$ if > > 1 ! F, $g_y \lesseqgtr g_y^*$ and $g_N \gtreqless g_N^*$ as $JR_D \lesseqgtr J_{rD}^*$ if > < 1 ! F.** [9]

While asymmetry in tax rates can induce differential growth rates when both countries adopt the residence principle, we note that the adoption of asymmetric international tax principles by different countries can also generate disparity in growth rates. Note also from equation (15.7) that, in cases intermediate between the source and residence principles (i.e., without complete exemption from taxes on foreign-source capital income to be paid to the domestic and/or foreign governments), the relative magnitudes of the tax wedges (with respect J_{rD}, J_{rA}, and J_{rN}) matter. In those cases, it will also be important to distinguish between the differential growth effects of the credit system that we have been assuming here and the alternative deduction system.

15.3 Tax Divergence in an Open Economy: Labor Mobility

Let us now turn to the other extreme case where labor is freely mobile but capital is not. Under perfect labor mobility, the absence of arbitrage opportunities ensures the equalization of after-tax marginal products of labor (MPH or wage rates) for any worker who can choose to work in either country. In particular, $(1 ! J_{wDt})MPH_t = (1 ! J^*_{wAt})(1 ! J_{wNt})MPH^*_t$, implying that $MPK^*_t = 7_t MPK_t$. The fundamental relative growth equation (15.7) can be rewritten as:

$$\hat{g}^{(1\&F)} \; ' \; \frac{1 \% (1\&J_{rD})(r_k\&*_k)}{1 \% (1\&J^{(}_{rD})(7 r_k\&*_k)}, \quad where\ 7 \; ' \left(\frac{1\&J_{wA}\&}{1\&J_{,}} \right. \tag{15.8}$$

Note that $J_{wD} = J_{wA}$ and $J_{wN} = 0$ (implying $7 = 1$) under the residence principle of wage taxation, and $J_{wD} = J_{wN}$ and $J_{wA} = 0$ (implying $7 \gtreqless 1$ as $J_{wD} \gtreqless J^*_{wD}$) under the source principle. The proposition below should be transparent.[10]

Under labor mobility and international labor income taxation:

(3) When both countries adopt the source principle, they will exhibit different rates of growth of per capita income and population in general. In particular, $g_y \gtreqless g^*_y$ and $g_N \lesseqgtr g^*_N$ as $(1 ! J_{rD})^{1!"}(1 ! J_{wD})^{"} \gtreqless (1 ! J^*_{rD})^{1!"}(1 ! J^*_{wD})^{"}$ if $> >$ $1 ! F$, $g_y \lesseqgtr g^*_y$ and $g_N \gtreqless g^*_N$ as $(1 ! J_{rD})^{1!"}(1 ! J_{wD})^{"} \gtreqless (1 ! J^*_{rD})^{1!"}(1 ! J^*_{wD})^{"}$ if $> <$ $1 ! F$.

(4) When both countries adopt the residence principle, they will exhibit different rates of growth of per capita income and population in general. In particular, $g_y \gtreqless g^*_y$ and $g_N \lesseqgtr g^*_N$ as $J_{rD} \lesseqgtr J^*_{rD}$ if $> >$ $1 ! F$, $g_y \lesseqgtr g^*_y$ and $g_N \gtreqless g^*_N$ as $J_{rD} \lesseqgtr J^*_{rD}$ if $> <$ $1 ! F$.

Contrary to what we find in the capital mobility case, (3) shows that the source principle is not necessarily growth-equalizing. Although the post-tax MPHs are equalized under territorial taxation, the post-tax MPKs are not unless the weighted tax wedges $(1-\tau_{rD})^{1-\alpha}(1-\tau_{wD})^{\alpha}$ are uniform across countries. So, in contrast to (1), wage tax asymmetry matters here as much as interest tax asymmetry. Like (2), though, (4) implies that asymmetry in capital tax rates can be a source of growth disparity under worldwide taxation. As before, we can show that asymmetry in the international income tax principle can also be another source of growth rate differences.

In Chapter 14, we have seen how labor mobility combined with knowledge spillovers (the Lucas-externality) can bring about convergence in income levels in the absence of tax differences. When tax rates do differ across countries, however, the resulting differences in growth rates (as shown in (3) and (4) above) imply that level convergence can no longer be achieved. We can therefore view cross-country tax asymmetry as both a growth-diverging and level-diverging force.

15.4 Summary

Let us first summarize the answers to the several questions posed in the introduction, and then make some concluding remarks.

(a) Is factor mobility a growth-equalizing force?

Yes, for *aggregate* income growth rates; but not necessarily, for *per capita* income growth rates.

(b) Are capital mobility and labor mobility perfect substitutes as growth-equalizing forces?

Yes, in the absence of international tax differences.

(c) Can tax-driven diversity in growth rates be preserved under factor mobility and international income taxation?

Yes, (i) under residence-based taxation with either capital or labor mobility; (ii) under source-based taxation if labor is mobile; (iii) if different countries adopt different international tax principles; and (iv) if different international tax principles are applied to capital incomes and labor incomes separately.

(d) Are the growth effects of taxes symmetric under capital mobility and labor mobility?

Yes, under the residence principle. No, under the source principle, or when different countries follow different international tax principles.

In a nutshell, we have identified two sources of disparity in income and population growth rates across countries. They are: (a) asymmetry in factor income tax rates, and (b) asymmetry in international income tax principles, as adopted by different countries or applied to different factors of production. We have also shown how the growth effects of capital mobility and labor mobility can differ under these two cases and how they are related to the relative bias in preferences towards quantity and quality of children. Although these differences can easily be eliminated if enough symmetry is assumed between the two factors (e.g., uniform taxation of incomes from both factors), we believe that the asymmetries examined here are very real. In fact, the unequal barriers to the cross-border movements of the two factors can be another real source of asymmetry that is nonetheless ignored in our analysis.

Appendix A: Derivation of the Tax-Growth Relation in the Closed Economy

In this appendix, we derive equations (15.2) and (15.3) in the text. The consumer's first order conditions with respect to c_t, e_t, K_{t+1}, and h_{t+1} are given by:

$$c_t^{-\sigma} = \mu_t, \tag{A.1}$$

$$\mu_{h_t} \gamma B e_t^{\gamma-1} = \mu_t S_{wt} w_t N_t, \tag{A.2}$$

$$\mu_t = \beta \mu_{t+1}[1 + S_{r_{t+1}}(r_{t+1} - \delta_k)], \tag{A.3}$$

$$[\mu_{h_{t+1}}[B e_{t+1}^{\gamma} + (1-\delta_h)] + \mu_{t+1} w_{t+1} N_{t+1}[S_{w_{t+1}}(1-e_{t+1}) + NJ \tag{A.4}$$

The Lagrange multipliers (μ for `mu'ltipliers) at time t associated with the consumer budget constraint and the law of motion of human capital are denoted by μ_t, and μ_{ht} respectively. Assuming that the capital income tax rate τ_{rt} applies uniformly to both financial and physical capital, the arbitrage condition implies that $r_t = r_{kt} - \delta_k$. The firm's first order conditions are

$$w_t = (1-\alpha) A \left(\frac{K_t}{H_t} \right)^{\alpha}, \text{ and} \tag{A.5}$$

$$r_t = \alpha A \left(\frac{K_t}{H_t} \right)^{\alpha - 1}. \tag{A.6}$$

The equilibrium conditions in the labor and capital markets are

$$N_t h_t = H_t^d, \text{ and} \tag{A.7}$$

$$K_t = K_t^d. \tag{A.8}$$

Substituting (13.1) and (A.5) into (A.2), we get,

$$\frac{\gamma \mu_{h_t}[h_{t+1}/h_t - (1-\delta_h)]}{e_t} = \frac{\alpha \mu_t S_{w_t} Y_t}{1-e_t}. \tag{A.9}$$

Along the balanced growth path, time allocations and tax rates are constant, i.e., $e_t = e_{t+1}$, $S_t = S_{t+1}$, $S_{wt} = S_{wt+1}$, and $S_{rt} = S_{rt+1}$, and human capital and consumption will grow at the same constant rate g_h and output at the rate $(1+g_N)(1+g_h)-1$ respectively, so that (A.1) and (A.9) imply that

$$\frac{\mu_{h_{t+1}}}{\mu_{h_t}} = \frac{\mu_{t+1} Y_{t+1}}{\mu_t Y_t} = (1+g_N)(1+g_h)^{1-\sigma}. \tag{A.10}$$

Multiplying (A.4) throughout by h_{t+1} and dividing the resulting expression by $\beta\mu_{ht+1}h_{t+2}$, we get

$$\begin{aligned} &\frac{\mu_{h_t} h_{t+1}}{\beta \mu_{h_{t+1}} h_{t+2}} = 1 + \left(\frac{\mu_{t+1} Y_{t+1}}{\mu_{h_{t+1}} h_{t+2}}\right)\alpha\left[S_w + \phi\delta_h\tau_w\left(\frac{1}{1-e}\right)\right], \\ &\text{where } \frac{\mu_{t+1} Y_{t+1}}{\mu_{h_{t+1}} h_{t+2}} = \left(\frac{\gamma}{\alpha S_w}\right)\left(\frac{1-e}{e}\right)\left(\frac{g_h+\delta_h}{1+g_h}\right). \end{aligned} \tag{A.11}$$

Combined with (A.10), this yields equation (15.2) in the text.

Equation (15.3) can be derived by combining (A.1) and (A.3) and imposing the steady state restrictions.

References

Azariadis, Costas, and Allan Drazen, "Threshold Externalities in Economic Development," *Quarterly Journal of Economics* 105 (1990), 501!26.

Becker, Gary S., and Robert J. Barro, "A Reformulation of the Economic Theory of Fertility," *Quarterly Journal of Economics* 103 (1988), 1!25.

Becker, Gary S., Kevin M. Murphy, and Robert Tamura, "Human Capital, Fertility, and Economic Growth," *Journal of Political Economy* 98 (1990), S12!S37.

Benhabib, Jess, and Roberto Perli, "Uniqueness and Indeterminacy: Transitional Dynamics in a Model of Endogenous Growth," *Journal of Economic THeory* (1993).

Easterly, William, and Sergio T. Rebelo, "Fiscal Policy and Economic Growth: An Empirical Investigation," *Journal of Monetary Economics* (1995), forthcoming.

Jones, Larry E., and Rodolfo E. Manuelli, "A Convex Model of Equilibrium Growth," *Journal of Political Economy* 98 (1990), 1008!38.

King, Robert G., and Sergio T. Rebelo, "Public Policy and Economic Growth: Developing Neoclassical Implications," *Journal of Political Economy* 98 (1990), S126!49.

Lucas, Robert E., Jr., "On the Mechanics of Economic Development," *Journal of Monetary Economics* 22 (1988), 3!42.

__________, "Supply-Side Economics: An Analytical Review," *Oxford Economic Papers* 42 (1990), 293!316.

Razin, Assaf, and Uri Ben-Zion, "An Intergenerational Model of Population Growth," *American Economic Review* 69 (1975): 923-33.

Razin, Assaf, and Chi-Wa Yuen, "Utilitarian Tradeoff between Population Growth and Income Growth," *Journal of Population Economics* 8 (1995a), 81!7.

__________, "Capital Income Taxation and Long Run Growth: New Perspectives," *Journal of Public Economics* (1995b), forthcoming.

__________, "Factor Mobility and Economic Growth: Tax-Driven Divergence," working paper, 1995c.

__________, "Factor Mobility and Income Growth: Two Convergence Hypotheses," working paper, 1995d.

Rebelo, Sergio T., "Long Run Policy Analysis and Long Run Growth," *Journal of Political Economy* 99 (1991), 500!21.

__________, "Growth in Open Economies," *Carnegie-Rochester Conference Series on Public Policy* 36 (1992), 5!46.

Stokey, Nancy L., and Sergio T. Rebelo, "Growth Effects of Flat Rate Taxes," *Journal of Political Economy* 103 (1995).

Xie, Danyang, "Divergence in Economic Performance: Transitional Dynamics with Multiple Equilibria," *Journal of Economic Theory* (1993).

Problems

1. Consider the closed economy model of Section 15.1 with an additional time-consuming activity, leisure (L_t). The time constraint is specified as: $L_t + n_t + e_t = 1$. The utility function is rewritten as:

$$U = \sum_{t=0}^{\infty} \beta^t N_t \left(\frac{(c_t^{\eta} L_t^{1-\eta})^{1-\sigma} - 1}{1-\sigma} \right).$$

Assume for simplicity that $\phi = 0$ and $\delta_h = 1$. Analyze the effects of capital and labor income taxes on the steady state growth rate of income.

2. Consider the closed economy model of Section 15.1 with an additional time-consuming activity, child-rearing (v_t), which gives rise to endogenous population growth. The time constraint is specified as: $v_t + n_t + e_t = 1$. The utility function is given by (15.5), and the law of motion of population by (15.6). Assume for simplicity that $\phi = 0$, $\delta_k = \delta_h = \delta_N = 1$, and $\tau_r = 0$. Analyze the effects of labor income tax on the investment in physical capital, human capital (child quality), and population (child quantity), and the steady state growth rate of income.

3. Consider the capital mobility model with endogenous population in Section 15.2. Explain how the effects of international capital income taxation on cross-country growth rates will change if the home country adopts the source principle while the rest of the world adopts the residence principle.

4. Consider the labor mobility model in Section 15.3. Explain how the effects of international labor income taxation on cross-country growth rates will change if the home country adopts the source principle while the rest of the world adopts the residence principle.

5. Suppose capital and labor are both internationally mobile. Examine how the effects of international capital and labor income taxation on cross-country growth rates will change if the source principle is applied to the taxation of labor income and the residence principle to the taxation of capital income in both countries.

Endnotes

1. Two other explanations include (a) multiple steady states—economies with different initial endowments can evolve along the same equilibrium growth path, but in different directions, thus converging to different long-run positions (see, e.g., Becker, Murphy, and Tamura (1990) and Azariadis and Drazen (1990)), and (b) multiple equilibria—economies with the same initial endowment can follow different equilibrium growth paths and converge to different long-run positions (see, e.g., Benhabib and Perli (1993) and Xie (1993)).

2. See, e.g., Rebelo (1991) and Jones and Manuelli (1990) for a qualitative analysis; Easterly and Rebelo (1995) for an empirical examination; and King and Rebelo (1990), Lucas (1990), and Stokey and Rebelo (1995) for a quantitative assessment, of the effects of tax changes on long run growth rates in models with capital formation (human and physical) as the source of growth.

3. This argument applies to substitution effect between education and work. The potential income effect of taxes is absent in this case due to the homotheticity of preferences.

4. The progressivity of income tax implies that the tax rate which could have been applied to forgone income is smaller than the tax rate which is actually applied to the increase in future labor earnings due to human capital investment. Subsidized health care in the form of tax-deductibility of medical expenses is equivalent to the depreciation allowance for human capital associated with health.

5. Another way to see this is to observe that, with zero capital flows, the number of unknowns falls short of the number of equilibrium conditions.

6. See Razin and Ben-Zion (1975) for similar setup. Note that, as in Becker and Barro (1988), the altruism parameter > dictates the extent to which the marginal utility of children diminishes as the number of children is increased. Note that F does not only reflect the elasticity of substitution in consumption, but also the preference weight attached to child quality (relative to quantity). We thus restrict F to be less than unity to ensure that children command positive marginal utility, which implies a different restriction on its value for the existence of steady state than that specified in Section 13.1. The utility function from previous chapters is altered by dropping the \` ! 1' from the numerator in order to ensure that, under endogenous population growth, consumption will grow at the same rate as human capital in the steady state. The objective function (1) can also be interpreted as a social welfare function. In terms of the utilitarian approach, it is a Millian (average utility) social welfare criterion when > = 0. When > = 1, it becomes a Benthamite (sum of utilities) criterion. See Razin and Yuen (1995a) for details.

7. This implication means that, in a small vs large economy world, the small economy may \`disappear' relative to the large economy in terms of population, but its aggregate income will still grow at the same rate as the latter's in the limit. This need not be true, though, if capital is not mobile across these two economies or if they do not have current account imbalances in the long run.

8. The tax rate J_{rD}, rather than the after-tax MPK, matters here because the cross-country MPKs will be equalized under the residence principle anyway.

9. We require tax symmetry across countries $J_{rD} = J^*_{rD}$, so $(1+ g_N)(1+ g_y) = (1+ g^*_N)(1+ g^*_y)$, for the existence of world steady state growth, if $> = 1 ! F$. This condition is not required, however, in (1) when $> = 1 ! F$.

10. We require $S = S^*$ in case (1) and $J_{rD} = J^*_{rD}$ in case (2), so $(1+ g_N)(1+ g_y) = (1+ g^*_N)(1+ g^*_y)$, for the existence of balanced growth, if $> = 1 ! F$.