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SEQUENTIAL PLAY AND CARTEL STABILITY IN A COURNOT OLIGOPOLY

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Abstract. We reconsider the problem of cartel stability in a linear symmetric Cournot oligopoly by assuming that every coalition of firms defecting from a cartel can choose its quantity before the remaining firms. We show that differently from Salant et al. (1983) the only profitable cartel includes all firms in the industry. This result is shown to be robust to non linearities in payoffs provided that the inverse demand function is not too log-concave.

Abstract. **Keywords:** *Cartel Stability*, *Cournot Oligopoly*
JEL: L0, L1, D4, C7.

1. Introduction

Within the wide and heterogeneous literature on cartels and horizontal mergers the seminal result by Salant et al. (1983), characterizing the smallest profitable cartel under Cournot oligopoly, has been recently shown to play an important role also in endogenous cartel formation games (see Bloch, 1997 and Yi, 1997 for surveys). One relevant case is the well known Hurt and Kurz (1988) gamma game of coalition formation, for which all stable coalition structures require the presence of a cartel of at least the minimal size s^* defined by Salant et al. (1983), all remaining $(n - s^*)$ firms playing independently. The prediction of such a wide set of stable coalition structures can be viewed as a limitation of the theory, at least in terms of policy indications.

The purpose of our paper is to show that, if firms defecting from a cartel set their new quantity as leaders with respect to the fringe of remaining firms, then cartels of the size s^* defined by Salant et al. (1983) are no longer stable, and the *only stable* cartel includes all firms in the market. Moreover, this result is shown to be robust to non linearities of the firm's payoff, provided that the inverse demand function is not too log-concave.

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2. The Model

Let us assume a finite n-firms symmetric oligopoly model with the usual inverse demand function defined by:

$$p(Q) = a - bQ$$

where $Q = \sum_{i \in N} q_i$ is the total market quantity and, for every firm $i \in N$, a cost function given by:

$$C_i(q_i) = cq_i.$$

The constraints on the parameters are:

$$a > c \geq 0 \text{ and } b > 0.$$

It is easy to compute the payoff of a cartel of size $s = |S| \leq |N|$ when the remaining firms are independent Cournot players:

$$\Pi_S = \sum_{i \in S} \pi_i \left(q_S^*, \left(q_j^* \right)_{j \notin S} \right) = \frac{(a-c)^2}{b(n-s+2)^2}. \tag{1}$$

Salant et al. (1983) define a profitable cartel as a coalition of firms that produce, by competing against the fringe of remaining firms, a per member profit which is higher than the profit of each firm in the absence of cartels. They reach the following result.

Proposition 1. *(Salant et al. (1983)). The minimum profitable cartel in a linear Cournot oligopoly has a size equal to* $s^* = \frac{2n+3-\sqrt{4n+5}}{2}$.

Proof. By normalizing $\frac{(a-c)^2}{b} = 1$, $\frac{\Pi_S}{s} = \frac{1}{s(n-s+2)^2} \geq \pi_i = \frac{1}{(n+1)^2}$ for $s^* \geq \frac{2n+3-\sqrt{4n+5}}{2} \geq 0,8n$. ■

The formation of a cartel, and, more in general, the switch by a group of firms to a new coordinated set of actions, may imply the exploitation of some sort of "first mover advantage". This is the case, for instance, of firms wishing to leave a cartel in order to reach a new collusive agreement, setting their new quantities before their defection is detected by the other firms. The payoff of a cartel acting as a Stackelberg leader against the fringe of remaining firms is given by:

$$\Pi_S^L = \sum_{i \in S} \pi_i \left(\widehat{q}_S, \left(q_j \left(\widehat{q}_S \right) \right)_{j \notin S} \right) = \frac{(a-c)^2}{4b(n-s+1)}, \tag{2}$$

where the function $q_j(q_S)$ is the equilibrium strategy of the representative fringe firm when q_S is played by the cartel, for all possible q_S. A profitable cartel can here be defined, as in Salant et al. (1983), as a cartel giving to its members a higher payoff than each of them would get by acting non cooperatively, that is, by leaving the cartel. The following proposition shows that, in this case, the whole industry is the unique profitable cartel.

Proposition 2. *When cartels' profits are given by (2), the only profitable cartel is the entire industry cartel.*

Proof. This result simply follows from expression (2). Under normalization,

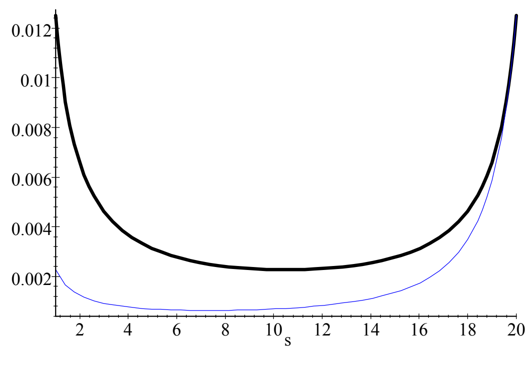
$$\frac{\Pi_S^L}{s} = \frac{1}{4s(n-s+1)} \tag{3}$$

and

$$\pi_i^L = \frac{1}{4n}. \tag{4}$$

It is easy to see that (3) is smaller than (4) for $1 < s < n$ and also that they coincide for $s = n$ (see also the figure below). This is equal to say that the only cartel that is not objectable by the most profitable deviation (that of just one firm) is the whole industry cartel, thus representing the only profitable cartel. ■

The meaning of the above result is that, when firms deviating from a cartel can pre-commit to a (collusive) joint strategy, anticipating that the fringe firms will react non cooperatively, only the entire industry cartel is profitable. This is illustrated through an example ($n = 20$ and $s = 1,..20$) in which the sequential equal-split payoff $\frac{\Pi_S^L}{s}$ (thick line) and the simultaneous one $\frac{\Pi_S}{s}$ (thin line) are compared. It is easy to see that, while $\frac{\Pi_S}{s}$ starts to be profitable (greater than π_i) for $s^* \geq 17 (\geq 0,8n)$, $\frac{\Pi_S^L}{s}$ $< \pi_i^L$ for $s \in [2, 19]$ and $\frac{\Pi_S^L}{s} = \pi_i^L$ for $s = n = 20$.



To check the robustness of this result, note that the whole industry cartel is still profitable in a symmetric Cournot oligopoly, provided that every firm's profit function does not decrease *too much* with the other firms' output. This property has been shown to depend on the degree of log-concavity of the inverse demand function $P(.)$ (see Amir, 1996 and Vives, 2000). As an example, let $P(Q) = (a - Q)^{\beta}$, with $a > Q$ and $c = 0$. This function is log-concave for $\beta \geq 0$, concave for $\beta \leq 1$ and convex for $\beta \geq 1$. The unique Cournot equilibrium is $q_i^* = \frac{a}{\beta+n}$ for every $i \in N$. Also, very simple algebra yields the following cartel profit function:

$$\Pi_S^L = a^{\beta+1}\beta^{2\beta}(\beta+1)^{-\beta-1}(\beta+n-s)^{-\beta}. \tag{5}$$

By computing the difference between the efficient equal-split allocation and what a single firm obtains, we get

$$\frac{\Pi_N^L}{n} - \pi_i^L = \beta^{\beta}(\beta+n-1)^{-\beta}n - 1 > 0 \Leftrightarrow \beta > 1. \tag{6}$$

It follows that the cartel including all firms in the industry is profitable only when $P(.)$ is convex, that is, for low degrees of log-concavity. This feature implies that best reply functions are not "too" decreasing, thus ensuring that deviating firms, acting as leaders, do not exploit too much their first mover advantage. When best reply functions are increasing (supermodular games), the above result can be also shown to be robust against every coalitional deviation (see Currarini and Marini (2002) for a more general framework).

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