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BONDS FUTURES: DELTA? NO GAMMA!

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ABSTRACT. Bond futures are liquid but complex instruments. Here they are analysed in a one-factor Gaussian HJM model. The *in-the-model delta* and *out-of-the-model delta and gamma* are studied. An explicit formula is provided for *in-the-model* delta. The out-of-the-model delta and gamma are equivalent to partial derivatives with respect ot discount factors. In particular cases the derivative can not be obtain by standard techniques. The same situations lead to cases where the gammas (second order partial derivatives) do not exists.

1. INTRODUCTION

Bond futures are very liquid instruments and are often consider as vanilla products, after all they are just *futures*. Behind this simplifying name a lot of complexity is hidden. One of them is the delivery option: the short side of the futures can deliver any bonds in a pre-defined basket. This embedded option creates a real complexity in the pricing and risk management of the product. Bond futures are exotic products.

With the EUR very long term rates close to 4%, the BUXL futures is almost *at-the-money*. The USD bond and notes rates have crossed the 5% level and their futures may also be soon at-the-money. It is probably the good moment to ensure that the delivery option embedded in the bond futures is well understood and managed.

This note should be read as a sequel to the article Henrard (2006a). A semi-explicit valuation formula for the delivery option in the Gaussian HJM one-factor model was proposed. Beyond the price of the instrument is their hedging. In a model with deterministic volatility, the theoretical hedging is done only with the *underlyings*. In term structure models the underlyings are (zero-coupon) bonds. As the model used here is one-factor any bond can be used to hedge any option. This hedging with the underlying is called *delta* hedging or, to emphasize the fact that the hedging is coherent with the model, *in-the-model* delta hedging. This risk measure is the only one theoritically justified.

There are other measures of rate or price dependency used in practice: the *out-of-the-model* delta and gamma. They correspond to the first and second order derivatives of the price with respect to the (zero-coupon) bond prices (or rates). Those figure are used in practice and easy to compute (approximatively) by symmetrical difference. Neverthless those figure have no theoretical justification and even their *existence* is not garanted.

The in-the-model delta is computed in the main theorem of the note. The formula is obtained in two different ways. The first one through partial derivatives computations. This is a standard approach for example used in (Lamberton and Lapeyre, 2000, Section 4.3.3) for Black-Sholes formula or in Henrard (2003) for the swaptions in the same model that is used here. In that proof one encounters some technical difficulties. It is not clear that the partial derivatives always exist. Generically they do but in some singular cases some part of the formula is not differentiable (infinite slope). The fact that some part of a composition is not differentiable does not prove that the total composition is non-differentiable but creates a significant technical difficulty. The theorem is proved when the technical difficulty does not appear.

A second proof using the structure of the model and no partial derivatives is provided. The proof is valid for all cases. The proof is using heavily the structure of the problem created by the model. It does not require technical conditions to be valid but require higher level view of the valuation formula and of the structure of the problem.

With the second proof the theoretical approach is cleaned but not the practical one. It is still not clear if the partial derivatives always exists. The definitive answer is not provided here. Nevertheless through the example of the last section it is shown that the technical condition of the first proof is not only technical but results from difficulties in the problem it-self. It is shown that in a specific example the second order partial derivative with respect to one of the prices does not exist. The example is quite simple as it involves only two bonds in the delivery basket with only one cash-flow each.

The conclusion of the example is that the *gamma* (which is always out-of-the-model) is not always well defined. It can tend to infinity in some points (even away from the expiry date) and can be discountinuous. This can be the start of thoughs on the usage of gamma as a risk measure and an indicator for limits.

2. FUTURES AND HJM MODEL

The notation and description are the same as in Henrard (2006a).

Suppose there are N bonds in the basket. Each of them ($1 \leq i \leq N$) has n_i coupons after the delivery date t_0; the cash flows amount are $c_{i,j}$ and are paid in $t_{i,j}$. Let A_i denote the accrued interests at delivery and K_i the convertion factor. The fixing take place in $\theta \leq t_0$. The price in t of a zero coupon bond with maturity u is denoted $P(t, u)$. The time t *futures price* is denoted by F_t

The futures price is

$$F_\theta = \min_{1 \leq i \leq N} \left(\sum_{j=1}^{n_i} \frac{c_{i,j}}{K_i} \frac{P(\theta, t_{i,j})}{P(\theta, t_0)} - \frac{A_i}{K_i} \right).$$

The valuation of the futures in this framework is done using the Gaussian HJM one-factor model. Preliminary results on that model can be found in the above paper.

When the discount curve $P(t, .)$ is absolutely continuous, which is something that is always the case in practice as the curve is constructed by some kind of interpolation, there exists $f(t, u)$ such that

(1) $$P(t, u) = \exp \left(- \int_t^u f(t, s) ds \right).$$

The idea of Heath et al. (1992) was to exploit this property by modeling f with a stochastic differential equation

$$df(t, u) = \mu(t, u) dt + \sigma(t, u) dW_t$$

for some suitable (stochastic) μ and σ and deducing the behavior of P from there. To ensure the arbitrage-free property of the model, a relationship between the drift and the volatility is required. Here the volatility, the drift, the rate and the Brownian motion are 1-dimensional. The model technical details can be found in the original paper or in the chapter *Dynamical term structure model* of Hunt and Kennedy (2004). The notation of the later is used.

The probability space is $(\Omega, \{\mathcal{F}_t\}, \mathcal{F}, \mathbb{P})$. The filtration $\mathcal{F}_t$ is the (augmented) filtration of the standard Brownian motion $(W_t)_{0 \leq t \leq T}$. To simplify the writing in the rest of the paper, the bond volatility is denoted

$$\nu(t, u) = \int_t^u \sigma(t, s) ds.$$

Let $N_t = \exp(\int_0^t r_s ds)$ be the cash-account numeraire with $(r_s)_{0 \leq s \leq T}$ the short rate given by $r_t = f(t, t)$. The equations of the model in the numeraire measure $\mathbb{N}$ associated to N_t are

$$df(t, u) = \sigma(t, u) \nu(t, u) dt + \sigma(t, u) dW_t$$

or

$$dP^N(t, u) = -P^N(t, u)\nu(t, u)dW_t$$

The notation $P^N(t, s)$ designates the numeraire rebased value of P, i.e. $P^N(t, s) = N_t^{-1}P(t, s)$.

Like in the case of swaption analysed in Henrard (2006b), a separability condition is used to obtain explicit results. The condition is similar to the condition to have Markovian short rate obtained by Carverhill (1994).

H: The function σ satisfies $\sigma(t, u) = g(t)h(u)$ for some positive function g and h.

The condition is satisfied by the extended Vasicek model.

The two following technical lemmas were presented in Henrard (2006b) for the Gaussian one-factor HJM. Similar formulas can be found in (Brody and Hughston, 2004, (3.3),(3.4)) in the framework of coherent interest-rate models and in Nunes and de Oliveira (2004) for multi-factor Gaussian HJM.

Lemma 1. *Let $0 \le t \le u \le v$. In HJM framework the price of the zero coupon bond is*

$$P(u, v) = \frac{P(t, v)}{P(t, u)} \exp\left(-\int_t^u (\nu(s, v) - \nu(s, u))\, dW_s - \frac{1}{2}\int_t^u \left(\nu(s, v)^2 - \nu(s, u)^2\right) ds\right).$$

The discount factor ratio in the futures price can be written as

$$\frac{P(\theta, t_{i,j})}{P(\theta, t_0)} = \frac{P(t, t_{i,j})}{P(t, t_0)} \beta_{i,j} \exp(-\alpha_{i,j}^2 - \alpha_{i,j} X)$$

with

$$\beta_{i,j} = \beta_{i,j}(t, \theta) = \exp\left(-\int_t^\theta \nu(s, t_0)(\nu(s, t_{i,j}) - \nu(s, t_0)) ds\right),$$

$$\alpha_{i,j}^2 = \alpha_{i,j}^2(t, \theta) = \int_t^\theta (\nu(s, t_{i,j}) - \nu(s, t_0))^2 ds,$$

and X a $\mathbb{N}$-standard normal random variable. The separability condition (H) is used to prove that the same variable X can be used for all bonds and coupons.

Using the notation

$$d_{i,j} = d_{i,j}(t, \theta) = \frac{c_{i,j}}{K_i} \beta_{i,j},$$

$e_i = A_i/K_i$, $t_{i,0} = t_0$ and $d_{i,0}(t, \theta) = -e_i$ the future price is

$$F_\theta = \min_{1 \le i \le N} \left(\sum_{j=0}^{n_i} d_{i,j}(t, \theta)\frac{P(t, t_{i,j})}{P(t, t_0)} \exp(-\frac{1}{2}\alpha_{i,j}^2(t, \theta) - \alpha_{i,j}(t, \theta)X)\right).$$

The functions in the minimum are denoted

$$f_i(x) = f_i(x; t, \theta)) = \sum_{j=0}^{n_i} d_{i,j}\frac{P(t, t_{i,j})}{P(t, t_0)} \exp(-\frac{1}{2}\alpha_{i,j}^2 - \alpha_{i,j}x).$$

3. Delta

In bond futures the value that should be *attained* is not the futures price but its *gain process*. The futures price is only a *screen number*. It drives the marginning process but the price is not a pay-off. The *pay-off* that should be attained in the hedging process is the gain associated to the futures denoted $G_t(F, N)$ (see (Hunt and Kennedy, 2004, Chapter 12)). The numeraire used here is the cash account (denoted ζ^{-1} in Hunt and Kennedy (2004)). With that numeraire the discounted gain process satisfies (Hunt and Kennedy, 2004, Equation 12.6)

$$G_t(F, N) = \int_0^t N_u^{-1}dF_u.$$

If the process is written as

$$G_t = \int_0^t \phi_u dB_u^N$$

for some asset B_u, then the random process ϕ is a hedging strategy. To obtain this the futures process is written as

$$dF_t = N_t\phi_t dB_t^N.$$

The hedging bond is described below and the hedging strategy is described in the following theorem.

Let B be a bond with cash-flow dates s_i $(1 \leq i \leq l)$ and cash-flow amounts b_i. The discounting value of the bond B_t^N satisfy the equation

$$dB_t^N = -\sum_{i=1}^{l} b_i P^N(t, s_i)\nu(t, s_i)dW_t.$$

Theorem 1. *In the separable one-factor gaussian HJM model, the hedging strategy in t for the bond futures is to hold the quantity*

$$\Delta_t = \frac{1}{P(t, t_0)}\frac{\sum_{i=1}^{k}\sum_{j=0}^{n_{m_i}} d_{m_i,j}P(t, t_{m_i,j})\left(N(\kappa_i + \alpha_{m_i,j}) - N(\kappa_{i-1} + \alpha_{m_i,j})\right)\nu(t, t_{m_i,j}) - F_t P(t, t_0)\nu(t, t_0)}{\sum_{i=1}^{l} b_i P(t, s_i)\nu(t, s_i)}$$

of the bond B.

Proof (Using partial derivatives). Let $x = P(t) = (P(t, t_0), \ldots, P(t, t_{m_i,j}), \ldots) = (\ldots, P_{(i,j)}, \ldots) = P$. The order in which the bonds $P_{(i,j)}$ $(1 \leq i \leq k, 1 \leq j \leq n_{m_i})$ are sorted in th vector is irrelevant for the argument that follows. The value of the bond is

$$F_t = F(t, P, \kappa).$$

Note that κ_i satifies $f_i(\kappa_i, P) = 0$ for

(2)
$$f_i(x, P) = \sum_{j=0}^{n_{m_i}} d_{m_i,j}P_{(i,j)}\exp(-\frac{1}{2}\alpha_{m_i,j}^2 - \alpha_{m_i,j}x) - \sum_{j=0}^{n_{m_{i+1}}} d_{m_{i+1},j}P_{(i+1,j)}\exp(-\frac{1}{2}\alpha_{m_{i+1},j}^2 - \alpha_{m_{i+1},j}x).$$

The dependency of F on P appears directly and indirectly through κ. It is first proved that

$$\frac{d}{dP_{(i,j)}}F(t, P, \kappa(P)) = \frac{\partial}{\partial P_{(i,j)}}F(t, P, \kappa).$$

The (total) derivative of F with respect to $P(t, t_0)$ is denote $D_{(0)}F$ and the one with respect to $P_{(i,j)}$ by $D_{(i,j)}F$.

Generically the quantity $D_1 f \neq 0$ and one can use the implicit function theorem to obtain the existence and derivability of κ as a function of P. One can then compute $D_{(i,j)}F$ by composition rule.

The result is proved for $p > 0$, the other case is similar. Using the explicit formula for the future price (Henrard, 2006a, Equation 2)

(3)
$$\begin{aligned}
D_{(p,q)}F &= \frac{1}{P(t, t_0)}\left(d_{m_p,q}N(\kappa_p + \alpha_{m_p,q}) - d_{m_p,q}N(\kappa_{p-1} + \alpha_{m_p,q})\right)\\
&+ \sum_{i=1}^{k-1}\sum_{j=0}^{n_{m_i}} d_{m_i,j}P_{(i,j)}N'(\kappa_i + \alpha_{m_i,j})D_{(p,q)}\kappa_i\\
&- \sum_{i=1}^{k-1}\sum_{j=0}^{n_{m_{i+1}}} d_{m_{i+1},j}P_{(i+1,j)}N'(\kappa_i + \alpha_{m_{i+1},j})D_{(p,q)}\kappa_i
\end{aligned}$$

The normal density can be split in

$$N'(\kappa_i + \alpha_{m_i,j}) = \exp(-\frac{1}{2}\alpha_{m_i,j}^2 - \alpha_{m_i,j}\kappa_i)\frac{1}{\sqrt{2\pi}}\exp(-\frac{1}{2}\kappa_i^2).$$

By using that split, grouping the factors of $D_{(p,q)}\kappa_i$ and the using Equation 2 one proves that only the first line of Equation 3 is non null.

The derivative is given by

$$D_{(p,q)}F = \frac{1}{P(t,t_0)}d_{m_p,q}\left(N(\kappa_p + \alpha_{m_p,q}) - N(\kappa_{p-1} + \alpha_{m_p,q})\right)$$

and

$$D_{(0)}F = -\frac{1}{P^2(t,t_0)}\sum_{i=1}^{k}\sum_{j=1}^{n_{m_i}}d_{m_i,j}P_{(i,j)}(N(\kappa_i + \alpha_{m_i,j}) - N(\kappa_{i-1} + \alpha_{m_i,j})) = -\frac{1}{P(t,t_0)}F_t.$$

Like in Henrard (2003) the hedging is given by

$$\phi_t = \frac{\sum_{i=1}^{k}\sum_{j=0}^{n_{m_i}}P_{(i,j)}\nu(t,t_{m_i,j})D_{(i,j)}F + P_{(0)}\nu(t,t_0)D_{(0)}F}{\sum_{i=1}^{l}P(t,s_i)\nu(t,s_i)b_i}$$

The result follows using the description of $D_{(i,j)}F$ and $D_{(0)}F$ given above. $\qquad\square$

There exists examples where the non-degenaracity condition $D_1 f_i(\kappa_i, P) \neq 0$ is not satisfied. In those conditions the proof given does not hold anymore. Also in this context a small perturbation of the initial conditions P can modify (substantially) the number of crossing.

The non-robustness too small perturbation is not in contradition with the strong robustness result proved in Henrard (2006a) for all movements admissible in the gaussian HJM model. Here the initial condition are perturbed in a way non compatible with the model. The individual derivatives computed $D_{(p,q)}F$ are *out-of-the-model* derivatives while the one in the proof on options on futures are *in-the-model* rate movements.

Proof (Using model structure). The discount factor ratios are

$$\frac{P(t,t_{i,j})}{P(t,t_0)} = \frac{P(0,t_{i,j})}{P(0,t_0)}\beta_{i,j}(0,t)\exp(-\frac{1}{2}\alpha_{i,j}^2(0,t) - \alpha_{i,j}(0,t)X_t)$$

with X_t a $\mathbb{N}$-normally distributed random variable.

For each value of X, the ratios are different. The sequences κ_i and m_i associated are also potentially different.

If the m_i and k of (Henrard, 2006a, Theorem 2) were actually different it would complicate substantially the task. One would have a number of terms in the sum that represents the value of the futures F_t which is stochastic.

With a technique similar to the one used in (Henrard, 2006b, Theorem 4) it can be proved that the number of terms k and their order m_i is constant for all value of X. The constance is proved by showing that the number and order of intersection points between all the f_i curves pairs are always the same.

The intersection between two curves is at point $\kappa_{p,q}(X_t)$ that satisfies

$$\sum_{j=0}^{n_{p,q}}d_{p,q,j}(t,\theta)\frac{P(0,t_{p,q,j})}{P(0,t_0)}\exp\left(-\frac{1}{2}\alpha_{p,q,j}^2(0,\theta) - \alpha_{p,q,j}(0,t)X_t - \alpha_{p,q,j}(t,\theta)\kappa\right) = 0.$$

The standard deviation coefficients can be written using (H) as

$$\alpha_{pq,j}^2(u,v) = H^2(t_{pq,j})(G(v) - G(u))$$

and

$$\alpha_{i,j}(0,t)X_t = \int_0^t \nu(s,t_{i,j}) - \nu(s,t_0)dW_s$$

$$\alpha_{i,j}(0,t)dX_t = ... + (\nu(t,t_{i,j}) - \nu(t,t_0))dW_t.$$

Let Λ be the X_t-independent solution(s) of (Henrard, 2006a, Equation 3).

The solution $\kappa_{pq}(X_t)$ of the initial equation can be written through

$$\kappa_{pq}(X_t) = \kappa(\Lambda_{pq}, X_t)$$

with

$$\kappa(\Lambda, X) = \frac{\Lambda - \sqrt{-G(0)}X}{\sqrt{G(\theta)}}.$$

All the $\kappa_{p,q}$ $(1 \le p, q \le k)$ have the same order as the $\Lambda_{p,q}$. This was proved in Theorem 4 in Henrard (2006a).

With all these ingredients in place the futures price can be written as

$$F_t = \sum_{i=1}^{k} \sum_{j=0}^{n_{m_i}} d_{m_i,j}(t,\theta) \frac{P(0,t_{i,j})}{P(0,t_0)} \beta_{m_i,j} \exp(-\frac{1}{2}\alpha_{m_i,j}^2(0,t) - \alpha_{m_i,j}(0,t)X_t)$$
$$\left(N(\kappa(\Lambda_i, X_t) + \alpha_{m_i,j}(t,\theta)) - N(\kappa(\Lambda_{i-1}, X) + \alpha_{m_i,j}(t,\theta)) \right).$$

The variation of F_t is then

$$dF_t = \cdots - \sum_{i=1}^{k} \sum_{j=0}^{n_{m_i}} d_{m_i,j}(t,\theta) \frac{P(0,t_{m_i,j})}{P(0,t_0)} \beta_{m_i,j} \exp(-\frac{1}{2}\alpha_{m_i,j}^2(0,t) - \alpha_{m_i,j}(0,t)X_t)$$
$$\left(N(\kappa(\Lambda_i, X_t) + \alpha_{m_i,j}(t,\theta)) - N(\kappa(\Lambda_{i-1}, X) + \alpha_{m_i,j}(t,\theta)) \right) \alpha_{m_i,j}(0,t)dX_t$$
$$+ \sum_{i=1}^{k-1} \sum_{j=0}^{n_{m_i}} d_{m_i,j}(t,\theta) \frac{P(0,t_{m_i,j})}{P(0,t_0)} \beta_{m_i,j} \exp(-\frac{1}{2}\alpha_{m_i,j}^2(0,t) - \alpha_{m_i,j}(0,t)X_t)$$
$$N'(\kappa(\Lambda_i, X_t) + \alpha_{m_i,j}(t,\theta))D_2\kappa(\Lambda_i, X_t)dX_t$$
$$+ \sum_{i=1}^{k-1} \sum_{j=0}^{n_{m_{i+1}}} d_{m_{i+1},j}(t,\theta) \frac{P(0,t_{m_{i+1},j})}{P(0,t_0)} \beta_{m_{i+1},j} \exp(-\frac{1}{2}\alpha_{m_{i+1},j}^2(0,t) - \alpha_{m_{i+1},j}(0,t)X_t)$$
$$N'(\kappa(\Lambda_i, X_t) + \alpha_{m_{i+1},j}(t,\theta))D_2\kappa(\Lambda_i, X_t)dX_t.$$

The dots indicate the constant and dt terms. As the future price is a martingale, the dt terms are nuls. Using the equations caracterising κ_i, the two last term of the equation are nul.

Rearranging the terms and using the caracterisitic of αdX_t given above lead to the result. $\square$

4. A SINGULAR EXAMPLE

An example where the non-degenaracy condition is not satisfied is provided. It is also proved that for the same example the second order partial derivative (gamma) does not exists.

The example is similar to the third example in Henrard (2006a). There are two bonds with one cash-flow each. The parameters are given in Table 1.

Singular cross				
i	j	e_i	$d_{i,j}$	$\alpha_{i,j}$
1	1	0	1	1
2	1	y_0	3	1/3
Table Note				

TABLE 1. Constant for two bonds with one singular intersection

The initial discount factors are chosen to be 1. The derivative to the discount factor associated to the maturity in the second bond is analysed. The constant y_0 is chosen in such a way that there is one degenerate cross-over point. It's value is $2 * \exp(1/6)$. With those constants one has

$$f_1(P,x) = \exp(-1/2 - x)$$
$$f_2(P,x) = 3P\exp(-1/18 - 1/3x) - y_0.$$

The two functions are equal in $x = -2/3$. This is the only intersection point and for the other points $f_1(1,x) > f_2(1,x)$.

It can be shown that for $P < 1$ there is no intersection and for $P > 1$ there are two intersections. The value of the futures is

$$P \leq 1 \qquad F_0(P) = 3P - y_0$$
$$P > 1 \qquad \bar{F}_0(P) = 3PN(\kappa_1 + \alpha_2) - y_0 N(\kappa_1) + (N(\kappa_2 + \alpha_1) - N(\kappa_1 - \alpha_1))$$
$$+ 3P((1 - N(\kappa_2 + \alpha_2)) - y_0(1 - N(\kappa_2)).$$

The function is continuous and $\lim_{P \to <1} F_0(P) = \lim_{P \to >1} \bar{F}_0(P) = 3 - y_0$.

The derivative of the two parts of the function can be computed. Using the details in the proof,

$$P \leq 1 \qquad F_0'(P) = 3$$
$$P > 1 \qquad \bar{F}_0'(P) = 3N(\kappa_1 + \alpha_2) + 3(1 - N(\kappa_2 + \alpha_2)).$$

Here also the left and right limits are equal with $\lim_{P \to <1} F_0'(P) = 3 = \lim_{P \to >1} \bar{F}_0'(P)$. This proves that the futures price is C^1 in P around $P = 1$.

Nevertheless this can not be deduced directly from the form of F_0' as the function κ_i are not well defined for $P \leq 1$. For $P > 1$ they exists but their derivative tend to infinity for $P \to 1$. The graph of the κ's is displayed in Figure 1(a).
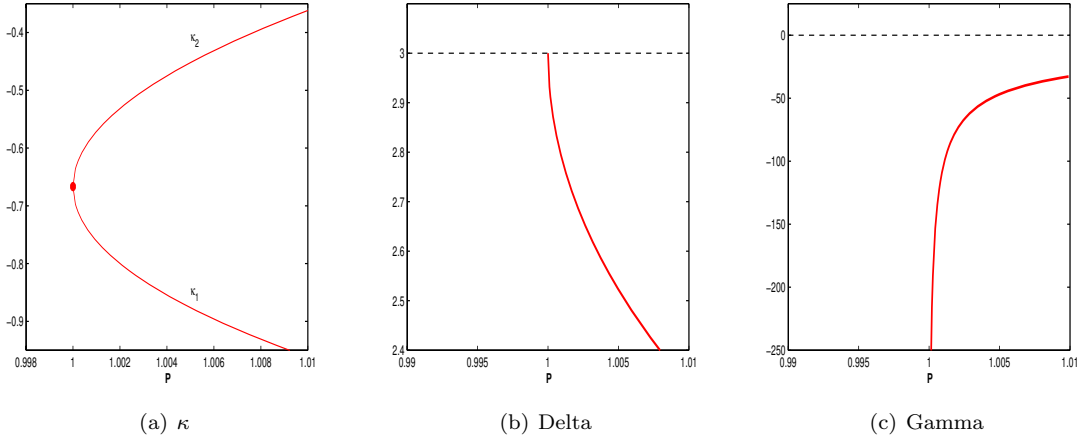

(a) κ (b) Delta   (c) Gamma

FIGURE 1. Price dependence of κ, delta and gamma.

The second derivatives are given by

$$P \leq 1 \qquad F_0''(P) = 0$$
$$P > 1 \qquad \bar{F}_0''(P) = 3N'(\kappa_1(P) + \alpha_2)\kappa_1'(P) - 3N'(\kappa_2(P) + \alpha_2))\kappa_2'(P).$$

But $\kappa_1'(P) \to -\infty$ and $\kappa_3'(P) \to +\infty$ as $P \to > 1$ and N' is positive. So F_0'' tends to $-\infty$ as $P \to > 1$. The graphs of the first and second order derivatives are given in Figure 1(b) and 1(c).

The example proves that the second order partial derivatives does not always exist. Several questions are still open. The first one is the existence of the first order derivatives. It was proved that for non degenerate κ they exist. From the example there exists cases where they exist even if the crossing is degenerate and the κ's are non differentiable. The general proof of the existence or the description of the conditions of existence is still an open problem. For the second order derivatives, the existence is proved for non-degenerate crossing points and an example of non-existence is given. The precise conditions of non-existence are still open.

5. Conclusion

The note discuss several rate risk measures for bond futures: *in-the-model* delta and *out-of-the-model* delta and gamma.

A general formula for the in-the-model delta is provided. The best proof of the general result does not use standard approach through partial derivatives. The stochastic differential with respect to the factor of the model is used instead. The proof is less standard but is robust and uses more the structure of the model.

The *out-of-the-model* delta and gamma or first and second order partial derivatives with respect to the zero-coupon prices are studied. Under a non-degeneracy condition of the crossing points those derivatives exists. A relatively simple example proves that in somes cases second order derivatives does not exists. Between those two results several existence questions in the degenerate case are still open.

Disclaimer: The views expressed here are those of the author and not necessarily those of the Bank for International Settlements.

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